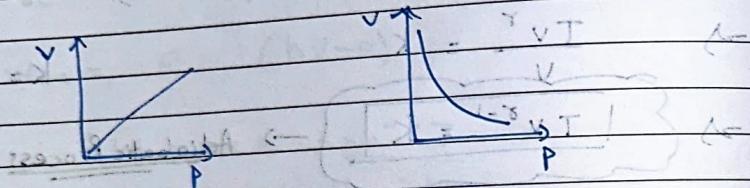


## \* Kinetic Theory Of Gases

- o Boyle's Law (only for isothermal)

$\nabla \propto \frac{1}{P}$  ( $T = \text{const}$ , isothermal)



$$PV = \text{constant}$$

- o Charles' Law (for isobaric)

$V \propto T$  ( $P = \text{const}$ )

- o Gay-Lussac's Law

$P \propto T$

## \* Ideal Gas Eqn

$$PV = nRT$$

$n = \text{no. of moles}$   
 $R = \text{Gas constant}$

from Charles' & Boyle's Law,

$$PV = T$$

$$PV \propto T$$

$$PV = RT$$

For  $n$  moles of gas,  $PV = nRT$

$$R = 8.314 \text{ J/molK}$$

The Eqn. for per molecule

$$\frac{R}{N_A} = K_B \rightarrow \text{Boltzmann} \rightarrow (1.38 \times 10^{-23} \text{ J/K}) \text{ Constant}$$

$$PV = n \cdot K_B \cdot N_A \cdot T \quad \text{Also } n = \frac{N}{N_A}$$

$$PV = N \cdot K_B \cdot T$$

(moles =  $\frac{\text{no. of molecules}}{N_A}$ )

(Assume 1 mole of gas contains  $N$  molecules)

$\rightarrow K_B = \text{Boltzmann constant}$

## o Assumptions of Kinetic Theory

- 1) All molecules of the gas are rigid sphere.
- 2) The size of molecules is negligible when compared to the distance between the molecules.
- 3) Collisions are perfectly elastic and there is no force of attraction among the molecules.
- 4) Molecules are in state of continuous random motion.

5) During the motion, molecules collide with each other and the walls of the container.

6) Between 2 collisions, molecule travels along a straight line.

7) The density of the gas remains uniform.

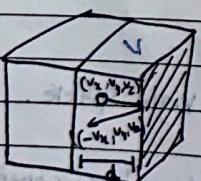
8) Collisions are instantaneous.

### \* Concept Of Pressure

According to Kinetic Theory of Gases, pressure is the no. of collisions of molecules with the wall of system.

Collision of molecule  $\rightarrow$  Momentum transferred to wall with wall  $\rightarrow$  (momentum of molecule changes)

Pressure developed  $\rightarrow$  force exerted per unit area on wall  $\rightarrow$  Force generated due to NLM 2nd Law ( $F = \frac{dp}{dt}$ )



$d$  = distance covered by molecule  $\rightarrow$  final momentum  $= mv_x$  of molecule

$\rightarrow$   $\Delta P$  of molecule  $= +mv_x - mv_x = 2mv_x$

Momentum transferred to wall  $= 2mv_x$  by one molecule

Velocity considers both +ve & -ve directions.

Now, total no. of molecules in volume:  $\sum n_i A v_i \Delta t$

$\therefore$  Total no. of molecules colliding with the wall in volume  $= \frac{1}{2} \times n_i A v_i \Delta t$

Total momentum transferred to wall by  $n$  molecules  $= \sum n_i A v_i \Delta t \times m v_i$

$$\text{Total momentum} = mn v_x^2 A \Delta t$$

Total force on wall =  $\infty \theta dp$  in  $x$  direction.

$$= \frac{\partial p}{\partial t} \frac{S}{2} = \frac{1}{2} mn v_x^2 A \Delta t$$

$$F = mn v_x^2 A$$

$$\therefore \text{Pressure on wall} = mn v_x^2$$

$m \rightarrow$  mass of one molecule  
 $n \rightarrow$  no. of molecules.

Since all molecules are moving with different velocities, Velocity = mean velocity ( $\bar{v}$ )

$$\therefore \text{Pressure} = mn \bar{v}_x^2$$

Also, if pressure is uniform all over body,  $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$

Total velocity =  $\frac{v_x^2 + v_y^2 + v_z^2}{3} \rightarrow \text{Net Velocity}$

∴  $s = \frac{m}{v} = \frac{m}{\text{Net Velocity}}$

for unit volume,  $s = mn$ .  
(v=1)

∴ Pressure exerted on walls by  $n$  molecules of mass 'm' with mean velocity  $\bar{v}$ .

\* Also, as  $P = \frac{1}{3} s \bar{v}^2$

$$\therefore P = \frac{2}{3} \times \frac{1}{2} s \bar{v}^2 = \frac{2}{3} \times KE.$$

$$\therefore P = \frac{2}{3} \times KE$$

### Kinetic Interpretation of Temperature

∴ We know that,  $P = \frac{1}{3} s \bar{v}^2$

$$\Rightarrow P = \frac{1}{3} \frac{m}{V} \bar{v}^2$$

$$\Rightarrow PV = \frac{1}{3} M \bar{v}^2$$

$$\Rightarrow PV = \frac{2}{3} \times \frac{1}{2} M \bar{v}^2$$

∴  $P$  By Boyle's Ideal Gas Eqn.,  $PV = nRT$ .

For 1 mole,  $PV = RT$ .

$$\therefore RT = \frac{2}{3} \times KE$$

$$\therefore KE = \frac{3}{2} RT$$

→ Here KE = mean kinetic energy (For 1 mole) is measurement of temperature.

∴ As molecules start moving faster, temperature increases

Also,  $KE_{avg} = \frac{3}{2} \frac{R}{N} T$

$$KE_{avg} = \frac{3}{2} K_B T$$

$K_B$  = Boltzmann Constant

→ Avg KE per molecule

o Imp!

$$\therefore \overline{KE} = \frac{3}{2} RT$$

$$\Rightarrow \frac{1}{2} m \bar{v}^2 = \frac{3}{2} RT$$

$$\Rightarrow \sqrt{\bar{v}^2} = \sqrt{\frac{3RT}{m}}$$

$$\therefore V_{rms} = \sqrt{\frac{3RT}{m}}$$

$$V_{rms} = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$V_{rms} = \text{root mean square}$

Result!!

$$V_{rms} \propto \sqrt{T}$$

$$V_{rms} \propto \frac{1}{\sqrt{m}}$$

Q. Calculate rms velocity at STP.  $S_{\text{air}} = 1.3 \text{ kg/m}^3$

$$\hookrightarrow \text{We know, } \frac{1}{3} S\bar{v}^2 = T \text{ P} \quad \text{at } 1 \text{ atm}$$

$$\Rightarrow \frac{1}{3} S\bar{v}^2 = 1 \times 10^5 \text{ N/m}^2 = T \text{ P}$$

~~$$\Rightarrow \frac{1}{3} \bar{v}^2 = \frac{10^6 \times 3}{4 \times 3.4} = 7.8$$~~

~~$$\Rightarrow \bar{v}^2 = \frac{10^6}{4}$$~~

~~$$\Rightarrow \sqrt{\bar{v}^2} \approx \frac{10^3}{2} = 5 \times 10^2 \text{ m/s}$$~~

$$\Rightarrow \bar{v}^2 = \frac{10^6 \times 3}{N_A \cdot 13} = 490 \text{ m/s}$$

Q. A vessel filled with gas of 76 cm pressure of mercury at certain temp. Mass of gas is increased by 50%. at same temp. find final pressure.

$$\hookrightarrow \text{We know, } PV = \frac{1}{3} M \bar{v}^2$$

$$\Rightarrow P \propto M$$

$\therefore M \rightarrow$  increases by 50%.  $P \uparrow 50\%$ .

$$\therefore P = 76 + \frac{1}{2} \times 76 = 114 \text{ cmHg}$$

Q. Calculate KE of 1 mole of argon at 127°C.

$$\hookrightarrow KE = \frac{3}{2} RT$$

$$T = 127 + 273 = 400 \text{ K}$$

$$= \frac{3}{2} \times \frac{4.15 \times 10^{-23}}{8.314} \times \frac{400}{127} = 1524 \text{ J/K}$$

$$= 14988 \text{ J}$$

Q. Calculate KE for molecule & RMS velocity of gas at 127°C

$$m = 6.4 \times 10^{-27} \text{ kg}$$

$$400 \text{ K}$$

$$\hookrightarrow KE = \frac{3}{2} k_B T$$

$$69$$

$$= \frac{3}{2} \times \frac{1.38 \times 10^{-23}}{1.98} \times 400$$

$$= 1.38 \times 10^{-23} \times 4 = 5.52 \times 10^{-23}$$

~~$$\text{Also, } KE = \frac{1}{2} m \bar{v}^2$$~~

~~$$\Rightarrow \bar{v}^2 = \frac{1.38 \times 10^{-23} \times 4}{6.4 \times 10^{-27}}$$~~

~~$$\therefore \bar{v}^2 = \frac{5.52 \times 10^{-23}}{6.4 \times 10^{-27}}$$~~

$$\bar{v}^2 = \sqrt{\frac{2 \times 8.28 \times 10^{-23}}{6.4 \times 10^{-27}}} = \sqrt{\frac{1656 \times 10^6}{6.4}} = 1.6 \times 10^3$$

$$[1.6 \times 10^3]$$

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Q. A flask contains Argon & chlorine in the ratio of 2:1 by mass. Temperature of mixture is 21°C

(1) Avg KE per molecule.

(2)  $v_{rms}$  of two gases.

$$\hookrightarrow (1) \rightarrow K_F = \frac{3}{2} k_B T$$

$$= \frac{3}{2} \times \frac{8.34 \times 10^{-23}}{100} \times 294 \text{ J/mol.} \\ = [6.09 \times 10^{-23} \text{ J}]$$

$$\begin{array}{r} 8 \\ 69 \\ \times 9 \\ \hline 621 \end{array}$$

$$(2) \frac{v_{rms} \text{ Ar}}{v_{rms} \text{ Cl}} = \frac{\sqrt{3k_B T}}{M_{Ar}} =$$

$$\frac{v_{rms} \text{ Ar}}{v_{rms} \text{ Cl}} = \frac{\sqrt{3k_B T}}{M_{Cl}}$$

$$\Rightarrow \frac{v_{rms} \text{ Ar}}{v_{rms} \text{ Cl}} = \frac{M_{Cl}}{\sqrt{M_{Ar}}} = \frac{70}{40} = \sqrt{1.75} \\ = 1.34$$

Q. 4 molecule have speeds; 2, 4, 6, 8 km/s. Calculate avg. speed & RMS speed.

$$\hookrightarrow \text{Avg speed} = 5 \text{ m/s}$$

$$\text{RMS} = \sqrt{\frac{4+16+36+64}{4}} = \sqrt{30} = 5.4 \text{ m/s}$$

Q. Oxygen molecule RMS speed at certain temp T. If temp is doubled, oxygen dissociates into atomic oxygen, what is new speed.

$$\hookrightarrow \sqrt{V_1} = \sqrt{\frac{3RT}{m}}$$

$$\hookrightarrow \sqrt{V_2} = \sqrt{\frac{3R(2T)}{m}} = \boxed{2\sqrt{\frac{3RT}{m}}}$$

Q. At what temp. RMS of H<sub>2</sub> molecule equal to that of oxygen molecule at 47°C

$$\frac{3RT'}{32} = \frac{3RT}{16}$$

$$\frac{T'}{16} = T$$

$$\frac{320}{16} = T \Rightarrow T = 20 \text{ K}$$

Q. Calculate RMS V of O<sub>2</sub> molecule at 57°C

$$\hookrightarrow v_{rms}^{O_2} = \sqrt{\frac{3 \times 6.344 \times 3 \times 8.314 \times 300}{32 \times 10^{-3}}}$$

$$\hookrightarrow 30 \times \sqrt{\frac{8.314}{32}} = 460 \text{ m/s}$$

Here, as we are talking about motion of particles there is no potential energy taken into account. Purely kinetic energy is considered.

M	T	W	T	F	S	S
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### \* Degree of freedom (DOF)

- The no. of independent coordinates to describe position & configuration of particles.

↳ This depends on types of motion (translational, rotational, vibrational).

that the object is undergoing  $(x, y, z)$  forward &  $(\theta, \phi, \psi)$ , type of motion  
Also on the no. of coordinates that can be used to describe the particle's position

$$\circ \text{DOF} = 3N - K$$

$N$  = No. of molecules.

$K$  = No. of independent (constraint) relation associated with a molecule.

### DOF for rigid body

$$\text{DOF} = 6$$

$$\hookrightarrow \text{DOF (translational)} = 3$$

$$\hookrightarrow \text{DOF (rotational)} = 3$$

### DOF for monoatomic gas (eg He, Ne, Ar) (only translational)

$$\rightarrow \text{DOF}_{\text{net}} = 3$$

$$\hookrightarrow \text{DOF (translational)} = 3$$

$$\text{DOF} = 3N - K$$

for monoatomic,  $N=1, K=0$

$$\therefore F = 3$$

Eg.  $H_2, N_2, O_2$  etc

M	T	W	T	F	S	S
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### DOF for diatomic gas

Rigid rotator, or low temperature

$$\hookrightarrow \text{DOF}_{\text{net}} = 5$$

$$\circ \text{DOF (translational)} = 3$$

$$\circ \text{DOF (rotational)} = 2 = \chi, \varepsilon = 1$$

$$\text{DOF} = 3N - K$$

(For diatomic,  $N=2, K=1$ )

$$F = 5$$

translational, rotational, vibrational

### DOF for diatomic gas [High Temperature]

$$\rightarrow \text{DOF}_{\text{net}} = 7$$

$$\circ \text{DOF (translational)} = 3$$

$$\circ \text{DOF (Rotational)} = 2$$

$$\circ \text{DOF (Vibrational)} = 2$$

### DOF for triatomic gas

① Non-linear molecule



$$\hookrightarrow F = 3N - K$$

For triatomic,  $N=3, K=3$

$$\left. \begin{array}{l} \text{DOF} = 6 \\ \downarrow \end{array} \right\}$$

$$\left. \begin{array}{l} \text{Translational} = 3 \\ \text{Rotational} = 3 \end{array} \right\}$$

$$\therefore F = 6$$

$C_p - C_v = h$

M	T	W	T	F	S	S
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o DOF

② Linear molecule



$$F = 3N - K$$

for triatomic,  $N=3, K=2$  (non-linear) 7 DOF

(1-2), S=1, i.e. linear

$$\therefore \text{DOF} = 7 \quad (3 \text{ rot}^h, 3 \text{ trans}^h) [1 \text{ extra}]$$

vibra<sup>b</sup>n nati  
tional

\* Specific Heat Ratio

$$\Gamma = (7/5) 700$$

o Law of Equipartition Energy

$$E = (1/2) k_B T$$

↳ for any system in thermal equilibrium, the energy is distributed equally among its various degrees of freedom

↳ Each degree of freedom contributes  $\frac{1}{2} k_B T$  per molecule

$$(\text{No. of molecules} \times \text{DOF} \times \frac{1}{2} k_B T = \text{System Energy})$$

o Specific Heat of Monoatomic Gas ( $\gamma$ )  $N_f = 3$

↳ DOF of monoatomic gas = 3 (constant vol)

$$\text{Total energy per molecule} = \frac{3}{2} k_B T = 3$$

$$\text{Total energy per mole} = \frac{3}{2} k_B N_A T = \frac{3}{2} RT$$

$$\therefore \text{Specific Heat at } (C_V) = \frac{dU}{dT} = \frac{3}{2} R$$

$$\text{Sp. Heat at constant } (C_P) = C_V + R = \frac{5}{2} R$$

$$\therefore \text{Sp. Heat Ratio} = \frac{C_P}{C_V} = \boxed{5/3}$$

\* Specific Heat Ratio of diatomic gas [low temp]

↳ DOF = 5

$$\text{Energy/molecule} = \frac{5}{2} k_B T$$

$$\text{Total internal energy} = \frac{5}{2} RT$$

$$C_V (\text{const vol}) = \frac{5}{2} R$$

$$C_P (\text{const. pressure}) = \frac{7}{2} R$$

$$\therefore \text{Sp. Heat Ratio} = \frac{C_P}{C_V} = \boxed{7/5}$$

\* Sp. Heat Ratio of diatomic gas [High temp]

↳ DOF = 7

$$\text{Energy/molecule} = \frac{7}{2} k_B T$$

$$\text{Total internal energy} = \frac{7}{2} RT$$

$$\therefore C_V = \frac{7}{2} R \quad (\text{v}) \quad \text{Total N.H. } q^2 \quad \text{excl. V term}$$

$$C_P = \frac{9}{2} R$$

$$\therefore \gamma = \frac{C_P}{C_V} = \boxed{\frac{9}{7}}$$

\*  $\gamma$  for triatomic (Non-Linear)  $\text{N}_3\text{H. } q^2$

↳ DOF = 6

$$\therefore \text{Energy/molecule} = 3K_B T$$

$$\therefore \text{Total internal energy} = 3RT = \text{Exclusion term}$$

$$\therefore C_V = 3R$$

$$\therefore C_P = 4R$$

$$\therefore \gamma = \frac{C_P}{C_V} = \boxed{\frac{4}{3}} \quad (\text{excl. V term})$$

\*  $\gamma$  for triatomic (linear)

↳ DOF = 7

$$\therefore \text{Energy/molecule} = \frac{7}{2} K_B T$$

$$\therefore \text{Total internal energy} = \frac{7}{2} RT$$

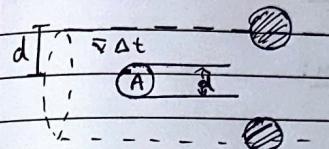
$$\therefore C_V = \frac{7}{2} R$$

$$\therefore \gamma = \frac{C_P}{C_V} = \boxed{\frac{9}{7}} \quad (\text{excl. V term})$$

$$\star \bar{\lambda} = \lambda_1 + \lambda_2 + \lambda_3 \dots \lambda_n$$

Total no. of collision

→ Assumption → Molecules are spheres of diameter  $d$ .



$$\text{Total collisions in time } \Delta t = \bar{\lambda} \times n$$

$$= \pi d^2 \bar{\lambda} \Delta t \times n$$

$$\therefore \text{Mean free path} : \bar{\lambda} = \frac{\bar{\lambda} \Delta t}{\pi d^2 \bar{\lambda} \Delta t \times n} = \boxed{\frac{1}{\pi d^2 n}}$$

o Correct formula

~~$$= \frac{1}{\sqrt{2} \pi d^2 n}$$~~

$$\therefore P = \frac{n \times m}{V}, \text{ since } V=1, S=m$$

$$\therefore n = \frac{S}{m}$$

$$\therefore \bar{\lambda} = \frac{m}{\sqrt{2} \pi d^2 S} \quad \text{--- (2)}$$

$$\therefore PV = RT$$

$$\Rightarrow P = \frac{RT}{V} \times \frac{N}{N} = \frac{RT}{N} \times \frac{N}{V}$$

$$\Rightarrow P = n \times K_B T$$

$$\Rightarrow n = \frac{P}{K_B T}$$

$$\therefore \bar{\lambda} = \frac{K_B T}{\sqrt{2} \pi d^2 P} \quad \text{--- (3)}$$

$$\begin{aligned} \rightarrow & \boxed{\bar{x} \propto \frac{1}{n}} \\ \rightarrow & \boxed{\bar{x} \propto T} \\ \rightarrow & \boxed{\bar{x} \propto \frac{1}{P}} \end{aligned}$$

$$\begin{aligned} \rightarrow & \boxed{\bar{x} \propto 1/s} \\ \rightarrow & \boxed{\bar{x} \propto m} \end{aligned}$$

# Avogadro Number :  $6.022 \times 10^{23}$

① To measure weight of one atom:

$$\begin{array}{|c|} \hline \text{Atomic weight (in grams)} \\ \hline N_A \\ \hline \end{array}$$

② To measure 1 molecular weight:

$$\begin{array}{|c|} \hline \text{Molecular Weight (in gm)} \\ \hline N_A \\ \hline \end{array}$$

③ No. of atoms present in a given mass:

$$\begin{array}{|c|} \hline N_A \quad \times \text{mass} \\ \hline \text{atomic wt.} \\ \hline \end{array}$$

④ No. of molecules present in a given mass:

$$\begin{array}{|c|} \hline N_A \quad \times \text{mass} \\ \hline \text{atomic wt.} \\ \hline \text{molecular} \\ \hline \end{array}$$

⑤ To find Total no. of molecules in a given molecule:

$$\begin{array}{|c|} \hline \frac{N_A}{22.4} \times N_A \\ \hline \end{array}$$

Q Estimate volume of water molecule. ( $s_w = 1000 \text{ kg/m}^3$ )

↳ Molecule in 18g =  $N_A$

$$\therefore 1 \text{ molecule wt} = \frac{18 \times 10^{-3}}{N_A} \text{ kg}$$

$$= \frac{18 \times 10^{-3}}{6 \times 10^{23}} = 3 \times 10^{-26} \text{ kg}$$

$$\therefore V = \frac{3 \times 10^{-26}}{10^3} = 3 \times 10^{-29} \text{ m}^3$$

## \* Oscillations

### o Periodic Functions

↳ Property  $\Rightarrow F(t) = \sin \omega t$   
 $\therefore = \sin \frac{2\pi}{T} t$

$$\begin{aligned} f(t+T) &= \sin \frac{2\pi}{T} (t+T) \\ &= \sin \frac{2\pi}{T} t + \frac{2\pi}{T} \times T \\ &= \sin \left( \frac{2\pi}{T} t + 2\pi \right) \\ &= \sin \frac{2\pi}{T} t \end{aligned}$$

$$\therefore f(t+T) = f(t).$$

i.e., the function repeats itself after time period  $T$ .

## \* Simple Harmonic Motion (SHM)

↳ Body performing l motion about fixed position in which displacement of body is directly proportional to restoring force.

$$F \propto x$$

$$\therefore F = -kx$$

[ $k$  = force constant]

$$\therefore ma = -kx$$

$$\therefore a = -\frac{k}{m} x$$

$$\therefore a \propto x$$

→ Acceleration is directly proportional to displacement

$$\therefore F \propto x$$

$$\therefore F = -kx$$

$$\therefore ma = -kx$$

$$\therefore \frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

→ object is oscillating

$$x = A \cos(\omega t + \phi)$$

Position Amplitude Freq Time

$$[x \cdot (t + T_0) \text{ and } \omega] \text{ and } \frac{\omega T}{2\pi} = \frac{2\pi}{T} = 1$$

\* Time Period

$$\text{By } x = A \cos(\omega t + \phi)$$

$$\text{Let } t = t + \frac{2\pi}{\omega}$$

$$\therefore x' = A \cos(\omega t + \frac{2\pi}{\omega} + \phi)$$

$$\therefore x' = A \cos(\omega t + 2\pi + \phi)$$

$$\therefore x = A \cos(\omega t + \phi)$$

$$\therefore T = \frac{2\pi}{\omega}$$

$$\text{or } T = 2\pi \sqrt{\frac{m}{k}}$$

$$(m \propto \frac{1}{k})$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \rightarrow \text{eqn}$$

$$x = A \cos(\omega t + \phi) \rightarrow \text{solt}$$

$$\therefore v = \frac{dx}{dt} = A \left[ -\sin(\omega t + \phi) \cdot \omega \right]$$

$$\therefore v = -A\omega \sin(\omega t + \phi) \rightarrow \text{Velocity of wave}$$

$$\text{Also, } a = \frac{d^2x}{dt^2} = -A\omega \left[ \cos(\omega t + \phi) \cdot \omega \right]$$

(After differentiating)

$$2) a = -A\omega^2 \underbrace{\cos(\omega t + \phi)}_{\substack{\downarrow \\ \omega^2}} \quad (\omega t + \phi) \rightarrow A = x \quad \checkmark$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$(\omega t + \phi) \rightarrow A = x \quad \checkmark$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \checkmark$$

Hence  
Proved

## \* Energy in SHM

$$TE = KE + PE$$

### (1) K.E.

$$KE = \frac{1}{2} mv^2$$

~~By~~  $x = A \cos(\omega t + \phi)$

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$

$$v^2 = A^2 \omega^2 \sin^2(\omega t + \phi)$$

$$\text{Now } \rightarrow A^2 \sin^2(\omega t + \phi) = A^2 (1 - \cos^2(\omega t + \phi))$$

$$= A^2 - x^2$$

$$\therefore v^2 = \omega^2 (A^2 - x^2)$$

$$\therefore KE = \frac{1}{2} mv^2 = \frac{1}{2} m \omega^2 (A^2 - x^2)$$

$$KE = \frac{1}{2} K (A^2 - x^2) \quad (m = \frac{k}{\omega^2})$$

(2) P.E

$$\hookrightarrow F = -kx$$

$\therefore$  Work done against restoring force,

$$dW = -F dx$$

$$= (-kx dx) \cdot sA = -\frac{1}{2} kx^2 sA$$

$$\boxed{PE = \frac{1}{2} kx^2}$$

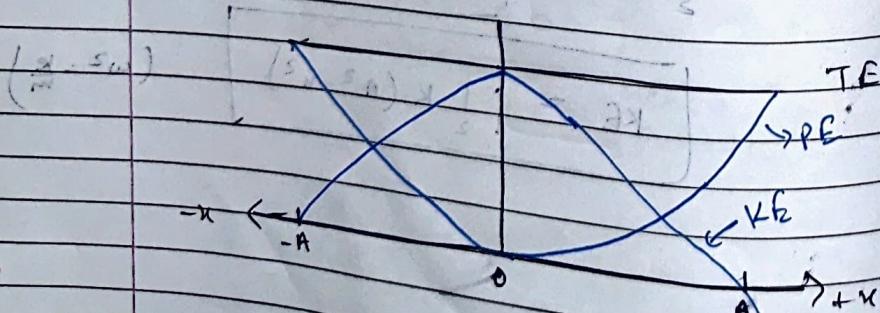
$$\therefore P.E. Energy = \frac{1}{2} m\omega^2 (x_0^2 - A^2) = S_A$$

$$(x_0 + \omega t)^2 - A^2 = (x_0 + \omega t)^2 - S_A$$

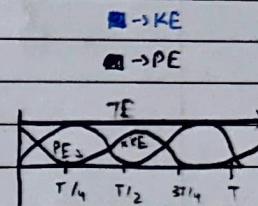
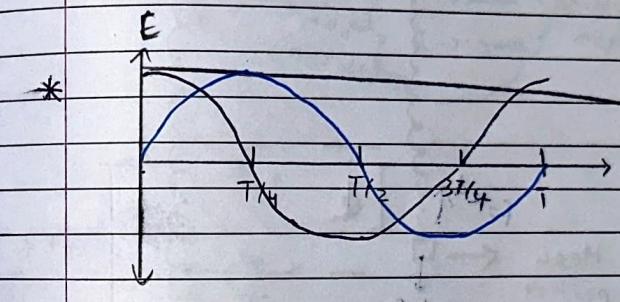
$$(3) T.E. \quad x - S_A =$$

$$\hookrightarrow TE = KE + PE$$

$$\boxed{TE = \frac{1}{2} KA^2} \quad \& \quad \boxed{TE = \frac{1}{2} m\omega^2 A^2}$$



$$\begin{aligned} KE &= 0 & x=0 & KE = 0 \\ PE &= \frac{1}{2} KA^2 & KE = \frac{1}{2} KA^2 & PE = \frac{1}{2} KA^2 \\ TE &= \frac{1}{2} KA^2 & PE = 0 & TE = \frac{1}{2} KA^2 \\ && TE = \frac{1}{2} KA^2 & \end{aligned}$$



\* Oscillations in loaded spring (horizontal)

$$\rightarrow \text{or } F = -kx \text{ (Hooke's law)}$$

$$\Rightarrow m\frac{d^2x}{dt^2} = -kx$$

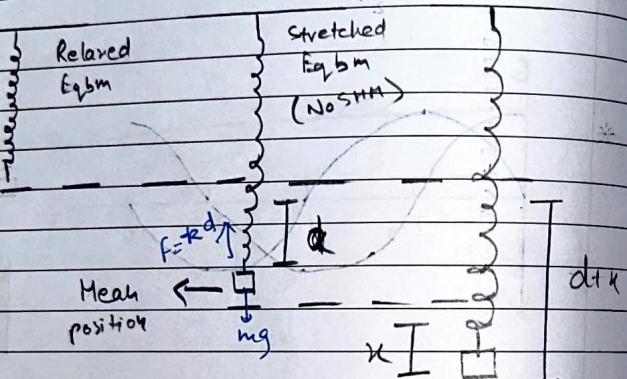
$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\Rightarrow \frac{d^2v}{dt^2} = -\omega^2 v$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \rightarrow \text{Proof that spring is SHM.}$$

$$\text{Time period of spring} = \frac{1}{2\pi} \sqrt{\frac{m}{k}}$$

### \* Oscillations in loaded spring (vertical)



Vertical spring is already stretched. ~~due to weight of mass.~~

Mean position will be changed. (d distance from relaxed)

2nd diagram  $\rightarrow mg = kd$

↑ Restoring force

Eqn for vertical oscillation:  $F = mg - kd - kx$

$$F = -kx$$

$$mg = -kx$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

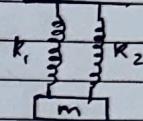
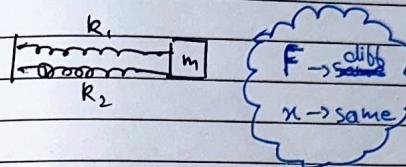
$$\therefore T = 2\pi \sqrt{\frac{m}{k}}$$

$$\omega = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Common connection  $\rightarrow$  parallel

In vertical oscillation, role of 'g' or weight of body.

### \* II combination in springs



$$F_1 = -k_1 x$$

$$F_2 = -k_2 x$$

$$\therefore F_{\text{res}} = F_1 + F_2$$

$$= - (k_1 + k_2) x$$

$$\therefore K_{\text{res}} = k_1 + k_2 \dots K_n$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$