1hp = 746 W

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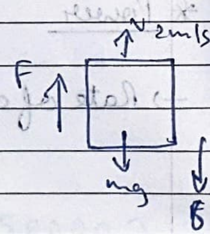
$P_1 = \frac{600000}{60} = 10,000\ W$

$P_2 = \frac{600000}{120} = \underline{5000\ W}$

→ Do cranes consume same fuel?

→ Same fuel as work done is same.

Q → A lift of 1800 kg moving up with 2 m/s. Friction = 4000 N. Calculate max power.



→ At equilibrium, F = mg + f

$\therefore F = 18000 + 4000$

$= \underline{22000 N}$

$\therefore P = F \times v$

$= 22000 \times 2 = 44000\ W$

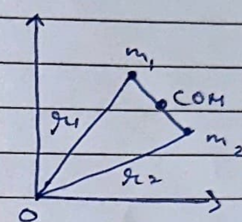
$= \underline{44\ kW}$

Date: 16/08/24

* Ch-6 : System of Particles & Rotational Motion

* Centre of Mass

→



→ $COM = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

∴, if n particles are there,

$$COM = \frac{m_1 r_1 + m_2 r_2 + \dots m_n r_n}{m_1 + m_2 + m_3 \dots m_n}$$

∴ Coordinates of COM for ~~two~~ 3 dimensional body:

$$X = \frac{m_1 x_1 + m_2 x_2 \dots m_n x_n}{m_1 + m_2 \dots m_n}$$

$$Y = \frac{m_1 y_1 + m_2 y_2 \dots m_n y_n}{m_1 + m_2 \dots m_n}$$

$$Z = \frac{m_1 z_1 + m_2 z_2 \dots m_n z_n}{m_1 + m_2 \dots m_n}$$

☆ If masses are same, i.e, mass distribution is uniform,

$COM = \frac{r_1 + r_2 \dots r_n}{n}$, i.e, at geometrical centre

☆ Ch - COM & Rotational Motion

→ $\vec{F}_{total} = M a_{cm}$

If $F_{total} = 0$, $M a_{cm} = 0$

$\Rightarrow \vec{a}_{cm} = 0$

$\Rightarrow \frac{d\vec{v}_{cm}}{dt} = 0$

$\Rightarrow \vec{v}_{cm} = \text{constant}$

→ $F_{total} = 0$

$\Rightarrow F_1 + F_2 + F_3 \dots F_n = 0$

$\Rightarrow m_1 a_1 + m_2 a_2 \dots m_n a_n = 0$

$\Rightarrow m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} \dots + m_n \frac{dv_n}{dt} = 0$

$\Rightarrow \frac{d}{dt}(m_1 v_1 + m_2 v_2 + m_3 v_3 \dots m_n v_n) = 0$

$\Rightarrow m_1 v_1 + m_2 v_2 \dots m_n v_n = \text{constant}$

$\Rightarrow P_1 + P_2 + \dots P_n = \text{constant}$

→ $\vec{R}_{cm} = \frac{m_1 r_1 + m_2 r_2 \dots + m_n r_n}{m_1 + m_2 + \dots m_n}$

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$$\Rightarrow \frac{dR_{cm}}{dt} = \frac{1}{M}\frac{d}{dt}\left[m_1 r_1 + m_2 r_2 \ldots m_n r_n\right]$$

$$= \frac{1}{M}\left[m_1\frac{dr_1}{dt} + m_2\frac{dr_2}{dt} \ldots m_n\frac{dr_n}{dt}\right]$$

$$V_{cm} = \frac{1}{M}\left[P_1 + P_2 + P_3 + P_4\right]$$

∴ Total Momentum of System = $\underbrace{M}_{\text{Total Mass}}\; V_{cm}$ → v of com

$$\boxed{V_{cm} = \frac{m_1v_1 + m_2v_2 \ldots + m_nv_n}{m_1 + m_2 \ldots m_n}}$$

5.5 ✓ 31/8/24

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★ Newton's Law of Motion, Revision

Q.1 Give magnitude & direction of force:

(1) Drop of rain falling down with constant speed.

↳ Magnitude = ~~mg~~ 0   ~~Direction = downwards~~ Weight of raindrop = Air Drag

(2) A cork of mass 10g floating on water.

↳ Net force = 0   Weight of cork = Buoyant force

(3) A kite held stationary in the sky.

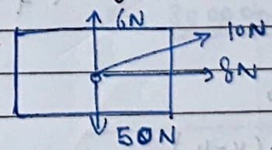
↳ Net force = 0   Tension in string = Force by flow of Air

(4) A car moving with constant speed 30 km/h on rough road.

↳ Net force = 0   Force of Engine = Friction.

Q.2 A body of mass 5 kg is acted upon by two perpendicular forces 8N & 6N. Give magnitude and direction of acceleration.

Ans 2



$F = ma$

$10 = 5 \times a$

$\underline{a = 2\,m/s^2}$ in direction of resultant force. i.e. 37° North of East.

Fnet = 0

$\Rightarrow 8 = f\cos\theta$

$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right) = 37^\circ$.

Q.3

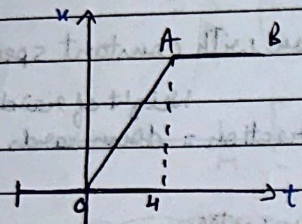


Fig. show x-t graph of particle of mass 4 kg. What is the:

(1) Force acting on particle for $t<0$, $t>>4s$, $0<t<4s$

(2) Impulse at $t=0$ & $t=4$

↳ (1) $t<0 \rightarrow$ 0N

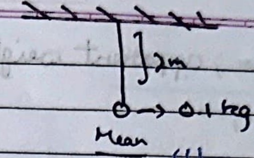
$t>>4s \rightarrow$ 0N as no change in velocity so $a=0$.

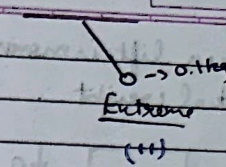
$0<t<4s \rightarrow$ 0N as acc $=0$

(2) $J = F\times t$
$= \Delta P$
$= mv - mu$
$= m(v-u) = 4\left(\frac{3}{4}\right) = \underline{3 \text{ kg m/s}}$

$J = \Delta P$
$= m(v-u)$
$= 4\left(0-\frac{3}{4}\right) = \underline{-3 \text{ kg m/s}}$

Q.4





Speed of bob at mean = 1 m/s

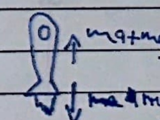
What is trajectory?
in fig (II)

→ At extreme, KE = 0.
∴ v = 0
∴ The bob will fall straight down.

→ At mean position, the bob will fall down in direction of velocity in projectile motion.

Q.5 A rocket with lift off mass 20,000 kg is moved upward with acceleration with 5 m/s². Calculate the initial thrust.

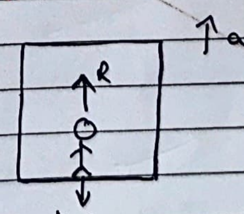
Ans 5 ~~Resultant force = ma − mg~~
~~= 20000×5 − 20000×10~~
~~= −100,000 N~~



To fly rocket
To raise rocket

Force to raise rocket up $= ma + mg$
$= 20000 \times 15$
$= \underline{300000 \text{ N}}$

Q.6

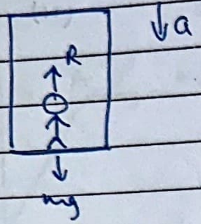


$\boxed{R - mg = ma \\ R = m(a+g)}$ — Lift Question

apparent weight

• When lift is moving upwards, apparent weight is higher than actual weight.

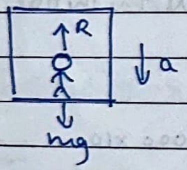
• When lift is moving downwards, apparent weight is less than actual weight.



$\Rightarrow mg - R = ma$

$\Rightarrow \boxed{R = m(g-a)}$ — apparent weight < actual weight

• When rope holding lift is cut, weightlessness is experienced.



Free fall is initiated.

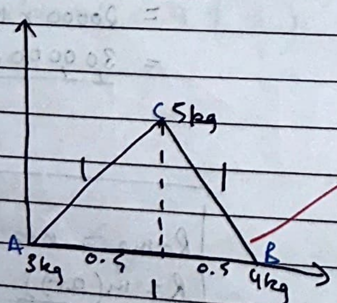
$\therefore a = g$

$\therefore \underline{\underline{R = 0}}$

x ———— x ———— x ———— x

★ **Rotational of Revision**

Q.1



$A = (0,0)$ $\quad C = (0.5, \frac{\sqrt{3}}{2})$

$B = (1,0)$

$$CM_x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{4 + 5 \times \frac{1}{2}}{12}$$

$$= \frac{13}{2} \times \frac{1}{12} = \boxed{\frac{13}{24}}$$

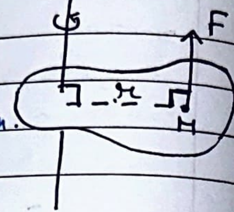
$$CM_y = \frac{\frac{\sqrt{3}}{2} \times 5}{12}$$

$$= \frac{5\sqrt{3}}{24}$$

$$\therefore CM = \left(\frac{13}{24}, \frac{5\sqrt{3}}{24}\right)$$

* MOMENT OF FORCE or TORQUE

→ Torque is defined as moment of force or cross product of force applied to ⊥ to centre of mass & perpendicular dist. in between.

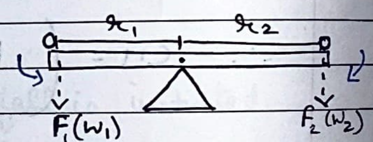


τ = Force × perpendicular dist.
$= r \times F$.

In vector form → $\vec{\tau} = \vec{r} \times \vec{F}$ → cross product
$= rF\sin\theta\, \hat{n}$

Unit
Nm

* Principle of Moments (Torque)



→ For clockwise rotation:
$\tau_1 = W_2 \times r_2$

→ For anticlockwise rotation:
$\tau_2 = W_1 \times r_1$

→ Let $\tau_1 = \tau_2$.

$\therefore W_1 \times r_1 = W_2 \times r_2$ → imp!

→ Used in lever principle.

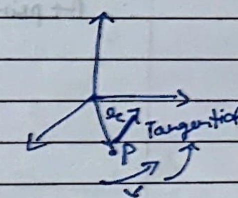
Eg $\underbrace{W_1}_{\text{Load}} \times \underbrace{r_1}_{\text{Load Arm}} = \underbrace{W_2}_{\text{Effort}} \times \underbrace{r_2}_{\text{Effort Arm}}$

→ If effort arm is larger load arm, less effort is reqd.

Imp! If Torques are equal & in same direction (both up or both down), body will not undergo rotational motion.

"Couple" → Pair of equal & opposite Torques (one up & one down)
↳ Necessary for Rotational Motion.

* Angular Momentum (Turning effect of body)



→ $P = mv$

$P \times r$ = Turning effect of body
= Angular momentum

$\therefore \underbrace{L}_{\text{Ang. Momentum}} = P \times r$

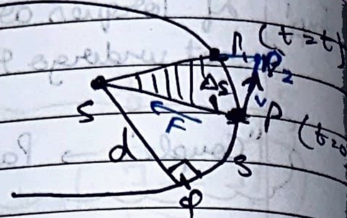
$\therefore \vec{L} = \vec{r} \times \vec{P}$ → cross product ~~cross product~~

* If $\theta = 0°$, $L = 0$.
If $\theta = 90°$, $L = max$

• **Conservation of Angular Momentum**

↳ If total Torque (τ_{ext}) acting on system is 0, then total angular momentum of the system is conserved.

Eg Planetary Motion



→ Planet at P at t = 0

Planet at P_1 at t = t

∴ In Δt time, Δs distance travelled

Let perpendicular distance be 'd'.

∴ At point P, $\vec{L} = \vec{r} \times \vec{P}$

$L = rp \sin\theta$

$= rp$

$= mvd$

$\tau = \vec{r} \times \vec{F}$

$= rF\sin\theta$

$= 0$

~~Again~~ As $\sin\theta = 0$

Both Force & distance are parallel to each other.

Area of $\Delta S\phi P = \frac{1}{2} \times s \times d$

→ In time Δt, planet p moves Δs distance.

∴ $\Delta SPP_1 \approx \Delta SPP_2$ (for $\lim \Delta t \to 0$)

∴ $\Delta A = \frac{1}{2} \times \Delta s \times d$

Taking timerate on both sides,

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \times \frac{\Delta s}{\Delta t} \times d$$

$$\Rightarrow \frac{\Delta A}{\Delta t} = \frac{1}{2} \times v \times d$$

Multiply by mass 'm' both sides,

$$\Rightarrow m\frac{\Delta A}{\Delta t} = \frac{1}{2} mvd$$

$$\Rightarrow \boxed{\frac{\Delta A}{\Delta t} = \frac{L}{2m}}$$

→ The line of action of force is passing through the sun so Torque is 0.

$$\therefore \boxed{\frac{dA}{dt} = \text{constant}}$$

Rotational Kinematics

(1) **Velocity Time Relation**

↳ $\alpha = \frac{d\omega}{dt}$

$\Rightarrow d\omega = \alpha\, dt.$

$\Rightarrow \int_{\omega_0}^{\omega} d\omega = \alpha \int_0^t dt$

$\Rightarrow (\omega - \omega_0) = \alpha t$

$\Rightarrow \boxed{\omega = \omega_0 + \alpha t}$ where ω → final & ω_0 → initial

(2) **Position time relation**

$\Rightarrow \omega = \frac{d\theta}{dt}$

$\Rightarrow \cancel{\omega dt = d\theta}$ $\quad d\theta = \omega\, dt$

$\Rightarrow \cancel{\int \omega . dt =}$ $\quad d\theta = (\omega_0 + \alpha t)\, dt$

$\Rightarrow \int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)\, dt$

$\Rightarrow \boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$

(3) Position velocity relation

$\rightarrow \alpha = \frac{d\omega}{dt}$

$= \frac{d\omega}{d\theta} \times \frac{d\theta}{dt}$

$\alpha = \omega \frac{d\omega}{d\theta}$

$\Rightarrow \int_0^\theta \alpha\, d\theta = \int_{\omega_0}^{\omega} \omega . d\omega$

$\Rightarrow \alpha\theta = \frac{\omega^2}{2} - \frac{\omega_0^2}{2}$

$\Rightarrow \boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$

Analogy

	Linear Motion	Rotational Motion
1	Position $\rightarrow x / s$	Ang. Displacement $= \theta$
2	Velocity 'v': $v = \frac{dx}{dt}$, $v = r\omega$	Ang. Velocity 'ω': $\omega = \frac{d\theta}{dt}$; $\omega = \frac{v}{r}$
3	Acceleration 'a'. $a = \frac{dv}{dt}$, $a = r\alpha$	Ang. Acceleration 'α' $\alpha = \frac{d\omega}{dt}$
4	Momentum $= mv$ (P)	Angular Mom $= L = I\omega = \vec{r} \times \vec{P}$
5	Mass $\rightarrow m$	Moment of Inertia $\rightarrow I$
6	Force $\rightarrow F = ma = \frac{dP}{dt}$	Ang. Force $= \tau = I\alpha = \vec{r} \times \vec{F} = \frac{dL}{dt}$
7	Kinetic Energy $= \frac{1}{2}mv^2$	Ang KE $= \frac{1}{2}I\omega^2$
8	Work $= Fs$	Work $= \tau\theta$
9	Power $= F \times v$	Power $= \tau\omega$

(10) $\omega = \omega_0 + \alpha t$ $s = ut + \frac{1}{2}at^2$ $2as = v^2 - u^2$	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $2\alpha\theta = \omega^2 - \omega_0^2$

★ $L = \vec{r} \times \vec{p}$

$\tau = \vec{r} \times \vec{F}$

$\rightarrow \frac{dL}{dt} = \frac{d}{dt}[\vec{r} \times \vec{p}]$

$= \frac{dr}{dt}\vec{p} + \vec{r}\frac{dp}{dt}$

$= \underbrace{\vec{v} \times \vec{p}}_{(as\ \theta = 0)} + \vec{r} \times \vec{F}$ $[v \parallel p]$

$$\boxed{\frac{dL}{dt} = \vec{r} \times \vec{F} = \tau}$$

Q. The angular speed of wheel increased from 1200 (20 rps) rpm to 3120 rpm (52 rps) in 16 s.

(1) What is α?

(2) How many revolutions?

a) $\rightarrow \alpha = \frac{2\pi(52 - 20)}{16}$

$= 4\pi \text{ rad/s}^2$

b) $\rightarrow$ ~~$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$~~

$\rightarrow$ ~~$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$~~

$\rightarrow$ ~~$\theta = 2\pi$~~

~~$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$~~

~~$\theta = 400\pi(16) + \frac{1}{2} \times 40\pi \times 256$~~

~~$\theta = 6400\pi + 5120\pi$~~

$\theta = 1280\pi$

~~$\therefore$ Total revolutions = 640 revolution~~

$\rightarrow \theta = 40\pi(16) + \frac{1}{2} \times 4\pi \times 256$

$\theta = 640\pi + 512\pi$

$= 1152\pi$

$\therefore$ Revolutions = 576

Q. Radius of the wheel is 0.4 m. Car accelerates from rest to $\alpha = 1.5 \text{ rad/s}^2$ in 20 s. How much distance the wheel covers in this time & what will be linear velocity?

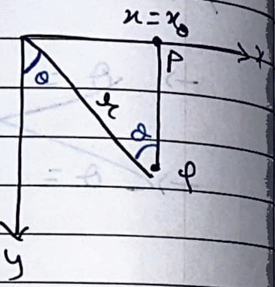
$\rightarrow a = 0.4 \times 1.5$ $\quad u = 0$

$= 0.6 \text{ m/s}^2$

$v = at \Rightarrow v = 0.6 \times 20 = 12 \text{ m/s}$

→ $s = \frac{1}{2} \times 0.6 \times \cancel{400}^{200} = \boxed{120\,m}$

Imp. Q. A particle of mass m is released from point P at $x = x_0$ on the x axis from origin, falls vertically along y axis as shown.



(1) Find the Torque acting on the particle at time t when it is at point Q w.r.t O.

→ $\tau = r\,mg \sin\theta$

$= \cancel{r}\,mg \frac{x_0}{\cancel{r}} = \boxed{mg\,x_0}$

(2) Find the angular momentum L about point O.

→ $R = r$, $v = gt$, $P = mgt$

$\therefore L = \cancel{r}\,mgt\,\frac{x_0}{\cancel{r}} = \boxed{\tau t}$

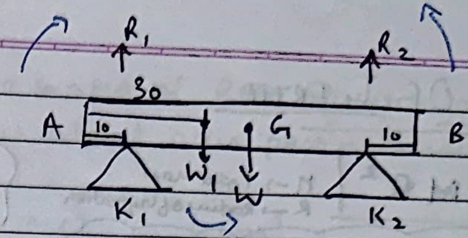
$= \boxed{mgtx_0}$

(3) Prove, $\tau = \frac{dL}{dt}$

→ $\frac{dL}{dt} = mgx_0 \frac{dt}{dt}$

$= mgx_0 = \tau$

Q.



Metal bar 70 cm long, 4 kg mass. K_1, K_2 placed 10 cm from each end. 6 kg weight is suspended at 30 cm from one end.
Find reaction at fulcrum ($g = 9.8$).

→ For translational equilibrium:

$W + W_1 = R_1 + R_2$ — (1)

$10 = R_1 + R_2$

$\therefore R_1 + R_2 = 98\,N$

For rotational eqbm:

Clockwise torque = Anticlockwise moment

$R_1 \times 25 = R_2 \times 25 + W_1 \times 5$

→ $25(R_1 - R_2) = 300$

→ $R_1 - R_2 = \frac{30^{6}}{25_{5}}$ $R_1 - R_2 = \frac{300}{25} = 12$

→ $R_1 = R_2 + \frac{6}{5}$ — (2) $R_1 = 12 + R_2$

$\therefore 2R_2 + \frac{6}{5} = 98$ $2R_2 + 12 = 98$

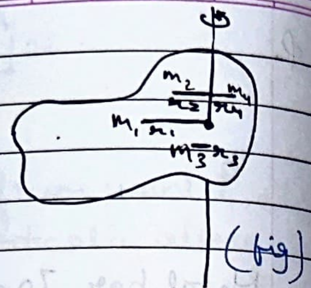
$2R_2 = 86$

$R_2 = 43$ $R_1 = 55$

$10R_2 = 490 - 6$

$R_2 = 48.4$ $R_1 \to 49.6$

Moment of Inertia



→ $\boxed{I = MR^2}$ M → Total Mass, R → Radius of Gyration

→ $I = m_1r_1^2 + m_2r_2^2 \ldots m_nr_n^2$

General

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

- Depends on mass of the body.
- Shape & Size of Body.
- Distribution of mass

Radius of Gyration

→ From fig, let $m_1 = m_2 = m_3 \ldots = m_n = m$

$$I = m(r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2)$$

$$= m \times n \left(\frac{r_1^2 + r_2^2 + r_3^2 \ldots r_n^2}{n}\right)$$

$$= m \times n \times K^2$$

$$= MK^2$$

where $K = \sqrt{\frac{r_1^2 + r_2^2 \ldots r_n^2}{n}}$ (root mean square)

K is radius of gyration

Q. Calculate the ratio of ROG of circular ~~wheel~~ ring & disk of same radius about axis passing through COM

Ans) $I_1 = Mr^2$ (ring)

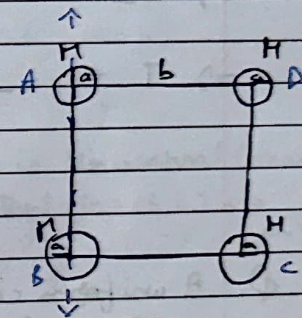
$I_2 = \frac{1}{2}MR^2$ (disc)

$K_1 = R$

$K_2 = \frac{1}{2}R$

$\therefore \frac{K_{ring}}{K_{disc}} = \frac{\sqrt{2}R}{R} = \boxed{\sqrt{2}:1}$

Q. Find MOI.



~~→ A = (0, b), B = (0, 0), C = (0, b), D = (a, a)~~

→ $I_A = \frac{2}{5}Ma^2$

$I_B = \frac{2}{5}Ma^2$

$I_C = Mb^2 + \frac{2}{5}Ma^2$

$I_D = Mb^2 + \frac{2}{5}Ma^2$

$I_{square} =$

Total I $= \left(4 \times \frac{2}{5}Ma^2\right) + (2Mb^2) = \frac{8}{5}Ma^2 + 2Mb^2$

$= \frac{2M}{5}(4a^2 + 10b^2)$

$\boxed{\pi^2 = 10}$

Q. A wheel is rotating with 1000 rpm. $KE = 10^6$ J. Calculate MOI of wheel.

$\Rightarrow KE = \frac{1}{2} I\omega^2$

$\Rightarrow I = \frac{2KE}{\omega^2}$ $\quad \omega = \frac{1000}{3600} \times 2\pi = \frac{100\pi}{3}$

~~$\Rightarrow I = \frac{2 \times 10^6 \times 3}{50\ 25}$~~

~~$\Rightarrow I = 12 \times 10^4$~~

$\Rightarrow I = \frac{2 \times 10^6 \times 9}{100\pi \times 100\pi}$

$= \frac{9 \times 10^2}{5} = \boxed{180}$

Q. A uniform circular disc of mass m is rolling on a smooth table with velocity v. Find Total KE.

↳ ~~$KE = \frac{1}{2} mr^2 \times \frac{v^2}{r^2} = \boxed{\frac{1}{2}mv^2}$~~

TKE = Rotational kinetic energy + Translational KE.

$= \frac{1}{2} I\omega^2 + \frac{1}{2}mv^2$

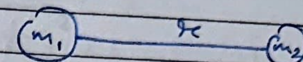
~~$= \frac{1}{2} mv^2 + \frac{1}{2} mv^2 = \boxed{1\,mv^2}$~~

$= \frac{1}{4} mr^2 \times \frac{v^2}{r^2} + \frac{1}{2}mv^2$

$= \frac{1}{4}mv^2 + \frac{1}{2}mv^2 = \boxed{\frac{3}{4}mv^2}$

★ Ch-7: Gravitation

★ Newton's Law of Gravitation



$\Rightarrow F \propto m_1 m_2$

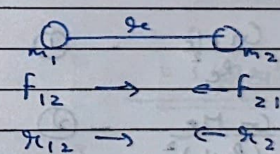
$\Rightarrow F \propto \frac{1}{r^2}$

$\therefore$,

$\Rightarrow F \propto \frac{m_1 m_2}{r^2}$

$F = \frac{G m_1 m_2}{r^2}$, where G is Universal Gravitation constant ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

→ Vector Form



$F_{12} = \frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}$ → in direction of 1 to 2.
↳ magnitude

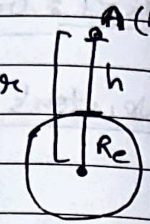
$F_{21} = \frac{G m_1 m_2}{r_{21}^2} \hat{r}_{21}$

$\Rightarrow \hat{r}_{12} = -\hat{r}_{21}$

$\therefore$, $\vec{F}_{12} = -\vec{F}_{21}$

Date:

★ Acceleration due to gravity (g)



→ $F = \frac{GmM_e}{r^2}$ $(r = R_e + h)$

Also, $F = mg$.

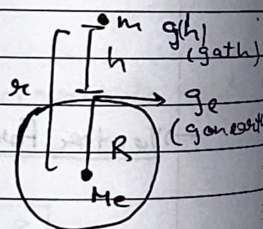
$\therefore mg = \frac{GmM_e}{r^2}$

⇒ $\boxed{g = \frac{GM_e}{r^2}}$

If $r = R_e$, ∴ $\boxed{g = \frac{GM_e}{R_e^2}}$ ⇒ $g = 9.8\,m/s^2$

* only on surface of Earth.

★ Variation of g with altitude



→ $g_e = \frac{GM_e}{R_e^2}$ — ①

$g_h = \frac{GM_e}{(R_e+h)^2}$ — ②

Dividing ② by ①,

$\cancel{\frac{g_h}{g_e} = \frac{(R_e + h)^2}{(R_e)^2}}$

⇒ $\cancel{\boxed{g_h = g_e\left(\frac{R_e + h}{R_e + h}\right)^2}}$ ⇒

⇒ $\boxed{\frac{g_h}{g_e} = \frac{R_e^2}{(R_e+h)^2}}$ → Main Eqn

(For large height)

$R_e = 6400\,km$

⇒ $\frac{g_h}{g_e} = \frac{1}{\left(1+\frac{h}{R_e}\right)^2}$

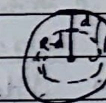
$= \left(1 + \frac{h}{R_e}\right)^{-2}$ → (Binomially expanding and not taking higher order terms) (When $h << R$)

$g_h = g_e\left(1 - \frac{2h}{R_e}\right)$

(For small height only)

↳ Height increases, value of g decreases

→ Variation of g with depth



→ $g_e = \frac{GM}{R^2}$ $\rho = \frac{M}{V}$

$\cancel{g_h} = \frac{GM}{(R-d)^2}$ $M = \rho V$

$= \rho \frac{4}{3}\pi r^3$ → Mass of Earth

$= \frac{G \rho \frac{4}{3}\pi r^3}{r^2}$

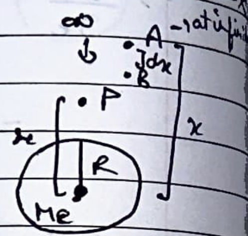
$= \frac{4}{3} G\rho\pi R$ — ①

$g_d = \frac{4}{3} G\rho(R-d)\pi$ — ②

$\frac{g_d}{g_e} = \frac{R-d}{(R-d)}$ ⇒ $\cancel{g_d = g_e \frac{R}{(R-d)}}$

$$\boxed{g_d = g_e\left(1 - \frac{d}{R}\right)} \rightarrow H \downarrow \; g \downarrow$$

Gravitational Potential Energy



→ From A to B,

$$F_A = \frac{Gm_1m_2}{x^2}$$

Workdone through AB (dw) = $F\,dx$

$$\rightarrow dw = \frac{Gm_1m_2}{x^2} \cdot dx$$

$$\rightarrow \int dw = \int_{\infty}^{R} \frac{Gm_1m_2}{x^2} \cdot dx$$

$$\rightarrow W = GM_1m_2\left[-\frac{1}{x}\right]_{\infty}^{R}$$

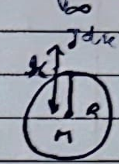
$$\rightarrow \boxed{W = -\frac{GMm}{R}}$$

$$\boxed{W = U = -\frac{GMm}{R}} \rightarrow \text{G.P.E}$$

→ -ve sign (indicates bound state) because external work needs to be done to bring the body away from the ~~body~~ Earth.

Escape Velocity

→ Max velocity with which a body can be thrown to remove the influence of Earth's gravitational field.



$$dw = f \cdot dx$$

$$\int dw = \int \frac{GMm}{x^2} \cdot dx$$

$$\rightarrow W = GMm\left[\frac{1}{x}\right]_{R}^{\infty}$$

$$\rightarrow \boxed{W = \frac{GMm}{R}}$$

→ At surface of Earth $x = R$

$$U = \frac{GMm}{R}$$

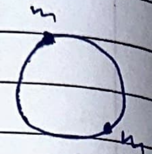
$$gR^2 = gM$$

$$\rightarrow \frac{GMm}{R} = \frac{1}{2}mv^2$$

$$\rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}} = \boxed{\sqrt{2gR}}$$

$$= 11.2 \text{ km/s}$$

Q. 2 particles each of mass 'm' go round a circle of radius r under the action of gravitational force. Find the speed of each particle.

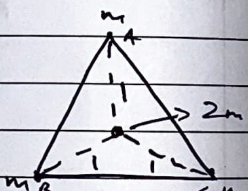


↳ $F = \frac{Gm^2}{4r^2}$

$\frac{mv^2}{r} = \frac{Gm^2}{4r^2}$

⇒ $\boxed{v = \sqrt{\frac{Gm}{4r}}}$

Q. Total force acting on mass 2m placed at centroid.



→ $F_G = F_{GA} + F_{GB} + F_{GC}$

$= \cancel{6Gm^2}$

$= (-F_{GA}) + F_{GB} + F_{GC}$

$= (-2Gm^2) + \sqrt{(2Gm^2)^2 + (2Gm^2)^2 - (2Gm^2)(2Gm^2)}$

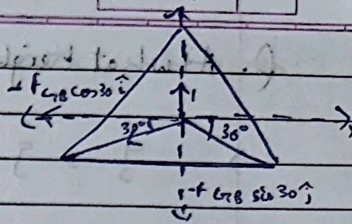
$= (-2Gm^2) + 2Gm^2$

$= \sqrt{}$

→ $F_{GA} = 2Gm^2\,\hat{j}$

→ $F_{GB} = 2Gm^2\,[-\hat{i}\cos 30^\circ - \hat{j}\sin 30^\circ]$

→ $F_{GC} = 2Gm^2\,[\hat{i}\cos 30 - \hat{j}\sin 30]$



$F_R = 2Gm^2\,\hat{j} - 2Gm^2\,\hat{j} - Gm^2\,\hat{j}$

⇒ $\underline{0}$

Q. At what height from the surface of the Earth the value of g reduces by 36% from the value at surface. $R = 6400\,km$

→ $g' = \frac{64g}{100}$ ($\frac{16}{25}$)

$g' = \frac{16g}{25}$

$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

⇒ $\frac{16}{25} = \frac{R^2}{(R+h)^2}$

⇒ $\frac{5}{4} = \frac{R}{R+h}$

⇒ $\cancel{5R+5h} = 4R$ $4R + 4h = 5R$

⇒ $h = \frac{6400}{4} = \boxed{1600\,km}$

Q. At what height is value of g half?

→ $\frac{g}{2} = g \frac{(R)^2}{(R+h)^2}$

$(R+h)^2 = 2R^2$

→ ~~$R^2 + h^2 + 2Rh = 2R^2$~~

→ ~~$h^2 + 12800h = 6400 \times 6400$~~

→ $(R+h) = \sqrt{2} R$

→ $h = (\sqrt{2}-1) R$

→ $h = (\sqrt{2}-1) 6400$
$= \underline{2649.6 \text{ km}}$

Q. Mount Everest is 8848 m (9 km) above sea level. Calculate g.

→ $g_h = 9.8 \left(1 - \frac{2 \times 8.8}{64000}\right)$

$= 9.8 \left(\frac{4000-11}{4000}\right)$

$= 9.8 \left(\frac{3989}{4000}\right)$

$= 9.8 (0.997)$

$= \boxed{9.77}$

Q. How much below surface of the Earth, g becomes 1% of its value at surface.

→ $\frac{g}{100} = g\left(1 - \frac{d}{R}\right)$

→ $\frac{1}{100} = 1 - \frac{d}{R}$

→ $R = 100(R-d)$

→ $100d = 99R$

→ $d = \frac{99 \times 6400}{100} = \boxed{6336 \text{ km}}$

Q. At what height the value of g is same as 80 km deep.

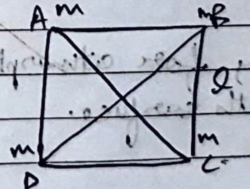
→ $\left(1 - \frac{2h}{R}\right) = 1 - \frac{d}{R}$

→ $\frac{2h}{R} = \frac{d}{R}$

→ $2h = d$ → Imp

$\underline{h = 40 \text{ km}}$

Q. find out pot energy of system.



$U_{net} = U_{AB} + U_{BC} + U_{CA} + U_{DA} + U_{AC} + U_{BD}$

Masses kept at vertices, Agar pure system ki ya aisa kuch pucha hai toh summation of individual quantities.

$U = -\frac{Gm_1m_2}{R}$

$U = -\frac{4Gm^2}{l} - \frac{2Gm^2}{\sqrt{2}l}$

$= -\frac{4Gm^2}{l} - \frac{\sqrt{2}Gm^2}{l}$

$= \boxed{-\frac{Gm^2}{l}\left(4+\sqrt{2}\right)}$

$= \boxed{\frac{-5.41\,Gm^2}{l}}$

Potential at centre

Potential → (V) = $\frac{\text{Work done}}{\text{mass}} = \frac{P.E}{m}$

$V = \boxed{\frac{-GM}{r}}$

$V_{centre} = V_A + V_B + V_C + V_D$

$= \frac{\sqrt{2}\,Gm}{l}$ $\boxed{-4\sqrt{2}\frac{GM}{l}}$

Q. Calculate the escape velocity for atmospheric particle 1600 km above Earth surface.

→ $v_e = \sqrt{2g_h(R+h)}$

$= \sqrt{2 \times g_c \frac{(R^2)}{(R+h)^2}(R+h)}$

⇒ $v_e = R\sqrt{2 \times \frac{9.8}{\frac{8000}{4000} \times 10^3}}$

⇒ $v_e = \frac{\overset{16}{6400}}{\underset{5}{20} \times 10^2}\sqrt{2 \times 4.9}$

Q. Determine escape velocity for moon. ($R = 1.76 \times 10^6\,m$, $m = 7.36 \times 10^{22}\,kg$)

→ $v_e = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{1.76 \times 10^6}}$

$= \sqrt{\frac{2 \times 6.67 \times 7.36 \times 10^5}{1.76}} = \boxed{2.37\,km/s}$

Q. Satellite revolves at 1000 km. Find orbital velocity & period of revolution. ($R = 6700\,km$, $M = 6 \times 10^{24}\,kg$)

→ $v_o = \sqrt{\frac{GM}{(R+h)}}$

$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7400 \times 10^3}} = \sqrt{\frac{6.67 \times 6 \times 10^8}{7412 \times 4}}$

$= \sqrt{\frac{6.67 \times 10^7}{4}}$

$= \boxed{7364\,m/s}$

$$T = \frac{2\pi(R+h)}{v_0}$$

$$= \frac{6.28 \times \cancel{7460}^{1.6} \times 10^3}{\cancel{7369}} = \boxed{6297\ sec}$$

Q. A satellite revolves very close to the Earth surface around density of planet $5.51 \times 10^3\ kg/m^3$. Calculate the time period.

$\rightarrow$ ~~$v_0 = \sqrt{gR}$~~

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{G \rho V}}$$

$$= 2\pi\sqrt{\frac{(R+h)^3}{G \frac{4}{3}\pi R^3 \rho}}$$

$$= \sqrt{\frac{\cancel{12}^{3}\pi^2 (R+h)^3}{G \cancel{4}\cancel{\pi} R^3 \rho}}$$

$$= \sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{3 \times 3.14 \times 10^{11}}{6.67 \times 5.51 \times 10^3}}$$

$$= \boxed{5062.7\ sec}$$

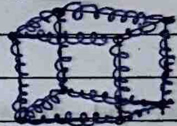
Ch-8 ⇒ Mechanical Properties of Solids

- **Elastic Behaviour:**

↳ Whenever force is applied on a solid body, it may deform. But once the force is released, the body retains its original state (shape & size). This is called elastic behaviour.

- **Plastic Behaviour:** → When force is applied on the body, it deforms. If, when the force is released, the body doesn't regain its shape & size, is called plastic behaviour. (when object stops regaining original shape & size, it enters plastic region)

Spring Model



→ Each atom is connected with a spring-like force (bond).

This result is origination of elastic behaviour.

→ Under effect of force, the body can compress & expand, and regain its original position.

- **Stress:** → $\frac{\text{Internal Restoring Force}}{\text{Area}}$ is defined as the stress.

$S = F/A$ → SI unit – N/m^2 Dim F → $[ML^{-1}T^{-2}]$

Types :-

!! Stress & Pressure are different.

(1) ~~Tangential~~ Tensile Stress / Longitudinal Stress

★ Length Increases

F ← [→F ———— F←] → F

↳ Inwards

→ Force opposing external force

Tensile → F/A

(2) Compressional Stress

★ Length Decreases

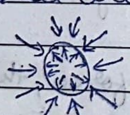
F → |←F ———— F→| ← F

↳ Outwards

$$C.\ Stress = \frac{F}{A}$$

(3) Volumetric / Hydrostatic Stress → (This is only considered as pressure.)

↳ When body is under ~~stress~~ forces from all the sides, restoring force develops all around the body.



Vol. Stress = F/A

↳ Considered the same as hydrostatic pressure as it is applied all over the body.

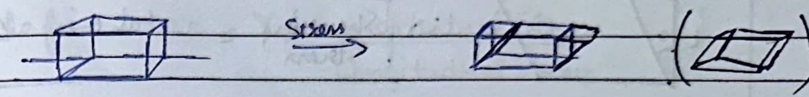
• Diff. b/w pressure & stress

Imp

↳ Stress is specific. We can apply stress to only change shape, its volume or its size etc. It can be applied in a direction oriented way.

But, pressure is force applied all over the body always. There is no category for pressure along the length, tangent, width etc.

(4) Shearing Stress (Tangential Stress)



↳ This kind of stress only & only changes the shape of the body. (No change in dimensions)

X ———— X ———— X ———— X

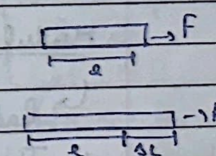
★ Strain (Deformation) → Dimensionless

↳ Strain is defined as: $\dfrac{\text{Change in Dimension}}{\text{Original Dimension}}$

○ Types:

(1) Longitudinal Strain: (Both tensile & compressional stress cause this)

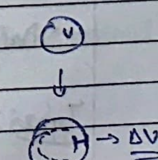
↳ Applying longitudinal stress to a rod of length l, causes its length to increase by Δl m.



$$\therefore \boxed{\text{Longitudinal strain} = \frac{\Delta L}{L}}$$

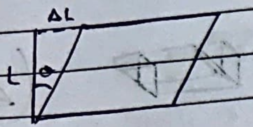
2) Volumetric Strain:

↳ Volumetric stress is applied on body causing volume to reduce by ΔV.



$$\boxed{\text{Volumetric Strain} = \frac{\Delta V}{V}}$$

(3) Shearing Strain: (Caused by shearing stress)



↳ $\boxed{\text{Shearing stress} : \tan\theta = \frac{\Delta L}{L}}$

★ Hooke's Law

↳ Hooke's law states that the strain observed in a body is proportional to the stress applied on it, provided it is within elastic limit.

$$\text{Stress} \propto \text{Strain}$$ (within elastic limit)

↳ $$\frac{\text{Stress}}{\text{Strain}} = \text{Modulus of Elasticity}$$

↳ Modulus of Elasticity → Constant
- ↳ Ratio of stress & strain.
- ↳ It is the factor which determines the max force a body can endure before it goes into plastic limit. Also determines the body's elastic limit.
- ↳ Depends on:
  1. Nature of material.
  2. Nature of deformation.

→ Doesn't depend on stress & strain! Only depends on

Types:

① Young's Modulus of Elasticity (Y)

↳ Defined as: $Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$

$$\therefore \boxed{Y = \pm \frac{F \times L}{A \times \Delta L}}$$ SI = N/m^2

→ + when length increase, − when length decrease

② Bulk Modulus of Elasticity (K)

↳ Defined as: $K = \frac{\text{Volumetric Stress (or Pressure) } \cancel{\text{Pressure}}}{\text{Volumetric Strain}}$

→ $K = \frac{P}{\frac{\Delta V}{V}}$

→ $\boxed{\therefore K = -\frac{PV}{\Delta V}}$ where P is pressure $P = \frac{F}{A}$

Why negative? → As volume decreases on increase in pressure. Thus negative sign is used. (doesn't indicate less than 0)

○ Compressibility → $\frac{1}{K}$ → Bulk Modulus

↳ It is reciprocal of bulk modulus.

★★ → Bulk modulus tells about how much stress should be given to compress the body. Also, it tells about compressibility of the body.

→ amt. of stress reqd. to change shape

3) Shear Modulus / Modulus of Rigidity (η)

↳ Defined as: $\eta = \dfrac{\text{shearing stress}}{\text{shearing strain}}$

$$= \frac{F/A}{L/\Delta L} = \boxed{\frac{Fl}{A\Delta L}}$$

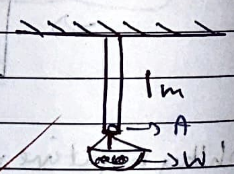
$$\boxed{\eta = \frac{Fl}{A\Delta L}}$$

↳ Relation b/w Young Modulus & Shear modulus:

$$\boxed{Y = 3\eta} \text{ (approx)}$$

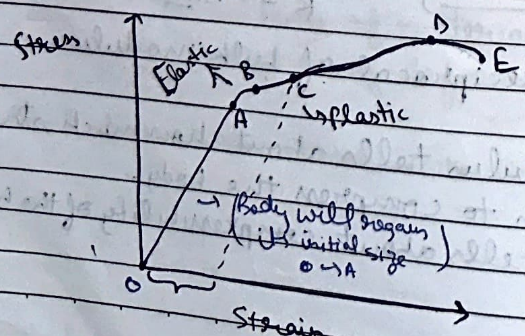
X ——— X ——— X

★ Stress - Strain Curve



↳ Rod of length 1m, area → A
Keep adding weight in pan using weights.

Calculate both longitudinal stress & strain till the rod breaks.



Kg	Δl
1	1.1
2	1.2
3	1.3
4	1.4
5	1.5
6	1.7

(Beyond elastic limit)

OA → Hooke's Law obeyed

A → Proportional Limit } Body can regain original length
B → Elastic Limit
↳ Yield point (Elasticity lost)

18/10/24

• Beyond yield point, body will never regain its original length. (Plastic behaviour)

C → Permanent Set / Deformation (Body has entered plastic region)

C - D region → Plastic Region

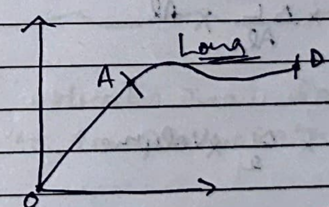
D → Tensile Strength (Ultimate Strength)
↳ Max load that can be put on before breaking.

• After point D, the object will break.

E → Fracture Point ⇒ The body breaks apart.

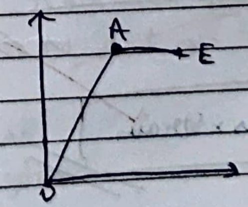
~~Brittle~~

• Ductile



→ Material takes long time to break. It holds plastic behaviour of very long.

• Brittle

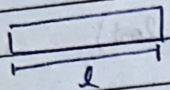


→ Material breaks very easily after elastic limit.

Work → necessary

work done on body → P.E 19/10/24

★ Elastic Potential Energy



↳

$F = 0$

$F \leftarrow$  $\rightarrow F$

$\rightarrow F \rightarrow$ ✓

Avg Restoring force $= \frac{0+F}{2} = \frac{F}{2}$

Work done $= \frac{F}{2} \cdot \Delta L$

$= \frac{F \Delta L}{2}$

⇒ $U = \frac{F}{2} \times \frac{\Delta L}{Al} \times Al$

→ $U = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{l} \times \text{Volume}$

⇒ $\boxed{U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}}$

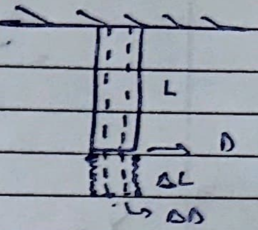
$\boxed{\text{Energy density} \rightarrow \frac{1}{2} \times \text{Stress} \times \text{Strain}}$

Stress = $Y \times$ Strain
↓
Young modulus

$\boxed{\text{Energy density} = \frac{1}{2} \times Y \times (\text{Strain})^2}$

X —— X —— X —— X —— X

★ Poisson's Ratio



↳ Longitudnal Strain $= \frac{\Delta L}{L}$

Let wire of diameter D be decreased by ΔD.

∴ Lateral Strain on wire $= -\frac{\Delta D}{D}$

~~∴ Poisson's Ratio (Σ) = Longitudnal~~

∴ Poisson's Ratio $(\sigma) = \frac{\text{Lateral Strain}}{\text{Long. Strain}} = \frac{-\Delta D/D}{\Delta L/L}$

★ → −ve sign indicates that lateral strain is developed opposite to longitudnal strain.

X —— X —— X —— X

Q. A steel rod has radius 10mm. Length → 1m. Force - 100 Kilo N, act on the rod. Find.

1. Stress
2. Elongation
3. Strain

$Y_{steel} = 2 \times 10^{11}\ N/m^2$

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$\hookrightarrow$ Stress $= \frac{F}{A}$

$= \cancel{\frac{100 \times 10^3}{\pi r^2 (\text{with})}} = \frac{100 \times 10^3}{\pi (10 \times 10^{-3})^2}$

$= \cancel{\frac{100 \times 10^3}{\pi (10^{-4})}} = \boxed{3.1 \times 10^8 \ N/m^2}$

$Y = \frac{F}{A} \times \frac{L}{\Delta L}$

$\Rightarrow \Delta L = \frac{3.1 \times 10^8 \times 1}{2 \times 10^{11}}$

$= \boxed{1.6 \times 10^{-3} \ m} \rightarrow \boxed{1.6 mm}$

Strain $= \frac{1.6}{1} = 1.6 \times 10^{-3}$

Q. A composite wire of uniform diameter 3mm, consisting of copper of $l = 2.2$ m & steel of 1.6m stretches under load by 0.7mm. Calculate the load.

$Y_{steel} = 2 \times 10^{11} \ N/m^2$ $\quad Y_{copper} = 1.1 \times 10^{11}$

$\hookrightarrow$ Let copper elongate by x.
steel by $0.7 - x$.

$\therefore Y_{cu} = \frac{F}{A} \times \frac{L}{\Delta L} \Rightarrow F = \frac{L}{Y A \Delta L Y_{cu}}$

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$\Rightarrow F = \frac{L}{A \Delta L Y_{steel}}$

Both forces are equal.

$\therefore \frac{L}{\Delta L \times Y_{cu}} = \frac{L}{\Delta L \times Y_{steel}}$

$\Rightarrow \cancel{\frac{2.2 \times 10^3}{2.2 (0.7 - x)}}$

$\Rightarrow \cancel{2.2 \times 10^3 (0.7 - x)(2 \times 10^{11}) = 1.6 \times 10^3}$

$\Rightarrow 2.2 \times (0.7 - x) \times 10^{-3} (2 \times 10^{11}) = 1.6 \times 10^{-3} \times (x)(1.1 \times 10^{11})$

$\Rightarrow 4.4 (0.7 - x) = 1.76 (x)$

$\Rightarrow 3.08 - 4.4x = 1.76x$

$\Rightarrow x = \frac{6.16}{3.08} = \frac{1}{2.2} = 0.4$

$\therefore \Delta L_c = 0.4$ mm

$\Delta L_s = 0.3$ mm

$\cancel{\therefore F = \frac{2.2 \times 10^3}{0.4 \times 1.1 \times 10^{11}} = 14.4 \times 10^{-8} N}$

$\therefore F = \frac{1.1 \times 10^{11} \times 0.4 \times 10^{-3}}{2.2 \times 10^3} = \boxed{176.8 N}$

44 × 7 = 308

4.40 + 1.76 = 6.16

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alt

~~to karna~~

Stress on Cu = Stress on steel

$\Rightarrow Y_{cu} \times \frac{\Delta l_c}{l_c} = Y_s \times \frac{\Delta L_s}{l_s}$

$\Rightarrow \frac{\Delta l_c}{\Delta l_s} = \frac{Y_s}{Y_c} \times \frac{l_c}{l_s}$

$= 2.5$

$\therefore 2.5 \Delta l_s + \Delta l_s = 0.7 \times 10^{-3}$

$\Delta l_s = 0.2 \text{ mm}$

$\Delta l_c = 0.5 \text{ mm}$

$F = \frac{Y_s A \Delta l_s}{l_s}$

$= 176.8 \text{ N}$

Q. Total mass of performer in act = 280 kg.

Mass of the performer lying on back = 60 kg.

Length of each thighbone = 50 cm

$Y = 9.4 \times 10^9 \text{ N/m}^2$

$R = 2 \text{ cm}$

Calculate compression in each thighbone.

$\hookrightarrow$ ~~m = 220~~ $F = 2200 \text{ N}$ $R = 2 \text{ cm} = 0.02 \text{ m}$ $L = 50 \text{ cm} = 0.5 \text{ cm}$

$\Rightarrow Y = \frac{FL}{A \Delta L}$

~~$\Rightarrow \Delta L = \frac{2200 \times 50 \times 10^{-2}}{\pi \times 4 \times 10^{-4}}$~~

$\Rightarrow \Delta L = \frac{\cancel{2200} \times 0.5 \times 10000}{\pi \times \cancel{0.02 \times 0.02} \times 9.4 \times 10^9}$

$= \frac{275 \times 10000 \times 5}{3.14 \times 9.4 \times 10^6} = \boxed{4.55 \times 10^{-5}}$

~~$\frac{6875}{30 \times 10^6} = 225$~~

Q. Pressure of medium change from 1.01×10^5 Pa to 1.165×10^5 Pa. Change in vol = 10%. Find Bulk Modulus.

$\hookrightarrow \Delta P = 0.155$ $\frac{\Delta V}{V} = 10\% = \frac{1}{10}$

$\Rightarrow \quad B = \frac{PV}{\Delta V}$

$= 0.155 \times 10^{10}$ ~~0.0155~~

$= 1.55 \times 10^{5}$ N/m²

Q. A sphere contracts in volume 0.01%, when taken 1 km deep into sea. Find K. $\rho = 10^3$ kg/m³.

$\Rightarrow \quad K = 10^7 \times \frac{10000}{0.01}$

$= \boxed{10^{11}}$ N/m²

Q. A square metal slab of side 50 cm. Thickness 10 cm. is subjected to a tangential force of 9×10^4 N. The lower edge is fixed. How much is upper edge displaced? $\eta = 5.6 \times 10^9$ Pa

$\Rightarrow \quad \eta = \frac{FL}{A\Delta L}$

$\Rightarrow \quad \Delta L = \frac{FL}{A\eta}$

$= \frac{9 \times 10^4 \times 50 \times 10^{-2}}{25 \times 5.6 \times 10^9} = \frac{90 \times 10^4 \times 50 \times 10^{-2}}{(50 \times 10^{-2})^2 \times 5.6 \times 10^9}$ ~~...~~

$= 1.6 \times 10^{-4}$

$\Delta L =$ ~~3.3×10^{-7} m~~ 1.6×10^{-4} m

Q. Load on wire increased from 3 kg to 5 kg the elongation increase from 0.61 mm to 1.02 mm. How much work is done.

$\Rightarrow$ ~~$W = \frac{1}{2} \times 2 \times 0.42 \times 10^{-3}$~~

$W_1 = \frac{1}{2} \times 3 \times 0.61 \times 10^{-3} = 8.7 \times 10^{-3}$ J

$W_2 = \frac{1}{2} \times 5 \times 1.02 \times 10^{-3} = 25 \times 10^{-3}$ J

$\therefore \Delta W = 16 \times 10^{-3}$ J

Q. $L = 2$ m $\quad \Delta L = 2$ mm $= 2 \times 10^{-3}$ m $\quad A = 4$ mm² $= 4 \times 10^{-6}$ m²

$Y_s = 2 \times 10^{11}$ Pa. Calculate U.

$\Rightarrow \quad Y = \frac{FL}{A\Delta l}$

$F = \frac{YA\Delta l}{L}$

$U = \frac{1}{2} \times Y \times (\text{Strain})^2 \times A l$

$= \frac{1}{2} \times 2 \times 10^{11} \times \left(\frac{2 \times 10^{-3}}{2}\right)^2 \times 4 \times 10^{-6} \times 2$

~~$\therefore U = \frac{1}{2} \times \frac{YA\Delta l \times \Delta l}{L}$~~

$= 10^{11} \times 8 \times 10^{-6} \times 10^{-6}$

$= 8 \times 10^{-1}$

$U = 0.8\,J$

Q. A rope of $L = 4.5\,m$, $d = 6\,mm = 6\times10^{-3}$

A monkey of 100N jumps to catch the free end & stays there. Find elongation.

$Y = 4.8\times10^{11}\,Pa$ Poisson's R $= 0.2$ (σ)

$\Rightarrow 0.2 = \dfrac{\Delta D/D}{\Delta L/L}$

$\Rightarrow 0.2 = \dfrac{\Delta D \times L}{\Delta L \times D}$

$\Rightarrow \dfrac{\Delta D}{\Delta L} = \dfrac{0.2 \times \cancel{6}^{2}\times10^{-3}}{\cancel{4.5}\,15}$

$= \dfrac{4}{15}\times10^{-3}$

$\dfrac{\Delta D}{\Delta L} = 0.26\times10^{-3}$

$$15\overline{)40} = 0.26$$
-30, 100, -90, 10

$\rightarrow Y = \dfrac{F\,L}{A\,\Delta L}$

$\Rightarrow \Delta L = \dfrac{100\times4.5}{\pi\times\left(\frac{3\times10^{-3}}{2}\right)^2\times4.8\times10^{11}}$

$= \dfrac{100}{\pi\times\cancel{4}\times10^{-6}\times4.8\times10^{11}}$

$= \dfrac{100}{\pi\times9.6\times10^{5}} = 0.31\times10^{-4}\,m$

$$22\overline{)70} = 0.31$$
-66, 40, -22, 18

$\Rightarrow \Delta D = 0.26\times10^{-3}\times0.31\times10^{-4}$

$= \boxed{0.806\times10^{-7}\,m}$

$31 \times 26 = 186 + 620 = 806$

Ch-9 → Fluids

* Pascal's Law

↳ "The pressure is same everywhere in the liquid, if gravity is ignored."

↳ Change of pressure applied to incompressible liquid is transmitted equally in all directions.
(undiminished)

⇒ Liquids don't compress when pressure is applied. Thus it will bear no pressure itself & will distribute that pressure equally & undiminished throughout the liquid body.
↳ (no loss)

Only if gravity is ignored.

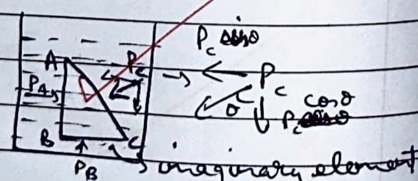
• Applications : (1) Automobile Industry
↳ ~~Disc~~ Hydraulic brakes on automobiles, jack for lifting vehicles

(2) Hydraulic Engineering
↳ Suspension, Cranes, Trucks use hydraulic press

(3) Medical Science
↳ Syringes, IV drips etc.

*Proof !!

↳ Let an imaginary triangle having sides ABC be present in a container of water.



Let F_A, F_B, F_c → Forces acting on A, B, C resp.
A_a, A_b, A_c → Area...

→ Now, resolving components, we have:

$F_A = F_c \sin\theta$ — (1) $\qquad A_a = A_c \sin\theta$ — (3)
$F_B = F_c \cos\theta$ — (2) $\qquad A_b = A_c \cos\theta$ — (4)

→ $\frac{(1)}{(3)}$ ⇒ $\frac{F_A}{A_a} = \frac{F_c \cancel{\sin\theta}}{A_c \cancel{\sin\theta}}$

⇒ $P_a = P_c$

↳ $\frac{(2)}{(4)}$ ⇒ $\frac{F_B}{A_b} = \frac{F_c \cancel{\cos\theta}}{A_c \cancel{\cos\theta}}$

⇒ $P_b = P_c$

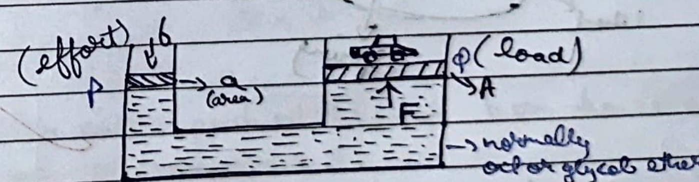
∴ $P_a = P_b = P_c$

∴ Pressure is equal throughout

X ——— X ——— X ——— X

* Hydraulic Lift (Pascal's Law Application)

↳



↳ Pressure at P (P_1) = $\frac{b}{a}$

↳ Pressure at ϕ = F