cw Ch - 1 → Units And Measurements :

- SIGNIFICANT FIGURES :

(1) All non-zero digits are significant.
Eg. 23.459 - 5 significant figures

(2) Zeroes between 2 non-zero digits are significant.
Eg - 10.25, 102.5, 1.025 } 4 significant figures

(3) If the number is without decimal, trailing zeroes are not significant.
Eg - 100 - 1 significant figure
25000 - 2 significant figure
300000 - 1 sig. fig.

(4) If number is with decimal, the zeroes to the right of non-zero digit, but to the left of the decimal are significant.

Eg - 250.56 - 5 sig. fig.
3800.60 - 5 sig. fig.

(5) If number is with decimal, ~~the zeroes to the right~~ side of decimal point are significant.
Eg - 2500.00 → because they indicate precision level of an instrument.

(6) If number is < 1, zeroes to the left but right of decimal point before non-zero are not significant.
Eg. .0012 → 2 significant figures

(7) Significant figures do not depend on system of unit.

25 km] 2 sig. fig
25000m

★ **Extra Q**

• Counting no. of significant figures:

① 0.0009
② 5.049 → 4
③ 3000 → 1
④ 0.020 m → 3
⑤ 0.0200 → 3
⑥ 201.0 → 4
⑦ 71 → 2
⑧ 4.50×10^3 → 3
⑨ 0.23×10^{-6} → 2
⑩ 2301.01×10^2 → 6

• ~~Count~~ Round off the following digits

① 18.35 (3 digits) → 18.4
② 18967 (3) → ~~19000~~ 19000
③ 18967 (2) → 19000
④ 248337 (3) → 248000
⑤ 321.135×10^3 (4) → 321.1×10^3
⑥ 101.55×10^6 (3) → 102×10^6
⑦ 0.7995 (1) → 0.8

★ **Addition & Subtraction in terms of significant figures:**

Q. 7.21 + 12.141 + 0.0028

→
7.2100
12.1410
0.0028
19.3538
→ $\boxed{19.35}$

Rule
★ We will look for the no. which has least digits after decimal and the ans. will have as many digits after decimal as that no.

6.80700
− 4.27153
2.53547
→ $\boxed{2.535}$

6.2000
+ 4.330
+ 17.456
$\boxed{27.986}$
→ ~~27.9~~ 28.0

★ **Division & Multiplication in S.F.**

Q. $\frac{5.74}{1.2}$ = ~~4.7813~~ ~~4.78~~

Rule
Select the least significant figures wala no. for final ans.

Q - Radius of sphere is 1.41 cm. Express its volume in app. significant digit

→ Volume $= \frac{4}{3}\pi r^3$

→ Vol $= \frac{4}{3} \times \frac{22}{7} \times \frac{141}{100} \times \frac{141}{100} \times \frac{141}{100}$

$= \boxed{11.7 cm^3}$

Q - $\frac{755}{10} \times \frac{1252}{10} \times \frac{051}{100}$

→ 4820.8260

→ 4800

Q - 6.2 + 4.33 + 17.456 Ab. Done

Q - 187.20 − 63.54 = 123.66

→ $\boxed{123.6}$

Q - $2.15 \times 10^4 - 3.9 \times 10^3$

Q $390 \times 10^4 - 2.5 \times 10^4 = 36.5 \times 10^4$

→ ~~36.0×10^4~~

Dimensional Formula (also known as MLT eqn)

• Symbols

1) Length → [L]

2) Mass → [M]

3) Time → [T]

4) Temperature → [K]

5) Current → [A]

6) Amount of substance → [mol]

7) Luminous intensity → [cd]

Eg → Force = mass × acceleration

$= \text{mass} \times \frac{\text{length}}{\text{time}^2} = kg\,m/s^2$ or N

Force $= [M^1 L^1 T^{-2}]$ → Dimensional formula for force.

TRY!! (1) Power = Wt

(2) Work = Fd

(3) Energy $= \frac{1}{2}mv^2$

(4) Momentum

(5) Plane angle $= \frac{\text{arc}}{\text{radius}}$

Date: 8/4/24

~~1) Power = W/t~~

1) Work = Force x displacemente

$= \frac{kgm}{s^2} \times m$

$= \frac{kgm^2}{s^2}$

$= [M^1 L^2 T^{-2}]$

(2) Power $= \frac{kgm^2}{s^2} \times \frac{1}{s}$

$= [M^1 L^2 T^{-3}]$

(3) Energy $= \frac{1}{2} \times kg \times \left(\frac{m}{s}\right)^2$

$= [M^1 L^2 T^{-2}]$

(4) Momentum = mv

$= \frac{kgm}{s}$

$= [M^1 L^1 T^{-1}]$

(5) Plane angle $= [M^0 L^0 T^0]$

(6) Electric charge = It

$= As$ $= [M^0 L^0 T^1 A^1]$

(7) Potential Difference = W/Q

$= \frac{kgm^2}{As^3} = [M^1 L^2 T^{-3} A^{-1}]$

Page No.: 09
Date: 8/4/24
YOUVA

(8) Resistance $= \frac{W}{Q}$

$= \frac{kgm^2}{A^2 s^3} = [M^1 L^2 T^{-3} A^{-2}]$

(9) Resistivity $= \frac{RA}{l}$

$= [M^1 L^3 T^{-3} A^{-2}]$

(10) Gravitational constant $= \frac{Fr^2}{m_1 m_2}$

$= \frac{kgm}{s^2} \times m^2 \times \frac{1}{kg^2} = [M^{-1} L^3 T^{-2}]$

(11) Heat Energy = VIt

$= \frac{kgm^2}{As^3} \times A \times s = [M^1 L^2 T^{-2}]$

(12) Solar Energy $= [M^1 L^2 T^{-2}]$

(13) Mechanical Energy $= [M^1 L^2 T^{-2}]$

★ Applications of Dimensional Formula

Appl-1) To check dimensional consistency of given eqn.

Eg.

RHS $= kg \times \left(\frac{m}{s}\right)^2$

RHS $= [M^1 L^2 T^{-2}]$

LHS = E

$E = Fd$

$= [M^1 L^2 T^{-2}]$

∴ LHS = RHS.

Eg. ① $F = m^2 \frac{v}{r}$

LHS = F
$= [M^1 L^1 T^{-2}]$

RHS = $[M^2 L^0 T^{-1}]$

∴, LHS ≠ RHS

Hence, the given equation is dimensionally inconsistent.

② $T = 2\pi\sqrt{\frac{l}{g}}$

⇒ LHS = $[M^0 L^0 T^1]$

⇒ RHS = $[M^0 L^0 T^1]$

∴, LHS = RHS, and thus eqⁿ is dimensionally consistent.

③ $v = \sqrt{\frac{P}{S}}$

⇒ LHS = $[M^0 L^1 T^{-1}]$

⇒ RHS = $\sqrt{\frac{\frac{M^1 L^{-1}}{L^2 T^2}}{\frac{M^1}{L^3}}}$

⇒ RHS = $\sqrt{\frac{L^2}{T^2}}$ $= [M^0 L^1 T^{-1}]$

∴, LHS = RHS, it is consistent.

④ $Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

LHS = $[M^1 L^1 T^{-2}][L^1]$
$= [M^1 L^2 T^{-2}]$

RHS = $\frac{1}{2}mv^2$
$= [M^1 L^2 T^{-2}]$

∴, As LHS = RHS, the eqⁿ is dimensionally consistent.

⑤ $\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

⇒ LHS = ν
$= \frac{1}{T} = [M^0 L^0 T^{-1}]$

⇒ RHS = $\frac{1}{L^1}\sqrt{\frac{M^1 L^1 T^{-2}}{M^1 L^{-1} T^0}}$

⇒ RHS = $\frac{1}{L} \times L T^{-1} = [M^0 L^0 T^{-1}]$

As LHS = RHS,

∴, the equation is dimensionally consistent.

Appl. 2 → To obtain the formula of one physical quantity in terms of other physical quantities.

Eg. T ∝ length of string
∝ mass of bob
∝ acc. due to gravity

$T \propto l^a m^b g^c$

$\Rightarrow T = K l^a m^b g^c$

$\Rightarrow \cancel{[M^0 L^0 T^1] = K l^a m^b g^c}$

$\Rightarrow [M^0 L^0 T^1] = K [M^0 L^1 T^0]^a [M^1 L^0 T^0]^b [M^0 L^1 T^{-2}]^c$

$\Rightarrow [M^0 L^0 T^1] = K [M^0 L^a T^0][M^b L^0 T^0][M^0 L^c T^{-2c}]$

$\Rightarrow [M^0 L^0 T^1] = K [M^b L^{a+c} T^{-2c}]$

So, on comparison, we get,

$b = 0$

$a + c = 0$

$-2c = 1$

$a = \frac{1}{2}$

$\rightarrow \boxed{c = -\frac{1}{2}}, \ a = \boxed{\frac{1}{2}}, \ \boxed{b = 0}$

∴, on putting in eqⁿ we get,

$T = K l^a m^b g^c$

$\Rightarrow T = 2\pi l^{1/2} g^{-1/2}$ $(K = 2\pi)$

$\Rightarrow \boxed{T = 2\pi \sqrt{\frac{l}{g}}}$

Practice Q

Q-1 (1) The velocity of water waves depends on wavelength (λ), density (ρ), and (g) acc. due to gravity.

$\rightarrow v \propto \lambda^a \rho^b g^c$

$\Rightarrow v = K \lambda^a \rho^b g^c$

$\Rightarrow [M^0 L^1 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^{-3} T^0]^b [M^0 L^1 T^{-2}]^c$ (for K = 1)

$\Rightarrow [M^0 L^1 T^{-1}] = [M^0 L^a T^0][M^b L^{-3b} T^0][M^0 L^c T^{-2c}]$

On comparison, we get,

$b = 0$

$a - 3b + c = 1$

$-2c = -1$

$\Rightarrow \boxed{c = \frac{1}{2}}, \ \boxed{a = \frac{1}{2}}, \ \boxed{b = 0}$

Putting values in eqⁿ,

$\Rightarrow v = \lambda^{1/2} \rho^0 g^{1/2}$

$\Rightarrow \boxed{v = \sqrt{\lambda g}}$

(2) Frequency (ν) is proportional to

$\rightarrow \nu \propto l^a \mu^b T^c$

$\Rightarrow \nu = K l^a \mu^b T^c$

$\Rightarrow [M^0 L^0 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^{-1} T^0]^b [M^1 L^1 T^{-2}]^c$ (For K = 1)

$\Rightarrow [M^0 L^0 T^{-1}] = [M^a L^a T^0][M^b L^{-3b} T^0][M^c L^c T^{-2c}]$

On comparison, we get:

$b + c = 0$

$a - 3b + c = 0$

$-2c = -1$

$\therefore \boxed{c = \frac{1}{2}, \; b = -\frac{1}{2}, \; a = -1}$

Putting values in eq^n,

$\nu = l^{-1} \mu^{-1/2} T^{1/2}$

$\boxed{\nu = \frac{1}{l}\sqrt{\frac{T}{\mu}}}$

(3) $T = K r^a M^b G^c$

$\Rightarrow [M^0 L^0 T^1] = [M^0 L^1 T^0]^a [M^1 L^0 T^0]^b [M^{-1} L^3 T^{-2}]^c$ (for $K = 1$)

$\Rightarrow [M^0 L^0 T^1] = [M^0 L^a T^0][M^b L^0 T^0][M^{-c} L^{3c} T^{-2c}]$

On comparison, we get:

$b - c = 0$

$a + 3c = 0$

$-2c = 1$

$\therefore \boxed{c = -\frac{1}{2}, \; b = -\frac{1}{2}, \; a = \frac{3}{2}}$

Putting in eq^n we get,

$\boxed{T = r^{3/2} M^{-1/2} G^{-1/2}}$ or $\cancel{T = \sqrt{\frac{r^3}{MG}}}$

3rd Appl $\rightarrow$ To convert unit of physical quantities from one system to other.

Formula $\rightarrow$ $\boxed{n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c}$

Eg. Convert 1 J to erg

Energy = $[M^1 L^2 T^{-2}]$

$\longrightarrow$ $a = 1, b = 2, c = -2$

$\Rightarrow n_2 = 1 \left[\frac{1\,kg}{1\,g}\right] \left[\frac{1\,m}{1\,cm}\right]^2 \left[\frac{1\,sec}{1\,sec}\right]^{-2}$

$\Rightarrow n_2 = 1 \left[\frac{1000\,g}{1}\right] \left[\frac{100\,cm}{1}\right]^2 \left[\frac{1}{1}\right]^{-2}$ (Convert big to small units)

$\Rightarrow n_2 = [1 \times 10^3] [\times 10^4] \times [1]$

$\Rightarrow n_2 = 10^7$

$\therefore \boxed{1\ \text{Joule} = 10^7\ \text{erg}}$

Date: 12/4/24

Q. The value of G in C.G.S is 6.67×10^{-8} dyne$\left(\frac{cm^2}{g^2}\right)$. Calculate its value in SI.

$\Rightarrow \cancel{n_2 = 6.67\times10^{-8}}$

$\Rightarrow G = [M^{-1} L^3 T^{-2}]$

$\Rightarrow n_2 = 6.67 \times 10^{-8} \left[\frac{1g}{1kg}\right]^{-1} \left[\frac{1cm}{1m}\right]^{3} \left[\frac{1s}{1s}\right]^{-2}$

$\Rightarrow n_2 = 6.67 \times 10^{-8} \left[\frac{1}{1000} g\right]^{-1} \left[\frac{1}{100}\right]^{3} [\ ]^{-2}$

$\Rightarrow n_2 = 6.67 \times 10^{-8} \times \cancel{1000}\, 10^3 \times 10^{-6}$

$\cancel{\Rightarrow n_2 = 6.67 \times 10}$ $\Rightarrow n_2 = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$\cancel{\Rightarrow n_2 = 66.7\,N}$

$\therefore \boxed{G = 6.67 \times 10^{-11}\ Nm^2/kg^2}$

Q. Convert 1 N into dyne.

$\Rightarrow F = [M^1 L^1 T^{-2}]$, $a = 1, b = 1, c = -2$

$\Rightarrow n_2 = 1 \times \left[\frac{1kg}{1g}\right]^1 \times \left[\frac{1m}{1cm}\right]^1 \left[\frac{1sec}{1sec}\right]^{-2}$

$\Rightarrow n_2 = 1 \times 10^3 \times 10^2$

$\Rightarrow \boxed{n_2 = 10^5}$

$\therefore$ ~~1N = 10^5 dyne~~ ~~1 dyne = 10^5 N~~ $1N = 10^5$ dyne

Page No.: 17
Date: 12/4/24

Q. Find the value of 60 J/m on a system that has 100g, 100cm and 1 min as fundamental units.

$\rightarrow 60\ J/m = \frac{60}{60}\ 1J/s$

$= \boxed{1J/s} = [M^1 L^2 T^{-3}]$

$n_2 = 1 \times \left[\frac{100g}{100}\right]^1 \left[\frac{100}{100}\right]^2 \left[\frac{1}{60}\right]^{-3}$

$\Rightarrow n_2 = 1 \times 10 \times 1 \times 60^3$

$\Rightarrow n_2 = 216 \times 10^3 \times (10)$

$= \boxed{2.16 \times 10^6 \text{ new units}}$

Date: 15/4/24

Ch - 2

Motion In Straight Line

Reference Point or Frame of Reference

1) 1D → x/y/z → 1 coord.
2) 2D → xy/yz/zx → 2 coord
3) 3D → xyz → 3 coord

Parameters

① Distance / path length (s/d)

- Amount of path covered by an object.
- SI unit : metres
- It is always positive.
- It is scalar quantity.

② Displacement / change in position

- Shortest path covered from initial to final position.
- SI unit : metres
- It may be positive or negative or zero.
- It is a vector quantity.

③ Speed

- $\frac{\text{Distance travelled}}{\text{time}} = \frac{m}{s}$ or $\boxed{m/s}$ - SI unit
- It is a scalar quantity
- It is always positive

Page No.: 19
Date: 16/4/24

④ Velocity

- $\frac{\text{Displacement}}{\text{Time}} = \frac{m}{s}$ or $\boxed{m/s}$ - unit
- It is a vector quantity.
- It can be positive, negative or 0.

★ In straight line motion, distance = displacement
speed = velocity

- Types of speed → Instantaneous Speed (at given instant of time) = $V_I = \frac{dx}{dt}$

(at given interval) Avg. Speed

$V_{Avg} = \frac{\text{Total Distance}}{\text{Total Time}}$

$\boxed{V_{Avg} = \frac{\Delta x}{\Delta t}}$

Note → $V = \frac{dx}{dt}$ ⇒ rate of change of position w.r.t. time

★ Uniform and Non Uniform Motion

- Uniform Motion → Position changes equally in equal intervals of time.
- Non - Uniform Motion → Position changes unequally in equal intervals of time.

Q- A body covers distance $x = 5t^2 + 3t + 2$ in t seconds. Calculate the speed at $t = 1$ second.

$\Rightarrow v = \frac{d}{dt}(5t^2 + 3t + t)$ $\quad \left[v = \frac{dx}{dt} \right]$

$\Rightarrow v = 10t + 3$

for $t = 1$,

$\Rightarrow v = 10 + 3 = \boxed{13 \text{ m/s}}$

Q- The position $x = 10t^4 + 4t^2 + 2t$, find speed at $t = 2$ sec

$\Rightarrow v = \frac{d}{dt}(10t^4 + 4t^2 + 2t)$ $\quad \left[v = \frac{dx}{dt} \right]$

$\Rightarrow v = 40t^3 + 8t + 2$

for $t = 2$ secs,

$\Rightarrow v = 40(2)^3 + 8(2) + 2$

$\Rightarrow v = 320 + 16 + 2$

$\Rightarrow \boxed{v = 338 \text{ m/s}}$

Q- $x = \frac{4}{5}t^3 - \frac{2}{5}t^2 + 9$ at $t = 1$ sec.

$\Rightarrow v = \frac{d}{dt}\left(\frac{4}{5}t^3 - \frac{2}{5}t^2 + 9\right)$ $\quad \left[v = \frac{dx}{dt} \right]$

$\Rightarrow v = \frac{12t^2}{5} - \frac{4}{5}t$

for $t = 1$,

$\Rightarrow v = \frac{12}{5} - \frac{4}{5}$

Page No.: 21 Date: 17/4/24

$\Rightarrow \boxed{v = \frac{8}{5} \text{ m/s or } 1.6 \text{ m/s}}$

Q- $x = 1.5t^2 + 2t$. Find v at $t = 3$ s.

$\Rightarrow v = \frac{d}{dt}(1.5t^2 + 2t)$

$= 3t + 2$

$= 9 + 2 = \boxed{11 \text{ m/s}}$ (for $t = 3$)

Q- The relation b/w time t & position x is $t = ax^2 + bx$ (a & b are const). Find out instantaneous velocity.

~~$\Rightarrow v = \frac{d}{dx}(ax^2 + bx)$ $\quad x = \frac{t - ax^2}{b}$~~

~~$= 2ax + b$~~

~~$\Rightarrow v = \frac{dt}{dx}$~~

~~$x = \frac{d}{dt}\left(\frac{t}{b} - \frac{ax^2}{b}\right)$~~

~~$\Rightarrow v = \frac{d}{dx}(ax^2 + bx)$~~

~~$\Rightarrow v = \frac{1}{b} - \frac{2ax}{b}$~~

~~$\Rightarrow \frac{dx}{dt} = 2ax + b$~~

~~$\Rightarrow v = \frac{1 - 2ax}{b}$~~

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(ax^2 + bx)$

$\frac{dt}{dx} = 2ax + b$

$\frac{dx}{dt} = \frac{1}{2ax + b}$

$\boxed{v = \frac{1}{2ax + b}}$ or $\boxed{v = (2ax + b)^{-1}}$

Date: 17/4/24

Q- $x = 8.5 + 2.5t^2$. v between t = 1 sec to t = 2 sec

→ ~~$v = \frac{d}{dt}(8.5 + 2.5t^2)$~~ $v = \frac{dx}{dt}$

~~$= 5t$~~

$\boxed{v = \frac{x_2 - x_1}{t_2 - t_1}}$ — (1)

~~For t = 1 sec, v = 5 m/s~~ When t = 1, $x_1 = 8.5 + 2.5$
$= 11\ m$

~~For t = 2 sec, v = 10 m/s~~ When t = 2, $x_2 = 8.5 + 10$
$= 18.5\ m$

$\therefore, v = \frac{18.5 - 11}{1} = \boxed{7.5\ m/s}$

AVERAGE SPEED & VELOCITY

Q- A body travels from A to B at 40 m/s and from B to A at 60 m/s. Calculate the avg. speed & avg. velocity.

→ A ⇄ B (40 m/s, 60 m/s)

~~Avg. speed = $\frac{60 + 40}{2} = \boxed{50\ m/s}$~~

~~Avg. velocity = 0, as displacement is zero.~~

Page No.: 23
Date: 18/4/24

→ Avg. speed = $\frac{\text{total distance}}{\text{total time}} = \frac{2AB}{\frac{AB}{v_1} + \frac{AB}{v_2}}$

$= \frac{2v_1v_2}{v_1 + v_2}$ → Harmonic Average

$= \frac{2 \times 40 \times 60}{100} = \frac{4800}{100} = \boxed{48\ m/s}$

★ If the distances are same, then avg speed is given by harmonic average :

$v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$

→ Velocity = $\frac{\text{displacement}}{\text{time}} = \boxed{0\ m/s}$

Q- On a 60 km track, a train travels first 30 km with speed of 30 km/h. How fast must the train travel the next 30 km so avg. speed for the entire trip becomes 40 km/h.

→ Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

A ← 30 (30 km/h) — 30 (v_2) → B

→ 40 km/h = $\frac{2v_1v_2}{v_1 + v_2}$ [Harmonic Avg.]

⇒ $40(30 + x) = 60x$ (Let v_2 be x km/h)

⇒ $1200 + 40x = 60x$

⇒ $1200 = 20x$

⇒ $\boxed{x = 60\ km/h}$

Q A train moves with a speed of 30 km/h in the first 15 min, with another speed of 40 km/h in next 15 min. Find average speed.

Ans → $V_{avg} = \frac{v_1 + v_2 \dots v_n}{n}$ [Arithmetic Avg]

A — 30 km/h, 15 min — 40 km/h, 15 min — B

$V_{avg} = \frac{30+40}{2} = \boxed{35 \text{ km/h}}$

★ If time intervals are same, we use arithmetic average.

$$V_{avg} = \frac{v_1 + v_2 \dots v_n}{n}$$

Q- A car travels along a straight line for the first half time with 50 km/h, second half time with 60 km/h. Calculate avg. speed.

→ ~~$V_{avg} = \frac{2v_1v_2}{v_1+v_2}$~~ $V_{avg} = \frac{50+60}{2}$

$= \boxed{55 \text{ km/h}}$

~~$= \frac{2 \times 50 \times 60}{50 \times 60} = \frac{6000}{} = 54$~~

Q- A body covers $\frac{1}{3}$ rd of journey with 20 km/h, next $\frac{1}{3}$ rd with 40 km/h and last $\frac{1}{3}$ rd of journey with 60 km/h. Avg. speed?

→ $V_{avg} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

$= \frac{3 \times 20 \times 40 \times 60}{20 \times 40 + 40 \times 60 + 60 \times 20}$

$\frac{3 \times 20 \times 40 \times 60}{800 + 2400 + 12\dots}$ $\frac{3 \times 20 \times 40 \times 60}{}$

→ $V_{avg} = \frac{360}{11}$

$= \boxed{32.72 \text{ km/h}}$

ACCELERATION

↳ Rate of change of velocity w.r.t. time

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$\boxed{a = \frac{dv}{dt}}$ → $a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$\boxed{= \frac{d^2x}{dt^2}}$ → 2nd order derivative

Q- $x = 7t^4 + 5t^3 - 3t^2 + 9$. Find a at t = 2.

$v = \frac{d}{dt}(7t^4 + 5t^3 - 3t^2 + 9)$

⇒ $v = 28t^3 + 15t^2 - 6t$

⇒ $a = \frac{d}{dt}(28t^3 + 15t^2 - 6t)$

⇒ $a = 84t^2 + 30t - 6$

for t = 2

⇒ $a = 84 \times 4 + 30(2) - 6$

⇒ $a = 336 + 60 - 6$

$= \boxed{390 \text{ m/s}^2}$

Sign 19/4/24

Page No.: 86
Date: 22/4/24

* Integration

Acceleration $\xrightarrow{\int dt}$ Velocity $\xrightarrow{\int dt}$ Position

Q- $v = \frac{3}{2}t^3 + 4t^2 + 3$

x btwn t = 1s & t = 2s.

$\rightarrow \int_1^2 \frac{3}{2}t^3 dt + \int_1^2 4t^2 dt + \int_1^2 3\, dt$

$\Rightarrow dx = \left[\frac{3}{8}t^4\right]_1^2 + \left[\frac{4}{3}t^3\right]_1^2 + \left[3t\right]_1^2$

for t = 2 $\rightarrow 6 + \frac{32}{3} + 6 = \frac{18+32+18}{3} = \frac{68}{3}$

for t = 1 $\rightarrow \frac{3}{8} + \frac{4}{3} + 3$

$\frac{9 + 32 + 72}{24} = \frac{113}{24}$

$x = \frac{68}{3} - \frac{113}{24}$

$x = \frac{544-113}{24} = \boxed{\frac{431}{24} m}$

Q- $v = 4.2t^2 + 2.1t$

Find x b/w t = 0 to t = 1

$\Rightarrow dx = 4.2\int_0^1 t^2 dt + 2.1\int_0^1 t\, dt$

$\Rightarrow dx = \frac{4.2t^3}{3} + \frac{2.1t^2}{2}$

Page No.: 87
Date: 22/4/24

for t = 0 $\Rightarrow$ dx = 0

for t = 1 $\rightarrow dx = 1.4 + \frac{2.1}{2}$

$dx = \frac{2.8 + 2.1}{2} = \frac{4.9}{2}$

$\boxed{2.45 m}$

$\therefore \boxed{x = 2.45m}$

Q.3 $a = 5t^5 + 4t^4 + 3t^3 + 2t + 1$

v b/w t = 1 & t = 2

$\Rightarrow dv = 5\int_1^2 t^5 + 4\int_1^2 t^4 + 3\int_1^2 t^3 + 2\int_1^2 t + \int_1^2 1$

$\Rightarrow dv = \left[\frac{5}{6}t^6\right]_1^2 + \left[\frac{4}{5}t^5\right]_1^2 + \left[\frac{3}{4}t^4\right]_1^2 + \left[t^2\right]_1^2 + \left[t\right]_1^2$

For t = 1, $dv = \frac{5}{6} + \frac{4}{5} + \frac{3}{4} + 1 + 1$

$= \frac{50 + 48 + 45 + 120}{60} = \frac{263}{60}$

For t = 2, $dv = \frac{5}{6}\times 64 + \frac{4}{5}\times 32 + \frac{3}{4}\times 16 + 4 + 2$

$\frac{320}{6} + \frac{128}{5} + 18$

$= \frac{1600 + 768 + 540}{30} = \frac{2908}{30}$

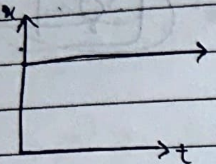
$v = \frac{2908}{30} - \frac{263}{60} = \frac{5816 - 263}{60} = \frac{5553}{60}$

$= \boxed{92.56}$

Date: 23/4/24

★ POSITION - TIME GRAPH

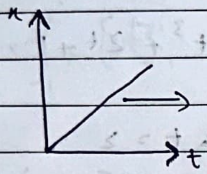
Inference - (1) About motion



— At Rest

(2) Motion

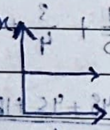
→ 1. Velocity



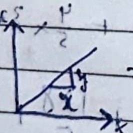
→ The slope will give velocity.

- If slope = 0, v = 0
- If slope > 0, v > 0 (+ve)
- If slope < 0, v < 0 (-ve)

Eg. (1). slope → 0, v ⇒ 0

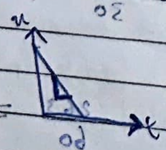


(2) Slope → +ve, v → +ve



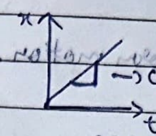
→ $\boxed{\frac{y}{x} = v}$

(3) Slope → -ve, v → +ve



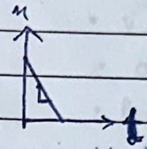
Page No.: 29
Date: 23/4/24

(4) Slope → constant & +ve, v ⇒ constant & +ve.

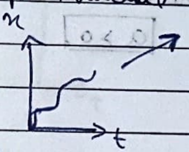


→ constant slope & +ve

(5) slope → constant & -ve, v → constant & -ve



(5) slope → variable, v → variable



~~Constant~~ 2. Slope

(1) distance - time $\xrightarrow{\text{slope}}$ speed

(2) Displacement - time $\xrightarrow{\text{slope}}$ Velocity

(3) Type of Motion

Uniform	TYPES	Non - Uniform
If line is straight		↓ If line is curved

→ Velocity - Constant
→ Slope - Constant
→ a = 0

→ Velocity - Variable
→ Slope - variable
→ a = Variable (depends on v)

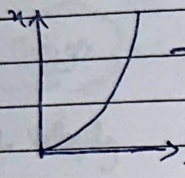
Date: 24/4/24

(4) Acceleration

→ Acceleration occurs only in non-uniform motion.

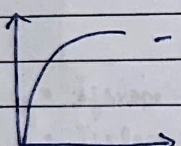
• slope - variable, velocity - variable.

$\underline{a \neq 0}$

①  - Curve up.

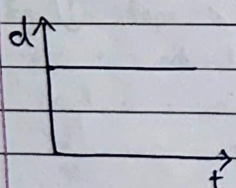
Velocity increases as slope increases.
So, this is accelerated motion.

$\boxed{a > 0}$

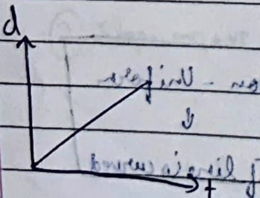
②  - Curve down.

Velocity decreases as slope decreases.
So, this is retarded motion.

$\boxed{a < 0}$

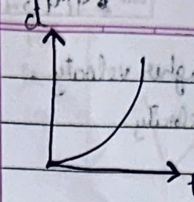
Φ - 

① The body is at rest.
② ~~Acceleration = 0~~, Velocity = 0, Slope 0
③ A = 0 ∵ curveness = 0
④ No motion as body is at rest.

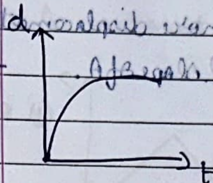
Φ - 

① The body is in motion.
② v = constant & pos. ∵ slope = const. posi
③ a = 0 ∵ curveness = 0
④ Uniform motion ∵ slope = straight line

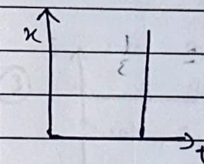
Page No.: 31
Date: 24/4/24

Φ - 

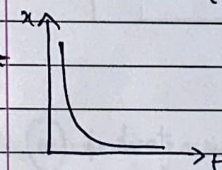
1) The body is in motion.
2) v = ~~variable~~ increasing & positive ∵ slope = ~~varia curve up~~ increasing
3) a > 0, ∵ graph is curve up. acceleration.
4) Body in non-uniform motion.

Φ - 

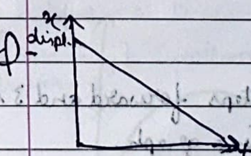
• Body in motion
• V is decreasing ∵ slope → decreasing
• a < 0, ∵ graph is curve down. retardation
• Body in non-uniform motion.

Φ - 

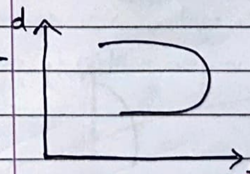
• This graph is not possible

Φ - 

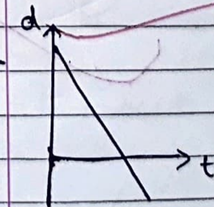
• ~~Body is in motion~~ This graph is not possible.
Distance can't decrease

Φ - 

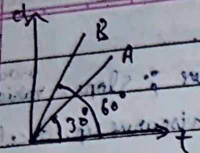
• Body is in motion.
• v → -ve as slope is negative.
• a = 0, as slope is constant.
• Body is in uniform motion.

Φ - 

• At an instant, there can't be two values of d for same time t. Also, time can't be reversed.
Thus, this graph doesn't exist.

Φ - 

• Body is in motion

Q. 

(1) Which body has higher velocity.

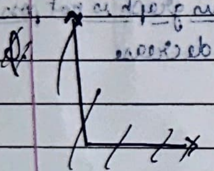
(2) Calculate ratio of velocity.

→ (1) B has a higher velocity as it has more displacement in less time, i.e, its slope is higher than slope of A.

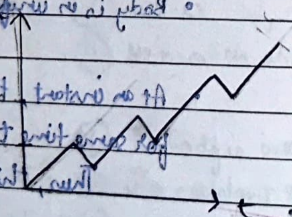
∴ $v_B > v_A$.

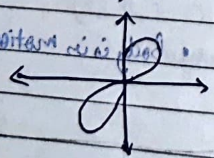
(2) $\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

$\boxed{\therefore V_A : V_B = 1 : 3}$

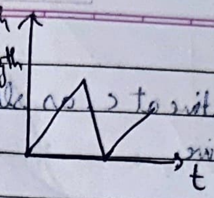
Q. 

Q- A person walking in a narrow lane takes 5 steps forward and 3 steps backward and so on. Plot the position-time graph.

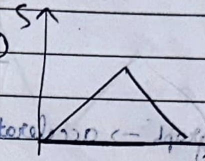


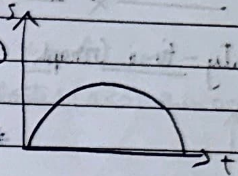
Q- 

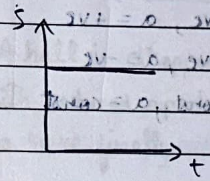
→ Not possible as time can't be negative.

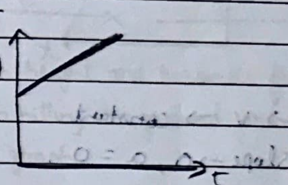
Q- 

→ Does not exist as distance can't decrease.

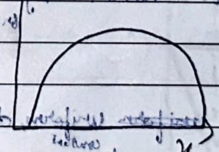
Q → (1) 

(2) 

(3) 

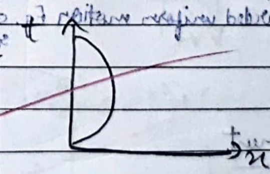
(4) 

→ (4) indicates uniform motion.

Q 

→ It is not possible to have two diff. positions for same time.

∴ This doesn't exist.

Q- 

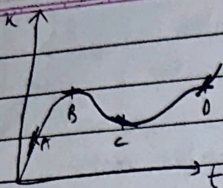
→ Body in motion

→ $V_{av} = 0$, as displacement = 0

→ $a > 0$, then $a = 0$.

→ Non-uniform motion.

Date: 25/4/24

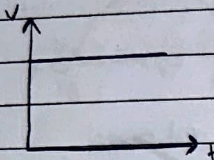
Q- 

v is negative at C, as slope is negative.

× ———— × ———— × ———— × ————

25/4/24

✱ Velocity - time Graph

Outcomes → (1) 

1. →

✱ Slope of v-t graph → acceleration

Slope = 0, a = 0

Slope = +ve, a = +ve

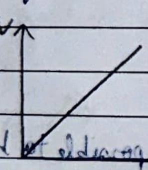
Slope = -ve, a = -ve

Slope = const., a = const

↳ v → constant

slope → 0, a = 0.

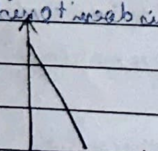
uniform motion

(2) 

→ v → ~~const~~ uniform, increases

slope - constant, a = constant, +ve

accelerated uniform motion. Eg. Increasing speed of motorcycle uniformly

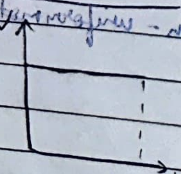
(3) 

→ v → ~~uniform~~ uniform, decreases

slope → negative, a = negative & constant

retarded uniform motion Eg. decreasing speed of motorcycle

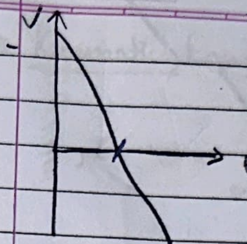
2. Area under v-t graph → Displacement

1) 

Area → v × t = displacement

Page No.: 35

Date: 25/4/24

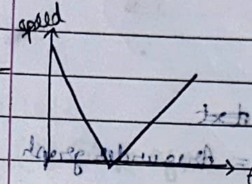
Q- 

→ slope = -ve, v ↓, constant

a < 0 & a → constant

→ v ↑ in opposite direction.

a > 0 & a → constant Eg. Throwing ball

Q- 

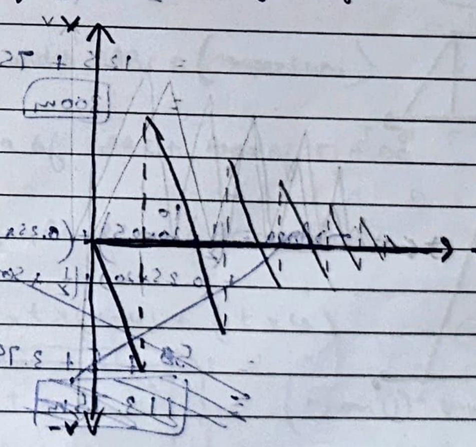
→ speed ↓ then ↑

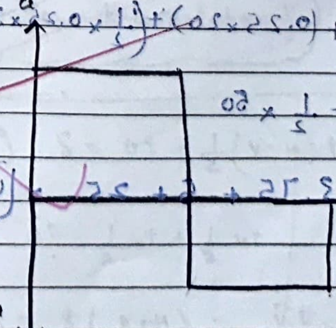
→ First decelerated (a<0) then accelerated (a>0) motion

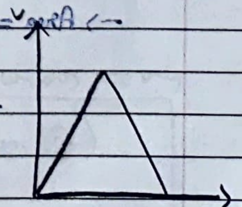
× ———— × ———— × ———— × ————

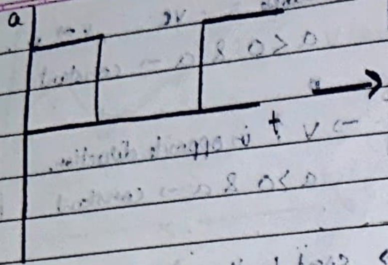
29/4/24

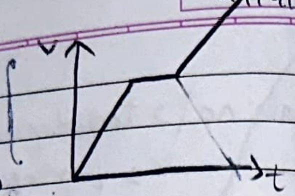
Q- A ball is dropped from a certain height and rebounds from the floor with reduced speed. It goes on hitting the ground and ball comes to rest finally. Plot velocity time graph

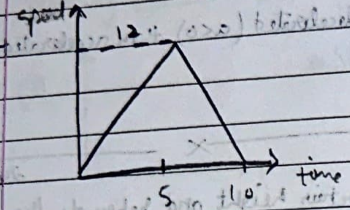
→ 

Q- 

→ 



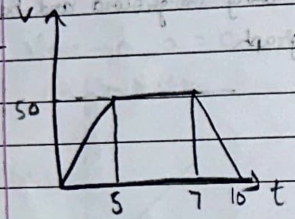




~~Area = s×t~~

Distance = Area under graph

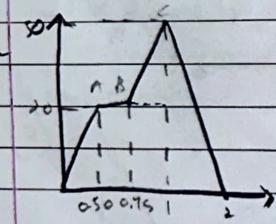
$= \frac{1}{2} \times 10 \times 12 = 60m$



→ Area $= \left(\frac{1}{2} \times 5 \times 50\right) + \left(\frac{1}{2} \times 3 \times 50\right) + 50 \times 2$

$= 125 + 75 + 100$

$= \boxed{300m}$



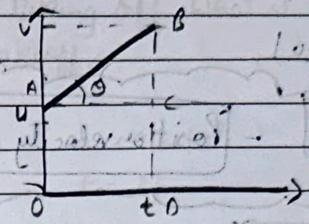
~~→ Area $= \left(\frac{1}{2} \times 20 \times 0.5\right) + (0.25 \times 20) + \frac{1}{2} \times (0.25) \times 30 + (0.25 \times 20) + \left(\frac{1}{2} \times 50 \times 2\right)$~~

~~$= 5 + 5 + 3.75 + 5 + 50 = \boxed{113.75m}$~~

→ Area $= \left(\frac{1}{2} \times 0.5 \times 20\right) + (0.25 \times 20) + \left(\frac{1}{2} \times 0.25 \times 30\right) + (20 \times 0.25) + \frac{1}{2} \times 50$

$= 5 + 5 + 3.75 + 5 + 25 = \boxed{43.75m}$

☆ Kinematics Equation of Motion for Uniform Acceleration



(1) Slope of AB $= \tan\theta = \frac{BC}{AC}$

$a = \frac{v-u}{t}$ — (1)

$\Rightarrow \boxed{v = u + at}$ — Velocity time Equation

(2) Area under OABCD (trapezium)

⇒ Area of ΔABC + Area of □ACDO.

$S = \frac{1}{2} \times AC \times BC + (AC \times CD)$

$S = \frac{1}{2} \times t \times (v-u) + (t \times u)$ — (2)

$\boxed{S = ut + \frac{1}{2}at^2}$ (From (1), $v-u = at$)

(Position - time relation)

For uniform acc only

(3) $S = ut + \frac{1}{2}(v-u)t$ (from (2))

$\Rightarrow S = \frac{1}{2}ut + \frac{1}{2}vt$

$\Rightarrow S = t\left(\frac{v+u}{2}\right) = \bar{v}.t$ ($\bar{v}$ = avg. velocity)

(4) from (3), $t\left(\frac{v+u}{2}\right)$

$$s = \frac{(v-u)(v+u)}{2\times a}$$

$\boxed{2as = v^2 - u^2}$ – Position velocity relation

30/4/24

• Distance covered in nth second

S_n

t=0   t=n-1   t=n

S_{n-1}

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow S_n = un + \frac{1}{2}an^2$$

$$\Rightarrow S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$\therefore S_n^{th} = S_n - S_{n-1}$$

$$= un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 - 2n + 1)\right]$$

$$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{1}{2}an^2} + an - \frac{1}{2}a$$

$$= u + an - \frac{a}{2}$$

$$\boxed{S_{n^{th}} = u + \frac{a}{2}(2n-1)}$$

(n = time)

Q- A jet plane starts from rest w/ $a = 3\,m/s^2$, makes a run of 35 sec, before taking off. What is length of runway & velocity of jet before takeoff.

$\rightarrow u = 0\,m/s$ $a = 3\,m/s^2$

$v = ?$ $t = 35$

$S = ?$

$v = u + at$	$S = ut + \frac{1}{2}at^2$
$\Rightarrow v = 0 + 3(35)$	$S = \frac{1}{2}\times 3 \times 35 \times 35$
$\Rightarrow \boxed{v = 105\,m/s}$	$S = \frac{3}{2}\times 1225$
	$\boxed{S = 1837.5\,m}$

Q- An electron travelling with speed of $5\times10^3\,m/s$ having $a = 10^{12}\,m/s^2$

(1) How long will it take to double its speed?

(2) What will the distance covered during this time?

$\rightarrow u = 5\times10^3\,m/s$ $a = 10^{12}\,m/s^2$

$s = ?$

$v = 2u$

(1) $v = u + at$

$$\Rightarrow t = \frac{v-u}{a}$$

$$\Rightarrow t = \frac{2u-u}{a}$$

$$\Rightarrow t = \frac{u}{a} = \frac{5\times10^3}{10^{12}} = \boxed{5\times10^{-9}\,s}$$

(2) $s = ut + \frac{1}{2}at^2$

$$\Rightarrow s = (5\times10^3)(5\times10^{-9}) + \frac{1}{2}(10^{12})(5\times10^{-9})^2$$

$s = 25\times10^{-6} + \frac{1}{2}(10)^{12}\times 25\times10^{-18}$

$\Rightarrow s = 25\times10^{-6} + \frac{1}{2}\times 25\times10^{-6}$

$\Rightarrow s = 25\times10^{-6}\left(1+\frac{1}{2}\right)$

$\Rightarrow \boxed{s = 37.5\times10^{-6}\ m}$

Q- A driver takes 0.20 sec to apply brakes after he sees a need for it. The driving speed of car is 54 km/h, & the deceleration is $6 m/s^2$ ($a = -6$), find distance travelled by car.

→ $t = 0.20$ $s = ?$

$u = 54\ km/h = 15\ m/s$ $a = -6\ m/s^2$

$v = 0$

~~$s = ut + \frac{1}{2}at^2$~~

~~$\Rightarrow s = 15(0.20) + \frac{1}{2}(-6)(0.20)^2$~~

~~$\Rightarrow s = 3 + (-3)(0.04)$~~

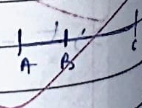
~~$\Rightarrow s = 3 - 0.12$~~

~~$\Rightarrow \boxed{s = 2.88 m}$~~

~~$s = \frac{v^2 - u^2}{2a}$~~

~~$s = \frac{0 - 225}{-12}$~~

~~$s = \frac{225}{12}$~~



→ Dist. travelled before braking (AB) $= 0.20\times15$ $\boxed{3m}$

Dist. travelled after braking = BC
(Using $v^2 - u^2 = 2as$)

$\Rightarrow$ BC → $0 - (15)^2 = 2(-6)s$

$\frac{225}{12} = s$

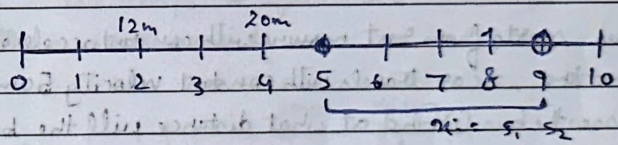
$\boxed{s = 18.75\ m}$

→ Total dist. travelled $= 18.75 + 3$
$= \boxed{21.75 m}$

Q- A body covers 12 m in 2nd second and 20 m in 4th second. How much distance will it in 4 second after 5th second.

→ $S_2 = 12m$ $S_5 \to S_2\ m$

$S_4 = 20m$ $S_9 \to S_1\ m$



$x = S_1 - S_2$

$S_n = u + \frac{a}{2}(2n-1)$ $S_4 = u + \frac{a}{2}(2n-1)$

$\Rightarrow 12 = u + \frac{a}{2}(2(2)-1)$ $\Rightarrow 20 = u + \frac{7a}{2}$ — (2)

$\Rightarrow 12 = u + \frac{3a}{2}$ — (1)

$\frac{7a}{2} + u = 20$

$-\frac{3a}{2} - u = -12$

$\frac{4a}{2} = 8$

$\boxed{a = 4 m/s^2}$

→ Substituting in (1),

$12 = u + \frac{12}{2}$

$\boxed{u = 6 m/s}$

$\therefore S_9 = ut + \frac{1}{2}at^2$

$\Rightarrow S_9 = 6(9) + \frac{1}{2} \times 4(9)^2$

$\Rightarrow S_1 = 54 + 162 = \boxed{216 m}$

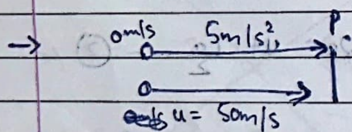
$\Rightarrow S_2 = 6(5) + \frac{1}{2} \times 4(5)^2$

$S_2 = 30 + 50$
$= 80 m$

$\Rightarrow x = 216 - 80$
$= \boxed{136 m}$

Q- A bus starts from rest moving with constant acceleration $5 m/s^2$. At same time, a car travels with constant velocity 50 m/s overtakes & passes the bus. (1) Find at what distance will the bus overtake the car.

(2) How fast the bus will be travelling?

→ 

Dist. travelled by bus $= ut + \frac{1}{2}at^2$

$S_B = 0 + \frac{1}{2}(5)(t)^2$

$\boxed{S_B = \frac{5t^2}{2}}$ $= 1000m$

Dist. travelled by car $= 50 \times t$
$= 50t$
$= 1000m$

Now, the dist travelled by car & bus to overtaking is equal:

$\therefore 50t = \frac{5t^2}{2}$

$\Rightarrow 100 = 5t$

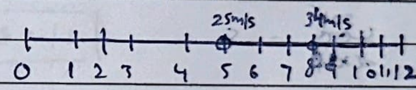
$\Rightarrow \boxed{t = 20 s}$

→ (2) $v = u + at$

$v = 0 + 5 \times 20$

$\boxed{v = 100 m/s}$

Q- An object is moving with uniform acceleration. Its velocity after 5 sec is 25 m/s & after 8 sec is 34 m/s. Find out dist. travelled in 12th second.

→ 

$\Rightarrow v_1 = u + at$ | $v_2 = u + at$

$25 = u + a(5)$ — (1) | $34 = u + 8a$ — (2)

$\Rightarrow u + 8a = 34$

$- u + 5a = -25$

$3a = 9$

$\boxed{a = 3 m/s^2}$

$\Rightarrow \boxed{u = 10 m/s}$

$\Rightarrow S_{12} = u + \frac{a}{2}(2(12) - 1)$

$\Rightarrow S_{12} = 10 + \frac{3}{2}(23)$

$S_{12} = 10 + \frac{69}{2}$

$S_{12} = \frac{89}{2} = \boxed{44.5m}$

Date: 2/5/24

Motion Under Gravity

1) $v = u + gt$ $(a = g)$

2) $s = ut + \frac{1}{2}gt^2$ "

3) $v^2 - u^2 = 2gs$

$g = 9.8 m/s^2$ or $10 m/s^2$

Note → In upward motion, g → negative, $v = 0$.
In downward motion, g → positive, $u = 0$.

Q- A ball is thrown vertically upwards with velocity 20 m/s from top of a building of height 25m. How high the ball will rise?

→ $v^2 - u^2 = 2as$
⇒ $0 - 400 = 2 \times \cancel{5} \times 9.8$
⇒ $\frac{400}{2 \times 10} = s$
⇒ $s = 20m$

x, $v = 0$, $u = 20 m/s$, 25m

Total dist. from ground = $20 + 25$
$= \boxed{45m}$

→ How long will it take the ball to hit the ground.

⇒ $s = ut + \frac{1}{2}at^2$
⇒ $45 = 0 + \frac{1}{2} \times 10 t^2$
⇒ $t = \sqrt{9}$
⇒ $\boxed{t_2 = 3s}$
⇒ $\boxed{t_f = 5s}$ $(t_1 + t_2)$

$t_1 = \frac{u - v}{g}$

$t_1 = \frac{20}{10}$ ⇒ $t_1 = 2s$

Date: 2/5/24

Imp

Q- A ball thrown up, caught by the thrower after 4s. How high did it go? What is initial velocity?

→ Time for upward motion = Time for downward motion

⇒ Total time taken = 4s

⇒ $t_d = 2s$ & $t_u = 2s$

$a = 9.8 m/s^2$, 4s

☆ Upward motion

↳ $u = ?$ $v = 0$
$g = -10 m/s$ $t = 2s$

→ $v = u + at$
⇒ $0 = u - 20$
⇒ $\boxed{u = 20 m/s}$

$v^2 - u^2 = 2as$
⇒ $-400 = -20s$
⇒ $\boxed{s = 20m}$

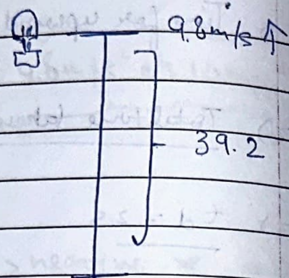
③ How far was it below the highest point 3 sec after it was thrown.

→ $s = ut + \frac{1}{2}gt^2$

⇒ $s = 0 + \frac{1}{2} \times 10 \times 1$

⇒ $\boxed{s = 5m}$

Q- A ballon is moving up at a rate of 9.8m/s. At a height of 39.2m above the ground a food packet is thrown. After how much time and with what velocity does it reach to the ground?



$\rightarrow S = ut + \frac{1}{2}at^2$

$\rightarrow S = 9.8t + \frac{1}{2} \times \overset{-4.9}{\cancel{9.8}} \times t^2$

$\rightarrow \cancel{39.2 = +9.8t - 4.9t^2}$

$\rightarrow \cancel{8 = +2t - t^2}$

$\rightarrow \cancel{t^2 + 2t - 8 = 0}$ $\Rightarrow 8 = 2t - t^2$

$\Rightarrow \cancel{t^2 + 4t - 2t - 8 = 0}$ $\Rightarrow t^2 - 2t + 8 = 0$

$\Rightarrow \cancel{t(t+4) - 2(t+4) = 0}$ $\Rightarrow t^2 - 4t$

$\Rightarrow \cancel{(t-2)(t+4) = 0}$

$\Rightarrow -39.2 = 9.8t - 4.9t^2$

$\Rightarrow -8 = 2t - t^2$

$\Rightarrow t^2 - 2t - 8 = 0$

$\Rightarrow t^2 - 4t + 2t - 8 = 0$

$\Rightarrow t(t-4) + 2(t-4) = 0$

$\Rightarrow (t-4)(t+2) = 0$

As time can't be negative, $\boxed{t = 4s}$.

$\rightarrow v = u + at$

$\Rightarrow v = +9.8 - 9.8(4)$

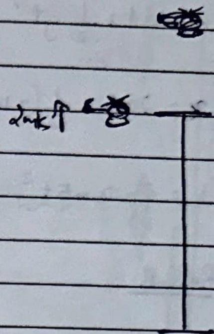
$\Rightarrow v = +9.8 - 39.2$

$\Rightarrow \boxed{v = -29.4 m/s}$

Q. A food packet is released from a helicopter which is moving steadily upwards with 2m/s. After 2s, what is velocity of packet? How far is it below the helicopter?

$t = 2s,\ u = 2m/s,\ g = -10m/s^2$

$S = ?$



$\rightarrow v = u + gt$

$\Rightarrow v = 2 + (-10)2$

$\Rightarrow \boxed{v = -18 m/s}$

$\rightarrow \cancel{v^2 - u^2 = 2as}$ $S = ut + \frac{1}{2}gt^2$

$\Rightarrow S = 2(2) + \frac{1}{2}(-10) \times 2 \times 2$

$\Rightarrow S = 4 - 10 = \boxed{-16m}$

(Neg. as direction is opposite)

$\cancel{S_2 = ut + \frac{1}{2}gt^2}$

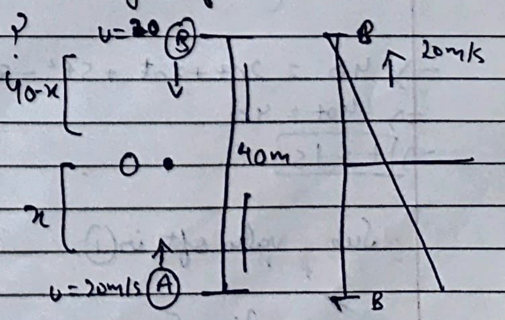
$\cancel{S_2 = 2(2) + \frac{1}{2}(}$

$\rightarrow$ Dist covered by helo in 2 secs $\rightarrow 2 \times 2$

$= 4m$

$\therefore$ Total distance $= 4 + 16 = \boxed{20m}$ (As direction is opposite)

Q. Two balls are thrown simultaneously. Ball A upward with 20m/s, Ball B downward 20m/s at a height of 40m along the same line. At what point they both collide?



Ball A

$\Rightarrow s = ut + \frac{1}{2} g t^2$

$\Rightarrow x = 20t + \frac{1}{2}(-10)t^2$

$\Rightarrow x = 20t - 5t^2$ — (1)

Ball B

$\Rightarrow s = ut + \frac{1}{2} g t^2$

$\Rightarrow 40 - x = 20t + \frac{1}{2}(10)t^2$

$\Rightarrow 40 - x = 20t + 5t^2$ → (2)

$\Rightarrow \cancel{x = -5t^2 - 20t - 40}$

~~Subtracting (2) from (1),~~

$\cancel{40 - x = 20t + 5t^2}$ From (1),

$\Rightarrow 40 - (20t - 5t^2) = 20t + 5t^2$

$\Rightarrow 40 = 20t + 20t + 5t^2 - 5t^2$

$\Rightarrow 40t = 40$

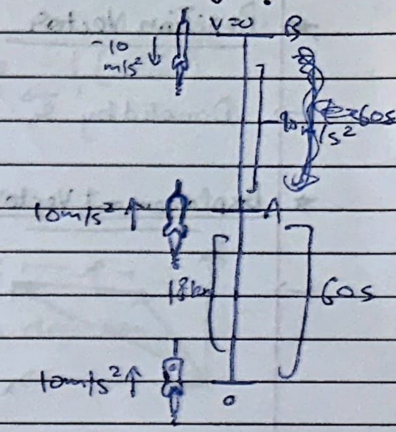
$\Rightarrow \boxed{t = 1 s}$

Sub. value of t in (1),

$x = 20 - 5$

$= \boxed{15 m}$

Q- A rocket is moving upward with $a = 10 m/s^2$. The fuel finish in 1 minute and it continues to up. What is the max height?



$\Rightarrow$ Dist OA $a = 10 m/s^2$ $u = 0$
$t = 60 s$

$\Rightarrow s = ut + \frac{1}{2} g t^2$

$\Rightarrow s = \frac{1}{2} \times 10 \times 3600$

$\Rightarrow s = 18000 m$ or 18 km

$\Rightarrow v = u + at$

$\Rightarrow v = 10 \times 60 = 600 m/s$

Dist AB $a = -10 m/s^2$ $u = 600 m/s$

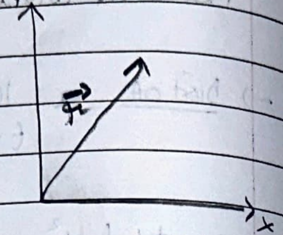
$\Rightarrow v^2 - u^2 = 2as$

$\Rightarrow -360000 = 2 \times -10 \times s$

$\Rightarrow -18000 = s$

Total dist $\Rightarrow 18000 + 18000 = 36000$ km

CW Ch-3 → MOTION IN A PLANE

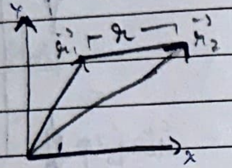


★ **Position Vector** ($\vec{r}$)

↳ Denoted by '$\vec{r}$'

vector symbol ($\longrightarrow$)
$\vec{A}$ or **A** → Bold

★ **Displacement Vector**



→ $\vec{r} = \vec{r_2} - \vec{r_1}$

Key Terms

(1) **Equal vectors**: Vectors having same magnitude and same direction are known as equal vectors.

$\vec{A}$ $\quad$ $\vec{B}$ $\quad$ → $\vec{A} = \vec{B}$

(2) **Parallel Vectors**: Vectors having same direction but magnitude need not be same are called parallel vectors.

$\left.\begin{array}{l}\vec{A}\\ \vec{B}\end{array}\right\}$ Parallel Vectors.

(3) **Negative of Vectors**: Opposite vector of a given vector.

$\left.\begin{array}{l}\vec{A}\\ \vec{B}\end{array}\right\}$ $B = -\vec{A}$

(4) **Modulus of vector**: $\vec{A} \rightarrow |\vec{A}| \rightarrow A$
To obtain magnitude of vector.

(5) **Unit vector**: Vector which has magnitude = 1 (unity)

Cap ← $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$\left.\begin{array}{l}\text{'x' unit vector} - \hat{i}\\ \text{'y' unit vector} - \hat{j}\\ \text{'z' unit vector} - \hat{k}\end{array}\right]$ To indicate direction

(6) **Null Vector**: $\vec{0}$, vector having zero magnitude.

Ex (1) At rest, $\vec{v} = \vec{0}$
(2) At origin, $\vec{r} = \vec{0}$
(3) Uniform velocity, $\vec{a} = \vec{0}$
(4) $\vec{A}$ & $\vec{B} = -\vec{A}$
$\vec{A} + \vec{B} = \vec{0}$

(7) **Multiplication of vector by real number**:

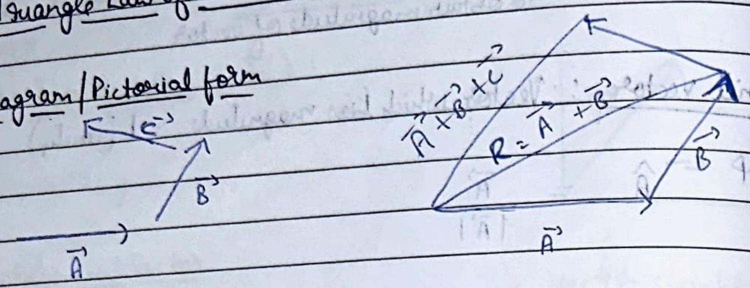
$\vec{A} \times 2 = 2\vec{A}$

$\times -2 = -2\vec{A}$

Date: 9/5/24

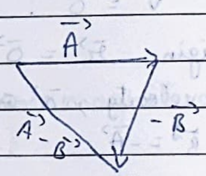
★ Triangle Law of Vector Addition

1) Diagram / Pictorial form

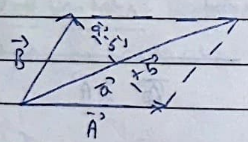


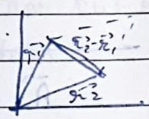
2) Subtraction form

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

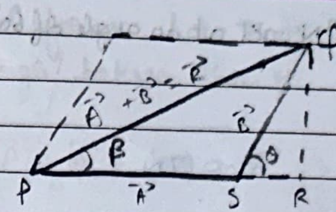


★ Parallelogram Law of Vector Addition





★ Vector Addition of Two Vectors using triangle / parallelogram law of Vector Addition (Sine & Cosine), (Magnitude & Direction of resultant vector).



For Δ SQR, → $\sin\theta = \dfrac{QR}{QS}$

$\Rightarrow QS \sin\theta = QR$

$\Rightarrow QR = B\sin\theta$ — (1)

$\rightarrow \cos\theta = \dfrac{SR}{QS}$

$\Rightarrow SR = B\cos\theta$ — (2)

In Δ PQR, $R^2 = QR^2 + (PR)^2$

$= QR^2 + (PS+SR)^2$

$\Rightarrow R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$

$\Rightarrow R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$

$\Rightarrow R^2 = B^2 + A^2 + 2AB\cos\theta$

magnitude → $\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$ (cosine law)

★ θ is angle b/w $\vec{A}$ & $\vec{B}$

★ Direction → $\boxed{\tan\beta = \dfrac{|\vec{B}|\sin\theta}{|\vec{A}| + |\vec{B}|\cos\theta}}$

Q- Two forces 5N & 7N act at an angle of 60° b/w them. Find resultant force.

→ $\vec{A} = 5N$ $\quad \vec{B} = 7N$

$\vec{R} = \sqrt{25+49+70\cos 60}$

$= \sqrt{74 + \overset{35}{\cancel{70}} \times \frac{1}{\cancel{2}}}$

$= \sqrt{109}$ N $= \underline{10.4N}$ approx

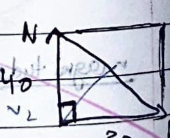
Direction = $\tan\alpha$

$= \dfrac{7 \times \frac{\sqrt{3}}{2}}{5 + 7 \times \frac{1}{2}}$

$= \frac{7\sqrt{3}}{2} \times \frac{2}{17} = \frac{7\sqrt{3}}{17}$ $\cancel{= \frac{12.124}{17}}$

$\alpha = \boxed{\tan^{-1}\left(\frac{7\sqrt{3}}{17}\right)}$

Q. A body has two velocities 30 m/s due East & 40 m/s due north. Find resultant velocity.



→ $\vec{R} = \sqrt{1600+900}$

$\vec{R} = 50$ m/s

$\tan\beta = \dfrac{v_2 \sin 90}{v_1 + v_2\cos 90} = \dfrac{40}{30}$

$\beta = \tan^{-1}\frac{4}{3} = \boxed{53°8'}$

Q. Two equal forces have their resultant equal to either. What is the angle between them?

Imp

→ $\vec{R} = \sqrt{\vec{P}^2 + \vec{Q}^2 + 2PQ\cos\theta}$

$\cos^{-1}\left(\frac{1}{2}\right) = 120°$

As all three forces are equal,

$\vec{R} = \sqrt{2\vec{R}^2 + 2\vec{R}^2\cos\theta}$

⇒ $\vec{R}^2 = 2\vec{R}^2 + 2\vec{R}^2\cos\theta$

⇒ $-\vec{R}^2 = 2\vec{R}^2\cos\theta$

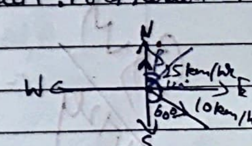
⇒ $\cos\theta = -\frac{\vec{R}^2}{2\vec{R}^2}$

⇒ $\cos\theta = -\frac{1}{2}$

⇒ $\theta = (\cos^{-1}) -\frac{1}{2}$

$= \boxed{120°}$

Q- A boat moving towards north with 25 km/hr & water flowing with 10 km/h in direction of 60° East of South. Find resultant velocity.



→ $\vec{R} = \sqrt{\vec{v}_1^2 + \vec{v}_2^2 + 2v_1v_2\cos\theta}$ $\quad (\theta = 120°)$

⇒ $\vec{R} = \sqrt{625+100+500\cos 120°}$

⇒ $\vec{R} = \sqrt{725 + \overset{250}{\cancel{500}} \times -\frac{1}{\cancel{2}}}$

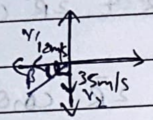
⇒ $\vec{R} = \sqrt{475} = \boxed{21.8 \text{ km/hr}}$

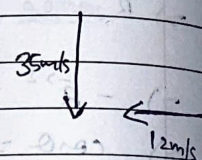
Date: 10/5/24

$\tan\beta = \dfrac{v_2 \sin\theta}{v_1 + v_2 \cos\theta}$

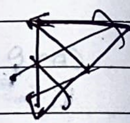
$\tan\beta = \dfrac{10 \times \frac{\sqrt{3}}{2}}{25 + 10 \times -\frac{1}{2}} = \boxed{\tan^{-1}\left(\frac{\sqrt{3}}{4}\right)}$

Q- Rain was falling vertically with 35m/s and wind is blowing in east to west with 12m/s. In which direction should a boy hold his umbrella





$\rightarrow \vec{R} = \sqrt{\vec{v_1}^2 + \vec{v_2}^2 + 2v_1v_2\cos\theta}$



$= \sqrt{144 + 1225 + 840 \times 0}$

$= \sqrt{144 + 1225} = \sqrt{1369}$

$= \boxed{37.1m}$

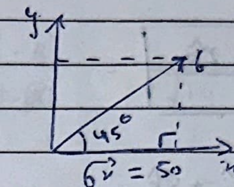
$\tan\beta = \dfrac{v_2 \sin 90}{v_1 + v_2 \cos 90}$

$= \dfrac{35}{12} = \boxed{\tan^{-1}\left(\frac{35}{12}\right)}$

10/5/24

Page No.: 57
Date: 10/5/24

Q- A force is at 45° to the horizontal ~~component~~. Its component in horizontal direction is 50N.
Find component in vertical direction.



$\rightarrow f_x = f\cos\theta$

$\Rightarrow f = \dfrac{f_x}{\cos\theta}$

$\Rightarrow f = \dfrac{50}{\frac{1}{\sqrt{2}}} = \boxed{50\sqrt{2}}$

$f_y = f\sin\theta$

$\Rightarrow f_y = 50\sqrt{2} \times \dfrac{1}{\sqrt{2}} = \boxed{50N}$

x —— x —— x —— x —— x

$\rightarrow \vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$ → magnitude of A

$\vec{B} = \hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{A} + \vec{B} = 4\hat{i} + 6\hat{j} + 7\hat{k}$

Magnitude of $\vec{A} = \sqrt{3^2 + 4^2 + 2^2}$
$= \sqrt{9 + 16 + 4} = \sqrt{29}$

$|\vec{B}| = \sqrt{1^2 + 2^2 + 5^2}$
$= \sqrt{30}$

Date: 10/5/24

$$|\vec{A}+\vec{B}| = \sqrt{4^2+6^2+7^2} = \sqrt{101}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \boxed{\frac{3\hat{i}+4\hat{j}+2\hat{k}}{\sqrt{29}}}$$

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{\hat{i}+2\hat{j}+5\hat{k}}{\sqrt{30}}$$

$$\hat{A}+\hat{B} = \frac{4\hat{i}+6\hat{j}+7\hat{k}}{\sqrt{101}}$$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

$\rightarrow \theta =$ angle b/w $\vec{A}$ & $\vec{B}$

- $\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0 = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

- $\hat{i}\cdot\hat{j} = |i|\,.\,|j|\,\overset{\cos 90^\circ}{\cancel{\ }} = 0$

$\hat{i}\cdot\hat{k} = 0$

$\hat{j}\cdot\hat{k} = 0$

Ex. $\vec{A} = 2\hat{i}+3\hat{j}+\hat{k}$

$\vec{B} = \hat{i}-\hat{j}-\hat{k}$

Page No.: 59
Date: 11/5/24

$$\vec{A}\cdot\vec{B} = (2i+3j+k)\cdot(\hat{i}-\hat{j}-\hat{k})$$

$$= 2-3-1 = \boxed{-2}$$

$$\rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,.\,|\vec{B}|} = \frac{-2}{\sqrt{14}\times\sqrt{3}}$$

$$= \frac{-2}{\sqrt{42}}$$

$$\therefore,\ \theta = \cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$$

Q- $\vec{A} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{B} = 2\hat{i}-\hat{j}+0\hat{k}$

$$\vec{A}\cdot\vec{B} = (\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}-\hat{j}+0\hat{k})$$

$$= 2-2+0 = \boxed{0}$$

Q- $\vec{A} = 5\hat{i}+7\hat{j}-3\hat{k}$ Find α

$\vec{B} = 2\hat{i}+2\hat{j}-\alpha\hat{k}$

$\rightarrow \cos\vec{A},\vec{B} = (5\hat{i}+7\hat{j}-3\hat{k})(2\hat{i}+2\hat{j}-\alpha\hat{k}) = 0$

$10+14+3\alpha = 0$

$3\alpha = -24$

$\boxed{\alpha = -8}$

Q. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

Find angle between A & B.

→ $|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2$

⇒ $A^2 + 2\vec{A}\cdot\vec{B} + B^2 = A^2 - 2\vec{A}\cdot\vec{B} + B^2$

⇒ $4\vec{A}\cdot\vec{B} = 0$

⇒ $4AB\cos\theta = 0$

⇒ $\cos\theta = 0$

∴ $\boxed{\theta = 90^\circ}$

Q. $\vec{P}, \vec{Q}$ & $\vec{R}$ have magnitudes 5, 12 & 13 resp. If P+Q=R find out angle between Q & R.

→ $\vec{P} = \vec{R} - \vec{Q}$

Squaring both sides,

⇒ $P^2 = R^2 + Q^2 + 2\vec{R}\cdot\vec{Q}$

⇒ $25 = 144 + 169 - 2\vec{R}\cdot\vec{Q}$

~~⇒ $25 = -25 + 2\vec{R}\cdot\vec{Q}$~~ ⇒ $25 = 313 - 2\vec{R}\cdot\vec{Q}$

~~⇒ $\vec{R}\cdot\vec{Q} = 25$~~ ⇒ $\vec{R}\cdot\vec{Q} = 144$

~~⇒ $RQ\cos\theta = 25$~~ ⇒ $RQ\cos\theta = 144$

~~⇒ $\cos\theta =$~~

⇒ $\cos\theta = \frac{\cancel{144}^{12}}{\cancel{12}\times 13}$

Q. $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$. Prove that A is perpendicular to B.

→ $\vec{A} + \vec{B} = \vec{C}$

Squaring;

⇒ $A^2 + B^2 + 2\vec{A}\cdot\vec{B} = C^2$

⇒ $\cancel{C^2} + 2\vec{A}\cdot\vec{B} = \cancel{C^2}$

⇒ $2AB\cos\theta = 0$

⇒ $\cos\theta = 0$

⇒ $\boxed{\theta = 90^\circ}$

★ Vector Product

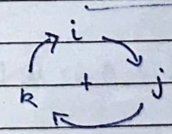
→ $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta\;\hat{n}$

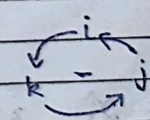
$\hat{n}$ = normal vector (perpendicular vector)

→ normal vector to vector $\vec{A}$ & $\vec{B}$ both

$\hat{i}\times\hat{j} = \hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$
$\hat{k}\times\hat{j} = -\hat{i}$
$\hat{i}\times\hat{k} = -\hat{j}$





$\hat{i}\times\hat{i} = 0$
$\hat{j}\times\hat{j} = 0$
$\hat{k}\times\hat{k} = 0$

Date: 11/5/24

Q- $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$

1. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 4 & -1 \end{vmatrix} = \hat{i}(2-4) - \hat{j}(-1-2) + \hat{k}(4-(-4))$

$= \boxed{-2\hat{i} + 3\hat{j} + 8\hat{k}}$

- Always left to right cross multiplication.

2. Angle b/w A & B → $\sin\theta = \dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}|\,|\vec{B}|}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{77}}{\sqrt{21}\,\sqrt{6}}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{\sqrt{77}}{\sqrt{21}\,\sqrt{6}}\right)$

3. Find normal vector → $\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$= \boxed{\dfrac{-2\hat{i} + 3\hat{j} + 8\hat{k}}{\sqrt{77}}}$

Q. $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$

$\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$

Prove A & B are parallel.

Page No.: 263

Date: 11/5/24

$\Rightarrow \vec{A} \times \vec{B} = \hat{i}(-9-(-9)) - \hat{j}(6-6) + \hat{k}(18-18)$

$= \boxed{0\hat{i} - 0\hat{j} + 0\hat{k}} = \boxed{0}$

$\therefore$ A $\parallel$ B, hence proved. parallel to

Q- $\vec{A} = 3\hat{i} + 4\hat{j} + 0\hat{k}$

$\vec{B} = -3\hat{i} + 7\hat{j} + 0\hat{k}$

→ $|\vec{A} \times \vec{B}|$

are the sides of parallelogram. Find area of ||gm.

$\Rightarrow \vec{A} \times \vec{B} = \hat{i}(0+0) - \hat{j}(0+0) + \hat{k}(21-(-12))$

$\Rightarrow \vec{A} \times \vec{B} = 33\hat{k}$

$\Rightarrow |\vec{A} \times \vec{B}| = \sqrt{33^2}$

$= \sqrt{1089} = \boxed{33}$ sq. unit

Q- $\vec{A}$ & $\vec{B}$ are sides of ||gm. Its area is $\dfrac{|\vec{A}||\vec{B}|}{2}$. Find angle between A & B.

$\Rightarrow |\vec{A} \times \vec{B}| = \dfrac{|\vec{A}||\vec{B}|}{2}$

$\Rightarrow \cancel{|\vec{A}|}\cancel{|\vec{B}|}\sin\theta = \dfrac{\cancel{|\vec{A}||\vec{B}|}}{2}$

$\Rightarrow \sin\theta = \dfrac{1}{2}$

$\Rightarrow \boxed{\theta = 30^\circ}$

Q. $\vec{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{B} = \hat{i} + \hat{j} - \hat{k}$

Find a vector whose length is 7 & perpendicular to A&B.

$\Rightarrow \vec{A}\times\vec{B} = \hat{i}(3-6) - \hat{j}(-2-6) + \hat{k}(2+3)$

$\Rightarrow \vec{A}\times\vec{B} = -3\hat{i} - 8\hat{j} + 5\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{9+64+25} = \sqrt{98}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

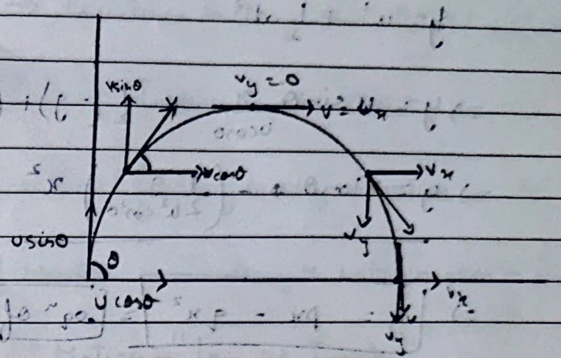
$$= \frac{-3\hat{i}+8\hat{j}+5\hat{k}}{\sqrt{98}}$$

$$= \frac{-3\hat{i}+8\hat{j}+5\hat{k}}{7\sqrt{2}}$$

$$\Rightarrow \cancel{7}\hat{n} = \frac{-3\hat{i}+8\hat{j}+5\hat{k}}{\cancel{7}\sqrt{2}}$$

$$\therefore \boxed{\hat{n} = \frac{-3\hat{i}+8\hat{j}+5k}{\sqrt{2}}}$$

Projectile Motion



⇒ Horizontal Direction → v_x → constant (always)
a_x → 0

velocity
• direction never changes

⇒ Vertical Direction → ~~v changes with direction~~
Direction of velocity changes

~~v~~	→	variable
a	→	uniform ~~variable~~

~~accelerated motion~~

Imp ★★ (1) Equation of Trajectory
(Displacement)

Horizontal Distance → $x = v_x t$

$x = u\cos\theta\, t$

$$\boxed{t = \frac{x}{u\cos\theta}} \quad \text{— (1)}$$

Vertical Displacement → $v_y = u\sin\theta$

$$y = ut + \frac{1}{2}at^2$$

$$\Rightarrow y = u\sin\theta \frac{x}{u\cos\theta} + \frac{1}{2}(-g) + \left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

Let $\tan\theta = p$

$\frac{g}{2u^2\cos^2\theta} = q$

$$\Rightarrow \boxed{y = px - qx^2} = \boxed{\text{eq}^n \text{ of parabola}}$$

② Time of maximum height (T_m)

For vertical, $v = u + at$

$\Rightarrow 0 = u\sin\theta - gt_m$

$$\boxed{t_m = \frac{u\sin\theta}{g}}$$

Ch 4 ★ Laws of Motion

* Inertia → Resistance to change in state of motion.

- of Rest - Tendency to stay at rest.
- of motion - Tendency to stay in motion
- of direction - Tendency to retain motion in one direction.

\# 1st Law Of Motion - "Laws of Inertia"

↳ Every body continues in the state of rest or state of uniform motion unless compelled by external force.

★ Projectile Motion

③ Time of flight

→ Displacement in vertical direction = 0

From $s_y = u_y t + \frac{1}{2}at^2$

$$0 = u\sin\theta t - \frac{1}{2}gt^2$$

$$\Rightarrow \boxed{t = \frac{2u\sin\theta}{g}} \text{ — Time of flight}$$

Also, time of max height $= \frac{u\sin\theta}{g}$

$\therefore$ Time of flight $= 2 \times$ Time of max height.