5) $z_2 = \dfrac{x^2-1+y^2}{x^2+1+2x+y^2}$

$= \dfrac{x^2-x^2-y^2+y^2}{x^2+x^2+2x+y^2+y^2} = 0$

$\therefore Re(z_2) = 0$

Q. In a competition there are 5 children participating. 3 prizes are to be given and no participant can get all three. In how many ways can this happen?

Choices for first = 5

Choices for second = 5

Choices for third = 5

$\therefore$ Total choices $= 5\times5\times5 - 5$ (As 5 children cannot win all three.)
$= \underline{120}$

CW Ch7- BINOMIAL THEOREM

(Recall)

↳ $(a+b)^2 = a^2+b^2+2ab$

↳ $(a+b)^0 = 1$

↳ $(a+b)^1 = a+b$

↳ $(a+b)^3 = a^3+b^3+3ab(a+b)$

↳ $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

★ $\underbrace{A^n B^m \rightarrow n+m}_{\text{Degree}}$

★ First term power decreases, second term power increases.

★ Observations

① In $(a+b)^n$ → new, no. of terms $= (n+1)$

Eg $(a+b)^{100}$ will have 101 terms.

② In $(a+b)^n$, degree of each term → n

③ In $(a+b)^n$, ~~power~~ powers of a keep reducing by 1 & powers of b keep increasing by 1.

④ In $(a+b)^n$, the coefficients are ~~${}^nC_0$~~ ${}^nC_0, {}^nC_1, {}^nC_2 \ldots {}^nC_n$.

$(T_1)\quad(T_2)\quad(T_3)\quad(T_4)\quad(T_5)$

$\rightarrow (a+b)^5 = 5C_0 a^5 + 5C_1 a^4 b + 5C_2 a^3 b^2 + 5C_3 a^2 b^3 + 5C_4 a b^4 + 5C_5 b^5$

(T_6')

For $(a+b)^n$,

$T_r = nC_{r-1}\ a^{n-r+1}\ b^{r-1}$

$\boxed{T_{r+1} = nC_r\ a^{n-r}\ b^r}$ $(r = 0, 1, 2 \ldots n)$

$T_{r+2} = nC_{r+1}\ a^{n-r-1}\ b^{r+1}$

☆ General Formulae

$\rightarrow (a+b)^n = [nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 \ldots nC_n a^0 b^n]$

Try!! Q. Write 4th term of $(x^2-y)^6$

$T_4 = 6C_3 x^6 (-y)^3$

$= -20x^6 y^3$

Q. Write 13th term of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$

$\rightarrow T_{13} = 18C_{12} (9x)^6 \left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$\frac{18\times17\times16\times14\times15}{120} \times (9)x^6 \ldots \frac{1}{(3\sqrt{x})^{12}}$

$= \boxed{18C_6 \times (9x)^6 \times \left(\frac{1}{3\sqrt{x}}\right)^{12}}$

$= 18C_6 \times (9)^6 \times \frac{1}{(3)^6} = \boxed{18C_6 \times 9^3}$

Q → The no. terms in $(1+2x+x^2)^{10}$.

$\rightarrow (a+b)^n = [nC_0 a^n b^0 + nC_1 a^{n-1} b^1 \ldots nC_n a^0 b^n]$

Taking $a = 1$ & $b = 1$.

$\therefore \boxed{2^n = [nC_0 + nC_1 + nC_2 \ldots nC_n]}$

Total sum of coefficients in binomial series.

→ Sum of every alternate coefficient is equal to sum of remaining.

Eg $(a+b)^3 \rightarrow 1\quad 3\quad 3\quad 1$ $\quad 1+3 = 3+1$ Equal

→ Taking $a = 1, b = -1$.

$0 = nC_0 - nC_1 + nC_2 - nC_3 + nC_4 \ldots + (-1)^n nC_n$

$\Rightarrow nC_1 + nC_3 + nC_5 \ldots = nC_0 + nC_2 + nC_4 \ldots$

→ Sum of odd coefficients = Sum of even coefficients

$= \underline{2^{n-1}}$

Try!! Evaluate $10C_1 + 10C_2 \dots 10C_{10}$

$= 2^{10} - 1 = 32 \times 32 - 1$

$= \underline{1023}$

$32 \times 32 = 64 + 960 = 1024$

→ $(a-b)^n = [nC_0 a^n b^0 + nC_1 a^{n-1}(-b)^1 + nC_2 a^{n-2}(-b)^2 \dots]$

Try!! Write exp. of $(x - 2y)^5$

→ $[5C_0 x^5 - 5C_1 x^4 2y + 5C_2 x^3 4y^2 - 5C_3 x^2 8y^3 + 5C_4 x\, 16y^4 - 5C_5\, 32y^5]$

→ $x^5 - 5x^4 2y + 10x^3(4y^2) - 10x^2(8y^3) + 5x(16y^4) - 32y^5$

★ Ex 8.1

Ans1 $(1-2x)^5$

$= [5C_0 - 5C_1(2x)^1 + 5C_2(2x)^2 - 5C_3(2x)^3 + 5C_4(2x)^4 - 5C_5(2x)^5]$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

Ans2 $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

$= [5C_0\left(\frac{2}{x}\right)^5 - 5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right)^1 + 5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 - 5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + 5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - 5C_5\left(\frac{x}{2}\right)^5$

$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^3}{8} - \frac{x^5}{32}$

Ans5 $\left(x + \frac{1}{x}\right)^6$

→ $6C_0 x^6 + 6C_1(x)^5\left(\frac{1}{x}\right) + 6C_2(x^4)\left(\frac{1}{x}\right)^2 + 6C_3(x^3)\left(\frac{1}{x}\right)^3$

$6C_4 x^2 \frac{1}{x^4} + 6C_5\, x \frac{1}{x^5} + 6C_6 \frac{1}{x^6}$

→ $x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

Ans6 $(96)^3 = (100-4)^3$

⇒ $3C_0(100)^3 - 3C_1(100)^2(4) + 3C_2(100)(4)^2 - 3C_3(4)^3$

⇒ $1000000 - 120000 + 4800 - 64 = \underline{884736}$

Ans10 $(1.01)^{10000}$ → $(1 + 0.1)^{10000}$

→ $[10{,}000 C_0 (0.1)^{10000} + 10000 C_1 (0.1)^1 \dots]$

→ $[1000 + 1 + \dots]$

Clearly $(1.01)^{10000} > 1000$.

Ans 11 $(a+b)^4 - (a-b)^4$

$\rightarrow [{}^4C_0 a^4 + {}^4C_1 a^3 b^1 + {}^4C_2 a^2 b^2 + {}^4C_3 ab^3 + {}^4C_4 b^4] - [{}^4C_0 a^4 - {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 - {}^4C_3 ab^3 + {}^4C_4 b^4]$

$\rightarrow 2[{}^4C_1 a^3 b + {}^4C_3 ab^3] = 8(a^3 b + ab^3)$

Putting $a = \sqrt{3}$ & $b = \sqrt{2}$,

$\rightarrow 2[4(\sqrt{3})^3(\sqrt{2}) + 4\sqrt{3}(\sqrt{2})^3]$

$\rightarrow 8[3\sqrt{6} + 2\sqrt{6}] = \boxed{40\sqrt{6}}$

Ans 12 $[{}^6C_0 x^6 + {}^6C_1 x^5 + {}^6C_2 x^4 + {}^6C_3 x^3 + {}^6C_4 x^2 + {}^6C_5 x + {}^6C_6] + [{}^6C_0 x^6 - {}^6C_1 x^5 + {}^6C_2 x^4 - {}^6C_3 x^3 + {}^6C_4 x^2 - {}^6C_5 x + {}^6C_6]$

$= 2[{}^6C_0 x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6]$

$= 2[x^6 + 15x^4 + 15x^2 + 1]$

Putting $x = \sqrt{2}$,

~~$\rightarrow 2\times8 + 30\times4 + 30\times2 + 1$~~

~~$\rightarrow 16 + 120 + 60 + 1 = 197$~~

$\rightarrow 2[8 + 60 + 30 + 1] = \underline{198}$

Imp

Ans 13 ~~$9^{n+1} = (1+8)^{n+1}$~~

~~$\rightarrow [{}^{n+1}C_0 8^{n+1} + {}^{n+1}C_1 8^n \dots {}^{n+1}C_{n+1}]$~~

$\rightarrow$

$9^{n+1} - 8n - 9$

$\rightarrow (1+8)^{n+1} - 8n - 9$

$\rightarrow [{}^{n+1}C_0 + {}^{n+1}C_1 8 + {}^{n+1}C_2 8^2 \dots {}^{n+1}C_{n+1} 8^{n+1}] - 8n - 9$

$\rightarrow [1 + (n+1)8 + 8^2[{}^{n+1}C_2 \dots]] - 8n - 9$

$\rightarrow \cancel{8n+9} + 8^2[{}^{n+1}C_2 \dots] - \cancel{8n - 9}$

$= 64 \times [{}^{n+1}C_2 \dots]$

Clearly, $9^{n+1} - 8n - 9$ is a multiple of 64.

$\therefore$ 64 divides $9^{n+1} - 8n - 9$.

Q:
PT $2^{3n} - 7n - 1 \div 49$ (Bhery Eazzz)

$\rightarrow 8^n - 7n - 1$

$\rightarrow (1+7)^n - 7n - 1$

$\rightarrow [{}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 \dots {}^nC_n 7^n] - 7n - 1$

$\rightarrow [\cancel{1 + 7n} + 7^2[{}^nC_2 \dots]] \cancel{- 7n - 1} =$

$= 49 \times [nC_2 \ldots]$

$\therefore$ as $2^{3n} - 7n - 1$ is a multiple of 49, it is divisible by 49.

Q.14 $\sum_{r=0}^{n} 3^r \, nC_r = 4^n$

$\rightarrow$ LHS $= [nC_0 + nC_1 3 + nC_2 3^2 \ldots + nC_n 3^n]$

RHS $= 4^n$

$= (1+3)^n$

$= [nC_0 + nC_1 3 + nC_2 3^2 \ldots + nC_n 3^n]$

$\therefore$ LHS = RHS

★ H/ex:

~~Ans~~ $nC_0 a^n = 729$

$a^n = 729$ — (1)

$nC_1 a^{n-1} b^1 = 7290$ — (2)

Q.4 $(a^n - b^n) \div (a-b)$

$\Rightarrow [(a-b)+b]^n - b^n$

$\rightarrow$

$\rightarrow [nC_0 (a-b)^n + nC_1 (a-b)^{n-1} b + nC_2 (a-b)^{n-2} b^2 \ldots \cancel{nC_n b^n}] - \cancel{b^n}$

$\rightarrow (a-b)[nC_0 (a-b)^{n-1} \ldots]$

$\therefore (a^n - b^n) \div a-b$

Q.2 $(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

HW

$\rightarrow$

Try!! $(a^2 + \sqrt{a^2-1})^4 + (a^2 - \sqrt{a^2-1})^4$

$\rightarrow 2(4C_0 x^4 + 4C_2 x^2 y^2 + 4C_4 y^4)$

$\rightarrow 2(a^8 + 6a^4(a^2-1) + (a^2-1)^2)$

$\rightarrow 2(a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2)$

$\rightarrow 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$

$\rightarrow 2(6C_1(\sqrt{3})^5(\sqrt{2}) + 6C_3(\sqrt{3})^3(\sqrt{2})^3 + 6C_5(\sqrt{3})(\sqrt{2})^5)$

$\rightarrow 2(6(9\sqrt{3})\sqrt{2} + 15(3\sqrt{3})(2\sqrt{2})$

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Q. $(0.99)^5$

$\rightarrow (1-0.01)^5$

$\rightarrow [1 - 5C_1(0.01) + 5C_2(0.01)^2 \ldots]$

$\rightarrow 1 - 0.05 + 10 \times 10^{-4}$

$\rightarrow 1 - 0.05 + 0.001 = \underline{0.951}$

Q. $\left(1+\left(\frac{x}{2}-\frac{2}{x}\right)\right)^4$

$\rightarrow \left[1 + 4\left(\frac{x}{2}-\frac{2}{x}\right) + 6\left(\frac{x}{2}-\frac{2}{x}\right)^2 + 4\left(\frac{x}{2}-\frac{2}{x}\right)^3 + \left(\frac{x}{2}-\frac{2}{x}\right)^4\right]$

$\rightarrow \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5$

x ——— x ——— x ——— x

Try!! ① Find T_4 (beginning) & T_4 (end) in expansion of $\left(x+\frac{2}{x}\right)^9$

$\rightarrow T_{r+1} = nC_r \, x^{n-r} y^r$

$\therefore T_{3+1}$ (beginning) $= 9C_3 \, x^6 y^3$

$= 84 x^6 y^3 = 84 \times x^6 \times \frac{8}{x^3}$

$= \underline{672x^3}$

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$\therefore T_{3+1}$ (end) $= 9C_6 \, x^3 y^6$

$= 84 \, x^3 \, \frac{64}{x^6} = \underline{\frac{5376}{x^3}}$

② Find the term independent of x (constant term) in $\left(2x+\frac{1}{3x^2}\right)^9$

$\rightarrow T_{r+1} = nC_r \, a^{n-r} b^r$

$T_{r+1} = 9C_r \, (2x)^{9-r} \frac{1}{(3x^2)^r}$

For ~~only coefficient~~,
~~$= 9C_r$~~

$T_{r+1} = 9C_r \frac{(2)^{9-r}}{(3)^r} \frac{(x^{9-r})}{(x^{2r})}$

$= 9C_r \frac{(2^{9-r})}{(3)^r} x^{9-3r}$

$\therefore$ For $n = 0$, $r = 3$.

$\therefore T_{3+1} = 9C_3 \frac{(2^6)}{(3)^3}$

$T_4 = \frac{84 \times 64}{27} = \boxed{\frac{5376}{27}}$

$= \boxed{\frac{1792}{9}}$

Q. If 4th term in $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, find a & n.

$\rightarrow$

$T_4 = {}^nC_3 (ax)^{n-3} \left(\frac{1}{x}\right)^3$

$\frac{5}{2} = {}^nC_3 (ax)^{n-3} \frac{1}{(x^3)}$

$\frac{5}{2} = {}^nC_3 \cancel{(ax)^{n-3}} (a)^{n-3} \; x^{n-6}$

As only constant, $n-6=0$

$\therefore n=6.$

$\frac{5}{2} = {}^6C_3 \; a^3$

$a^3 = \frac{5}{2} \times \frac{1}{204}$

$a^3 = \frac{5}{2} \times \frac{6}{6\times5\times4}$

$a^3 = \frac{1}{8}$

$\boxed{a = \frac{1}{2}}$

$a = \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$

Q. Find the positive value of m for which coefficient of x^2 in $(1+x)^m$ is 6.

$\rightarrow T_{r+1} = {}^mC_r (1)^{m-r} (x)^r$

For x^2, $r=2$,

$T_3 = {}^mC_2 \; x^2$

$\Rightarrow 6\cancel{x^2} = {}^mC_2 \cancel{x^2}$

$\Rightarrow 6 = \frac{m(m-1)}{2}$

$\Rightarrow m^2 - m - 12 = 0$

$\Rightarrow m^2 + 3m - 4m - 12 = 0$

$\Rightarrow m(m+3) - 4(m+3) = 0 \qquad \therefore \underline{m=4}$

Q. If the middle term in the expansion of $\left(\frac{1}{x} + x\sin x\right)^{10}$ is $\frac{63}{8}$, find x where $x \in [0, 2\pi]$.

$\hookrightarrow$ Total no. of terms = 11.

$\therefore$ Mid term = 6.

$\therefore T_6 = {}^{10}C_5 \left(\frac{1}{x}\right)^5 (x\sin x)^5$

$\frac{63}{8} = {}^{10}C_5 (\sin x)^5$

$\frac{63}{8} = 252 (\sin x)^5$

$(\sin x)^5 = \frac{63}{4\,252\times8}$

$(\sin x)^5 = \frac{1}{32}$

$\sin x = \frac{1}{2} \qquad \therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

Note:

Q. Does the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$ contain any term involving x^9.

Try!! Q. Find a if the coeffs of x^2 & x^3 in $(3+ax)^9$ are equal

$\hookrightarrow \quad T_{1+1} = {}^9C_2 \, (3)^7 (ax)^2$

$T_{2+1} = {}^9C_3 \, (3)^6 (ax)^3$

$\rightarrow \quad {}^9C_2 (3)^7 (a)^2 = {}^9C_3 (3)^6 (a)^3$

$\rightarrow \quad 36 \times 3 = \frac{9 \times 8 \times 7}{6} \times a$

$\rightarrow \quad a = \frac{36 \times 3}{84} = \boxed{\frac{9}{7}}$

A1 $\rightarrow \quad T_{r+1} = {}^{20}C_r \, (2x^2)^{20-r} \left(\frac{1}{x}\right)^r$

$= {}^{20}C_r \, (2)^{20-r} \, \frac{x^{40-2r}}{x^r}$

$= {}^{20}C_r \, (2)^{20-r} \, x^{40-3r}$

$9 = 40 - 3r$

$3r = 31$

$r = \frac{31}{3}$

$\therefore$ No x^9 term.

Q. Find a, if T_{17} & T_{18} $(2+a)^{50}$ are equal.

$\rightarrow \quad T_{17} = {}^{50}C_{16} \, (2)^{34} (a)^{16}$

$T_{18} = {}^{50}C_{17} \, (2)^{33} (a)^{17}$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r}$

$\rightarrow \quad \frac{{}^{50}C_{16} \times 2}{{}^{50}C_{17}} = a$

$\rightarrow \quad \frac{17}{34} \times 2 = a$

$\rightarrow \quad \cancel{a = \frac{34}{33}} \quad \boxed{a = 1}$

Q. If coeff of 2nd, 3rd, & 4th terms in expansion of $(1+x)^n$ are in AP. Find n.

$\rightarrow \quad T_2 = {}^nC_1 (x)^1$

$T_3 = {}^nC_2 (x)^2$

$T_4 = {}^nC_3 (x)^3$

$\rightarrow \quad 2 \times {}^nC_2 = {}^nC_1 + {}^nC_3$

$\rightarrow \quad 2 \times {}^nC_2 = {}^nC_1 + {}^nC_3$

$\rightarrow \quad \frac{2 \times n(n-1)}{2} = n + \frac{n(n-1)(n-2)}{3 \cdot 2}$

$\rightarrow \quad n(n-1) = \frac{3n + n(n-1)(n-2)}{3}$

$\rightarrow \quad 3n(n-1) = 3n + n(n-1)(n-2)$

$\rightarrow \quad 3(n-1) = 3 + (n-1)(n-2)$

$\Rightarrow 3n-3 = 3+n^2-3n+2$

$\Rightarrow n^2-6n+8=0$

$\Rightarrow n^2-4n-2n+8=0$

$\Rightarrow n(n-4)-2(n-4)=0$

$\therefore \underline{n=4}$ as n can't be 2.

alt

$$2 = \frac{{}^nC_1}{{}^nC_2} + \frac{{}^nC_3}{{}^nC_2}$$

$$2 = \frac{2}{n-1} + \frac{n-2}{3}$$

$$\Rightarrow 2 = \frac{6+(n-2)(n-1)}{3(n-1)}$$

$\Rightarrow 6(n-1) = 6+(n-2)(n-1)$

$\Rightarrow 6n-6 = 6+n^2-3n+2$

$\Rightarrow n^2-9n+14=0$

$\Rightarrow n^2-7n-2n+14=0 \Rightarrow (n-7)(n-2)=0$

$\Rightarrow \underline{n=7}$

Q. If the mid term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

$\rightarrow T_5 = {}^8C_4 \left(\frac{p}{2}\right)^4 (2)^4$

$$\Rightarrow 1120 = \frac{8\times7\times6\times5}{24} \times p^4$$

$$\Rightarrow \frac{1120\times24}{8\times7\times6\times5} = p^4$$

$\rightarrow \boxed{p=2}$

Q. $(1-3x+x^2)^{111}$. Find sum of coeff.

$\rightarrow$ Putting $x=1$, $(-1)^{111} = \underline{-1}$

Q. Find the value of α for which the coeff of mid terms of $(1+\alpha x)^4$ and $(1-\alpha x)^6$ are equal.

$\rightarrow T_3 = {}^4C_2 (\alpha x)^2$

$T_4 = -{}^6C_3 (\alpha x)^3$

${}^4C_2\, \alpha^2 = -{}^6C_3 (\alpha)^3$

$6\alpha^2 = -20\alpha^3$

$\Rightarrow (15\alpha^4 + 6\alpha^2) = 0$

$\Rightarrow \alpha^2(20\alpha+6)=0$

Either $\boxed{\alpha=0}$ or $\boxed{\alpha = -\frac{3}{10}}$

Q. If coeff of 2nd, 3rd & 4th terms in expansion of $(1+x)^{2n}$ are in AP, show that $2n^2-9n+7$ is 0

$\rightarrow$ $T_2 = {}^{2n}C_1\, x^1$

$T_3 = {}^{2n}C_2\, x^2$

$T_4 = {}^{2n}C_3\, x^3$

$\Rightarrow$ $2 = \dfrac{{}^{2n}C_1}{{}^{2n}C_2} + \dfrac{{}^{2n}C_3}{{}^{2n}C_2}$

$2 = \dfrac{2}{2n-1} + \dfrac{2n-2}{3}$

$6(2n-1) = 6 + (2n-2)(2n-1)$

$12n-6 = 6 + 4n^2 - 6n + 2$

$\Rightarrow 4n^2 - 18n + 14 = 0$

$2n^2 - 9n + 7 = 0$ H.P

$n = \frac{7}{2}, 1$ $\therefore \boxed{n = \frac{7}{2}}$

Eg Find coefficient of x^6y^3 in the expansion of $(x+2y)^9$

$\Rightarrow$ $T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$

$T_{r+1} = {}^9C_r\, x^{9-r}\, y^r\, 2^3$

For x^6y^3, $r = 3$.

$T_4 = {}^9C_3\, x^6 y^3$

$= \dfrac{9 \times 8 \times 7}{6} \times 8 = 21 \times 32 = 672$

Seen.

★ Ch 8 → Sequences & Series

Q. What is a sequence?

↳ It is a list of entries.

Eg $f(n) = n+1$, $n \in N$ $\quad f(1) = 2 - a_1$, $f(2) = 3 - a_2$, $f(3) = 4 - a_3$

∴ 2, 3, 4 . . . is a sequence of nos

∘ $f: N \to R$

Defn ↳ A function $f: N \to R$, defined as:

$$f(n) = a_n$$ is called a sequence.

Eg 2, 4, 6, 8 . . .
2, 3, 5, 7 . . .

→ Sequence which follows certain rules/patterns is called a progression, i.e, they can be expressed through some rule or formula.

Note:- 1. All progressions are sequences but all sequences are not progressions !

Eg. List of Prime nos. is a sequence but not a progression

∘ Sequences having finite terms, is called finite sequence (1st & Last terms are known)

∘ Sequences having infinite terms, is called infinite sequence (Last term unknown)

★ Constant Sequence → All terms equal. (common diff = 0)

★ Ex 8.1

① $a_n = n(n+2)$

$a_1 = 3$, $a_2 = 8$, $a_3 = 15$, $a_4 = 24$, $a_5 = 35$

② $a_n = \frac{n}{n+1}$

$a_1 = 1/2$, $a_2 = 2/3$, $a_3 = 3/4$, $a_4 = 4/5$, $a_5 = 5/6$

③ $a_n = 2^n$

$a_1 = 2$, $a_2 = 4$, $a_3 = 8$, $a_4 = 16$, $a_5 = 32$

④ $a_n = \frac{2n-3}{6}$

$a_1 = -1/6$, $a_2 = 1/6$, $a_3 = 1/2$, $a_4 = 5/6$, $a_5 = 7/6$

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(5) $a_n = (-1)^{n-1} 5^{n+1}$

$a_1 = 25$
$a_2 = -125$
$a_3 = 625$
$a_4 = -3125$
$a_5 = 15625$

(6) $a_n = \frac{n(n^2+5)}{4}$

$a_1 = 3/2$
$a_2 = 9/2$
$a_3 = 21/2$
$a_4 = 21$
$a_5 = \frac{150}{4} = 75/2$

(9) $a_n = (-1)^{n-1} n^3$

$a_9 = 729$

(10) $a_n = \frac{n(n-2)}{n+3}$

$a_{20} = \frac{360}{23}$

- A sequence in which terms are dependent on the previous term / terms, is called recursive series.

★ Series

Let $a_1, a_2, a_3 \ldots a_n$ be a sequence.

The expression $a_1 + a_2 + a_3 \ldots + a_n$ is called the series corresponding to given sequence.

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All terms have to be non zero!!

★ Geometric Progression

Eg $2, (-6), 18, (-54)$

↳ This a G.P. → a → first term
→ r → common difference

$\frac{a_2}{a_1} = -3, \quad \frac{a_3}{a_2} = -3, \quad \frac{a_4}{a_3} = -3$

$\therefore r = -3$
↳ common ratio of G.P.

a → First Term   r → common ratio.

↳ $a, ar, ar^2, ar^3, ar^4 \ldots ar^n$ ($T_1, T_2, T_3, T_4, T_5, T_n$)
is called Geometric Progression.

$\boxed{T_n = ar^{n-1}}$ → nth term of G.P.

Constant sequences (same no. throughout) are both AP as well as G.P. Eg 1, 1, 1, 1, ...

Ex 9.3

(1) $a = \frac{5}{2}, r = \frac{1}{2}$

$\therefore a_{20} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{19}$
$= \frac{5}{2^{20}}$

$a_n = \frac{5}{2^n}$

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(2) $ar^7 = 192$

$\Rightarrow a(2)^7 = 192$

$\Rightarrow a = \frac{192}{2 \times 8 \times 16 \times 2} = \frac{3}{2}$

$a_{12} = a(r)^{11}$

$= \frac{3}{2} \times (2)^{11}$

$= 3 \times 2^{10}$

$= \underline{3072}$

$32 \times 32 = 64 + 960 = 1024$, $1024 \times 3 = 3072$

(3) $T_5 = ar^4 = p$ $\quad T_{11} = ar^{10}$

$T_8 = ar^7 = q$

$\frac{T_8}{T_5} = r^3$ $\quad \frac{T_{11}}{T_8} = r^3$

$T_8^2 = \frac{T_{11}}{T_5} \times T_5$

$\Rightarrow \boxed{q^2 = p \times s}$

alt

$LHS = \cancel{(ar)^4}(ar^7)^2 = a^2 r^{14}$ | $RHS = (ar^4)(ar^{10}) = a^2 r^{14}$

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(4) $T_4 = (T_2)^2$ $\quad a = -3$

$\cancel{ar^3 = ar^2}$

$\cancel{r = 1}$

$r^3 = a^2 r^2$

$\Rightarrow a = r$

$\therefore r = -3$

$a_7 = (-3)(-3)^6$

$= \boxed{-2187}$

$81 \times 9 = 729$, $729 \times 3 = 2187$

(5) a) $128 = 2(\sqrt{2})^n$

$(\sqrt{2})^n = 64$

$n = 12$

$\therefore$ 13th term

b) $\frac{1}{19683} = \frac{1}{3}\left(\frac{1}{3}\right)^n$

$\Rightarrow \left(\frac{1}{3}\right)^n = \frac{1}{6561}$

$\Rightarrow n = 8$

$\therefore$ 9th term

$81 \times 81 = 6561$

a, b, c

AP: $2b = a+c$

GP: $b^2 = ac$

(6) $x^2 = \cancel{\ldots}$

$x = \sqrt{1} = \boxed{\pm 1}$

(7) In a GP of +ve terms, if any term is equal to the sum of the next 2 terms. Find the common ratio of the G.P.

Try!!

~~Let a, b, c be in GP.~~

~~$\therefore b^2 = ac$~~

~~Also, given, $a = b+c$~~

~~$\therefore b^2 = (b+c)c$~~

~~$b^2 = bc + c^2$~~

~~$b(b-c) =$~~

$\rightarrow a_n = a_{n+1} + a_{n+2}$

Also, $ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow r^{n-1} = r^n + r^{n+1}$

$\Rightarrow 1 = r + r^2$

$\Rightarrow r^2 + r - 1 = 0$

Roots $\rightarrow \dfrac{-1 \pm \sqrt{5}}{2}$

$\therefore \boxed{r = \dfrac{-1+\sqrt{5}}{2}}$ as GP is of +ve terms

Q. If $x, 2x+2, 3x+3$ are in GP. Find the 4th term.

$\rightarrow (2x+2)^2 = (3x+3)(x)$

$\Rightarrow 4x^2+4+8x = 3x^2+3x$

$\Rightarrow x^2+5x+4 = 0$

$\Rightarrow x(x+4)+1(x+4) = 0$

$\therefore x = -1$ or $x = -4$

Either GP $\rightarrow -1, 0, 0$

or

$\rightarrow \cancel{4, 10, 15}$

$\rightarrow -4, -6, -9$

$\therefore r = \dfrac{3}{2}$

$\therefore$ 4th term $= -4\left(\dfrac{3}{2}\right)^3$

$= -4 \times \dfrac{27}{8} = \boxed{-\dfrac{27}{2}}$

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Sn for GP

$\Rightarrow S_n = a + ar + ar^2 + ar^3 \ldots + ar^{n-1}$ — (1)

$r.S_n = ar + ar^2 + ar^3 \ldots ar^n$ — (2)

Subtracting (2) from (1),

$\Rightarrow$ ~~$S_n - r.S_n$~~

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

$\because r \neq 1$

If $r=1$, GP becomes AP.
$\therefore$ formula is not valid.

For $r=1$, GP $\rightarrow a, a, a, a \ldots$
$S_n = na$

Ans 12 ~~$a, ar, ar^2 \rightarrow 3$ terms~~

Let the 3 nos. be $\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = 1$

$a = 1$

$\frac{1}{r} + 1 + r = \frac{39}{10}$

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$\Rightarrow \frac{1+r+r^2}{r} = \frac{39}{10}$

$\Rightarrow 10r^2 + 10r + 10 = 39r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r-5) - 2(2r-5) = 0$

$\Rightarrow (5r-2)(2r-5) = 0$ $\quad r = \frac{2}{5}$ or $\frac{5}{2}$

$\therefore$ GP $\rightarrow \frac{5}{2}, 1, \frac{2}{5}$

GP $\rightarrow \frac{2}{5}, 1, \frac{5}{2}$

Ans 13 ~~$120 = 3(3)^{n-1}$~~

$120 = \frac{3(1-3^n)}{(1-3)}$

$120 = \cancel{\frac{3(1-3^n)}{-2}}$

$\Rightarrow \frac{240}{3} = (3^n - 1)$

$\Rightarrow 3^n = 81$

$\boxed{n = 4}$

$\therefore$ Sum of 4 terms.

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(14) → ~~Let first three term~~
~~Let the terms be $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$~~

~~Let the GP have first term a & common diff r.~~

~~$S_3 = \frac{a(1-r^n)}{(1-r)}$~~

~~$16(1-r) = a(1-r^3)$~~

~~$16 = a(1-r^2)$~~

(14) Let the terms be a, ar, ar^2; ar^3, ar^4, ar^5

$a + ar^2 + ar^3 = 16$
$a(1 + r^2 + r^3) = 16$ — (1)

$ar^3 + ar^4 + ar^5 = 128$
$ar^3(1 + r + r^2) = 128$ — (2)

Dividing (2) by (1),

$r^3 = \frac{128}{16} = 8$

$\boxed{r = 2}$

∴ $a(1 + 2 + 4) = 16$
$\boxed{a = \frac{16}{7}}$

$S_n = \frac{16}{7} \frac{(2^n - 1)}{1} = \boxed{\frac{16}{7}(2^n - 1)}$

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(16) $a + ar = -4$
$a(1+r) = -4$ — (1)

$ar^4 = 4ar^2$
$r^2 = 4$
$\boxed{r = \pm 2}$

For $r = +2$, $a = \frac{-4}{3}$ → GP: $\frac{-4}{3}, \frac{-8}{3} \ldots$

For $r = -2$, $a = 4$ → GP → 4, −8, 16...

× ——— × ——— × ——— ×

★ M/ex: 17/10/24

Q.15 Let no. of terms be 2n. Let r > 1.

GP → a, ar, ar^2 ... ar^{2n-1}.

→ $5(a + ar^2 + ar^4 \ldots ar^{2n-2}) = S_n$
↳ no. of terms n

→ $5\left[\frac{a(r^{2n}-1)}{r^2-1}\right] = \left(\frac{a(r^{2n}-1)}{r-1}\right)$

→ $5(r^{2n}-1)(r-1) = (r^{2n}-1)(r-1)(r+1)$

→ ~~$5(r^{2n}-1) = (r^{2n}-1)(r+1)$~~

→ ~~$r = 4$~~

$\Rightarrow 5(r^{2n}-1) = (r^{2n}-1)(r+1)$

$\Rightarrow \therefore r = 4$

Q.12 $\cancel{a+ar} + ar^2 + ar^3 = 56$

$a(1+r+r^2+r^3) = 56$

Let no. of terms be 'n'.

$\Rightarrow \cancel{ar^{n-1}+ar'}\ ar^{n-1} + ar^{n-2} + ar^{n-3} + ar^{n-4} = 112$

$\Rightarrow a(\cancel{r^{n+1}+r^{n-2}}$

$\Rightarrow ar^n\left(\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}\right) = 112$

Q.13 ~~Let no. of terms be 'n'.~~

$\Rightarrow \frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

$\Rightarrow \frac{a}{bx} = \frac{b}{cx} = \frac{c}{dx}$ (By componendo & dividendo)

$\Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ $\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ ①

$\Rightarrow ac = b^2$ ②, $bd = c^2$ ③, $ad = bc$ ④

From ①, ②, ③ & ④

$\therefore a, b, c, d$ are in G.P

Q.12 Let no. of terms be 'n'.

$\therefore a + ar + ar^2 + ar^3 = 56$

$\Rightarrow a(1+r+r^2+r^3) = 56$ — ①

$\therefore ar^{n-1} + ar^{n-2} + ar^{n-3} + ar^{n-4} = 112$

$\Rightarrow ar^n\left(\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}\right) = 112$

$\Rightarrow ar^n\left(\frac{r^3+r^2+r+1}{r^4}\right) = 112$

$\Rightarrow ar^n\left(\frac{56}{ar^4}\right) = 112$

$\Rightarrow r^{n-4} = 2$

★ Ex 9.3

Q.18 S_n for 8, 88, 888 ...

↳ It is neither GP nor AP.

~~(9-1), (90-2), 900-~~

~~$S_n = 8 + 88 + 888 \ldots + 8 - n$ places.~~

~~$S_{n+1} = 0 + 8 + 88 \ldots + 8 - n$ times $+ 8 - n$ times~~

$\rightarrow 8[1 + 11 + 111 + 1111 + \ldots]$

$\rightarrow \frac{8}{9}[9 + 99 + 999 + 9999 \ldots]$

$\rightarrow \frac{8}{9}[(10-1) + (100-1) + (1000-1) \ldots]$

$\rightarrow \frac{8}{9}[(10 + 100 + 1000 \ldots) - n]$

$\rightarrow \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

Q. $S_n = 0.6 + 0.66 + 0.666 \ldots$

$= 6[0.1 + 0.11 + 0.111 \ldots]$

$= \frac{6}{9}[0.9 + 0.99 + 0.999 \ldots]$

$= \frac{6}{9}[[1-0.1] + [1-0.01] + [1-0.001] \ldots]$

$= \frac{6}{9}\left[n - \left[0.1\left(\frac{1-0.1^n}{1-0.1}\right)\right]\right]$

$\rightarrow \frac{6}{9}\left[n - \frac{0.1}{0.9}(1 - 0.1^n)\right]$

$\rightarrow \frac{2}{3}\left[n - \frac{(1-0.1^n)}{9}\right] = \frac{2}{3}\left[n - \left(\frac{1-10^{-n}}{9}\right)\right]$

★ Result

$\rightarrow GP_1 = a + ar + ar^2 \ldots$ (FT = a, C.R = r)

$GP_2 = A + AR + AR^2 \ldots$ (FT = A, C.R = R)

$GP_{12} = Aa + AaRr + Aar^2R^2 \ldots$

New GP → $FT_{12} = FT_1 \times FT_2$

→ C. Ratio = $R_1 \times R_2$

✗ ——— ✗ ——— ✗ ——— ✗ ——— ✗

★ H/ex:

Q. GP(1): 2, 4, 8, 16, 32 ...

GP(2): 128, 32, 8, 2, $\frac{1}{2}$ ...

→ GP → 256, 128, 64, 32, 16 ...

$a = \frac{2 \times 128}{a_1 \times a_2}$, $r = \frac{1}{2}$ $= \frac{1}{4} \times 2$

$a_2 \times a_1$

$\Rightarrow \cancel{S_5 = 256\left(2(2^5-1)\right)}$

$\Rightarrow S_5 = 256 \times 2 ($

$\Rightarrow S_5 = 256\left(\frac{1-(\frac{1}{2})^5}{1-\frac{1}{2}}\right)$

$\Rightarrow S_5 = \frac{\cancel{256}^{8} \times 31 \times 2}{\cancel{32}} = 496$

$31 \times 16 = 186$, $31\,8$

Q. Find GP: $a_3 > a_1$ by 9

$a_2 > a_4$ by 18.

$\Rightarrow a_3 = a_1 + 9$ — (1)

$a_2 = a_4 + 18$ — (2)

$\Rightarrow ar^2 = a + 9$ — (1)

$ar^3 = ar + 9r$

$ar = ar^3 + 18$ — (2)

$\Rightarrow \cancel{a(1+r^2) = 9}$

$\Rightarrow ar(a - ar^2) = 18$ $\Rightarrow \cancel{ar} = \cancel{ar} + 9r + 18$

$\Rightarrow r(a - a - 9) = 18$ $\Rightarrow \boxed{r = -2}$

$\Rightarrow -9r = 18$ ~~GP not possible~~

$\Rightarrow a(4) = a + 9$

$\Rightarrow 3a = 9$

$\Rightarrow \boxed{a = 3}$

GP $\rightarrow$ 3, −6, 12, −24

Imp Result

$P^2 R^n = S^n$ → Sum of n terms

P → Product of n terms

R → sum of reciprocals of n terms

Q. F.T of GP = a

nth term = b

P = product of n terms

PT: $P^2 = (ab)^n$

Soln: Let GP be $a, ar, ar^2 \ldots b\,(ar^{n-1})$

$P = a^n (r \cdot r^2 \cdot r^3 \ldots r^{n-1})$

$= a^n r^{(1+2+3+4+5 \ldots n-1)}$

$= a^n r^{\left(\frac{n-1}{2}\right)[n]}$

$\Rightarrow \cancel{P^2 = a^{2n} r^{\frac{n-1}{2}(n) \cdot 2}}$

$\Rightarrow P^2 = \left(a^n \cdot r^{n\left(\frac{n-1}{2}\right)}\right)\left(a^n \cdot r^{n\left(\frac{n-1}{2}\right)}\right)$

$\Rightarrow P^2 = (a^n)\left(a^n \cdot r^{n(n-1)}\right)$

$\Rightarrow P^2 = (a^n)(ar^{n-1})^n$

$\Rightarrow \boxed{P^2 = a^n b^n}$ Hence Proved $[b = ar^{n-1}]$

Result

Q. a, b, c, d are in G.P.

$b^2 = ac \quad c^2 = bd \quad ad = bc$

$a = a, \; b = ar, \; c = ar^2, \; d = ar^3$

$(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$

$\Rightarrow$ ~~LHS = $(a^2+c^2+ac)(ac+c^2+bd)$~~

$\Rightarrow LHS = (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$

$= r^2(a^2 + a^2r^2 + a^2r^4)^2$

$= a^4r^2(1+r^2+r^4)^2$

$\Rightarrow RHS = (a^2r + a^2r^3 + a^2r^5)^2$

$= a^4r^2(1+r^2+r^4)^2$

$\therefore$ LHS = RHS

Try! $a_k = 5^k$

$\Rightarrow S_n = \frac{5(5^n-1)}{4}$

Try! $S_n = 11, 102, 1003, 10004 \ldots$

$\Rightarrow (10+100+1000\ldots) + (1+2+3\ldots)$

$\Rightarrow \frac{10(10^n-1)}{9} + \frac{n}{2}[2+(n-1)]$

$\Rightarrow \boxed{\frac{10(10^n-1)}{9} + \frac{n(n+1)}{2}}$

★ Geometric Mean — A.M ≥ G.M

↳ Let a & b be two +ve nos.

Then the geometric mean of a & b is defined as:

$GM = \sqrt{ab}$

→ How many GM b/w A & B are possible? → Infinite

Eg. $a = 9, \; b = 576$ — 5 geometric means

$9, a_1, a_2, a_3, a_4, a_5, 576$

$a = 9$

$ar^{6} = 576$

$r^{6} = \frac{576}{9} = 64$ — $r = \pm 2$

$$\begin{array}{l} \quad\quad\quad g_1 \quad g_2 \quad g_3 \quad g_4 \quad g_5 \\ GP \rightarrow 9, 18, 36, 72, 144, 288, 576 \end{array}$$

Q. $\underset{a}{3}, \underset{b}{81} \rightarrow$ 2 nos.

↳ $3, 9, 27, 81$

Q $\underline{a, b} \rightarrow GM = \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

↳ $\sqrt{ab} = \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow ab = \dfrac{(a^{n+1}+b^{n+1})^2}{(a^n+b^n)^2}$

$\Rightarrow ab(a^n+b^n)^2 = (a^{n+1}+b^{n+1})^2$

$\Rightarrow ab(a^{2n}+b^{2n}+2(ab)^n) = a^{2n+2}+b^{2n+2}+2(ab)^{n+1}$

$\Rightarrow a^{2n+1}b + b^{2n+1}a = a^{2n+2}+b^{2n+2} \quad \Rightarrow a^{2n+1}(b-a) = b^{2n+1}(b-a)$

↳ $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$\Rightarrow a^{n+1}+b^{n+1} = \sqrt{a}\sqrt{b}(a^n+b^n)$

$\Rightarrow a^{n+1}+b^{n+1} = \sqrt{a}\,a^n\sqrt{b} + \sqrt{b}\,b^n\sqrt{a}$

$\Rightarrow a^{n+1} - \sqrt{a}\,a^n\sqrt{b} = \sqrt{b}\,b^n\sqrt{a} - b^{n+1}$

$\Rightarrow \sqrt{a}\,a^n(\sqrt{a}-\sqrt{b}) = \sqrt{b}\,b^n(\sqrt{a}-\sqrt{b})$

$\Rightarrow \dfrac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}} = \left(\dfrac{a}{b}\right)^0$

$\Rightarrow \therefore \boxed{n = -\dfrac{1}{2}}$

★ **Note:** $a>0, b>0$

AM: $\dfrac{a+b}{2}$ GM: $\sqrt{ab}$

Q. If AM = 5, GM = 4 find a & b.

↳ $\sqrt{ab} = 4$

$\Rightarrow ab = 16$ — (1)

↳ $\dfrac{a+b}{2} = 5$

$\Rightarrow a+b = 10$ — (2)

$\Rightarrow \dfrac{16}{b} + b = 10$

$\Rightarrow 16 + b^2 - 10b = 0$

$\Rightarrow b^2 - 8b - 2b + 16 = 0$

$\Rightarrow b(b-8) - 2(b-8) = 0$

$\therefore b = 2$ & $b = 8$ If $b = 2$, $a = 8$; $b = 8$, $a = 2$

★ Direct formula :

$$a = A + \sqrt{A^2 - G^2}$$

$$b = A - \sqrt{A^2 - G^2}$$

★ Derivation

↳ A = AM of a & b
G = GM of a & b

$A = \frac{a+b}{2} \Rightarrow a+b = 2A$

$G = \sqrt{ab} \Rightarrow G^2 = ab$

Assuming a & b are roots,

Eqn → $\boxed{x^2 - 2A + G^2 = 0}$

Roots = $\frac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$

$= \boxed{A \pm \sqrt{A^2 - G^2}}$

★ Reln. b/w AM & GM

↳ AM ≥ GM

$\Rightarrow (\sqrt{a} - \sqrt{b})^2 \geq 0$

$\Rightarrow \boxed{\frac{a+b}{2} \geq \sqrt{ab}}$

Q Try!! $f(x) = x + \frac{1}{x}$ Range = ?

↳ $\frac{x + \frac{1}{x}}{2} \geq \sqrt{x \cdot \frac{1}{x}}$

$\Rightarrow x + \frac{1}{x} \geq 2$ $R_f = [2, \infty)$

Xtra!! Imp If the lengths of 3 unequal sides of a rectangular sides are in GP, volume of the block is 216 cm³. TSA = 252 cm². find the ~~sides~~ longest side.

↳ $\frac{a}{r}, a, ar$ are sides. (l, h, b)

$a^3 = 216$

$\underline{a = 6}$

↳ $252 = 2(lh + bh + lb)$

$\Rightarrow 126 = \left(\frac{a^2}{r} + a^2 r + a^2\right)$

$\Rightarrow 126 = 36\left(\frac{1}{r} + r + 1\right)$

$\Rightarrow \frac{7}{2} = \frac{126}{36} = \frac{r + r^2 + 1}{r}$

$\Rightarrow 7r = 2r + 2r^2 + 2$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow 2r(r-2) - 1(r-2) = 0$

$\therefore r = 2$ or $\frac{1}{2}$

Longest side = 12 cm

Q. Find min value of $f(x) = 4^x + 4^{1-x}$

Q. If $x, 2y, 3z$ are in AP, where distinct nos. x, y, z are in GP. Find the common ratio.

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$\Rightarrow f(x) = 4^x + 4^{1-x}$

$\Rightarrow f(x) = 4^x + \frac{4}{4^x}$

$\Rightarrow \left(4^x + \frac{4}{4^x}\right) \geq 2\sqrt{4^x \cdot \frac{4}{4^x}}$

$\Rightarrow 4^x + \frac{4}{4^x} \geq 4$

$\therefore f(x) \geq 4$

$\Rightarrow x, y, z \rightarrow GP$ $\quad y^2 = zx$

$\Rightarrow \frac{y}{z} = \frac{x}{y}$

$\Rightarrow \frac{z}{y} = \frac{y}{x}$

$\Rightarrow x, y, 3z \rightarrow AP.$

$\Rightarrow 2y = \frac{x+3z}{2}$

$\Rightarrow \cancel{4y = x + 3z}$

$\Rightarrow 4 = \frac{x}{y} + \frac{3z}{y}$

$\Rightarrow 4 = \frac{y}{z} + \frac{3z}{y}$

$\Rightarrow 4yz = y^2 + 3z^2$

$\Rightarrow y^2 + 3z^2 + 4yz = 0$

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Q. Let the 2 nos. be a & b.

$a + b = 6\sqrt{ab}$

$\Rightarrow \frac{a+b}{2} = 3\sqrt{ab}$

$\Rightarrow A.M = 3.GM$

$\rightarrow$ We know that, if $A \rightarrow A.M$, $G \rightarrow G.M$:

$a = A + \sqrt{A^2 - G^2}$

$b = A - \sqrt{A^2 - G^2}$

$\frac{a}{b} = \frac{3GM + \sqrt{9G^2 - G^2}}{3GM - \sqrt{9G^2 - G^2}}$

$\boxed{\frac{a}{b} = \frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}}$

Q. Given $\rightarrow A = 8$, $G = 5$

Let the two roots be a & b.

$\therefore A = \frac{a+b}{2}$

$\Rightarrow a + b = 2A = 16$

$G^2 = ab$

$ab = 25$

$\therefore$ Eqn $\Rightarrow x^2 - 16x + 25 = 0$

Q. 4, 16, 64 ...

$a = 4$, $r = 4$, $n = 8$

$\rightarrow a_n = ar^{n-1}$

$\Rightarrow a_8 = 4 \times (4)^7$

$= 64 \times 64 \times 16 \times$

$=$

64 × 64 = 4096; 4096 × 16 = 65536; 65536 × 50 = 3276800

Q. 4, 16, 64 ...

$a = 4$ $r = 4$ $n = 8$

$\Rightarrow \cancel{a_8 = ar^{8-1}}$

$\cancel{= ar^7}$

$= 4^8 =$

$\Rightarrow S_8 = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{4(4^8 - 1)}{3} = \dfrac{4(\cancel{65535}^{21845})}{\cancel{3}}$

$= 87380$

1024 × 64 = 4096

$\therefore$ Total cost $= 87380 \times 50 = 4369000$ paise $\rightarrow$ ₹43690

Q4 15625; $\dfrac{15625 \times 8^4}{10^4} = 12500$

$\Rightarrow$ 15625, 12500 ...

$a = 15625$, $r = \dfrac{84}{105}$

$a_5 = 15625 \times \dfrac{4}{5} \times \dfrac{64}{125} = \boxed{6400}$

$\Rightarrow a_6 = \dfrac{15625 \times 64 \times 16}{125 \times 625} = \boxed{5120}$

64 × 80 = 5120

Try!!

Q. If $S_n = 3 - \dfrac{3^{n+1}}{4^{2n}}$ for a G.P., find r.

$\Rightarrow S_n = \dfrac{a(r^n - 1)}{r - 1}$

Also, $S_n = \dfrac{3(1 - 3^n)}{2^{4n}}$

$\therefore$

$\Rightarrow S_n = 3 - \dfrac{3^{n+1}}{4^{2n}}$

$S_1 = 3 - \dfrac{9}{16} = \dfrac{39}{16}$

$S_2 = 3 - \dfrac{27}{256} = \dfrac{1041}{256}$

$S_2 - S_1 = \dfrac{1041}{256} - \dfrac{39}{16} = \dfrac{1041 - 624}{256} = \dfrac{417}{256}$

256 × 3 = 768; 768 − 27 = 741; 39 × 16 = 624

$r = \frac{a_2}{a_1}$

$= \frac{3}{256} \times \frac{16}{3} = \boxed{\frac{3}{16}}$

Imp (2) If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$, PT. x, y, z are in AP.

→ a, b, c → GP → $\frac{b}{a} = \frac{c}{b}$

→ Given → $a^{1/x} = b^{1/y} = c^{1/z} = k$

~~$a^{1/x} = (\sqrt{ac})^{1/y} = c^{1/z}$~~

~~$a^{1/x} = (\sqrt{a})^{1/y} (\sqrt{c})''$~~

∴ $a^{1/x} = k$, $b^{1/y} = k$, $c^{1/z} = k$

$a = k^x$, $b = k^y$, $c = k^z$

Also, $b^2 = ac$,

∴ $k^{2y} = k^x k^z$

⇒ $x + z = 2y$ ∴ x, y, z are in AP

(3) If a, b, c are in AP as well as in GP, PT. $a = b = c$.

→ Given a, b, c are in GP.

∴ $b^2 = ac$ — (1) ⇒ $\frac{b}{a} = \frac{c}{b}$

Also, a, b, c are in AP.

∴ $2b = a + c$ — (2)

~~$\frac{2b}{a} = 1 + \frac{c}{a}$~~ → $\left(\frac{a+c}{2}\right)^2 = ac$

~~$\frac{2c}{b} = 1 + \frac{c}{a}$~~ → $(a+c)^2 = 4ac$

→ $a^2 + c^2 + 2ac = 4ac$

→ $a^2 + c^2 - 2ac = 0$

→ $(a-c)^2 = 0$

⇒ $a = c$ — (3)

from (3) & (2), $2b = 2a$

∴ $b = a$ — (4)

from (3) & (4), $a = b = c$

(4) First 3 of 4 nos are in GP and last 3 are in AP, with common diff = 6. If first & 4th are equal, find 1st no.

→ ~~Let nos. be a, a+d, a+2d, a+3d.~~

~~⇒ a = a+3d~~

$\hookrightarrow$ Let the nos. be x, a, b, x.

$a+6 \quad a+12$

$a^2 = bx$ — (1) $\underline{b = a+6}$

$\underline{2b = a+x}$

$2(a+6) = a+x$ $a+12, a, a+6, a+12$

$2a+24 = a+x$

$a+24 = x$ — (2) $a^2 = (\ x\ )$

$a^2 = (a+6)(a+24)$

$a^2 = a^2 + 72 + 6a + 12a$

$\Rightarrow 18a = 72$

$\Rightarrow \underline{a = 4}$ $\underline{b = 10}$

$\Rightarrow a^2 = a^2 + 30a + 144$

$\Rightarrow a^2 = (a+6)(a+24)$

$\Rightarrow a^2 =$

$\Rightarrow a = -4, \quad \underline{x = 8}$

* $\underline{\text{Infinite GP \& its Sum}}$

$\hookrightarrow$ GP = $a, ar, ar^2 \ldots$

Eg (1) (Increasing series): $2, 4, 6, 8 \ldots$

$S_n = 2+4+6+8 \ldots = \infty$ (very large value)

(2) (Decreasing series): $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

$\hookrightarrow S_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$

$a = 1, \quad r = \frac{1}{2}$

$\therefore S_n = \frac{a(1-r^n)}{1-r} \rightarrow$ as $r \in (0,1)$, r^n is a very small value ≈ 0

$\boxed{S_n = \frac{a}{1-r}} \rightarrow$ when Infinite GP has $-1 < r < 1$.

$\therefore S_n = \frac{1}{1-\frac{1}{2}} = \underline{\underline{2}}$

* $|r| < 1$
Dist. of r from 0 is less than 1

Q. $S_n = 1 + \frac{1}{3} + \frac{1}{9} \ldots$

$\hookrightarrow S_n = \frac{3}{2}$

$\frac{1}{1-\frac{1}{3}}$

Q. $5, \frac{20}{7}, \frac{80}{49}$

$\rightarrow S_n = \frac{5}{1-\frac{4}{7}}$

$= \boxed{\frac{35}{3}}$

Q. $-\frac{3}{4}, \frac{3}{16}, -\frac{3}{64}$.

$\rightarrow S_n = \frac{-\frac{3}{4}}{1+\frac{1}{4}}$

$= \boxed{-\frac{3}{5}}$

Q. PT. $3^{1/2} \times 3^{1/4} \ldots = 3$

$\rightarrow LHS = 3^{1/2+1/4+1/8} \ldots$

$S_n = \frac{\frac{1}{2}}{1-\frac{1}{2}}$

$S_n = 1$

$LHS = 3^1 = 3$ Hence Proved

Q. $x = 1+a+a^2 \ldots$

$y = 1+b+b^2 \ldots$

PT. $1 + ab + a^2b^2 \ldots = \frac{xy}{x+y-1}$

$\rightarrow$ GP, $\Rightarrow x = 1+a+a^2 \ldots$

$y = 1+b+b^2 \ldots$

$\therefore x \cdot y = 1 + ab + a^2b^2 \ldots$

$\rightarrow S_x = \frac{1}{1-a} \qquad S_y = \frac{1}{1-b}$

$\therefore$ In RHS $\Rightarrow \frac{\left(\frac{1}{1-a}\right)\left(\frac{1}{1-b}\right)}{\left(\frac{1}{1-a}\right)+\left(\frac{1}{1-b}\right)-1}$

$\cancel{\Rightarrow RHS = \frac{\frac{1}{1-b-a+ab}}{\frac{a-ab+b-ab-(1-b-a+ab)}{1-b-a+ab}}}$

$\rightarrow RHS = \frac{\frac{1}{(1-a)(1-b)}}{\frac{1-b+1-a-(1-a)(1-b)}{(1-a)(1-b)}} = \frac{1}{1-b+1-a-(1-a-b+ab)}$

$= \frac{1}{1-b+1-a-1+a+b-ab}$

$\cancel{\rightarrow RHS = \frac{1}{2-ab-1-b-a+ab} = \frac{1}{1-2b-2a}}$

$= \frac{1}{1-ab}$

$LHS = \frac{1}{1-ab}$

$\therefore LHS = RHS$