Ch-9 → Fluids

* Pascal's Law

↳ " The pressure is same everywhere in the liquid, if gravity is ignored. "

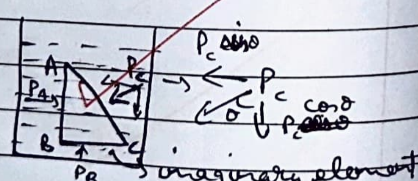
↳ Change of pressure applied to incompressible liquid is transmitted equally in all directions.
(undiminished)

∴ Liquids don't compress when pressure is applied. Thus it will bear no pressure itself & will distribute that pressure equally & undiminished throughout the liquid body.
↳ (no loss)

Only if gravity is ignored.

- Applications : ① Automobile Industry
  ↳ Hydraulic brakes on automobiles, jack for lifting vehicles
  ② Hydraulic Engineering
  ↳ Suspension, Cranes, Trucks use hydraulic press
  ③ Medical Science
  ↳ Syringes, IV dripets.

★ Proof !!



↳ Let an imaginary triangle having sides ABC be present in a container of water.

Let F_A, F_B, F_c → Forces acting on A, B, C resp.
A_a, A_b, A_c → Areas ...

→ Now, resolving components, we have:

$F_A = F_c \sin\theta$ — ① $A_a = A_c \sin\theta$ — ③
$F_B = F_c \cos\theta$ — ② $A_b = A_c \cos\theta$ — ④

→ $\frac{①}{③}$ ⟹ $\frac{F_A}{A_a} = \frac{F_c \cancel{\sin\theta}}{A_c \cancel{\sin\theta}}$

⟹ $P_a = P_c$

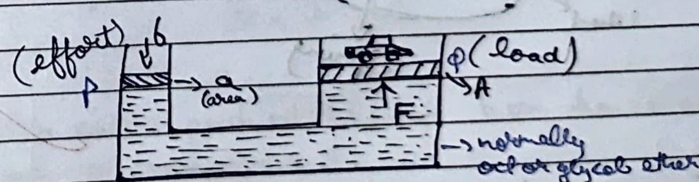
↳ $\frac{②}{④}$ → $\frac{F_B}{A_b} = \frac{F_c \cancel{\cos\theta}}{A_c \cancel{\cos\theta}}$

⟹ $P_b = P_c$

∴ $P_a = P_b = P_c$

∴ Pressure is equal throughout

* Hydraulic Lift (Pascal's Law Application)



↳ Pressure at P $= \frac{b}{a}$
(P₁)

↳ Pressure at Φ = F

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↳ Force on larger piston $\Rightarrow F = P \times A$

$$\Rightarrow \boxed{F = f \times \frac{A}{a}}$$

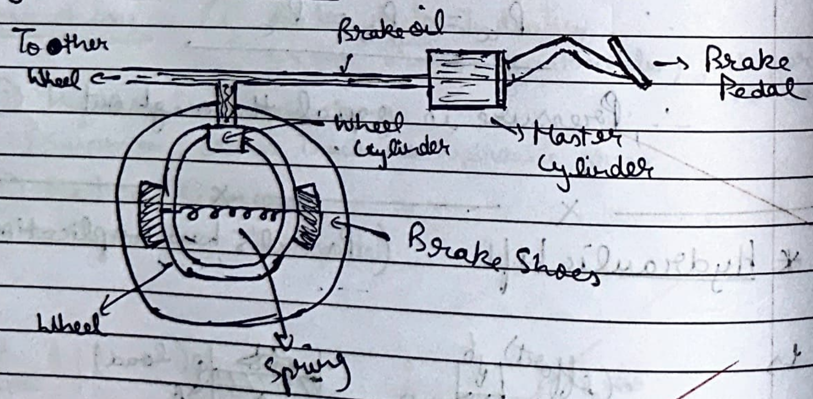
From eqn.

↳ We know that $A >> a$ ∴ Value of F is very much greater than f.

→ If we apply small amount of force at effort end, ~~we can turn~~ the Pascal's law causes the force to increase in magnitude, which makes it easier for us to lift heavier things.

∴, With lesser effort, heavier things can be lifted.

(2) Hydraulic Brakes



Q. In a car lift a force F_1 exert on a small piston of $r = 5$ cm. Pressure transmitted to the 2nd piston of 15 cm. Mass of car = 1350 kg. Calculate F_1,

↳ ~~1350 N~~ $P_1 = \frac{F_1}{25\pi}$

$P_2 = \frac{\cancel{13500}^{60}}{\cancel{225}\pi}$

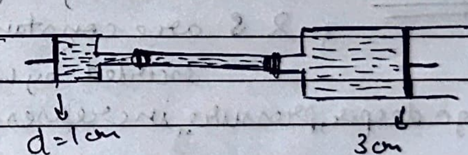
$P_2 = \frac{60}{\pi}$

↳ $\frac{60}{\pi} = \frac{F_1}{25\pi}$

$\Rightarrow F_1 = 1500\,N$

$P = \frac{1500 \times 7}{25 \times 22 \times 10^{-4}} = 1.9 \times 10^5$ Pa (N/m²)

Q.



force on larger piston when 10 N on smaller.

↳ $F = 10 \times \frac{\pi \times 1.5 \times 1.5}{\pi \times 0.5 \times 0.5} = 90\,N$

Q. If smaller piston pushed through 6 cm how much does larger piston move out.

↳ $\pi R^2 h = \pi r^2 h$

$\Rightarrow 9h = 1 \times 6 \times 10^{-2}$

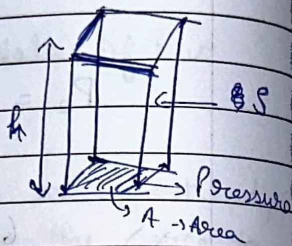
$\Rightarrow h = \frac{6}{900} = \frac{1}{150} = \frac{0.066}{10}$

$= \frac{2.6}{...} \times 10^{-2} = \boxed{0.67 \times 10^{-2}\,m} \Rightarrow \boxed{0.67\,cm}$

* Pressure due to liquid column

↳ As you go deeper in a body of water, the water exerts an increasing pressure on your body.

↳ $P = \dfrac{\text{Force}}{\text{Area}}$ → due to weight of water column



~~= mg~~

$W = mg$
$= \rho V g$ → (Area x height = Volume)
$= \rho A h g$

$\boxed{P = \rho g h}$ → Hydrostatic Pressure

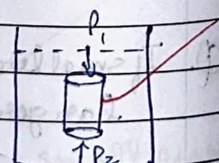
*Note: $P \propto h$. Pressure only depends on h as g & ρ are constant.

∴ As we go deeper, pressure excerted by water increases.

This is also why we need swimsuits to go into deep water.
Also why ~~we need~~ mountaineers experience nosebleeds.

* Effect of Gravity on $P = \rho g h$

↳ Let an open container of water be taken.



Let us imagine imaginary cylindrical body inside water body.

∴ By Pascal's law, pressure same everywhere (effect of gravity ignored)
But $P_2 > P_1$, as it is deeper.

Not imp but padh lena

*↳ Force on cylindrical body will be from all sides.
Here, volumetric strain is found. (Bulk Modulus)

Now, $P_1 = \dfrac{F_1}{A}$ $\qquad P_2 = \dfrac{F_2}{A}$

$\Rightarrow F_1 = P_1 A$ —(1) (downwards), $\qquad F_2 = P_2 A$ —(2) (upwards)

→ $W = Ah\rho g$ —(3) (Downwards) (Weight of water)

Assuming equilibrium,

⇒ $F_1 + W = F_2$
(Dward Force) = (Upward Force)

⇒ $F_2 - F_1 = W$

⇒ $P_2 A - P_1 A = A h \rho g$

⇒ $\boxed{P_2 - P_1 = h\rho g}$

→ ~~$\Delta P = h\rho g$~~ face

Now, when cylindrical body is brought to top,
$P_2 = P$
$P_1 = P_a$ (Atmospheric Pressure)

$\boxed{P - P_a = h\rho g}$ → Gauge Pressure

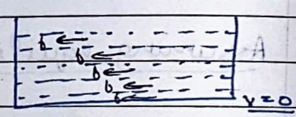
$\boxed{P = P_a + h\rho g}$ → Absolute Pressure differentiation w.r.t x → gradient

↳ Pascal's Law → "Pressure is same everywhere inside the liquid, but only at same depth."
in Reality

Gauge Pressure only in closed system.

* Viscosity

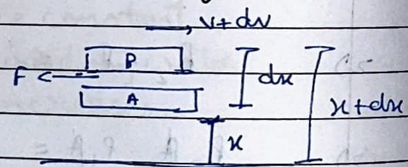
↳ Internal friction between every two layers of liquid.



↳ Bottommost layer is in direct contact with ground, so it is at rest.

↳ As we go higher up in the water, velocity of flow increases.

↳ Viscosity depends on area!



↳ $F \propto A$

$F \propto \frac{dv}{dx}$ → velocity gradient
↳ higher gradient more difference in velocities

$F \propto A$

↳ $\boxed{F = -\eta A \frac{dv}{dx}}$

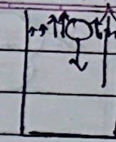
η → coefficient of viscosity.

−ve sign as it always opposes motion

SI unit :- Pa s (Pascal-second)

CGS → poise

Layers close to the body travel with same velocity.
But distant layers travel with less velocity

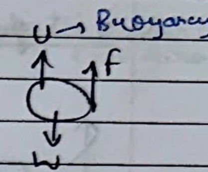


* Stokes' Law

↳ $\boxed{F = 6\pi\eta r v}$

Depends on → viscosity, radius of body, velocity.

◦ Terminal velocity



↳ ~~Max safe velocity~~
↳ The max possible velocity with which an object falls in liquid.

↳ Assuming equilibrium,

↳ $W = mg = \rho V g$ (ρ → density of solid)

$= \frac{4}{3}\pi r^3 \rho g$

↳ $U = mg$ (σ = density of liquid)

$= \sigma V g$

$= \frac{4}{3}\pi R^3 \sigma g$

↳ $W = U + F$

⇒ $\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi R^3 \sigma g + 6\pi\eta r v$

⇒ $\frac{4}{3}\pi R^{3} g(\rho - \sigma) = 6\pi\eta R v$

⇒ $R^2 g(\rho - \sigma) = \frac{9}{2}\eta v$

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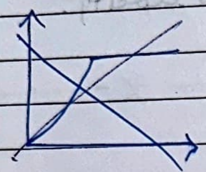
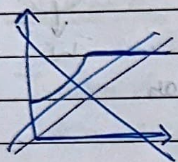
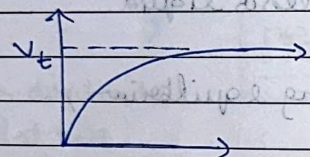
$$\therefore v = \frac{2}{9}\frac{R^2 g(\rho - \sigma)}{\eta}$$

Terminal Velocity

↳ v-t graph







* Flow

○ Streamline

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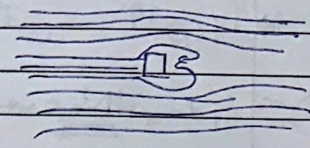
○ Turbulent Flow



$$v_c = \frac{K\eta}{\rho D}$$

Critical Velocity

K - constant (Reynold's No.)
η - viscosity
ρ - density
D - diameter of cross section

↳ Critical Velocity (v_c)

→ It the max velocity of liquid upto which it undergoes laminar flow.
If velocity of water is greater than critical velocity it undergoes turbulent flow.

x ——— x ——— x ——— x ——— x

☆ Bernoulli Principle

↳ "Sum of pressure energy, PE & KE per unit volume for non-viscous, incompressible liquid flow remains constant."

↳ $\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}}$

↳ If height of tube is kept same, h = 0.

$$\therefore \boxed{P + \frac{1}{2}\rho v^2 = \text{const}}$$

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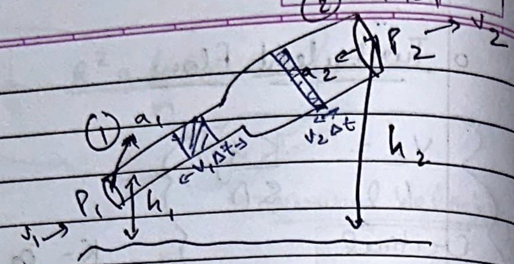
→ Dig →



→ Amt. of mass of water entering = Amt. of mass of water leaving

$$m_1 = m_2$$

$$\Rightarrow \rho V_1 = \rho V_2$$

$$\Rightarrow \cancel{\rho} a_1 v_1 \cancel{\Delta t} = \cancel{\rho} a_2 v_2 \cancel{\Delta t}$$

$$\Rightarrow \boxed{a_1 v_1 = a_2 v_2} \quad \text{Eqn of continuity}$$

↳ For any area, area × velocity = constant

↳ velocity $\propto \frac{1}{a}$

* Change in kinetic energy $= KE_{②} - KE_{①}$

$$= \frac{1}{2} m (v_2^2 - v_1^2) \rightarrow ①$$

$$= \frac{1}{2} a_1 v_1 \Delta t \rho (v_2^2 - v_1^2) - ①$$

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* Change in potential energy $= PE_{②} - PE_{①}$

$$= mg(h_2 - h_1) \Rightarrow ②$$

$$= a_1 v_1 \Delta t \rho g (h_2 - h_1) - ②$$

* Work done $= W_1 - W_2$

$$= F_1 v_1 \Delta t - F_2 v_2 \Delta t$$

$$= P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t$$

$$= a_1 v_1 \Delta t (P_1 - P_2) \quad [\because a_1 v_1 = a_2 v_2]$$

Also by WET,

$$W_{net} = \text{Change in energy}$$

$$\Rightarrow \cancel{a_1 v_1 \Delta t} (P_1 - P_2) = \frac{1}{2} \cancel{a_1 v_1 \Delta t} \rho (v_2^2 - v_1^2) + \cancel{a_1 v_1 \Delta t} \rho g (h_2 - h_1)$$

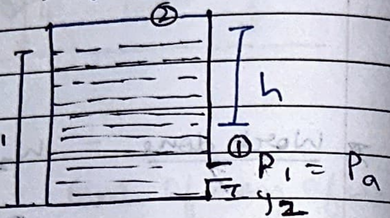
$$\Rightarrow P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (h_2 - h_1)$$

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\therefore P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

★ Applications

∘ Torricelli's Theorem (Velocity of Efflux)



At ① $P_1 = P_a$ (Pressure)
v_1 = velocity of efflux
a_1 = area of cross section

At ② $P_2 = P$
v_2 = velocity of flow
a_2 = area of cross section

By Bernoulli's theorem,

$$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

By eqn of continuity, $a_1 v_1 = a_2 v_2$

As $a_2 >> a_1$,
$v_2 \approx 0$. $\Rightarrow \frac{1}{2}\rho v_2^2 = 0$

$$\Rightarrow \frac{1}{2}\rho v_1^2 = P_2 - P_1 + \rho g (y_2 - y_1)$$

$$\Rightarrow \frac{1}{2}\rho v_1^2 = P_2 - P_1 + \rho g h$$ [∵ $y_2 - y_1 = h$]
↳ Height dist b/w hole & water level.

$$\Rightarrow v_1^2 = \frac{2}{\rho}(P_2 - P_1) + 2gh$$

$$\Rightarrow v_1 = \sqrt{2gh + \frac{2}{\rho}(P_2 - P_1)}$$

$$\Rightarrow \boxed{v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}}$$ → velocity of efflux

★ Cases:

① $P = P_a$ (Open container)

∴ $v = \sqrt{2gh}$ → Torricelli's Theorem

② $P >> P_a$ (Rocket Propulsion)

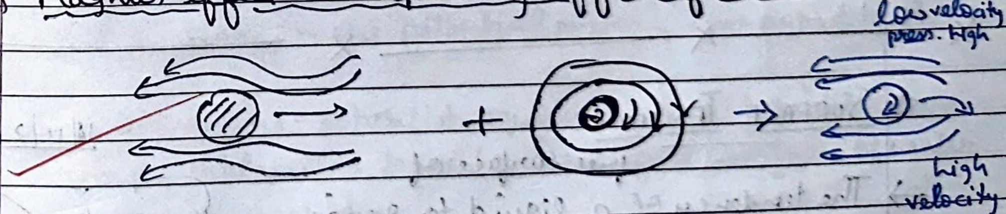
Let us ignore $2gh$

∴ $v = \sqrt{\frac{2(P - P_a)}{\rho}}$

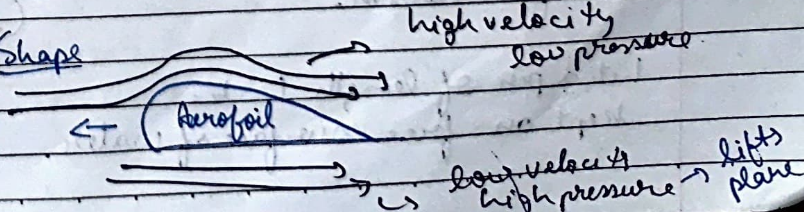
X —— X —— X —— X

★ Simple Applications

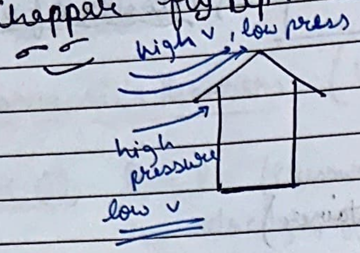
(1) Magnus effect – spinning effect of ball



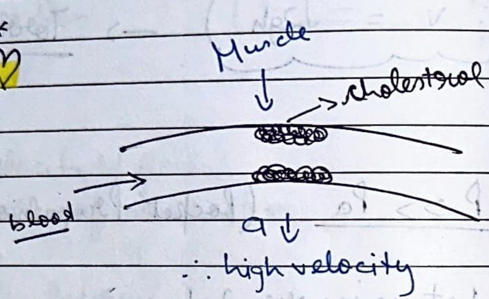
(2) Aerofoil Shape



(3) Chappar fly up.



♡* *
(4) **Heart Attack** ♡



↳ Clot due to cholestrol.
↳ Area decreases so high velocity
∴ pressure decreases (low blood pressure)

Also, muscle pressure increases. So velocity again increases

This keeps repeating till 100% block.

X ——— X ——— X ——— X

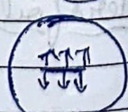
☆ SURFACE TENSION

11/11/24

→ The tendency of free surface of a liquid to regain minimum surface area is known as surface tension.

~~free surface~~ — surface of liquid in direct contact with air

Let a pin of length L be kept on free surface of water



Liquid apna area कम करना चाहता है हमेशा ! object का बढ़ाना चाहता है।

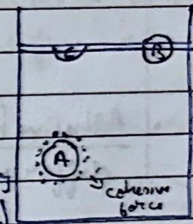
The molecules of water will exert a force on the pin, tangential to free surface of the object.

∴ This force per unit length is known as surface tension

$$\text{Surface Tension } (\sigma) = \frac{F}{l} \qquad \text{Unit} = N/m$$

☆ Free surface of liquid behaves as stretched elastic membrane.

o Molecular Theory Of Surface Tension



↳ A is in equilibrium. (Because same cohesive force is applied all over)

B is not in equilibrium. (75% is experiencing cohesive force, other is in contact with air)

C is not in equilibrium (50% is experiencing cohesive, rest is in air).

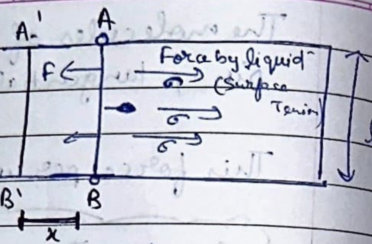
↳ ∴ B & C have more potential energy compared to A.

In order to re-establish equilibrium; liquid reduces ~~area~~ so molecules have to leave free surface of water and move inwards.

o SURFACE ENERGY

→ →

↳ Let us take a rectangular frame of length l with a movable wire.



Dip the apparatus in soap soln. & move the wire by 'x' m.

∴ Surface energy is developed as work is done against surface tension.

∴ Surface Energy = Work done to increase surface area.

$$S.E = \boxed{\frac{W}{\Delta A}} \quad \frac{\text{Workdone}}{\text{Increasing Area}}$$

- **Adhesive Forces**: Force of attraction b/w particles of different ~~mats~~ substances.
- **Cohesive forces**: Force of attraction b/w particles of same substance.

★ **Derivation**

↳ $W = F.d$
$= F.x$ ---(i)

$\sigma = \frac{F}{l}$

$\therefore F = 2\sigma.l$ (because film has $2\sigma l$)

Area & Work find करने में free surface se multiply karna!

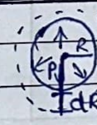
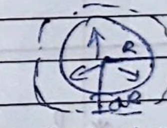
$\therefore W = 2\sigma l x$

$\Delta Area = 2x.l$ (length × breadth)
↳ (2 free surfaces)

$$\therefore \text{Surface energy} = \frac{W}{\Delta A} = \boxed{\sigma}$$

↳ ∴ value of surface energy = Surface tension acting on liquid.

X —— X —— X —— X —— X

★ Excess Pressure in Liquid Drop (1 free surface)	Excess Pressure in Liquid Bubble (2 free surface)
↳ Let us take a liquid drop of radius R. 	Let us take a bubble of radius R 
∴ Due to excess pressure radius of drop increases by 'dR'	~~Due~~ ∴ Due to excess pressure radius of bubble increases by 'dR'.
→ Initial Surface Area $= 4\pi R^2$ —①	→ Initial SA $= 4\pi R^2$ —①
Final Surface Area $= 4\pi(R+dR)^2$ $= 4\pi(R^2+dR^2+2RdR)$	→ Final SA $= 4\pi(R^2+dR^2+2RdR)$
→ FSA $= 4\pi R^2 + 4\pi dR^2 + 8RdR$ $= 4\pi R^2 + 8\pi RdR$ —② (dR^2 very small can be ignored)	∴ FSA $= 4\pi R^2 + 8\pi RdR$ —②
∴ Change in Area (2-1) $= 8\pi RdR$	∴ Change in Area (2-1) $= 2\times 8\pi RdR$ (As ~~drop~~ bubble has 2 free surfaces)

Workdone = Surface Tension × ΔA
$= 8\pi R dR . \sigma$ — ③

Workdone = F.xd
$= P \times A \times d$
$= P \times 4\pi R^2 \times dR$ — ①

As both work done are ~~equal~~ same.
③ = ④.

$\therefore P \times \cancel{4}\pi R^{\cancel{3}\,2} \times \cancel{dR} = \cancel{8}^{2}\cancel{\pi}\cancel{R}\cancel{dR}.\sigma$

$\Rightarrow \boxed{P = \frac{2\sigma}{R}}$

↳ Excess pressure in liquid drop

$W = \sigma \times \Delta A$
$= 16\pi R dR . \sigma$ — ③

$W = F \times d$
$= P \times A \times d$
$= P \times 4\pi R^2 \times dR$ — ④

As both work done are same
③ = ④

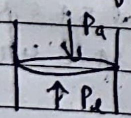
$\therefore P \times 4\pi R^2 \times dR = 16\pi R dR \sigma$

$\Rightarrow \boxed{P = \frac{4\sigma}{R}}$

↳ Excess pressure in liquid bubble.

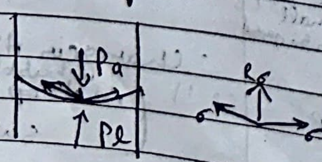
* Pressure across the curved surface

(1) Plane Surface (Meniscus)



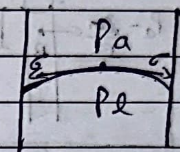
$\Rightarrow \boxed{P_L = P_a}$

(2) Concave Surface



$\Rightarrow \boxed{P_a > P_l}$

(3) Convex Surface

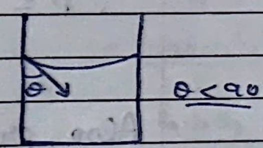


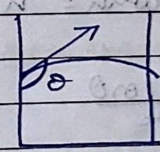
$\Rightarrow \boxed{P_L > P_a}$

* Angle of Contact

↳ Angle between tangent to the point of contact and solid inside the liquid.

→ Angle of contact = θ

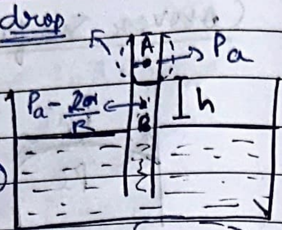




→ If θ < 90, liquid wets solid.
If θ > 90, liquids doesn't wet solid.

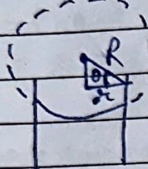
Depends on: ① Nature of liquid & solid in contact.
② Medium above surface of liquid
③ Temperature
④ Cleanliness of surface.

★ Capillarity



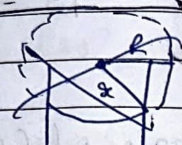
~~Pressure~~ :

Excess pressure at B $= \frac{2\sigma}{R} = P$ — ①



Imagine the curved surface as a part of a liquid drop.

∴ Excess pressure inside $= \frac{2\sigma}{R}$



Also, $R = \frac{r}{\cos\theta}$

$$\therefore P = \frac{2\sigma\cos\theta}{r} \quad \text{— ②}$$

Also, pressure in liquid column $= h\rho g$.

Comparing, $h\rho g = \frac{2\sigma\cos\theta}{r}$

$$\boxed{\therefore h = \frac{2\sigma\cos\theta}{r\rho g}}$$

x ——— x ——— x

Q. Pressure on a swimmer 10m below the surface of the lake? (1×10^5) Pa

↳ $P = \rho g h + P_a + 10^5$

$= 1000\times10\times10$ ~~$= 10^5$ Pa~~ $= 2\times10^5$ Pa

$= 2$ atm

Q. At a depth of 1000m: 1) Absolute Pressure
2) Gauge Pressure
3) Find force acting on 20cm/20cm area.

↳ 1) → $10^7 + 10^5 =$ ~~$10^5($~~ 101×10^5 Pa $= 101$ atm

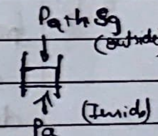
2) $\rho g h = 10^7$ Pa $= 100$ atm

~~3) $F = P\times \text{Area} = 101\times\left(\frac{20}{100}\right)^2 = 101\times\frac{2}{10}\times\frac{2}{10}\times10^5 = 4.04$ N~~

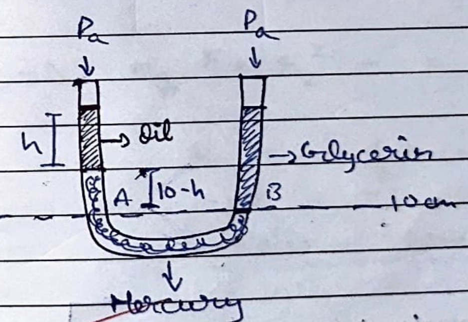
$= 404000$ N

3) $F = 100\times10^5\times\frac{1}{25} = 4\times10^5$ N

(Pressure acting on window = gauge pressure)



Q.



$l_{(glycerin)} = 10$ cm

$\rho_{(glycerin)} = 1.3\ gm/cm^3$

$\rho_{(oil)} = 0.8\ gm/cm^3$

$\rho_{(Hg)} = 13.6\ gm/cm^3$

What is column length of oil.

↳ By Pascal Law, $P_A = P_B$

$\Rightarrow P_{oil} + P_{mercury} = P_{glycerin}$

$\Rightarrow (P_a + \rho g h)$ ~~$+ (\rho g h)$~~ $\rho g(10-h) = P_a + (\rho g h)$

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$\Rightarrow \rho g h + \rho g(10-h) = 10\rho g$

$\Rightarrow$ ~~(13)~~

$\Rightarrow (0.8\times1000\times h) + (13.6\times1000\times(10-h)) = 10\times1.3\times1000$

$\Rightarrow 0.8h + 136 - 13.6h = 13$

$\Rightarrow 12.8h = 123$

$\Rightarrow h = \frac{1230}{128} \approx \underline{9.6 \text{ cm}}$

$128 \overline{)1230}\ 9$
-1152

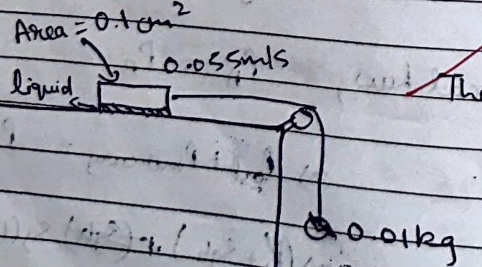
Q. A square metal plate, 10 cm side, moving with 10 cm/s parallel to another plate in water. Viscous force is 200 dyne. $\eta_{(water)} = 0.01$ poise. Calculate their distance apart.

$\rightarrow F = \eta A \frac{dv}{dx}$

$\Rightarrow dx = \frac{\eta A\, dv}{F}$

$\Rightarrow dx = \frac{0.01\times10\times10\times10}{200} = \boxed{0.05 \text{ cm}}$

Q.



Thickness of film = 0.3 mm

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YOUVA

$\hookrightarrow F = \eta A \frac{dv}{dx}$

$\Rightarrow \eta = \frac{F \cdot dx}{A\, dv}$

$\Rightarrow \eta = \frac{\cancel{0.1} \times 0.3\times10^{-3}}{\cancel{0.1}\times \cancel{10} \times 0.055}$

$\frac{0.3}{0.55} \times 10^{-3} = \boxed{3.45\times10^{-3} \text{ Pa s}}$

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Q. Water flows through horizontal pipe whose diameter is 2 cm, with speed 1 m/s. What should be diameter of nozzle if water comes out with 4 m/s.

$\hookrightarrow a_1 v_1 = a_2 v_2$

$\Rightarrow r_2^2 = \frac{1\times1}{4}$

$\rightarrow r_2 = \frac{1}{2}$ cm

$\therefore$ Diameter = 1 cm, 0.01 m

Q. Pressure in closed pipe is 3.5×10^5 N/m². On opening the valve the pressure reduced to 3×10^5. Calculate speed of water.

$\hookrightarrow 3.5\times10^5 + \frac{1}{2}\rho u^2 = 3\times10^5 + \frac{1}{2}\rho v^2$ ($u^2 \to 0$)

$\Rightarrow 0.5\times10^5 = \frac{1}{2}\rho v^2$

$\Rightarrow v = \sqrt{\frac{1\times10^5}{10^3}} = \boxed{10 \text{ m/s}}$

Q. Calculate minimum pressure to force blood from heart to top of head. (Dist = 50 cm)

Density of blood = 1.04 g/cc

$\hookrightarrow$ $P_1 + \rho g h_1 = P_2 + \rho g h_2$

$\hookrightarrow$ $P_1 - P_2 = \rho g (h_2 - h_1)$

$= 1.04 \times 1000 \times 50$

$= \underline{52000 \text{ dyne/cm}^2}$

Q. Area of water pipe at baseline is 4×10^{-4} m^2 & pressure 3×10^5 Pascal, speed 2m/s. Area at 2nd floor 2×10^{-4} at height 8m, calculate speed & pressure.

$\Rightarrow$ $v_2 = \frac{4 \times 10^{-4} \times 2}{2 \times 10^{-4}} = 4$ m/s

$\therefore$ $3 \times 10^5 + \frac{1}{2} \times 1000 \times 4 = P_2 + 1000(10)(8) + \frac{1}{2} \times 1000 \times 16$

$\Rightarrow$ $P_2 = 3 \times 10^5 - 8 \times 10^4 - 2 \times 10^3$

$\Rightarrow$ $P_2 = 10^3 [300 - 80 - 2]$

$= \boxed{2.18 \times 10^5 \text{ Pa}}$

Q. At what velocity does water coming from a hole in a tank in which $P = 3 \times 10^5$ before the flow starts.

$\hookrightarrow$ $P = 3 \times 10^5$

$\therefore h = \frac{3 \times 10^5}{10^4} = 30$ m

$\therefore v = \sqrt{2 \times 10 \times 30} = \underline{10\sqrt{6} \text{ m/s}}$

$= \underline{23.8 \text{ m/s}}$

Q. Calculate work done in blowing a soap bubble from 2cm radius to 3cm. $\sigma = 30$ dyne/cm.

$\hookrightarrow$ ~~$P = \frac{25}{R} = \frac{60}{1} = 60$ N/m~~

Surface energy $= \frac{W}{\Delta A}$

$W =$ surface energy $\times \Delta A$

$\Rightarrow W = 2 \times 30 \times 4\pi (9 - 4)$

$\Rightarrow W = 1200 \times 3.14$

$= \boxed{3770.4 \text{ erg}}$

Q. A mercury drop of radius 1cm is split into 10^6 droplets of equal size. Calculate energy expanded.

$\sigma = 32 \times 10^{-2}$ N/m

$\hookrightarrow$

$\Rightarrow W = SE \times \Delta A$

$= 10^6 \times 32 \times 10^{-2} \times \cancel{\ldots}$

Volume of large drop = $10^6 \times$ volume of small drops

$\Rightarrow \frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$

$\Rightarrow r = \frac{R}{10^2}$ — (1)

$= 10^{-4}$ m

$\therefore \cancel{W = 10^6 \times 32 \times 10^{-2} \times 4\pi\left(R^2 - \frac{R^2}{10^4}\right)}$

$\cancel{4\pi(10^{-4} \times 10^6)}$

$\Delta A = A_2 - A_1$

$= 10^6 \times 4\pi r^2 - 4\pi R^2$

$= 4\pi(10^{-2} - 10^{-4})$

$= 4\pi\, 10^{-4}(100 - 1)$

$= 4\pi\, 10^{-4}(99)$

$\therefore W = 32 \times 10^{-2} \times 4\pi\, 10^{-4}(99)$

$= \boxed{4 \times 10^{-2}\ J}$

Q. 2 soap bubbles of radii 3 cm & 4 cm form a single bubble. What is radius of new bubble?

$\Rightarrow W_1 = 2\sigma\Delta A$

$= 8\pi r_1^2 \sigma$

$W_2 = 8\pi r_2^2 \sigma$

$W_3 = 8\pi r_3^2 \sigma$

By conservation of energy,

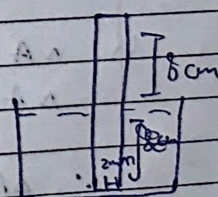
$W_1 + W_2 = W_3$

$\Rightarrow 8\pi\sigma(r_1^2 + r_2^2) = 8\pi\sigma r_3^2$

$\Rightarrow 9 + 16 = r_3^2$

$\Rightarrow \boxed{r_3 = 5}$

Q. The lower end of capillary tube of diameter 2mm & deep 8 cm below surface of water. What is pressure req'd to blow hemispherical bubble at its end in water?



$\Rightarrow P = P_{outside} + \text{Excess pressure}$

$= P_a + h\rho g + \frac{2\sigma}{R}$

$= 1 \times 10^5 + 8 \times 10^3 \times 10^{-2} \times 10 + \frac{2 \times 7.3 \times 10^{-2}}{1 \times 10^{-3}}$

$= 10^3$ Pa

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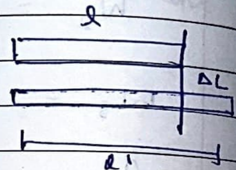
☆ Ch-10 : Thermal Properties of Matter

☆ Heat & Temperature

- Thermal expansion in solids

☆ Linear Expansion



↳ $l' - l(\Delta l) \propto l$

$l' - l \propto \Delta T$

↳ $\Delta l \propto l \Delta T$

$\Delta l = \alpha l \Delta t$

$\Rightarrow \alpha = \dfrac{\Delta l}{l \Delta t}$

α = coeff of linear expansion

Unit → $/°C$ or $/K$

☆ Areal Expansion

↳ $\Delta A \propto \Delta T$ ....

$\Delta A \propto A$

$\therefore \Delta A \propto A \Delta T$

$\therefore \Delta A = \beta A \Delta T$

$\therefore \beta = \dfrac{\Delta A}{A \Delta T}$ → Coeff of areal expansion

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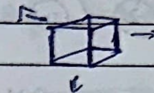
☆ Volume expansion



↳ $\Delta V \propto V$

$\Delta V \propto \Delta T$

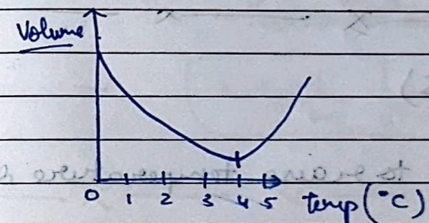
$\therefore \Delta V = \gamma V \Delta T$

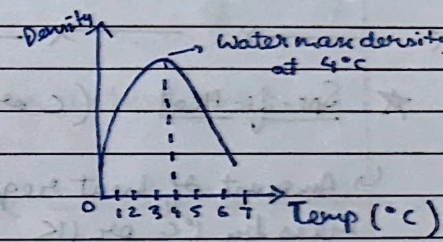
$\Rightarrow \gamma = \dfrac{\Delta V}{V \Delta T}$ → Volume Expansion Coefficient

☆ Expansion in Liquid

(Anomalous behaviour of water)

↳





Water expands after 4°C

Application : Survival of Aquatic Wildlife.

☆ Water as liquid is more dense than solid at 4°C

☆ Expansion in Gases

↳ $PV = nRT$ —①

at const. P

$P\Delta V = nR\Delta T$ —②

↳ ② / ①

$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta T}{T}$

$\Rightarrow \frac{\Delta V}{V \Delta T} = \frac{1}{T}$

$\therefore \boxed{\gamma = \frac{1}{T}}$ → Coeff of volume expansion inversely proportional to temperature

X —— X —— X —— X —— X —— X

★ Specific Heat (c or S)

↳ Amount of heat reqd. to raise temperature of unit mass by 1°C or 1K

$c / SH = \frac{\Delta Q}{M \Delta T}$ Unit → J/kg K

★ Molar Specific Heat

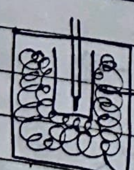
↳ $C_m = \frac{\Delta Q}{n \Delta T}$ Unit → J/mol K

X —— X —— X —— X —— X —— X

★ Calorimetry

↳ Principle ⇒ Heat Gain = Heat Loss

• Calorimeter



★ Amount of heat reqd. to change phase of a substance is called latent heat.

Temp. Change → Specific Heat   Phase Change → Latent Heat

• Solid → Liquid : Latent heat of fusion (L_f)

• Liquid → Gas : Latent heat of vaporisation (L_v)

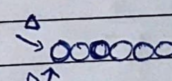
$Q = mL$

$\rightarrow \boxed{L = \frac{Q}{m}}$ → Latent Heat

X —— X —— X —— X —— X —— X

★ Mode of Heat Transfer

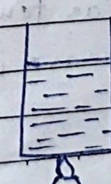
1) Conduction : No movement of particle. Happens in solids only.



Heat energy provided, molecule starts vibrating about its mean position and transmits vibration to other molecules. Eventually, all particles begin vibrating.

2) Convection : Movement of particles involved. Happens in liquids & gases.

↗ Natural
↗ Forced

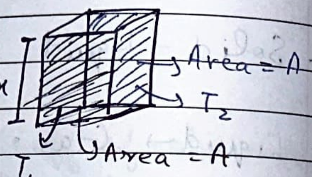


→ Lower layer heats, becomes less dense and moves upwards. Upper cold & more dense layer is pushed downwards

3) Radiation : No role of intermediate medium (solid, liquid or gas) (particles)

Heat travels in form of waves.

• Eqn for conduction



Let $T_1 > T_2$

Heat travels from $T_1 \to T_2$.

1) $Q \propto T_1 - T_2$

2) $Q \propto t$ (time)

3) $Q \propto A$

4) $Q \propto \frac{1}{x}$

$$\therefore Q \propto \frac{A(T_1 - T_2)t}{x}$$

$$Q = \frac{K \cdot A(T_1 - T_2)t}{x}$$

where K is coeff of thermal conduction

↳ $\boxed{H = -KA\frac{dT}{dx}}$ −ve sign as temp decreases with increase in distance

★ Newton's Law of Cooling

↳ Rate of loss of heat $\propto$ Temp diff. b/w system and surroundings.

$$\frac{dQ}{dt} = K(T - T_0) \quad -(1)$$

$$= mc\,dT$$

$$\frac{dQ}{dt} = \frac{mc\,dT}{dt}$$

Substituting $\frac{dT}{dt}$ in (1),

$$\Rightarrow mc\frac{dT}{dt} = K(T - T_0)$$

$$\Rightarrow \frac{dT}{dt} = \frac{K}{mc}(T - T_0)$$

$$\Rightarrow \frac{dT}{dt} = -P(T - T_0) \quad \text{where } P = \frac{K}{mc}$$

$$\Rightarrow \int \frac{dT}{T - T_0} = \int -P\,dt$$

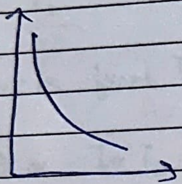
$$\Rightarrow \log_e(T - T_0) = -Pt + c$$

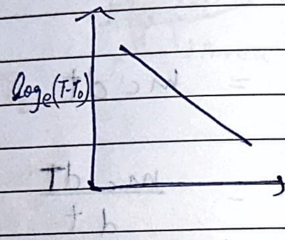
$$\Rightarrow T - T_0 = e^{-Pt + c}$$

$$T - T_0 = e^{c} \cdot e^{-Pt}$$

$$T - T_0 = Ce^{-Pt}$$

$$\boxed{T = T_0 + Ce^{-Pt}}$$ where $C = e^{c}$



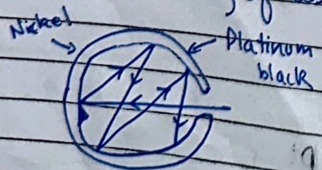


* Black Body Radiation

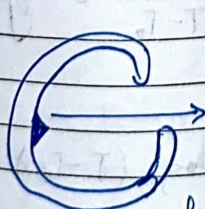
↳ The body which neither reflects nor transmits but absorbs fully heat radiation incident on it is known as the black body.

→ Absorption power is unity.
↳ 100%

↳ In practise, a surface coated with lamp black or platinum black, absorbs 95% - 97% heat radiation. But when it is heated it doesn't emit full radiation spectrum. This is known as black body of absorption.



Bl. Bo. of Absorption



black body of emission

↳ The wavelengths of radiation is independent of material & only depend on temperature of the material.

* Stefan - Boltzmann's Law

↳ The total heat energy emitted by perfect black body per second per unit area is ~~directly~~ directly proportional to 4th power of its surface temperature.

$$E \propto t^4$$

$$\boxed{E = \sigma t^4}$$

σ ← Stefan-boltzmann constant
↳ 5.67×10^{-8} kg m^2 s^{-6}

* Wien's Displacemnt Law

↳ The wavelength corresponding to which the energy emitted by ~~heat~~ black body is maximum is inversely proportional to the temperature.

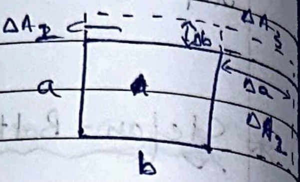
$$\lambda_m \propto \frac{1}{T}$$

$$\rightarrow \boxed{\lambda_m = \frac{b}{T}}$$

b = Wien's constant
↳ 2.9×10^{-3} m Kelvin

Temperature ↓ → Light emitted whose wavelength is high.

Q. Show that coefficient of area expansion of rectangular sheet is twice its linear expansion.



→ T.P → $\beta = 2\alpha$

$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$

$\Delta A = a\Delta b + b\Delta a + \Delta a \Delta b$ — (1)

Now, $\alpha = \dfrac{\Delta l}{l\Delta t}$

Also, $\alpha = \dfrac{\Delta a}{a\Delta t}$ & $\alpha = \dfrac{\Delta b}{b\Delta t}$

$\Rightarrow \Delta a = a\Delta t\,\alpha$; $\Delta b = \alpha b \Delta t$

$\therefore \Delta A = a(\alpha b\Delta t) + b(a\Delta t\alpha) + (a\Delta t\alpha)(\alpha b\Delta t)$

$= \alpha ab\Delta t + \alpha ab\Delta t + \alpha^2 ab\Delta t^2$

$\beta A\Delta T = 2\alpha A\Delta t + A\alpha^2\Delta t^2$

$\Rightarrow \beta = 2\alpha + \alpha^2\Delta T$

↳ This can be ignored as it is very small.

$\therefore \boxed{\beta = 2\alpha}$

Q. An iron ring of diameter 5.231 m have to fix on wooden wheel of diameter 5.243 m. Kept at 27°C. To what temp should the ring be heated so that it fits the wheel. $\alpha = (1.20\times10^{-5})$

↳ $\alpha = \dfrac{\Delta L}{L\Delta T}$; $\beta = 2\alpha$

$\Rightarrow \Delta T = \dfrac{\Delta L}{L\alpha}$; $\Delta T = \dfrac{\Delta A}{A\beta}$

$\Delta T = \pi\left(\dfrac{(5.243)^2}{4} - \dfrac{(5.231)^2}{4}\right)$

↳ $\Delta T = \dfrac{\Delta L}{L\alpha}$

$= \dfrac{0.12}{5.231\times1.20\times10^{-5}}$

$= \dfrac{400}{5.250\times10^{-3}\times21}$

$21\overline{)40}$ = 0.19
-21
190
-189
1

$T_2 - 27 = 0.19\times10^3$

$T_2 = 190 + 27$

$= \boxed{218^\circ C}$

Q. If volume of block changes by 0.12 % when heated by 20°C what is α?

↳ $\gamma = 3\alpha$

$\therefore \gamma = \dfrac{\Delta V}{V\Delta T}$

$\gamma = 0.12$

$\rightarrow \gamma = \dfrac{12}{2\times10^5}$

$= 6\times10^{-5}$

$\therefore \boxed{\alpha = 2\times10^{-5}/°C}$

Water is best coolant as it has ~~very~~ high specific heat.

Q. An aluminium spear of 0.047 kg at 100°c placed into 0.14 kg copper calorimeter, containing 0.25 kg water at 20°C. The temperature rises and attains steady state at 23°C. Calculate specific heat of aluminium.
~~(Sp. of~~ ($C_w = 4.18 \times 10^3$ J/kgK) ($C_{copper} = 0.38 \times 10^3$ J/kg
Sp. Heat of water)

→ Final temp of Al, Cu & H_2O → $\underline{23°C}$

Al mass = ~~0.47 kg~~ 0.047 kg   Initial T = 100°C
Cu mass = 0.14 kg
H_2O mass = 0.25 kg   ] Initial T = 20°C

As. $\underset{(aluminium)}{MS\Delta T} = \underset{(Cu)}{MS\Delta T} + \underset{(Water)}{MS\Delta T}$

→ $0.047 \times S \times (23-100) = \overset{0.25}{\cancel{0.14}} \times 4.18 \times 10^3 \times 3 + \overset{0.14}{\cancel{0.25}} \times 0.38 \times 10^3$

⇒ $S \times 0.047 \times 77 = 0.25 \times 4.18 \times 3 \times 10^3 + 0.4 \times 0.38 \times 10^3 \times 3$

⇒ $S = \dfrac{0.25 \times 4.18 \times 3 \times 10^3 + 0.4 \times 0.38 \times 10^3 \times 3}{0.047 \times 77}$

$= \underline{0.91 \times 10^3}$ J/kg K

Q. Calculate the heat reqd. to convert 3kg of ice at -12°C to steam at 100°C.

$C_{ice} = 2100$ J/kgK
$C_w = 4186$ "
$L_f = 3.35 \times 10^5$ J/kg
→ $L_v = 2.25 \times 10^6$ J/kg

→ ~~Q = msΔT + mL~~

$Q = \underbrace{ms\Delta T + mL}_{fusion} + \underbrace{ms\Delta T + mL}_{vapour}$

⇒ $Q = 3(2100(12) + 3.35 \times 10^5) + 3(4186(100) + 2.25 \times 10^6)$

$= 3(25200 + 335000) + 3(418600 + 2250000)$

$= 3(3028800)$

$= 3 \times 3.0 \times 10^6$

$= \underline{9.084 \times 10^6}$ ~~J/kg~~ J

21
×12
42
210
252

125200
335000
418600
2250000
3028800

Q. When 0.15 kg of ice at 0°C mix with 0.30 kg of water at 50°C, the resulting temp is 6.7°C. Calculate latent heat of fusion of ice. $C_w = 4186$ J/kgK

50.0
-6.7
43.3

→ $ms\Delta T + mL = ms\Delta T$

⇒ $(0.15)(4186)(6) + mL = (0.30)(4186)(43.3)$

→ $mL = (0.30)(4186)(\overset{42}{8}) - (0.15)(4186)(6)$

$= (0.15)(4186)(6)[14 - 1]$

$= 0.9(4186)13$

$= \dfrac{489762}{} = 4.8 \times 10^{4}$ J/kg

Q. $L = \dfrac{0.33 \; 4.8 \times 10^4}{15 \times 10^{-2}} = \boxed{3.3 \times 10^5 \text{ J/kg}}$

4186
×117
29302
41860
418600
489762

★ Latent Heat of fusion always less than Latent Heat of vaporisation.

★ Relation between α, β & γ.

↳ Reln between γ & α:

$V = l^3$

$V' = l'^3$

$\Rightarrow l' = l + \Delta l$

$\Rightarrow l' = l + \alpha l \Delta t$

$\Rightarrow l' = l(1 + \alpha \Delta t)$

$V' = l'^3$

$= [l(1+\alpha\Delta t)]^3$

$= l^3 [1 + \underbrace{\alpha^3\Delta t^3 + 3\alpha\Delta t + 3\alpha^2\Delta t^2}] \circ$ can be ignored as very small.

$\therefore V' = V[1 + 3\alpha\Delta t]$ — ①

Also, we know that $\gamma = \frac{\Delta V}{V\Delta t}$

$\Rightarrow \Delta V = \gamma V \Delta t$

$\therefore V' = V + \Delta V$

$= V + \gamma V \Delta t$

$= V(1 + \gamma\Delta t)$ — ②

Comparing ① & ②, we get $\boxed{\gamma = 3\alpha}$

$\rightarrow \boxed{\beta = 2\alpha \quad \& \quad \gamma = 3\alpha}$

$\therefore \boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$

• $A = l^2$

$A' = l'^2$

$l' = l + \Delta l$

$= l(1 + \alpha\Delta t)$

$\therefore A' = l^2 (1 + (\alpha\Delta t)^2 + 2\alpha\Delta t)$ — very small is ignored

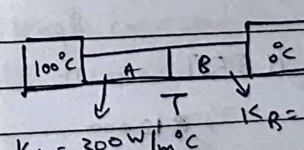
$A' = A(1 + 2\alpha\Delta t)$ — ①

Also, $A' = A + \Delta A$

$= A + \beta A \Delta T$

$= A(1 + \beta\Delta T)$ — ②

On comparison of ① & ②, $\boxed{\beta = 2\alpha}$

Q. 

What will be temperature of interface.

$\Rightarrow Q = \frac{KA(T_1 - T_2)t}{x}$

$\Rightarrow \frac{Q_A}{t} = \frac{Q_B}{t}$

$\Rightarrow \frac{K_A A(T_1 - T_2)}{\cancel{x}} = \frac{K_B A(T_1 - T_2)}{\cancel{x}}$

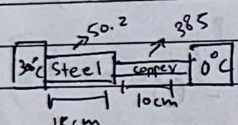
$\Rightarrow K_A(100 - T) = K_B \cancel{(100-T)} (T - 0)$

$\Rightarrow 30000 - 300T = \cancel{20000 - 200T}\ 200T$

$\Rightarrow \cancel{10000}\ 30000 = 500T$

$\Rightarrow T = \frac{\overset{60}{\cancel{30000}}}{\cancel{500}} = 60°C$

Q.



$A_{steel} = 2A_{Cu}$

$\Rightarrow \frac{K\,2A(300 - T)}{15 \times 10^{-2}} = \frac{K A(T - 0)}{10 \times 10^{-2}}$

$\Rightarrow \frac{200(300 - T)}{3} = \frac{385T}{2}$

$\Rightarrow 120000 - 2T = 1155T$

$\Rightarrow T = \frac{\cancel{120000}}{\cancel{1155}} = \frac{\cancel{12000}\ 60000}{\cancel{1355}\ 277} = 44°C$

Q. Calculate T at which a perfect black body radiate energy at rate of 5.67 W/cm²

$\Rightarrow E = \sigma t^4$

$\Rightarrow t = \sqrt[4]{\frac{5.67 \times 10^{4}}{5.67 \times 10^{-8}}}$

$\Rightarrow t = 10^{3}°C$

Q. Luminosity (Energy/Area) of star is ~~1000~~ 1700 times that of our sun. If surface temp of sun is 6000K what is temp of star? (orion)

$\Rightarrow E_{sun} = 5.67 \times 10^{-8} \times (6000)^4$

Now, $E_{star} = 1700 \times (6000)^4 \times 6$

$\sigma t^4 = 1700 \times (6000)^4 \times 6$

$\Rightarrow t = \sqrt[4]{1700 \times (6000)^4}$

$\Rightarrow t = 6000\sqrt[4]{17000}\ K$

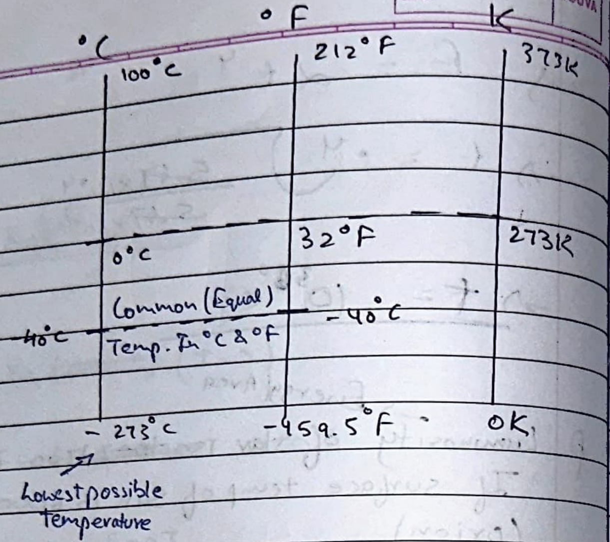
$= 68520\ K$

Q. The wave length corresponding to E_{max} for moon is 14 μm. Calculate temperature of moon.

$\Rightarrow T = \frac{\overset{0.2}{2.9 \times 10^{-3}}}{14 \times 10^{-6}} = 206\ K$

★ Steam Point
(upper fixed point)

Ice Point
(lower fixed point)



★ formula

→ °C → °F ⇒ $\boxed{F = \frac{9}{5}C + 32}$

→ °C → K ⇒ $\boxed{K = C + 273}$

★ Ganda Formula → $\boxed{\frac{\text{Temp of 1 scale} - LFP}{UFP - LFP} = \frac{\text{Temp on other scale} - LFP}{UFP - LFP}}$

Q. A faulty thermometer has fixed point marked at 5°C & 95°C. Temp of body measured is 59°C. Correct the measurement in °C scale.

↳ $\frac{59-5}{90} = \frac{x-0}{100}$

⇒ $5400 = 90x$

⇒ $\underline{x = 60°C}$

Q. A faulty thermometer reads 5°C in melting ice & 99°C in steam. Calculate correct temp in °F. $\underline{52°C}$

↳ $\frac{52-5}{99-5} = \frac{x-32}{180}$

$\frac{212}{-32}$
$\overline{180}$

⇒ $(x-5)\,180 = 95(x-32)$ ⇒

⇒ $2x - 10 = x - 32$

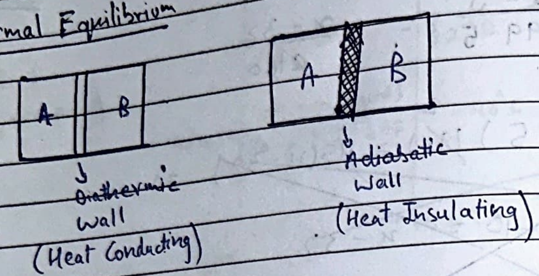
⇒ $x - 32 = \frac{47}{95} \times 180$

⇒ $\frac{47 \times 90}{47} = x - 32$

⇒ $\boxed{x = 122}$

* Thermodynamics

* Thermal Equilibrium



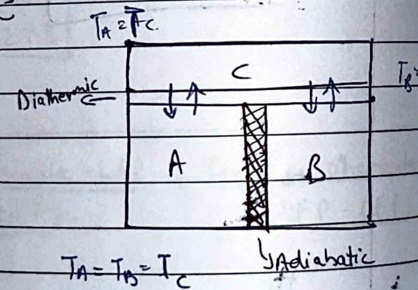
In system 1, heat exchange takes place till $T_A = T_B$.

In system 2, no heat exchange takes place due to adiabatic wall.

* Zeroth Law of Thermodynamics

=> If $T_A = T_C$ & $T_B = T_C$

Then $T_A = T_B$ also.



↳ Temp. is the quantity which develops Thermal ~~Equilib~~ Equilibrium

* Internal Energy (U)

↳ $IE = \underline{K.E} + \underline{PE}$ of a system.

• Real gas : All molecules attracted by each other.

• Potential energy in real gas is developed due to attraction b/w molecules.

→ Volume increases ~~→ ForA~~ Distance increases => P.E

PE = fn of Volume

• If heat energy is responsible for KE.

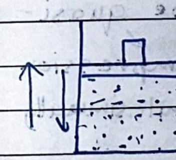
T ~~KE~~ = fn. of KE

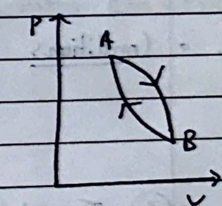
* Processes

1) Cyclic Process

↳ Any process in which the system returns to its initial state after undergoing a series of changes.

Eg





for isothermal,

↳ Piston pushed down, volume decreases, pressure increases

↳ ~~Piston~~ force removed, gas pushes piston up and returns to initial P, V, ~~&~~ T

Imp : 1) P-V graph will be closed loop.

2) The work done per cycle is numerically equal to area of the loop.

3) If closed loop traced in a clockwise direction, work done is +ve (area is +ve), it is work done by system.

4) If the loop is in anticlockwise direction, area is -ve. (work done -ve), (work done on system).

② Reversible Process

↳ Any process which can be proceeded in a reverse direction by variation in its conditions such that any change occuring in any part of direct process is exactly reversed in reversible process.

↳ At the end of the reversible process, both system & surrounding must return to its initial states.

• Conditions : ① The process must be quasi-static process of compression or expansion infinitesimally slow.

② The dissipative forces like viscosity, friction should be absent.

③ Irreversible Process

↳ Any process which can't be exactly reversed or retraced is called irreversible process.

• Most processes in nature are irreversible.
Eg popping, mixing etc.
sudden expansion of gases

* Second law of Thermodynamics (

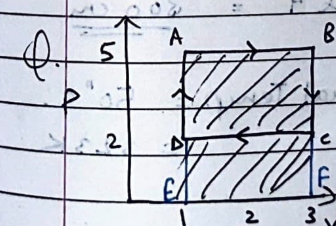
↳ It is impossible to construct an engine which will produce no effect other than absorbing heat from a reservoir and performing an equivalent amount of work.

↳ It is applicable to the heat engine
↳ Efficiency is never 100%. (unity)

↳ Clausius Statement : It is impossible for a self acting machine which transfers heat from low temperature reservoir to high temperature reservoir, without input of energy by work.

• The efficiency of heat engine can never be unity.
• Coefficient of performance for refrigerator can never be ∞.
↳ (External work done never zero)

Q.



↳ W_{AB} = Area of ABFE = $5 \times 2 = 10$ J

↳ W_{BC} = 0 J (Since no change in volume)

↳ $W_{CD} = 2 \times 2 = -4$ J

↳ W_{DA} = 0 J (ΔV=0)

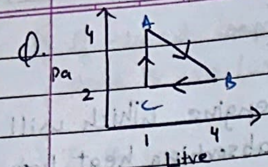
Only each line ki direction dekhna hai.

Total work done = ~~10~~ 6 J

Slowly means isothermal

~~Sto~~ Suddenly means adiabatic

Q.



$W_{AB} = \frac{1}{2} \times 3 \times 2 \times 10^{-3} + 2 \times 3 \times 10^{-3}$

$= 3 \times 10^{-3} + 6 \times 10^{-3}$

$= 9 \times 10^{-3}$ J

$W_{BC} = -6 \times 10^{-3}$ J (Anticlockwis)

$W_{CA} = 0$ ($\Delta V = 0$)

$\therefore$ Total work = 3×10^{-3} J

Q. If 50°c & 75 cm of mercury pressure, a gas is compressed : ① Slowly ② Suddenly.
(isothermal) (adiabatic)

What will be the final temp & pressure of the system if final volume is $\frac{1}{4}$ th of initial volume ; $\gamma = 1.5$.

↳ ① $P_1 V_1 = P_2 V_2$

$\Rightarrow P_2$ (final pressure) $=$ ~~$75 \times 13.6 \times$~~ $75 \times 4 = 300$ cm

Isothermal $\Rightarrow \Delta T = 0$. $\therefore$ final Temp = 50°c = 323 K

② Also, $PV^\gamma =$ constant. — ①

$\Rightarrow P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma = 75 (4)^{1.5}$

$= 75 \times 8 = 600$ cm

↳ We know that $PV = RT$

$\therefore P = \frac{RT}{V}$

$\therefore \frac{RT}{V}(V^\gamma) = K$ (from ①)

$\Rightarrow \frac{TV^\gamma}{V} = K$ $K = \frac{k}{R}$

$\Rightarrow \boxed{TV^{\gamma-1} = K}$ → Adiabatic Process

$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\Rightarrow T_2 = 50 (4)^{0.5} = 100°c$ / 373 K

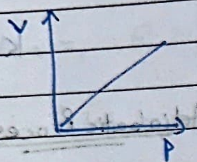
Result !!

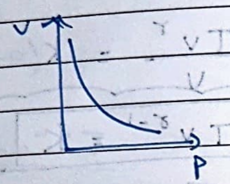
↳ $TV^{\gamma-1} =$ constant → for Adiabatic Process.

* Kinetic Theory Of Gases

∘ **Boyle's Law** (only for isothermal)

↳ $V \propto \frac{1}{P}$ (T = const, isothermal)





∴ $\boxed{PV = \text{constant}}$

∘ **Charles' Law** (for isobaric)

↳ $V \propto T$ (P = const)

∘ **Gay Lusac's Law**

↳ $P \propto T$

★ Ideal Gas Eqn

$$\boxed{PV = nRT}$$

n = no. of moles
R = Gas constant

from Charles' & Boyle's Law,

$$V \propto \frac{T}{P}$$

↳ $PV \propto T$

{ $R = 8.314$ J/molK }

↳ $\boxed{PV = RT}$

For n moles of gas, $\boxed{PV = nRT}$ → This is per mole

∴ ~~the~~ Eqn. for per molecule

$\frac{R}{N_A} = K_B$ → Boltzmann Constant → (1.38×10^{-23} J/K)

↳ $PV = n \cdot K_B \cdot N_A T$ Also $n = \frac{N}{N_A}$

(moles = $\frac{\text{no. of molecules}}{N_A}$)

↳ $\boxed{PV = N \cdot K_B \cdot T}$

↳ N = no. of molecules

↳ K_B = Boltzmann constant

∘ **Assumptions of Kinetic Theory**

1) All molecules of the gas are rigid sphere.
2) The size of molecules is negligible when compared to the distance between the molecules.
3) Collisions are perfectly elastic and there is no force of attraction among the molecules.
4) Molecules are in state of continuous random motion.