Vertical Displacement → $v_y = u\sin\theta$, $a = -g$

$$y = ut + \frac{1}{2}at^2$$

$$\Rightarrow y = u\sin\theta \frac{x}{u\cos\theta} + \frac{1}{2}(-g) + \left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

Let $\tan\theta = p$

$\frac{g}{2u^2\cos^2\theta} = q$

$\Rightarrow \boxed{y = px - qx^2} = \boxed{\text{eq}^n \text{ of parabola}}$

② Time of maximum height (T_m)

For vertical, $v = u + at$

$\Rightarrow 0 = u\sin\theta - g t_m$

$$\boxed{t_m = \frac{u\sin\theta}{g}}$$

Ch 4 ★ Laws of Motion

* **Inertia** → Resistance to change in state of motion.
  - ↳ of Rest - Tendency to stay at rest.
  - ↳ of motion - Tendency to stay in motion
  - ↳ of direction - Tendency to retain motion in one direction.

1st Law Of Motion — "Laws of Inertia"

↳ Every body continues in the state of rest or state of uniform motion unless compelled by external force.

★ Projectile Motion

③ Time of flight

→ Displacement in vertical direction = 0

From $s_y = u_y t + \frac{1}{2}at^2$

$$0 = u\sin\theta t - \frac{1}{2}gt^2$$

$\Rightarrow \boxed{t = \frac{2u\sin\theta}{g}}$ — Time of flight

Also, time of max height $= \frac{u\sin\theta}{g}$

$\therefore$ Time of flight $= 2 \times$ Time of max height.

④ Max Height $v_y = 0$

⇒ Using $v^2 = u^2 + 2as$

⇒ $0 = u^2\sin^2\theta - 2gs$

⇒ $\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$ — for Max Height

⑤ Horizontal Range

→ Using $d = s \times t$

⇒ $R = u\cos\theta \times \frac{2u\sin\theta}{g}$

⇒ $\boxed{R = \frac{u^2\sin 2\theta}{g}}$ — Horizontal Range

★ $R_{max} = \frac{u^2}{g}$ — when $\theta = 45°$.

★ Angles different, same range → Complementary angles

→ $R = \frac{u^2\sin 2\theta}{g}$

Replacing θ with $90-\theta$,

⇒ $R' = \frac{u^2\sin(180-2\theta)}{g}$

$R' = \frac{u^2\sin 2\theta}{g} = R$

∴ Range for complementary angles is equal

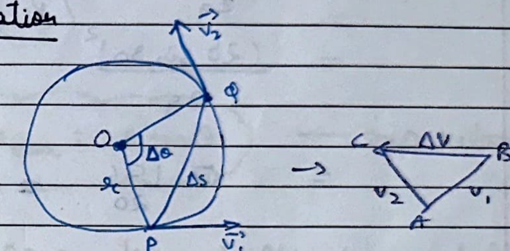
★ Whenever velocity changes direction, acceleration is always ~~drawn~~ perpendicular to velocity

Uniform Circular Motion

↳ When an object moves with constant speed, it is in uniform circular motion.

↳ It is an accelerated motion, acceleration - centripetal acceleration (variable)

★ Centripetal Acceleration



→ In ΔABC & ΔPOQ, $\frac{BC}{AB} = \frac{PQ}{OP}$

⇒ $\frac{\Delta v}{v} = \frac{\Delta s}{r}$

⇒ $\Delta v = \frac{v}{r}\Delta s$

⇒ $\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$ (Differentiating w.r.t. to time)

⇒ $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$

⇒ $\frac{dv}{dt} = \frac{v}{r}\frac{ds}{dt}$

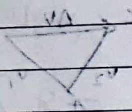
⇒ $\boxed{A_c = \frac{v^2}{r}}$ — Centripetal acceleration

Q.1 A cricket ball thrown at a speed of 28 m/s in direction of 30° with horizontal. Calculate max height, time of flight & range.

Ans 1 ~~Max height = $\frac{(28\sin 30)^2}{2\times 10}$~~

$H_{max} = \frac{u^2\sin^2\theta}{2g}$

$= \frac{(28\sin 30)^2}{2g}$



$= \frac{196}{20} = \underline{9.8\,m}$

$T_f = \frac{2u\sin\theta}{g}$

$= \frac{2\times 14}{10} = \underline{2.8\,s}$

$R = \frac{u^2\sin 2\theta}{g} = \frac{28\times 28\times \frac{\sqrt{3}}{2}}{10} = \underline{39.2\sqrt{3}\,m}$

Q.2 Projectile range of 50m reach to max height of 10m. Calculate projectile angle.

Ans 2 $50 = \frac{u^2\sin 2\theta}{g}$

$\Rightarrow 50 = \frac{2u^2\sin\theta\cos\theta}{g}$ — ①

$10 = \frac{u^2\sin^2\theta}{2g}$

$10 = \frac{u^2\sin^2\theta}{2g}$ — ②

$u^2 = \frac{10\times 2g}{\sin^2\theta}$

From ① & ②, $50 = 2\times\frac{10\times 2g}{\sin^2\theta}\times\frac{\sin\theta\cos\theta}{g}$

$\Rightarrow \frac{50}{40} = \cot\theta$

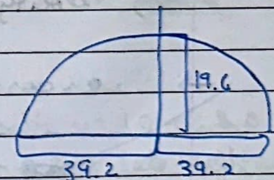
$\Rightarrow \tan\theta = \frac{4}{5}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{5}\right)$

$\boxed{4H_{max} = R\tan\theta}$ — imp relation

Q.3 A boy stands at 39.2m from a building throws a ball which passes through a window, landing point 19.6m above the ground. Calculate velocity of projectile.

Ans 3 Range = 39.2 × 2 = 78.4 m
Height of window = 19.6m



∴ Using $4H = R\tan\theta$,

$\Rightarrow \tan\theta = \frac{4\times 19.6}{78.4} = \tan\theta$

~~$\Rightarrow \tan\theta = 16$~~

~~$\therefore \theta = \tan^{-1}(16)$~~

$\therefore \tan\theta = 1$

$\underline{\theta = 45°}$

$\Rightarrow$ Using $R = \frac{u\sin 2\theta}{g}$

$\therefore R = \frac{u}{10}$

$\Rightarrow u = \sqrt{784}$

$= \underline{28\,m/s}$

Q.4 Find the angle of projection for H = R.

Ans4 $H = R$

$\Rightarrow \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow \sin^2\theta = 4 \sin\theta \cos\theta$

$\Rightarrow \sin\theta = 4\cos\theta$

$\Rightarrow \tan\theta = 4$

$\Rightarrow \theta = \tan^{-1}(4)$

Q. $u = 20\,m/s$, $\theta = 60^\circ$

1) Position after 0.5 sec.
2) Velocity after 0.5 sec.

$\rightarrow$ ~~$u\cos\theta = 20\,m/s$~~

~~$v_y = u\sin\theta - gt$~~

~~$= 10\sqrt{3} - 5$~~

~~$v = \sqrt{\;}$~~

$\rightarrow$ Position after 0.5 secs:

$P_x = u\cos\theta \times t$

$= \frac{20}{2} \times 0.5 = 5m$

$P_y = u\sin\theta\, t - \frac{1}{2} g t^2$

$\Rightarrow P_y = 10\sqrt{3} \times 0.5 - \frac{1}{2} \times 10 \times 0.25$

$\Rightarrow P_y = 5\sqrt{3} - 1.25\,m$

$= 8.5 - 1.25$

$= 7.25\,m$

2) $\vec{V_x} = \vec{U_x} = 10\,m/s$

$v_y = u_y + a_y t$

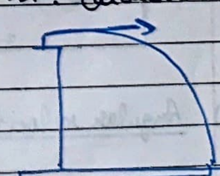
$= 10\sqrt{3} - 5$

$= 12\,m/s$

$\therefore v = \sqrt{100+144}$

$= \sqrt{244} =$

Q. A ball is thrown from top of tower with $u = 10\,m/s$ & $\theta = 30^\circ$. It hit the ground at 17.3 m. Calculate height of the tower.



$\Rightarrow$ ~~Using $4H = R\tan\theta$~~

~~$\Rightarrow H = \frac{24.6}{4} \times \frac{1}{\sqrt{3}}$~~

$\rightarrow R = u\cos\theta \times t$

$\Rightarrow$ ~~26.4 = $5 \times t$~~

$\Rightarrow t = \frac{26.4}{5}$

$\rightarrow R = \cancel{u \sin\theta}\, u\cos\theta \times t$

$17.3 = 5\sqrt{3} \times t$

$\Rightarrow t = \frac{17.3}{5\sqrt{3}} = \underline{\underline{2 secs}}$

$\rightarrow$ Now max height $= ut + \frac{1}{2}gt^2$

$= \cancel{\frac{5 \times 17.3}{5\sqrt{3}}} + \frac{1}{2} \times 10 \times \cancel{17.3}$

$= 5 \times 2 - \frac{1}{2} \times 10 \times 4$

$= 10 - 20 = \boxed{-10m}$

$\therefore$ Distance = 10m

★ **Important Equation for UCM.**

① Relation b/w linear displacement & angular displacement.

$\rightarrow \boxed{\Delta\theta = \frac{\Delta s}{r}}$

② Angular velocity (ω)

$\rightarrow \boxed{\omega = \frac{d\theta}{dt}}$

③ Time period (T)

$\rightarrow$ Time taken to complete one revolution.

④ Frequency (ν) 'nu'

$\rightarrow$ No. of revolutions per unit time. Unit $\rightarrow$ Hertz

$\Rightarrow \boxed{\nu = \frac{1}{T}}$

⑤ Relation between ω and ν.

$\Rightarrow \boxed{\omega = \frac{2\pi}{T}}$ & $\boxed{\omega = 2\pi \cdot \nu}$

⑥ Relation between v and ω

$\rightarrow \boxed{v = r\omega}$ – Scalar

$\boxed{\vec{v} = \omega \times \vec{r}}$ – Vector

⑦ Angular acceleration (α)

$\rightarrow a = \frac{dv}{dt} \rightarrow \alpha = \frac{d\omega}{dt}$ (Unit → rad/s²)

⑧ Relation between α & a

$\boxed{a = r\alpha}$

$\boxed{\vec{a} = \vec{r} \times \vec{\alpha}}$

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Q. A body of $m_1 = 10$ kg revolves in circle of diameter 0.40m making 1000 revolutions. Calculate linear velocity and centripetal acceleration.

Ans → $m_b = 10$ kg $r = 0.20$ m

For $v = \frac{\cancel{100}0}{3\cancel{60}} = \frac{50}{3}$ rev/s

Linear velocity $= r \times 2\pi \times \frac{50}{3}$

$= 0.20 \times 2\pi \times \frac{50}{3}$

$= \frac{20}{100} \times 2\pi \times \frac{50}{3} = \frac{20\pi}{3}$ m/s

$A_c = \frac{v^2}{r}$

$= \frac{400\pi^2}{9} \times \frac{100}{20} = \frac{2000\pi^2}{9}$ m/s^2

Q. The radius of earth ~~or width~~ around the sun is 1.5×10^{11} m. Calculate angular & linear velocity.

→ $\omega = \frac{2\pi}{365 \times 76400}$

$= \frac{2\pi}{3.14 \times 10^7} \approx$ ~~0.2×10^{-4} m/s~~

$= \frac{2}{10^5} \approx 2 \times 10^{-7}$ rad/s (approx)

365 × 864: 1460, 21900, 292000, 31438000

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→ $v = 1.5 \times 10^{11} \times 2 \times 10^{-7}$

$v \approx \underline{3 \times 10^4}$ m/s

Q. The ω of particle is moving along circle of $r = 50$cm increased in 5 min from 100 rev/min to 400 rev/min.

Find α & a.

→ $t = 5$ min $= 300$s $F_1 = \frac{100}{60} = \frac{5}{3}$ rev/s

$r = 50 \times 10^{-2}$ m $F_2 = \frac{400}{60} = \frac{20}{3}$ rev/s

$\omega_1 = \frac{10\pi}{3}$ $\omega_2 = \frac{40\pi}{3}$

⇒ ∴ $\omega_2 = \omega_1 + \alpha t$

⇒ $\alpha = \frac{\omega_2 - \omega_1}{t}$

⇒ $\alpha = \frac{10\pi}{300} = \frac{\pi}{30}$ rad/s^2

$a = \frac{50}{100} \times \frac{\pi}{30} = \frac{5\pi}{300}$ m/s^2

$= \boxed{\frac{\pi}{60} \text{ m/s}^2}$

2nd Law of Motion

→ $F \propto \frac{dP}{dt}$ $\boxed{F = ma}$

⇒ $= \frac{dmv}{dt}$

$= \frac{m\,dv}{dt}$

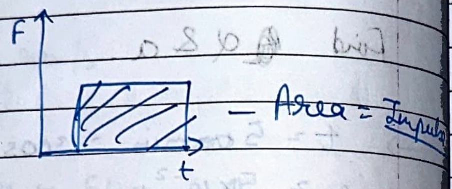
3rd Law of Motion

→ Every action has equal and opposite reaction.

★ **Impulse** – Force during very short period of time.
↳ J

→ $I = F \times \Delta t$

$\boxed{I = \Delta P}$



LAW OF CONSERVATION OF MOMENTUM

→ For an isolated system, the total momentum remains constant if :-

$F_{total} = 0$

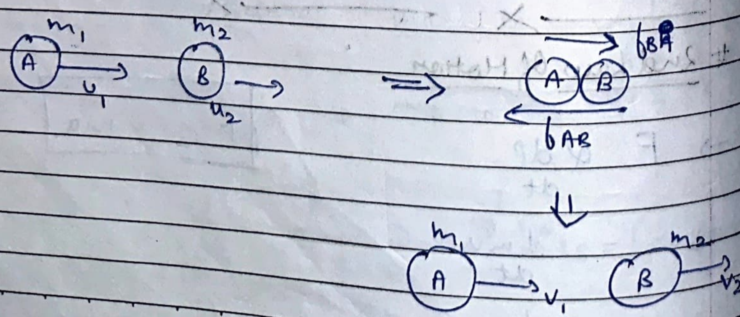
→ $F = \frac{dp}{dt}$

→ $0 = \frac{dp}{dt}$

⇒ $dt = dp$

∴ $\boxed{p = constant}$

Derivation :



→ Force acting on A ⇒ F_{AB}

Impulse of A = $F_{AB} \times \Delta t = \Delta P_A$

$= m_1 v_1 - m_1 u_1$ — (1)

→ Force acting on B ⇒ F_{BA}

Impulse of B = $F_{BA} \times \Delta t = \Delta P_B$

$= m_2 v_2 - m_2 u_2$

By Newton's third law; $F_{AB} \Delta t = -F_{BA} \Delta t$

~~∴, $m_1 u_1 + m_2 u_2 = m_2 v_2$~~

∴, $m_1 v_1 - m_1 u_1 = -(m_2 v_2 - m_2 u_2)$

⇒ $m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$

⇒ $\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$

i.e.,

Total momentum before collision = Total momentum after collision

★ **Application of Conservation of Momentum**

(1) **Recoil of Gun**

→ Total momentum of G+B before firing = Total momentum of G+B after firing

• random velocities alway give vector addition.

$$0 = \underset{\text{bullet}}{mv} + \underset{\text{gun}}{MV}$$

→ $\boxed{V = \frac{-mv}{M}}$ — Recoil Velocity

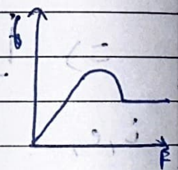
* Concurrent Forces

→ The forces which passes through a same point on the body are known as concurrent forces.

→ The body will remain in equilibrium under the concurrent forces.

→ for The body under the concurrent forces,

$$\boxed{\vec{F}_{net} = 0}$$



* Friction

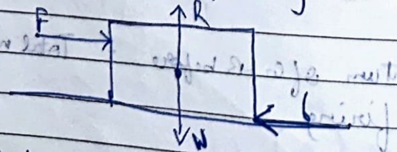
• Law of Static Friction (f_s)

1 → Static friction depends on condition of surface or the state of polish.

& 2 → It is independent of area of contact.

3 → Static friction prevents impending motion.

4 →



Frictional force ~~applied~~ ... solid

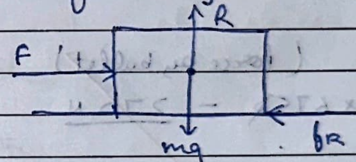
force and tangential to contact point.

→ $\boxed{f_s \propto R}$ — Equation for static friction.

$\boxed{f_s = \mu_s R}$ — μ_s coeff. of static friction.
— μ_s is always b/w 0 to 1

• Law of Kinetic Friction (f_k)

→ 1 from static friction.
→ 2 from static friction.
→ ~~3 from sta~~
→ It is independent of velocity.



→ $f_k \propto R$

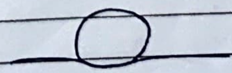
$f_s > f_k$
$\therefore \mu_s > \mu_k$

⇒ $\boxed{f_k = \mu_k R}$

$\mu_k = 0 - 1$

• Rolling friction

$f_r \propto R$ — 1
⇒ $f_r = \mu_r R$ — 1



$f_r \propto \frac{1}{r}$ — 2

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From ① & ②, $\boxed{f_r = \frac{\mu_r R}{r}}$ $\mu_r \rightarrow$ metre

Q. A bullet of mass of 0.04kg moving with 90 m/s enters a wooden block and stop after 60cm. What is the force exerted on bullet?

→ Acceleration of bullet

Using $v^2 - u^2 = 2as$

$8100 = 2 \times a \times 0.6$

$\Rightarrow a = \frac{8100 \times 100}{2 \times 6} = 6750 \ m/s^2$

$F = ma$ (force by bullet)

$\Rightarrow F = \frac{0.04 \times 6750}{100} = 270 \ N$

By Newton's third law of motion,

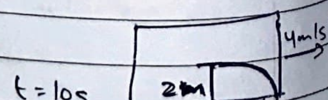
Force exerted by block on bullet = −(force of bullet on block)

= −270N.

Q. A bus starts from rest accelerate uniformly with $4 m/s^2$. At t = 10s, a stone is dropped from the bus 2m high.

① Magnitude of velocity at 10.2 s.

② Acc at 10.2 sec.



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Ans ① $V_x = u + at$
$= 0 + 4 \times 10$
$= 40 m/s$ (In horizontal direction v is constant)

~~$v_y = u \sin\theta t + \frac{1}{2} at^2$~~

∴ v in horizontal direction at t = 10.2 secs is 40m/s.

For vertical direction, (t = 10.2 − 10 = 0.2s)

$\Rightarrow v_y = u_y + at$

$\Rightarrow v_y = 10 \times 0.2$

$\Rightarrow v_y = 2 m/s$

$V_{net} = \sqrt{1604} = 40 m/s$ (approx)

② ~~Acceleration = $\frac{40}{0.2}$~~

② In projectile motion, acceleration is only in vertical direction is ~~units~~ g ($9.8 m/s^2$)

Acceleration in horizontal direction $= 0 m/s^2$.

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Q. A batsman hit a ball without changing its speed of 12 m/s. mass of the ball is 0.15 kg. Calculate impulse.

$\rightarrow I = \Delta p$

$= P_f - P_i$

$= mv_f - mv_i$

$= 0.15(-12-12)$

$= 0.15 \times -24 = \underline{-3.60 N}$

$24 \times 15 = 120 + 240 = 360$

$\therefore$ Impulse $= \underline{-3.6 N}$

Q. A rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20s. Calculate the velocity at the end of 20s.

Ans $\rightarrow$ $I = F \times \Delta t$ & $I = \Delta P$

$\Rightarrow I = 5\times10^5 \times 20$ & $I = mv$ $(u=0)$

$\Rightarrow I = 10^7$ N

$\therefore mv = 10^7$

$\Rightarrow v = \frac{10^7}{2\times10^4} = \boxed{0.5\times10^3 m/s}$

$= \boxed{500 m/s}$

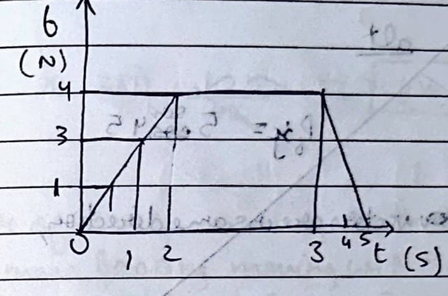
Q. A machine gun fires bullet of mass 40g with 1200 m/s. A person can exert 144N force on it. What is no of bullet fired per second?

Ans $\rightarrow$ $F\Delta t = \Delta P \times n$ - no. of bullets

$\Rightarrow 144 = mv \times n$

$\Rightarrow n = \frac{\cancel{144}^{3} \times 10^2}{\cancel{4} \times 1200} = \underline{3 \text{ bullets}}$

Q.

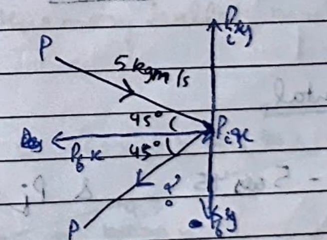


(Impulse)

Area $= \left(\frac{1}{2} \times 4 \times 2\right) + \left(\frac{1}{2} \times 4 \times 2\right) + (4 \times 1)$

$= \underline{12 N}$

Q. Ball moving with 5 kgm/s strike on a wall at an angle of 45°, reflected at the same angle. Calculate change in momentum.



$\Rightarrow |\vec{V_1} + \vec{V_2}| = \sqrt{\ldots}$

$P_i \rightarrow P_{ix} = 5\cos45$ & $P_{iy} = 5\sin45°$

$P_f \rightarrow P_{fx} = -P_{ix}$ & $P_{fy} = -P_{iy}$

$= -5\cos45$ $= -5\sin45$

→ $P_f = 5$ m/s

$P_f - P_i = \sqrt{25+25}$

$= \sqrt{50} = \boxed{5\sqrt{2}\ kgm/s}$

alt

$P_{iy} = 5\sin45$ $P_{fy} = 5\sin45$

As both vectors are same direction,

$\Delta p =$

For vertical,

→ $P_{iy} = 5\sin45$ & $P_{fy} = 5\sin45$

$\Delta P_y = 5\sin45 - 5\sin45$

$= 0$ m/s → No change in momentum

→ For horizontal,

$P_{fx} = -5\cos45$ & $P_{ix} = 5\cos45$

$\Delta P = -5\cos45 - 5\cos45$

$= -5\sqrt{2}$ kgm/s → Change in horizontal

$P_f = \sqrt{\frac{50}{2}} = 5$ m/s

∴ Δp overall $= \sqrt{25+25} = \sqrt{5\ldots} = 5\sqrt{\ldots}$ m/s

Q. A 30kg bomb flying at 48m/s when explodes one part of 18 kg stops, while remaining parts fly on. Find the velocity.

Let velocity be 'x'.

Ans → Momentum of bomb = Momentum of 18kg + Momentum of 12kg

⇒ $30 \times 40 = 18 \times 0 + 12 \times x$

⇒ $x = \cancel{\frac{120}{12}} = \cancel{10\ m/s}$ $\frac{30 \times 48}{12} = \boxed{120\ m/s}$

Q. Car of 1000kg moving with 32 m/s collides with truck of mass 8000kg moving with 4m/s. After collision, car bounced back with 8m/s. Find velocity of truck.

⇒ $(1000 \times 32) + (8000 \times 4) = (1000)(-8) + (8000 \times v)$

⇒ $32000 + 32000 = -8000 + 8000v$

⇒ $64000/8000 = -1 + v$ ⇒ $\boxed{v = 9\ m/s}$

Q. Hunter fires 50g bullet with 150m/s. A 60kg tiger jump with 10m/s at him. How many bullets must be fired by hunter to stop him.

Ans ~~$(50 \times 0) + (60 \times 0) = (60 \times 10) - (50 \times 10^{-3} \times 150) \times n$~~

⇒ ~~$0 = -600 + 4.5n$~~

⇒ ~~$n = \frac{6000}{45} = \frac{400}{3}$~~

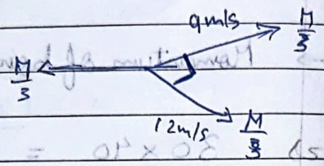
$n \times$ (Momentum of bullet) = Momentum of tiger

⇒ $n \times (50 \times 10^{-3} \times 150) = 60 \times 10$

⇒ $n = \frac{600 \times 10^3}{4500} = \frac{6000}{45}$

$45 \times 8 = 360$

Q. A bomb explodes in 3 parts of equal mass. ~~to~~ 2 fly off right angle to each other with 9m/s & 12m/s. What is speed of 3rd part?



⟹ Let mass of bomb = M

Each part mass = $\frac{M}{3}$

$\Rightarrow 0 = \left(\frac{M}{3}\times 9\right) + \left(\frac{M}{3}\times 12\right) + \left(\frac{M}{3}\times x\right)$

$\Rightarrow \left(-\frac{M}{3}\times x\right) = \left(\frac{M}{3}\times 9\right) + \left(\frac{M}{3}\times 12\right)$

$\Rightarrow -\frac{M\vec{x}}{3} = 3\vec{M} + 4\vec{M}$ (Momentum is vector)

$\Rightarrow \left|-\frac{M}{3}x\right| = \sqrt{9M + 16M}$ $(\theta = 90°)$

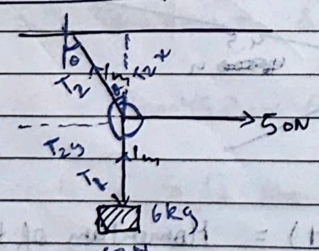
$\Rightarrow -\frac{M}{3}x = 5M$

$\Rightarrow -Mx = 15M$

$x = -15\,m/s$ (direction opp to resultant)

Q. Concurrent forces

1)



Tension?
Angle?

~~$\Rightarrow T_2\cos\theta = 60$ & $T_2\cos\theta = 50$~~

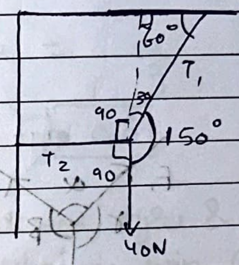
$\Rightarrow T_2\cos\theta = 60N$ — ① $\quad T_2\sin\theta = 50N$ — ②

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{5}{6}$

$\Rightarrow \tan\theta = \frac{5}{6}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{5}{6}\right)$

2)



By Lami's theorem,

$\frac{T_1}{\sin 90} = \frac{T_2}{\sin(150)°} = \frac{40}{\sin 120}$

$\Rightarrow T_1 = \frac{T_2}{\sin 30} = \frac{40}{\cos 30}$

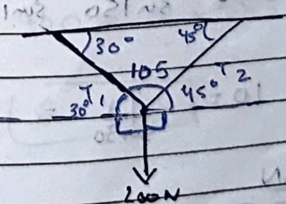
~~$\Rightarrow T_1 = \frac{40}{6}\times\sqrt{3} = 20\sqrt{3}N$~~

~~$\Rightarrow T_2 = \frac{40\times\sqrt{3}}{2}\times\frac{1}{2} = 10\sqrt{3}N$~~

$\Rightarrow T_1 = \frac{80}{\sqrt{3}}N$

$\Rightarrow T_2 = \frac{80}{\sqrt{3}}\times\frac{1}{2} = \frac{40N}{\sqrt{3}}$

3)



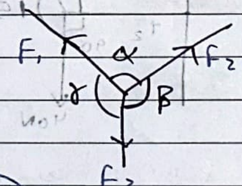
By Lami theorem,

$\frac{T_1}{\sin(135)°} = \frac{T_2}{\sin(120)} = \frac{200}{\sin(105)}$

$$\Rightarrow \frac{T_1}{\sin 45} = \frac{T_2}{\sin 60} = \frac{200}{\sin 75}$$

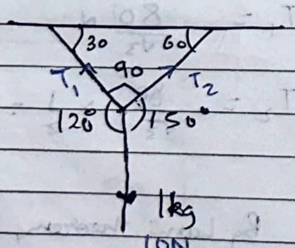
$$\Rightarrow T_1\sqrt{2} = \frac{2T_2}{\sqrt{3}} = \frac{200}{\sin 75}$$

★ Lami's Theorem



$$\rightarrow \frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

Q.



By Lami's Theorem,

$$\rightarrow \frac{10}{\sin 90} = \frac{T_1}{\sin 150} = \frac{T_2}{\sin 120}$$

$$\Rightarrow 10 = \frac{T_1}{\sin 30} = \frac{T_2}{\sin 60}$$

$$\therefore T_1 = 10 \times \frac{1}{2} = 5N$$

$$T_2 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\,N$$

Q. A body rolls on ice with 8 m/s comes to rest after travelling 4 m. Calculate coeff of friction.

$\rightarrow u = 8$, $v = 0$, $s = 4$, $t = ?$
$a = ?$

$$f = \mu R = \mu mg$$

Using $v^2 - u^2 = 2as$
$-64 = 2 \times a \times s$
$\boxed{-8 = a}$

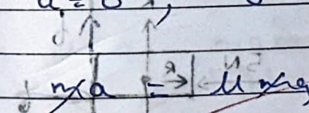
$$\Rightarrow \cancel{m}a = \mu \cancel{m}g$$

$$\Rightarrow \mu = \frac{a}{g}$$

$$\Rightarrow \mu = \boxed{0.8}$$

Q. μ between tyre & road is 0.5. If car start from rest what is min. distance in which it can acquire a speed of 72 km/h.

$\rightarrow u = 0$, $v = 20$ m/s, $\mu = 0.5$



$$\cancel{m}a = \mu \cancel{m}g$$

$$\Rightarrow a = 0.5 \times 10 = 5\,m/s^2$$

* Outcome *

$$\boxed{\mu = \frac{a}{g}}$$

$\rightarrow$ Using $v^2 - u^2 = 2as$,

$$\Rightarrow 400 = 2 \times 5 \times s$$

$$\Rightarrow \boxed{s = 40m}$$

Q. Determine max acceleration of train in which a box lying on its floor remains stationary.

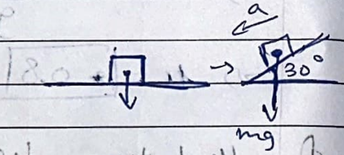
$\mu = 0.15$

$\rightarrow \boxed{a = 1.5 m/s^2}$

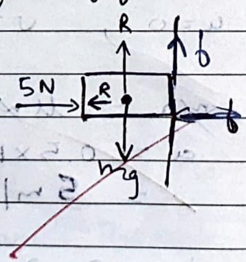
Q. A 4kg block rests on horizontal plane. The plane is gradually inclined till 30°, the begins to slide. Find μ.

$\rightarrow \mu = \tan\theta$

$= \boxed{\frac{1}{\sqrt{3}}}$



Q. A block of 0.1kg is held against a wall by force of 5N. $\mu \rightarrow 0.5$. Calculate force of friction.



$\rightarrow$ ~~5N~~

~~$5 = \mu R$~~

~~$f = \mu R$~~

$\rightarrow$ ~~$f = 0.5 \times$~~

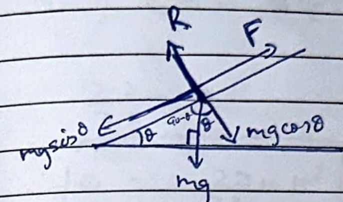
$\rightarrow f = mg$

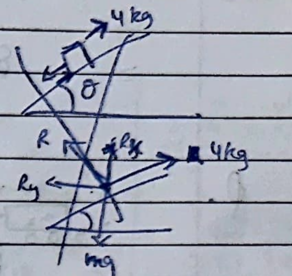
$= 0.1 \times 10 = \boxed{1N}$

Q. 4kg box rest upon an incline plane. The angle is gradually increased when slope becomes 1 in 3 the box starts sliding down. (1) Find μ

(2) F parallel to the plane just makes it move up?

Ans $\rightarrow$



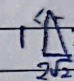


$\sin\theta = \frac{1}{3}$, $m = 4kg$

90+ 90-θ+x = 180
x = θ

$\rightarrow f = \mu R$

$\rightarrow$ ~~$mg \sin\theta = \mu mg \sin\theta$~~ $\quad f = mg\sin\theta$
$R = mg\cos\theta$



$\Rightarrow mg\sin\theta = \mu \, mg\cos\theta$

$\Rightarrow \mu = \frac{mg \sin\theta}{mg\cos\theta}$

$\Rightarrow \boxed{\mu = \frac{1}{2\sqrt{2}}} = \boxed{0.35}$

(2) ~~$F = mg\sin\theta + f$~~

~~$= \frac{40}{3} + = \boxed{\frac{80}{3}} N = \frac{40}{3} + 0.35 \times \frac{30}{2\sqrt{2}}$~~

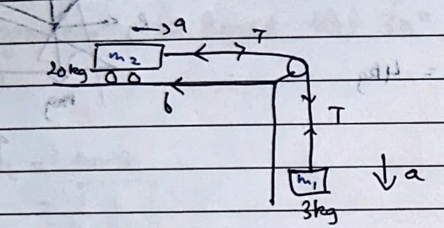
~~$= \frac{40}{3} +$~~

(2) $F = mg(\sin\theta + \mu\cos\theta)$

$= 40\left(\frac{1}{3} + \frac{1}{3}\right)$

$= \frac{80}{3} = \boxed{26.6\,N}$

Q.



$\Rightarrow m_1 g - T = m_1 a$ — (1)

$T - f = m_2 a$ — (2)

$\Rightarrow m_1 g + f = m_1 a + m_2 a$

$\Rightarrow 10(20+3) = a(20+3)$

$\Rightarrow 230 = 23a$

$\Rightarrow \boxed{a = 10\,m/s^2}$

$\mu_k = 0.04$

$f = \mu_k \times R$

$ma = 0.04 \times mg$

$a = 0.4$

$\Rightarrow m_1 g - f = m_1 a + m_2 a$

$\Rightarrow$

$\Rightarrow m_1 g - f = m_1 a + m_2 a$

$\Rightarrow 30 - \mu_k m_2 g = a(20+3)$

$\Rightarrow 30 - 8 = 23a$

$\Rightarrow \frac{22}{23} = a$

$\therefore \boxed{a = \frac{22}{23}\,m/s^2}$

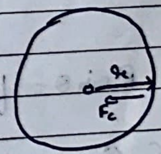
$\frac{4}{100} \times 20 \times 10$

$\Rightarrow T = m_1(g - a)$

$= 3\left(10 - \frac{22}{23}\right)$

$= \frac{3 \times 208}{23} = \boxed{27.1\,N}$

★ Centripetal Force (F_{cp})

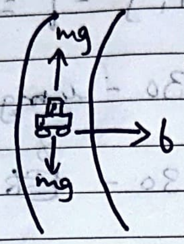


$\Rightarrow F_c = ma$

$$\boxed{F_c = \frac{mv^2}{r} = m\omega^2 r}$$

Eg. 1) Planetary motion. → Gravitational force is F_c.
2) Motion in vertical circle (Swinging stone tied to string)
3) Motion of electron in orbit. → Electrostatic force is F_c
4) Motion of car in roundabout → Friction is F_c

* Motion of car on plane road



$\rightarrow f = \mu R$

$f = mg R$ — ①

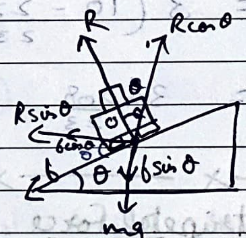
$\frac{mv^2}{r} \leq f$

Min friction reqd. $\rightarrow \frac{mv^2}{r} \leq f$

$\Rightarrow \frac{mv^2}{r} \leq \mu mg$

$\Rightarrow \boxed{v \leq \sqrt{\mu g r}}$ — Max velocity for passing on circular road

* Motion on inclined (banked) road



$\rightarrow R\cos\theta = f\sin\theta + mg$ ①

$\rightarrow R\cos\theta - f\sin\theta = mg$

$\rightarrow R\sin\theta + f\cos\theta \geq \frac{mv^2}{r}$

Eq ② divide by Eq ①,

$\Rightarrow \frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{v^2}{rg}$

(Dividing both by $R\cos\theta$)

$\rightarrow \frac{\tan\theta + f/R}{1 - \frac{f}{R}\tan\theta} = \frac{v^2}{rg}$

$\Rightarrow \frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$

$\Rightarrow \boxed{v \leq \sqrt{\frac{\tan\theta + \mu}{1 - \mu\tan\theta} \times rg}}$ — Max safe speed for banked road.

* Optimum speed $\rightarrow \mu = 0$

$v \leq \sqrt{\tan\theta \cdot rg}$

Q. A level road of $r = 100m$, $\mu = 0.8$.

$\rightarrow v = \sqrt{0.8 \times 1000}$

$= 10\sqrt{8} = \underline{20\sqrt{2}\ m/s}$

Q. For $g = 9.8$, $v = \underline{28 m/s}$

Q. cyclist with $v = 18\ km/h$ make sharp turn road of radius 3m $\mu = 0.1$

$\rightarrow v = 5\ m/s$, ~~$v_{max} = \sqrt{3}\ m/s$~~

As cyclist's velocity is ~~lesser~~ greater than, he ~~can~~ can't pass safely.

Q. Track of $r = 300m$ is bent at 30°. $\mu = 0.2$. Find

① optimum speed. $\rightarrow v = \sqrt{\frac{300 \times 10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}} = \sqrt{1000\sqrt{3}}$

$= 10(10\sqrt{3})^{1/2}\ m/s$

$= \underline{170 m/s}$

② max speed.

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→ Optimum speed = $\sqrt{\frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}}$

$= \sqrt{1000\sqrt{3}} \approx 40 \text{ m/s}$

→ Max speed = $\sqrt{\frac{\frac{1}{\sqrt{3}} + 0.2}{1 - \frac{0.2}{\sqrt{3}}} \times 3000}$

$= \sqrt{\frac{1 + 0.2 \times 1.7}{1.7 - 0.2} \times 300}$

$= \sqrt{\frac{1.34}{1.5} \times 300} = \sqrt{\frac{1340}{5}}$

$= \sqrt{2.68}$

$\approx \boxed{16.6 \text{ m/s}}$

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* Work, Energy, Power

- Concept of Work

→ $W = \vec{F} \cdot \vec{d} \cos\theta$ → Scalar Product of $\vec{F}, \vec{d}$

Types:

1. Positive work → $W = Fs\cos\theta$
   $W = Fs$ ($\cos\theta = 1$)

∴ If body moves in direction force ($\theta = 0$), work is positive!

2. Negative work → $W = Fs\cos\theta$
   $W = -(Fs)$ ($\cos 180 = -1$)

∴ If body moves in direction opposite of force (antiparallel), work is negative.

3. Zero work done → $W = Fs\cos\theta$
   $W = 0$ ($\cos 90 = 0$)

∴ If force is applied perpendicular to body.

∴ If force is 0, work is 0.

∴ If displacement is 0, work is 0.

* Energy

→ Energy is the work done on the body.

$u=0$ O ——→ O $v=v$, d

Types: (1) **Kinetic Energy**

→ $E = Fs$

$E = mas$

$v^2 - u^2 = 2as$

$\boxed{a = \frac{v^2}{2s}}$

$\boxed{KE (W) = \frac{1}{2}mv^2}$ — imp!

→ Energy possessed by a body by virtue of its motion.

* In Momentum terms → $KE = \frac{1}{2m}m^2v^2$

$\boxed{KE = \frac{p^2}{2m}}$ — imp!

◦ **Work Energy Theorem by constant force**

→ $v^2 - u^2 = 2as$ (Multiply by $\frac{1}{2}m$)

⇒ $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(2as)$

⇒ $\boxed{K_f - K_i = Fs}$ — Imp

⇒ $\boxed{\Delta K = W}$

∴ Work = Change in K.E.

* Work done by variable Force

$W = Fd\cos\theta$

→ $\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} dW = \int dW$ = Area under F-x graph

Imp * **W.E.T for variable force**

→ $dW = F\,dx$

$= ma\,dx$

$= m\frac{dv}{dt}\,dx$

$dW = mv\,dv$

⇒ $\int dW = \int_u^v mv\,dv$

⇒ $W = m\int_u^v v\,dv$

⇒ $W = m\left[\frac{v^2}{2}\right]_u^v$

⇒ $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = K_f - K_i$

⇒ $\boxed{W = \Delta K}$ → Hence Proved!

★ **Potential Energy** → Energy possessed by an object by virtue of its position or configuration.

Types: (1) Gravitational P.E. (U)
(2) Elastic P.E.
(3) Electrostatic P.E.

★ **G. P. E**

→

PTO

Gravitational force → Conservative force

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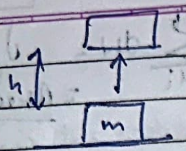

$\Rightarrow W = F \times s$

$\Rightarrow \boxed{W = mgh = G.P.E}$ Imp

$\rightarrow \left\{ F = -\frac{dU}{dx} \right\}$ → Potential gradient
→ conservative force.

⊛ Conservation of Mechanical Energy

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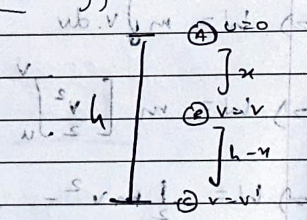
$M.E = KE + PE$

At Point A

$\rightarrow T.E_A = KE_A + PE_A$

$= \frac{1}{2}mv^2 + mgh$

$T.E_A = mgh$ — ①



At Point B

$\rightarrow T.E_B = KE_B + PE_B$

$= \frac{1}{2}mv^2 + mg(h-x)$

$= \frac{1}{2}m(\sqrt{2gx})^2 + mgh - mgx$

$= \frac{1}{2}m \, 2gx + mgh - mgx$

$\therefore TE_B = mgh$ — ②

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At point C

$\rightarrow TE_C = KE_C + PE_C$

$= \frac{1}{2}mv'^2 + mg(0)$

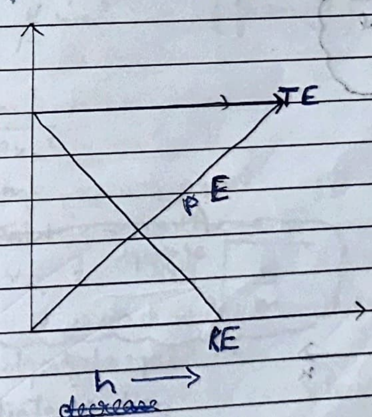
$= \frac{1}{2}m(\sqrt{2gh})^2$

$TE_C = mgh$ — ③

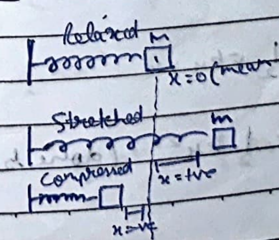
→ Clearly ①, ②, ③ are equal

∴, total energy is always conserved.

also ⇒ {Gain in Kinetic Energy = Loss in Potential Energy}



★ Elastic Potential Energy of Spring



$\rightarrow F \propto x$ → Hooke's law (→ Restoring force)

$\Rightarrow F = -kx$ (→ spring constant)

$\rightarrow dw = \int F\,dx$

$\Rightarrow W = \int_{x_i}^{x_f} -kx\,dx$

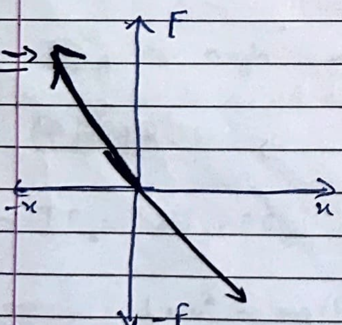
$\Rightarrow W = -\frac{k}{2}(x_f^2 - x_i^2)$

$\Rightarrow \boxed{W = \frac{k}{2}x_i^2 - \frac{k}{2}x_f^2}$ → work done

$\Rightarrow \boxed{E = \frac{1}{2}kx^2}$ (for no initial & final position)

Elastic Potential Energy

graph



→ Area of graph → Work

→ Slope of graph - Force constant

Area $= \frac{1}{2} \times F \times x$

$= \frac{1}{2} \times kx \times x$

$W_{spring} = -\frac{1}{2}kx^2$ (done by spring)

W done on spring $= +\frac{1}{2}kx^2$

Conservation of Energy in Spring

(1) At extreme position

$PE = \frac{1}{2}kx_m^2$

$KE = 0$

$T.E = \frac{1}{2}kx_m^2$ — (1)

(2) At mean

$PE = 0$

$KE = \frac{1}{2}mv_m^2$

$TE = \frac{1}{2}mv_m^2$ — (2)

$\rightarrow \frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$ (from (1))

$\Rightarrow mv_m^2 = kx_m^2$

$\Rightarrow v_m = \sqrt{\frac{kx_m^2}{m}}$

$\therefore v_m = x_m\sqrt{\frac{k}{m}}$ Velocity at mean position (v_{max})

(3) At intermediate position

$KE = \frac{1}{2}mv^2$

$PE = \frac{1}{2}kx^2$

$\Rightarrow TE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

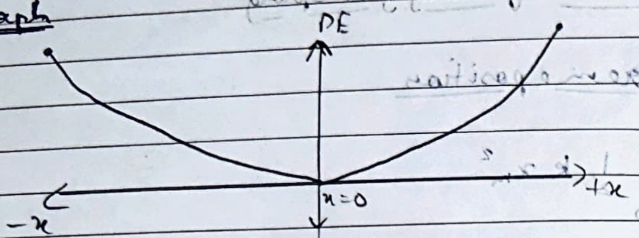
From (1), $\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$\Rightarrow k(x_m^2 - x^2) = mv^2$

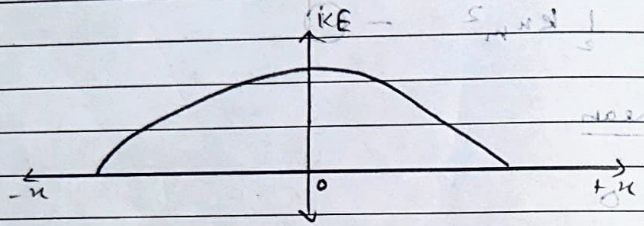
$\Rightarrow \boxed{v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}}$ (v at any instant)

Graph

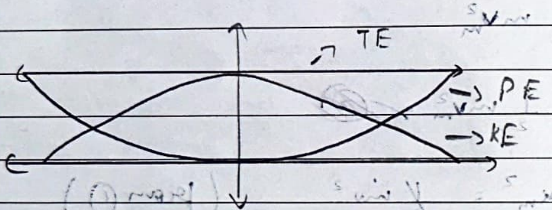
Pot. En



Kin E



TE & all



* Q/A:

Q. Cyclist stops in 10m. During process, force of 200N act on cycle by road.

① How much work done by road on cycle?

② By cycle on road?

① $\theta = 180^\circ$

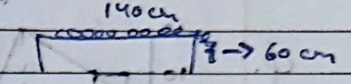
$W = Fd\cos\theta$

$= 200 \times 10 \times -1$

$= -2000\,J$

② As displacement of road is 0, work done on road is 0.

Q. Chain of length 2m kept table such that 60cm hangs freely from the edge. Total mass 4kg. Calculate work done in pulling the entire chain.



→ Mass of 60cm $= \frac{4}{\cancel{200}\,5} \times \cancel{60}$

$= 1.2\,kg$

COM of 60cm mass is at 30cm

$F = 1.2 \times 10 = 12N$

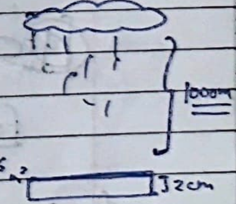
~~W = 12 × 140/100~~

$W = Fd\cos\theta$

$W = 12 \times \frac{30}{100} \times \cos 0$

$\underline{W = 3.6\,J}$

Q. A cloud at height of 1000m burst, rain fell to cover $10^6\,m^2$ with depth 2cm. How much work would have been done in raising the water to the height of cloud?



→ $W = Fd$

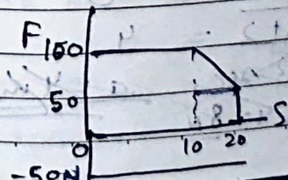
Volume $= 2\times10^4\,m^3$

Mass $= 2\times10^7\,kg$ (as density of water is $10^3\,kg/m^3$)

$\therefore W = 2\times10^7 \times 10 \times 10^3$

$= \boxed{2\times10^{11}\,J}$

Q.



~~$W_1 = (F-f)d$~~

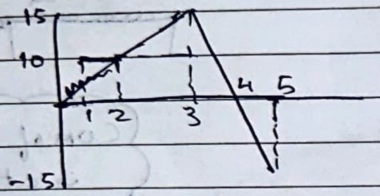
~~$W_1 = 950\times10 = 950$~~

Work done by woman = $(100\times10) + (\frac{1}{2}\times\overset{25}{\cancel{50}}\times10) + (10\times50)$

$= 1000 + 250 + 500 = \boxed{1750\ J}$

Work done by frictional force $= -50\times20 = \boxed{-1000\ J}$

Q.



$\to W = 10 + 10 + (\frac{1}{2}\times5) + (\frac{1}{2}\times15) + \frac{1}{2}\times-15$

$= 20 + 10 - \frac{15}{2}$

$= \frac{60-15}{2} = \frac{45}{2} = \boxed{22.5\ J}$

Q. at $F = 7 - 2x + 3x^2$

Integrating with respect to x

$\int F.dx = \left[7x - x^2 + x^3\right]_0^5$

For $x = 5$, $35 - 25 + 125 = 135\ J$

Q. $x = \frac{t^3}{3}$; $m = 2\ kg$ Calculate work done in first 2 sec

$\to \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^3}{3}\right)$

$\Rightarrow v = t^2$

$\Rightarrow a = 2t$

v at 0 = 0 $a = 2\ m/s^2$

v at 2 = 4

~~For t = 2 secs~~ $\cancel{a = 4\ m/s^2}$

$\cancel{F = 2\times4 = 8\ N}$

$\cancel{= \frac{8}{3}}$

$\cancel{\therefore W = \frac{8\times8}{3} = \frac{64}{3}\ J}$

$\to a = 2t$ $v = \frac{dx}{dt} = t^2$

$= dx = t^2\,dt$

$\to W = Fd$

$= m\,2t\times dx$

$= m\,2t\times t^2\,dt$

$\int W = 2\int 2t^3\,dt$

$\to W = 2\left[\frac{2\times16}{4}\right] = 16\ J$

Q. A toy rocket of mass 0.1kg has fuels of 0.02 kg which burn out in 3 seconds. Starting from rest, it gets a speed of 20m/s after the fuel is burnt out. What is the thrust? Energy per unit mass of fuel?

$\to$ Mass = 0.1kg   Mass of fuel = 0.02 kg
$v = 20\ m/s$ $u = 0\ m/s$ $t = 3$ secs.

$F = ma$

$= m\left(\frac{20}{3}\right)$

$\boxed{F = \frac{2}{3}\ N}$

$\to KE = \frac{1}{2}mv^2$

$= \frac{1}{2}\times\frac{1}{10}\times20\times20 = 20\ J$

Energy per unit mass $= \frac{20}{0.02} = \boxed{1000\ J/kg}$

Q. Bullet of 10g fired with 800m/s after passing through wall of 1m thick, v decreases to 100m/s. Find resistance offered by wall.

→ Mass = 10×10^{-3} kg ; $u = 800$ m/s
$v = 100$ m/s

→ By WET,

$KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$F = \frac{1}{2}\times 10\times10^{-3}\times 100\times100 - \frac{1}{2}\times 10\times10^{-3}\times 800\times800$

⇒ $F = 50 - 3200$

⇒ ~~$F = -2.5$ N~~ $\boxed{F = -3150\text{ N}}$

Q. Bullet of 50 g fired with 200m/s on a plywood of 2cm thickness. The bullet emerge with 10% of initial Kinetic energy. Calculate final kinetic energy & speed.

→ $M = 50\times10^{-3}$ kg $u = 200$ m/s

$KE_i = \frac{1}{2}\times 50\times10^{-3}\times 200\times200$
$= 1000$ J

$KE_f = \frac{10}{100}\times 1000 = \boxed{100\text{ J}}$

$\frac{1}{2}mv^2 = 100$

$v = \sqrt{\frac{100\times2}{50\times10^{-3}}}$ ⇒ $v = \sqrt{4\times10^3}$ $= \boxed{20\sqrt{10}\text{ m/s}}$
≈ 63.2 m/s

Q. Raindrop mass → 1g = 10^{-3} kg
h → 1000 m
v → 50 m/s

→ $\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
$= \frac{1}{2}\times10^{-3}\times 50\times50 = 1250\times10^{-3}$
$= 1.25$ J

$\Delta U = mgh = 10^{-3}\times10\times1000$
$= 10000\times10^{-3} = 10$ J

→ $W_g + W_r = \Delta KE$
→ $W_r = 1.25 - 10$
⇒ $\boxed{W_r = -8.75\text{ J}}$

Q. KE is increased by 300%. By what % linear momentum increases.

→ $P_i = \sqrt{2MK}$ —(1)

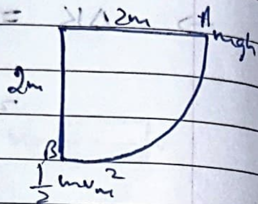
$KE_i = K$ & KE_f } initial & final resp.
P_i & P_f

$KE_f = 4K$

→ $P_f = \sqrt{2M4K}$
$= 2\sqrt{2MK}$
$= 2P_i$

∴ Momentum increases by 100%.

Q. A pendulum of 2m long. It pulled sideways so that string becomes horizontal then it is released. What will be speed at lower position. 10% of initial energy dissipated.



→ Energy at A(PE) = mgh
Energy at B (KE) = $\frac{1}{2}mv^2$

KE = PE

$\Rightarrow \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$\Rightarrow v^2 = \frac{9\cancel{0}}{1\cancel{00}} \times 2 \times 1\cancel{0} \times 2$

$\Rightarrow \boxed{v = 6 m/s}$

Q. 2 spring of k_1 & k_2 ($k_1 > k_2$). Which spring more work is done if:

1. They stretch by same force.
2. They stretch by same amount.

① → Force is same.

$F = k_1 x_1 = k_2 x_2$

$\frac{W_1}{W_2} = \frac{\cancel{\frac{1}{2}} k_1 x_1^2}{\cancel{\frac{1}{2}} k_2 x_2^2} = \frac{k_1 x_1^2}{k_2 x_2^2}$

$= \frac{k_1 x_1 \cdot x_1}{k_2 x_2 \cdot x_2}$

$\frac{W_1}{W_2} = \frac{\cancel{F} x_1}{\cancel{F} x_2} = \frac{x_1}{x_2}$

$\Rightarrow \frac{W_1}{W_2} = \frac{F}{k_1} \times \frac{k_2}{F}$

$\Rightarrow \frac{W_1}{W_2} = \frac{k_2}{k_1}$

$\Rightarrow \boxed{W_1 < W_2}$

∴ Spring 2 does more work.

② $F_1 = k_1 x_1$
$F_2 = k_2 x_2$

$\frac{W_1}{W_2} = \frac{k_1 x_1^2}{k_2 x_2^2}$

$\frac{W_1}{W_2} = \frac{k_1}{k_2} \Rightarrow W_1 > W_2$

∴ Spring 1 does more work.

Q. PE of spring is 10 J, when displaced by 'x' m. What is the work done on it to stretch through an additional distance 'x'?

→ ATQ, $PE = \frac{1}{2}kx^2$ (For distance x)

$10 J = \frac{1}{2}kx^2$

Now distance is increased to 2x.

∴ $PE = \frac{1}{2}kx^2$

$PE = \frac{1}{2}k \cdot 4x^2$

$PE = 4 \times 10 = \boxed{40 J}$

∴ Work done to displace spring $= 40 - 10 = 30 J$

* Collisions

(1) Elastic Collision → Kinetic energy remains conserved.
→ Total Energy conserved.
→ Momentum is conserved.
→ No energy lost (Dissipative force absent)

(2) Inelastic Collision → Kinetic energy is not conserved.
→ Total energy is conserved.
→ Momentum is conserved.
→ Dissipative forces present.

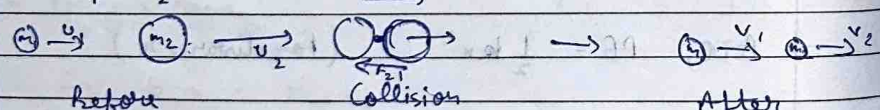
- Perfectly Inelastic Collision

→ If two objects joined together after collision.

- Head-on Collision → Both object travel in same direction after collision.

★ Elastic Collision in 1D

$u_1 > u_2$



By LCLM,

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ — (1)

$\Rightarrow m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2)$ — (2)

→ By Conservation of K.E.,

$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Rightarrow m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$

$\Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

$\Rightarrow m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$ — (3)

→ Eq (3) divided by (2)

$\Rightarrow u_1 + v_1 = v_2 + u_2$

$\Rightarrow \boxed{v_2 = u_1 + v_1 - u_2}$ — (4)

From (1),

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$

$\Rightarrow \cancel{m_1(v_1 + v_2}$

$\Rightarrow v_1(m_1 + m_2) = \cancel{v_1(m_1 + m_2)}\ m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$

$\Rightarrow \boxed{v_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2}}$ — Final Velocity of particle responsible for collision.

→ From (4), $v_1 = v_2 + u_2 - u_1$

We get $\boxed{v_2 = \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2 + \left(\frac{2m_1}{m_1+m_2}\right)u_1}$

↳ Velocity of other particle in collision

o Cases

① Case-1 → If masses of bodies are same - m

$$\boxed{v_1 = u_2 \quad \& \quad v_2 = u_1}$$

② Case-2 → $u_2 = 0$ & masses are same.

$$\boxed{v_1 = u_2 = 0 \quad \& \quad v_2 = u_1}$$

③ Case-3 → $m_1 > m_2$, $u_2 = 0$

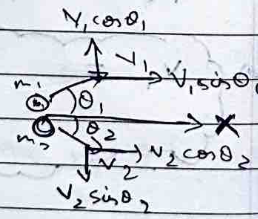
$$\boxed{v_1 = u_1 \quad \& \quad v_2 = 2u_1}$$

④ Case-4 → $m_2 >> m_1$, $u_2 = 0$

$$v_1 = -u_1 \qquad v_2 = 0$$

* Elastic Collision in 2D

(m_1) u_1 → $\quad$ (m_2) u_2



→ By LCLM → X axis

$$m_1u_1\cancel{\cos\theta_1} + m_2u_2\cancel{\cos\theta_2} = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

$$\Rightarrow m_1u_1\cancel{\cos\theta_1} = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \quad \text{— (1)}$$

→ By LCLM → Y axis

$$\Rightarrow 0+0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

$$\Rightarrow m_2v_2\sin\theta_2 = m_1v_1\sin\theta_1 \quad \text{— (2)}$$

→ KE ⇒ $\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ — (3)

Case-1 → $m_1 = m_2 = m$ $\qquad u_1^2 = v_1^2 + v_2^2$

$$\Rightarrow m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$\Rightarrow u_1 = v_1 + v_2$$

$$\Rightarrow u_1 \cdot u_1 = (v_1+v_2)\cdot(v_1+v_2)$$

$$\Rightarrow u_1^2 = v_1^2 + v_2^2 + 2v_1\cdot v_2$$

$$\Rightarrow \cancel{u_1^2} = \cancel{u_1^2} + 2v_1\cdot v_2$$

$$\Rightarrow v_1\cdot v_2 = 0$$

$\therefore \theta = 90^\circ$ i.e. $v_1 \perp v_2$

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Q. A railway compt. of 9000kg moving with 36 km/h (10 m/s) collides with a stationary compt. of same mass. After collision they both get coupled & move together. What is their common speed.

→ $m = 9000 kg$

$u_1 = 10 m/s$ $u_2 = 0$

$V_1 = V_2 = V$

By LCLM,

⇒ $mu_1 = mv + mv$

⇒ $u_1 = 2v$

⇒ $10 = 2v$

⇒ $\boxed{v = 5 m/s}$

Initial KE = $\frac{1}{2} m u_1^2$

$= \frac{1}{2} \times 9000 \times 100 = 450000 J$

Final KE = ~~$\frac{1}{2} \times 9000 \times 25 = 125000 J$~~

$= mv^2 = 9000 \times 25 = 225000 J$

→ Inelastic Collision

Q $m_1 = m_2 = m$

2 bodies collide with equal speed in opp. direction to each other (Elastic). Predict outcomes. Calculate v_1 & v_2

→ $m_1 = m_2 = m$

$u_1 = u$, $u_2 = -u$

→ $v_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2}$

⇒ $v_1 = -u$

→ $v_2 = \frac{u_2(m_2 - m_1) + 2m_1u_1}{m_1 + m_2}$

⇒ $v_2 = u_1 = u$ (u_2)

∴ $v_1 = -u$ m/s & $v_2 = u$ m/s

* Power

→ Rate of doing work.

* $\boxed{P = \frac{W}{t}}$ * $\boxed{P = \frac{dW}{dt}}$

⇒ $P = \frac{d(F \cdot dx)}{dt}$

* $\boxed{P = Fv}$ $\boxed{P = \text{force} \cdot \text{Velocity}}$

SI-unit → J/sec = Watt hp – horsepower

Q. A car of 2000kg lifted up by 30m. in 1 minute by crane 1. Another crane does same work in 2 minute. Calculate power.

→ $W = mgh$

$= 2000 \times 10 \times 30 = 600000 J$

$\boxed{1hp = 746\,W}$

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$$P_1 = \frac{600000}{60} = 10{,}000\,W$$

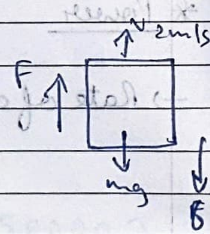
$$P_2 = \frac{600000}{120} = 5000\,W$$

→ Do cranes consume same fuel?

→ Same fuel as work done is same.

16/8/24

Q→ A lift of 1800kg moving up with 2m/s. Friction = 4000N. Calculate max power.



→ At equilibrium, $F = mg + f$

$\therefore F = 18000 + 4000$

$= 22000N$

$\therefore P = F \times v$

$= 22000 \times 2 = 44000\,W$

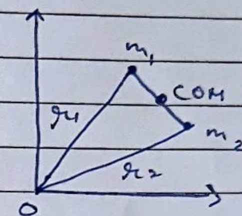
$= 44\,kW$

Date: 16/08/24

* Ch-6 : System of Particles & Rotational Motion

* Centre of Mass

→



→ $COM = \dfrac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

∴, if n particles are there,

$$\boxed{COM = \frac{m_1 r_1 + m_2 r_2 + \cdots m_n r_n}{m_1 + m_2 + m_3 \cdots m_n}}$$

∴ Coordinates of COM for ~~two~~ 3 dimensional body:

$$X = \frac{m_1 x_1 + m_2 x_2 \cdots m_n x_n}{m_1 + m_2 \cdots m_n}$$

$$Y = \frac{m_1 y_1 + m_2 y_2 \cdots m_n y_n}{m_1 + m_2 \cdots m_n}$$

$$Z = \frac{m_1 z_1 + m_2 z_2 \cdots m_n z_n}{m_1 + m_2 \cdots m_n}$$

* If masses are same, i.e, mass distribution is uniform,

$COM = \dfrac{r_1 + r_2 \cdots r_n}{n}$, i.e, at geometrical centre