

**Q1 Team name**

0 Points

Kerberos

**Q2 Commands**

10 Points

List the commands used in the game to reach the ciphertext.

```
'enter' 'enter' 'pick' 'c' 'c' 'back' 'give'
'back' 'back' 'thrnxtzy' 'read'
```

**Q3 Analysis**

50 Points

Give a detailed analysis of how you figured out the password? (Explain in less than 500 words)

Using % as modulus operator

using p = prime number = 19807040628566084398385987581

We solved the question mainly using the modular arithmetic and one brute force method to calculate the value of g and then password.

---> Given the pairs (a, b) of the form  $b = \text{password} * g^a$ .

$$11226815350263531814963336315 = \text{password} * g^{324} \quad \text{-eqn 1}$$

$$9190548667900274300830391220 = \text{password} * g^{2345} \quad \text{-eqn 2}$$

$$4138652629655613570819000497 = \text{password} * g^{9513} \quad \text{-eqn 3}$$

---> Dividing one equation with other

$$(g^{A1} / g^{A2}) = (B1 / B2) \% p$$

$$\Rightarrow g^{(A1-A2)} = (B1 * B2^{-1}) \% p$$

$$\Rightarrow g^{(A1-A2)} = (B21) \% p$$

So the equation we got were

$$g^{2021} = 7021284369301638640577066679 \% p \quad \text{-eqn 21) Dividing 2 by 1}$$

$$g^{7168} = 6339248851737327508924059257 \% p \quad \text{-eqn 32) Dividing 3 by 2}$$

$$g^{9189} = 3426347385144995225825016781 \% p \quad \text{-eqn 31) Dividing 3 by 1}$$

---> "BRUTE FORCE/ HIT AND TRIAL FUNCTION" --

Assuming that if by dividing or multiplying in some order the above three expression, if the exponent on g becomes 1 we can get value of g directly. So i tried to find possible set of values(i, j, k) such that  $a31 * i + a32 * j + a21 * k = 1$  using a hardcoded program.

And we found many sets of such values.

We picked one set (139,0,632) as it had a 0 for a32 which made calculation easier  
 $-9189 \cdot 139 + 632 \cdot 2021 = 1$

---> Therefore after appropriate multiplication and division, we got 2 new expressions

From 31

$$g^{(a_{31} \cdot 139)} = (g^{a_{31}})^{139} = b_{31}^{139} \pmod{p}$$

$$\Rightarrow g^{1277271} = 3426347385144995225825016781^{139} \pmod{p} = 17064457453994872811494067145$$

From 21

$$g^{(a_{21} \cdot 632)} = (g^{a_{21}})^{632} = b_{21}^{632} \pmod{p}$$

$$\Rightarrow g^{1277272} = 7021284369301638640577066679^{632} \pmod{p} = 9145714735161140899390199931$$

Dividing above two equations

$$g^{(1277272 - 1277271)} = (9145714735161140899390199931 \cdot (17064457453994872811494067145^{-1}) \pmod{p}) \pmod{p}$$

$$\Rightarrow g = 192847283928500239481729$$

\*\*\* Here we matched this g with the given g = 1\_\_4\_\_2\_\_\_\_0\_\_94\_\_\_\_9 with missing entries and it matched.

---> Finally we computed password using the eqn 1 by substituting value of g

$$\text{password} = (b_1 \cdot (g^{a_1})^{-1}) \pmod{p}$$

$$\text{password} = 3608528850368400786036725$$

--NOTE: I may have missed %p in some places. Please forgive for missing %p in those places for the mathematical equations.

## Q4 Password

10 Points

What was the final command used to clear this level?

3608528850368400786036725

## Q5 Codes

0 Points

Upload any code that you have used to solve this level.

▼ Assignment\_3.ipynb

Download

In [37]:

```
#Input Given Pair of (a, pass*g^a) as (a,b)
# P is the prime number belonging to the field Zp*
```

```
p = 19807040628566084398385987581
(a1,b1) = (324, 11226815350263531814963336315) #--1
(a2,b2) = (2345, 9190548667900274300830391220) #--2
(a3,b3) = (9513, 4138652629655613570819000497) #--3
```

```
In [4]: # Simplifier : Dividing one equation with other
# g^a1 / g^a2 (mod p) = b1 / b2 (mod p)
# => g^(a1-a2) (mod p) = (b1 * b2^-1) mod p

def eqndivider( a1,a2,b1,b2 ):
    a = a1-a2
    b = ( b1 * pow(b2,-1,p) ) %p
    return(a,b)
```

```
In [38]: # Computing equation for all possible pair from given 3 sets of
points
# aij means ai/aj

a21,b21 = eqndivider(a2,a1,b2,b1)
a32,b32 = eqndivider(a3,a2,b3,b2)
a31,b31 = eqndivider(a3,a1,b3,b1)
```

```
In [40]: # PRINT
a21,a32,a31,b21,b32,b31
```

```
Out [40]: (2021,
7168,
9189,
7021284369301638640577066679,
6339248851737327508924059257,
3426347385144995225825016781)
```

```
In [ ]:
```

```
In [45]: # ----- Hit and Trial -----
# My aim was to somehow make the exponent in g^e e = 1.
# So I tried to search for set of values ( i, j, k) such that
# any combination of the +- e1 * i +- e2 * j +- e3 * k becomes 1.
# Luckily i found many. Below is the code
# -
# Also stopped the execution after getting sufficient information

for i in range(1000):
    for j in range(1000):
        for k in range(1000):
            if( -a31 * i + a32 * j + a21 * k == 1 ):
                print(i, ' ',j, ' ',k)
                break
```

```
139    0    632
140    1    633
141    2    634
142    3    635
143    4    636
144    5    637
145    6    638
146    7    639
147    8    640
148    9    641
149   10    642
150   11    643
151   12    644
152   13    645
153   14    646
```

```

154 15 647
155 16 648
156 17 649

```

```

-----KeyboardInterrupt                                Traceback (most
recent call last)<ipython-input-45-9269dfc3af53> in <module>
      4     for j in range(1000):
      5         for k in range(1000):
----> 6             if( -a31 * i + a32 * j + a21 * k == 1 ):
      7                 print(i, ' ', j, ' ', k)
      8                 break
KeyboardInterrupt:

```

```

In [60]: # Since a21 * 632 - 139 * a31 = 1
# Adding 632 and 139 as exponents in both side for each equation
# makes
# g ^ ( a31 * 139 ) = (g ^ a31)^ 139 = b31 ^ 139 = num31
# g ^ ( a21 * 632 ) = (g ^ a21)^ 632 = b21 ^ 632 = num21

num31 = pow(b31, 139, p)
num21 = pow(b21, 632, p)

print('num31 = ', num31, '\nnum21 = ', num21)

num31 = 17064457453994872811494067145
num21 = 9145714735161140899390199931

```

```

In [61]: # Finally dividing two updated equation
# num31 / num 21 = g ^ ( a21* 632 - a31*139 ) = g

g = ( num21 * pow(num31,-1,p) ) %p
print('g = ', g)

g = 192847283928500239481729

```

```

In [62]: # Verifier that g matches the partial characters given in question

# Calculating password using equation 1

password = (b1 * pow( pow(g,a1,p) , -1, p ) ) %p
print('Password = ', password)

Password = 3608528850368400786036725

```

```

In [ ]:

```

GROUP  
SUMIT SINGH SHEORAN  
MANI KANT KUMAR  
AMAN ARYAN  
 [View or edit group](#)

TOTAL POINTS  
**70 / 70 pts**

QUESTION 1  
[Team name](#) **0 / 0 pts**

QUESTION 2  
[Commands](#) **10 / 10 pts**

QUESTION 3  
[Analysis](#) **50 / 50 pts**

QUESTION 4  
[Password](#) **10 / 10 pts**

QUESTION 5  
[Codes](#) **0 / 0 pts**