

Prob 4

Solving using the two dimensional version of Newtons method

Our equations are:

$$3x^2 - 1y^2 = 0$$

$$-1x^2 + 3xy^2 - 1 = 0$$

The first iteration of Newtons methdo using a guess of [1,1] gives us:

[0.631578947368421, 0.8947368421052632]

using the infinity norm our error is:

0.368421052632

...

Iteration 2:

[0.5147475360559554, 0.8686913766633854]

Error: 0.130576283232

...

Iteration 3:

[0.5223831442795319, 0.9054436885714463]

Error: 0.0423076743886

...

Iteration 4:

[0.5204292071132371, 0.9014124892667191]

Error: 0.00445218113021

...

Iteration 5:

[0.5208678548198985, 0.9021695863387289]

Error: 0.00083990080127

...

Iteration 6:

[0.5207670846235098, 0.901995049477007]

Error: 0.000193463473348

...

Iteration 7:

[0.5207901245087303, 0.9020349557292424]

Error: 4.42422076025e-05

...

Iteration 8:

[0.5207848509316706, 0.9020258216258374]

Error: 1.0126108026e-05

...

Iteration 9:

[0.5207860576928399, 0.9020279117974953]

Error: 2.3171971442e-06

Solving by using the one dimensional method

For this case $f(x)$ is first equation minus the second equation
for the first iterations we will guess the x value is 1, which
makes the $f(1) = 0.915554226641$
which gives us the $x_1: 0.664883894507$

using the infinity norm our error is:
0.335116105493

...

Iteration 2:

$f(x_1) = 0.301337066928$

$x_2 = 0.531894517046$

Error: 0.200018948511

...

Iteration 3:

$f(x_2) = 0.0246140765581$

$x_3 = 0.520863934851$

Error: 0.25002960775

...

Iteration 4:

$f(x_3) = 0.000174045913882$

$x_4 = 0.52078586871$

Error: 0.000149900651395

...

Iteration 5:

$f(x_4) = 7.96710600826e-08$

$x_5 = 0.520785832975$

Error: 6.86187734287e-08

...

Iteration 6:

$f(x_5) = 3.2717273335e-11$

$x_6 = 0.52078583296$

Error: 2.81786447192e-11

...

Iteration 7:

$f(x_6) = 1.3433698598e-14$

$x_7 = 0.52078583296$

Error: 1.15118422076e-14

...

Iteration 8:

$f(x_7) = 2.22044604925e-16$

$x_8 = 0.52078583296$

Error: 2.13182263103e-16

...

Iteration 9:

$f(x_8) = -1.11022302463e-16$

$x_9 = 0.52078583296$

Error: 0.0

For this problem we first expanded the polynomial.py file from our first practice homework to include polynomials with 2 variables and an evaluate function to solve the polynomial for a given x and a derivative function for polynomials with 2 variables.

Then we created the newtons2d function that when given 2 of the above polynomials and a guess vector, produces the next iteration of newtons method. We also created the newton function that does the same thing for single variable functions. Finally we made functions to check the tolerance of our guesses.

I was really surprised at how quickly the single variable versions error shrunk. After 9 iterations it is 10^{-16} away from 0. The two variable version, while still very fast, was much slower. After 9 iterations the error is still $2 \cdot 10^{-6}$.



