Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875

    B. 0.2676

    C. 0.5

    D. 0.6987

ANS:

Option B. 0.2676. Refer python file.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

    A. More employees at the processing center are older than 44 than between 38 and 44.

ANS: False. Refer python file for details.

    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS: True. Refer python file.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS:

The main difference between 2X1 and X1 + X2 is that the variance of 2X1 is twice the variance of X1 + X2. This is because 2X1 is essentially the same as adding X1 to itself, which doubles the variance. On the other hand, X1 + X2 is adding two independent random variables, which only increases the variance by a factor of 2.

In other words, 2X1 is more spread out than X1 + X2. This can be seen by comparing their standard deviations, which are the square roots of their variances. The standard deviation of 2X1 is 2σ, while the standard deviation of X1 + X2 is sqrt(2)σ. Both 2X1 and X1 + X2 are normally distributed random variables with mean 2μ. However, the variance of 2X1 is twice the variance of X1 + X2. This means that 2X1 is more spread out than X1 + X2.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

ANS: D. 48.5, 151.5, refer python file.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1$ ~ N(5, 3^2) and $Profit_2$ ~ N(7, 4^2) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS: Range in Rs.= (99.00810347848784, 980.9918965215122) millions. Refer python file.

    B. Specify the 5^{th} percentile of profit (in Rupees) for the company

ANS: 5th percentile of profit in Rs.= 170.0 millions.

    C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS: Probability of devision1 going into loss is more than the second.