N 1

$M[\xi] = \frac{\Theta}{2}$

$M[\xi^2] = \int_0^\Theta x^2 \frac{1}{\Theta}\, dx = \frac{\Theta^2}{3}$

$D[\xi] = \frac{\Theta^2}{3} - \frac{\Theta^2}{4} = \frac{\Theta^2}{12}$

а) 1) $M\left[\frac{2}{n}\sum_1^n x_i\right] = \frac{2}{n}\sum M[x_i] = 2M[\xi]$ –

$\left\{ \begin{array}{l} x_i \sim R(0,\Theta) \\ x_i = \xi \end{array} \right\}$ – несмещен.

дост. усл.

$D[\tilde\Theta_1] = D\left[\frac{2}{n}\sum x_i\right] = \frac{4}{n^2}\sum D x_i =$

$= \frac{4}{n} D[\xi] = \frac{\Theta^2}{3n} \xrightarrow[n\to\infty]{} 0$

$\Rightarrow \tilde\Theta_1$ – сост. по д.у.

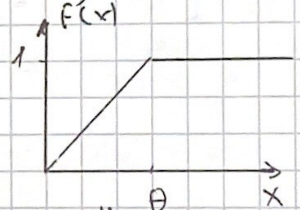
2) $\xi \sim F(x)$ независим. $\Rightarrow$

$\Rightarrow \min(\xi_1 \dots \xi_n) \sim \underbrace{1-(1-F(y))^n}_{\Phi(y)}$

$\varphi(y) = \Phi'(y)$

плотн. распр. min

$\Rightarrow n(1-F(y))^{n-1} p(y)$



$p(y) = \frac{1}{\Theta}\ (0,\Theta)$

$M[\tilde\Theta_2] = \int_0^\Theta y\,\varphi(y)\,dy = \int_0^\Theta yn\left(1-\frac{y}{\Theta}\right)^{n-1}\frac{1}{\Theta}\,dy =$

$= \left\{ \begin{array}{l} t = y/\Theta \\ dt = \frac{1}{\Theta}dy \end{array} \right\} = \int_0^1 tn(1-t)^{n-1}\Theta\, dt = n\Theta B(2,n) =$

$= n\Theta\frac{\Gamma(2)\Gamma(n)}{\Gamma(n+2)} = \left\{ B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \right\}$ $(n+1)n\Gamma(n)$ $= \frac{n\Theta}{n(n+1)} = \frac{\Theta}{n+1}$ – смещенная

Возьмем $\tilde{\Theta}_2' = (n+1)\tilde{\Theta}_2 = (n+1)x_{min}$
несмещ.

$$M[\tilde{\Theta}_2] = \int_0^{\Theta} y^2 n \left(1-\frac{y}{\Theta}\right)^{n-1}\frac{1}{\Theta}\,dy =$$

$$= \int_0^1 t^2(1-t)^{n-1}\Theta^2 n\,dt = n\Theta^2 B(3,n) =$$

$$= n\Theta^2 \frac{\Gamma(3)\Gamma(n)}{\Gamma(n+3)} = \frac{2\Theta^2}{(n+1)(n+2)}$$

$$D[\tilde{\Theta}_2] = 2\Theta^2/(n+1)(n+2) - \Theta^2/(n+1)^2 =$$

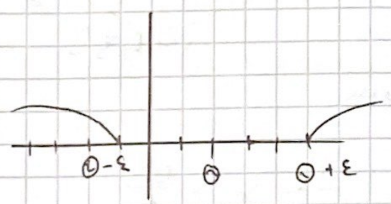
$$= \Theta^2\frac{2n+2-n-2}{(n+2)(n+1)^2} = \Theta^2 \frac{n}{(n+2)(n+1)^2}$$

$$\Rightarrow D[\tilde{\Theta}_2'] = D[(n+1)\tilde{\Theta}_2] =$$

$$= \Theta^2\frac{n}{n+2} \xrightarrow[n\to\infty]{} \Theta^2 \neq 0 \Rightarrow$$ д.у. не работает

по опред.: $\tilde{\Theta}_2' \xrightarrow{p} \Theta$

$$\forall \varepsilon > 0 \quad P(|\tilde{\Theta}_2' - \Theta| \geq \varepsilon) \xrightarrow[n\to\infty]{} 0$$



$$P(|\tilde{\Theta}_2' - \Theta| \geq \varepsilon) \geq P(\tilde{\Theta}_2' \geq \Theta+\varepsilon) =$$

$$= P((n+1)\tilde{\Theta}_2 \geq \Theta+\varepsilon) =$$

$$= P\left(\tilde{\Theta}_2 \geq \frac{\Theta+\varepsilon}{n+1}\right) =$$

$$= P\left(x_1 \geq \frac{\Theta+\varepsilon}{n+1}, \dots, x_n \geq \frac{\Theta+\varepsilon}{n+1}\right) =$$

$$= \prod_i P\left(x_i \geq \frac{\Theta+\varepsilon}{n+1}\right) = \left(P\left(\xi \geq \frac{\Theta+\varepsilon}{n+1}\right)\right)^n =$$

$$= \left(1 - F\left(\frac{\Theta+\varepsilon}{n+1}\right)\right)^n = \left(1 - \frac{\Theta+\varepsilon}{(n+1)\Theta}\right)^n \xrightarrow[n\to\infty]{} 0$$ при $n\to\infty$)

$$\xrightarrow[n\to\infty]{} e^{-\frac{\Theta+\varepsilon}{\Theta}} > 0$$ (противореч.) => несостоят.)

3) $\tilde{\Theta}_3 = x_{max}$

$\xi \sim F(x) \Rightarrow \max(\xi_1 \dots \xi_n) \sim (F(z))^n =$

$= \Psi(z)$

$\Psi(z)$ - мощность

$$\psi(z) = nF^{n-1}(z)p(z) = n\left(\frac{z}{\Theta}\right)^{n-1}\cdot\frac{1}{\Theta}\ (0,\Theta)$$

$M[\hat{\Theta}_3] = \int_0^{\Theta} z n \left(\frac{z}{\Theta}\right)^{n-1} \frac{1}{\Theta} dz =$

$= n\Theta \int_0^1 t^n dt = \frac{n}{n+1}\Theta$ — смещенная

$\hat{\Theta}_3' = \frac{n+1}{n} \cdot x_{max}$ — несмещ.

$M[\hat{\Theta}_3^2] = \int_0^{\Theta} z^2 n \left(\frac{z}{\Theta}\right)^{n-1} \frac{1}{\Theta} dz = \int_0^1 n\Theta^2 t^{n+1} dt =$

$= \frac{n}{n+2}\Theta^2$

$D[\hat{\Theta}_3] = \frac{n}{n+2}\Theta^2 - \frac{n^2}{(n+1)^2}\Theta^2 =$

$= \Theta^2 \left[\frac{n(n+1)^2 - n^2(n+2)}{(n+2)(n+1)^2}\right] = \dots = \frac{n\Theta^2}{(n+2)(n+1)^2}$ —

— D смещенной

$D[\hat{\Theta}_3'] = D\left[\frac{n+1}{n}\hat{\Theta}_3\right] = \frac{\Theta^2}{n(n+2)} \xrightarrow[n\to\infty]{} 0$ —

— по д.у. состоят.

4) $\hat{\Theta}_5 = x_1 + \frac{1}{n-1}\sum_{k=2}^{n} x_k$

$M[\hat{\Theta}_5] = \underbrace{M[x_1]}_{M[\xi]} + \frac{1}{n-1}(n-1)\underbrace{M[x_2]}_{M[\xi]} =$

$= \frac{\Theta}{2} + \frac{\Theta}{2} = \Theta$ — несмещ.

$\hat{\Theta}_5 = x_1 + \frac{1}{n-1}\sum_{k=2}^{n} x_k \xrightarrow{p} x_1 + \frac{\Theta}{2}$ — несост.

$\begin{cases} \xi_n \xrightarrow{p} \xi \\ \eta_n \xrightarrow{p} \eta \end{cases} \Rightarrow \xi_n + \eta_n \xrightarrow{p} \xi + \eta$; $\Theta/2$ т. Хинчина (ЗБЧ)

б) Выберем самую эффективную из

$\hat{\Theta}_1$ и $\hat{\Theta}_3'$:

$D[\hat{\Theta}_1] = \Theta^2/3n$ $\Rightarrow$ $\hat{\Theta}_3'$ — более эффективная

$D[\hat{\Theta}_3'] = \frac{\Theta^2}{n(n+2)}$