I задание Мат. статистика

N 3

$$p(x) = \begin{cases} \frac{e^{-x/\theta}}{\theta}, & x \geq 0 \\ 0, & x < 0 \end{cases} \qquad \theta > 0$$

а) Несмещенность:

1) $\tilde{\theta}_1 = \bar{x}$

$$M[\tilde{\theta}_1] = \frac{1}{n} \sum_{i=1}^{n} M[\xi] = \int_0^{\infty} \frac{x}{\theta} e^{-x/\theta} dx =$$

$$= \theta \int_0^{\infty} t e^{-t} dt = \theta \Rightarrow \tilde{\theta}_1 \text{ - несмещ.}$$

2) $\tilde{\theta}_3 = x_{(2)}$

$$M[\tilde{\theta}_3] = \int_0^{\infty} 3x C_2^1 \left(1 - e^{-x/\theta}\right) e^{-x/\theta} \frac{e^{-x/\theta}}{\theta} dx =$$

$$= 6/\theta \left(\frac{5\theta^2}{36}\right) = 5/6 \; \theta \Rightarrow \tilde{\theta}_3 \text{ - смещ.}$$

Исправим: $\tilde{\theta}_3' = \frac{6}{5} x_{(2)}$

b) Эффективность:

$$D[\tilde{\theta}_1] = \frac{1}{9} \sum D[\xi] = \frac{1}{3} D[\xi] =$$

$$= \frac{1}{3} \left(\int_0^{\infty} x^2 \frac{e^{-x/\theta}}{\theta} dx - \left(\int_0^{\infty} x \frac{e^{-x/\theta}}{\theta} dx \right)^2 \right) =$$

$$= \frac{1}{3} \theta^2$$

$$D[\tilde{\theta}_3'] = \frac{36}{25} \theta^2 \left(12 \left(\frac{1}{3} - \frac{1}{27} \right) - \frac{25}{30} \right) =$$

$$= 13/25 \; \theta^2$$

$\Rightarrow \theta_1$ - наиболее эфф.

в) Неравенство Крамера-Рао:

$$D[\tilde{g}(\vec{x}_n)] \geq \frac{g'^2(\theta)}{nI(\theta)}$$

$$g(\theta) = \theta \Rightarrow g'(\theta) = 1$$

$$I(\theta) = -M\left[\frac{\partial^2 \ln p}{\partial \theta^2}\right] = -M\left[\frac{\partial^2 \left(-\frac{x}{\theta} - \ln \theta\right)}{\partial \theta^2}\right] =$$

$$= M\left[\frac{2x}{\theta^3} - \frac{1}{\theta^2}\right] = \frac{2}{\theta^3} M[\xi] - \frac{1}{\theta^2} = \frac{1}{\theta^2}$$

$$\frac{g'^2(\theta)}{n I(\theta)} = \frac{1}{3}\theta^2$$

$D[\tilde{\theta}] = \frac{1}{3}\theta^2 \Rightarrow \tilde{\theta}_1 = \bar{x}$ – самая эффективная оценка.