№ 10

а) $n = 1$; α

Нейман - Пирсон

$l = \frac{p_1}{p_0} = \frac{e}{e-1} e^{-x} \geq C$

$e^{-x} \geq B$

$G: x \leq A$ по Нейману - Пирсону

$P(x \leq A \mid H_0) = \alpha$

$\int_0^A 1 \, dx = \alpha$

$A = \alpha$

$G: x \leq \alpha$

$\alpha_1 = P(H_1 \mid H_0) = \alpha$

$W = P(x \leq A \mid H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx =$

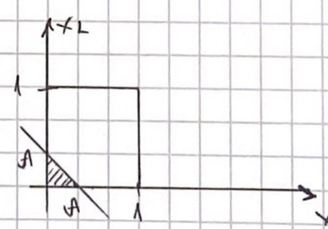
$= \frac{e}{e-1} e^{-x} \Big|_\alpha^0 = \frac{e}{e-1} (1 - e^{-\alpha})$

$\alpha_2 = 1 - W = 1 - \frac{e}{e-1} (1 - e^{-\alpha})$

б) $n = 2$

Н - П : $l = \frac{\alpha_1}{\alpha_0} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} \cdot e^{-x_2}}{1 \cdot 1} \geq C$

$e^{-x_1 - x_2} \geq B$



$G: x_1 + x_2 \leq A$

$P(x_1 + x_2 \leq A \mid H_0) =$

$= A^2/2 = \alpha \quad \Rightarrow A = \sqrt{2\alpha}$

$G: x_1 + x_2 \leq \sqrt{2\alpha}$

$\alpha_1 = \alpha$

$W = P(x_1 + x_2 \leq A \mid H_1) = \int_0^A dx_1 \int_0^{A - x} \frac{e}{e-1} e^{-x_1 - x_2} dx_2 =$

$$= -\frac{e^2}{(e-1)^2}\left(1 - e^{-\sqrt{2\alpha}} - \sqrt{2\alpha}\, e^{-\sqrt{2\alpha}}\right)$$

$$\alpha_2 = 1 - W = 1 + \frac{e^2}{(e-1)^2}\left(1 - e^{-\sqrt{2\alpha}} - \sqrt{2\alpha}\, e^{-\sqrt{2\alpha}}\right)$$

c) $l = \frac{L_1}{L_0}$ = (отн. правдоподобия)

$= \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq c$ (прин. Неймана - Пирсона)

$P(l \geq c \mid H_0) = \alpha$

$\ln l = \sum_{i=1}^{n} \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i} \Rightarrow$ ЦПТ

$$\frac{\overbrace{\sum \eta_i}^{\ln l} - nM[\eta_i]}{\sqrt{nD[\eta_i]}} \sim N(0,1)$$

Найдем M, D:

H_0: $M[\eta_i] = M\left[\ln \frac{e}{e-1} e^{-x_i}\right] =$

$= M\left[\ln \frac{e}{e-1} - x_i\right] = \ln \frac{e}{e-1} - \frac{1}{2}$

$D[\eta_i] = D\left[\ln \frac{e}{e-1} e^{-x_i}\right] =$

$= D\left[\ln \frac{e}{e-1} - x_i\right] = D[x_i] = 1/12$

ЦПТ: $P(\ln l \geq \ln c \mid H_0) = P\left(\frac{\sum \eta_i - n \cdot M[\eta]}{\sqrt{nD[\eta]}} \geq \right.$

$\left. \geq \frac{\ln c - nM[\eta]}{\sqrt{nD[\eta]}}\right) = \alpha$

$$\frac{\ln c - n\left(\ln \frac{e}{e-1} - 1/2\right)}{\sqrt{\frac{n}{12}}} = u_{1-\alpha}$$

$\ln c = n\left(\ln \frac{e}{e-1} - \frac{1}{2}\right) + u_{1-\alpha}\sqrt{\frac{n}{12}}$

$\ln l = \sum_{i=1}^{n} \ln\left(\frac{e}{e-1} e^{-x_i}\right) = n \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i$

G: $\ln l \geq \ln c$

$$-\sum_{i=1}^{n} x_i \geq -\frac{n}{2} + u_{1-\alpha}\sqrt{\frac{n}{12}}$$

$G: \bar{x} \leq A$, где $A = \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$ – крит. область;
$\alpha_1 = \alpha$

$$W = P(\vec{x}_n \in G \mid H_1) = P(\bar{x} \leq A \mid H_1)$$

ЦПТ: $$\frac{\bar{x} - M\xi}{\sqrt{D\xi}}\sqrt{n} \sim N(0,1)$$

$$H_1: M\xi = \int_0^1 x \frac{e}{e-1} e^{-x}\,dx = \frac{e}{e-1}\left[-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right] =$$
$$= \frac{e}{e-1}\left[-e^{-1} + 1 - e^{-1}\right] = \frac{e-2}{e-1}$$

$$M[\xi^2] = \int_0^1 x^2 \frac{e}{e-1} e^{-x}\,dx = \frac{e}{e-1}\left[-x^2e^{-x}\Big|_0^1 +\right.$$
$$\left. + 2\int_0^1 x e^{-x}\,dx\right] = \frac{e}{e-1}\cdot(-1)e^{-1} + 2\frac{e-2}{e-1} = \frac{2e-5}{e-1}$$

$$D\xi = M[\xi^2] - M^2[\xi] = \frac{e^2 - 3e + 1}{(e-1)^2}$$

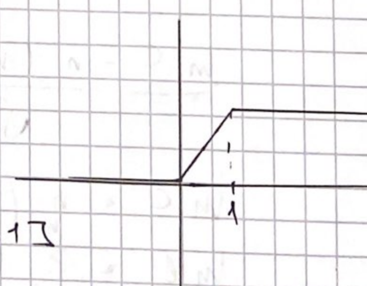
$$W = P(\bar{x} \leq A \mid H_1) = P\left(\frac{\bar{x} - M\xi}{\sqrt{D\xi}}\sqrt{n} \leq \frac{A - M\xi}{\sqrt{D\xi}}\sqrt{n}\right) =$$
$$= \int_{-\infty}^{\frac{A-M\xi}{\sqrt{D\xi}}\sqrt{n}} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

$$\alpha_2 = 1 - W$$

d) $G: x_{min} < C$

$P(x_{min} < C \mid H_0) = \alpha$

$$H_0: F_0(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases}$$



$$F_{min}(x) = (1 - F_0(x))^n - P(x_{min} \geq C \mid H_0)$$

$$P(x_{min} < C \mid H_0) = 1 - F_{min}(C) = \alpha$$

$$1-(1-c)^n = \alpha$$

$$\sqrt[n]{1-\alpha} = 1-c \Rightarrow c = 1-\sqrt[n]{1-\alpha}$$

$$W = P(x\min < c \mid H_1)$$

$$H_1 = F_1(x) = \int_0^x \frac{e}{e-1} e^{-t} dt = \frac{e}{e-1}(1-e^{-x})$$

$$W = 1 - F\min(c) = 1-(1-F_1(c))^n =$$
$$= 1-\left(1-\frac{e}{e-1}+\frac{e}{e-1}\cdot e^{-c}\right)^n = 1-\left(\frac{-1}{e-1}+\frac{e}{e-1}e^{-c}\right)^n =$$
$$= 1+\frac{1}{(e-1)^n}\left(1-e^{1-c}\right)^n$$

$$\alpha_1 = \alpha, \quad \alpha_2 = 1-W$$

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$$l = \frac{L_1}{L_0} = \frac{p_1(x_1)\,p_1(x_2)}{p_0(x_1)\,p_0(x_2)} \geq C$$

" 4 " - 6 $\qquad p_1 = \frac{1}{4}\delta(x-1)+\frac{1}{4}\delta(x-2)+\frac{1}{4}\delta(x-3)+\frac{1}{4}\delta(x-4)$

" 3 " - 4 $\qquad p_0 = \frac{1}{4}\delta(x-1)+\frac{1}{4}\delta(x-2)+\frac{1}{6}\delta(x-3)+\frac{1}{3}\delta(x-4)$

" 2 " - 6

" 1 " - 6

всего - 24

l:

$x_2 \backslash x_1$	1	2	3	4
1	1	1	3/2	3/2
2	1	1	3/2	3/4
3	3/2	3/2	9/4	9/8
4	3/4	3/4	9/8	9/16

H_0:

$x_2 \backslash x_1$	1	2	3	4
1	1/16	1/16	1/24	1/12
2	1/16	1/16	1/24	1/12
3	1/24	1/24	1/36	1/18
4	1/12	1/12	1/18	1/9

H_1:

$x_2 \backslash x_1$	1	2	3	4
1	1/16	1/16	1/16	1/16
2	1/16	1/16	1/16	1/16
3	1/16	1/16	1/16	1/16
4	1/16	1/16	1/16	1/16

$P(l \geq C \mid H_0) = 0{,}2$ ($\leq 0{,}2$, т. к. дискр. вели.)

$$1/36 \leq 0{,}2$$

$$\underbrace{\frac{1}{36}+\frac{1}{24}\cdot 4}_{\approx 0{,}194} \leq 0{,}2$$

Дальше $> 0{,}2 \Rightarrow$ Вкр: выпадение $(1,3),(2,3),$
$\alpha_1 \approx 0{,}194$ $\qquad (3,2),(3,1),(3,3)$

$$\alpha_2 = 1-W = 1-\underbrace{P(l \geq C \mid H_1)}_{\frac{1}{16}\cdot 5 = W = 0{,}3125} = \frac{11}{16} = 0{,}6875$$

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$n = 25$

$\xi \sim N(\Theta_1, \Theta_2^2)$

$S^2 = 0{,}2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \langle x \rangle)^2$

$H_0 : \Theta_2^2 = 0{,}1$

$\quad\quad \Theta_2^2(0)$

$H_1 : \Theta_2^2 > 0{,}1$

$\Delta = \frac{(n-1) S^2}{\Theta_2^2} \sim \chi^2(n-1)$

$G_{\text{крит}} : \Delta \geq c \; ; \; \tilde{\Delta} = \frac{(n-1) S^2}{\Theta_2^2(0)} = \frac{24 \cdot 0{,}2}{0{,}1} = 48$

1) Решение о гип. H_0 принимаем при помощи:

$p\text{-value} = P(\Delta \geq \tilde{\Delta}) = P(\Delta \geq 46) =$

$= \int_{48}^{\infty} \chi^2(\underset{n-1}{24})(x)\, dx \approx 0{,}0025$

$\Rightarrow H_0$ отвергается (то есть дисп не равна 0,1)

2) Определение и исслед. крит. области мощ. и ошибок

Н-П:

$G_{\text{крит}} : \Delta \geq c \; ; \; \Delta \sim \chi^2(24)$

$\alpha_1 = P(\Delta \geq c \mid H_0) = \alpha = 0{,}05$

То есть $\int_{c}^{+\infty} p_{\chi^2(24)}(x)\, dx = 0{,}05$

$c \approx 36{,}4$

След. $G_{\text{крит}} : \frac{24 \cdot S^2}{0{,}1} \geq 36{,}4$

Крит. обл.: $S^2 \geq 0,152$

Функция мощности:

$W = P(\bar{X}_n \in G \mid H_1)$

$W(\Theta_2^2) = P(S^2 \geq 0,152 \mid H_1) =$

$= P\left(\frac{S \cdot 24}{\Theta_2^2} \geq \frac{0,152 \cdot 24}{\Theta_2^2}\right) =$

$= \int_{u} p_{\chi^2(24)}(x)\, dx.$

$\left\{ u = \frac{0,152 \cdot 24}{\Theta_2^2} = \frac{3,648}{\Theta_2^2} \right.$

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$\vec{x}_n, \vec{y}_m$

$\xi \sim N(a, \sigma_x^2)$; $\sigma_x^2 = 2$; $n = 3$

$\eta \sim N(b, \sigma_y^2)$; $\sigma_y^2 = 1$; $m = 2$

$x = \{-1,11; -6,10; 2,42\}$; $\bar{x} = -1,597$

$y = \{-2,29; -2,91\}$, $\bar{y} = -2,6$

$H_0: a = b$; $H_1: a \neq b$, $a > b$, $a < b$; $\alpha = 0,05$

Т.к. $\sigma_x \neq \sigma_y$ воспольз. стат. критерием.

$$Z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}}$$

$$Z_{набл.} = \frac{-1,597 - (-2,6)}{\sqrt{\frac{2}{3} + \frac{1}{2}}} \approx \frac{1,003}{1,080} \approx 0,929$$

$H_1: a \neq b$ $\Phi(Z_{крит}) = \frac{1-\alpha}{2} = 0,475 \Rightarrow Z_{крит} = 1,96$

$H_1: a > b$, $a < b$: $\Phi(Z_{крит.}) = \frac{1-2\alpha}{2} = 0,45 \Rightarrow Z_{кр.} \approx 1,65$

$H_1: a \neq b$: $-1,96 < 0,929 < 1,96 \Rightarrow$ нет основ. отвер. H_0

$H_1: a > b$: $0,929 < 1,65 \Rightarrow$ нет осн отверг. H_0

$H_1: a < b$: $-1,65 < 0,929 \Rightarrow$ нет осн. отвр. H_0