BMME 201 Summer 2021
PA6: Stress-Strain Analysis
Due: Friday, 6/11, at 8 pm

Background

Stress-strain curves give information about the mechanical properties of a metal. Knowing the stress a material can tolerate before deforming or failing is a critical factor when designing a component. Akin to the critical stresses are the resilience and toughness, which respectively define how much energy the material can absorb before deforming permanently or failing entirely.

Stress-strain curves can be measured with a tension test in a load frame, where a dogbone-shaped specimen is stretched uniaxially until it fractures. The load frame has a displacement sensor to measure how far the sample has stretched from its initial position and a load cell measures the force required to maintain that displacement. "Force" and "displacement" depend on the dimensions of the sample and are thus very specific to a sample. To generalize to all geometries of the same material, force and displacement must be converted to the equivalent material properties: stress and strain.

Problem

You have run a tension test on a sample of aluminum and saved your force-displacement data in *aluminum.xlsx*. The first column is the displacement (m), which is zeroed before the test. The second column is the tensile force (N) applied by the load cell to maintain the corresponding displacement. Your sample has a gage length of 38 mm and a circular cross-section with radius of 1.5 mm.

Note that 1 Pa = 1 N/m^2, and that 1 MPa = 10^6 Pa and 1 GPa = 10^9 Pa.

Write a program that imports the data and extracts the following properties:

- Proportional stress (MPa) and corresponding strain
- Yield stress (MPa) and corresponding strain using 0.002 offset rule.
- Ultimate stress (MPa) and corresponding strain
- Fracture stress (MPa) and corresponding strain
- Elastic modulus (GPa)
- Stress (MPa) to hundredths place at a strain of **EXACTLY** 0.100000000000000. Make sure that this point is a variable that we can adjust to find any strain, and make sure that your `fprintf` displays both the value of strain and that of the corresponding stress.

Print the values to the command window with `fprintf`. Stress should be to the nearest hundredth of MPa, and strain should be to the ten thousandths place. See the Theory section for details about how to calculate these.

Finally, your code should plot stress vs. strain and show some type of mark at the proportional, yield, ultimate, and fracture stresses. Make sure that your stress is in MPa. You should **NOT** have a 10^8 factor on the y axis. Remember that strain is dimensionless, so you do not have to convert its units or report them somehow.

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Your code should be able to handle another data set of a similar properties (i.e., ductile metal, not an elastomer or very brittle material) but with a different force-displacement data set. We **WILL** test it with another data set.

Scoring

Doing everything above will earn an 80/100. For a higher grade, your code should also do the following:

- (5 pts) Calculate the resilience and toughness in MJ/m^3. Look at the numerical integration functions or devise your own method. We will cover this later.
- (5 pts) Use the `text` function to label the critical points on your graph as *Proportional*, *Yield*, *Ultimate*, and *Fracture*. Make sure it works with different data sets and that the writing doesn't cover the data points.
- (5 pts) Shade the area under the curve in red corresponding to the elastic region (triangular region to the **LEFT** of the yield point; shown as resilience in the graph later). Shade the area under the curve in cyan corresponding to plastic region (larger region to the **RIGHT** of the yield point; shown as toughness minus resilience later). See the `area` function's documentation.
- (5 pts) Smooth your force data **BEFORE** converting it to stress with a 25-point span moving average. See the function `smooth`.

Theory

Stress (σ) and strain (ε) are analogous to force (F) and displacement (Δl), respectively:

$$\sigma = \frac{F}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta l}{l}$$

where A is the cross-sectional area and l is the specimen gage length.

A typical stress-strain curve has a linear region at the beginning followed by a non-linear region that increases slowly to a maximum, then decreases until fracture. The slope of the linear region is the **elastic modulus, *E***. The end of the linear regime is called the **proportional limit**, and it has (x,y) coordinates of (ε_p, σ_p).

Unfortunately, the proportional limit is difficult to detect exactly; instead, the **yield point** is often used as the end of the linear region. Its coordinates are (ε_y, σ_y). Starting along the x axis at a strain value of $\varepsilon = 0.002$ and following a line with a slope of E, the intersection of this line with the stress-strain curve is the yield point. Any stress exceeding the yield stress will cause permanent, or plastic, deformation; stresses lower than the yield stress cause non-permanent, or elastic, deformation that is recovered upon unloading.

Continuing past the yield point, the stress reaches a maximum value at its **ultimate point**. This point has coordinates of (ε_u, σ_u). Beyond the ultimate point, the stress decreases until the material fails at its **fracture point**, which has coordinates of (ε_f, σ_f).

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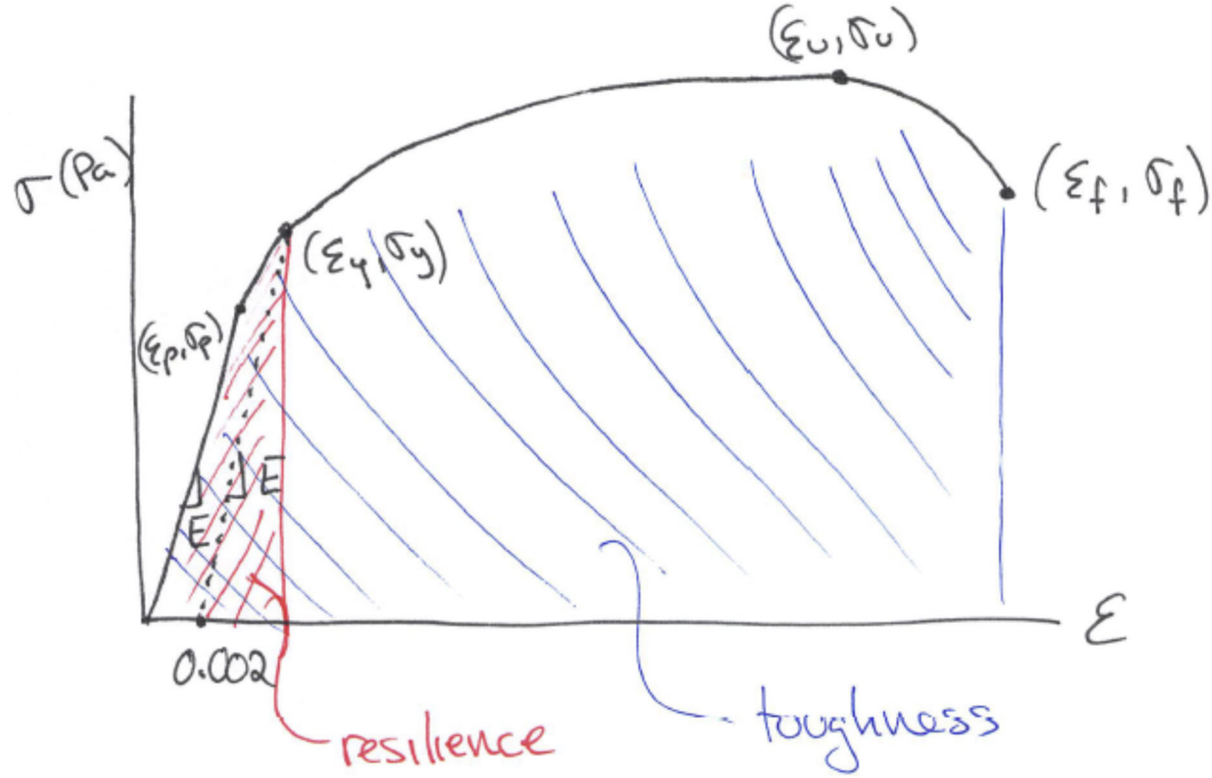

The SI units of stress are pascals, Pa, which are N/m^2. This is equivalent to one unit of energy per volume, or J/m^3. Thus, the area under the stress-strain curve has units of J/m^3. The **resilience** is the amount of energy a material can tolerate before it yields, while the **toughness** is the total energy absorbed before fracture.

Hints

- To find the proportional limit, you can fit a line to increasing amounts of data until you find the data that give the best correlation coefficient, *r*. Set up a `for` loop and calculate *r* between the strain data from `strain(1)` to `strain(k)` and the corresponding stress data from `stress(1)` to `stress(k)`. The variable `k` is an increment that goes from 3 (or else you're fitting a line to only two data points) to some value well beyond the anticipated proportional point. Use the command

```
r = corr(xData,yData);
```

  Look for the increment *k* that has *r* closest to 1. Fit a line to this point from the origin to get the slope, which is the elastic modulus.
- Once you have the elastic modulus, find the yield point from the intercept between the data with a line starting at an x-intercept of 0.002 with a slope of E. You may already know how to find the intercept from the trebuchet programming assignments. If not, it involves the functions `abs` and `min`.
- A common mistake that produces a confusing error comes from using a bad choice of variable names that overwrites a useful function. Be careful.
- When using exponents, 1e6 = 10^6 and 10e6 = 10^7. Careful.
- You can look up some of these values for aluminum to check whether your answers are in the correct ballpark.