Report: Viscek Model

- How to compile and run:

Module: viscek.c   Header: viscek.h   Main: run-viscek.c

USAGE: make

./run-viscek integer number time steps: for example: ./run-viscek 3000

To see the results:

graph -T png -C -m 3 f1.txt -m 1 f2.txt -m 2 f3.txt -m 4 f4.txt -m 3 f5.txt -m 1 f6.txt -m 2 f7.txt -m 4 f8.txt -m 2 f9.txt -m 4 f10.txt > ***.png

display ***.png

- Aim of the program:

Simulation of Viscek model as a continuous-continuous system, and evaluating its phase transitions considering two different type of noise.

-Definition of the model and algorithm:

This model consists of N particles on a 2D square box of sides L with periodic boundary conditions. Each particle state, let us say the nth particle, is characterized with three numbers: (x_n, y_n, θ_n), where each of the is a function of time. At t=0 these parameters are determined randomly using ranlxs, random number generator. The magnitude of velocity of all particles in all the times is a constant like v_0, but their direction like their positions evolve in time. In other words, all the particles move with some speed v_0, what changes from one particle to another is the direction of the motion, given by the angle θ_n.

In order to determine the interaction rules between particles mathematically, we shall define the circular neighborhood with radius r_0 centered at point (x_n, y_n), and k_n as the number of particles within this neighborhood. Let specify $\boldsymbol{V_n}$ be the average velocity of the particles within each neighborhood and $\boldsymbol{v_n}$ the velocity of each particle:

$$\boldsymbol{V_n}(t) = \frac{1}{k_n(t)} \sum_{j:(x_j,y_j)\ \epsilon\ neighborhood} \boldsymbol{v_j}$$

We can formulate interaction and evolution of the system in two different ways:

First, Interaction between the particles with *intrinsic noise*, which is given by the simultaneous updating of all the velocity angles and the all the particles, states according to the following rules:

$$\theta_n(t+\Delta t) = Angle\big(\boldsymbol{V_n}(t)\big) + \eta \xi_n(t)$$
$$\boldsymbol{v_n}(t+\Delta t) = v_0 e^{i\theta_n(t+\Delta t)}$$
$$x_n(t+\Delta t) = x_n(t) + \Delta t . v_n(t+\Delta t)$$

Second, Interaction between the particles with *extrinsic noise,* which is given by the simultaneous updating of all the velocity angles and the all the particles, states according to the following rules:

$$\theta_n(t+\Delta t) = Angle\big(V_n(t) + \eta e^{i\xi_n(t)}\big)$$
$$v_n(t+\Delta t) = v_0 e^{i\theta_n(t+\Delta t)}$$
$$x_n(t+\Delta t) = x_n(t) + \Delta t . v_n(t+\Delta t)$$

Where in both flow equations $\xi_n(t)$ is a random variable distributed in the interval $[0, 2\pi]$, and η is the noise density, which is positive number. We will see the role of noise density is similar to the role of Temperature in the Magnets.

It is natural to interpret the average velocity $\boldsymbol{V_n}(t)$, as the signal the nth particle receives from its neighborhood, and the Angle function as the decision-making mechanism. Now we can easily say, in intrinsic noise case, particle receives the signal from its neighborhood, computes the angle of the motion, but then decides to move in a different direction. In contrast, for the extrinsic noise case, the particle receives a noisy signal from neighborhood, but once the signal has been received, the particle calculates the angle from received signal, and follows it.
In analogy with what we know from magnets, we can define order parameter as following:

$$\psi_\eta(t) = \frac{1}{Nv_0}\left|\sum_{n=1}^{N} \boldsymbol{v_n}(t)\right|$$

η Subscript indicates that the order parameter in the system depends on the noise intensity. After long time the system reaches a steady state, and it's possible to take average the order parameter $\psi_\eta(t)$ over time, to find stationary order parameter:

$$\psi(\eta) = \lim_{T\to\infty}\frac{1}{T}\int_0^T \psi_\eta(t)$$

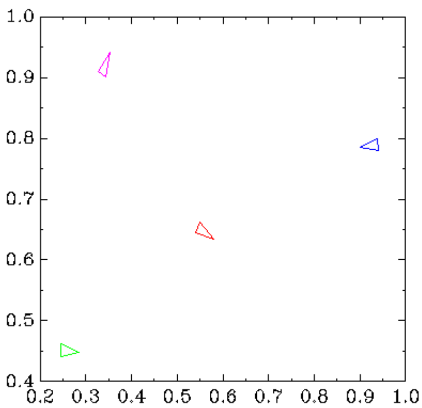
Evaluation of order parameter in terms of η leads to find some information about the phase transitions.

- Simulation Results:

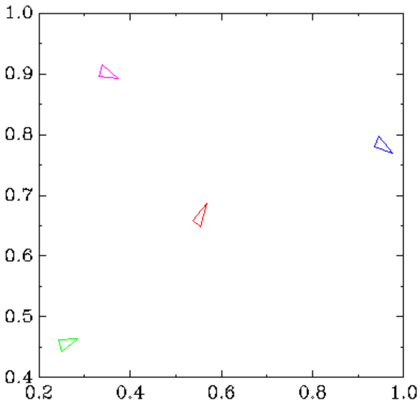
1. Evaluation of the system in absence of the noise:

After evolution of the system noiseless case, the system falls in the state with maximum order parameter: $\psi(\eta) = 1$. It means that all the particles after long time move in the same direction. Let's see it for two different cases explicitly.

Case 1: $\Delta t = 1, N = 4, L = 1, v_0 = 0.1, r_0 = 0.5, \eta = 0$

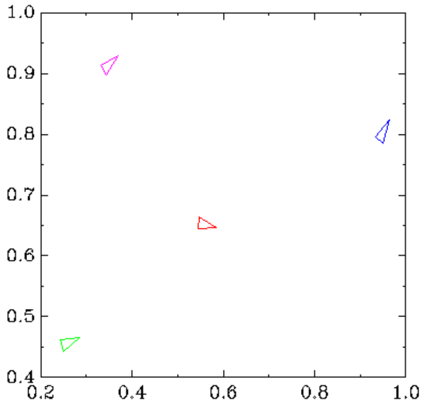


$t = 0$



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 1$



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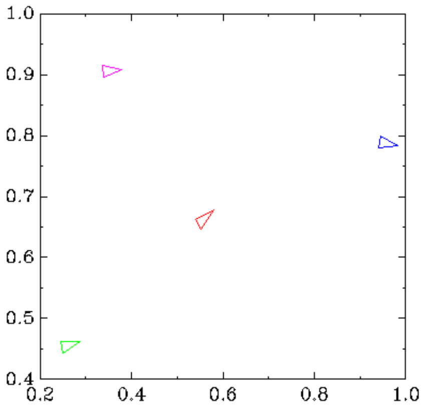
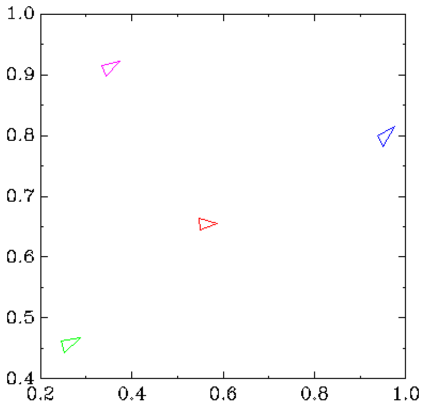
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 2$



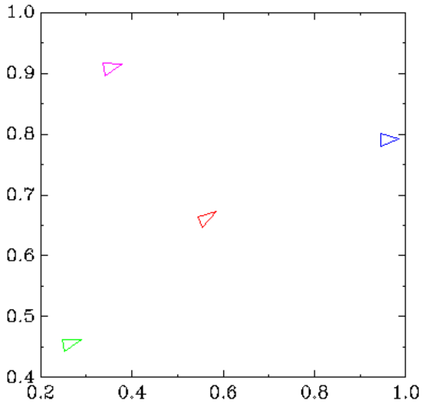
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 3$



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 4$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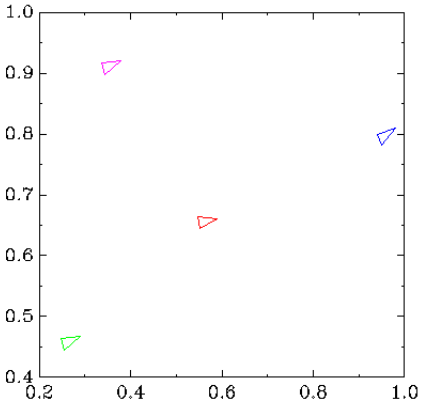
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 5$


1.0
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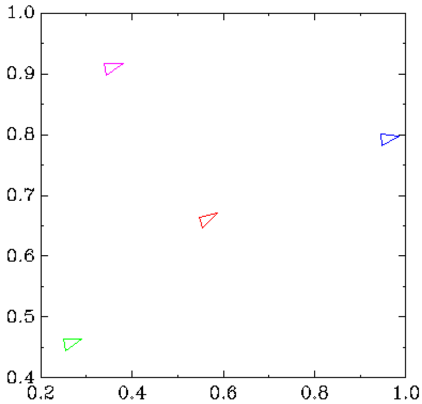
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

$t = 6$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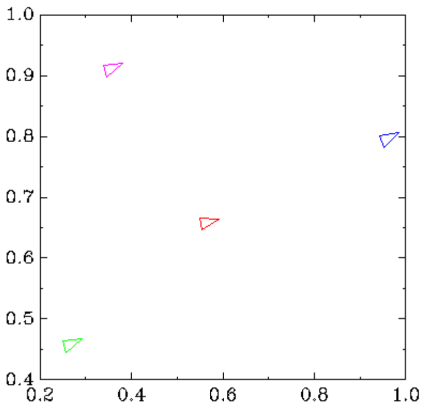
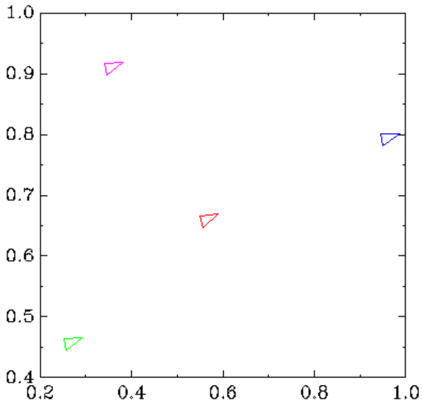
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

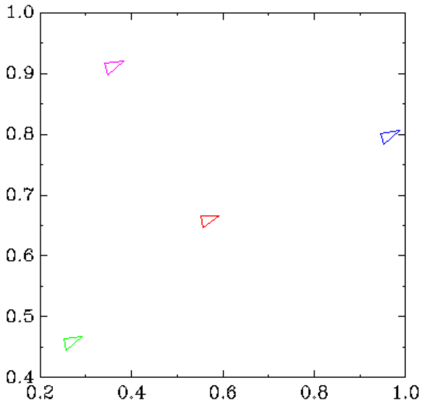
$t = 7$


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1.0

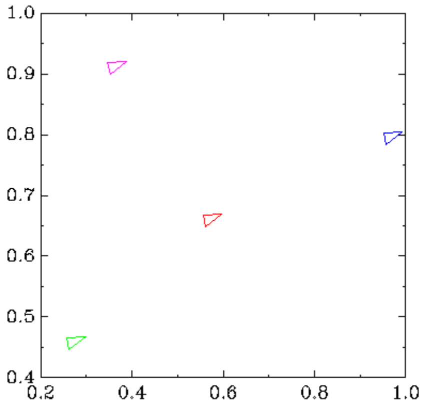
$t = 8$



$t = 9$



$t = 10$



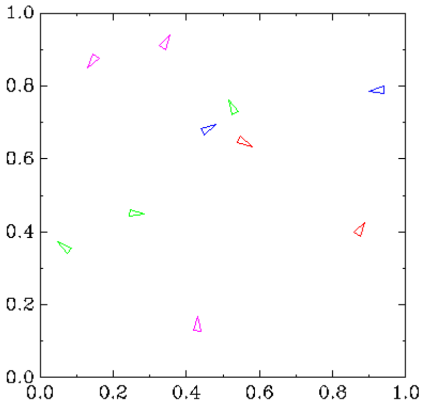
$t = 15$

It is deserved to say:

$$\psi_\eta(0) = 0.339182$$
$$\psi_\eta(1) = 0.842336$$
$$\psi_\eta(2) = 0.993936$$

We see order parameter tends to 1 very rapidly, as we can see it graphically in images.

Case 2: $\Delta t = 1, N = 10, L = 1, v_0 = 0.1, r_0 = 0.5, \eta = 0$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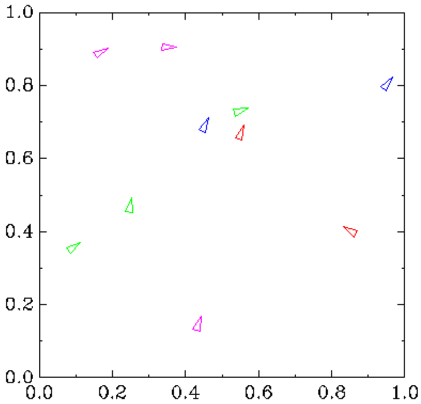
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
$t = 0$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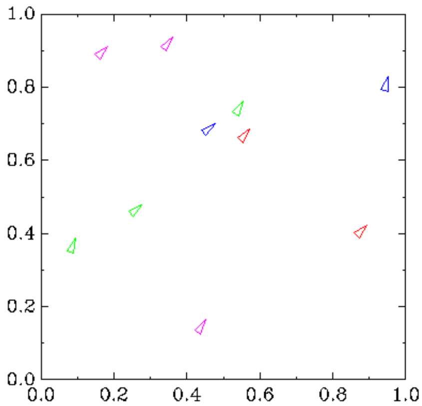
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
$t = 1$


1.0
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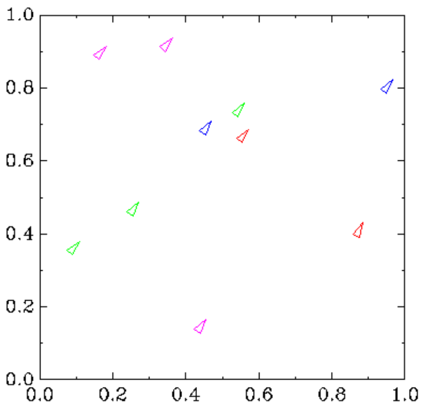
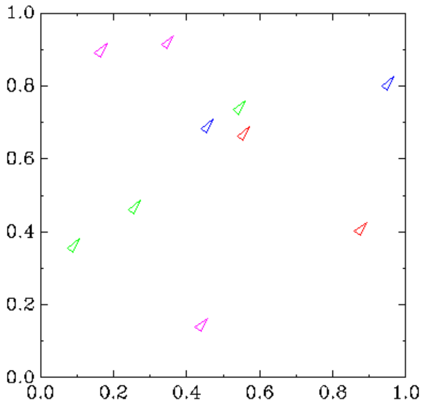
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
$t = 2$


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
$t = 3$



$t = 4$

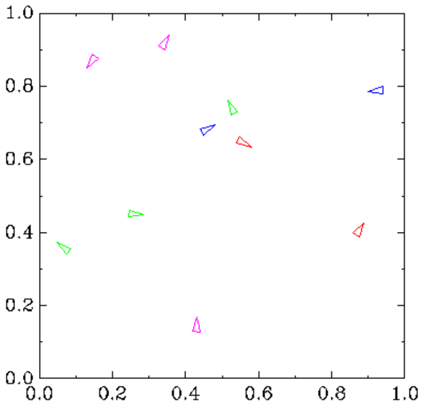
Let us denote that:

$$\psi_\eta(0) = 0.337922$$
$$\psi_\eta(1) = 0.799399$$
$$\psi_\eta(2) = 0.973318$$

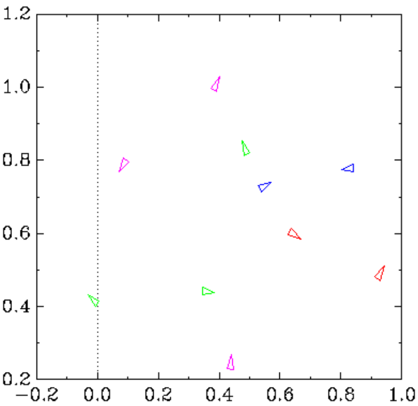
The same behavior as we have seen in the first case.

Case 3: $\Delta t = 1, N = 10, L = 1, v_0 = 0.1, r_0 = 0.05, \eta = 0$

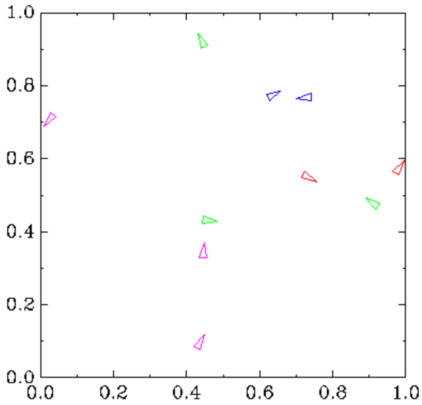
In this case the interaction neighborhood is much smaller in comparison with two first cases. As we expect the time takes for the system to reach to the stationary state increases striking.



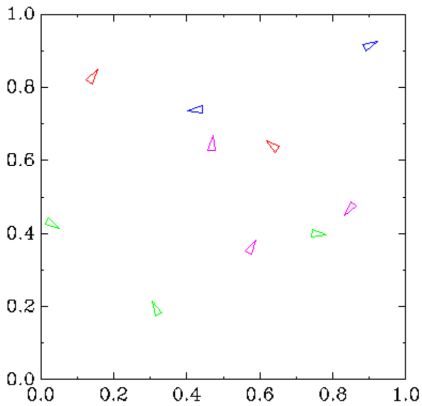
$t = 0$



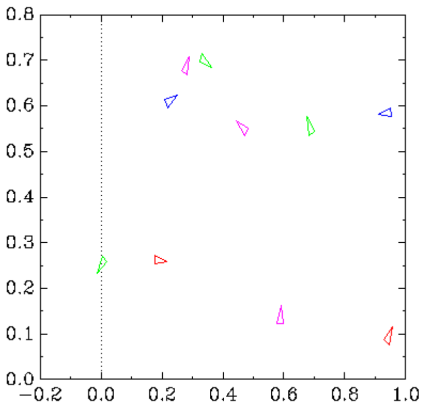
$t = 1$


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0

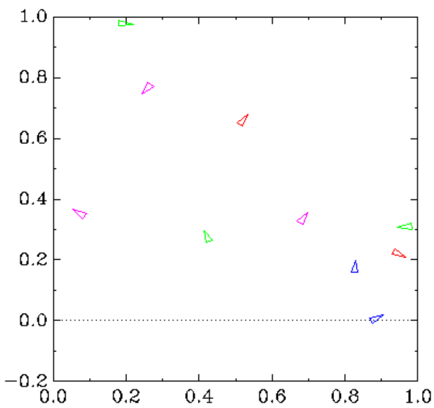
$t = 2$


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0

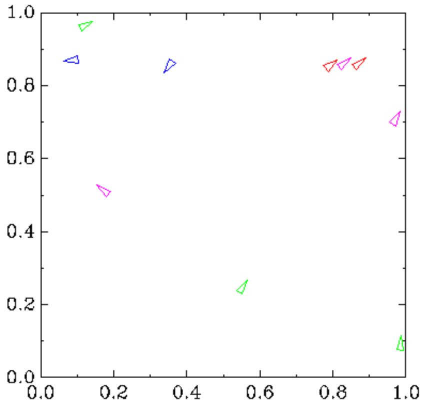
$t = 5$


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.2 0.0 0.2 0.4 0.6 0.8 1.0

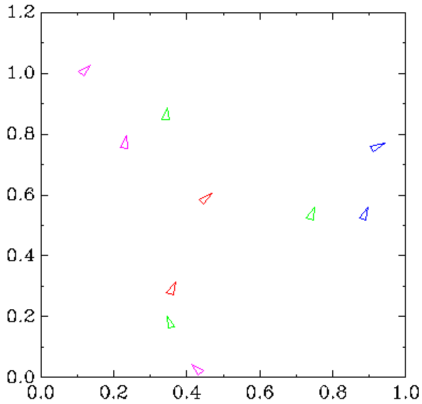
$t = 20$


1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0 0.2 0.4 0.6 0.8 1.0

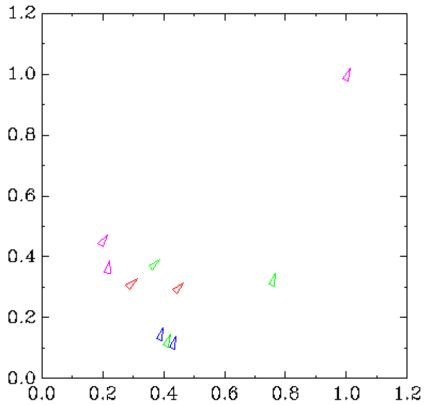
$t = 50$



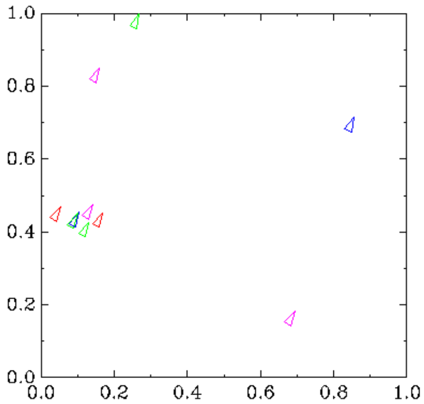
$t = 200$



$t = 500$



$t = 1000$



$t = 2000$

In this case we have:

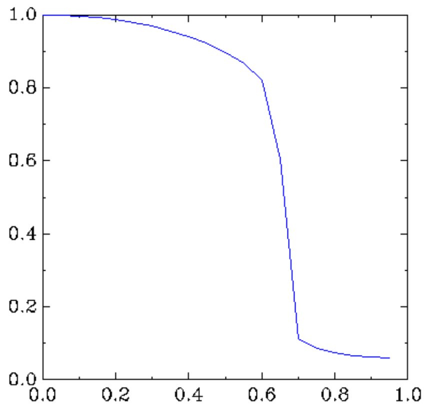
$$\psi_\eta(0) = 0.337922$$
$$\psi_\eta(157) = 0.337922$$
$$\psi_\eta(158) = 0.437709$$
$$\psi_\eta(200) = 0.509589$$
$$\psi_\eta(500) = 0.879535$$
$$\psi_\eta(1000) = 0.975051$$
$$\psi_\eta(2000) = 0.999531$$

2. Evaluation of the system in presence of the noise: Phase Diagrams

In this section we will see the behavior of the system when we switch on the noise in the system in two different cases: extrinsic and extrinsic noise. We observe that the system exhibits two different behaviors. In both cases we have computed the stationary order parameter for different values of η, noise intensity. In both cases the values of η has been changed from 0 to 1 with length steps 0.05 and also:

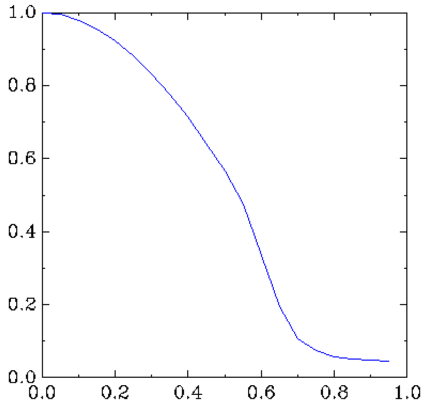
$$\Delta t = 1, N = 400, L = 10, v_0 = 1.0, r_0 = 1.0$$

For the *extrinsic noise*: (vertical axis: order parameter, horizontal axis: noise density)



So, for the intrinsic case about $\eta = 0.7$ we have discontinuity and it suggests the existence of the *first order phase transition.*

For the *intrinsic noise*: :(vertical axis: order parameter, horizontal axis: noise density)



This diagram suggests that there is a *second order phase transition.* (There are some debates about the type and origin of the intrinsic case, however for the extrinsic case there are no doubts that the phase transition is discontinuous and there is a plenty numerical and theoretical evidence supporting this fact.)