Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

Integrated Bachelor's-Master's Program

End-Semester (Sem-II) Examination-Spring 2020

Subject:Liner Algebra and Multivariable Calculus

Subject Code: MCS 1201 A

Full marks: 50 ***Time Alloted: 3 hours***

1. Suppose $f : [0,1] \to [0,1]$ is a continuous function. Then show that f has a fixed point. I.e. there is an $x \in [0,1]$ such that $f(x) = x$. [10]

2. Show that the function $f : [0,\infty) \to \mathbb{R}$ given by $f(x) = \sqrt{x}$ is uniformly continuous on $[0,\infty)$. [10]

3. Suppose $f : [0,2] \to \mathbb{R}$ is a continuous function on $[0,2]$ and differentiable on $(0,2)$. Suppose $f(0) = 0$, $f(1) = 1$ and $f(2) = 1$. Show that there is a $c \in (0,2)$ such that $f'(c) = \frac{1}{3}$. [10]

4. Suppose $f : [0,1] \to \mathbb{R}$ is a continuous function with the property that $\int_0^x f = \int_x^1 f$ for all $x \in [0,1]$. Show that $f(x) = 0$ for every $x \in [0,1]$. [10]

5. Show that
$$\lim_{n\to\infty}[\frac{1}{n^{p+1}}\sum_{k=1}^{n} k^p] = \frac{1}{p+1}.$$
[10]