Indian Association for the Cultivation of Science
(Deemed to be university under the *de novo* category)
Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's Program/PhD Course

End-Semester Examination-Spring 2025

Subject: Electricity, Magnetism, and Optics | **Subject Code(s): PHS 1201**
Full Marks: 50 | **Time Allotted: 3 h**

Electricity and Magnetism: Answer any one question out of two.

1. **Stationary EM**

   (a) Two long parallel wires carrying steady currents I_1 and I_2 in the same direction are initially situated a distance d apart. If the magnetic attraction between the wires is to be balanced by an electrostatic repulsion due to coating the insulation of the wires with electric charges smeared uniformly along their length, what is the ratio of the charge densities per unit length that must be used? Ignore any interaction between the fixed charges and the currents. [**4**]

   (b) Charges $\pm q$ are placed at the ends of a straight line of length r_0. Another charge, $-Q$, is placed on the perpendicular bisector of the joining line, at a distance y from the midpoint. What is the potential energy of the system? What is the net force on $-q$ in magnitude and direction? [**2+2**]

   (c) The electric potential on the walls of an empty room is ϕ_0. Suppose there are two solutions $\phi_1(\vec{r})$ and $\phi_2(\vec{r})$ of the Laplace equation which have the same value ϕ_0 on the walls. Is $\phi_1 = \phi_2$ everywhere inside the room? Why? [**1+3**]

2. **EM Waves**

   (a) Start with the homogeneous wave equations for the scalar and vector potentials and the transversality condition. Using the definitions of the electric and magnetic fields in terms of the potentials, show that the derived electric and magnetic fields themselves obey the wave equation. Taking plane wave solutions, show that $\vec{E}, \vec{B}$ are both transverse to the direction of propagation and also to each other. [**4+2**]

(b) If stars in the sky all emitted plane EM waves, would you expect the night sky to be dark like it is, or would we have been dazzled by extremely bright starlight? Why? Can you guess how the amplitude of the vector potential solution of the wave equations has to be modified, so that the night sky is dark as it is observed to be? **[2+3]**

Optics (Answer all questions)

3. An air bubble (refractive index $n_{\text{air}} = 1.00$) is trapped inside water (refractive index $n_{\text{water}} = 1.33$). Assume the air bubble is spherical with a radius of $R = 2\,\text{mm}$.

   A narrow beam of light strikes the bubble from water at normal incidence (i.e., along the center).

   (a) Find the **focal length** of the air bubble acting as a spherical lens.

   (b) Comment on whether the bubble behaves as a **converging** or **diverging** lens.

   (c) Find the **position of the image** if a point source is placed 5 cm away from the bubble. (Assume paraxial approximation — small angles.) **[4]**

4. A thin convex lens of focal length $f = 20\,\text{cm}$ is placed in air. An object is located 30 cm to the left of the lens.

   Using the ray transfer matrix method:

   (a) Write down the system matrices for:

      (i) Propagation from the object to the lens,

      (ii) Transmission through the thin lens,

      (iii) Propagation from the lens to the image plane.

   (b) Multiply the matrices to find the overall system matrix.

   (c) Using the matrix formalism, determine:

      (i) The position of the final image,

      (ii) The magnification.

   (d) Comment whether the image is real or virtual, and whether it is erect or inverted.

   (Assume paraxial approximation and neglect lens thickness.) **[5]**

5. In a Young's double-slit experiment, coherent light of wavelength $\lambda = 600\,\text{nm}$ is used. The two slits are separated by $d = 0.3\,\text{mm}$, and the screen is placed at a distance $L = 1.5\,\text{m}$ from the slits.

   (a) Find the fringe spacing Δx on the screen.

(b) Now, a thin transparent sheet of thickness $t = 2.0\,\mu\text{m}$ and refractive index $n = 1.5$ is placed in front of **one slit only**. Find the lateral shift of the fringe pattern on the screen. [**4**]

6. In a Michelson interferometer, monochromatic light of wavelength 600 nm is used. One of the mirrors is moved slowly. As a result, the observer sees fringes crossing the field of view. If 100 fringes cross the field of view, calculate the distance the mirror has moved. [**2**]

7. A Fabry-Perot etalon consists of two partially reflecting mirrors separated by a distance 1.5 cm. The reflectivity of each mirror is 0.9, and the setup is operated with monochromatic light of wavelength 600 nm. Find the smallest resolvable wavelength. [**3**]

8. A plane wave of monochromatic light with wavelength 600 nm passes through a small circular aperture of radius 0.5 mm. If the screen is placed at a distance of 2 m from the aperture, find the radius of the first dark ring of the Fraunhofer diffraction pattern on the screen. [**2**]

9. A diffraction grating with $N = 5000$ lines per centimeter is illuminated by monochromatic light of wavelength $\lambda = 600$ nm. The diffraction pattern is observed on a screen placed at a distance $D = 2$ m from the grating.

   (a) Calculate the angular positions θ_m of the first-order ($m = 1$) and second-order ($m = 2$) maxima.

   (b) Determine the linear distance Δy between the first- and second-order maxima on the screen.

   (c) If the light wavelength changes to 500nm, how will the angular positions of the maxima change qualitatively? [**5**]

10. The primary focal length of a zone plate is 20 cm for light of wavelength 500 nm. Calculate the radiuss of the central zone on the zone plate. [**2**]

11. A beam of linearly polarized light of initial intensity $I_0 = 10.0\,\text{mW/cm}^2$, with the electric field vector lying along the horizontal axis, passes through two ideal polarizers and one quarter-wave plate arranged as follows:

    - The light first encounters Polarizer A, whose transmission axis is at 0° (horizontal).
    - It then passes through a quarter-wave plate whose fast axis is at 30° to the horizontal.
    - Finally, it passes through Polarizer B, whose transmission axis makes an angle θ with the horizontal.

    Answer the following:

    After Polarizer A:

    (a) What is the intensity I_1 of the light emerging from Polarizer A?

    (b) What is the state of polarization of this beam?

    After the Quarter-Wave Plate (QWP):

(a) Determine the polarization state (linear, circular, or elliptical) of the beam exiting the QWP. Also, determine the orientation of the polarization after passing the QWP.

Through Polarizer B:

(a) Using Malus's law, derive an expression for the transmitted intensity $I(\theta)$ as a function of θ.

(b) Evaluate $I(\theta)$ numerically for $\theta = 0°$, $30°$, $60°$, $90°$.

Extinction Condition:

(a) Find all angles θ in the range $[0°, 180°)$ for which $I(\theta) = 0$.

(b) Explain physically why extinction occurs at these angles given the combined action of the QWP and Polarizer B. [**9**]

12. A compact disc (CD) stores data as a series of microscopic pits and lands (flat areas) arranged in a spiral track. A laser beam is used to read the information from the surface. Suppose:

- The wavelength of the laser used to read the CD is $\lambda = 780\,\text{nm}$.
- The minimum separation between pits is approximately $d = 1.6\,\mu\text{m}$.

Calculate the numerical aperture (NA) of the optical system assuming the focal length of the lens 5 mm. Consider that Blu-ray discs use a laser with a wavelength of 405 nm and a numerical aperture (NA) of 0.85. Comment on the relation between the storage capacity and the spot size. [**2**]