Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Integrated Bachelor's-Master's Program

Mid-Semester (Sem I) Examination-Autumn 2022

Subject: Introductory Classical and Quantum mechanics ***Subject Code(s): PHS 1101***
Full marks: 25 ***Time allotted: 2 hr***

1. (a) If $\vec{A}$ is any vector field then show that $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$ **(2 marks)**

   (b) If $\phi(x, y, z) = 5x^2yz - x^3z^2y$ is a scalar function then find $\vec{\nabla}\phi$ at the point $(1, 2, -1)$ **(2 marks)**

   (c) Show that $\vec{\nabla}\phi$ is perpendicular to $\phi = constant$ hypersurface **(2 marks)**

2. (a) Show that a central force $\vec{F}$ can be expressed in terms of a scalar potential V as $\vec{F} = -\vec{\nabla}V$ **(2 marks)**

   (c) Show that for motion in a central force total mechanical energy is conserved . Is angular momentum also conserved ? Explain . **(2 marks)**

3. (a) Force F on a particle of mass 1 Kg moving along a line is given as $F = -k_1x - k_2\frac{dx}{dt}$, where $k_1 = 4\ Newton/m$ and $k_2 = 6\ Newton\ sec/m$. The particle starts from rest at $x = 2\ m$. Find the position of the particle after 10 seconds. What is the nature of the motion of the particle ? Is total mechanical energy conserved in this motion **(3 marks)**

   (b) Find the moment of inertia of a uniform circular disc of radius R and mass M about an axis perpendicular to the disc and passing though it's centre **(2 marks)**

4. (a) Find the expressions for the radial and cross-radial accelerations a_r and a_θ for motion in a plane . Show that for a uniform circular motion $a_r = -\frac{v^2}{r}$ and $a_\theta = 0$ **(3 marks)**

   (b) Explain why the motion of a planet is confined in a plane when it moves in the gravitational field of the sun ? **(2 marks)**

5. Show that the path of an object moving in a repulsive central inverse square force is a hyperbola **(5 marks)**