Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

Mid-Semester Examination-2024 (Autumn Semester-I)

Subject: Mathematics I ***Subject Code(s): MAT 1101***

Full marks: 25 ***Time allotted: 2 hrs***

Answer **all** questions. Each question carries 5 marks.

---

1. Let A, B and C be non-empty sets. Let $f : A \to B$ and $g : B \to C$ be functions.

   (a) Show that if $g \circ f$ is injective, then f is injective.

   (b) Show that if $g \circ f$ is surjective, then f is surjective.

2. Let X be a non-empty set. Let f and g be defined on X and have bounded ranges in $\mathbb{R}$. Show that

$$|\inf_{x\in X} f(x) - \inf_{x\in X} g(x)| \leq \sup_{x\in X} |f(x) - g(x)|.$$

3. Let the real numbers x_n be defined as follows: $x_1 = 1, x_2 = 2$, and $x_{n+2} := \frac{1}{2}(x_{n+1} + x_n)$ for all $n \in \mathbb{N}$. Show that $1 \leq x_n \leq 2$ for all $n \in \mathbb{N}$.

4. Let $x_1 = 1$ and $x_{n+1} = \sqrt{2 + x_n}$ for $n \geq 1$. Show that $\{x_n\}$ is convergent and find the limit.

5. Show that if $\{x_n\}$ and $\{y_n\}$ are convergent sequences, then the sequences $\{u_n\}$ and $\{v_n\}$ defined by $u_n = \max\{x_n, y_n\}$ and $v_n = \min\{x_n, y_n\}$ are also convergent.