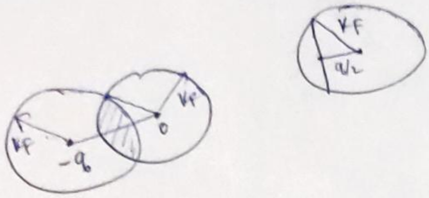

$\frac{4}{3}\pi$

Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

Master's/Integrated Master's – PhD program

End-Semester (Semester I) Re-examination- Spring 2022

Subject: Advanced Quantum Mechanics | *Subject Code: PHS4201*

Full marks: 50 | *Time allotted: 3 hrs*

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1) Consider an electric field of amplitude $\mathcal{E}_0$ giving rise to a scalar potential $V(r) = -e\mathcal{E}_0 r$ where r is the radial coordinate. Show that the application of such a field to a Dirac electron leads to a spin-orbit coupling term given by

$$H_{SO} = c_0 \vec{L} \cdot \vec{S}$$

where $\vec{L} = \vec{r} \times \vec{p}$ is the orbital angular momentum operator and $S = \hbar\vec{\sigma}/2$ is the spin and $\vec{\sigma}$ are the Pauli matrices. Hence find c_0 in terms of $\mathcal{E}_0$. [20]

2) Consider a system of weakly interacting electrons in the presence of a small Zeeman field B (we assume that the field has no orbital effect). Find out the first order correction to the energy due to an interaction term given by

$$H_{\text{int}} = V_0 \sum_{\vec{k}\vec{k}'\vec{q}} \sum_{\sigma\sigma'=\uparrow,\downarrow} f(q) c^{\dagger}_{\vec{k}+\vec{q},\sigma} c^{\dagger}_{\vec{k}'-\vec{q},\sigma'} c_{\vec{k}',\sigma'} c_{\vec{k},\sigma}$$

$\frac{E}{N} = \frac{E_0}{N} + \frac{E_1}{N} + \frac{E_2}{N}$

where $f(0) = 0$ and V_0 is the interaction amplitude. How does it depend on B? [20]

3) Consider the Fermi surface of non-interacting electrons in two-dimensions (2D) whose energy dispersion is $\epsilon(k_x, k_y) = A \cos k_x a$ with a is the lattice spacing, A is a constant, and $-\pi/a \leq k_x, k_y \leq \pi/a$. Find out the Fermi energy if the system is half-filled i.e. there are half the number of electrons as the number of available energy states. What is the shape of the 2D Fermi surface? [10]

$\epsilon \sim k_F^2$

$\epsilon \sim \frac{1}{k_F^3}$

$\frac{1}{(-k_F)^3} = -\frac{1}{k_F^3}$