



Mid-Semester Examination-2023 (Autumn Semester-I)

*Subject Code(s): MCS 2101A*

*Time allotted: 2 hrs*

Answer all questions. Each question carries 5 marks.

- ✓ 1. The probability  $p_k$  that a family has  $k$  children is given by  $p_0 = p_1 = a$ ,  $p_k = (1 - 2a)2^{-(k-1)}$  ( $k \geq 2$ ). It is known that a family has two boys. What is the probability that:  
(a) the family has only two children?  
(b) the family has two girls as well?
- ✓ 2. A lot of transistors contains 0.6 percent defectives. Each transistor is subjected to a test that correctly identifies a defective but also misidentifies as defective about 2 in every 100 good transistors. Given that a randomly chosen transistor is declared defective by the tester, compute the probability that it is actually defective.
3. Let  $X$  be a continuous random variable with cdf  $F_X$  which is strictly increasing. Let  $Y = F_X(X)$ . Show that  $Y$  is uniformly distributed.
4. Suppose that  $N$  people throw their hats in a box and then each picks one hat at random. What is the expected value of  $X$ , the number of people that get back their own hat?
- ✓ 5. Consider a quiz game where a person is given two questions and must decide which question to answer first. Question 1 will be answered correctly with probability 0.8, and the person will then receive as prize Rs. 1000, while Question 2 will be answered correctly with probability 0.5, and the person will then receive as prize Rs. 2000. If the first question attempted is answered incorrectly, the quiz terminates, i.e., the person is not allowed to attempt the second question. If the first question is answered correctly, the person is allowed to attempt the second question. Which question should be answered first to maximize the expected value of the total prize money received?

$\frac{1}{2} \times \frac{10}{5} = \frac{2}{5} \Rightarrow 3000$        $80\% \rightarrow 1000$   
 $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10} \Rightarrow 2000$        $50\% \rightarrow 1000$   
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