Indian Association for Cultivation of Science
B. S. - First Year
End-Semester Examination 23-24, Semester II
Subject: Linear Algebra and Multivariate Calculus

Subject Code: MAT 1201 A — Full Marks:—50

date: May 2, 2024 — Time Allotted:—3 hours

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1. The paper carries 60 Marks. You can answer as many questions as you wish. If you score X, your final score will be $\min\{X, 50\}$.

2. You are free to use any theorem that is taught to you by me. However you must state them at least once in your answer-scipt because they carry credits.

3. $\mathbb{R}$ *will denote the field of all real numbers. All Vector spaces are over the field of real numbers.*

4. Part-wise scores for each question is shown at the end of the question.

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(1) Determine if the following matrix is non-singular or not.

$$A = \begin{pmatrix} \frac{1}{5} & \sqrt{2} & 17 \\ 0 & \frac{1}{4} & 5 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

If The matrix A is non-singular, determine its inverse also.

[2 + 3]

(2) Show that the vectors $(4, 2, 2)$, $(0, 1, 2)$ and $(1, 0, 3)$ are linearly independent. (Give a complete answer.)

[10]

(3) Show that every vector space of dimension n is linearly isomorphic to $\mathbb{R}^n$.

[10]

(4) Let $f : \mathbb{R}^2 \setminus \{(0, 0)\} \to \mathbb{R}$ be a function such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and equals 0 at all the points of the domain of f. Show that f is a constant function.

[10]

(5) Let $f(x, y) = \cos(x^2 + y^2)$, $(x, y) \in \mathbb{R}^2$. Show the following:

(a) f is differentiable at all points.

(b) Find the total derivative (or differential) $df((x, y); \overline{h})$ of f at all (x, y).

(c) For any (x, y) and any $|\overline{u}| = 1$, compute the directional derivative $D_{\overline{u}} f(x, y)$.

[6 + 6 + 3]

(6) Let $f : [0, 1] \times [0, 1] \to \mathbb{R}$ be a continuous function. Define the function $I : [0, 1] \to \mathbb{R}$ by

$$I(x) = \int_0^1 f(x, y)dy, \; 0 \leq x \leq 1.$$

Show that the function $I : [0, 1] \to \mathbb{R}$ is continuous.

[10]