Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

Master's/Integrated Master's – PhD/PhD program

Mid-Semester (Sem-II) Examination-Spring 2021

Subject: Linear Algebra & Multivariable Calculus ***Subject Code: MCS 1201A***

Full marks: 25 ***Time allotted: 2 hrs***

---

*Answer any **four** questions. The maximum you may score is 25.*

---

1. Consider the following system of equations:

$$ax + by = 1$$
$$cx + dy = 3.$$

   (i) Show that the solution to the above system is unique, if $ad - bc \neq 0$.

   (ii) Analyse the cases when $ad - bc = 0$.

(3+4)

2. Check whether the following subsets of $\mathbb{R}^2$ are vector subspaces or not. Answer **Yes** or **No** and justify your answer.

   (i) $S = \{(x, y) \in \mathbb{R}^2 : ax + by = 0 \text{ where } a, b \in \mathbb{R}\}$.

   (ii) $S = \{(x, y) \in \mathbb{R}^2 : |x| = |y|\}$.

   (iii) $S = \{(x, y) \in \mathbb{R}^2 : ax + by = c \text{ where } a, b \in \mathbb{R} \text{ and } c \neq 0\}$.

(2+2+2)

3. Let U_1 and U_2 be two distinct subspaces of $\mathbb{R}^5$. Then prove or disprove the following:

   (i) $U_1 \cup U_2$ is always a vector subspace of $\mathbb{R}^5$.

   (ii) $U_1 \cap U_2$ is always a vector subspace of $\mathbb{R}^5$.

   (iii) $U_1 + U_2 = \mathbb{R}^5$ whenever $U_1 \cap U_2 = \{0\}$.

(2+2+2)

4. Let $A = \{v_1, v_2, \ldots, v_m\}$ be a collection of $m-$linearly independent vectors of a finite dimensional vector space V. Prove or disprove the following:

   (i) $A \cup \{w\}$ is a collection of linearly independent vectors whenever $w \notin \text{Span } A$.

   (ii) The dimension of Span $A \cup \{w\} = m + 1$ for every $w \in V$.

   (iii) The set $A_w := \{v_1 + w, v_2 + w, \ldots, v_m + w\}$ is a collection of linearly dependent vectors for every $w \in \text{Span } A$.

(2+2+3)

5. Let $P_d[x] :=$ {polynomials with real coefficients in one variable of degree d} for every $d \geq 0$ and $d \in \mathbb{N}$. Then answer the following:

   (i) Prove that $P_d[x]$ is a vector space for every $d \geq 0$.

   (ii) Construct a basis for $P_d[x]$ and hence conclude the dimension of $P_d[x]$.

   (iii) Let $A = \{2x^2 + 3x + 1, 3x^2 + x, 1\}$. Prove that A is a basis for $P_2[x]$.

(1+2+3)