



Indian Association for the Cultivation of Science
(Deemed to be University under *de novo* Category)
Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's
Program/PhD Course
Mid-Semester Examination-Autumn 2023

Subject: Quantum and statistical physics
Full Marks: 25

Subject Code(s): PHS2101
Time Allotted: 2 h

Q1. By any means establish that $\Omega(E) \propto V^N E^{3N/2-1}$ where the symbols have their usual meaning.
[5]

Q2. Consider a system of three fixed spin $\frac{1}{2}$ particles. The spins can point either up or down and there is an external magnetic field H along the Z axis. Write down the states accessible to the system using the quantum numbers m_1, m_2, m_3, \dots

When the total energy of the system is $+\mu H$ are the $+$ and $-$ states equally probable? If not what is the reason?
[3 + 1 + 1 = 5]

Q3. An isolated system consists of a very large number N of weakly interacting spin $\frac{1}{2}$ particles in an external magnetic field H . Write down the expressions for $\ln \Omega(E)$ and draw a qualitative plot of $\ln \Omega(E)$ versus E . What is the degeneracy factor for the state with $E = 0$? Symbols have their usual meaning.
[2 + 1 + 2 = 5]

Q4. If P_r is the canonical probability and Z is the partition function, how are they related?

Prove that

$$\overline{(\Delta E)^2} = -\frac{\partial \bar{E}}{\partial \beta} = \frac{\partial^2 \ln Z}{\partial \beta^2}$$

the symbols having usual meaning.

[1 + 2 + 2 = 5]

Q5. Demonstrate that in the process of absorbing heat the probability P_r of a state changes.

What happens when the system quasistatically changes volume?

Prove that the entropy is given by

$$S = -k \sum_r P_r \ln P_r$$

$$\bar{E} = \sum_r E_r P_r$$

$$d\bar{E} = \sum_r E_r dP_r + \sum_r P_r dE_r$$

$$dQ = d\bar{E} + dW$$

$$E^{3N/2-1}$$

$$dW = \sum_r E_r dP_r$$

$$\left(\frac{\bar{E}}{E} \right)^2 + \left(\frac{E}{\bar{E}} \right)^2 = 2$$

$$\frac{d\bar{E}}{E} = \frac{dE}{E}$$

$$dW = p dV$$

$$[1 + 1 + 3 = 5]$$

$$\sum_r (-dP_r) P_r = -p dV$$