$$ct' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \gamma\left(ct - \frac{vx}{c}\right)$$

$$x^{0'} = \gamma(x^0 - \beta x^1)$$

Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BSMS/Integrated MSc-PhD Programme

End-Semester Examination-2022 (Spring Semester)

Subject: Electromagnetic Theory ***Subject Code(s):PHS 4203***

Full marks: 50 ***Time allotted: 3 hrs***

Answer Q1 and any four out of the remaining six questions

$$ma^a = \frac{q}{c}F^{ab}u_b$$

1. *Answer each part of this question as precisely and briefly as you can*

   (a) What fundamental principle forbids the addition of a Lorentz-invariant mass term of the type $(1/2)m_A^2c^2A_aA^a$ to the Maxwell Lagrange density ? Explain briefly how this works.

   (b) If the frequency ω and the wave vector $\vec{k}$ are combined into a 4-vector $\mathbf{k}$, what physical effect does the Lorentz boost property of ω exhibit ? Why ?

   (c) Can a real scalar field carry electric charge ? Why ?

   (d) Do you think it is possible to move a toy sailboat by shining powerful laser beams on the sail ? Why ?

   (e) For a constant, uniform magnetic field $\vec{B} = B\vec{e}_y$, compute the non-vanishing components of the 3-vector potential $\vec{A}$?

   [5 ×2]

xyz
xzy
zxy

$$\left(\vec{E} + \vec{\beta}\times\vec{B}\right)^2 - (\vec{\beta}\cdot\vec{E})^2$$

2. Synchrotron Radiation

   (a) A relativistic particle of charge q and mass m initially moves in a plane perpendicular to a uniform constant magnetic field $\vec{B} = B\vec{e}_y$. Show that the particle moves in a circle whose radius you are to compute in terms of q, m, B, β_0 where β_0 is the initial speed. Using the formula $dP^a/d\tau = -(2/3c^4)q^2a^2u^a$, derive the rate of emission of radiant energy by the particle. **[3+3]**

   (b) The LHC has a facility for collision of ultrarelativistc protons moving in a circular tunnel under the influence of strong magnetic fields B_p. Protons travel at speeds (in light units) of β_p. It also has a facility for collision of relativistic heavy ions of mass number 200, driven by a magnetic field B_I. These ions travel at speeds of β_I. What should be the ratio of the magnetic fields such the heavy ion tunnel is about 10% in perimeter of the proton tunnel ? Can you estimate which of the two types of charged particles lose energy by synchrotron radiation at a faster rate ? **[2+2]**

$$\vec{D}_\alpha \equiv \epsilon_{x\beta}Q_{\alpha\beta}$$

3. Multipole radiation

   (a) A non-relativistic charge system has vanishing dipole moments, both electric and magnetic. It's electric quadrupole moment $Q_{\alpha\beta} \neq 0$. Derive the vector potential for such a system, and the radiant $\vec{E}, \vec{B}$ fields. **[2+3]**

$$f(x+h) = f(x) + \left.\frac{df(x+h)}{dx}\right|_{h=0}(h) + \ldots$$

(b) Derive the total integrated radiant intensity for the quadrupole moment. [5]

4. Radiation from an accelerated charge

(a) Starting with the formula

$$\frac{dI}{d\Omega} = \frac{q^2}{4\pi c}\left[\frac{\dot{\beta}^2}{(1-\vec{\beta}\cdot\vec{e}_R)^4} + \frac{2(\vec{\beta}\cdot\dot{\vec{\beta}})(\vec{e}_R\cdot\dot{\vec{\beta}})}{(1-\vec{\beta}\cdot\vec{e}_R)^5} - \frac{(\vec{e}_R\cdot\dot{\vec{\beta}})^2}{\gamma^2(1-\vec{\beta}\cdot\vec{e}_R)^6}\right]$$

for the radiant intensity angular distribution due to an arbitrarily moving point charge, compute the ratio of the radiant intensity angular distribution for the same charge and mass moving relativistically with the velocity parallel to the acceleration, and moving non-relativistically. Does a non-relativistic accelerated charge exhibit the phenomenon of relativistic 'beaming' ? Why ?[**3+2**]

(b) A non-relativistic electric dipole of moment d_0 rotates in a plane with an angular frequency ω_0. Does a co-planar observer detect any radiation from the dipole ? Why ? What is the integrated radiant intensity of the particle in the nth harmonic ? [**2+3**]

5. Energy-momentum tensor

(a) Starting with the expression for the radiant energy-momentum tensor due to a relativsitic point charge q, show that it explicitly vanishes everywhere if space is one dimensional. Why does this vanishing not happen in space of dimensionality higher than one ? [**5+1**]

(b) For a plane EM wave, show that there is a simple relation between the energy density and the Poynting vector ? If this relation looks familiar, can you state why that is so ? [**3+1**] T^{a0}

6. Scalar fields interacting with EM

(a) If ϕ is a complex scalar field decomposed as $\phi = \rho(x)\exp i\Theta(x)$, how do gauge transformations act on $\rho(x)$ and $\Theta(x)$? Show that the gauge-covariant derivative $D_a\phi$ has the following actions on ρ and Θ : $D_a\rho = \partial_a\rho$; $D_a\Theta = \partial_a\Theta + A_a$. [**2+3**]

(b) Prove the 'Ricci Identity' $[D_a, D_b]\phi = iF_{ab}\,\phi$. [**5**]

7. Special Relativity

(a) A particle moves with a steady velocity $\vec{\beta}_p$ in the lab frame. An observer moves on a cart inside the lab with a velocity $\vec{\beta}_O$, carrying a detector. What is the velocity of the particle as detected by that detector ? (hint : Resolve $\vec{\beta}_p = \vec{\beta}_{p||} + \vec{\beta}_{p\perp}$ where $\vec{\beta}_{p\perp}\cdot\vec{\beta}_O = 0$) [**6**]

(b) Two mobile detectors, D_1 and D_2, equipped with transmitters, are stationary in the same frame in a lab. D_1 is struck by a projectile and moves at a constant relativistic speed v_1 in the lab frame. After some time T_0, D_2 sends an EM pulse towards D_1 which responds to the pulse immediately on receipt. D_2 receives D_1's response T seconds after its initial transmission. What is v_1 ? [**4**]

$(\ddot{d})^2 = \cancel{q\vec{r}}\ \cancel{q\frac{\vec{v}}{c}}$ $= q^2c^2(\dot{\beta})^2 = q^2c^2\dot{\beta}^2$

$T^{ab}_{(R)} = ?$

$\frac{q^2\dot{\beta}^2}{4\pi c}\sin^2\theta$

$= \frac{(\ddot{d})^2}{4\pi c^3}\sin^2\theta$

$\frac{(\ddot{\vec{d}}\times\vec{e}_x)^2}{4\pi c^3}$