Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

Mid-Semester Examination-2025 (Spring Semester-II)

Subject: Mathematics II | *Subject Code(s): MAT 1201*

Full marks: 25 | *Time allotted: 2 hrs*

Answer **all** questions. Each question carries 5 marks.

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1. Suppose that $f^{(n)}(a)$ and $g^{(n)}(a)$ exist. Prove Leibniz's formula:

$$(fg)^{(n)}(a) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(a) g^{(n-k)}(a).$$

2. Suppose that f satisfies

$$f''(x) + f'(x)g(x) - f(x) = 0$$

for some function g. Prove that if f is 0 at two points then f is 0 on the interval between them.

3. Given n real numbers $a_1, a_2, \ldots, a_n$, let

$$f(x) = \sum_{i=1}^{n} (x - a_i)^2.$$

Show that the least value of $f(x)$ is attained when x is the arithmetic mean of $a_1, a_2, \ldots, a_n$.

4. Suppose that $f : [a,\, b] \to \mathbb{R}$ and that $f(x) = 0$ except for a finite number of points $c_1, c_2, \ldots, c_n$ in $[a,\, b]$. Show that f is integrable over $[a,\, b]$ and that $\int_a^b f = 0$.

5. Find the Taylor polynomial of the function $f(x) = e^{\sin x}$ of degree 3 at 0.