Indian Association for the Cultivation of Science
(Deemed to be university under de novo category)
Integrated Bachelor's-Master's Program
End-Semester Examination-2019 (Semester-I)

Subject: Introductory Classical and Quantum Mechanics **Paper Code: PHS 1101**
Full Marks: 50 **Time Allotted: 3hrs**

Section-A

Answer any three *questions*

1. What is a central force? What are the conserved quantities associated with the motion of a particle under the central force? Can a particle move in a non-uniform circular motion under the action of a central force? Justify. **(5 marks)**

2. The motion of a particle moving along a line is given by $x(t) = Ae^{-\alpha t}\sin(t\sqrt{\omega^2 - \alpha^2}+\epsilon)$, where A, α, ω and ϵ are all constants with $\omega > \alpha$. Find the force acting on the particle. Explain the motion due to such a force when $\omega < \alpha$ and $\omega = \alpha$ with appropriate plots. **(5 marks)**

3. In a planar motion show that the radial and cross-radial components of acceleration are, $\ddot{r} - r\dot{\theta}^2$ and $r\ddot{\theta} + 2\dot{r}\dot{\theta}$ respectively. Here an 'overdot' denotes derivative with respect to time. **(5 marks)**

4. Show that, if work done in a force field $\mathbf{F}$ is independent of the path then, $\nabla \times \mathbf{F} = 0$. If $\mathbf{F} = kf(r)\hat{\mathbf{r}}$, where k is a constant and $f(r)$ is an arbitrary function of r with $\hat{\mathbf{r}}$ being the unit vector along radial direction, then show that $\nabla \times \mathbf{F} = 0$. Show that from this one can arrive at the concept of a scalar potential. **(5 marks)**

Section-B

Answer any seven *questions*

1. Given $\hat{x} = x$ and $\hat{p}_x = -i\hbar(d/dx)$, show that $[\hat{x}, \hat{p}_x] = i\hbar$. Also find out $[\hat{x}^3, \hat{p}_x]$. Can you generalize this to $[\hat{x}^n, \hat{p}_x]$? **(5 marks)**

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2. From the relation $[\hat{x}, \hat{p}_x] = i\hbar$, show that $[\hat{L}_i, \hat{L}_j] = i\hbar\epsilon_{ijk}\hat{L}_k$. Also show that $[\hat{L}^2, \hat{L}_z] = 0$.
(5 marks)

3. Show that $i(d/dx)$ is a hermitian operator. Show that the energy eigenvalues of a particle of mass 'm' confined within two infinite walls, are discrete. Find the energy eigenvalues.
(5 marks)

4. The wave function of a free particle is given by $\psi(x) = A\sin(4x)$. What are the possible momentum eigenvalues? What are the probability of measuring each one of them. What are the energy eigenvalues associated with this wave function? **(5 marks)**

5. Write the Hamiltonian of a one dimensional harmonic oscillator. Without explicitly solving the Schrödinger equation, show that the energy eigenvalues are positive definite. Also show that the minimum energy is non-zero.
(5 marks)

6. Given $(\Delta A)^2 \equiv \langle \left(\hat{A} - \langle A \rangle\right)^2 \rangle$ and $(\Delta B)^2 \equiv \langle \left(\hat{B} - \langle B \rangle\right)^2 \rangle$, with $[\hat{A}, \hat{B}] = i\hbar$. Show that, $\Delta A \Delta B \geq (\hbar/2)$.
(5 marks)

7. Show that for a quantum particle moving in a potential $V(x)$ in one dimension, the following result holds, $(d\langle \hat{p}_x \rangle/dt) = -\langle \left(\partial \hat{V}/\partial \hat{x}\right) \rangle$.
(5 marks)

8. The wave function of a particle is given by $\psi(x) = Ae^{-\alpha x}$, for $-a \leq x \leq a$, where A, α and a are constants. Find the expectation value of the position $\hat{x}$ of the particle. Also find $(\Delta x)^2 = \langle (\hat{x} - \langle \hat{x} \rangle)^2 \rangle$.
(5 marks)

9. Define the creation operator $\hat{a}^\dagger$ and annihilation operator $\hat{a}$ associated with the Hamiltonian of the harmonic oscillator with frequency ω and mass m. Hence find out the position-momentum uncertainty associated with the nth excited state of the harmonic oscillator. What can you comment about the uncertainty of the ground state?
(5 marks)