Indian Association for the Cultivation of Science

(Deemed to be University under *de novo* Category)

Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's Program/PhD Course

Mid-Semester Examination-Autumn 2024

Subject: PHYSICS ***Subject Code(s): PHS 2101***

Full Marks: 25 ***Time Allotted: 2 h***

Answer Q1 and any two of the other questions.

1. Say whether the following statements are true or false. (No justification is needed for your answer.)

   (a) **3 marks** The momentum operator on a wavefunction acts as the differential operator $-i\hbar d/dx$. For the state vector $|\psi\rangle$, this means

   $$\langle x\,|p|\,\psi\rangle = \left\langle x\left|-i\hbar\frac{d}{dx}\right|\psi\right\rangle .$$

   (b) **3 marks** For a particle in a 1D potential $V(x) = bx^4$, the expectation value of x is zero in any energy eigenstate.

   (c) **3 marks** The Fourier transform of the Dirac δ-function is $F(k) = 1$.

2. A matrix M is given by:

$$M = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

(a) **1 mark** Is this matrix Hermitian? (Just a yes/no answer would be sufficient.)

(b) **3 marks** Find its eigenvalues.

(c) **4 marks** Find the eigenvectors. (No need to normalize them.)

3. Consider the double potential barrier given by

$$V(x) = \begin{cases} V_1 & \text{for } a \leqslant x \leqslant b \\ V_2 & \text{for } c \leqslant x \leqslant d \\ 0 & \text{otherwise,} \end{cases}$$

with $0 < a < b < c < d$.

(a) **2 marks** Make a rough sketch of the potential.

(b) **3 marks** For an incident wave of the form e^{ikx} coming from the left, what should be the forms of the solutions in the different regions for an energy eigenvalue E?

(c) **3 marks** What are the boundary conditions that the coefficients introduced in the previous part should satisfy? (You must not try to solve the equations.)

4. (a) **2 marks** Write the time-independent Schrödinger equation for a simple harmonic oscillator.

(b) **4 marks** The equation can have a solution of the form

$$\psi(x) = Nxe^{-x^2/2\ell^2},$$

where N and ℓ are constants. Find ℓ.

(c) **2 marks** Find the energy eigenvalue corresponding to this solution.