Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's Program/PhD Course

End-Semester Examination-Autumn 2023

Subject: Introductory Classical and Quantum Mechanics *Subject Code(s): PHS1101*
Full marks: 50 *Time allotted: 3 hr*

Answer any *five* questions

1. (a) Show that $i(\partial/\partial x)$ is a hermitian operator. Find its eigenfunctions. (5 marks)

   (b) Show that the eigenvalues of a hermitian operator are real. (5 marks)

2. (a) If $[\hat{x}, \hat{p}_x] = i\hbar$, then find $[\hat{x}^3, \hat{p}_x]$. (3 marks)

   (b) Find the de-Broglie wavelength of a particle having mass of 10^{-27} Kg moving with a speed of 10^8 m/sec. (2 marks)

   (c) Using the expression

$$\langle A \rangle = \int \Psi^*(x) \hat{A} \Psi(x) dx \ ,$$

   show from the Schrödinger's equation that,

$$\frac{d}{dt}\langle p_x \rangle = -\langle \frac{\partial V}{\partial x} \rangle$$

   (5 marks)

3. (a) Find the energy eigenfunctions and eigenvalues of a particle of mass m confined within two infinite walls separated by a distance L. (4 marks)

   (b) Using the results derived in the above problem, if $m = 10^{-31}$ Kg, $L = 10^{-10}$ m, find out the energy of the second excited state. (3 marks)

   (c) A free particle is in a state

$$\Psi(x) = A \sin(3x) \ .$$

   Find the possible outcome of measurement of momentum. What are the probabilities of each of these outcome? (3 marks)

4. (a) A particle of mass m travelling along a line towards positive x-axis encounters a potential barrier of height V_0 and width a. If the energy of the particle is E and $E < V_0$, find the tunnelling probability through the barrier. (8 marks)

   (b) What happens if the barrier width decreases? (2 marks)

5. (a) If $\hat{A}$ is a hermitian operator, show that $\langle \hat{A}^2 \rangle$ is always positive. (4 marks)

   (b) The operator for the z component of angular momentum is $\hat{L}_z = -i\hbar(\partial/\partial\phi)$. Show that the eigenvalues of $\hat{L}_z$ are integer multiples of $\hbar$. (3 marks)

   (c) What is the zero point energy of a linear harmonic oscillator? Explain. (3 marks)

6. (a) Write down the Hamiltonian of (i) a linear harmonic oscillator, and (ii) electron in a Hydrogen atom. (2 marks)

   (b) Using uncertainty relation

$$\Delta x \Delta p_x \sim \hbar \ ,$$

   estimate the order of the minimum energy for case (i) and (ii) above. (8 marks)