Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

Integrated Bachelor's-Master's Program

Mid-Semester Examination – Autumn Semester 2024

Subject: Probability and Statistics *Subject Code: MCS 2101A*

Full marks: 25 *Time allotted: 2 hrs*

- GROUP A has three questions with equal marks each and carries a total of 6 MARKS. You are to attempt ALL THREE.
- GROUP B has five questions with 7 MARKS each. Answer as much as you can. You may score a MAXIMUM of 19 MARKS in GROUP B.
- You do NOT NEED to give a simplified final answer, but if you do, the steps leading to it MUST BE SHOWN.

Group A

1. If A and B are two events with $P(A) = 0.8, P(B) = 0.6$ and $P(A \cap B) = 0.35$, then what is the probability that **exactly one** of the two events occur? [2]
2. A, B and C are three events with $P(A) = 0.6, P(B) = P(C) = 0.8, P(B \cap C) = 0.7$ and $P(A \cap B) = P(A \cap C) = 0.5$. What can be the **maximum possible value** of $P(A \cap B \cap C)$? **Why?** [2]
3. If A, B and C are three **independent** events, each with probability $1/3$, then what is the probability that **none** of these events occur? [2]

Group B

4. A touring fifteen member football team, that includes Rishav and Zakir, check into a hotel where they are randomly placed in fifteen adjacent single occupancy rooms. What is the probability that the two rooms where Rishav and Zakir check into are (i) **adjacent** rooms, (ii) separated by **exactly three** rooms? $(3+4) = [7]$
5. A box contains ten red cards numbered $1, 2, \cdots, 10$ and ten black cards numbered $1, 2, \cdots, 10$. If eight cards are drawn at random from the twenty cards in the box, what is the probability that the drawn cards have (i) **no pair** of cards with the same number (ii) **exactly two pairs** of cards with same numbers? $(3+4) = [7]$
6. Sixteen tosses of a coin resulted in 10 Heads and 6 tails. Assuming that all possible sequences in which the Heads and Tails occured are equally likely, find the probability that there are (i) **exactly five** runs of **Heads** (ii) **a total of nine** runs of **Heads and Tails** combined. $(3+4) = [7]$

7. Doctors at a hospital in the city of Bern consist of 50% Swiss, 30% German and 20% French. Of the doctors at the hospital from these three nationalities, the proportion who can speak English is 1/3 among Swiss, 1/2 among Germans and 1/4 among French. If I run into an unknown doctor at this hospital and find that she **cannot speak English**, what is the probability that the lady doctor is German? [7]

8. (a) From $1, 2, \cdots, 9$, we draw two distinct digits at random. Let A be the event that the digit 4 is among the two drawn and B the event that the larger of the two digits drawn is 7. Examine if A and B are independent events.

   (b) If A and B are two events such that $P(A \mid B) = P(A \mid B^c)$, then prove that the events A and B **must be** independent. $(4 + 3) =$[7]