Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

Mid-Semester Examination-2023 (Spring Semester-II)

Subject: Linear Algebra and Multivariable Calculus *Subject Code(s): MCS 1201A*

Full marks: 25 *Time allotted: 2 hrs*

Answer all questions.

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1. Let V be a finite dimensional vector space and S a subspace of V. Prove each of the following statements.

   (a) S is finite dimensional and $dim(S) \leq dim(V)$.

   (b) $dim(S) = dim(V)$ if and only if $S = V$.

   (c) Every basis for S is part of a basis for V.

   (d) A basis for V need not contain a basis for S.

   [6]

2. In the real vector space $C(1, e)$, define an inner product by the equation

$$(f, g) = \int_1^e (log x) f(x) g(x) dx.$$

   (a) If $f(x) = \sqrt{x}$, compute $\|f\|$.

   (b) Find a linear polynomial $g(x) = a + bx$ that is orthogonal to the constant function 1.

   [6]

3. Let A be a matrix such that $A^2 = \overset{A}{\cancel{I}}$. Prove that

$$(A + I)^k = I + (2^k - 1)A.$$

   [3]

4. Given an $n \times n$ matrix A with real entries such that $A^2 = -I$. Prove the following statements about A.

(a) A is nonsingular.

(b) n is even.

(c) A has no real eigenvalues.

(d) $\det A = 1$.

[4]

5. Suppose that A is a square matrix. Let the trace of A, denoted by $tr(A)$, be the sum of all diagonal elements of A. Prove that

(a) Traces of similar matrices are equal;

(b) $tr(A) =$ the sum of eigenvalues of A.

[6]