Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Integrated Bachelor's-Master's Program

End Semester Examination - Autumn 2022

Subject: Introductory Classical and Quantum Mechanics | *Subject Code(s): PHS 1101*
Full marks: 50 | *Time allotted: 3 hrs*

Answer all questions

1. (a) Write the hamiltonian operator $\hat{H}$ of a free particle of mass m moving in one dimension . Using $[\hat{x}, \hat{p_x}] = i\hbar$, Calculate $[\hat{x}, \hat{H}]$. **(4 marks)**

   (b) Calculate $[\hat{x}, \hat{L_z}]$, where $\hat{L_z}$ is the z-component of angular momentum operator . **(4 marks)**

2. (a) From Schrodinger equation show that $\frac{d}{dt} < \hat{p_x} > = - < \frac{\partial \hat{V}}{\partial x} >$.
   (Symbols have their usual meaning). **(4 marks)**

   (b) Show that the eigenvalues of a hermitian operator are real numbers.
   Also show that $\frac{d^2}{dx^2}$ is a hermitian operator. **(4 marks)**

3. (a) Show that the number of modes of waves between frequency ν and $\nu + d\nu$ inside a closed cavity is proportional to $\nu^2 d\nu$. **(4 marks)**

   (b) The stopping potential for a light of wavelength λ incident on a metallic surface is 4 volt. The stopping potential becomes 1 volt if the wavelength is doubled. Find λ . **(4 marks)**

4. (a) Find the energy difference between the first two energy eigenstates for a particle of mass $10^{-31} kg$ enclosed between two infinite walls separated by a distance 10^{-10} m .
   (Derive the necessary expression of energy eigenvalues for this purpose). **(4 marks)**

   (b) Show that the eigenvalue of the squares of a hermitian operator is positive. From this justify that the eigenvaues of a one dimensional harmonic oscillator is positive. **(4 marks)**

5. (a) Write the expression of the energy eigenvalues of a one dimensional harmonic oscillator . Explain , why the minimum energy is not zero .
   Consider a mass of 1 Kg , attached to a massless spring of spring constant k = 196 N/m . Find the energy spacing between two successive energy levels. **(4 marks)**

   (b) From the expression of hamiltonian of a one dimensional harmonic oscillator, estimate the expression of energy using the uncertainty principle $\Delta x \Delta p_x \sim \hbar$. **(4 marks)**

6. A particle of mass m , moving along a line encounters a potential barrier of height V_0 and width a . If the energy E of the incident particle is less than V_0 then find the expression of it's tunnelling probability and show that the tunnelling probability decreases with increase in V_0 . **(7 marks)**

(b) A one dimensional harmonic oscillator is in the ground state wave function $\psi(x) = Ae^{-\alpha x^2}$, where A and α are constants . Find the expectation value of it's position x. **(3 marks)**