Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

Final Examination-2023 (Spring Semester-II)

Subject: Linear Algebra and Multivariable Calculus Subject Code(s): MCS 1201A

Full marks: 50 *Time allotted: 3 hrs*

Answer all questions.

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1. If

$$f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2} \quad \text{when} \quad (x,y) \neq (0,0).$$

How must $f(0,0)$ be defined so as to make f continuous at the origin? [4]

2. Let f be a scalar field continuous at an interior point a of a set S in $\mathbb{R}^n$. If $f(a) \neq 0$, prove that there exists an $r > 0$ such that f has the same sign as $f(a)$ in $B(a, r)$. [5]

3. (a) Prove that there is no scalar field f such that $f'(a; y) > 0$ for a fixed vector a and every non-zero vector y.

   (b) Give an example of a scalr field f such that $f'(x; y) > 0$ for a fixed vector y and every vector x.

[6]

4. Let $f(x,y) = \frac{2xy}{x^2+y^2}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f has partial derivatives everywhere, but f is not continuous at $(0,0)$. [5]

5. Find the points (x, y) and the directions for which the directional derivative of $f(x,y) = 3x^2 + y^2$ has its largest value, if (x, y) is restricted to be on the circle $x^2 + y^2 = 1$. [5]

6. Find a pair of linear Cartesian equations for the line which is tangent to both the surfaces $x^2 + y^2 + 2z^2 = 4$ and $z = e^{x-y}$ at the point $(1, 1, 1)$. [5]

7. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ a vector field defined as follows:

$$f(x, y) = e^{x+2y}\mathbf{i} + \sin(2x + y)\mathbf{j}.$$

Compute the Jacobian matrix $Df(x, y)$. [5]

8. Let f be a scalar field which is C^1 in an $n-$ball $B(a)$. Assume f has a local maximum/minimum at a. Show that $\nabla f(a) = 0$. [5]

9. Find the stationary points and state their nature, for the function

$$f(x, y, z) = x^2(y-1)^2(z+\frac{1}{2})^2.$$

[6]

10. Find the maximun value of the function $f(x, y) = x^2y^2$ subject to the condition $x^2 + y^2 = c^2$, where c is a constant. [4]