Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

Integrated Bachelor's-Master's Program

End-Semester Examination – Autumn Semester 2024

Subject: Probability and Statistics | *Subject Code: MCS 2101A*

Full marks: 50 | *Time allotted: 3 hrs*

- GROUP A has **FOUR** questions with **2 MARKS EACH**. You are to attempt **ALL FOUR** in GROUP A.
- GROUP B has **SIX** questions with **10 MARKS EACH**. Answer as much as you can. You may score a **MAXIMUM of 42 MARKS** in GROUP B.
- A simple answer, when not obvious, **MUST INCLUDE STEPS** leading to it.

Group A

1. A, B and C are three events with $P(A) = 0.7, P(B) = 0.5, P(C) = 0.6, P(B \cap C) = 0.3$ and $P(A \cap B) = P(A \cap C) = 0.4$. What is the **maximum possible value** of $P(A \cap B \cap C)$? **Why?** [2]
2. If A, B and C are three **independent** events, each with probability $1/3$, then what is the probability that **exactly two** of these three events occur? [2]
3. For a random variable X with $E(X) = -1$, what is the **minimum possible value** of $E(X(X-2))$? **Why?** [2]
4. A density $f(x)$ on $(0, 2)$ satisfies $f(x) = f(2-x)$ for all $x \in (0, 2)$. What is the value of $F(1)$, where F is the distribution function? [Picture graph of f!] [2]

Group B

5. Cards are randomly drawn, one after another and without replacement, from a standard deck of cards. Find the probability that
   (a) the first time a card of spades appears is at the 10th draw.
   (b) no two jacks appear at consecutive draws. $(4+6) =$**[10]**
6. You are given two puzzles to solve. Chances are 70% that you will solve the first one. Chances that you will solve the second puzzle are 70%, if you can solve the first one, while they are 20%, if you cannot solve the first one.
   (a) Find the probability that you solve solve at least one puzzle.
   (b) Given that you solve exactly one, find the probability that it is the second one.
   $(5+5) =$ **[10]**

7. In each of the following two random experiments, you have to examine and say whether the stated events A and B are independent or not. Your answer must be **mathematically justified**.
   (a) A box contains five red cards numbered $1, \cdots, 5$ and five black cards numbered $1, \cdots, 5$. Two cards are drawn at random without replacement. A is the event that the first drawn card is a red card, while B is the event that the second drawn card is numbered 2.
   (b) Ten students in a class are ranked according to their writing skills. Assume that all possible rankings are equally likely and that there are no ties. A is the event that student X ranks better than student Y, while B is the event that student X has rank 4. $(5+5) = $ **[10]**

8. A ten-faced die has its faces marked with $1, 2, \cdots, 10$ and each time it is thrown, exactly one of its ten faces shows up, each with equal probability.
   (a) If the die is thrown 8 times and X denotes the number of times a face with a prime number appears, describe the distribution of X.
   (b) The die is thrown repeatedly until faces with prime numbers appear four times. Describe the distribution of the number of throws needed. $(5+5) = $ **[10]**

9. A box has a total of 50 balls of five different colours with 10 balls of each colour. Balls are randomly drawn, one after another with replacement.
   (a) Let X be the number of different colours that appear in the first n draws. Express X as a sum of binary random variables and use that to give a simple formula for $E(X)$.
   (b) Find the expected number of draws needed to have all five colours appear. Your answer must be **properly justified**. [You may use the fact that mean of a Geometric distribution with parameter p equals $\frac{1}{p}$] $(5+5) = $ **[10]**

10. Let X be a continuous random variable with density $f(x) = C(1+x)^{-3}$, $x \in (0, \infty)$.
   (a) Find the constant C and the probability $P(1 < X < 3)$.
   (b) Find the density of the random variable $Y = 1/X$.
   (c) Describe the distribution of the discrete random variable $Z = [X]$.
   $(4+4+2) = $ **[10]**