Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

Mid-Semester Examination-2022 (Autumn Semester-I)

Subject: Calculus of One Variable ***Subject Code(s): MCS 1101A***

Full marks: 25 ***Time allotted: 2 hrs***

Answer **all** questions. Each question carries 5 marks.

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1. Let $x_1 = 1$ and $x_{n+1} = \sqrt{2 + x_n}$ for $n \geq 1$. Show that $\{x_n\}$ is convergent and find the limit.

2. Suppose that a function f satisfies $f(x+y) = f(x) + f(y)$ for all x, y, and that f is continuous at 0. Show that f is continuous everywhere.

3. Suppose that f and g are continuous on $[a, b]$ and that $f(a) < g(a)$, but $f(b) > g(b)$. Prove that $f(x) = g(x)$ for some $x \in (a, b)$.

4. Let $f : [a, b] \to \mathbb{R}$ be continuous. Show that the range of f is a closed bounded interval.

5. Suppose that a function is one-one on an interval. Show that f is either strictly increasing or strictly decreasing.