CS 351 - Artificial Intelligence

Clustering and Recommendation Systems

Assignment 03

September, 2021

Group members: Altaf Shaikh & Maheen Anees
Student IDs: as05016, ma05156

Q2.1 Computing gradients

$$\begin{aligned}\Delta p_{ik} &= \frac{\partial e_{ij}^2}{\partial p_{ik}} \\ &= \frac{\partial (r_{ij} - \sum_k p_{ik} q_{kj})^2}{\partial p_{ik}} \\ &= 2(r_{ij} - \sum_k p_{ik} q_{kj})(-q_{kj})\end{aligned}$$

$$\Delta p_{ik} = -2 e_{ij} q_{kj}$$

$$\begin{aligned}p'_{ik} &= p_{ik} - \alpha \Delta p_{ik} \\ &= p_{ik} - \alpha(-2 e_{ij} q_{kj}) \\ &= p_{ik} + \alpha 2 e_{ij} q_{kj}\end{aligned}$$

$$\begin{aligned}\Delta q_{kj} &= \frac{\partial e_{ij}^2}{\partial q_{kj}} \\ &= \frac{\partial (r_{ij} - \sum_k p_{ik} q_{kj})^2}{\partial q_{kj}} \\ &= 2(r_{ij} - \sum_k p_{ik} q_{kj})(-p_{ik})\end{aligned}$$

$$\Delta p_{ik} = -2e_{ij}p_{ik}$$

$$\begin{aligned} q'_{kj} &= q_{kj} - \alpha\Delta q_{kj} \\ &= q_{kj} - \alpha(-2e_{ij}p_{ik}) \\ &= q_{kj} + \alpha 2e_{ij}p_{ik} \end{aligned}$$

Q2.2 Adding bias

$$e_{ij}^2 = (r_{ij} - bu_i - bi_j - \sum_k P_{ik}q_{kj})$$

$$\begin{aligned} \Delta p_{ik} &= \frac{\partial e_{ij}^2}{\partial p_{ik}} \\ &= \frac{\partial (r_{ij} - bu_i - bi_j - \sum_k P_{ik}q_{kj})^2}{\partial p_{ik}} \\ &= 2e_{ij}(-q_{kj}) \\ &= -2e_{ij}q_{kj} \end{aligned}$$

$$p'_{ik} = p_{ik} + \alpha 2e_{ij}q_{kj}$$

$$\begin{aligned} \Delta q_{kj} &= \frac{\partial e_{ij}^2}{\partial q_{kj}} \\ &= 2e_{ij}(-p_{ik}) \\ &= -2e_{ij}p_{ik} \end{aligned}$$

$$q'_{kj} = q_{kj} + \alpha 2e_{ij}p_{ik}$$

$$\begin{aligned} \Delta bu_i &= \frac{\partial e_{ij}^2}{bu_i} \\ &= 2e_{ij}(-1) \\ &= -2e_{ij} \end{aligned}$$

$$bu'_i = bu_i + \alpha 2e_{ij}$$

$$
\begin{aligned}
\Delta bi_j &= \frac{\partial e_{ij}^2}{bi_j} \\
&= 2e_{ij}(-1) \\
&= -2e_{ij}
\end{aligned}
$$

$$
bi'_j = bi_j + \alpha 2e_{ij}
$$