1a. $A + \varepsilon_1 = 2$

$B + \varepsilon_2 = 1$

$A + B + \varepsilon_3 = 4$

$A + \varepsilon_4 = 2$

$B + \varepsilon_5 = 2$

1b. $RSS = (\varepsilon_1)^2 + (\varepsilon_2)^2 + (\varepsilon_3)^2 + (\varepsilon_4)^2 + (\varepsilon_5)^2$

$RSS = (2-A)^2 + (1-B)^2 + (4-A-B)^2 + (2-A)^2 + (2-B)^2$

$0 = \frac{\partial RSS}{\partial A} = -2(2-A) + 0 - 2(4-A-B) - 2(2-A) + 0$

$= -2(8-3A-B)$

$0 = \frac{\partial RSS}{\partial B} = 0 - 2(1-B) - 2(4-A-B) + 0 - 2(2-B)$

$= -2(7-A-3B)$

$3A + B = 8 \qquad A + 3B = 7$

$4A + 4B = 15$

$A + B = \frac{15}{4}$

$3A + B - (A+B) = 8 - \frac{15}{4} \qquad A + 3B - (A+B) = 7 - \frac{15}{4}$

$2A = \frac{17}{4} \qquad 2B = \frac{13}{4}$

$\hat{A} = \frac{17}{8} \qquad \hat{B} = \frac{13}{8}$

1.c.

```
> y_1 <- c(2, 1, 4, 2, 2)

> a_1 <- c(1, 0, 1, 1, 0)

> b_1 <- c(0, 1, 1, 0, 1)

> mod_1 <- lm(y_1 ~ a_1 + b_1 -1)

> mod_1

Call:
lm(formula = y_1 ~ a_1 + b_1 - 1)

Coefficients:
  a_1    b_1
2.125  1.625
```

2.a. $A+\varepsilon_1=2$

$B+\varepsilon_2=1$

$A+B+\varepsilon_3=4$

$A+\varepsilon_4=2$

$B+\varepsilon_5=2$

$A+B+\varepsilon_6=5$

2.b. $RSS=(\varepsilon_1)^2+(\varepsilon_2)^2+(\varepsilon_3)^2+(\varepsilon_4)^2+(\varepsilon_5)^2+(\varepsilon_6)^2$

$$RSS=(2-A)^2+(1-B)^2+(4-A-B)^2+(2-A)^2+(2-B)^2+(5-A-B)^2$$

$$0=\frac{\partial RSS}{\partial A}=-2(2-A)+0-2(4-A-B)-2(2-A)+0-2(5-A-B)$$

$$=-2(13-4A-2B)$$

$$0=\frac{\partial RSS}{\partial B}=0-2(1-B)-2(4-A-B)+0-2(2-B)-2(5-A-B)$$

$$=-2(12-2A-4B)$$

$4A+2B=13 \quad 2A+4B=12$

$6A+6B=25$

$A+B=\frac{25}{6}$

$4A+2B-(2A+2B)=13-2\left(\frac{25}{6}\right) \quad 2A+4B-(2A+2B)=12-2\left(\frac{25}{6}\right)$

$2A=\frac{14}{3} \quad 2B=\frac{11}{3}$

$\hat{A}=\frac{7}{3} \quad \hat{B}=\frac{11}{6}$

2.c.

```
> y_2 <- c(2, 1, 4, 2, 2, 5)

> a_2 <- c(1, 0, 1, 1, 0, 1)

> b_2 <- c(0, 1, 1, 0, 1, 1)

> mod_2 <- lm(y_2 ~ a_2 + b_2 -1)

> mod_2

Call:
lm(formula = y_2 ~ a_2 + b_2 - 1)

Coefficients:
  a_2    b_2
2.333  1.833
```

3. $A+\varepsilon_1=2$

$B+\varepsilon_2=1$

$A+B+\varepsilon_3=4$

$A+\varepsilon_4=2$

$B+\varepsilon_5=2$

$A+B+\varepsilon_6=5$

$A+B+\varepsilon_7=5$

$RSS=(\varepsilon_1)^2+(\varepsilon_2)^2+(\varepsilon_3)^2+(\varepsilon_4)^2+(\varepsilon_5)^2+(\varepsilon_6)^2+(\varepsilon_7)^2$

$RSS=(2-A)^2+(1-B)^2+(4-A-B)^2+(2-A)^2+(2-B)^2+(5-A-B)^2+(5-A-B)^2$

$0=\frac{\partial RSS}{\partial A}=-2(2-A)+0-2(4-A-B)-2(2-A)+0-2(5-A-B)-2(5-A-B)$

$=-2(18-5A-3B)$

$0=\frac{\partial RSS}{\partial B}=0-2(1-B)-2(4-A-B)+0-2(2-B)-2(5-A-B)-2(5-A-B)$

$=-2(17-3A-5B)$

$18=5A+3B \qquad 17=3A+5B$

$35=8A+8B \qquad \frac{35}{8}=A+B$

$5A+3B-(3A+3B)=18-3\left(\frac{35}{8}\right) \qquad 3A+5B-(3A+3B)=17-3\left(\frac{35}{8}\right)$

$2A=\frac{39}{8} \qquad 2B=\frac{31}{8}$

$\hat{A}=\frac{39}{16} \qquad \hat{B}=\frac{31}{16}$

4a. $A+\varepsilon_1=2$

$B+\varepsilon_2=1.5$

$C+\varepsilon_3=4.5$

$$RSS=(\varepsilon_1)^2+(\varepsilon_2)^2+(\varepsilon_3)^2$$

$$=(2-A)^2+(1.5-B)^2+(4.5-A-B)^2$$

$$0=\frac{\partial RSS}{\partial A}=-2(2-A)+0-2(4.5-A-B)$$

$$=-2(6.5-2A-B)$$

$$0=\frac{\partial RSS}{\partial B}=0-2(1.5-B)-2(4.5-A-B)$$

$$=-2(6-A-2B)$$

$6.5=2A+B \qquad 6=A+2B$

$12.5=3A+3B \qquad \frac{25}{6}=A+B$

$2A+B-(A+B)=6.5-\frac{25}{6} \qquad A+2B-(A+B)=6-\frac{25}{6}$

$\hat{A}=\frac{7}{3} \qquad \hat{B}=\frac{11}{6}$

4.b. The results are the same, so when calculating least squares with the same number of measurements for each weight (A, B, A+B) we can calculate using the mean of each weight.