

Persistence - Models of Local Interaction Distribution Systems in Nature and Society

Alex Stokes

May 31, 2020

Abstract

All forms of life rely on the distribution of resource to allow them to grow and persist. There are a number of ways of classifying, describing and modelling these systems. Understanding of such systems can increase human knowledge and also be applied in technology.

The topic of resource distribution systems is multi-disciplinary which covers many subjects including computer science, life science, the humanities and engineering. In order to explore such a broad topic, numerical simulation models are used.

The thesis looks firstly at space optimising algorithms. Different algorithm types are employed to see which lead to the optimal arrangement for distribution systems.

A resource distribution system is then added to these space optimising algorithms to explore the growth, persistence, scaling and the affect of resource creation in such systems.

Finally, models of agent heterogeneity are explored to further understand how agents with different preferences change the properties of the system as a whole such as inequality and system size.

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1 Introduction

This thesis is on the general topic of how systems of many agent systems distribute amongst each other in nature and society. Nature has evolved solutions for resource distribution problems in order to persist; likewise societies have evolved institutions to enable their persistence in a stable manner.

The topic is explored within the general field of complex systems; a multidisciplinary field of study which generally studies how systems of many agents interact through non-linear interactions. Empirical and qualitative information is collated during the literature review on a number of human and biological systems. Abstract simulation models have then been built in order to find underlying algorithms and networks which may govern laws seen in nature and the stability of such systems.

There are commonalities between natural and social systems but also many differences. For example a *protocognitive biological system* will rely on local interactions (and also memory) in order to process information and distribute resource, and the same may be the case for traditional (small-scale) societal systems. However modern social systems and other systems in nature such as the nervous system have innovated ways to transfer and create resource through non-local interactions.

If local interactions are the dominant mode of information and resource transfer then the spatial structure of the system is likely to have properties critical to supporting the system. The universal allometric scaling law in biological systems West et al. (1997) is one such property which appears to be constant across all biological systems. This law may also be true for traditional societal systems. Systems which transfer information through non-local interactions appear to have different laws of scaling Bettencourt et al. (2007). If this is the case, then it has profound implications for the ways in which we manage our societies, most notably that notions of stability in traditional and natural systems cannot provide all the answers to how we manage our current societies and plan for the future.

The thesis is broken down into the following sections:

- Research questions, aims and objectives.
- Literature review.

- Space Optimising Algorithms
- The distribution of resources in Living Systems
- Distribution and Growth in Resource Creating Systems
- Resource Inequality in Multi-Agent Systems

2 Research Questions

This section summarises the general research themes and questions explored during this thesis. As mentioned in Section 1, the main research method applied is toy simulation models. These models are guided by the following research questions:

1. Section 5:

- Living systems are known to grow following a sigmoidal curve with $3/4$ scaling in relation to body mass and metabolic rate West et al. (1997). Will a toy simulation model using only self-organising rules set in the initial conditions have the same features?

2. Section 6:

- Systems undergoing regime shift are known to exhibit phenomena such as an increase in variance and flickering Scheffer et al. (2001). Under what conditions and changes in initial parameters (if any) will a resource creating system exhibit this phenomena?

3. Section 7:

- Most data on inequality in human societies is on the macro-scale, usually taken at the national level. This rarely takes into account mechanisms at a lower level which is likely to be a primary driver of this data. In an effort to bridge the gap, different models of agent interaction are explored in the resource distribution model and the results collated to provide general information on how and why system may grow, collapse, increase or decrease in inequality.

3 Literature Review

In an attempt to create a logical order for the reader to follow, the literature review has been broken down into sections. Section 3.1 firstly introduces the general field of Complex Systems, and focuses on previous theoretical and conceptual work on classifying such systems. A sub-field of Complex Systems is then introduced in Section 3.2 - "Complex Adaptive Systems" which is more relevant to the topic of this thesis, as it tends to focus on living systems. This includes sub-topics of spatial and non-spatial CAS, cognitive and protocognitive CAS and an introduction to scaling in such systems.

Sections 3.3 and 3.4 introduce previous literature on complexity and stability in complex adaptive systems and the related topic of CAS as networks respectively.

Section 3.9 then introduces methods for modelling Complex Systems and previous examples of modelling techniques used.

Finally, Section 3.12 provides a review of previous empirical and qualitative studies on complex systems of interest to this thesis.

3.1 The Field of Complex Systems

The field of complex systems tends to be an ill defined concept, most likely due to the generality of the subject. A complex system may be defined as a system with many degrees of freedom or many interacting components, which interact in a non-linear fashion. The three body problem in the physical sciences may be seen as the starting point of when a system becomes complex as there is no closed form solution for this as with two bodies Šuvakov and Dmitrašinović (2013). Instead methods to solve the three body problem under particular conditions tend to involve numerical simulations with certain parameters Li et al. (2017). The three body problem in itself is a large topic of study which this thesis will not delve into. However, it does serve as an example of how complex, chaotic systems can arise from the simplest of systems. In reality, most complex systems have many more interacting bodies than in the three body problem, but a lot of the same theoretic work still applies in terms of describing such systems.

Bar-Yam (1998) states the field of Complex Systems seeks to increase our ability to understand the universality that arises when systems are highly

complex and defines a complex system as one which consists of interconnected or interwoven parts. To put this into context, a simple or complicated system is one in which the whole is the sum of the parts; they act in a linear fashion. Therefore to describe the system behaviour, each part can be summed together. In a complex system, not only do the parts need to be understood, but their interaction with one another too due to the inherent non-linearity present.

Most if not all natural systems are complex, but many assumptions are made such as linearising the interactions of the components in order to simplify them. This is usually valid for explaining the behaviour of systems within given constraints but not applicable when the system is close to a regime shift such as a pendulum swinging out of control or a social-ecological system nearing collapse. Upon undergoing a regime shift, the system will often re-organise in order to persist Scheffer et al. (2001), or risk extinction.

General characteristics of complex systems is focused around the following four components Bar-Yam (1998):

Space The structure and spatial extent of the system. The structure may relate to its formation. It may also relate to its boundaries and how it interacts with other systems.

Time The length of time that dynamical processes take in the system. The ways in which the complex system changes in response to changes in their environment.

Organisation is present across all disciplines and a key concept/observation in any scientific endeavour. Two theoretical extreme fixed points can be thought of when thinking in terms of organisation. A self-organised system is a system which organises without any intervention. A planned system is one that is organised completely through external or centralised control. Neither of these pure forms exist in nature but any system lies on a spectrum between the two. Another spectrum involves top-down and bottom-up organisation, the degree to which bottom-up and top-down processes control the organisation of the system.

Complexity To what degree is the system complex. There are many definitions of complexity; one common definition is the amount of information

needed in order to describe the system Bar-Yam (1998).

These general characteristics are important properties to take into account when studying complex systems, but they are very general, and need to be built upon when studying a particular complex system, for instance when looking at a particular social-ecological system, the different variables in the systems need to be accounted for and the time-scales over which they operate.

And additionally,

Scale and Levels This may form part of the component *Organisation* but it is important to mention that many systems operate at a range of scales and if a particular system is being analysed at one scale there may be commonalities and differences with how it operates at other scales, and there may also be influences in behaviour from one scale to another. Levels are often thought of in terms of hierarchy for instance in a company or even academic organisation. However, they may also be thought of in terms of a "container view" such as minutes are a finer level to hours, but form part of hours and also an "emergent view" whereby processes arise at a higher level through interactions taking place at a lower level, (see Wilensky and Resnick (1999) for a more detailed discussion on this.)

3.2 Complex Adaptive Systems (CAS)

This thesis focuses on a subset of complex systems - Complex Adaptive Systems (CAS). CAS can be thought of as a subset of Complex Systems which is generally synonymous with the life sciences Gell-Mann (1995); Holland (1992); Lansing (2003).

3.2.1 Spatial or Non-Spatial Systems

Within CAS, a further subdivision can be made based on whether the system is spatially constrained West et al. (1997), such as biological systems (plants, slime moulds, chemical reaction-diffusion), or networked, where the spatial constraint isn't necessarily required such as social systems, or wireless systems (the internet, modern economic systems) Bettencourt et al. (2007).

3.2.2 Scaling in Complex Adaptive Systems

Many CAS are scalable – that is the same laws operate over many scales. Quarter power allometric scaling relations are characteristics shared by all organisms West et al. (1997). A common mechanism which underlies this relation is thought to be that ‘living things are sustained by the transport of materials through linear networks that branch to supply all parts of the organism’ West et al. (1997), a prime example of this is the branching structure of trees. Due to this branching, these systems are fractal in nature. A difference between biological systems and sociological systems is that the scaling exponent (β) in biological systems tends to be <1 and >1 in social systems Bettencourt et al. (2007). This leads to biological systems showing a sigmoidal growth curve and sociological systems showing boom/collapse type growth Bettencourt et al. (2007). As mentioned in West et al. (1997) this may be due to the physical constraints on biological systems which only allows for growth up to $\beta < 1$. One main contrast between a social-ecological system, or traditional economic system and a modern social-economic system is this difference in growth of the system which may have a long term effect on the stability of such systems.

The equation for scaling is as follows:

$$y = kX^\beta \tag{1}$$

where k is a constant, and β is the scaling exponent, different growth rates can be plotted. Figure 1 shows equation 1 plotted with $k=1$ and $0.05 < \beta < 0.5$, showing sigmoidal curves. Figure 2 shows equation 1 plotted with $k=1$ and $2.0 < \beta < 3.0$, showing accelerating growth rates.

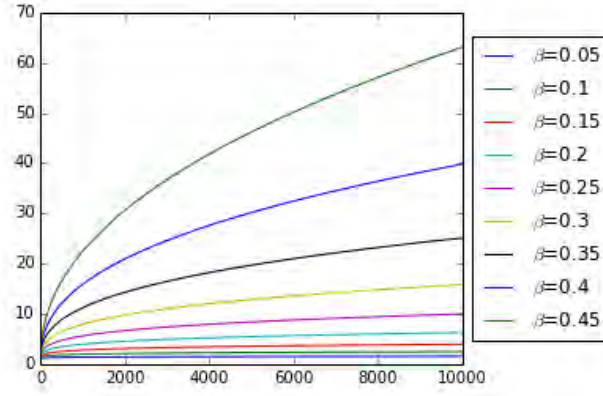


Figure 1: Plot of equation 1 with $k=1$, and $0.05 < \beta < 0.5$.

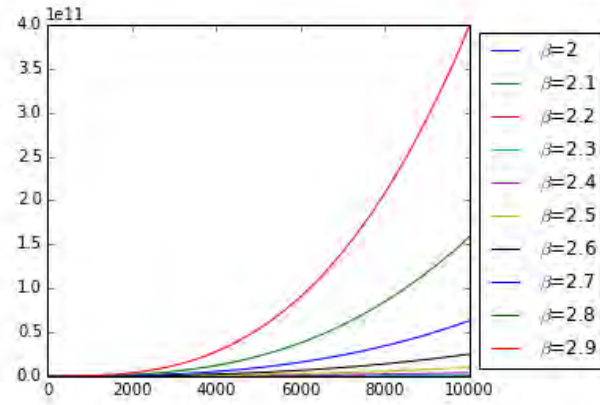


Figure 2: Plot of equation 1 with $k=1$, and $2.0 < \beta < 3.0$.

3.2.3 Cognition or Protocognition

Another way of subdividing CAS is based on the cognitive process of the system. For instance *Protocognition* is defined as cognition without a nervous system, which means that the system relies upon local interactions generally in the form of chemical diffusion to adapt its behaviour to its environment. Protocognitive forms of life include the same type of systems as defined in Section 3.2.1 as *Spatially Constrained*.

On the other hand electrical cognition uses electrical signals through neurons to transfer information at a much quicker rate than chemical diffusion. In nature, organisms which rely on electrical cognition such as animals also have protocognitive processes, and therefore rely on local interactions as well. This is both within individual organisms and between them (groups or organisms, indigenous human societies).

However recent technological innovations (The internet, wireless systems) have allowed humans to no longer need to rely on local interactions, which may have a large impact on the scaling nature of such systems. Also artificial life doesn't seem to require the protocognitive component at all in order to function such as in robotics and AI systems.

This differentiation is important for analysis of the models presented in the later sections of this report as it allows classification based on how information is processed.

3.3 Complexity and Stability in Complex Adaptive Systems

Whether increasing diversity leads to increased stability is an open question in ecology and complexity science McCann (2000). Complexity tends to be synonymous with diversity, for instance a more diverse ecosystem requires more information to describe it which is the definition of complexity in Section 3.1.

Early empirical work found that more diverse ecosystems were more stable MacArthur (1955). However theoretical mathematical work found that larger increasingly interconnected systems were more likely to be unstable May (1972). But this result is based on a number of species interacting with one another, with each species having a probability of being unstable. As connectedness and interaction increases the system is more likely to become

unstable. This is generally saying that as a system of connected species interacts, if this interaction has high strength and high connectedness, then if only a small proportion of the system is unstable, this will lead to the whole system becoming unstable. Although this simple model is elegant and clear with its conclusions, it doesn't represent what has been observed in the empirical data. It is possible that this model does not represent all the interactions and adaptabilities which are inherent in living systems. For instance, if one particular species is unstable, instead of this instability cascading through the entire ecosystem, perhaps it leads to a loss of connections as other species *adapt* in order to persist. This is similar to more recent work showing that CAS often re-organise in order to persist Scheffer et al. (2001). So the model lacks the presence of evolutionary principles which if added may show that the system will fluctuate between stability and instability as it evolves and adapts. This school of thought has also been applied to real world human systems such as the financial industry to explain that the reasons for financial crisis are due to highly interconnected high strength interactions May et al. (2008).

Following on from this, May (1972) also states that the organisation of the system is of importance for increasing stability. That is, if a system is arranged into blocks (is modulated) then the probability of stability is increased.

Generally, recent research suggests that increases in diversity, on average, gives rise to ecosystem stability McCann (2000). Diversity for the sake of diversity doesn't seem to increase stability May (1972). Instead it seems that a system which is able to maintain novel redundancies which are in turn capable of differential response dependent on changing conditions increases stability.

Themes on stability and complexity within CAS are important for the thesis as it will be important when designing and analysing the models in the later sections.

3.4 Complex Adaptive Systems as Networks

This section bears a lot of commonalities to the previous Section 3.3. For example, the model in May (1972) can be represented as a network.

Network theory is a fast growing discipline which has become increasingly

useful for both analysing and modelling systems. When simulating a CAS which by definition consists of a number of agents interacting in a non-linear fashion, a framework incorporating networks is often most appropriate. In fact the use of networks has greatly aided recent discoveries in complex systems, for example, 'small-world' networks appear to be ubiquitous in nature and society Watts and Strogatz (1998). Although when simulating a CAS using agents connected in a network, assuming a small-worlds network can be problematic due to the wide statistical definition of 'small-world' De Caux et al. (2014).

The notion of social and ecological networks has been present for a long time in qualitative social science and in an abstract form of mathematics in graph theory, however only recently has mathematics been applied to ecological and social networks Watts and Strogatz (1998). The simple nature of graph theory makes it very easily applicable, especially since it relies on reasonably simple mathematics. For instance a network can be modelled as an interaction matrix in which a number of properties are defined about the interaction of different agents in the system. Networks tend to not be studied spatially, although this doesn't have to be a limiting factor, as this can be added as an additional component in any system under study.

In terms of CAS, networks have been used in conceptual ecological systems such as food networks, and to an extent in system dynamics modelling. Attempts at modelling ecological systems with networks tend to be fraught with difficulty due to the large amount of interacting factors present. However tools such as system dynamics allow for a system's network to be built and visualised with little difficulty although calibrating and including the relevant variables may prove difficult.

Relating back to Section 3.3, stability and network connections is explored in the ecological/sociological theory of panarchy Holling (2001). This is a qualitative framework for exploring Complex Adaptive Systems. It can be generally thought of as a network based model, which relates to how systems expand and collapse with changes in 'potential' and 'connectedness'. But also to the scaling of the system, and how these cycles at different levels in the system interact with one another. When modelling resource distribution systems with altering levels of connectivity, it might be possible to see these cycles occurring within the system under study.

3.5 Common Pool Resources and Public Goods

As this thesis is concerned with the distribution of resource amongst many agent systems the topic of Common Pool Resources (CPRs) is relevant. In general it describes the allocation of pooled resources within a population and is most often studied in the humanities such as in social science and economics. It is not often referred to in the life sciences, but it can also be readily applied to natural systems. For example, a common resource has to be allocated to different parts of a plant to enable it to function properly in a stable manner.

A public good is one which all enjoy in common in the sense that each individual's consumption of such a good leads to no subtractions from any other individual's consumption of that good Samuelson (1954). The key issue of Common Pool Resources and Public Goods is whether individuals can suppress their self interest for an end result in which all individuals benefit to a greater degree than if acting independently Levin (2014). Evolutionary thinking can aid in helping societies address some of its greatest issues such as system sustainability as many common pool problems have been solved previously in biological and small-social systems Levin (2014).

Following on from this, the famous essay 'The Tragedy of the Commons' Hardin (1968) discusses the problem of overpopulation of humans and the effect of it on our CPR, the planet. The essay states that in order for humans to overcome the problem of overpopulation they need to abandon their freedom to breed and this should be enacted with mutual coercion, mutually agreed upon by the people affected. Many small-scale societies overcome the Tragedy of the Commons through the establishment of norms. Ostrom (1990) famously defined a number of common rules between societies that avoid the Tragedy of the Commons.

'The Tragedy of the anti-Commons' has been proposed Heller et al. (1998) as an opposite effect of the Tragedy of the Commons. In this scenario, over protection by individual players leads to the resource not being used at all.

Brede and Boschetti (2009) defines a spectrum between tragedy of the commons and tragedy of the anti-commons. Brede and Boschetti (2009) then explore this in a game-theoretic framework and an evolutionary simulation. The simulation found two stable fixed points, one being the tragedy of the commons, the other being the anti-tragedy of the commons. In-between

the two are locally stable fixed points where a few individuals initiate obstructive policies which stop overuse of the resource. These fixed points are however fragile to perturbations such as a population increase.

Common pool resources and the tragedy of the commons are key topics in this thesis. As stated in Section 2, each of the questions is about the allocation of resources in many agent systems which is directly related to the general theme of Common Pool Resources.

3.6 Gaia and notions of homoeostasis

The previous sections introduce stability, complexity and networks in Complex Adaptive Systems. This section introduces a related area of study - the Gaia Hypothesis.

Gaia is defined as "the thermodynamically open system at the Earth's surface comprising life (the biota), atmosphere, hydrosphere, dead organic matter, soils, sediments and that part of the lithosphere that interacts with the surface processes" Lenton and van Oijen (2002). Gaia theory, originally named the Gaia Hypothesis states that 'once life was established on Earth, it acquired control of the planetary environment and this homoeostasis by and for the biosphere has persisted since' Lovelock and Margulis (1974). This definition is often termed 'strong Gaia', in that the biosphere is the dominant processes in controlling the global environment. However, there is disagreement, the term 'weak Gaia' is often used to describe past geological eras when physical earth system processes tended to be dominant Leeder (2007).

In reality there must be interplay between both biotic and abiotic processes in controlling the nature of the global earth system, for example a large enough abiotic perturbation could remove the biotic component entirely. When comparing Earth to other planetary systems such as Mars or Venus which as far as we know do not support life at present and definitely not to the level of complexity on Earth, the biotic component of Earth does appear to be a dominant mechanism in shaping earth surface and atmospheric processes on the planet. Margulis and Lovelock (1974) provide further support for the Gaia Hypothesis with evidence that the Earth's atmosphere would have a different composition if life had not evolved, and adding comparisons to Mars and Venus. The presence of active tectonics is also a mechanism not present on other planets without life. Indeed, many theories relating to the origin of life point to hydrothermal vents as the starting point of biology

Martin et al. (2008); itself a abiotic process and the product of a dynamic earth. An interesting question would be to ask if life would continue on Earth in the absence of active plate tectonics.

Gaia theory relates to the planet as a whole, and this thesis is concerned with smaller systems, many of the underlying themes of Gaia theory and its subsequent embodiment in the form of Daisy world Watson and Lovelock (1983) are concerned with earth system science and the relationship between social and ecological systems; see section 3.9.3.

Philosophical schools of thought on the same themes of homoeostasis, balance, a living earth and Gaia theory have a long history in human societies across the planet. Greek philosophical Gaian views of a living earth date back to 2,500 years to the sixth century B.C. Scofield (2004). Eastern traditional philosophy also shares many commonalities with Gaian homoeostasis, specifically the principal of yin-yang Klinger (2004).

Criticisms of Gaia Theory

- Criticisms of Gaia theory relate to the fact that there is only one Planet Earth, so it is impossible to test theories on an alternative system, or collect data across different planetary systems which support complex life. This provides an argument for a complex system simulation based approach as this can provide further support for Gaia Theory. Daisy world is the first example of a mathematical model of Gaia Theory 3.9.3. Given the current increase in computing power, more elaborate models are possible, however this does not necessarily provide greater support as more complex models require many more assumptions.
- Gaia theory also doesn't offer an explanation for how complex intelligent life such as humans are having a large detrimental impact on the biosphere. But, if Gaia theory is defined as a biological theory, and humans are defined as sociological or socio-economic, then Gaia theory may no longer be applicable to humans, for example West et al. (1997) showed that sociological systems have different growth and rates when compared to biological systems, see section 3.2. However, if Gaia theory is established as a political movement within human societies then it may then aid in maintaining an optimal environment for the biosphere to continue, at least with humans present as well.
- It has been suggested that Gaia theory is 'teleological' McDonald-

Gibson et al. (2008), which implies that the biosphere has foresight and planning in order to maintain regulation. This seems impossible given the primitive nature of life at its origin, unless there is a creator. But if one of the first mechanisms which life creates is to find homoeostasis with its environment in order to persist, and if in fact it is able to do this then an observer may conclude that it has foresight and planning, or 'anticipation', which has been argued as defining quality of Complex Adaptive Systems Holland (2012).

- Evolutionary biologists will also argue that under classical Darwinian natural selection, why would a system which benefits others evolve. Gaia theory is actually a form (possibly extreme example) of group selection Nowak (2006), which is often seen as controversial amongst biologist, although there are many examples of it in nature and there are established models for how it may evolve in tandem with individual selection Wilson (1975). Wilson (1975) tries to find a balanced argument by stating 'the traditional concepts of group and individual selection appear to be two extremes of a continuum, with systems in nature operating in the interval in between'. Multi-level selection theory seeks to show that selection operates at many levels Traulsen and Nowak (2006); Nowak et al. (2010). Gaia theory can also be viewed as a public good 3.5 of the biosphere, and comparisons can be drawn with public goods and common pooled resources in economic systems.

In summary, there is strong evidence for the notion that biological or biotic processes have played an important role in changing the composition of the atmosphere, which has then enabled further biological processes to flourish in a positive feedback. This can be seen as an additional level of complexity that is established in the primordial chemical soup. The underlying meaning for why life does this is uncertain. Maximum Entropy Production (MEP) is one proposed reason Dyke et al. (2011), which would adhere the second law of thermodynamics, however there is no universally accepted derivation of MEP Bartlett and Bullock (2014). This would mean that if life does emerge in a particular system, then the reason for it is to increase the rate entropy production.

The Gaia Hypothesis has been introduced in this subsection as it is viewed as a continuation of themes from the previous sections. The Daisyworld model was intended to be a model of homoeostasis on earth allowing for the continuation of life in spite of changing inputs. However due to the abstract nature of it, the theory and model can be readily applied to many

other living systems. So Gaian thinking can aid in understanding why other systems in nature and society might be able to persist through finding a balance between contradictory forces within the system. The Flask model introduced by Williams and Lenton (2008) is an example of Gaia theory applied to a more complicated network/agent model. The model uses evolutionary computing to simulate a number of networked microbial systems which microbes can evolve to alter their environment in different ways. If the microbes evolve to improve the environment then they are able to support a greater population and spread to other microbial systems and the opposite is true, if the microbes evolve to degrade their environment then it reduces their population making them susceptible to invasion. The spread of environment improving microbes leads to the global environment being altered towards growth.

The models introduced on heterogeneous agents on resource sharing systems in Section 7 shares some commonalities with the flask model introduced by Williams and Lenton (2008).

3.7 Niche Construction Theory

Following on from the previous section on Gaia Theory, this section looks more detailed at Niche Construction Theory. Niche Construction Theory is defined as the process whereby organisms actively modify their own and each other's evolutionary niches Odling-Smee et al. (2003). Matthews et al. (2014) propose two criteria to test the presence of niche construction:

1. An organism must significantly modify environmental conditions and organism-mediated environmental modifications must influence selection pressures on a recipient organism.
2. For Niche Construction to affect evolution there must be an evolutionary response in at least one recipient population caused by the environmental modification.

Laland et al. (2016) provides a modern overview of Niche Construction theory as well as the wider implications in ecology and the human sciences. Niche Construction can be viewed as a theory in biology, and as such can be applied across all life science disciplines and scales. Recently, the notion of social niche construction has been popularised Powers (2010) which is the application of niche construction to social science.

The application of Niche Construction Theory is relevant to the simulations in Section 7, as these Sections model heterogeneous agents. In Section 5, the agents are treated as homogeneous so niches are unable to form.

3.8 Social Norms

Social norms can be viewed as the 'memes' which drive Social Niche Construction. They are representative or typical patterns and rules of behaviour in a human group Ehrlich and Levin (2005). Norms are of great interest in social science, environmental science and policy research due to the effect that human behaviour has on both social systems and ecological systems. Government policy can aid in changing human behaviour to avoid less desirable social and ecological states Kinzig et al. (2013). However such analysis and subsequent policies may have detrimental affect on individual freedom.

Research has also been carried out on the effect of tipping in social systems Schelling (1971); Kinzig et al. (2013); Hartnett et al. (2016). The large amount of plastic waste produced by societies in the current 'throwaway world' is a global norm which is having a detrimental effect on the planet and natural environment Jambeck et al. (2015). Relating back to Gaia, such a large amount of non-biodegradable waste is likely to have a large impact on the natural world, and reduce Gaia's ability to keep the planet within an appropriate state for life.

In order to model such behaviour, simple cellular automata can be used to abstractly model the development of social norms Schelling (1971); Hartnett et al. (2016); Ehrlich and Levin (2005). Figure 3 shows the basic idea of modelling norm changes in social systems. An agent with a low threshold is one which will change their state with even a low number of neighbours with a different state. A high threshold is an agent which won't change their state even if all their neighbours have an alternative state. This example looks at the changing of norms based on the opinions of neighbouring cells. But a similar methodology can be used to look at models of segregation in humans Schelling (1971). This is a related methodology to that used in game theory simulations Axelrod and Hamilton (1981); Nowak and May (1992).

3.9 Methods for modelling Complex Systems

The previous sections provided general explanations of complex systems and related fields which are important for the topic of this thesis. Methods are

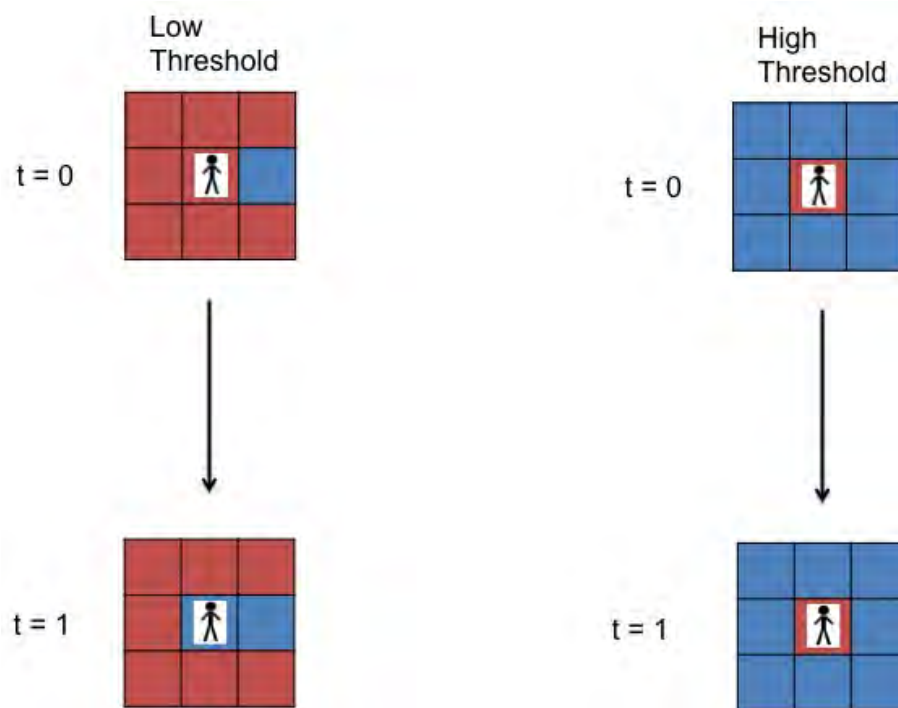


Figure 3: Basis of modelling thresholds using cellular automata.

touched upon briefly in some of the sections. The following sections give further details of methodological approaches to modelling complex systems and in particular the .

3.9.1 Modelling and its limitations

The seminal paper 'Why Model?' Epstein (2008) focuses on some of the largest misconceptions about modelling. Prediction and validation are often thought to be key attributes of a robust model. Whilst prediction is often thought of as the foremost reason to model, Epstein (2008) gives many more reasons to model other than to predict. These include:

- Explaining;
- Guide Data Collection;
- Illuminate Core Dynamics;
- Suggest Dynamical Analogies;
- Discover New Questions;
- Illuminate Core Uncertainties;
- Challenge the Robustness of Prevailing Theory Through Perturbations;
- Offer Crisis Options In Near-Real Time;
- Train Practitioners;
- Education;
- Reveal the Simple to be Complex and Visa-Versa.

The additional reasons to model should be further highlighted in the modern world where high powered computers and large datasets are readily available. Data driven modelling has its limitations in that it relies on the past in order to model the future. However, as chaos theory has shown, the simplest of systems are not predictable (See Section 3.9.2). Powerful simple models which offer general solutions to a range of problems allow the investigator to identify common features in systems and interpret possible scenarios which the system may take. In the current extremely dynamic modern world, large datasets may become obsolete due to the regime shifting between states. When the system shifts regime, the previous data may

become invalid, however due to hysteresis, the previous state may still be detectable in the system. Ezequiel et al. (2000) reiterate an alternative non-predictive use of models as *opaque thought experiments*.

Models are used in both qualitative and quantitative social science. In qualitative social science, modelling is implicit - it is undertaken via thought experiments. In quantitative social science, it is explicit, assumptions are laid out in detail so we can study what they entail Epstein (2008). This means that validation and analysis can be undertaken in a much more robust systematic manner.

3.9.2 Mathematical Origin

The origin of the modern field of complex systems can be traced back to the work of Henri Poincare Chenciner (2012). The three body problem stands as one of his most important fundamental contributions to the field which finds that for a system consisting of three bodies or more, there is no general closed form analytical solution to find the positions and velocities at any point in time without using numerical or simulation based methods. This forms the basis of chaos theory, in which chaos is defined as deterministic, sensitive to initial conditions and unpredictable. The need to model or simulate complex systems can be seen to come from this problem of not being able to find closed form solutions to problems.

The three body problem is particularly important for this research as many of the qualitative ways of analysing dynamic systems originate from studies on three body systems ?. As scholars realised that there was no closed form solution for the interaction of three or more bodies, a shift in analysing such systems occurred from trying to find the exact positions of each of the bodies in time, to asking more general questions about the system such as "Will the system be stable forever?" ?. This expanded into dynamic systems theory which can be in turn applied to many more much more complex systems. Seminal applications of stability analysis of many body systems include the seminal work on dissipative hydrodynamic flow by Edward Lorenz Lorenz (1963) and more recently of ecological systems Scheffer et al. (2012). This is an active research field

There are many ways to model complex systems, and in particular social-ecological systems Verburg et al. (2015). This section outlines general tools used to model complex systems. All the modelling methods discussed below

can be thought of as numerical models. Their differences lie in the different applications of mathematics and whether they rely on data or not. Models may be theoretical (which tend to be more general) or applied, when attempting to model a specific system.

Modelling techniques include:

- Equation based models;
- Deterministic process-based biophysical models;
- Cellular Automata Models;
- Simple Toy social-ecological models;
- Network Models;
- System Dynamic Models.

3.9.3 Equation based Models

Equation based models in life science do not have as strong a foundation as in physical science and engineering May (2004). May (2004, 1976) provide overviews of some efforts to build simplified equation based models which can capture observed patterns and processes seen in reality without being lost in the detail. Many of the most powerful and also most fundamental models in biology are the simplest and also the most general in that they can be applied universally. These simple models offer building blocks and clarity on which to build theory and applications for the real world.

Examples of equation based models used in the life sciences include:

Lotka-Volterra Equations The work of Lotka and Volterra was one of the first examples of equation based models applied to the life sciences. This explored, in the form of two coupled non-linear, first order differential equations how two species interact in a predator-prey model. A variation of the equations can be summarised as:

$$\frac{dx}{dt} = ax - \beta xy \quad (2)$$

$$\frac{dy}{dt} = \delta xy - \gamma y \quad (3)$$

where x and y are the two populations and a , β , δ and γ are constants. Sayama (2015) provides a clear summary on how such equations can be derived using simple mathematical functions.

These equations have since been extended to look at more complex ecosystem dynamics with many more agents and interactions Gilpin and Ayala (1973).

Daisy World Another example of coupled differential equations is the Daisy World model Watson and Lovelock (1983). The Daisy World model is one of the simplest examples of the emergence of homeostasis in a system. It was originally coined as a formalisation of the Gaia Hypothesis (see Section 3.6 - a hypothesis that the Earth is a self-stabilising system which allows life to continue despite environmental fluctuations. Daisy World, in its simplest form consists of two types of daisy - black and white which grow on a grey planet which orbits a star that gradually increases in luminosity over time. The two daisies have the same optimum temperature to grow and reproduce in but have an opposite albedo given their differences in colour. An increase in black daisies will lead to an increase in planetary temperatures as they absorb heat and an increase in white daisies will lead to a decrease in planetary temperatures as they reflect heat. Given the slow driving increase in temperature emitted by the nearby star, the planet will increase in temperature. However the presence of the two types of daisy forming a coupled system keeps the planet conditions desirable for life for longer periods than without their presence. Fascinating extensions to the Daisy World model have looked at the interaction between evolving species in coupled biosphere-climate models Baldocchi et al. (2005); Ackland and Wood (2010); Wood et al. (2006).

Within the field of evolutionary biology, there are criticisms regarding how a system such as Daisy World may evolve and that the species in the model are pre-selected to allow for regulation to occur Wood et al. (2008). In this regard the model is viewed as static and it should introduce the generation of new traits to allow for evolution to occur. Whilst these criticisms have a basis, Daisy World is a simplistic model which has its limits to explain biological behaviour and focuses on one aspect of biology - homeostasis. Another criticism of Daisy World is that there is no over-population, competition for resources, or ecological overshoot in the model. Although it could be argued that these features are present implicitly in the model, in

terms of feedbacks between the environment and the daises.

Interaction Matrices For modelling ecosystems consisting of a large number of components interaction matrices are often used May (1972). These have been useful in studying the stability of complex systems in relation to size and connectivity May (1972); Allesina and Tang (2011), an area of interesting study in both ecology and economics May et al. (2008). May (1972) and initially Gardner and Ashby (1970) find that large ($n \gg 1$) connected systems are inherently unstable. This has similarities to later work on how a virus spreads across a network in that the probability of becoming unstable can be thought of as similar to the probability of an infection spreading across the network May (2004). The notion of increased instability with increased connectivity was later explored and theorised qualitatively in Panarchy theory Holling (2001). More recent studies within the theoretical field have looked at combining models of ecosystems with a large number of components with models of homoeostasis to understand how regulation might emerge in such large systems Dyke and Weaver (2013).

Interaction matrices can be thought of as a generalised form of the Lotka-Volterra equations Malcai et al. (2002). They can also be used for modelling the interaction between individuals in micro-economics Lade et al. (2013). This can then be analysed with linear stability analysis to provide information about when a particular system is stable or unstable Lade et al. (2013); May (1972). In fact, further to the point made in Section 3.3 regarding whether May (1972) captures all the dynamics present between living systems, Lade et al. (2013) includes ostracism which allows the system to persist by removing interactions with individuals which can cause the system to become unstable.

Iterative Maps An iterative map of a single real variable is an example of a discrete simple mathematical model whereby the output is updated each time step based on the previous output and other variables in the equation May (1976). The following non-linear Iterative Map can be used to explore a simple biological system of a seasonally breeding population in which generations do not overlap.

$$X_{t+1} = rX_t(1 - X_t) \quad (4)$$

where,

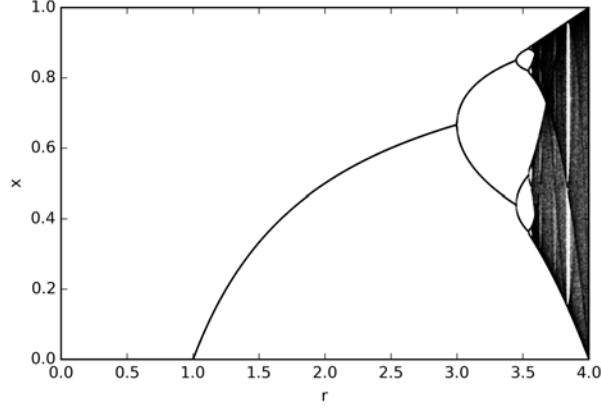


Figure 4: Bifurcation plot showing the different values of x when the constant r is changed in equation .

$0 < X_{t=0} < 1$ and the constant $0 < r < 4$.

By varying the value of the constant r , the model will iterate through different regimes, including one stable state, bi-stability, multiple states, limit cycles and chaos, figure 4. The different regimes which emerge from the model shows how complex behaviour can arise from simple interactions Bar-Yam (1998).

3.10 Spatial Models

Most systems in the real world tend to interact spatially. The Section 3.2.3 explains how systems which are spatially constrained can be categorised differently to those where other processes are more dominant. In this section the divide is made between cognitive and protocognitive systems. A cognitive system being one where information is transferred at such a high rate that the spatial configuration isn't necessarily important. Whereas, in a protocognitive system, the information transfer is much slower so the spatial structure is more important - local interactions are important in such a system. The degree to which the spatial component of a system . Simulation models such as cellular automata and agent based models also allow for a spatial dimension of the system to be added, which isn't always possible in equation based systems. Computational fluid dynamics which is the implementation of the Navier-Stokes equations to a spatial environment is

an example of such simulations. Whole research topics are devoted to computational fluid dynamics - which have been very successful in modelling systems such as climate and the ocean system which is very useful for the design and management of infrastructure.

A simplistic example of a fluid dynamics model can be implemented using cellular automata over a topography.

Cellular Automata Cellular Automata (CA) are one of the simplest frameworks in which issues related to complex systems, dynamics and computation can be studied Mitchell et al. (1993). At its most basic level, a CA consists of a spatial lattice of cells, each of which, at time t , can be in one of k states. The CA has a fixed rule which is used to update each cell based on its nearest neighbours Mitchell et al. (1993). They can be used to explore abstract phenomena such as 1-dimensional CA Wolfram (1984) or applied problems such as river basin development or flood dynamics Coulthard et al. (2002)).

Simple simulations can be undertaken to explore the evolution of different behaviours from initial conditions based on the above methodology. From an initial random configuration figure 5 different thresholds can be tested. If all agents are treated as homogeneous, a low threshold for all agents (from figure 3) after many iterations leads to a constantly dynamic random simulation, ie the same as the initial configuration figure 5. However, if all agents have a high threshold, the simulation remains static, and the same as the initial configuration, figure 5. Interesting dynamics occur at a moderate threshold, figure 6. At this point an agent will change their state with a probability of a half if half of their neighbours have a different state to them. A video of this simulation can be found at <https://youtu.be/HokoPnIPvas>. Figure 7 shows the dynamics of the system after 500 time steps. This has a lot of similarities to self-organising behaviour in other chemical and biological systems, which is usually termed morphogenesis Turing (1952). Another well known example is stripes on a zebra. Following the same methodology, but instead using a Von Neumann neighbourhood for the cellular automata, stripes form rapidly <https://youtu.be/jrBRsrAAGSk>.

Recent examples of more complex pattern formation processes using a cellular automata framework include implementations of the Grey-Scott equa-

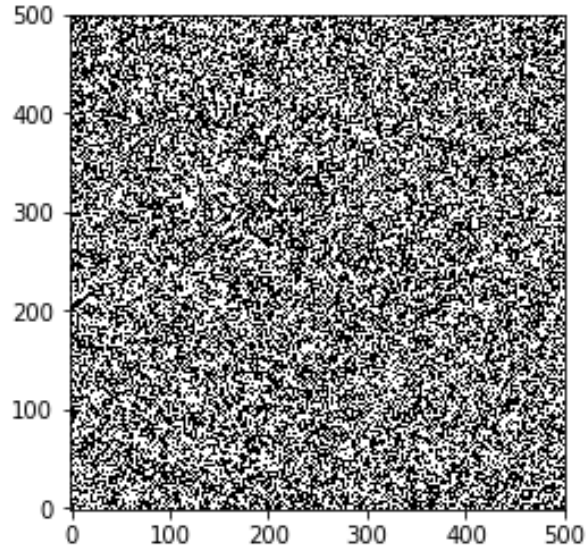


Figure 5: Initial configuration for a social norms simulation with two states - black or white.

tions (which is a simple reaction-diffusion model) Gray and Scott (1985); Bartlett and Bullock (2015); Adamatzky (2018) and the Navier-Stokes equations (the flow of incompressible fluids) Wolf-Gladrow (2000).

Agent Based Models Many of the models in this thesis use what is known as 'Agent Based Models' (ABMs). ABMs can be viewed as an extension of Cellular Automata in that each agent is an entity which is updated based on its own rule set and other agents in their neighbourhood. ABMs however offer greater flexibility as the neighbourhood doesn't necessarily have to be immediately adjacent to them. The term ABM is usually applied more readily to models in biological and social systems which require conscious decision making.

In terms of object-orientated computer science, ABMs can be thought of in terms of "Classes". Each instance of a class is an agent which stores a number of unique values about that particular agent, and after each time step the agent's state is updated based on the rules of the system and the unique values of that particular agent. ABMs offer great power in exploring systems of many interacting (heterogeneous) agents, which would not

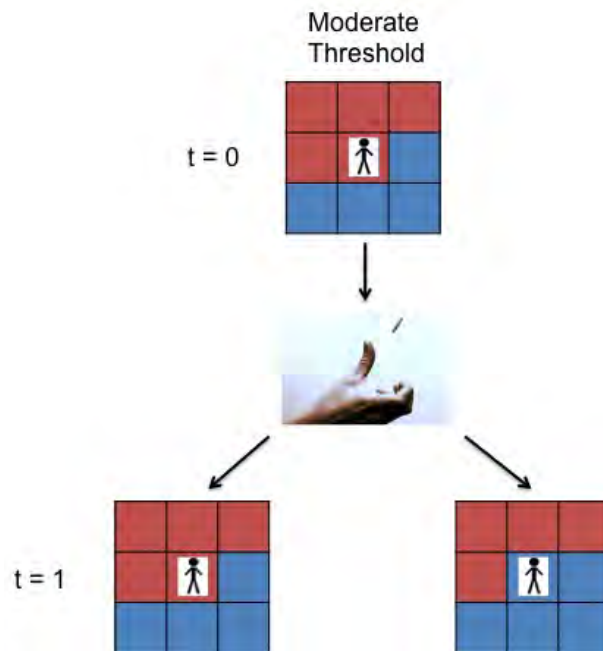


Figure 6: Moderate threshold scenario - Agents will switch behaviour to that of the majority of their neighbours. At the point where they have an equal number neighbours in opposition camps, their choice will depend on a coin toss.

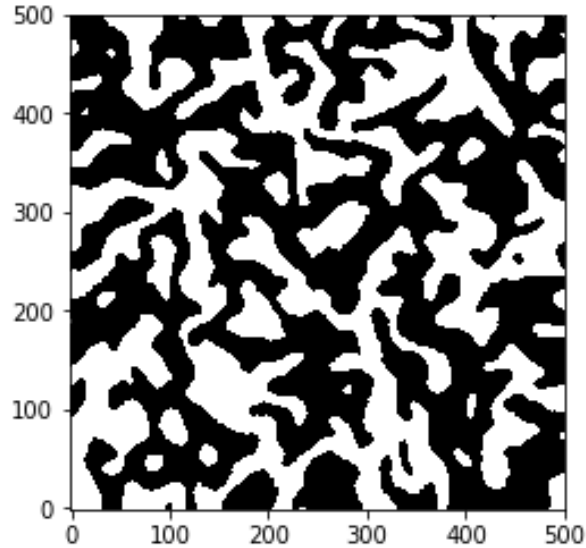


Figure 7: Final configuration after 100 time steps for a social norms simulation with two states - black or white.

be able to be explored in qualitative models, thought experiments or even using equation based models. Agent based models differ from system dynamics models in that they deal with individual agents and not aggregate behaviour Gilbert (2008). In a sense higher level behaviour emerges from the interaction between agents at a lower level, they are *Bottom-up models*.

Simple Toy Social-ecological models These are generally conceptual models with no direct empirical input. They can be described as theoretical and exploratory. They are useful for developing theories and underlying rules of complex systems that can then be applied in more empirical models. Examples of such models are evolutionary selfish-cooperative prisoners' dilemma games Axelrod and Dion (1988); Nowak and May (1992), Models of Segregation Schelling (1969), the sugarscape model of artificial societies Epstein and Axtell (1996), the Daisy World model of planetary homoeostasis Watson and Lovelock (1983), small-world networks Watts and Strogatz (1998), voter decision making models Ehrlich and Levin (2005); Hartnett et al. (2016) and collective behaviour Reynolds (1987).

Many of these models have similarities on the theme of local interactions

and global behaviour. The formalisation of small-world networks Watts and Strogatz (1998) and its prevalence in many systems (biological, technological and social networks) gave theoretical foundations for the need to exploring models with more complex interactions to tie them to real-world observations. Generally toy models don't use real data and are conceptual. This has led some authors to concluding that they are of low value, however they have great power in explaining general behaviour of systems and so can aid policy makers in decision making and also help with interdisciplinary thinking.

3.11 Discussion of approaches

Anderies (1998) states one downside of simulation based computer models is the difficulty to understand the structure and stability properties and simple dynamic models allow for stabilising and destabilising forces to be analysed more clearly.

3.12 Case Studies

This section provides a number of case studies of biological and sociological systems. By producing a knowledge base of different systems this provides inputs, legitimacy and validation for the simulation models produced.

3.12.1 Resource Collection and Resource Distribution Systems in Nature

This section reviews the previous literature on resource collection and distribution systems in nature.

Resource Collection Systems One of the most studied and well known examples of resource collection systems is that of leaf venation patterning Dimitrov and Zucker (2006); Runions et al. (2005); Nelson (1997); Katifori et al. (2010). This topic has been studied extensively throughout the 20th Century, with auxin canalisation, in which cells experiencing a higher auxin flux differentiate into specific cells for auxin transportation being the most widely accepted theory on their formation Lee et al. (2014). There is debate over whether a prepatter exists in such systems, a template which expands and adapts based on the conditions of the system, however, models have shown that no prepatter is necessary for emergent properties Dimitrov and Zucker (2006). Dimitrov and Zucker (2006) provide a model which initially

consists of homogeneous cells all producing auxin. But as time passes, cells specialise into canals as the flow of auxin increases leading to a branching vein structure forming. However recent studies of dynamically growing leaf systems show that additional rules have to be added to maintain canalisation and stop sinks from forming due to changing spatial arrangement of cells Lee et al. (2014).

There are many commonalities between this biological process, and other collection processes in nature such as natural drainage basins. Water concentrating on a terrain will induce feedbacks leading to channelisation. Experiments in uniform sediments such as sands are likely to show similar results as channelisation in leaf veins. However there are many more factors affecting natural drainage basins, so 'pre-pattern' factors are likely to play a larger role in natural drainage basins.

Resource Distribution Systems Resource distribution systems are likely to have many commonalities to resource collection systems, in terms of morphology and structure. In fact, in the life sciences the same network is often used for both resource distribution and resource collection.

3.12.2 Resource distribution systems in human societies

The following sections focus on resource distribution systems within human societies. There are many factors to be taken into account when defining resource distribution in human societies such as size and structure. According to complex systems theory, many distribution systems should be scale-free meaning that the same properties should be exhibited at different scales, particularly when analysed as a network (see Section 3.4. If this is the case then analysis of a small-scale system can provide solutions for larger more complex systems. Two types of distribution system are introduced in the following section, traditional small-scale distributions and modern large distribution systems. The aim of the following sections are to define both of these systems and provide a summary of key similarities and differences between such systems.

3.12.3 Traditional Distribution Systems

As noted in the previous Section 3.2.3, Complex Adaptive Systems can be subdivided into cognitive or protocognitive systems. Traditional distribution systems can be defined as protocognitive, in the sense that they are

spatially constrained and rely on nearest neighbour interactions. Indigenous irrigation systems (IIS) are a type of traditional distribution system which focus on the distribution of water to individuals in the system.

Indigenous Irrigation Systems can be defined as Groenfeldt (2004):

1. The physical structure of water capturing devices (diversion weirs, dams, or wells), conveyance devices (canals, aqueducts, tunnels, flumes), and control structures (gates, outlets, dividers) by which water is delivered to agricultural fields.
2. The management arrangements for designing, constructing and maintaining the physical works, allocating and distributing water among the users, resolving disputes, and addressing emergencies or other unforeseen circumstances.

Both the physical structure of the system and the management arrangements are important for analysing and understanding why a particular system has evolved to a given state, and what has enabled it to continually persist.

Indigenous can be defined as 'Born or produced naturally in a land or region; native or belonging naturally to (the soil, region, etc.)' Oxford University Press (2015). Indigenous Irrigation Systems represent an applied aspect of Indigenous Knowledge. Indigenous Knowledge (IK) is a systematic body of knowledge acquired by local people through accumulation of experiences, informal experiments and intimate understanding of the environment in a given culture Rajasekaran et al. (1993). These have often been labelled as unscientific but there have often been difficulties when introducing modern management structures where previous indigenous irrigation schemes were present IFAD (2005).

The archetypical example of an ISS is a system which has organised without outside intervention to reach a homoeostatic state with the social-ecological drivers operating on that system. In reality there are probably no systems which organise without outside intervention, but by understanding and modelling this archetype, strong emergent features can be found which may not be as obvious in real-world systems. Yu et al. (2015) describes ISS as the most ancient and ubiquitous example of public infrastructure systems.

It is thought that ISS can evolve through decentralised bottom-up self-organisation or set up through a centralised top-down bureaucracy Palerm-

Viqueira (2006). ISS have been of interest to a number of authors for gaining insight into how humans organise and cooperate over common pool resources Ostrom (1990). There is still debate regarding what factors or drivers lead to what structures forming and in particular whether large irrigation systems can self-organise without centralised bureaucracies Palerm-Viqueira (2006). Sengupta (1985) suggests a distinction between indigenous and modern systems is a recent one which happened with the advent of large-scale irrigation projects financed by national, generally colonial governments in the late 1800s and early 1900s. This may be seen as part of the onset of modern globalisation.

The interaction between top-down and bottom-up processes extends through many if not all scientific disciplines for example, product design, computer science, management, sociology Valverde and Solé (2007), political science Sabatier (1986), ecology Hunter and Price (1992) and engineering Wolf and Holvoet (2005) making it a key sub-discipline in the inter-disciplinary field of complex systems.

Most indigenous systems today use a mixture of indigenous and modern practices, and are linked by varying degrees to national, regional and global markets Mabry and Cleveland (1996). Due to the inherent complexity of studying real-world systems, particularly in the modern globalised highly interconnected world, simulation can play a more important role in developing simplified models of how social systems of many agents organise. The power of this method over others is that the simulation can be thought of as being alive and evolving. Hypotheses can be tested on the simulation which wouldn't be possible or ethical in real systems. Similarities can be drawn between the simulation and real-world systems to offer policy based insights into drivers of change.

3.12.4 Qanat System, Middle East

Qanats are one of the most significant hydraulic technologies of the pre-modern Middle East English (1998). The Qanat is an underground pipe dug to transfer water from highlands to villages for agricultural use. The main source of the Qanat, a well shaft usually 50m deep, is sunk into the groundwater recharge zone Bonine (1996). Most Qanats flow as a direct response to precipitation, reflecting the permeability into which the groundwater recharge occurs. Figure 8 shows a schematic example of the Qanat system redrawn from Bonine (1996).

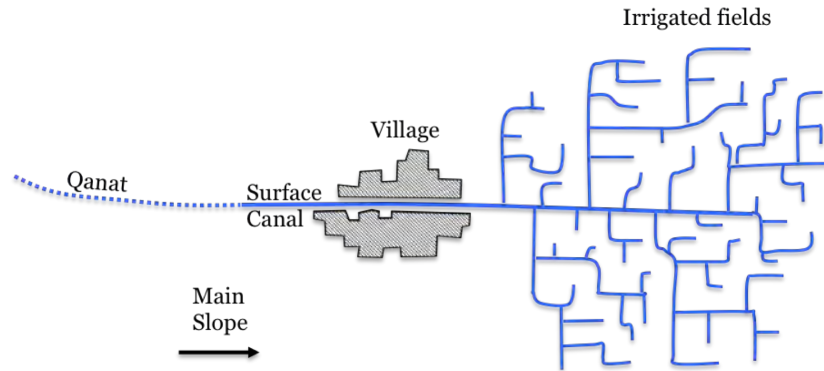


Figure 8: A schematic example of the Qanat system.

English (1998) outlines the change that is happening in the region with respect to the Qanat system. Population growth and agricultural expansion have heightened demand for water. This has led governments to abandoning the indigenous irrigation practices for more productive (but of questionable sustainability) modern pumping technologies and dams. In particular the Qanats are largely disappearing and being replaced by deep wells.

3.12.5 The Subak Irrigation System, Bali

The Balinese Subak irrigation system is thought to have been in operation for over 1000 years Lansing (1996). The main staple food in Bali, rice, is grown in paddy fields fed by irrigation which in turn is dependent on rainfall ?. Direct evidence for rice cultivation in Bali is from rice phytoliths (rigid, microscopic structures made of silica, found in some plant tissues and persisting after the decay of the plant) found in the sediments dated at AD 1 Lansing et al. (2009). There are two main technologies used for irrigation expansion in pre-colonial Bali. Small weirs are constructed upstream, and larger storage dams are created downstream Lansing et al. (2009). It has been extensively studied Janssen (2007); Lansing and Miller (2005); Wijermans and Schlüter (2014) and is thought to provide an example of a sustainable, resilient decentralised agricultural system (J. Stephen Lansing, 1996). However this is disputed, Wittfogel (1957) suggests central bureaucratic organisations are necessary to coordinate a network, whilst Lansing (1991) suggests they can self-organise through local interactions.

The work of Stephen Lansing in Bali provides some of the first and best examples of the application and usefulness of simulation for gaining insight into social-ecological systems in particular indigenous agricultural systems. His work mainly focuses on the social aspects of how cooperation can emerge in a system without any centralised control mechanism Lansing and Miller (2005). The simulations took place at the 'Subak' scale, where a Subak is a local-level farmer's association. These are the level at which decisions over water allocation are made Lansing (1987). For instance the district of Badung, an area 115 km by 40 km consists of 151 Subaks.

The strong dependence on rainfall for rice cultivation leads to difficulties given its variability both spatially and temporally on Bali (probably partly due to the steep topography present on the island). Rice pests can also have a strong effect on rice yields. Stability of these two factors seems to be the main reason for the emergence of the system Lansing and Miller (2005). If adjacent Subaks synchronise their fallow periods they reduce the amount of pests thus reducing the damage they could cause. Due to constraints on the water available, only a limited number of Subaks can synchronise their fallow periods. This leads to a decentralised system of localised coordination of fallow periods. ? constructed a simulation to show this coordination emerging. Janssen (2007) re-implemented this model giving a fuller account of the results and how it works. Janssen (2007) questions whether the simple model is a peculiarity of Bali or if it can be reapplied to other scenarios Lansing and Miller (2005) went on to construct a simple game-theoretic model of the trade-off between water sharing between upstream-downstream farmers and pest reduction. This simple model showed that cooperation over water increased with increased pressure of pests.

To understand how such a system might evolve, Lansing et al. (2009) propose a budding model whereby canal irrigation systems expand downstream as a result of local initiatives. Lansing et al. (2009) describe it as being an example of a Complex Adaptive System (CAS) where as the system expands it reacts to environmental and social processes. The size of the system means that downstream Subaks suffer losses from percolation and evaporation, however downstream Subaks routinely take advantage of excess flows from neighbours and local springs Lansing et al. (2009). In order to test the budding model genetic data was collected from farmers from different Subaks. If the irrigation system evolved by a budding model type process then small communities should be located along the irrigation system with oldest settlements located nearest ancient weirs. The results of the genetic

data found that Subaks located further upstream in their irrigation system contained greater genetic diversity than those downstream suggesting they were created first, giving support to the budding model.

3.12.6 The state of Indigenous Irrigation Systems in the modern world

Given the far reaching effect of globalisation and increased connectivity of the modern world, there are few indigenous places left untouched. This makes it increasingly difficult to study indigenous cultures in the present. Alternative methods to study such systems exist such as looking at the past archaeological and anthropological evidence and through simulation modelling.

3.12.7 Modern Distribution Systems

Modern distribution systems can be defined as cognitive Complex Adaptive Systems (see Section 3.2.3) as they are not spatially embedded and do not rely on local interactions for their distribution. Resource (in particular monetary) can be transported over very long distances in very fast times using modern technology, meaning local interactions are not necessary. This difference in properties in comparison with traditional societies may have a large impact on the structure and management arrangements of the system. However physical goods or biological processes still require local interactions (as teleportation has not been invented yet). For example research on the spread of contagions across a modern complex network such as air travel has found such spreads can be reduced to a simple, homogeneous wave propagation model as found in protocognitive systems Brockmann and Helbing (2013). Therefore modern distribution systems encompass both protocognitive processes (biological and physical interactions) and cognitive processes (transfer of information and monetary resources through electrical signals).

This creates an interesting problem when faced with growing issues in the current globalised world of limited resources, increased levels of environmental degradation and increased economic inequality. Modelling and predicting such systems requires a combination of methods at multiple time-scales to take into account the varying speed of interactions. It is important to model such systems as a sudden increase of financial or information can lead to large consequences to local protocognitive systems which may be operating on longer time-scales and prone to collapse with the sudden influx of

resources.

3.12.8 Comparisons between traditional systems and modern systems

The following sections of this thesis investigate resource distribution in a number of simulation models. There are many of the same problems face both traditional and modern systems such as inequality, collapse, availability of limited resources. However, the large variation in time-scale of resource transfer in modern systems may lead to completely different emergent features. It is therefore important to highlight the limitations of applying the outcomes of one model say for a traditional system to a modern system.

As highlighted in the Section 3.5 there has been a large amount of research conducted on common pool resources and public good. Given the transparency, reduced complexity and *more* defined boundaries of indigenous systems, they have been the focus of much research. However, much of this research fails to take account of the increased interaction with modern systems. Ostrom (1990) finds a number of common rules across traditional societies which allow for their persistence through time. However the legitimacy of such rules in modern and combined modern-traditional systems may not apply.

4 Space Optimising Algorithms

4.1 Initial Set Up

This section introduces the first modelling part of this thesis. In order to model the flow of resources in an expanding system, an algorithm is required to grow the system which chooses the most efficient path for the distribution of resources to cells in the system. To explore the best algorithm for attempting to model such a system, a cellular automata square array is used. Two types of cell are introduced - *Distribution Cells* and *Receiver Cells*. Distribution Cells pass resource through the system, and receiver cells take and use the resource. This is analogous to how different sorts of cells specialise in a leaf. Some cells specialise in transportation whilst others specialise in resource use. The aim of the simulation is to optimise the number of receiver cells in contact with distribution cells.

Consider at first a 30 by 30 square two dimensional grid with a single starting cell at the centre (Coordinates: (15, 15)) at time step $t=0$, Figure 9.

To keep the model simple, each distribution cell can only pass resource directly to its nearest neighbour receiver cells. For the initial distribution cell, it will therefore have eight cells which it can distribute resource to, Figure 10. The ratio of distribution cells to receiver cells at this point is 1:8. From this simple set up, there are many ways in which the system can grow in order to optimise the number of receiver cells to distribution cells.

4.2 Growing the System

For the system to grow, additional Distribution Cells have to be added. For this model, it is assumed that distribution cells can only expand directly adjacent horizontally and vertically (not diagonally), also known as the Von Neumann neighbourhood. A number of different algorithms can be used in order to select the direction in which the distribution cells are added.

At $t=1$, whichever way the system grows, the ratio of distribution cells to receiver cells will be the same, Figure 11. In this instance there are two Distribution Cells and ten Receiver Cells, giving a ratio of 1:5.

Four alternative models of growing the system are now explored. These are as follows:

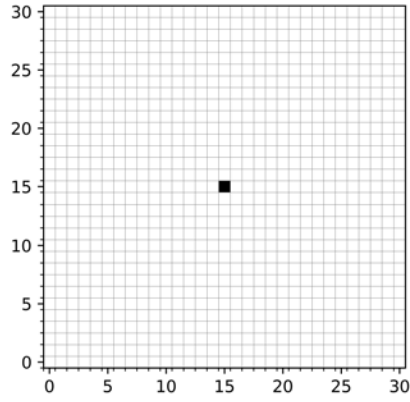


Figure 9: Initial distribution cell of space optimising algorithm on a 30 by 30 grid with single starting point in the centre $(15, 15)$ at time step 0.

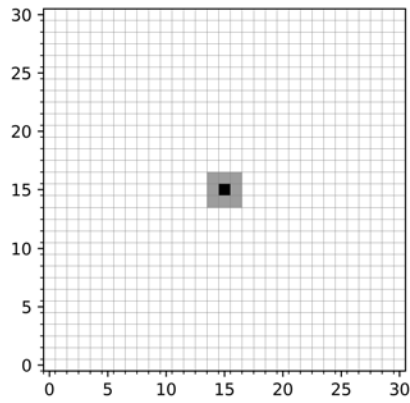


Figure 10: Initial configuration of single Distribution Cell (black) and Receiver Cells (grey).

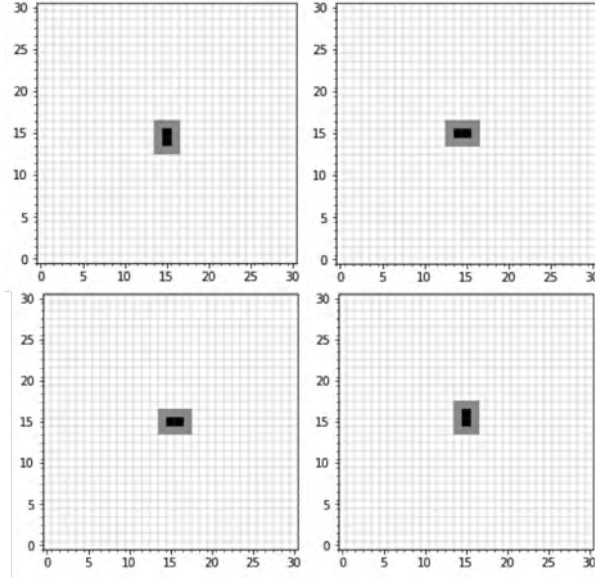


Figure 11: Initial configuration of single Distribution Cell (black) and Receiver Cells (grey).

- Deterministic and age dependent
- Stochastic and age dependent
- Global stochastic cell growth
- Global selection with local stochastic growth

The following models are allowed to grow to fifty Distribution Cells to allow for greater comparison between the number of Distribution to Receiver Cells of each.

4.2.1 Deterministic and Age Dependent

A deterministic system is one which is repeatable (without any stochastic processes) and by being age dependent the system preferentially allows older cells to grow. This is the simplest growing scenario explored as it produces the same result each time. A preferential growth direction is added to the left of the current distribution cell to allow for scenarios where the maximum amount of Receiver Cells can be added in multiple directions. This creates slight asymmetry in the system structure as can be seen in Figure 12. For

50 Distribution Cells, 82 Receiver Cells are possible for this model. This will be the same for all repeated runs of the simulation.

4.2.2 Stochastic and Age Dependent

This model is very similar to that outlined in the previous paragraph. The only difference being that in scenarios where there are multiple growth directions for a given Distribution Cell, to next Distribution Cell is chosen at random. This produces six alternatives for fifty Distribution Cell systems, Figure 13. All these models give the same Distribution to Receiver Cell ratio of 50 to 82; the same as for the Deterministic and age dependent model.

4.2.3 Global Stochastic Cell Growth

The global stochastic growth model disregards any preferential growth to older cells in the system. The model also has local stochastic processes similar to the previous paragraph. The outcomes of nine simulations are shown in Figure 14. In terms of ratios between Distribution and Receiver Cells outcome of this model produces much more variability than the previous two, Figure 15. It is normally distributed with a maximum Distribution:Receiver Cells ratio of 50:103 was found and a minimum of 50:81 was found.

4.2.4 Global Selection with Local Stochastic Growth

The Global Selection Model loops through all current Distribution Cells at each time step and selects Distribution Cells which will increase the Receiver Cells by the maximum amount. If there are multiple Distribution Cells which give the same maximum Receiver Cells, then one is selected at random. It is locally stochastic, so if for one particularly Distribution Cell there are multiple directions to grow which will give the same maximum amount of Receiver Cells, then one is chosen at random. The outcomes of nine simulations are shown in Figure 16. The highest ratio of Distribution to Receiver Cells is found from this model of 50:106. This appears to be the maximum ratio possible. Over multiple simulation runs, the variation is much lower than with the previous model, Figure 17. The Distribution:Receiver ratio is also asymmetrical distributed, with 50:106 being the modal value with a gradual decrease to lower ratios. A minimum ratio of 50:94 was found.

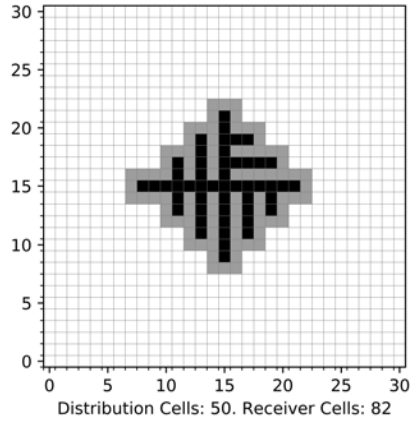


Figure 12: Deterministic and age dependent model with 50 Distribution Cells.

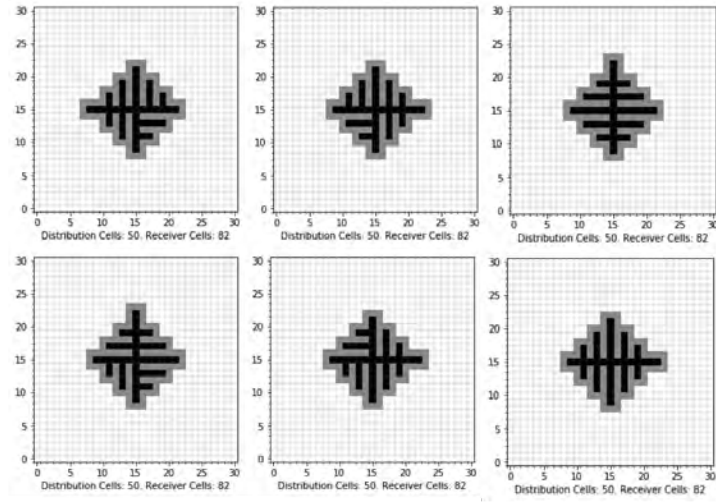


Figure 13: Six alternative scenarios for the stochastic and age dependent model with 50 Distribution Cells.

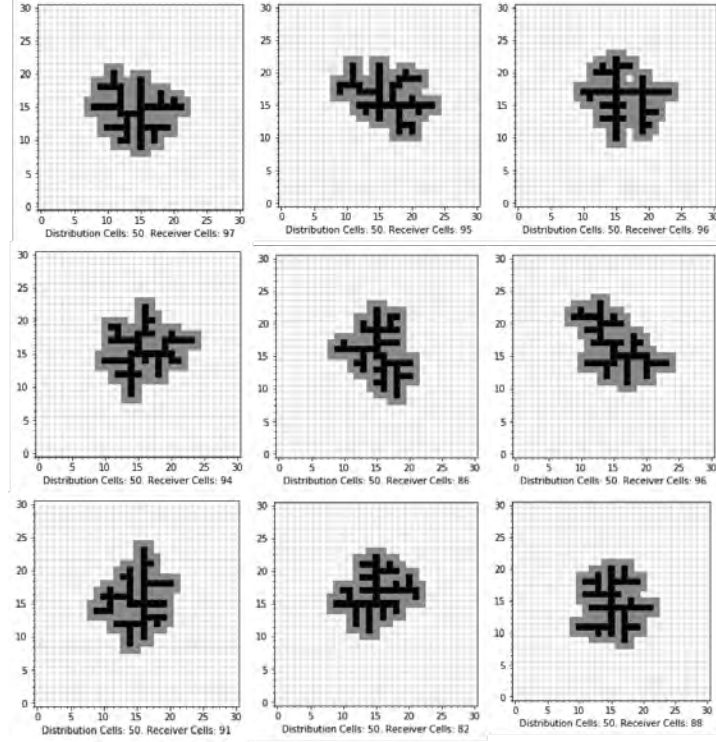


Figure 14: Nine alternative scenarios for the globally stochastic model with no age preference and 50 Distribution Cells.

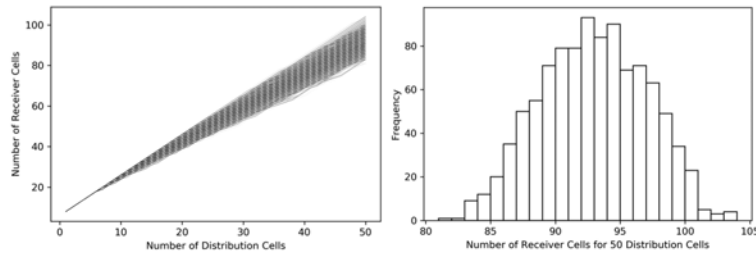


Figure 15: 1000 simulations for the Global Stochastic Cell Growth Model with fifty Distribution Cells. Left - Plot showing the number of Receiver Cells to Distribution Cells. Right - Histogram showing the variation in number of Receiver Cells.

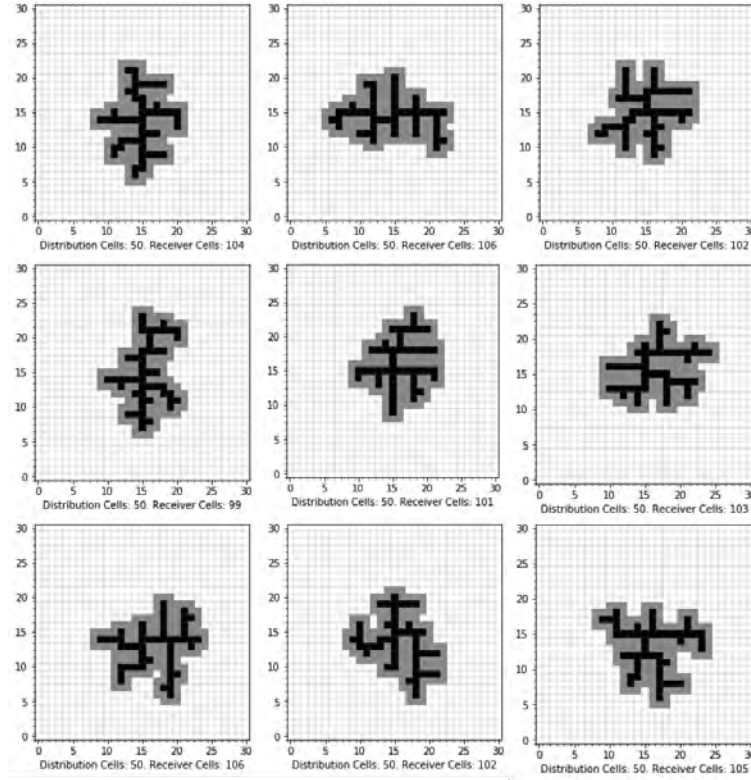


Figure 16: Nine alternative scenarios for the Global selection with local stochastic growth model with 50 Distribution Cells.

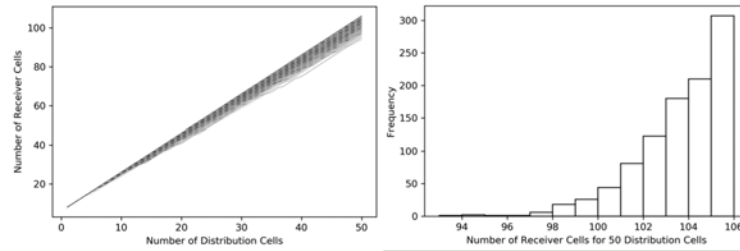


Figure 17: 1000 simulations for the Global Selection with Local Stochastic Growth Model with fifty Distribution Cells. Left - Plot showing the number of Receiver Cells to Distribution Cells. Right - Histogram showing the variation in number of Receiver Cells.

4.3 Discussion

This section of the thesis presents a number of simple models which offer explanations for the ways in which living systems grow in order to distribute and collect resource. Many of the patterns which emerge from the models can be seen in nature. For example, the ordered and symmetrical structure of the deterministic and locally stochastic systems with age dependence has a lot of commonalities with structure and certain types of plants and trees which have strong symmetry. However the ratio of Distribution to Receiver Cells for these models is the lowest.

The globally stochastic and global selection models bear a lot of resemblance with one another. Both models produce more irregular, even chaotic patterns. The ratio is greatest, and also most consistent for the global selection model. Again the pattern formed by the models has commonalities with real life living systems. Many trees have a rather irregular shape as they adapt to their environment in order to increase and maintain their exposure to sunlight.

By growing the global selection model to 2000 distribution cells, the resulting pattern can be further analysed, Figure 18. The Distribution:Receiver Cell ratio is 2000:3927 for this model, which is equivalent to 1:1.96. This is lower than the maximum ratio found for 50 distribution cells of 50:106 or 1:2.12. It is likely that increased simulations may increase the ratio for the larger system, although it is likely to reach the same ratio as found in the small system.

Whilst each of the models seem to produce patterns which are observable in nature, it should also be discussed whether the process undertaken taken by each model is observable in nature. For example, whilst the Global Selection Model produces the most consistent and highest Distribution:Receiver Cell ratio, it relies on a non-spatial information collection system to gather this data. It can therefore be thought of as a cognitive process within a protocognitive system. A protocognitive algorithm could be implemented which gathers all the information through nearest neighbour processes giving the same result as the current Global Selection Model, although this would take much more processing time.

Many living systems in nature are sub-optimal. They are not necessarily trying to just optimize in their current environment. Optimisation for the

sake of optimisation may in fact lead to stagnation and disrupt a path towards complexity Lehman and Stanley (2008). Novelty search is one search approach used in machine learning which doesn't attempt to optimise a system for a given task but instead explores for increased novelty Lehman and Stanley (2008). It is likely that living systems can be understood to follow a similar process. This therefore calls into question whether using the Distribution:Receiver Cell ratio has any basis for the selection of a particular model when attempting to simulate living systems. But of course, if the aim is to find space optimising algorithms, then the best one is the one with the highest ratio.

Whilst not explored in this section, the notion of *Persistence* should be introduced as this can be viewed as another pillar of evolutionary processes alongside optimisation and novelty. Since life started on earth 4 billion years ago, it has persisted. This can therefore be thought of as an underlying mechanism of life. Persistence occurs simultaneously with optimisation and novelty and can lead to other processes such as homoeostasis, niche construction, adaption, competition and cooperation. Persistence also requires a constant input of energy or resource, this is explored in the later sections when a resource is added to the system.

4.4 Combining the Growth Models

Real life living systems are likely to be on a spectrum between regular deterministic processes and stochastic irregular processes Raj and van Oudegaarden (2008). The models introduced in the previous section can also be combined to look at the effect of this on the structure and Distribution:Receiver Cell ratio. This may further explore the notion of novelty within these models.

In order to combine the models, an evolutionary computing approach is taken. This allows for the model to learn over the course of many simulations to select the appropriate growth model at which point.

4.5 Conclusion

The models introduced in this section form the basis for the models presented in later sections. The exploration of space optimising algorithms is an open-ended topic which can be explored indefinitely. This section has not provided all the answers to how living systems grow in certain ways but it

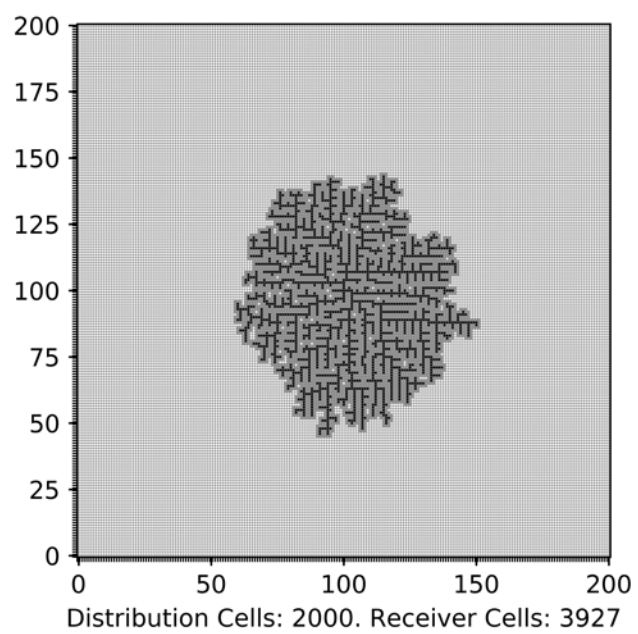


Figure 18: Global Selection with local stochastic growth model with 2000 distribution cells.

has given initial conclusions about underlying processes which lead certain patterns seen in nature. These include deterministic ordered processes which lead to well ordered symmetrical systems and stochastic chaotic processes which lead to irregular systems. The models presented show stochastic, global selection based systems give the greatest and most consistent Distribution:Receiver Cell ratio, but this may be due to limitations of the models themselves. These models are also limited by the fact that no resource is actually modelled, the growth limit is set by the number of distribution cells.

5 The Distribution of Resources in Living Systems

As mentioned in Section 2, the general research question in this section is as follows:

Living systems are known to grow following a sigmoidal curve with 3/4 scaling in relation to body mass and metabolic rate West et al. (1997). Will a toy simulation model using only self-organising rules set in the initial conditions have the same features?

5.1 Introduction

West et al. (1997) states that 'Living things are sustained by the transport of materials through linear networks that branch to supply all parts of the organism'. West et al. (1997) further describes a general model in which all living organisms share 3/4 power law allometric scaling relations for the metabolic rates in relation to the mass of the living system (Equation 5). This model is based on the following assumptions:

1. A space filling fractal network is required.
2. The final branch of the network is a size-invariant unit.
3. The energy required to distribute the resource is minimal.

$$Y = Y_0 M^b \tag{5}$$

where, Y is the biological variable, M is the mass, Y_0 is a constant that is characteristic of the organism and b is the scaling exponent. West et al. (1997) also states many more relationships between body mass and other biological variables using Equation 5 (see Table 1 in West et al. (1997)). These include the number of branches (capillaries) in the system, density of branches (capillaries) and many more.

Theoretical models of different supply networks have been attempted previously Banavar et al. (2010). The branching structure, coupled with the scaling, means that these systems are fractal in nature. Living, biological systems should also show a sigmoidal (long-term population limit) type growth curve Bettencourt et al. (2007), this is commonly modelled using the logistic model, Equation 6, Verhulst (1845) and has been extensively analysed in the life sciences Tsoularis and Wallace (2002). A plot of Equation 6 can be found in Figure 19.

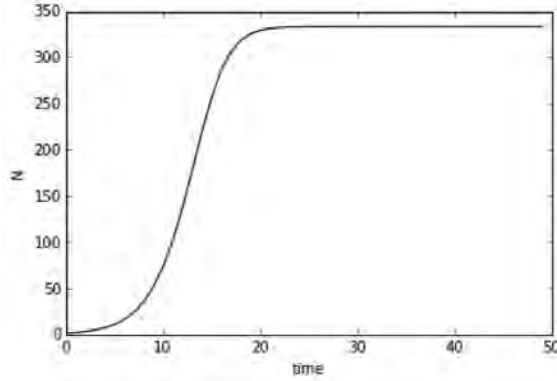


Figure 19: Plot of Equation 6 using $N_0=1$, $r=1.5$ and $K=1000$

$$\frac{dN}{dt} = \frac{rN(K - N)}{K} \quad (6)$$

where, N is the population, r is the rate of population growth and K is the carrying capacity.

The iterative map paragraph in Section 3.9.3 explains and explores of a variation of Equation 6.

Banavar et al. (2010) introduce two models, a radial explosion model and a hierarchical branching model to explore the quarter-power scaling in animals following on from the work of West et al. (1997). These models are equation based with no simulation component.

The work outlined in the following sections is original work with the aim of creating an agent based model which organises based on predefined rules set out producing a system which has similar properties to both the West et al. (1997) and Banavar et al. (2010) models.

5.2 Selection of the Space Optimising Algorithm

Section 4 introduces models which optimise space between distribution and receiver cells. This section uses these models in order to model the flow of resource. As the research question states, the aim of this section is to reproduce the 3/4 power law scaling in relation to body mass and metabolism.

All the models show commonalities with natural living systems although the models vary in the amount of stochastic and deterministic processes present. When discussing gene expression, Raj and van Oudenaarden (2008) describe life as 'a study in contrasts between randomness and determinism'. This is increasing apparent when studying universal scaling in biological systems. The $3/4$ power law logically leads to the conclusion that biological processes to a degree must be deterministic, although the biomolecular interactions are seemingly stochastic. As life displays both randomness and determinism, all the models may have relevance.

To first look at scaling in biological systems, measurements can be taken from the space optimising algorithm. The number of receiver cells in the system can be used as a proxy for the metabolism of the system and the combined number of distribution cells and receiver cells can be used as a proxy for the body mass of the system. Figure 20 shows the patterns formed by the different growth models analysed. Figure 21 shows the scaling of these systems. As can be seen, the logarithmic scaling of Receiver Cells:(Receiver and Distribution Cells) is between 0.9 and 0.95 for all the systems. This is greater than the observed $3/4$ relationship seen in nature West et al. (1997). The reason for this is likely to be due to the lack of levels in the current model. As stated in West et al. (1997), the size of the branches downstream from upstream branches is area-preserving, this is not the case in this model, as the area is the same for all distribution cells and receiver cells. In order to preserve area, levels need to be added to the model, increasing it's complexity.

5.3 Modelling the flow of resources

This section seeks to extend the algorithm to add a flow of resources through the network. There are different methodological options for modelling the flow through the system. This is not surprising given the vast variation of distribution systems in nature. Whilst designing a space optimising algorithm is relatively straight forward, when adding the flow of resources to the system, many additional rules have to be added, especially, as in nature if the expansion of the system is reliant on the availability of resources.

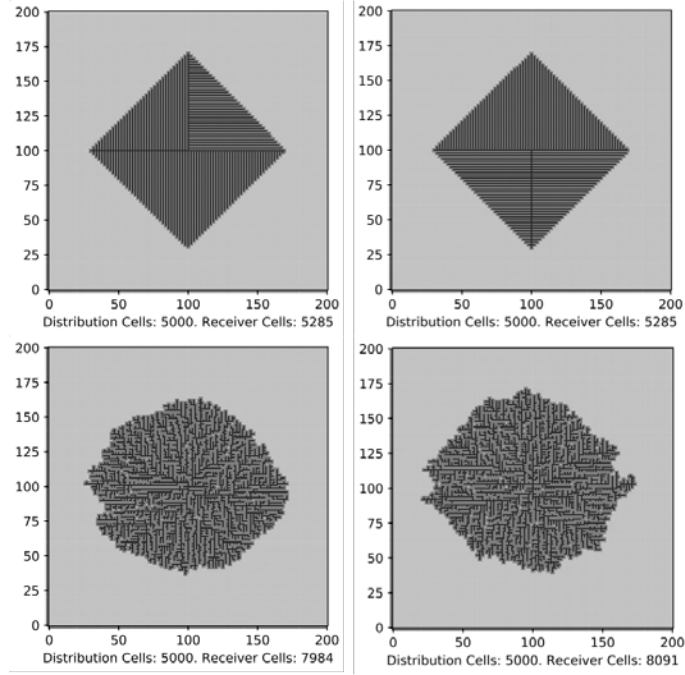


Figure 20: Figure showing the different growth patterns formed by the space optimising models for 5000 Distribution Cells in Section 4. *Top Left:* Deterministic Growth Model, *Top Right:* Locally Stochastic Model, *Bottom Left:* Globally Stochastic Model and *Bottom Right:* Global Selection model.

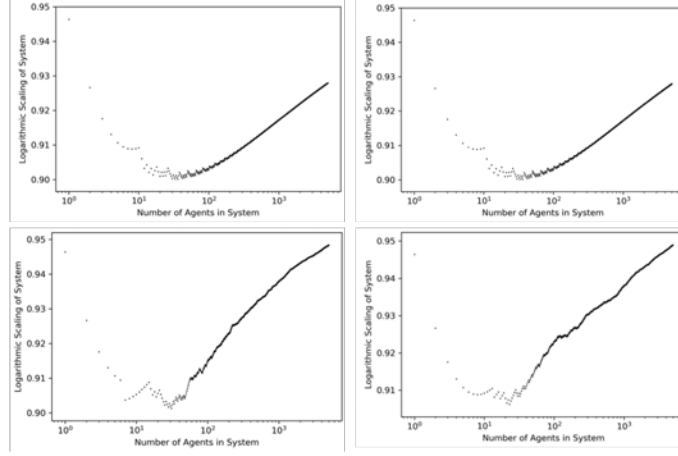


Figure 21: Proxy Logarithmic Scaling of Body mass in relation to metabolic rate for Models shown in Figure 20. *Top Left*: Deterministic Growth Model, *Top Right*: Locally Stochastic Model, *Bottom Left*: Globally Stochastic Model and *Bottom Right*: Global Selection model

5.3.1 Commonalities to Real World Systems

This model is based loosely on the resource distribution system found in plant systems. Notaguchi and Okamoto (2015) state that plant vascular systems are constructed by specialised cells known as xylem and phloem. The xylem cells form thickened cell walls which can survive long after the death and acts as conduits for transferring water to all parts of the plant. The phloem cells however form consist of complex living cells which transfer photosynthetic assimilates as well as gene products to other parts of the organism and may form the basis of a long-distance communication system.

This model is largely based on the transfer of water through the xylem cells and does not account for the phloem cells, at least in this iteration of the model.

The original space optimising algorithm was able to produce symmetrical patterns through deterministic rule sets and highly space optimised systems by systematically choosing the next location to expand the system into. The expansion of the system was not dependent on the availability of resources, but instead on the number of agents which are permitted (set out in the

initial conditions). As will be shown in this section, when the expansion of the system is dependent on the availability of resources, the shape which the system takes is hugely dependent on the functions which distribute resource to parts of the system.

5.3.2 Cell Specialisation

There are two main types of cell which are introduced in this model - Distribution Cells and Receiver Cells. The rules regarding where these cells can be placed are the same as in Section 4. So Distribution Cells can be added in a Von Neumann neighbourhood of any other Distribution Cell and Receiver Cells can be added in a Moore neighbourhood of any Distribution Cell. As in Section 4, the addition of Distribution and Receiver Cells is dependent on whether adding them will increase the ratio of Receiver Cells to Distribution Cells, but also the availability of resource in the Distribution Cell.

5.3.3 Distribution Cells (DCs)

The variables assigned to each Distribution Cell has are shown in Table 1. Most of the information in the table is self-explanatory. The *Need Type* variable will switch state between *Maintenance*, *Add Cell* and *Add DC* dependent on its requirements. *Maintenance* is prioritised over *Add Cell* over *Add DC*, which means that a distribution cell will maintain existing receiver cells it passes resource to, over adding new receiver cells, over adding new distribution cells.

5.3.4 Receiver Cells (RCs)

Resource is passed through a network of Distribution Cells to Receiver Cells. The variables assigned to each Receiver Cell are shown in Table 2. In this model, when a Distribution Cell is added, the Receiver Cells associated with it are added too. However, a construction cost is added for each Receiver Cell which has to be paid for resource to be passed to the cell. The *Add Cell* variable in the Distribution Cell completes this transaction when it is required. Each time step, the RCs use a resource. If the resource in a RC reaches zero, then the construction cost has to be paid again.

5.3.5 General Parameters of the Model

There are also general parameters for the model. These are presented in Table 3. The cost to construct parameter relates to the fact that in nature

DC Variable	Explanation
Position	Coordinates of the DC
Previous Agent	Link to 'upstream' DC
Creation	Time step which DC was created on
Extension	Binary value on whether DC can add additional DCs
Extension Amount	Amount of DCs which can be added to this DC
Keep	Amount of Resource DC is holding for it's cells
Pass	Amount of Resource DC is holding to pass to other DCs
Cells	Links to Receiver Cells this DC is passing resource to
Need Type	Current prioritised need of DC
Maintenance	DC need to maintain Receiver Cells
Add Cell	DC need to add Receiver Cells
Add DC	DC need to add DC cells
Collated Need	Summed need of DC and DCs downstream
Downstream DCs	Link to downstream DCs and their needs

Table 1: Explanation of Variables used by Distribution Cells

RC Variable	Explanation
Position	Coordinates of the RC
DC	Link to Distribution cell providing resource
DC Location	Coordinates of Distribution Cell
Resource	Total amount of resource held by RC
Construction Cost	Binary value for construction cost of RC

Table 2: Explanation of Variables used by Receiver Cells

there is likely to be a cost to construct cells as well as use them. In this model the Distribution and Receiver Cells are homogeneous, so the Optimum Resource for each Receiver Cell is the same and the Resource used per Time Step is the same for all Receiver Cells. Regarding the allocation of resources, each distribution cell will distribute resources proportionally to the number of cells which are downstream of it.

5.3.6 Explanation of Model Functions

This model is built in Python, which is a high level, object orientated coding language. The previous subsections explain the parameter inputs and variables of the different cell types in the model. This section explains the

Model Parameters	Explanation
Array Size	Initialises the size of the system (X,X). Systems with a larger input generally need a larger array size
Initial Coordinates	Coordinates of the initial Distribution Cell
Time steps	Number of time steps the simulation will run for
Resource added per time step	Amount of resource added to initial cell
Initial Resource per DC	Resource held by each DC each time step
Resource used per Time step	Resource which is used by each RC per time step
Cost to add RC	Cost required to add each RC
Cost to add DC	Cost required to add each DC
Optimum Resource	Optimum Resource for RCs
Growth Type	As per Section 4, the model can grow in a deterministic or stochastic manner

Table 3: General Parameters in the model

functions which pass both information about resource allocation and the resource distribution itself. Table 4 provides a summary of the functions used in the model. Figure 22 shows the order in which the functions operate in the Model.

This model is a simplified version of a living resource distribution system. The Receiver Cells (RCs) in the model collect the resource and then use it each time step. This is similar to how plant systems use water in photosynthetic processes. A cost is incurred to adding both distribution cells and receiver cells. The reason behind this is that work and energy is required to add these cells.

Model Function	Purpose
Main	This is the main function which calls all other functions
Initialise	Initialises the system. It builds the array based on the parameters set out and adds the initial Distribution Cell and it's Receiver Cells
Add Resource	Adds the resource to the system each time step
Select Need	Calls three further functions which calculate the need of each DC based on maintenance needs, cost to add Receiver Cells, cost to add Distribution Cells. This function then assigns the need type to each DC
Pass Need	Passes each of the need of the DCs in the system up to the initial Distribution Cell which receives the resource
Pass Down Resource	Based on the needs of each DC, resource is passed down through the system. Resource is also passed from a distribution Cell's <i>Pass</i> variable to it's <i>Keep</i> variable.
Pass Resource to RC	Passes resource from DCs to RCs in order to maintain them
Add RC to DC	Adds additional RCs to DCs if required
Add DC	Adds additional DCs if required
Add RC	Pays the cost to allow the use of an RC
Use Resource	Each RC uses a fixed amount of resource per time step
Remove RCs	Removes RCs which no longer have any resource
Remove DCs	Removes DCs which no longer have RCs

Table 4: Main functions in the model explained

5.3.7 The *Pass Need* Function

As stated in Table 4, the *Pass Need* function passes the needs of each DC in the system back up through the system to the initial input cell.

Communication within plants is often described in the form of information *cascades* conveying the effect of abiotic and biotic factors on the plant Huber and Bauerle (2016). There are many abiotic stresses such as salt, water, light, and heat and biotic stresses such as herbivore and plant-pathogen interactions. These may combine resulting in these information cascades Huber and Bauerle (2016), which will then feedback and change the patterns of distribution within the plant. The simple sandpile model Bak et al. (1987) is a well-known example of cascading of information although this is a model of a physical system where the cascades reflect shallowing of the oversteepened sandpile sides. However there might be commonalities in the underlying information transfer mechanisms, in particular the sandpile model is a computer simulation, and this section of the thesis is attempting to model information transfer in plants so the modelling concepts can be cross applied.

In this iteration of the model, information is passed up through the system using a nested *for loop* and *while loop* which will loop over each agent and pass the information up through the system to the initial distribution in turn. Whilst this model does employ local interaction for passing information, it does not rely on cascades to pass information. The main reason for this being that each function completes all its 'tasks' each time step, making the model easy to follow. With the addition of cascading information flows, the functions wouldn't necessarily complete their tasks each time step, increasing the complexity of the model. This may be more realistic and worth exploring in future research.

5.3.8 The *Pass Down Resource* Function

The model in its current form will expand based on the resource available. As stated in Section 5.3.3, resource will be prioritised for maintenance over adding Receiver Cells, over adding Distribution Cells. However the system can still grow to a size where there is more demand in the system than supply. If the demand outstrips supply, the *Pass Down Resource* function distributes resource proportionally to the needs of downstream Cells. On the other hand if a given cell has a greater amount of resource than is required

downstream, then it will hold onto the additional resource. The implications of this are shown in both the Results and Discussion sections.

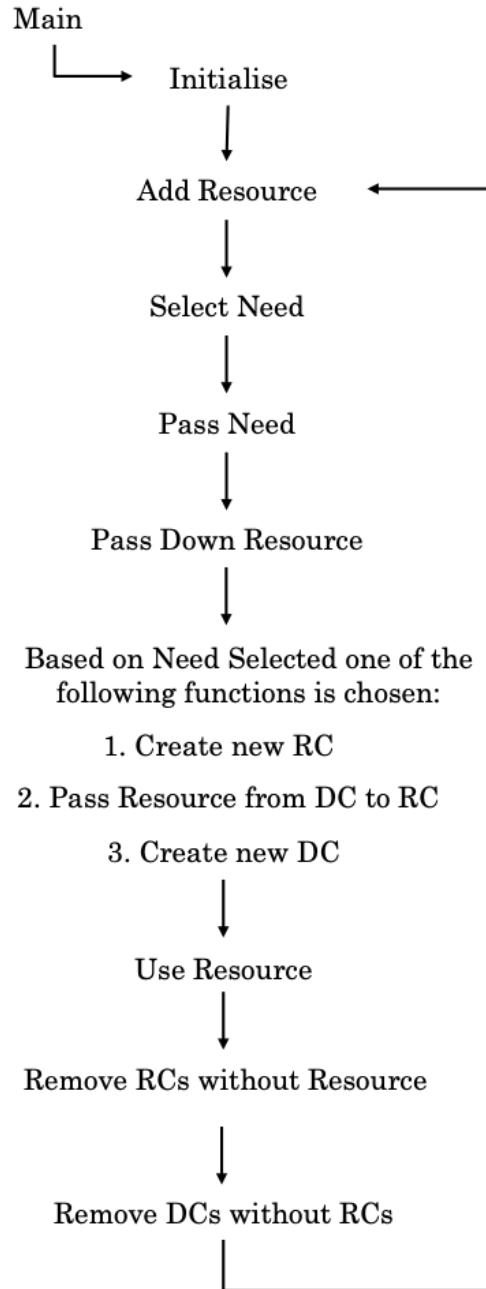


Figure 22: Flow chart showing the order of the functions in the Model.

5.4 Results

This section summarises the findings of a number of different simulation runs of the model. A number of system inputs, and growth types are explored.

5.4.1 Initial Simulation

The parameters used for this simulation are shown in Table 5. As stated in Table 5, this simulation uses a Deterministic Growth Type. The algorithm explained in Section 4.2.1, which adds additional distribution cells based on their age (the oldest distribution cells will grow first), however although additional DCs will be added based on age, the availability of resources is also a factor. The implications of this are discussed further in the following sections.

Model Parameters	Value
Array Size	(50,50)
Initial Coordinates	(25,25)
Time steps	200
Resource added per time step	300
Initial Resource per DC	0
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	3
Growth Type	Deterministic

Table 5: General Parameters in the model

The growth curve of the simulation is shown in Figure 23. The curve shows a similar sigmoidal curve as in Figure 19, although there is quite high variance. The animation <https://youtu.be/5h04iHLnga0> shows how the system initial grows out in all directions in a symmetrical fashion. Following this growth phase, parts fo the system expand whilst others contract. This is likely due to how the algorithm distributes resource to parts of the system proportionally. As parts fo the system which are expanding are connected closer to the source, it leads to parts of the system which are connected further away from the source to contract. Printouts of the system at different time steps are shown in Figure 24.

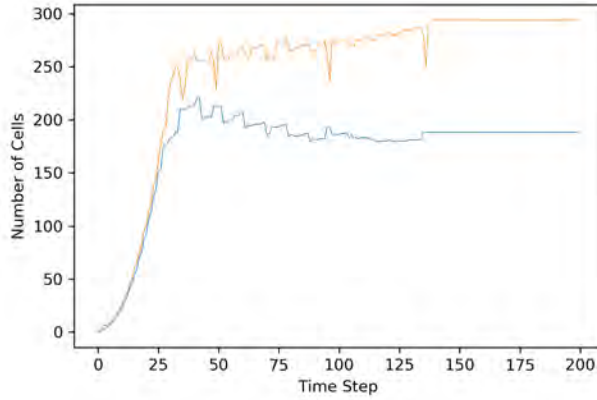


Figure 23: Figure showing the growth of the system for Simulation 1. The blue line is the number of Distribution Cells and the orange line is the number of Receiver Cells. An animation of the growth can be found here: <https://youtu.be/5h04iHLnga0>

5.4.2 Exploring the *Resource added per time step* Parameter

By changing the amount of resource added into the system each time step, the size to which the system grows changes. This also allows the original hypothesis of this section to be tested:

Living systems are known to grow following a sigmoidal curve with $3/4$ scaling in relation to body mass and metabolic rate West et al. (1997). Will a toy simulation model using only self-organising rules set in the initial conditions have the same features?

Section 5.2 showed that for a pattern forming system, without a resource input, the ratio Receiver Cells: (Receiver Cells + Distribution Cells) found a scaling relationship of 0.9-0.95. This is greater than the relationship observed in nature West et al. (1997). This section measures the scaling of Receiver Cells to Total Resource in the system as a measurement of scaling. Table 6 shows the parameters used in the simulation. A total of 18 simulations were undertaken. Figure 25 shows the growth of all the simulations and Figure 26 shows the scaling of total cells and total mass of the system verses total cells in the system. As stated on the figure the mean scaling value is 0.923, which is significantly greater than the proposed

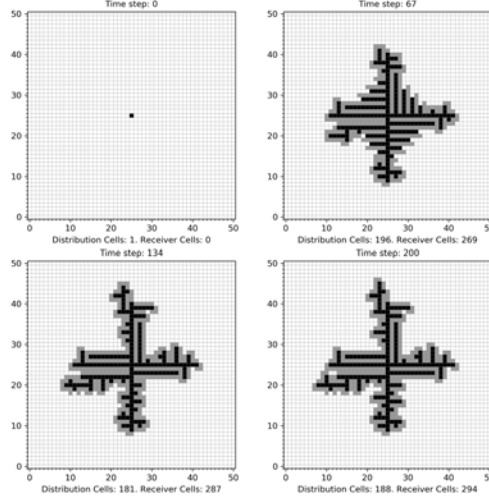


Figure 24: Figure showing the system at different time steps.

0.75 as found in the literature. The total mass of the system is calculated as:

Each Receiver and Distribution cell in the system is assumed to have uniform mass. The Total Resource in Cells is taken from the mass of the system as the *Optimum RC Resource* was assigned in an arbitrary manner and may affect the scaling parameter.

The reason for the mean scaling parameter being greater than the value found in the literature is likely to be the same reason as it being greater for the distribution cells: receiver cells ratio - the lack of levels in the simulation model.

Model Parameters	Value
Array Size	(100,100)
Initial Coordinates	(50,50)
Time steps	200
Resource added per time step	100, 125, 150...600
Initial Resource per DC	0
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	3
Growth Type	Deterministic

Table 6: General Parameters in the model

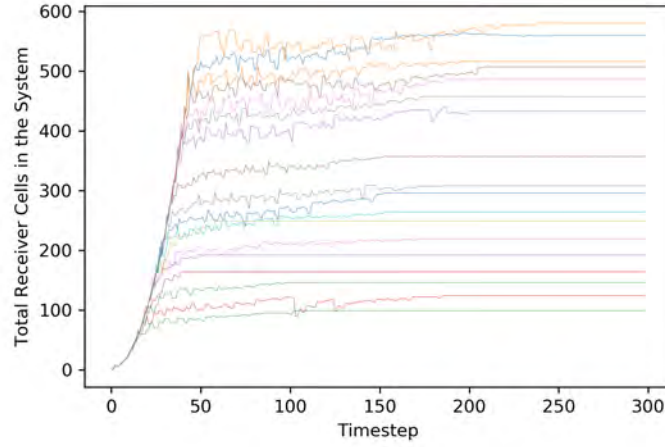


Figure 25: Growth of System when changing the resource input. Based on parameters in Table 6.

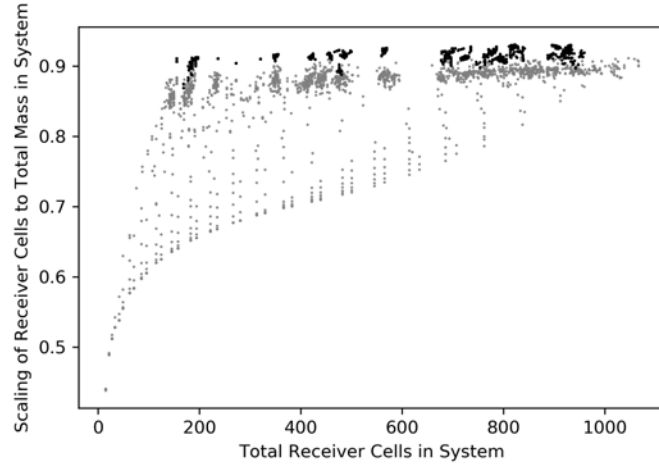


Figure 26: Scaling Relationship between Resource Use per Time step and total system mass. The grey dots represent the relationship during the growth phrase of the system and the block dots represent the relationship after time step 125 - when all systems have reached a stable state. The mean scaling relationship for the black dots is measured as 0.923. Based on parameters in Table 6.

5.4.3 Exploring the *Growth Type* Parameter

The different types of space optimising algorithm as outlined in Section 5.2 are also explored to see the effect of algorithm type on the system created.

The different Growth Types are as follows:

- Deterministic
- Locally Stochastic
- Globally Stochastic
- Global Selection

All the simulations tested so far in this section use the Deterministic Growth Type. This section looks at the affect of the resource distribution algorithm on the space optimising algorithm. The following simulations use all the same parameters as shown in Table 7 apart from a different Growth Type.

Model Parameters	Value
Array Size	(100,100)
Initial Coordinates	(50,50)
Time steps	200
Resource added per time step	500
Initial Resource per DC	0
Resource added to each DC per time step	0
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	10
Growth Type	All types

Table 7: General Parameters in the model for Section 5.4.3

The results show that the changing growth type given these parameters leads to very little change in the growth curve of the system, see Figures 27, 28, 29 and 30. Morphologically the systems appear quite different as shown in Figures 31, 32, 33 and 34. The reason for this is the resource distribution algorithm is controlling the shape of the system formed more than the space optimising algorithm, and based on the rules of the algorithm if resource isn't available in a certain part of the system then it cannot grow. There are two

ways to create a structure more similar to those seen in Section 4. Firstly, the space optimising algorithm which provides information on needs of different parts of the system could be changed to allow for preferential flow to different parts of the system based on what the space optimising algorithm prioritises, and secondly if there is more resource in the system than demand then the shape of the system will simply follow the space optimising algorithm. The former is explored below, and the later is explored in Section 6 which looks at resource creating systems

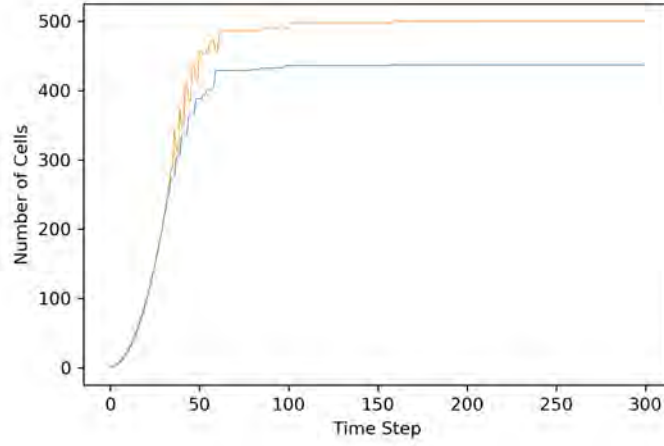


Figure 27: Number of DCs and RCs for the Deterministic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

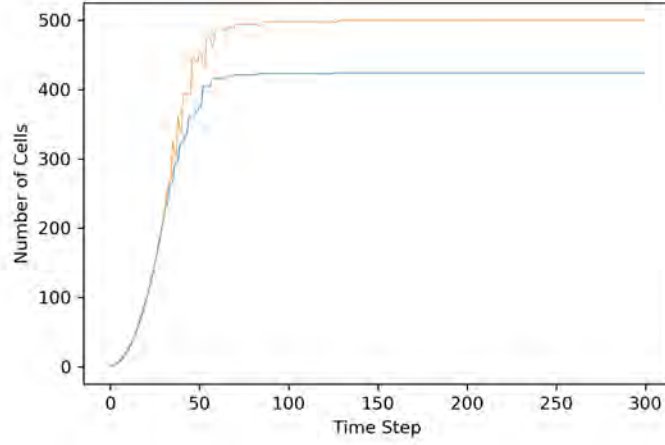


Figure 28: Number of DCs and RCs for the local stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

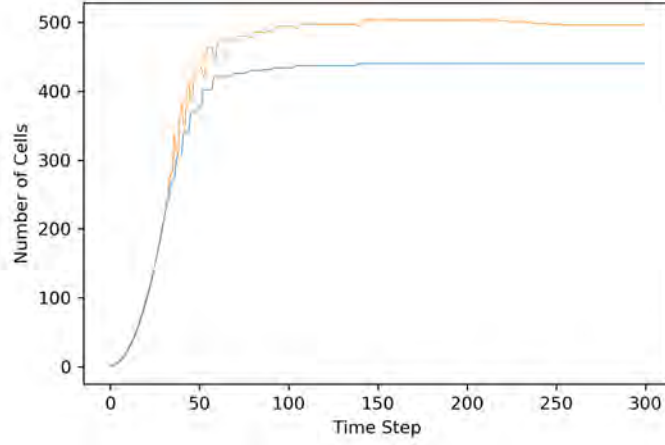


Figure 29: Number of DCs and RCs for the global stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

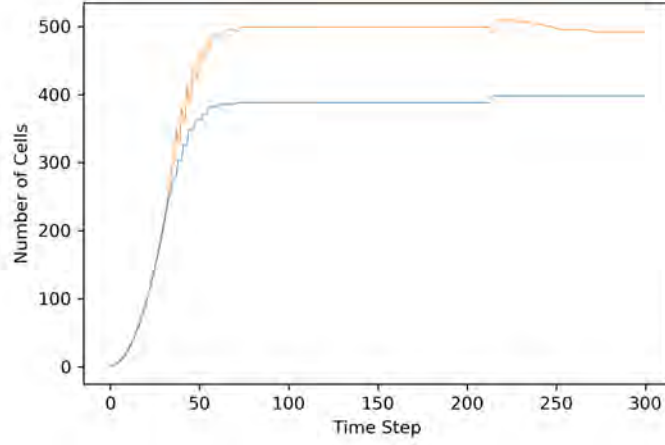


Figure 30: Number of DCs and RCs for the global selection growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

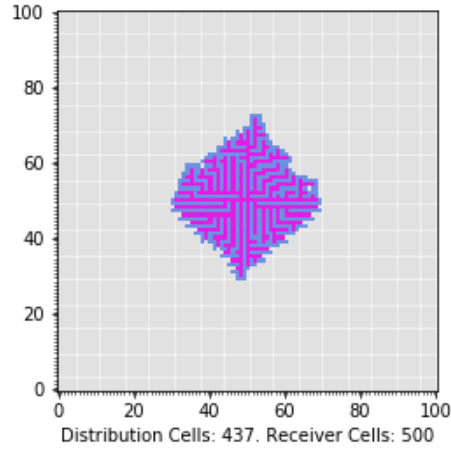


Figure 31: Number of DCs and RCs for the Deterministic growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

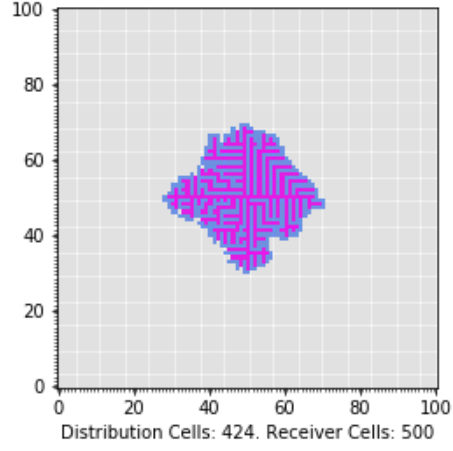


Figure 32: Number of DCs and RCs for the local stochastic growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

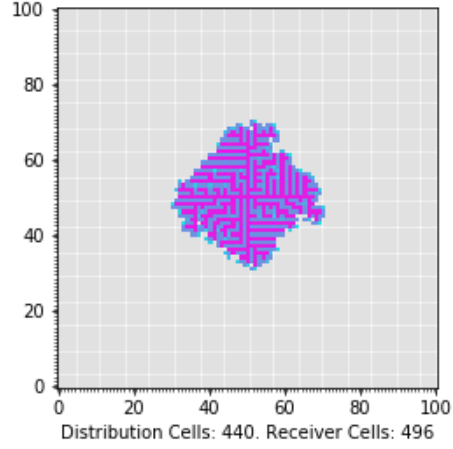


Figure 33: Number of DCs and RCs for the global stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

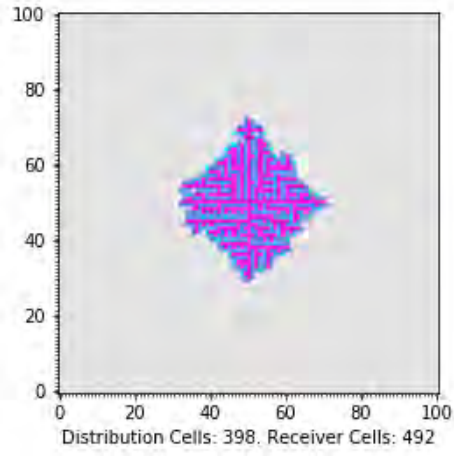


Figure 34: Number of DCs and RCs for the global selection growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

5.4.4 Preferential Growth

As outlined above, preferential growth is added to attempt to give similar results to the space optimising algorithm in (Section 4) once the resource distribution algorithm is added. The preferential growth function works by finding the distribution cells which will grow by the maximum amount each time step and then only allowing the DCs with the same maximum to grow each time step. However this function does not rely on nearest neighbour interactions, which in this model passes all the information back up through the model and distributes it based on this information. It therefore undermines the process-based nature of the model. Constructing a function which undertakes this task in a process-based manner could be used but would take much more processing power. As mentioned in Section 5.3.1, a model which passes information via concentrations would have more similarities to real world systems, which could be a way of passing information about preferential growth through the system. The results of using a non-nearest neighbour preferential growth model for each of the growth types are shown in the following figures. The parameters in Table 7 are used.

As can be seen the preferential growth function increases the number of receiver cells to distribution cells. For the Global Selection model, the morphology of the system created is much more similar to the space optimising algorithm, Figure 42. For the other growth types, the morphology of the system is quite different, Figures 39, 40, 41 and 42.

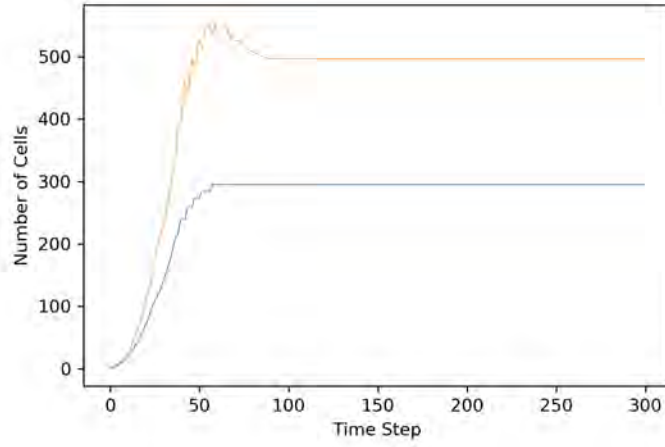


Figure 35: Number of DCs and RCs for the Deterministic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

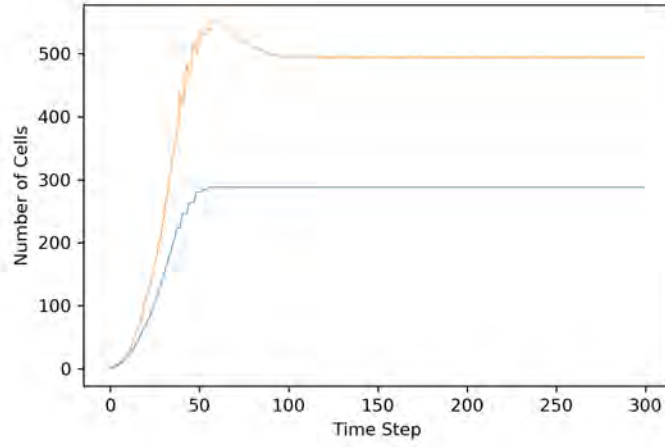


Figure 36: Number of DCs and RCs for the local stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

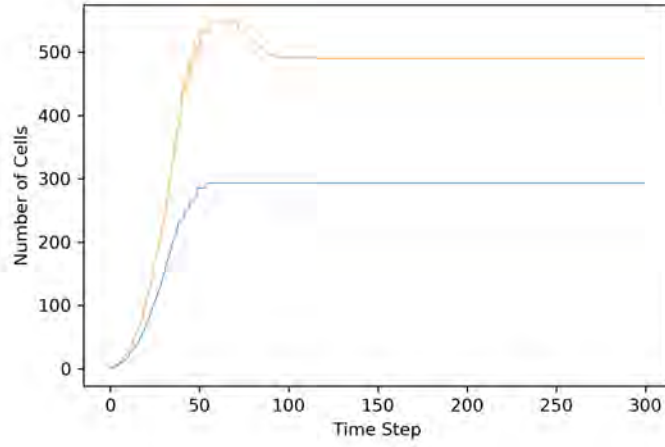


Figure 37: Number of DCs and RCs for the global stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

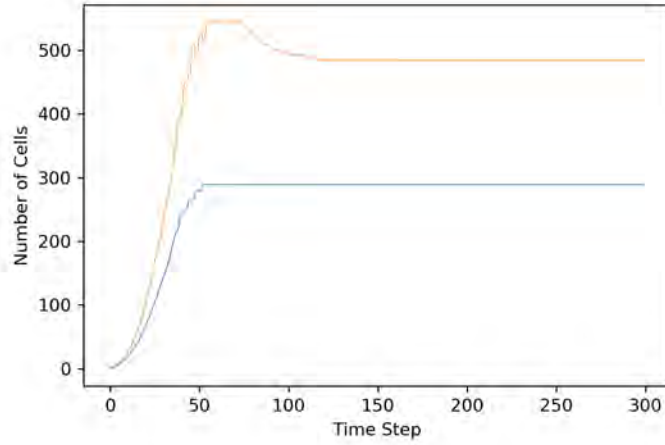


Figure 38: Number of DCs and RCs for the global selection growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

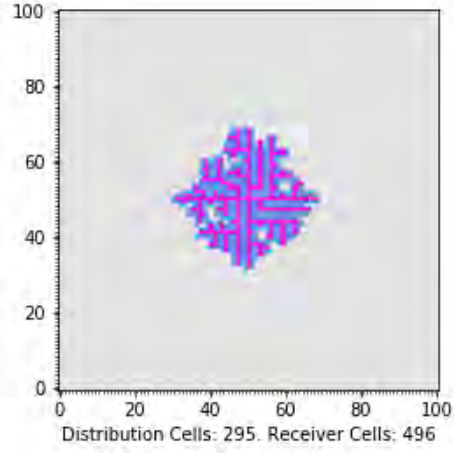


Figure 39: Number of DCs and RCs for the Deterministic growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

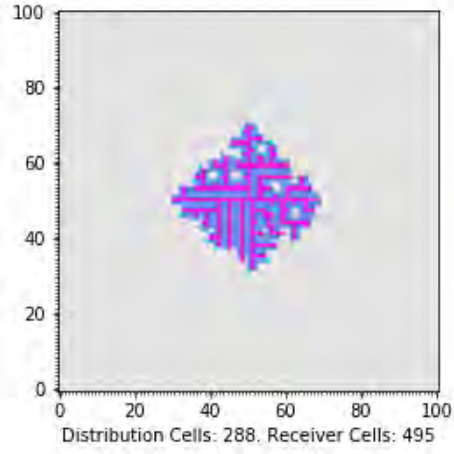


Figure 40: Number of DCs and RCs for the local stochastic growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

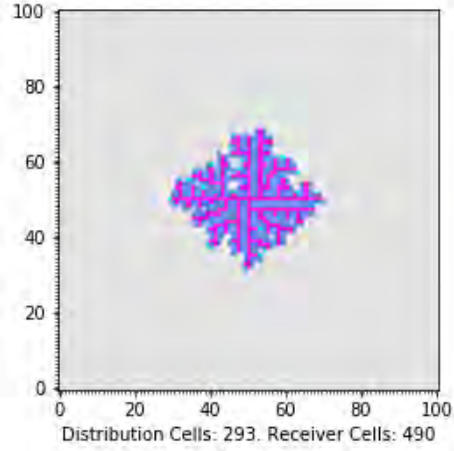


Figure 41: Number of DCs and RCs for the global stochastic growth model in Section 5.2. The orange line is the number of RCs and the blue line is the number of DCs. The parameters used are shown in Table 7.

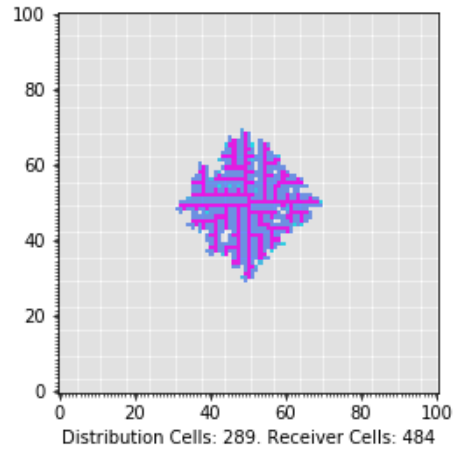


Figure 42: Number of DCs and RCs for the global selection growth model in Section 5.2. The pink cells are the DCs and the blue cells are the RCs. The parameters used are shown in Table 7.

5.5 Discussion

The model presented in this section builds upon the space optimising algorithm presented in Section 4. The additional functions in the model lead it to becoming a 'process based model'. By adding in these resource distribution features, the growth and structure of the system are no longer just dependent on the optimal placement of cells based on the rules set out in the algorithm, but instead on the availability of resource.

As shown in Simulation 5.4.1, using the deterministic space optimising algorithm, alongside a resource distribution algorithm, the shape of the system created is generally similar to the original space optimising algorithm, however there is some asymmetry, which is likely to be due to there not always being enough resource, at which point there are slight differences in the resource allocation. The growth curve is also sigmoidal which is to be expected for biological systems.

As shown in Section 5.4.2, the model doesn't show a reasonably similar power law scaling relationship between body mass and metabolic rate to the $3/4$ relationship found in nature. This model has also been built in a much different way to previous models such as West et al. (1997), which is an equation based model.

The models presented in this Section and also in the previous Section 4 assume that all cells are equal in size in the whole system. This is of course not the case in natural systems, especially for the distribution cells which decrease in volume each time they branch. This decrease in volume is reflected in the resource which is passed between distribution cells but not in the size of the cell.

This system is also grown in two dimensions whereas the majority of biological systems are three dimensional. This may affect the scaling laws observed.

5.6 Conclusion

This section presents a model for growth and maintenance of a biological system given a constant input. The model doesn't show $3/4$ power law scaling of metabolic rates and body mass, which is likely due to the lack of levels in the system. This model adds to previous models by allowing the

system to self-organise based on an initial rule set. Given that this model does not set out any sigmoidal type growth or scaling parameters in its initial conditions, but are still observed in the results shows that this model follows similar fundamental properties found in living systems. The model is also very adaptable, it is an agent based model in which all the different agents (both distribution and receiver cells) can have different parameters or characters meaning that a population of heterogeneous agents can be explored. This provides many more avenues in which the model can be explored. The following sections build upon this model, adding a range of new ways to explore it.

5.7 Analysis of Code Performance

This section contains a *Meta-Analysis* of the simulation performance itself. This is both useful for optimising the code and also understanding how the simulation relates to the system it is trying to model. Optimisation of the code is important for scaling up the simulation as it reduces the time taken by each simulation.

The parameters used in this simulation are shown in Table 8. The total time taken for this simulation was calculated as 1305 seconds. Figure 43 shows the time taken for each function in the simulation. Most of the functions take a small amount of time indicated by the flat horizontal line. The functions which follow the sigmoidal curve relate to the 'Select Need' function, which is used to calculate the need of each distribution cell. The shape of the curve isn't surprising, as it shows a proportional increase in the function time with an increase in the number of cells.

A plot of the time taken for each function excluding the 'Select Need' functions is shown in Figure 44. The time taken by these functions is very fast. The large fluctuations of the 'add_agent_add_agent' function is during the growth phase of the system when during which more agents are being added. The other functions show a generally linear increase in time taken with increased system size.

A plot of the sub-functions within the 'select_need' function are shown in Figure 45. This shows that the 'calc_growth_need' function is the cause of increased time taken by the 'select_need' function. The 'calc_growth_need' function calculates the number of receiver cells which would be added if a distribution cell is added, and in which adjacent cell that distribution cell

should be added. A way to decrease the amount of time used by this function is to only implement it when it is required.

Model Parameters	Value
Array Size	(200, 200)
Initial Coordinates	(100, 100)
Time steps	100
Resource added per time step	1000
Initial Resource per DC	0
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	10
Growth Type	Deterministic

Table 8: General Parameters in the model for Section 5.7

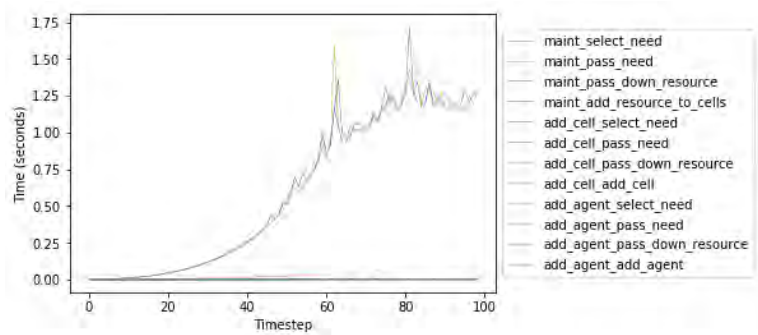


Figure 43: Time taken for each function in the simulation.

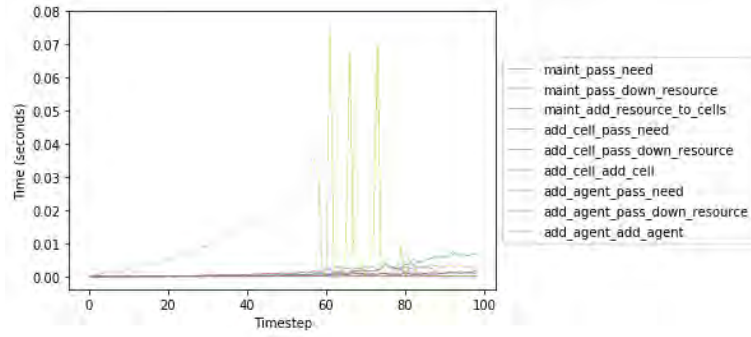


Figure 44: Time taken for each function in the simulation, excluding the 'Select Need Functions'.

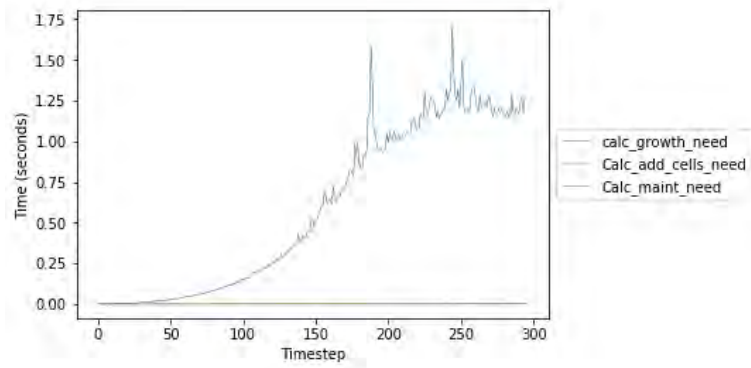


Figure 45: Time taken for each function in the simulation, excluding the 'Select Need Functions'.

6 Distribution and Growth in Resource Creating Systems

The model presented in the previous section introduced a resource distribution system, where the only source of resource was added to the initial distribution cell in the system. The resource is then distributed to the receiver cells which use the resource. This section introduces a modification to the model to explore resource creating systems. In this model each Distribution Cell creates resource itself. The main research area which can be explored using this model relates to how systems which are resource creating or undergoing transitions where additional resource is available behave. Examples of real-life systems which exhibit such behaviour are modern economic/sociological systems Banavar et al. (2010), critical transitions in biology Szathmary (2015). In the case of critical transitions in biology Scheffer et al. (2001), resource cannot be created as this would violate the laws of physics, instead most transitions originate from an innovation which then leads to a resource being allocated in a more efficient manner. In the case of economic and social systems, these aren’t necessarily linked to physical quantities so resource can be created out of thin air.

A number of variations of resource creating systems are explored. These include:

- Systems with Distribution Cells initialised with resource.
- Systems with Distribution Cells creating resource per timestep.
- Maximum growth systems.

The main area of interest in studying these systems is related to the following research question:

Systems undergoing regime shift are known to exhibit phenomena such as an increase in variance and flickering Scheffer et al. (2001). Under what conditions and changes in initial parameters (if any) will a resource creating system exhibit this phenomena?

The following sections introduce variations of the model presented in previous sections. Minor alterations to the model are introduced and explained to facilitate with answering the research question.

6.1 Systems with Distribution Cells creating resource per timestep

This model simply looks at the effect of each Distribution Cell creating resource per time step. The resource added to each Distribution Cell per time step is altered and the results collated. The parameters explored in this simulation are shown in Table 9.

Model Parameters	Value
Array Size	(200, 200)
Initial Coordinates	(100, 100)
Time steps	500
Resource added per time step	20
Initial Resource per DC	0
Resource created by DC per timestep	0-2.2
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	10
Growth Type	Deterministic

Table 9: General Parameters in the model for Section 6.1

A plot of the total number of cells (Distribution and Receiver Cells) per time step for a range of resource creating systems is shown in Figure 46. The plot shows that for *Resource created by DC per time step* above 1.6, exponential increase in total cells occurs. Figure 47 shows the increase in cells for resource creating systems between 1.0 and 1.6. For values below 1.5, the system will grow to a fixed point. For values between 1.4 and 1.6 the system will grow linearly at an increasing rate. For values between 1.0 and 1.4 the system follows sigmoidal growth similar to in the previous section, Figure 48. Under these conditions, the system doesn't shows any interesting characteristics as it switches to a resource creating system. The system is a gradual change from a sub-linear relationship to a linear relationship and finally to a super-linear relationship. This is the same change as shown in the scaling relationships in Section 3.2.2, using the equation:

$$y = kX^\beta \quad (7)$$

However in this system under study the switch from a sublinear *maintenance* type curve to a super-linear *exponential* growth occurs once the system is resource creating. As stated previously a noticeable linear increase in the total number of cells occurs when each DC is creating 1.4 resource per time step.

For the systems with a resource creation value above 1.6 (that is the growth is super-linear), the pattern formed can be quite unique as shown in Figure 49. With greater resource creation values, parts of the system begin to 'die', as the Receiver Cells cannot be sustained by the resource input. However this loss of cells is not shown in the Total Number of Cells in System plot (Figure 46).

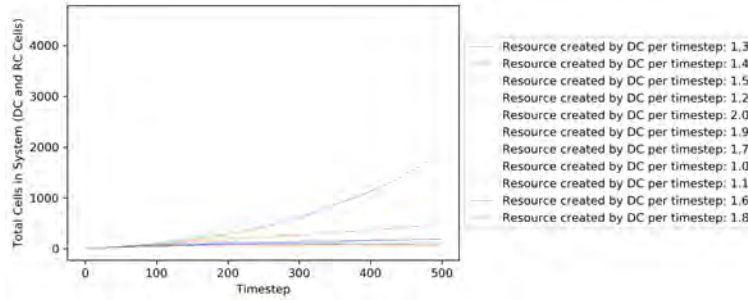


Figure 46: Total Number of Cells in System for Resource created by DC per timestep values between 1.0 and 2.2.

6.1.1 Discussion

This section looked at how a system responds to each distribution cell creating a resource each time step. This model has limited growth conditions as was the case in Section 5. In this scenario, each distribution cell can grow a maximum of one cell each time step. This found that under these conditions, the type of transition which takes place from a sub-linear growing system to a super-linear growing system is *gradual*. Even by having slight increases in the resource creation per distribution cell found a slow transition from sub-linear through linear growth to super-linear growth as shown in Figures 47 and 48. It is likely that the limited growth conditions lead to this type of transition. This system therefore does not seem to exhibit phenomena such

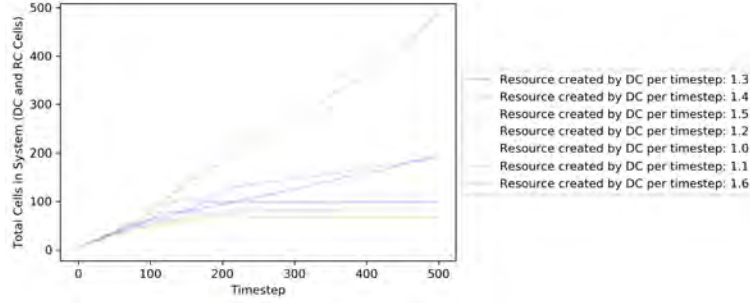


Figure 47: Total Number of Cells in System for Resource created by DC per timestep values between 1.0 and 1.6.

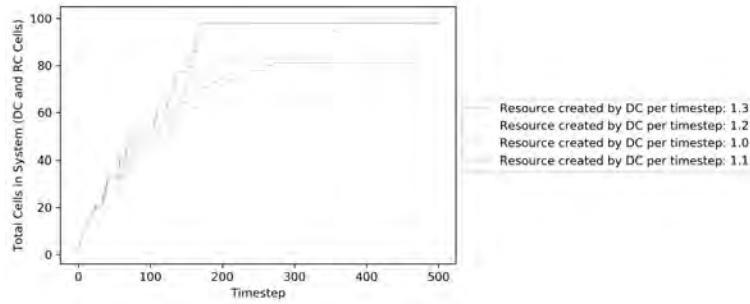


Figure 48: Total Number of Cells in System for Resource created by DC per timestep values between 1.0 and 1.3.

as increased variance or flickering as it transitions state as outlined in the research question.

6.2 Systems with Distribution Cells initialised with resource

This section explores a variation of the model presented in previous sections. The main difference between this model and those introduced previously is that this model only has distribution cells which also use the resource. This has been developed in order to test a more simplistic model in more detail on the effect of initial resource of each distribution on the transition point between resource sustaining and resource creating systems.

The model builds upon the main research question of this section exploring what characteristics a system exhibits when transitioning from a sub-linear system to a super-linear system.

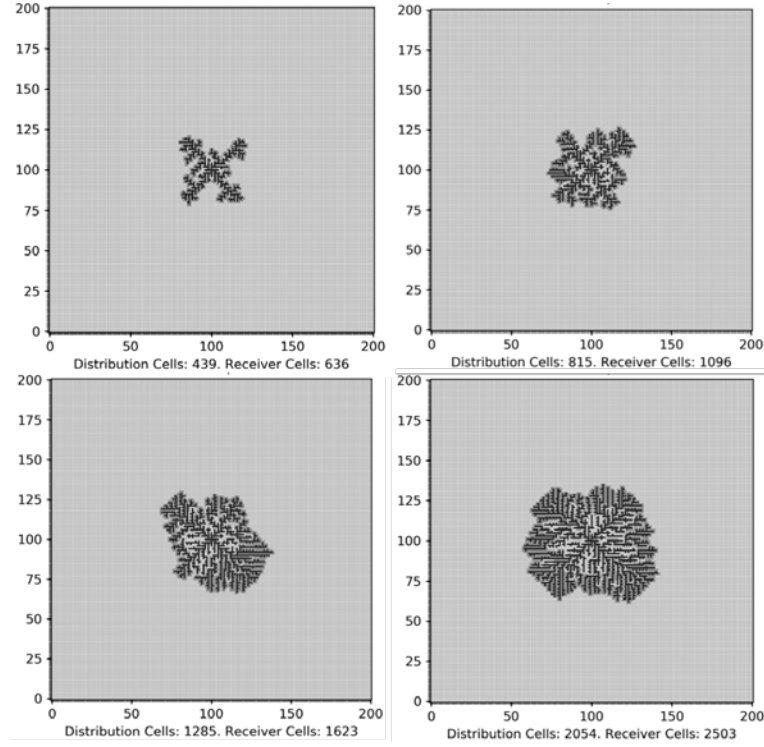


Figure 49: System morphology at 500th time step for Resource creating system. Parameters for system shown in Table 9. *Top Left*: Resource created per agent per time step - 1.7, *Top Right*: Resource created per agent per time step - 1.8, *Bottom Left*: Resource created per agent per time step - 1.9, *Bottom Right*: Resource created per agent per time step - 2.0.

The parameters of the model are shown in Table

Model Parameters	Value
Array Size	(200, 200)
Initial Coordinates	(100, 100)
Time steps	500
Resource added per time step	200
Initial Resource per DC	0
Resource created by DC per timestep	0-2.2
Resource used per Time step	1
Cost to add RC	5
Cost to add DC	5
Optimum RC Resource	10
Growth Type	Deterministic

Table 10: General Parameters in the model for Section 6.1

6.3 Maximum Growth Systems

This model uses a variation of the previous model but instead looks at a maximum growth system. The aim of this model is to see if the transition from a resource sustaining system to a resource creating one will show features similar to those outlined in the research question of this section, for example flickering or increase in variance.

This model is a slight variation of those previously introduced. The main difference being that this model only has distribution cells which also use the resource, so no receiver cells are present. This has been developed in order to test a more simplistic model in more detail on the effect of initial resource of each distribution on the transition point between resource sustaining and resource creating systems. This also allows the system studied to grow at a maximum rate as there is no need to add receiver cells. It studies what can be described as an extreme system which is growing at it's maximum possible rate with any resource available being used at all times.

This model uses the same space optimising algorithm as used in previous sections, with the exception that the space isn't filled with receiver cells and can form both deterministic and stochastic systems as shown in Figures 50 and 51. The colour on the figures denotes the time step which each cell was created on.

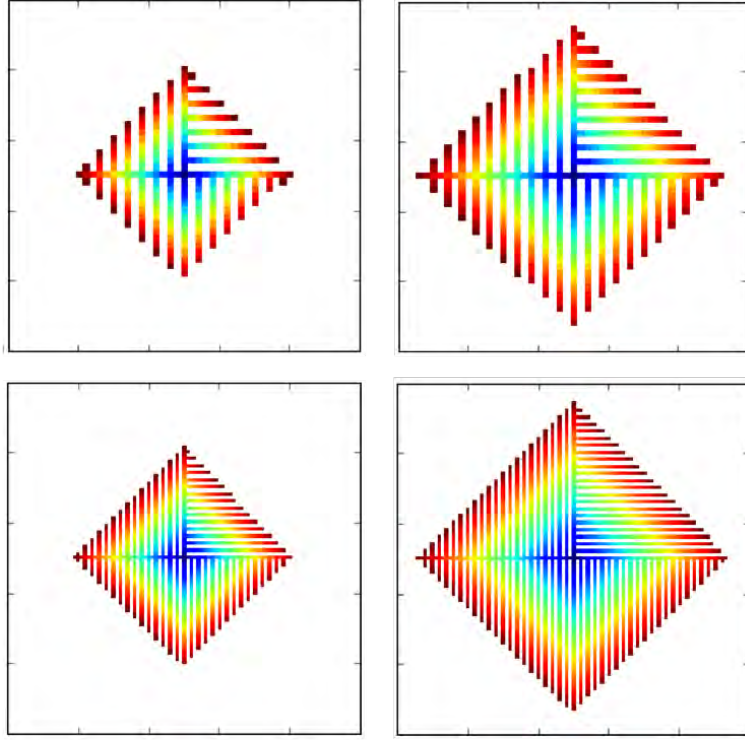


Figure 50: Systems of 250, 500, 1000 and 2000 Distribution Cells respectively (left to right, row by row), following deterministic rules.

The resource distribution algorithm is almost identical to that used in Section 5. It undertakes the following tasks:

- Add a resource per time step.
- Distribute the resource through the system based on the needs of the cells in the system.
- Expansion of the system only occurs if resource is available.
- Resource is consumed by each cell each time step.
- Cells are removed from the system if their resource drops to zero.

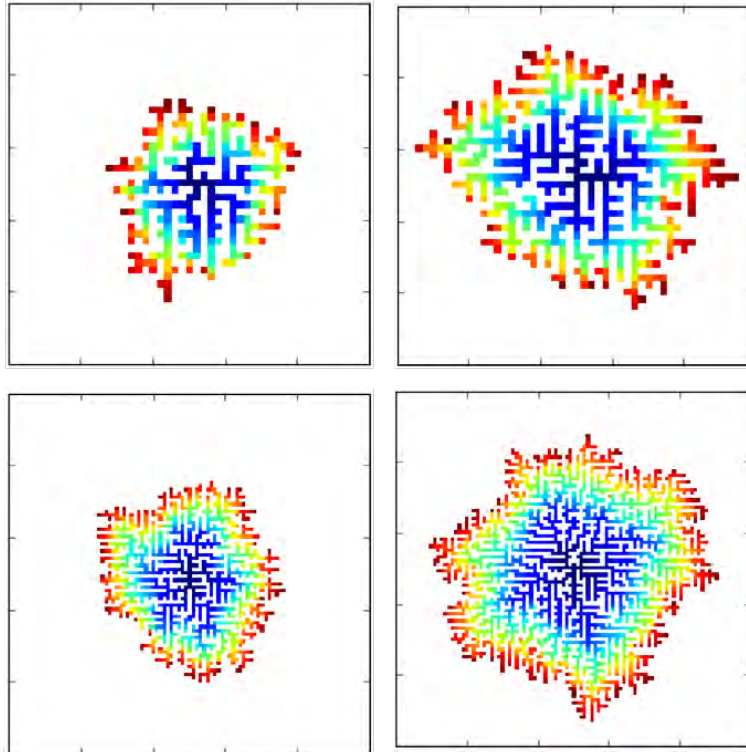


Figure 51: Systems of 250, 500, 1000 and 2000 Distribution Cells respectively (left to right, row by row), following stochastic rules with respect to the cell selected to expand.

- The system can either grow in a *normal* fashion whereby each cell can add a maximum of one per time step or in a *maximum growth* scenario whereby it adds as many cells as it can as long as the resource is available.

As this section is interested in exploring maximum growth systems, these are the only systems which have been tested. The first simulation tested is non-resource creating using the parameters in Table 11. This leads to a system which grows and reaches a stable optimum, Figure 52. The growth of the system is very quick, the graph shows a steep incline with no hockey stick growth. This is due to the fact that in the maximum growth model, all cells can add new cells, including the cells created in the particular time step.

Model Parameters	Value
Array Size	(100, 100)
Initial Coordinates	(50, 50)
Time steps	2000
Resource added per time step	500
Initial Resource per DC	0
Resource used per Time step	1
Cost to add DC	50
Optimum RC Resource	10
Growth Type	Deterministic
Maximum Growth?	Yes

Table 11: General Parameters in the model for Section 6.3

The simulation is now explored with DCs having an initial resource when they are added to the system. The parameters which have been used in the simulations are shown in Table 12. A total of 31 simulations were carried out with initial DC resource values between 50 and 61. Plots showing all the simulations is shown in Figures 53 and 54. Similar to resource creating systems in Sections 6 and 6.2, the simulations show a change from growth to a stable point and then switch to exponential growth at a certain point. However, in these maximum growth systems the switch from sigmoidal stable maintenance to continual growth is much more pronounced. Figure 53 shows this increase in variance with increased initial resource per DC. In fact it appears that for this case the switch follows the same (or similar) switch as

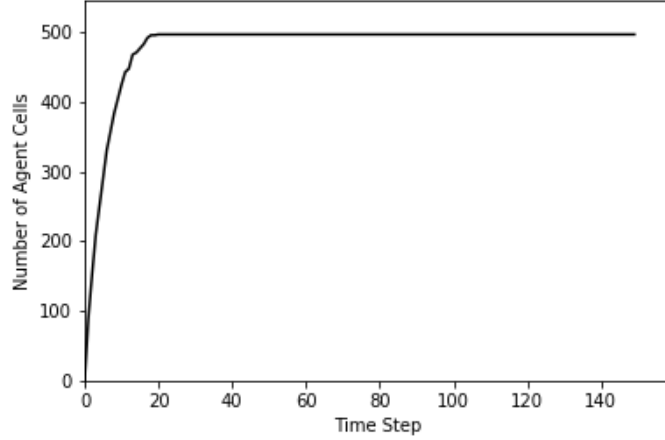


Figure 52: System with parameters from Table 11.

the iterative map equation in Section 3.9.3. The equation is reprinted below 8 and shown in Figure 55.

$$X_{t+1} = rX_t(1 - X_t) \quad (8)$$

where,

$0 < X_{t=0} < 1$ and the constant $0 < r < 4$.

Figure 54 seems to show that the increase in variation before the system switches to divergence has a pattern. It shows that at this boundary of increased variance, if it is broken down and analysed, it is possible to see a system switching from monotonic growth, to bi-stability, to limit cycles and chaos prior to divergent growth. This is the same as what is observed in Figure 55. It is uncertain why this behaviour is only observed in maximum growth systems. But is likely to be due to the maximum growth system being very efficient at allocating resources which leads to more obvious emergent properties.

Model Parameters	Value
Array Size	(100, 100)
Initial Coordinates	(50, 50)
Time steps	2000
Resource added per time step	400
Initial Resource per DC	50-61
Resource used per Time step	1
Cost to add DC	50
Optimum RC Resource	10
Growth Type	Deterministic
Maximum Growth?	Yes

Table 12: General Parameters in the model for Section 6.3

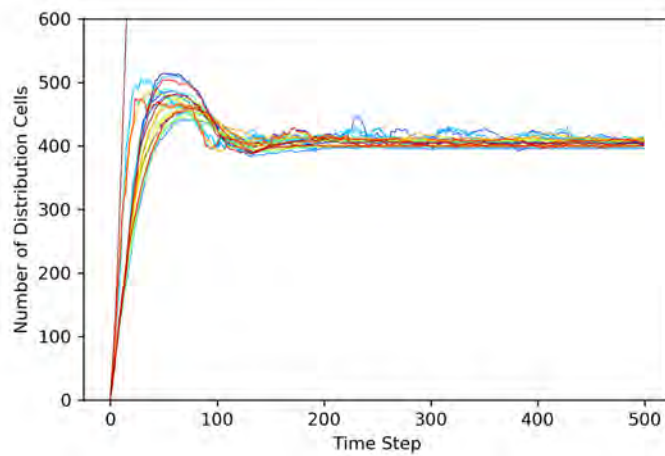


Figure 53: System with parameters from Table 12.

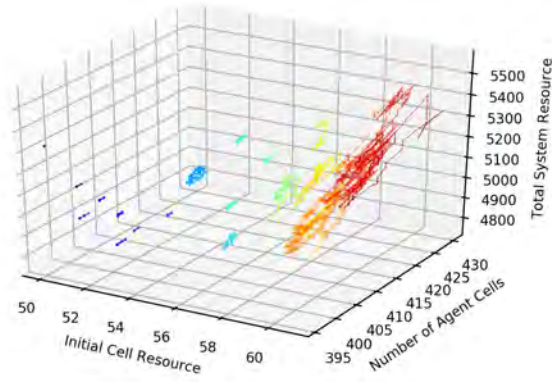


Figure 54: System with parameters from Table 12.

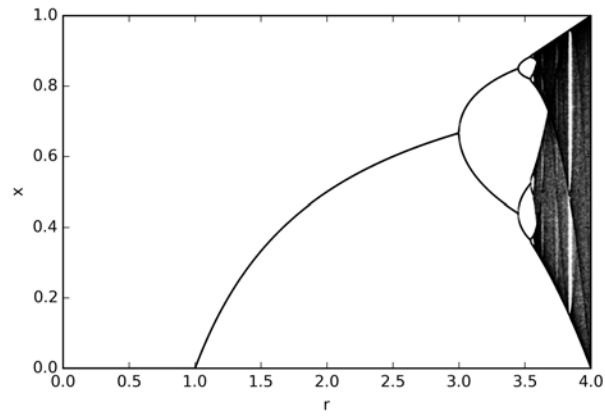


Figure 55: Bifurcation plot showing the different values of x when the constant r is changed in equation 8 .

7 Resource Inequality in Multi-Agent Systems

7.1 Introduction

This section explores another variation of the previous models, but with agent heterogeneity. The general topic of this section is to explore how inequality can develop and change in multi-agent systems which share a resource. There is a lot of empirical research which looks at changing levels of inequality both internationally between nations and within nations Roser and Ortiz-Ospina (2019). The right level of inequality for each nation and/or group of people is subjective and views are often polarized within different countries/continents and income levels about what the right amount is The World Bank (2016), Figure 56. This is also an important area for research given the increase in advanced technology which could automate many tasks in society which could in turn lead to increased inequality Dellot et al. (2019).

Smith and Choi (2006) formalises these differing opinions on inequality into two categories - the benefits which hierarchy brings to all which is termed *mutualism* and the costs being exploitation or coercion by one segment of society against the interests of the remaining members (elites exploiting commoners or slaves). Smith and Choi (2006) goes on to suggest that these scenarios are useful when best viewed as a continuum rather than a dichotomy, which is what is generally seen in the data in Figure 56.

The United Kingdom has one of the most comprehensive datasets on inequality through time Roser and Ortiz-Ospina (2019). This shows a general trend of a Gini Index between 50-60 from the 1680s to 1911 and from post World War 2 to the present day between 30-40, Figure 57. There has also been a lot of research looking at the share of income amongst different parts of the population. Typically these datasets look at the share of income the top 1% earn compared to the rest of the population, Figure 58. The two figures show a disparity between english speaking developed countries and non-english speaking developed countries. Over the twentieth century english-speaking countries have shown a U-shaped fall from 1920 to 1980m followed by a rise to the current day, however non-english speaking countries have shown a gradual decline over the course of the twentieth century. Obviously these are just datasets and do not provide any information on the underlying reasons for these trends. The underlying reasons for these changes may be due to cultural reasons, networking between countries, and

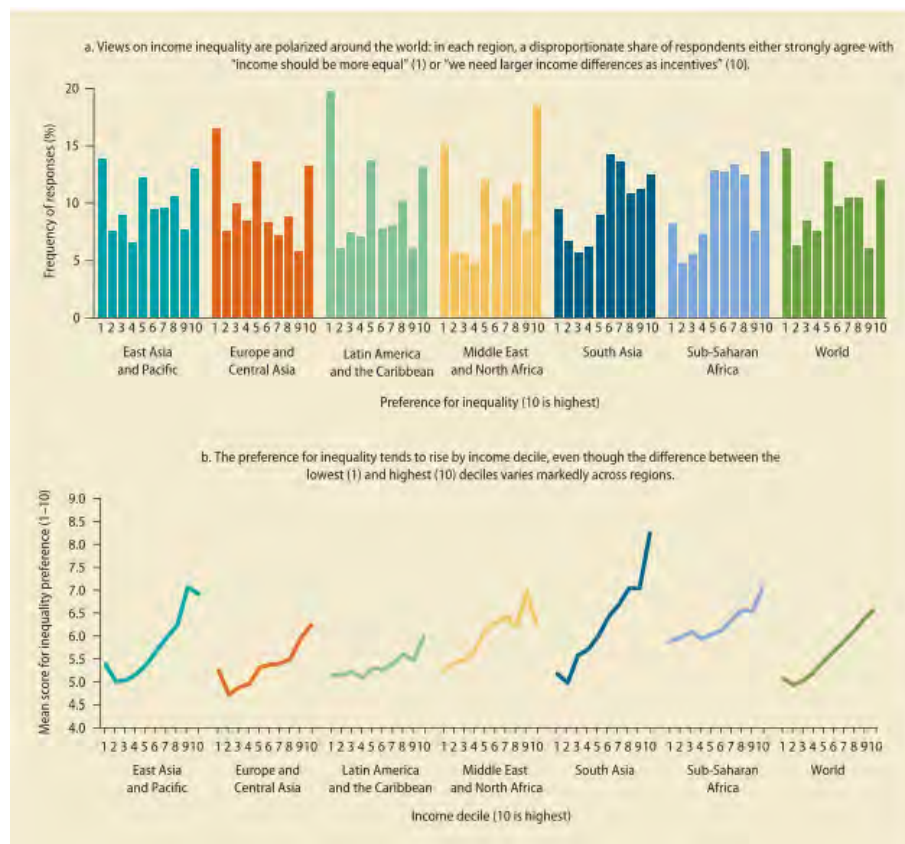


Figure 56: Differing opinions on the preferred level of inequality in a nation. Taken from Figure 1.7 in The World Bank (2016) .

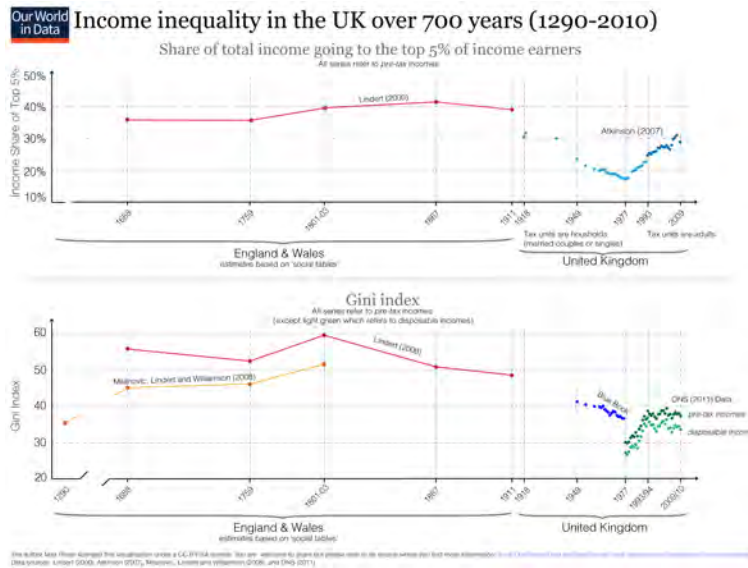


Figure 57: Income Inequality in the United Kingdom over 700 years. Taken from <https://ourworldindata.org/income-inequality> .

global conflict for instance the second world war.

Gini Coefficient (Normalised values of the Gini Index between) data from The World Bank has been collated for the years 1980-2015 for 161 countries and territories, 59. The full dataset can be found at <https://data.worldbank.org/indicator/SI.POV.GINI>. This is the largest amount of data available from this particular source. Whilst the dataset shows a lot of noise, it importantly shows general boundaries all *stable* societies appear to have to exist in. The mean gini coefficient for all the data is 0.39. The maximum gini coefficient is 0.66 and the minimum is 0.16. In general one standard deviation above and below the mean are between 0.3 and 0.55.

The data introduced so far provides macro-scale aggregate information on income inequality of large-scale societies through time. It does not provide information on the underlying interactions, behaviours or societal structures that lead to inequality changing through time. It is also of interest to understand whether commonalities can be drawn between modern large-scale

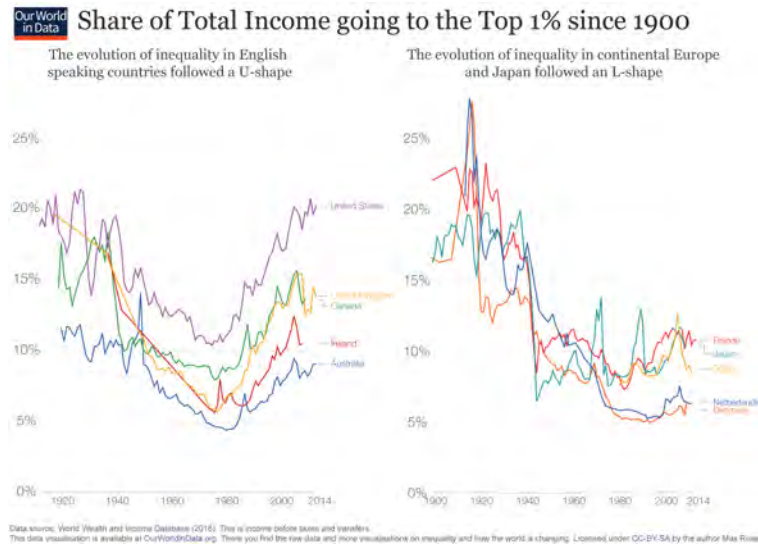


Figure 58: Share of Total Income going to the Top 1% since 1900. Taken from <https://ourworldindata.org/income-inequality>.

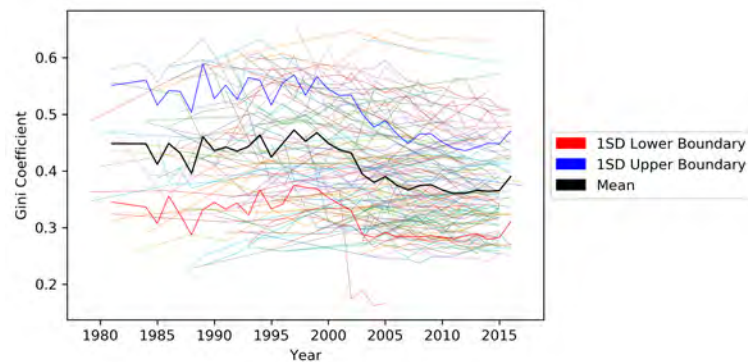


Figure 59: Collated data on Gini Coefficient for 161 countries and territories between 1980-2015. The mean and one standard deviation are shown in bold.

societies and small-scale traditional societies in terms of inequality. Section 3.12 gives examples of a number of traditional societal resource distribution systems and attempts to discuss the commonalities and differences between traditional and modern distribution systems and more abstract distribution systems in general.

Borgerhoff Mulder et al. (2009) offers a comparison of intergenerational inequality between different traditional small-scale societies and modern large-scale societies in terms of three types of wealth - material, relational and embodiment. They conclude that traditional horticultural and hunter-gather societies have lower material intergenerational inequality than pastoral and agricultural societies, which is principally due to material wealth being more easily transferred in pastoral and agricultural societies. Borgerhoff Mulder et al. (2009) also offer a comparison between traditional small-scale societies and modern large-scale societies finding that horticultural and hunter-gatherer societies have similar intergenerational inequality to Nordic social democratic countries such as Denmark, Sweden and Norway whereas pastoral and agricultural societies have similar intergenerational inequality to more unequal nations such as the United States and Italy.

As mentioned at the start of the section, this model uses a variation of the models in the previous sections with additional agent heterogeneity with the aim of exploring how inequality can develop in such a system. Smith and Choi (2006) introduces a number of scenarios for exploring the emergence of inequality in small-scale societies based on game theory and evolutionary ecology. These models look at the interaction between different agents which have different roles and how their interaction leads to stable equilibria. It is looking at the very origin of hierarchy and inequality in small-scale societies which can be readily applicable to understanding how people interact in groups in the modern world such as in the workplace. ? provides a model to look at the emergence of cooperation within what can be described as a spatial hierarchy, an indigenous irrigation system in Bali. This puts agents into a fixed physical geography which creates a hierarchy, but the social institutions in the model lead to an equitable society evolving. The model includes interactions between adjacent agents in the form of imitation. For example a farmer will copy an adjacent agents irrigation practise if it is more successful than theirs. The model presented in this section follows such a model.

This model is developed and influenced by a lot of literature from social and network science. Section 3.8 gives a brief overview of research looking

at social influence in both networks and grids. The simple model introduced shows how patterns can form on a random grid based on how each cell will change their state based on their neighbourhood of cells. Variations of this pattern are shown to occur in many biological systems, most visibly on the skin animals such as lizards Edelstein-keshet (2017). More relevant to this research is how the neighbourhood of a particular cell can affect it's behaviour. Over the course of the 20th century there has been a lot of research on social networks which is thought to aid in linking micro-social behaviour with macro-social trends Granovetter (1973). The strength of the tie between two actors is thought to have importance Granovetter (1973); Marlow et al. (2012). The strength of ties presents a dichotomy, with a large amount of research showing stronger ties (peers and friendships) lead to greater influence on behaviour, such as the uptake of smoking in social groups Mercken et al. (2010), however because of the strong ties, the behaviour of the individuals is likely to be similar. Weak ties have a lower influence on behaviour but the behaviour between the individuals is likely to be more contrasting, possibly leading to tipping points or regime shifts within groups when a change occurs.

7.2 Research Questions

Section 7.1 introduces the topic on income inequality in societies, both from empirical data and simple simulation models. The aim of this section is to build a model small-scale society which can then be tested to understand how minor changes in the model lead to changes in the income inequality at a global scale within the model. The following sections build the model in steps with a description of changes added to the model and the affect these have on results.

7.3 The Model

The previous models were built to optimise space and the maximum number of receiver cells to distribution cells (Section 4), apply a resource distribution system to such models (Section 5) and finally allow the system to create resource (Section 6). As stated in the Research questions, this model aims to look at how inequality can develop in a system. This is done in this model by allowing heterogeneity, that is each distribution cell has a different resource need.

Firstly the model uses the same space optimising algorithm as in Section

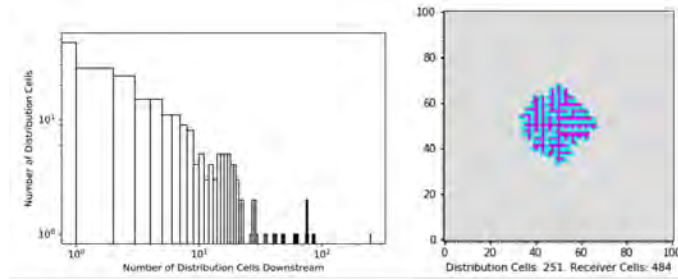


Figure 60: *Left* - Histogram showing the power law number of distribution cells downstream. *Right* - The system analysed.

4. The number of distribution cells each has downstream is found to be a power law, Figure 60.

7.3.1 Agent Individual Preference

In order to model agent heterogeneity, a statistical distribution is used. Figure 61 shows a histogram of randomly selected points from a normal distribution. The mean is 1.0 and the standard deviation is 0.1. One value is randomly selected from this dataset, the *agent individual preference* which is assigned to each agent added to the simulation. The value is used to calculate each agents wealth need by multiplying the agent preference by their own or their neighbours wealth dependent on the simulation. This particular simulation is purely random, with no *path dependence* Mahoney (2000). Figure 62 shows the result of a system with 40 agents with random sampling from the same distribution each time step. As can be expected, it produces a noisy dataset which echoes the distribution parameters input into the model.

Path dependence can be added quite simply by using the value of the agent in the previous time step as the mean for the normal distribution calculation of the agent to be added. Figure 63 shows the changing agent preference with path dependence. A histogram of the spread of the data at the final path data point is shown in Figure 64. As can be shown, the distribution has a greater spread due to each agent being influenced by a normal distribution and its path through time. The result has commonalities with the well-known concept in computer science - *a random walk*.

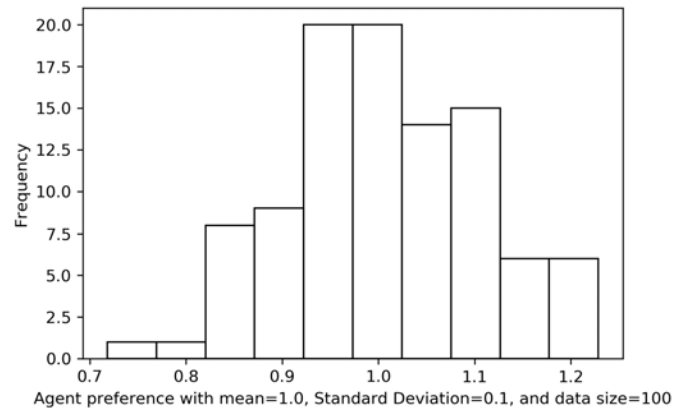


Figure 61: Normal distribution taken from random data points with each value being mutually exclusive.

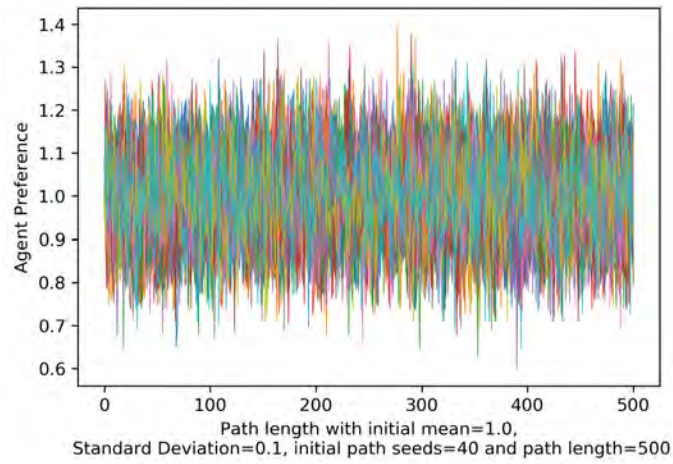


Figure 62: Normal distribution taken from random data points with each value being mutually exclusive.

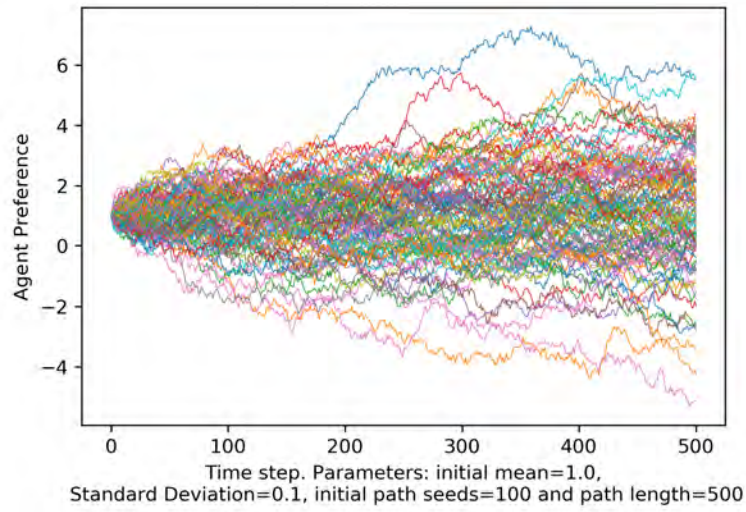


Figure 63: 100 simulations for agents with path dependence on their individual past state.

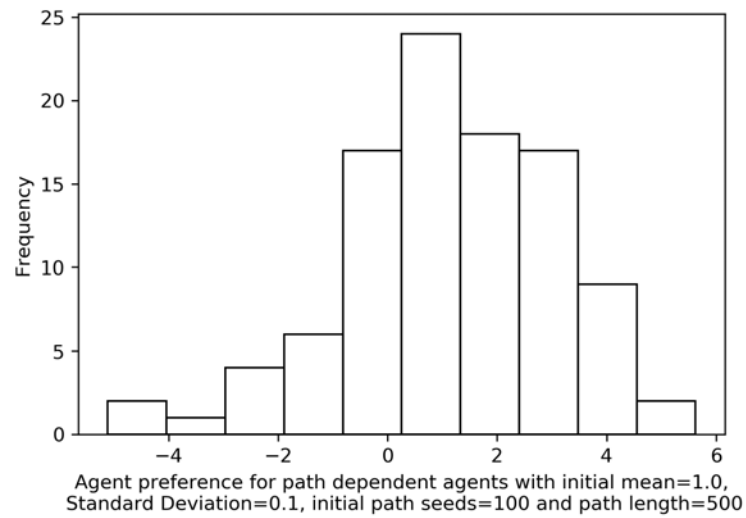


Figure 64: Final path data point for all agents in Figure 63.

7.3.2 Agent Network Influence

Agent preference can also be derived from the influence of other agents within a particular agent's network. Obviously there are many ways in which agents may interact with each other, this is a topic which has its own field of study *Network Science*. In this case an example is shown of agents being influenced by a maximum of two other agents and themselves. Each agent uses the same initial parameters for a normal distribution as previously (mean=1.0 and standard deviation=0.1). And each time step the agent preference is calculated by finding the mean of its own previous preference in addition to the nearest preference higher and lower. Obviously there must be one agent with the highest preference and one with the lowest and these will only depend on their previous preference and the nearest higher or lower. Figure 65 shows the result of this simulation for 100 agents over 500 time steps. The distribution of all the agent's preferences for the final time step is shown in Figure 66. The figure shows a stark difference in behaviour when compared to an agent only dependent on its own preference - Figure 63. The system seems to behave more as one with any large shifts moving the whole system and not isolated to one particular agent. This is not surprising given that by networking each agent to its nearest in terms of preference, it essentially networks them all together. The behaviour of the system can be thought of as being similar to *Flocking*, which has been modelled in simulations of 'boids' Reynolds (1987).

7.3.3 The Formation of Groups

Sections 7.3.1 and 7.3.2 explore two extremes or fixed points of a phase space of interaction, one being agents which do not interact with one another and the other being agents with strong interconnectivity. Figure 62 explores the other extreme of no dependence whatsoever. In between these extremes are likely to be systems of small group formation. An attempt at looking at this has been made by adding an influence threshold to each agent. In this scenario, agents will only depend on their previous preference for their future preference, unless other agents are within their preference threshold. The result of this simulation shows a system which under the same parameters can produce a lot of diversity, such as normally distributed data, bimodal distributed data which can change through time. Figures 67 and 68 shows an example of network influence where there appears to be two dominant groups. The emergence of groups is more obvious in simulations of a smaller number of agents, typically with an influence threshold slightly higher than

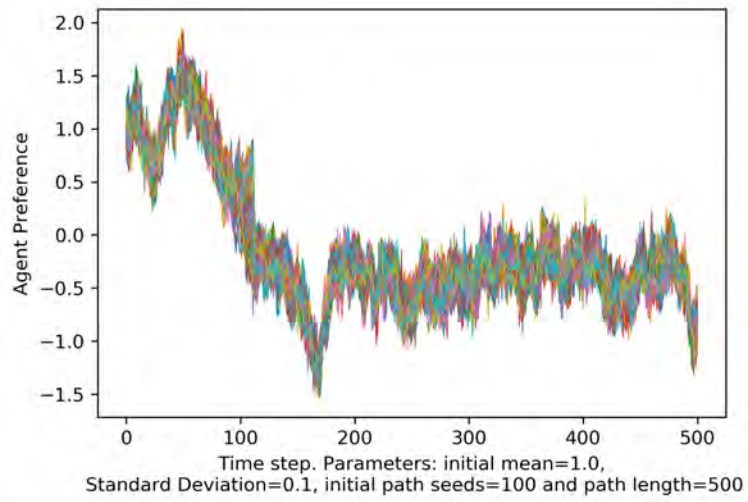


Figure 65: 100 agents with nearest preference networking.

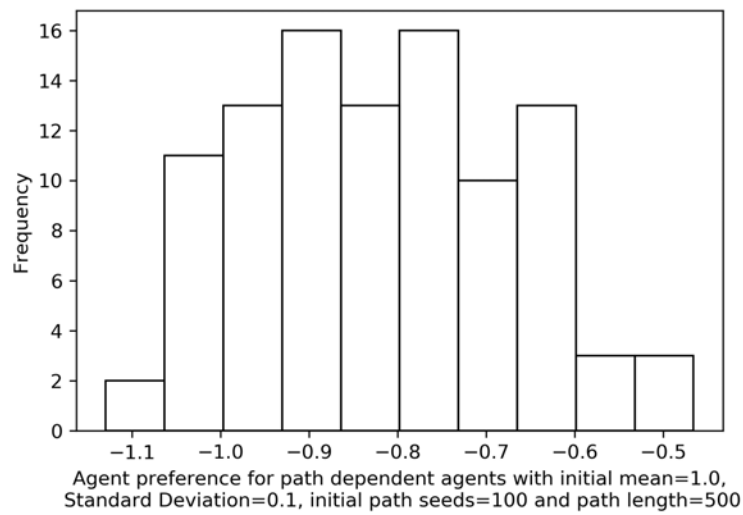


Figure 66: Final path data point for all agents in Figure 65.

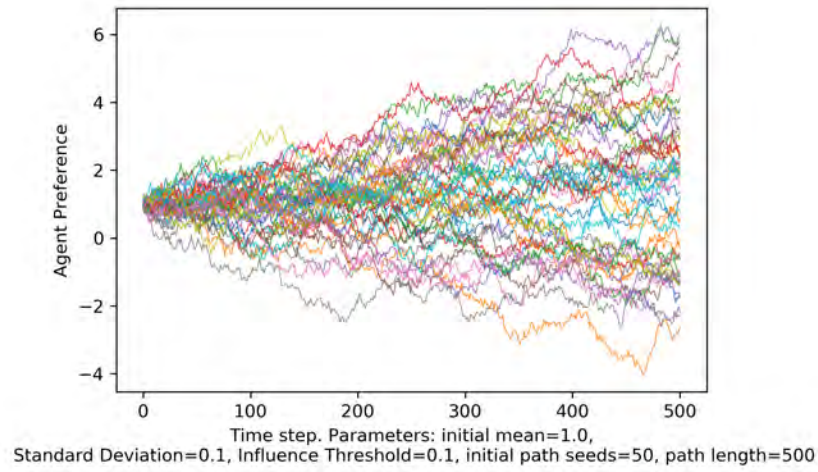


Figure 67: 100 agents with an influence threshold.

the standard deviation (eg, $SD=0.1$ and influence threshold = 0.15), Figures 69 and 70.

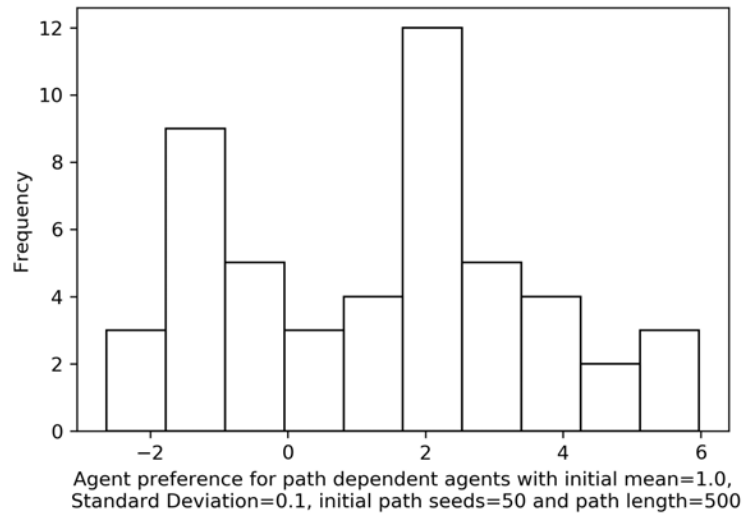


Figure 68: Final path data point for all agents in Figure 67.

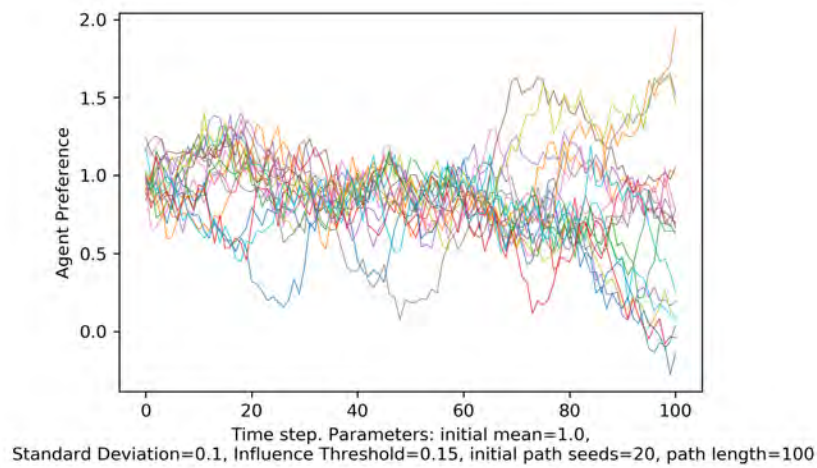


Figure 69: 20 agents with an influence threshold.

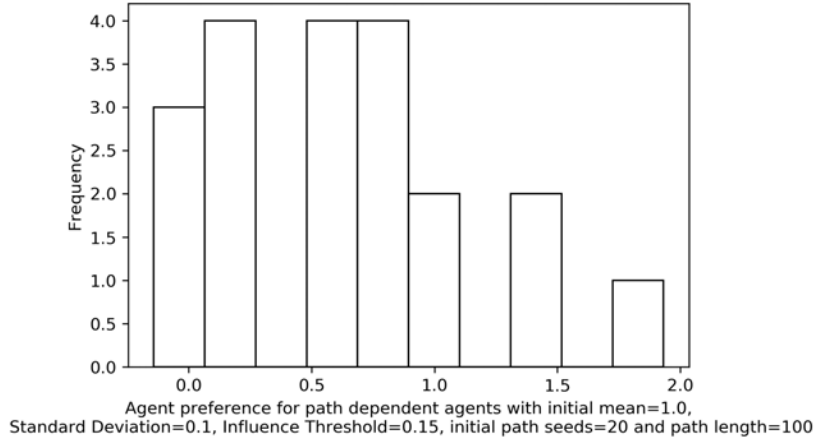


Figure 70: Final path data point for all agents in Figure 69.

There is an infinite number of possibilities to explore the different ways in which agents interact to form their preferences even with this abstract method. The following sections explore these different ways of agents interacting within the resource distribution model. The agent preference affects each agent's *need* and is multiplied by the agent's wealth or the agents neighbourhood wealth (dependent on the model) and added to the need already required by the agent for resource, adding RCs and DCs.

7.4 Results

The following sections explore the effect of different models of interaction between agents on the overall resource distribution system through time.

All the simulations use the same basic parameters as shown in Table 13.

7.4.1 Agent Individual Preference

The Agent Individual Preference model is based on the model of agent interaction as set out in Section 7.3.1. In this model each agent isn't influenced by any agent in it's network. Instead, each agent's initial preference is taken from a normal distribution of the previous agent with the mean being the previous agent's preference and the standard deviation taken as 0.1. Figure 63 shows the effect of individual path preference overtime without any distribution of resource.

Model Parameters	Value
Array Size	(50, 50)
Initial Coordinates	(25, 25)
Time steps	300
Resource added per time step	300
Initial Resource per DC	0
Initial Resource per RC	0
Resource used per RC per Time step	1
Cost to add DC	5
Cost to add RC	5
Optimum RC Resource	3
Growth Type	Deterministic
Maximum Growth?	No
Mortal Cells?	Yes, DCs are removed if their RCs have no resource for 10 time steps
Agent preference?	Yes, but dependent on the model.

Table 13: General Parameters in the model for Section 7.4

Forty simulations for the agent individual preference model were run. As can be seen the majority of the models follow the pattern of an initial increase in both DCs and RCs at the start of the simulation followed by a quick contraction and reduction in the number of cells, Figure 71. A plot of the Gini Coefficient for all the models shows on average the Gini Coefficient is very high (after the initial growth), with a mean of 0.82 , Figure 72.

The reason for the type of growth observed is likely to be due to initially each agent not having much resource, which reduces the agent need to normal levels (need for expansion and maintenance), and after a number of time steps the agents begin to gain wealth depending on their individual preferences. In turn this reduces the amount of resource passed down through the system which leads to its contraction and reduction in number of cells. So there is a positive feedback of resource increase per agent or at least for agents with a preference for increased resource. The agents which persist until the end of the simulation, are the agents closest to the source, and it appears that their preference does not matter, all that matters is their

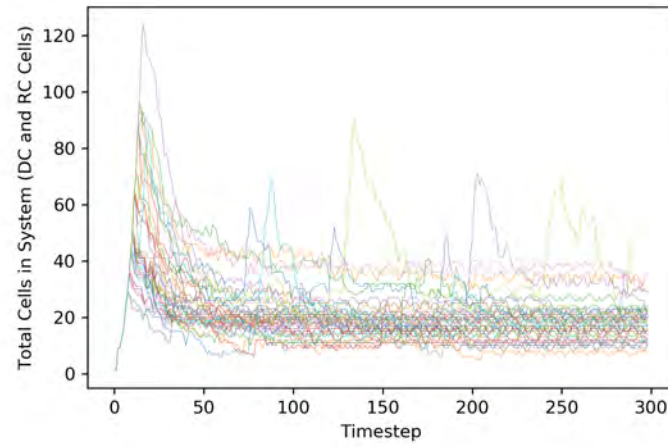


Figure 71: Forty simulation runs showing the total resource in the data number of cells in the system.

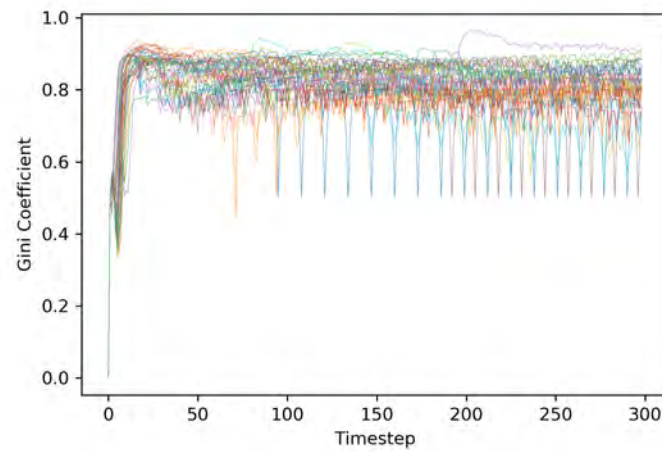


Figure 72: Forty simulation runs showing the Gini Coefficient through time.

position in the system.

There are a few anomalies in this model where the simulations show large fluctuations in the number of agents. An example is shown in Figure 73. Analysis of the mean individual agent preference and total preference shows correlation between the drop in number of cells and drop in agent preference, Figure 74. A plot showing all the agent preferences over the course of the simulation is shown in Figure 75. The plot shows a number of agents which persist over the course of the simulation, these are likely to be the agents closer to the source. Over the course of the simulation a number of offshoots occur from these main agents, but not many of these persist. As can be seen the preference of the persistent agents varies widely over time indicating that the preference isn't important for their survival. The path of these agents is also similar to those seen in Figure 63 which is to be expected given the same interaction model is used. An animation of the model can be found at <https://youtu.be/yXU0tqeIwTA>. The animation shows the sudden growth at time step 130 is synchronised, it occurs across the whole systems, whereas the growth at time step 250 is asynchronous, it occurs in one part of the system. Looking at the agent preference data in Figure 75, it shows a general trend before time step 130 in reducing the agent preference across the system. This does not occur before the sudden growth at time step 250. It is therefore likely, that although there is no influence between the agents, by chance all the agents reduced their preference at the same time leading to more resource being passed down through the system at this moment.

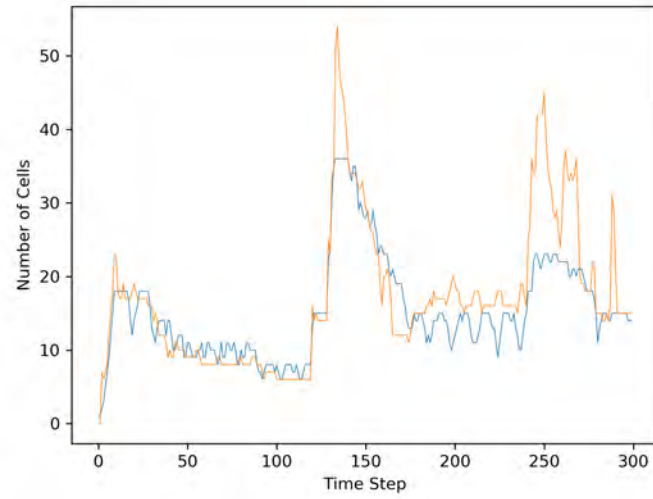


Figure 73: Number of Distribution Cells (blue) and Receiver Cells (orange) for the agent individual preference model showing fluctuating behaviour.

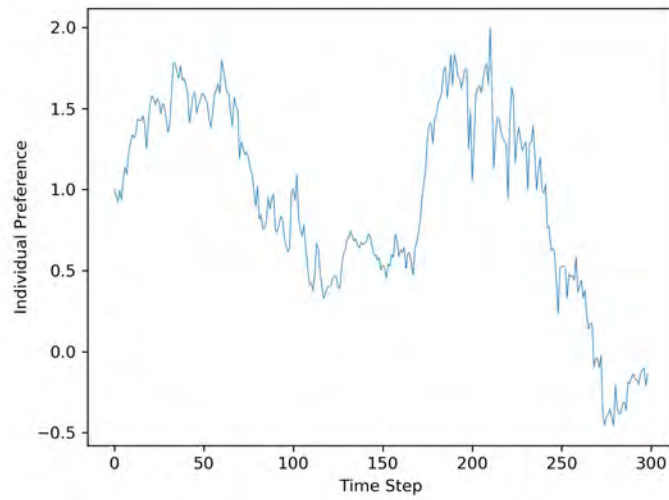


Figure 74: Mean individual preference of the system through time for the simulation in Figure 73.

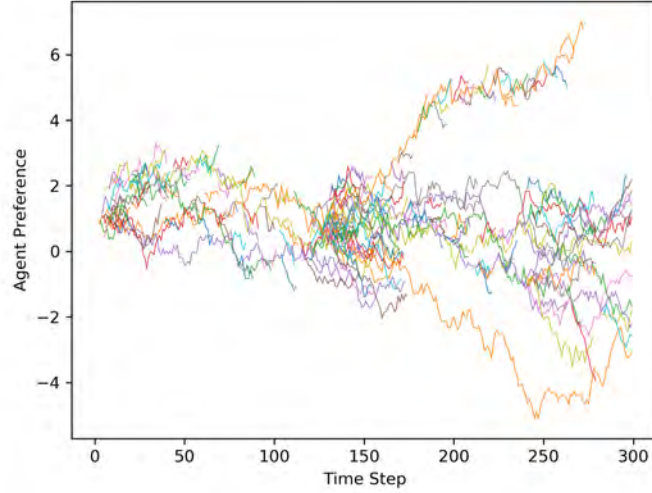


Figure 75: Individual Preference data for the system through time for the simulation in Figure 73.

7.4.2 Agent Network Influence

The Agent Network Influence model introduced in Section 7.3.2 shows how a system of agents which influence each other can behave. With each agent connected to those with the nearest higher and lower preference, the system becomes closely connected with little noise. When exploring such interactions on a spatial model such as the resource distribution model there are many different possibilities of agent interaction which can be explored, for example the distance with which agents can connect with one another. In order to explore how such behaviour can affect the whole system many simulation runs using different parameters were undertaken. The general parameters are the same as shown in Table 13. Details on the specific simulations explored are shown in Table 14. The *Number of Neighbours* is the maximum number of neighbours each agent can be influenced by. The influence of neighbours affects both the individual agent preference and the individual agent need. To find the agent preference, the mean of the neighbour's preference is calculated and then calculated from a normal distribution. The individual agent need is calculated as the mean wealth of the neighbours multiplied by the individual agent preference. This way of calculating has been done to take into account the spatial aspect of the model. The num-

ber of neighbours is also affected by the *Neighbourhood Size* which is the distance in each direction which agent is can influence a specific agent; the *agent's neighbourhood size*.

Model Parameters	Value
Number of Neighbours	1 - 9
Neighbourhood Size	5 and 10
Number of Repeat Simulations	20

Table 14: Parameters changed for each simulation for Section 7.4.2

Using the parameters in Table 14 generated 360 unique simulation models. This created over 1GB of data including graphs, animations and text files. This was subsequently analysed which is reported in the rest of this section.

Figure 76 shows the total number of distribution cells per time step for all 360 simulations. A histogram of the total number of distribution cells at time step 300 is shown in Figure 77. Across all the simulations a trend of smaller systems is observed - the majority of simulations have around 50 distribution cells. This is likely due to agents increasing their individual wealth, decreasing the size which the system can expand to. There is also a small amount of simulations with over 150 distribution cells. The Gini Coefficient has also been plotted for all 360 simulations, Figure 78. The mean Gini Coefficient for all simulations over all time steps is 0.64. A histogram of the Gini Coefficient at time step 300 is shown in Figure 79. A general trend of decreasing Gini Coefficient with increased number of neighbours per agent is found, Figure 82 and increased system size with increased number of neighbours, Figure 83.

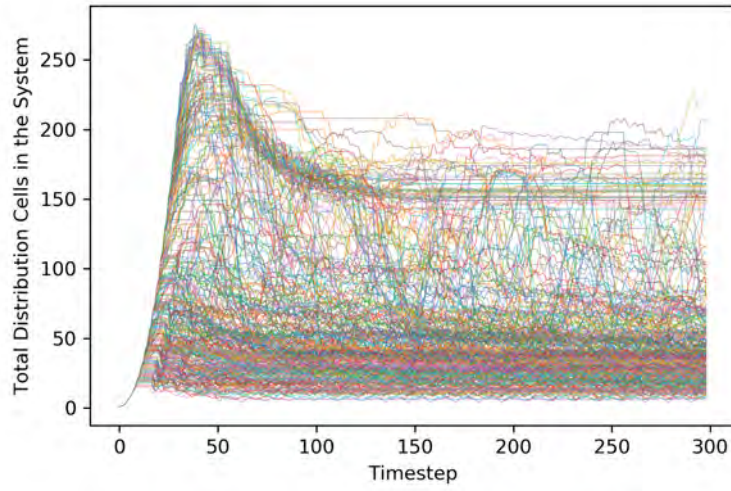


Figure 76: Number of Distribution Cells per time step for all 360 simulation in Table 14.

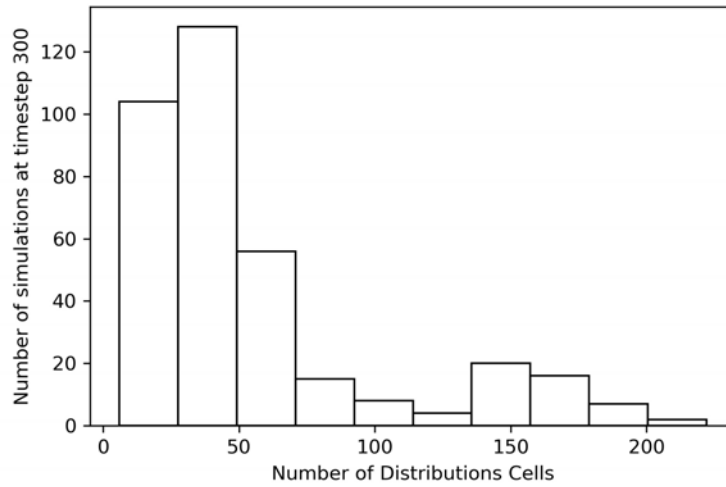


Figure 77: Number of Distribution Cells at time step 300 for all 360 simulation in Table 14.

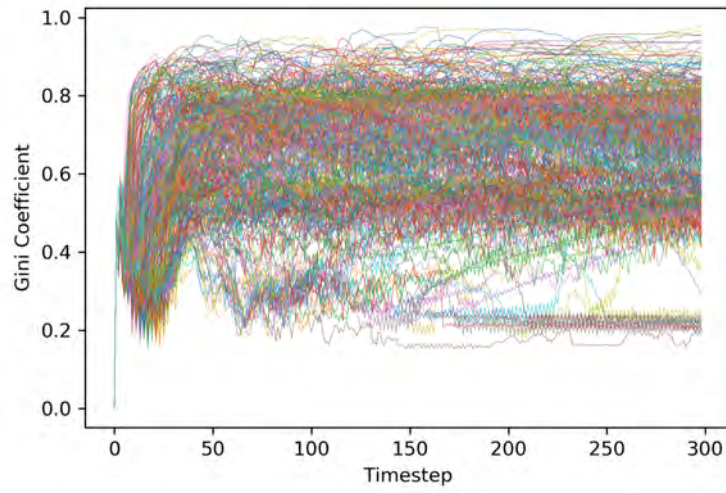


Figure 78: Gini Coefficient per time step for all 360 simulation in Table 14.

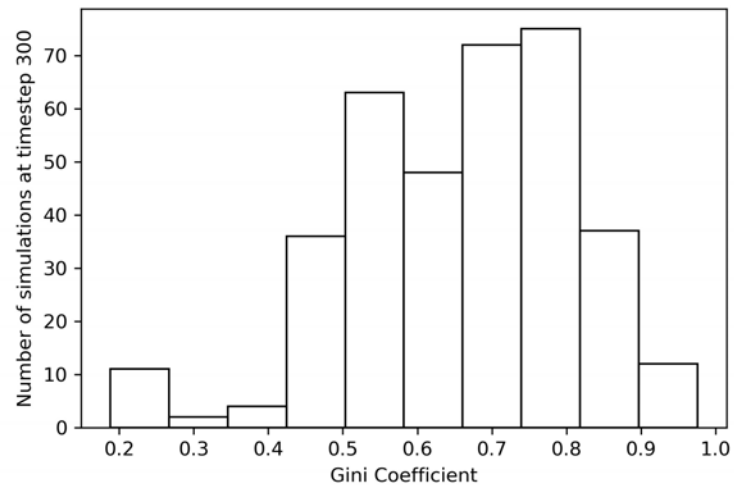


Figure 79: Gini Coefficient per time step for all 360 simulation in Table 14.

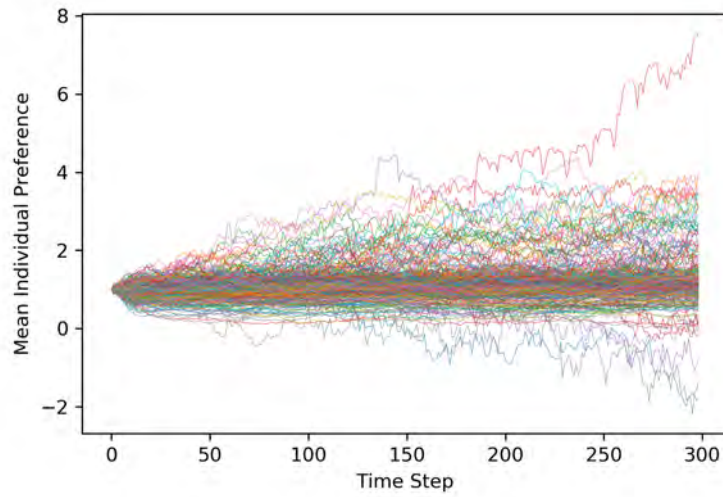


Figure 80: Mean agent preference per time step for all 360 simulation in Table 14.

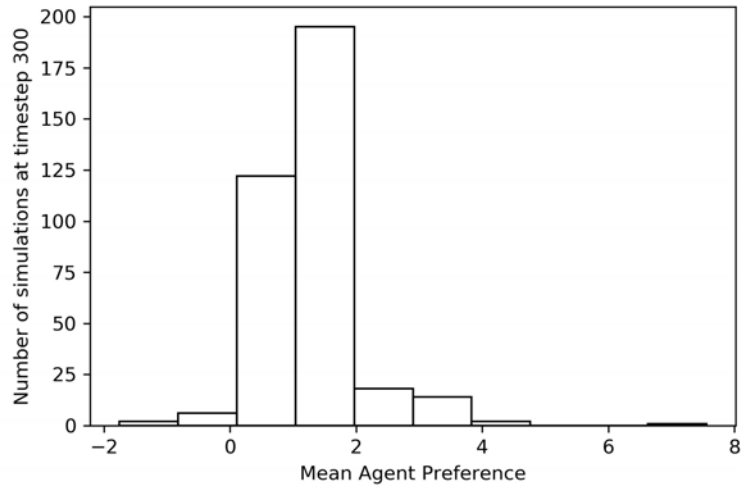


Figure 81: Histogram of mean agent preference for final time step for all 360 simulation in Table 14.

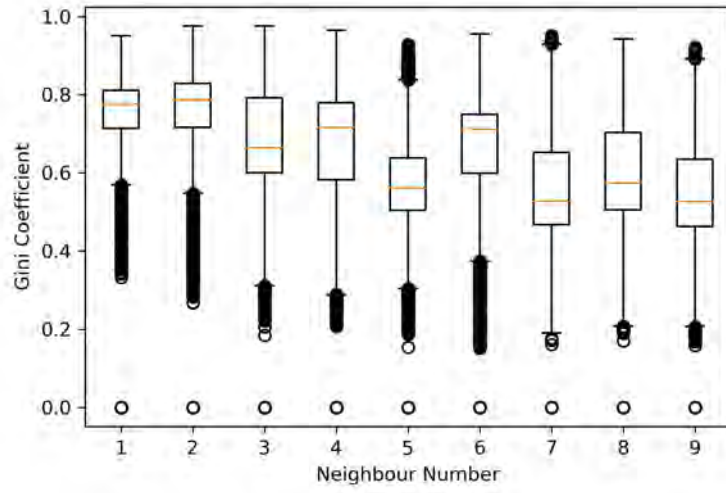


Figure 82: Box Plots for all 360 simulation showing the changing Gini Coefficient for Agent Neighbourhood Number in Table 14.

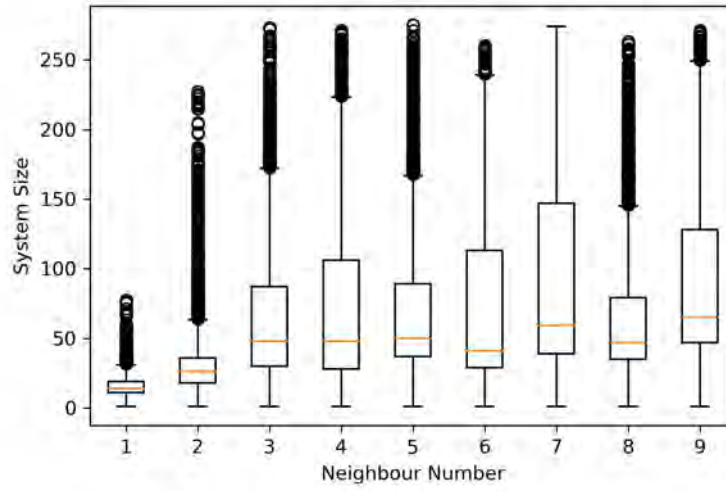


Figure 83: Box Plots for all 360 simulation showing the changing system size for Agent Neighbourhood Number in Table 14.

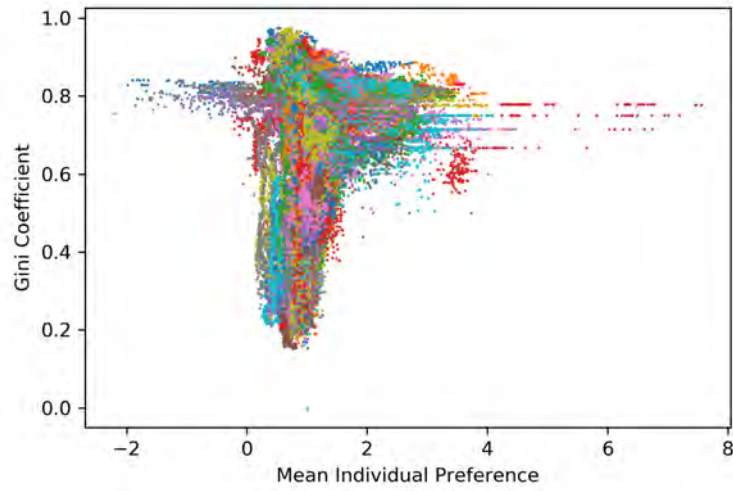


Figure 84: Gini Coefficient verses mean individual preference for all 360 simulation in Table 14.

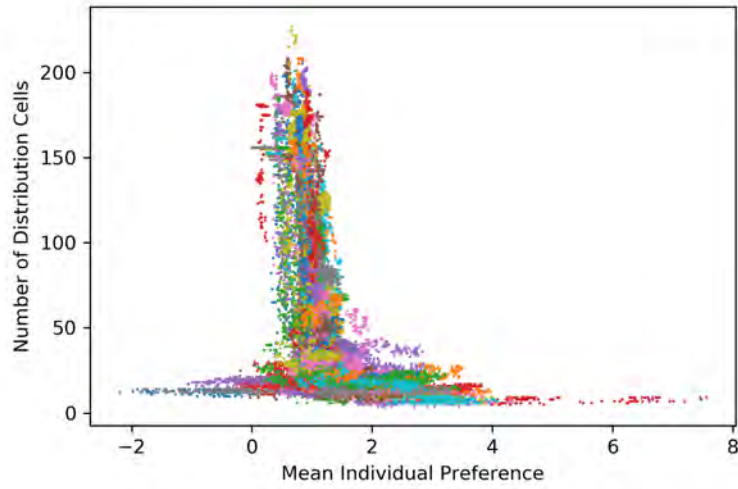


Figure 85: Number of Distribution Cells verses mean individual preference for all 360 simulation in Table 14.

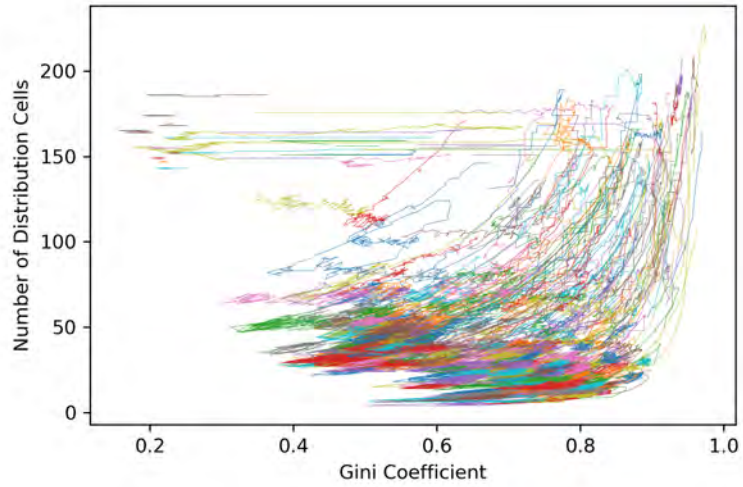


Figure 86: Gini Coefficient plotted against the Number of Distribution Cells between time steps 200 and 300 for all 360 simulation in Table 14.

Further simulations were undertaken with larger neighbour number and neighbourhood size. The parameters are shown in Table 15, leading to a further 300 simulations. The results are shown in Figures 87, 88, 89, 90, 95, 96 and 97.

Model Parameters	Value
Number of Neighbours	10-19
Neighbourhood Size	10, 11 and 15
Number of Repeat Simulations	10

Table 15: Parameters changed for each simulation for Section 7.4.2

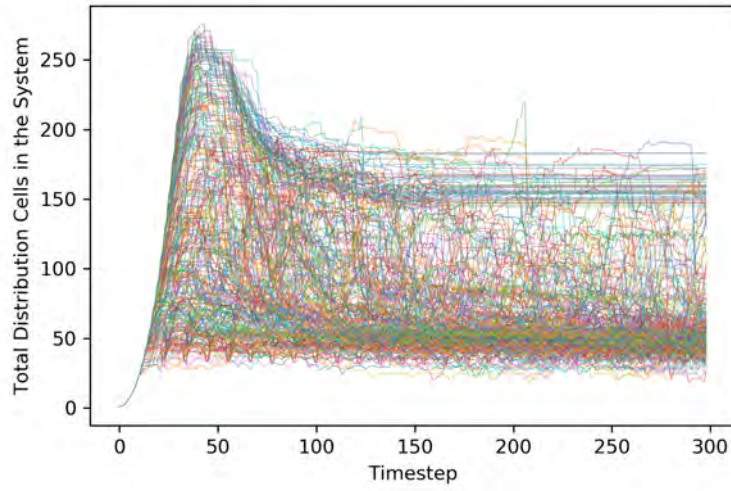


Figure 87: Number of Distribution Cells per time step for all 300 simulation in Table 15.

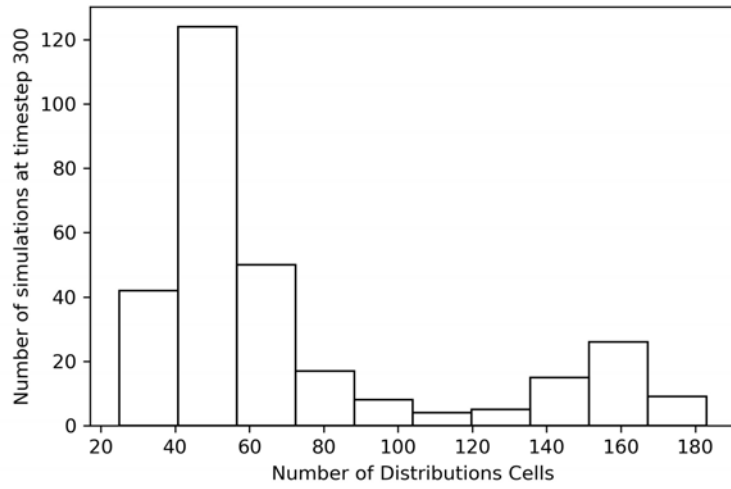


Figure 88: Number of Distribution Cells at time step 300 for all 300 simulation in Table 15.

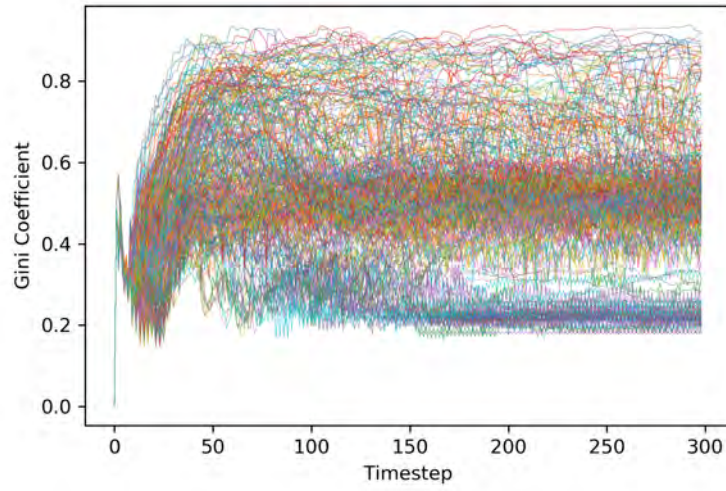


Figure 89: Gini Coefficient per time step for all 300 simulation in Table 15.

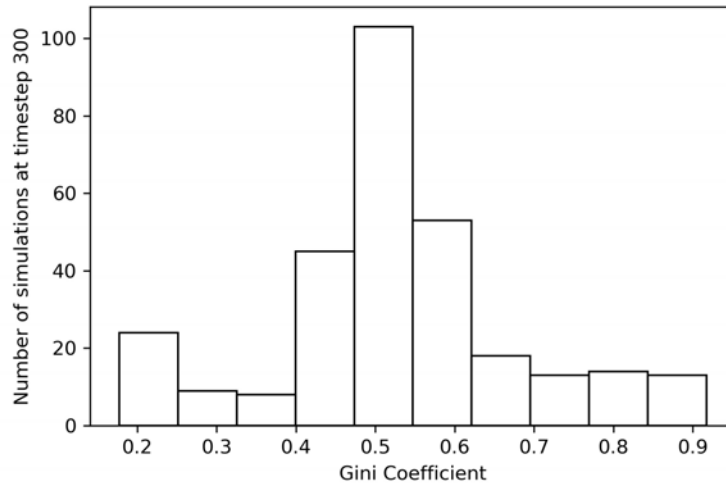


Figure 90: Gini Coefficient per time step for all 300 simulation in Table 15.

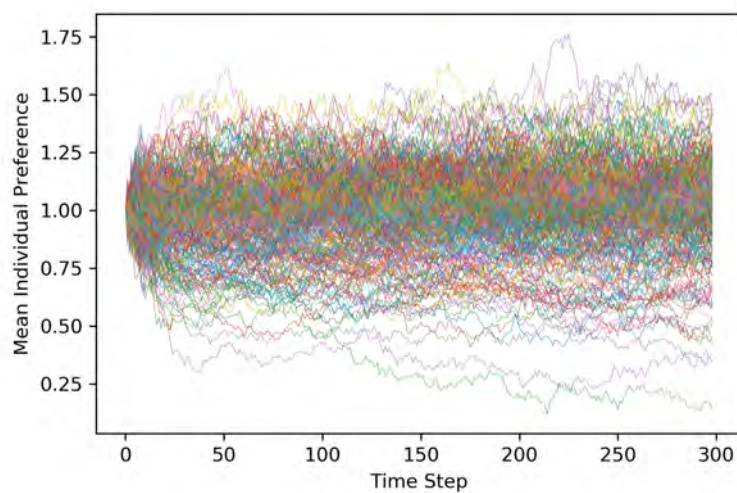


Figure 91: Mean agent preference per time step for all 300 simulation in Table 15.

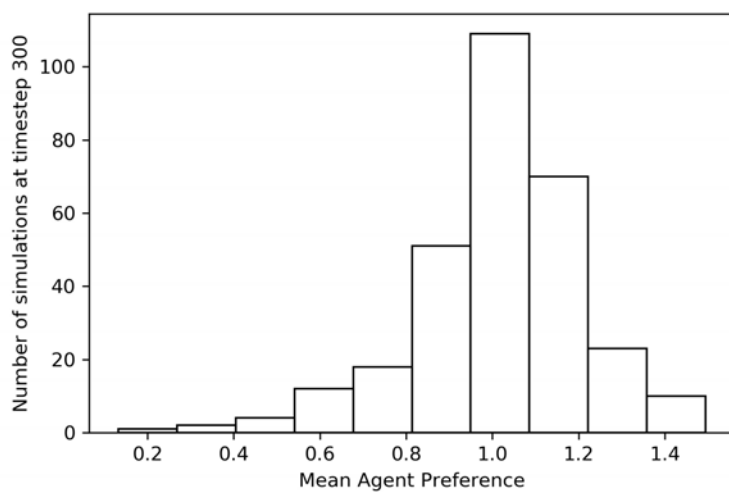


Figure 92: Histogram of mean agent preference for final time step for all 300 simulation in Table 15.

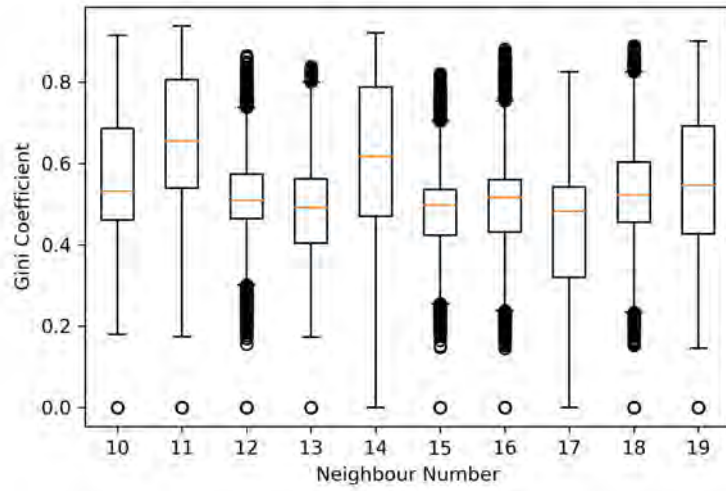


Figure 93: Box Plots for all 300 simulation showing the changing Gini Coefficient for Agent Neighbourhood Number in Table 15.

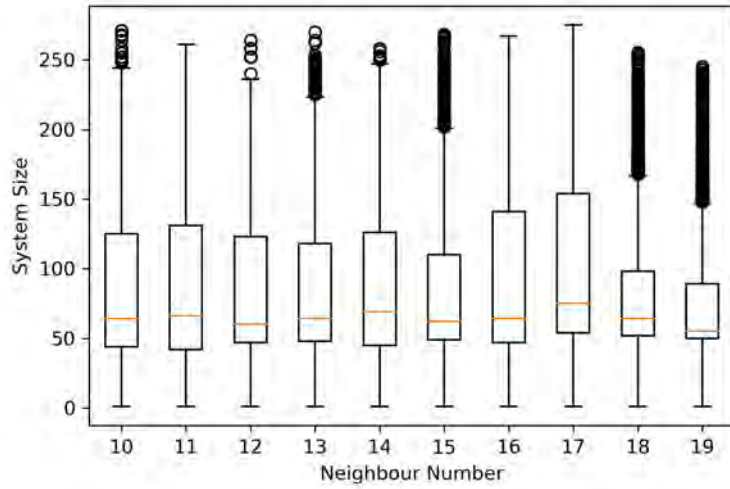


Figure 94: Box Plots for all 300 simulation showing the changing system size for Agent Neighbourhood Number in Table 15.

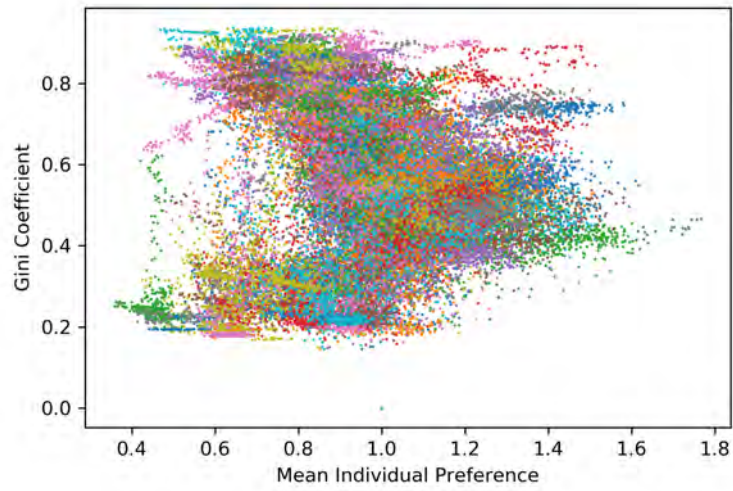


Figure 95: Gini Coefficient verses mean individual preference for all 300 simulation in Table 15.

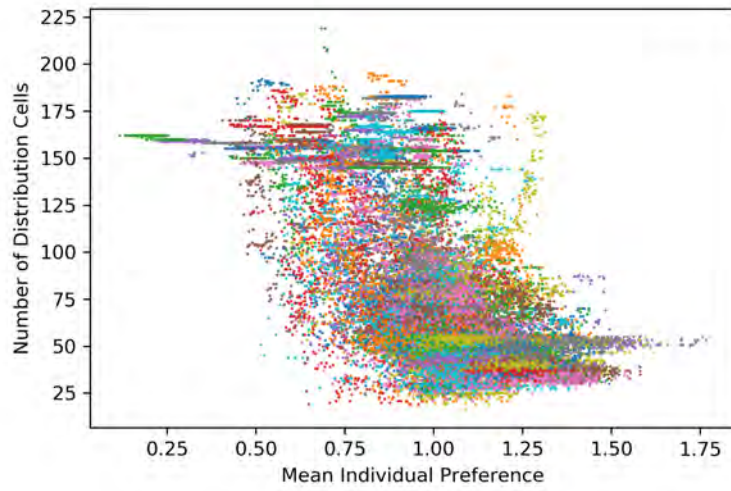


Figure 96: Number of Distribution Cells verses mean individual preference for all 300 simulation in Table 15.

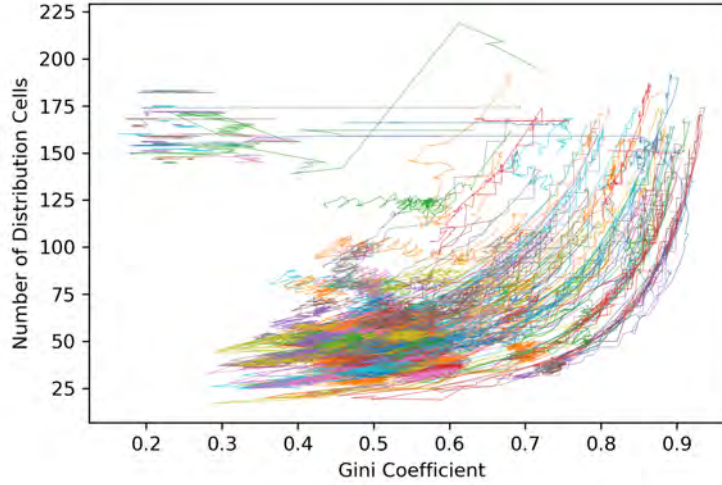


Figure 97: Gini Coefficient plotted against the Number of Distribution Cells between time steps 200 and 300 for all simulations in Table 15.

Discussion The Gini Coefficients found from these simulations seem to generally be greater than those observed in empirical data for modern societies (See Section 7.1). Although the data appears to be more similar to those observed in traditional societies, as can be seen in the data on the UK over the past 700 years, Figure 57. However as the neighbourhood size and neighbour number is increased for each agent the Gini Coefficient is reduced as shown in the box plots for increased neighbour number, Figures 82 and 93. This increased connectivity also leads to a more stable larger population, Figures 83 and 94. The mean agent preference also seems to show a slight increase in agent preference over time, Figure 81 and 92.

The reason for the Gini coefficient being comparable to traditional societies might be due to the rigid societal structure of these systems as this model has strong spatial characteristics which is similar to perhaps an indigenous irrigation system. The greater connectivity leading to larger more stable populations might be due to the decrease in extreme agent preferences as shown by the differences between Figures 80 and 91. The slight increase in agent preference over time might be due to a selection pressure operating on the system. With increased agent preference, agents will need more resource leading to proportionally more resource being passed to them. As shown in

the histograms in Figures 81 and 92 the agent preference is slightly asymmetrical with more agents having a preference greater than 1. This is very subtle, indicating that this or at least the mean agent preference is not a dominant mechanism in changing the system size or Gini Coefficient. Plots of the Gini Coefficient against the mean individual preference are shown in Figure 84 and 95. For lower connectivity there appears to not be a significant relationship between mean individual preference and Gini Coefficient, Figure 84. However the plot does show the possibility of more extreme agent preference being linked to great Gini Coefficient values. For systems with greater connectivity (Figure 95), the relationship between Gini Coefficient and mean individual preference seems to show moderate levels of inequality (0.4-0.6 Gini Coefficient) being linked to greater a mean individual preference above 1, and very low levels and very high levels of inequality being linked with generally lower than 1 individual mean preference. To further understand the reasons behind this trend more detailed is required. However it is thought that the reason for this trend might be that for high levels of inequality only a few agents are required to have high individual preferences which would not be reflected in the mean individual preference.

Analysis of Grouping in Results The plots showing the Gini Coefficient and Number of Distribution Cells for the final 100 time steps of each simulation provide information on possible groupings or basins of attraction which the system may grow towards (Figures 86 and 97).

Three main groups are identified as follows and referred to in Figure 98:

1. Small system (generally between 25 and 75) with a Gini Coefficient between 0.35 and 0.7.
2. Large systems (150 to 175) with a low Gini Coefficient between 0.2 and 0.3.
3. Fluctuating systems with changing size between 50 and 175 and Gini Coefficient between 0.7 and 0.9. Systems appears to follow an ordered path in Group 3 and may pass to and from Group 1.

To understand further the processes leading to the formation of these different groups, example systems from each group are analysed further.

Group 1 The parameters for this example are those shown in Table 13 with the simulation specific parameters shown in Table 16. Figure 99 shows

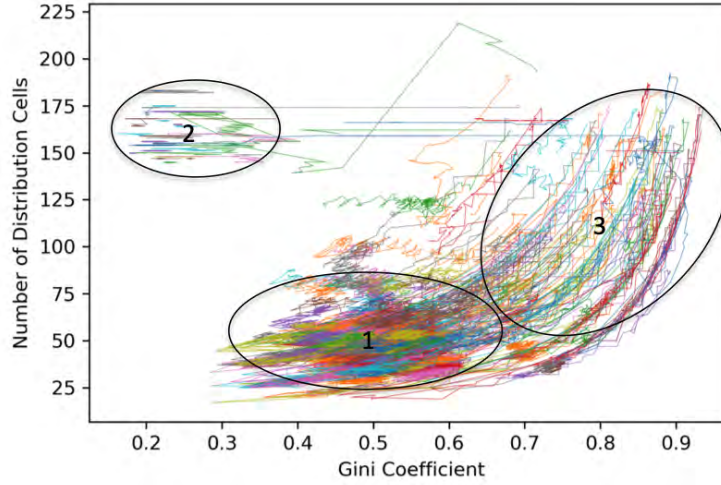


Figure 98: Labelled Groupings for Distribution Cells and Gini Coefficient. For simulations from Table 15.

it is located within Group 1 and Figure 100 shows the number of cells per time step. The number of Cells shows a rapid growth then fall which is common in all the simulations. The reason for this has already been explained previously as it takes time for agents to accumulate resource and increase their need based on their preference and their neighbours. For the majority of the simulation, the number of cells is reasonably constant with minor fluctuations. All the individual agent preferences for each time step are shown in Figure 101. Whilst there is a lot of noise in this figure, group formation can be seen. This can be thought of as similar to the concept of niche construction as mentioned in the literature review in sections 3.7 and 3.8 with each of the clusterings of agents being a separate niche. This clustering of preferences is also reflected in the resource per agent per time step, Figure 102. Two distinct groups are formed which have similar resource which is likely the reasons for the low Gini Coefficient. The networking within the model between the groups has not been analysed. This is likely to play a dominant role in the formation and maintenance of groups.

Model Parameters	Value
Number of Neighbours	19
Neighbourhood Size	10

Table 16: Parameters selected for Group 1 simulation

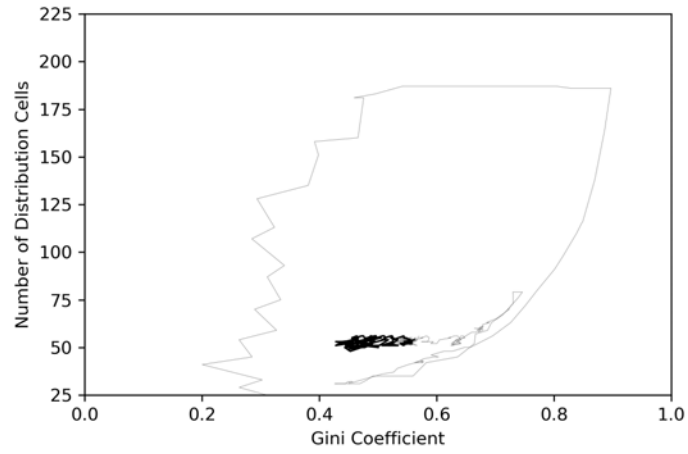


Figure 99: Number of Distribution Cells verses the Gini Coefficient for Group 1 example. Parameters from Table 16.

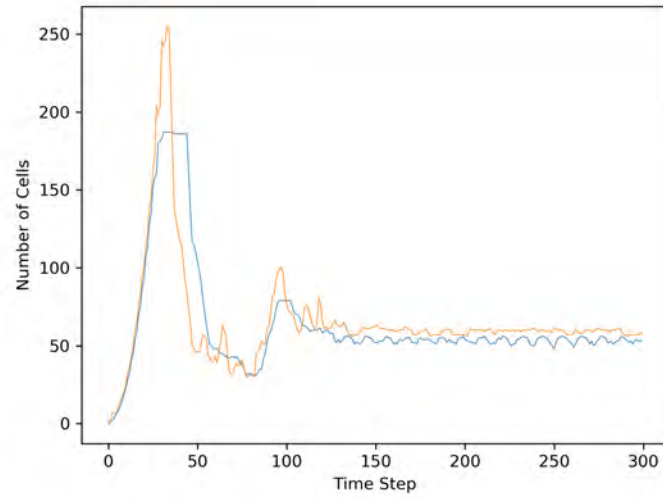


Figure 100: Number of Distribution and Receiver Cells per time step for Group 1 example. Parameters from Table 16.

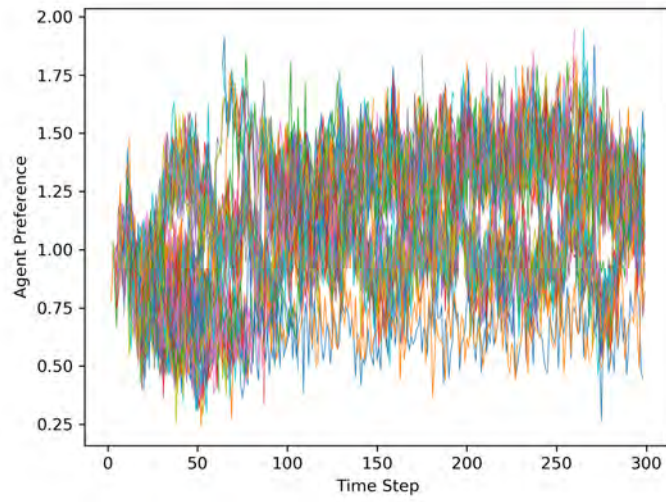


Figure 101: Individual agent preferences over the course of the simulation for the group 1 example. Parameters from Table 16.

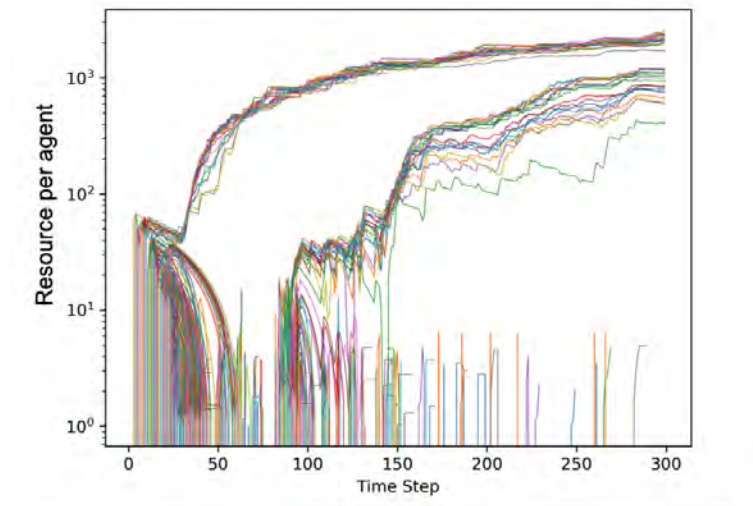


Figure 102: Resource per Agent for Group 1 example. Parameters from Table 16.

Group 2 The parameters for this example are those shown in Table 13 with the simulation specific parameters shown in Table 17. The changing number of distribution cells and Gini Coefficient per time step is shown in Figure 103. The black area is the location of the simulation in the final 100 time steps which is within Group 2. Figure 104 shows the number of Distribution Cells and Receiver Cells per time step. After the initial growth, the simulation seems to reorganise in order to maximise the number of receiver cells to distribution cells. The reason it is doing this is uncertain. The individual agent preferences each time step are shown in Figure 105. Whilst the group 1 preferences showed the formation of niches, the group 2 agent preferences show one group formed with little variance. This indicates a highly connected agent population. The resource per agent per time step is shown in Figure 106. This shows a gradual reduction in resource per agent for each time step.

Model Parameters	Value
Number of Neighbours	10
Neighbourhood Size	10

Table 17: Parameters selected for Group 2 simulation

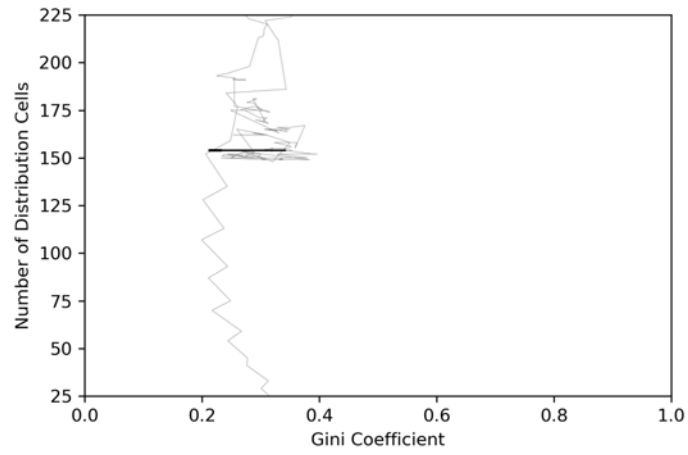


Figure 103: Path of Distribution Cells versus the Gini Coefficient for Group 2 example. Parameters from Table 17.

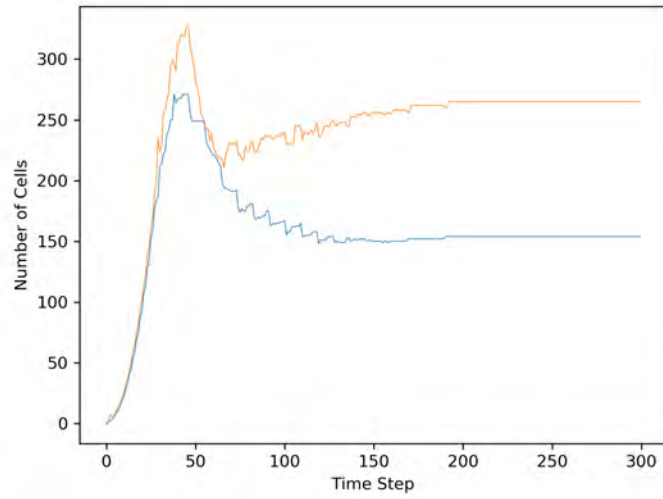


Figure 104: Number of Distribution and Receiver Cells per time step for Group 2 example. Parameters from Table 17.

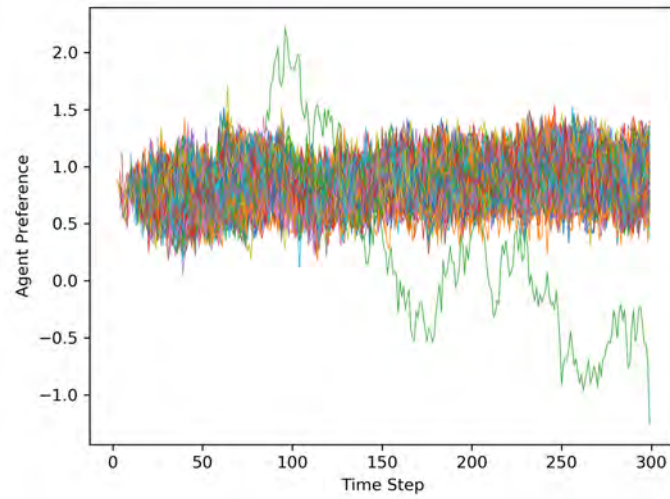


Figure 105: Individual agent preferences over the course of the simulation for the group 2 example. Parameters from Table 17.

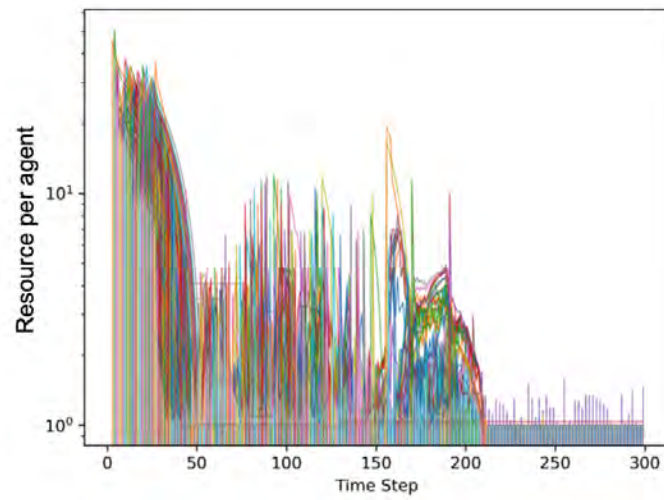


Figure 106: Resource per Agent for Group 2 example. Parameters from Table 17.

Group 3 The parameters for this example are those shown in Table 13 with the simulation specific parameters shown in Table 18. The changing number of distribution cells and Gini Coefficient per time step is shown in Figure 107. The black area is the location of the simulation in the final 100 time steps which is within Group 3. Figure 108 shows the number of Distribution Cells and Receiver Cells per time step. This shows the oscillating fluctuating behaviour which was previously described for group 3. The individual agent preference per time step shows a lack of niches formed, but more fluctuating behaviour than for Group 2, Figure 109. This fluctuating also seems to correlate with that seen in the number of distribution cells. The resource per agent data (Figure 109) shows the possible formation of *classes* with some agents persisting and obtaining increased amounts of resource whilst others being removed from the system and new agents added when the system grows again. This is likely to relate to the agents closer to the source. The ones closer to the source are able to persist for longer periods.

Model Parameters	Value
Number of Neighbours	16
Neighbourhood Size	15

Table 18: Parameters selected for Group 3 simulation

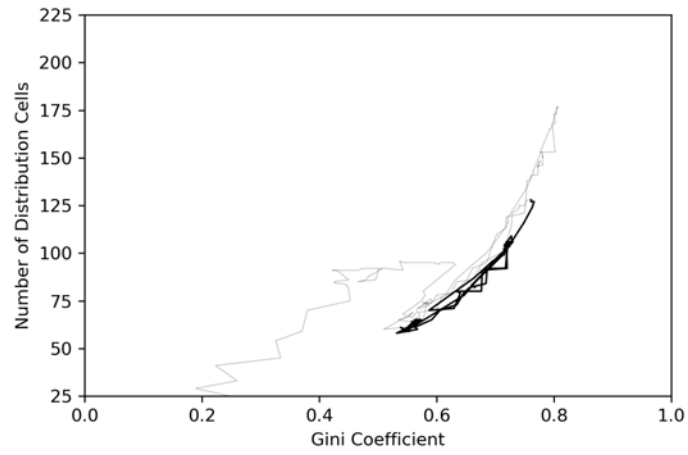


Figure 107: Path of Distribution Cells versus the Gini Coefficient for Group 3 example. Parameters from Table 18.

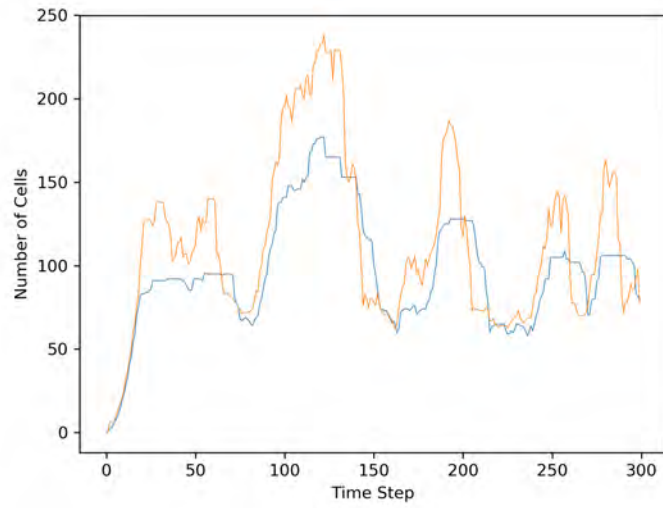


Figure 108: Number of Distribution and Receiver Cells per time step for Group 2 example. Parameters from Table 18.

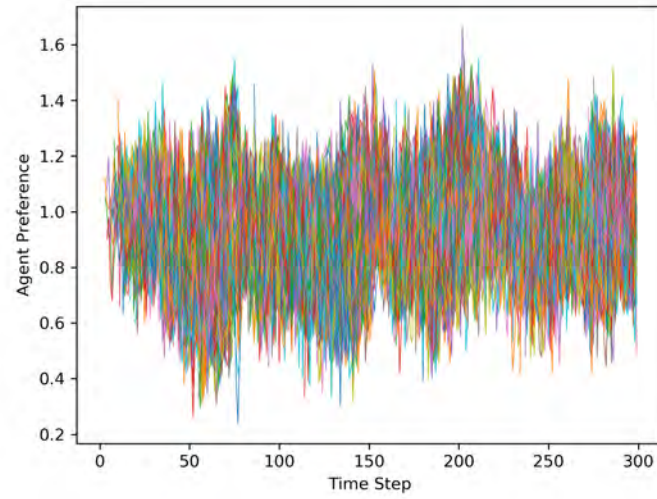


Figure 109: Individual agent preferences over the course of the simulation for the group 3 example. Parameters from Table 18.

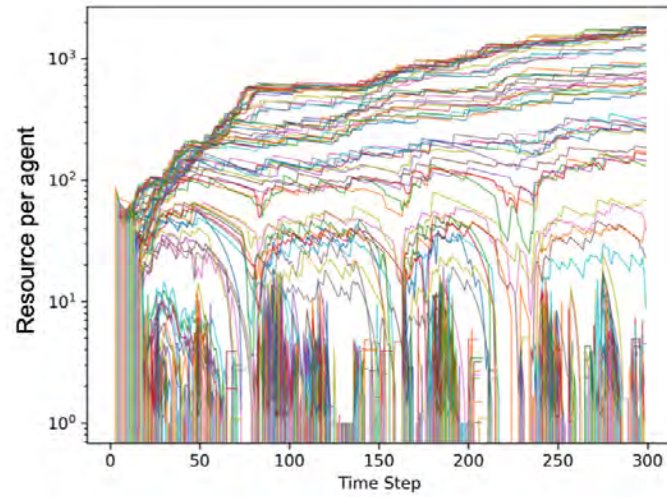


Figure 110: Resource per Agent for Group 3 example. Parameters from Table 18.

Discussion of Groups By looking more in depth at the behaviour of examples from each of the groups (1,2 and 3) a deeper understanding of the differences and possible mechanisms leading to their behaviour has been found. The number of neighbours and neighbourhood size do not seem to be dominant parameters in leading to a certain group type. Instead the networking of the system and stochastic choices of agents may play a role.

The data showing the individual agent preferences through time and the individual agent resource through time shed light on the degree of connectivity in the system which seems to be affecting the inequality and size of the system.

Before drawing conclusions on whether the behaviours observed in the examples above are dominant in different groups, additional examples need to be analysed. To further understand how connectivity, agent preference and agent location in the system lead to different system level behaviour, further analysis and visualisation of many simulations is required. This may also require more systematic and automated forms of analysis such as Principle Component analysis or network analysis.

It is hypothesised that horizontal (favourable networks with other agents at similar distance from the source) and vertical networking (favourable networking with other agents further or closer to the source) may play an important role. The dynamics of this may be quite complicated and counter-intuitive. For instance initially increased vertical networking may be viewed as leading to a lower Gini Coefficient but it may increase instability. Further modelling of this networking is required to test these theories.

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