CS411 Theory of Computation
Lecture 3

Sergey Kitaev

University of Strathclyde

27 September 2017

Recall:

Definition

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ where Q, Σ, Γ are all finite sets and

(1) Q is the set of states

(2) Σ is the input alphabet not containing the blank symbol $\sqcup$

(3) Γ is the tape alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$

(4) $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function

(5) $q_0 \in Q$ is the start state

(6) $q_{\text{accept}} \in Q$ is the accept state

(7) $q_{\text{reject}} \in Q$ is the reject state where $q_{\text{reject}} \neq q_{\text{accept}}$.

- A Turing Machine is formally described by specifying each of its seven parts.
- Going to that level of detail can be cumbersome.
- Consequently we will not spend much time giving such detailed descriptions, and instead give only higher level descriptions. These higher level descriptions are precise enough for our purposes.
- However, always remember that every higher level description is just shorthand for its formal counterpart.
- Today we will consider the Turing machine M_2.

The Turing Machine M_2

Let M_2 be the Turing machine that decides $A = \{0^{2^n} | n \in \{0, 1, 2, \ldots\}\}$. A is the language which consists of all strings of zeros whose length is a power of 2.

$$A = \{\ 0,\ 00,\ 0000,\ 00000000,\ \ldots\ \}.$$

Description of M_2: Given input string w,

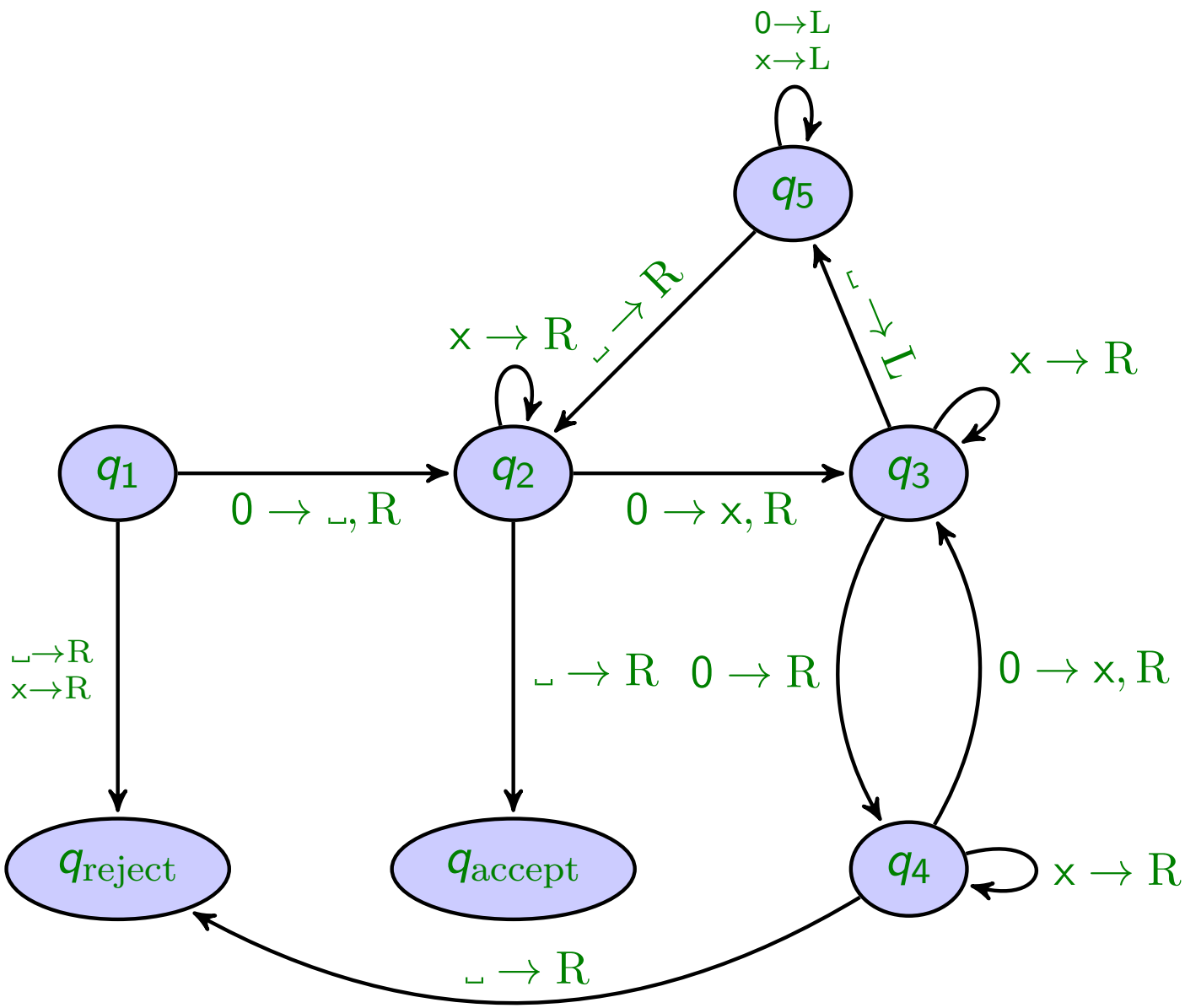
1. Sweep left to right across the tape, crossing off every other 0 (i.e. the 2nd, the 4th, ...).
2. If in Stage 1 the tape contained a single 0, then accept.
3. If in Stage 1 the tape contained more that a single 0 and the number of 0s was odd, reject.
4. Return the head to the left-hand end of the tape.
5. Go to Stage 1.

Check this for the following three cases;
(i) $w = 0$, (ii) $w = 000$, (iii) $w = 0000$.

Formal description of M_2

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{accept}}, q_{\text{reject}}\}$.
- $\Sigma = \{0\}$.
- $\Gamma = \{0, \text{x}, \sqcup\}$.
- δ is given in the diagram on the next slide.
- The start, accept and reject states are $q_1, q_{\text{accept}}, q_{\text{reject}}$.

Transition function δ for M_2



Some notes on the labels in the δ diagram of M_2

- The label $0 \to \sqcup, \mathrm{R}$ that appears on the transition from state q_1 to q_2 signifies that, when in state q_1 with the head currently reading 0, the machine goes to state q_2, write $\sqcup$, and move the head one place to the right, i.e. $\delta(q_1, 0) \to (q_2, \sqcup, \mathrm{R})$.
- The label $0 \to \mathrm{R}$ in the transition from state q_3 to state q_4 signifies the same as the previous point, except that the 0 that that tape is currently reading remains unchanged, i.e. $\delta(q_3, 0) = (q_4, 0, \mathrm{R})$.
- The machine begins by writing $\sqcup$ over the leftmost 0, this is in order to be able to find the left hand end of the tape. Other symbols could have been used but it was done this way (in this TM) so that the diagram is small.

Running M_2 on different inputs

On the board we will consider the outcome of the following inputs to M_2:

1. $w = 0$
2. $w = 00$
3. $w = 000$
4. $w = 0000$

Transition function δ for M_2