Contents

1 Math

1.1 快速冪

```
// 根據費馬小定
    理，若 a p 互質，a^(p-2) 為 a 在 mod p 時的乘法逆元
int fast_pow(int a, int b, int mod)
{
  // a^b % mod
  int res = 1;
  while(b)
  {
    if(b & 1) res = (res * a) % mod;
    a = (a * a) % mod;
    b >>= 1;
  }
  return res;
}
```

1.2 擴展歐幾里得

```
int gcd(int a, int b)
{
  return b == 0 ? a : gcd(b, a % b);
}

int lcm(int a, int b)
{
  return a * b / gcd(a, b);
}

pair<int, int> ext_gcd
    (int a, int b) //擴展歐幾里德 ax+by = gcd(a,b)
{
  if (b == 0)
    return {1, 0};
  if (a == 0)
    return {0, 1};
  int x, y;
  tie(x, y) = ext_gcd(b % a, a);
  return make_pair(y - b * x / a, x);
}
```

2 Graph

2.1 Tarjan SCC

```
class tarjan{
    //  1-base
    int time = 1;
    int id = 1;
    stack<int> s;
    vector<int> low;
    vector<int> dfn;
    vector<bool> in_stack;
    void dfs(int node, vector<vector<int>> &graph){
      in_stack[node] = true;
      s.push(node);
      dfn[node] = low[node] = time++;
      for(auto &j : graph[node]){
        if(dfn[j] == 0){
          dfs(j, graph);
          // 看看往下有沒有辦法回到更上面的點
          low[node] = min(low[node], low[j]);
        }
        else if(in_stack[j]){
          low[node] = min(low[node], low[j]);
        }
      }
      vector<int> t; // 儲存這個強連通分量
      if(dfn[node] == low[node]){
        while(s.top() != node){
          t.push_back(s.top());
          in_stack[s.top()] = false;
          scc_id[s.top()] = id;
          s.pop();
        }
        t.push_back(s.top());
        scc_id[s.top()] = id;
        in_stack[s.top()] = false;
        s.pop();
        id++;
      }
      if(!t.empty()) ans.push_back(t);
    }
  public:
    vector<int> scc_id;
    vector<vector<int>> ans;
    // ans ans[i] 代表第 i 個強連通分量裡面包涵的點
    // scc_id[i] 代表第 i 個點屬於第幾個強連通分量
    vector
        <vector<int>> scc(vector<vector<int>> &graph){
      int num = graph.size();
      scc_id.resize(num, -1);
      dfn.resize(num, 0);
      low.resize(num, 0);
      in_stack.resize(num, false);
      for(int i = 1; i < num; i++){
        if(dfn[i] == 0) dfs(i, graph);
      }
      return ans;
    }
};
```

2.2 2 SAT

```
// 用
    下面的 tarjan scc 算法來解 2 sat 問題，若 事件 a 發
    生時，事件 b 必然發生，我們須在 a -> b 建立一條有向
// 用
     cses 的 Giant Pizza 來舉例子，給定 n 個人 m 個配料
    表，每個人可以提兩個要求，兩個要求至少要被滿足一個

// 3 5
// + 1 + 2
// - 1 + 3
// + 4 - 2

// 以這
    個例子來說，第一個人要求要加 配料1 或者 配料2 其中
    一項，第二個人要求不要 配料1 或者 要配料3 其中一項
// 試問能不能滿足所有人的要求，我們可以把 要加
    配料 i 當作點 i ，不加配料 i 當作點 i + m(配料數量)
// 關於第一個人的要求 我們可以看成若不加 配
    料1 則必定要 配料2 以及 若不加 配料2 則必定要 配料1
// 關於第二個人要求 可看做加了 配料
    1 就必定要加 配料3 以及 不加 配料3 就必定不加 配料1

// 以這些條件建立有像圖，並且
    找尋 scc ，若 i 以及 i + m 在同一個 scc 中代表無解
// 若要求解，則若 i 的 scc_id
     小於 i + m 的 scc_id 則 i 為 true ，反之為 false
// tarjan 在下面
cin >> n >> m;

vector<vector<int>> graph(m * 2 + 1);
function<int(int)> tr = [&](int x){
  if(x > m) return x - m;
  return x + m;
};

for(int i = 0; i < n; i++){
  char c1, c2;
  int a, b;
  cin >> c1 >> a >> c2 >> b;
  // a 代表 a 為真，m + a 代表 a 為假
  if(c1 == '-') a += m;
  if(c2 == '-') b += m;
  graph[tr(a)].push_back(b);
  graph[tr(b)].push_back(a);
}

tarjan t;
auto scc = t.scc(graph);

for(int i = 1; i <= m; i++){
```

```
  if(t.scc_id[i] == t.scc_id[tr(i)]){
    cout << "IMPOSSIBLE\n";
    return 0;
  }
}

for(int i = 1; i <= m; i++){
  if(t.scc_id[i] < t.scc_id[tr(i)]){
    cout << '+';
  }
  else cout << '-';
  cout << ' ';
}
cout << '\n';
```

2.3 AP/Bridge

```
// adj[u] = adjacent nodes of u
// ap = AP = articulation points
// p = parent
// disc[u] = discovery time of u
// low[u] = 'low' node of u

int dfsAP(int u, int p) {
  int children = 0;
  low[u] = disc[u] = ++Time;
  for (int& v : adj[u]) {
    if (v == p) continue; //
        we don't want to go back through the same path.
                          // if we go back is because
                              we found another way back
    if (!disc
        [v]) { // if V has not been discovered before
      children++;
      dfsAP(v, u); // recursive DFS call
      if (disc[u] <= low[v]) // condition #1
        ap[u] = 1;
      low[u] = min(low[u],
           low[v]); // low[v] might be an ancestor of u
    } else // if v was already
        discovered means that we found an ancestor
      low[u] = min(low[u], disc[v]); // finds
           the ancestor with the least discovery time
  }
  return children;
}

void AP() {
  ap = low = disc = vector<int>(adj.size());
  Time = 0;
  for (int u = 0; u < adj.size(); u++)
    if (!disc[u])
      ap[u] = dfsAP(u, u) > 1; // condition #2
}

// br = bridges, p = parent

vector<pair<int, int>> br;

int dfsBR(int u, int p) {
  low[u] = disc[u] = ++Time;
  for (int& v : adj[u]) {
    if (v == p) continue; //
        we don't want to go back through the same path.
                          // if we go back is because
                              we found another way back
    if (!disc
        [v]) { // if V has not been discovered before
      dfsBR(v, u);  // recursive DFS call
      if (disc
          [u] < low[v]) // condition to find a bridge
        br.push_back({u, v});
      low[u] = min(low[u],
           low[v]); // low[v] might be an ancestor of u
    } else // if v was already
        discovered means that we found an ancestor
      low[u] = min(low[u], disc[v]); // finds
           the ancestor with the least discovery time
  }
}

void BR() {
  low = disc = vector<int>(adj.size());
  Time = 0;
  for (int u = 0; u < adj.size(); u++)
    if (!disc[u])
      dfsBR(u, u)
}
```

2.4 Max flow

```
#define int long long

// Edmonds-Karp Algorithm Time: O(VE^2) 實際上會快一點

class edge{
  public:
    int next;
    int capicity;
    int rev;
    edge(int _n, int _c
        , int _r) : next(_n), capicity(_c), rev(_r){};
};

vector<vector<edge>> graph;

void add_edge(int a, int b, int capacity){
  graph[
      a].push_back(edge(b, capacity, graph[b].size()));
  graph[b].push_back(edge(a, 0, graph[a].size() - 1));
}

int dfs(int now, int end
    , int flow, vector<pair<int, int>> &path, int idx){
  if(now == end) return flow;
  auto &e = graph[now][path[idx + 1].second];
  if(e.capicity > 0){
    auto ret = dfs(e.next
        , end, min(flow, e.capicity), path, idx + 1);
    if(ret > 0){
      e.capicity -= ret;
      graph[e.next][e.rev].capicity += ret;
      return ret;
    }
  }
  return 0;
}

vector<pair<int, int>> search_path(int start, int end){
  vector<pair<int, int>> ans;
  queue<int> q;
  vector
      <pair<int, int>> parent(graph.size(), {-1, -1});
  q.push(start);
  while(!q.empty()){
    int now = q.front();
    q.pop();
    for(int i = 0; i < (int)graph[now].size(); i++){
      auto &e = graph[now][i];
      if(e.
          capicity > 0 and parent[e.next].first == -1){
        parent[e.next] = {now, i};
        if(e.next == end) break;
        q.push(e.next);
      }
    }
  }
  if(parent[end].first == -1) return ans;
  int now = end;
  while(now != start){
    auto [node, idx] = parent[now];
    ans.emplace_back(node, idx);
    now = node;
  }
  ans.emplace_back(start, -1);
  reverse(ans.begin(), ans.end());
  return ans;
}

int maxflow(int start, int end, int node_num){
  int ans = 0;
  while(1){
    vector<bool> visited(node_num + 1, false);
    auto tmp = search_path(start, end);
    if(tmp.size() == 0) break;
    auto flow = dfs(start, end, 1e9, tmp, 0);
    ans += flow;
  }
  return ans;
}
```

3 String

3.1 Hash

```
vector<int> Pow(int num){
  int p = 1e9 + 7;
  vector<int> ans = {1};
  for(int i = 0; i < num; i++)
    ans.push_back(ans.back() * b % p);
  return ans;
}

vector<int> Hash(string s){
  int p = 1e9 + 7;
  vector<int> ans = {0};
  for(char c:s){
    ans.push_back((ans.back() * b + c) % p);
  }
  return ans;
}

// 閉區間[l, r]
int query
    (vector<int> &vec, vector<int> &pow, int l, int r){
  int p = 1e9 + 7;
  int length = r - l + 1;
  return
       (vec[r + 1] - vec[l] * pow[length] % p + p) % p;
}
```

3.2 Zvalue

```
vector<int> z_func(string s1){
  int l = 0, r = 0, n = s1.size();
  vector<int> z(n, 0);
  for(int i = 1; i < n; i++){
    if(i
        <= r and z[i - l] < r - i + 1) z[i] = z[i - l];
    else{
      z[i] = max(z[i], r - i + 1);
      while(i + z
          [i] < n and s1[i + z[i]] == s1[z[i]]) z[i]++;
    }
    if(i + z[i] - 1 > r){
      l = i;
      r = i + z[i] - 1;
    }
  }
  return z;
}
```

4 Geometry

4.1 Static Convex Hull

```
#define mp(a, b) make_pair(a, b)
#define pb(a) push_back(a)
#define F first
#define S second

template<typename T>
pair<T, T> operator-(pair<T, T> a, pair<T, T> b){
    return mp(a.F - b.F, a.S - b.S);
}

template<typename T>
T cross(pair<T, T> a, pair<T, T> b){
    return a.F * b.S - a.S * b.F;
}

template<typename T>
vector<pair
    <T, T>> getConvexHull(vector<pair<T, T>>& pnts){
    sort(pnts.begin
        (), pnts.end(), [](pair<T, T> a, pair<T, T> b)
    { return
         a.F < b.F || (a.F == b.F && a.S < b.S); });
    auto cmp = [&](pair<T, T> a, pair<T, T> b)
    { return a.F == b.F && a.S == b.S; };
    pnts.erase(unique
        (pnts.begin(), pnts.end(), cmp), pnts.end());
    if(pnts.size()<=1)
        return pnts;
    int n = pnts.size();
    vector<pair<T, T>> hull;
    for(int i = 0; i < 2; i++){
        int t = hull.size();
        for(pair<T, T> pnt : pnts){
            while(hull.size() - t >= 2 &&
                cross(hull.back() - hull[hull.size() -
                2], pnt - hull[hull.size() - 2]) <= 0){
                hull.pop_back();
            }
            hull.pb(pnt);
        }
        hull.pop_back();
        reverse(pnts.begin(), pnts.end());
    }
    return hull;
}
```