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## 1 Basic

### 1.1 Default Code

```
#include <bits/stdc++.h>
#define int long long
#define endl '\n' // 如果是互動題要把這個註解掉
#define de(x) cout << #x << '=' << x << ", "
#define dd cout << '\n';
// #pragma GCC target("popcnt")
// #pragma GCC optimize("O3")
using namespace std;
int tt = 1;

void pre() {
    cout.tie(nullptr); // 輸出加速
    cin >> tt; // 多筆輸入
}

void solve() {}

signed main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
#ifdef LOCAL
    // g++ -DLOCAL -std=c++17 <filename> && ./a.out
    freopen("input.txt", "r", stdin);
    // freopen("output.txt", "w", stdout);
#endif // LOCAL
    pre();
    while (tt--) { solve(); }
    return 0;
}
```

### 1.2 PBDS

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
```

```
template
<class T> using Tree = tree<T, null_type, less<T>,
>, rb_tree_tag, tree_order_statistics_node_update>;

/*
如果有 define int long long 記得拿掉
Tree<int> t 就跟 set<int> t 一樣, 有包好 template
rb_tree_tag 使用紅黑樹
第三個參數 less<T> 為由小到大, greater<T> 為由大到小
插入 t.insert(); 刪除 t.erase();
t.order_of_key
(k); 從前往後數 k 是第幾個 (0-base 且回傳 int 型別)
t.find_by_order(k);
從前往後數第 k 個元素 (0-base 且回傳 iterator 型別)
t.lower_bound
(); t.upper_bound(); 用起來一樣 回傳 iterator
可以用 Tree<pair<int, int>> T 來模擬 multiset
*/
```

### 1.3 int128 Input Output

```
// 抄 BBuf github 的
#include <bits/stdc++.h>
using namespace std;

void scan(__int128 &x) // 輸入
{
    x = 0;
    int f = 1;
    char ch;
    if((ch = getchar()) == '-') f = -f;
    else x = x*10 + ch-'0';
    while((ch = getchar()) >= '0' && ch <= '9')
        x = x*10 + ch-'0';
    x *= f;
}

void print(__int128 x) // 輸出
{
    if(x < 0)
    {
        x = -x;
        putchar('-');
    }
    if(x > 9) print(x/10);
    putchar(x%10 + '0');
}

int main()
{
    __int128 a, b;
    scan(a);
    scan(b);
    print(a + b);
    puts("");
    print(a*b);
    return 0;
}
```

### 1.4 Python

```
## Input
# p q 都是整數, 中間以空白分開輸入
p, q = map(int, input().split())

# 輸入很多個用空
白隔開的數字, 轉成 float 放進陣列, s 是 input 字串
arr = list(map(float, s.split()))

# 分數用法 Fraction(被除數, 除數)
from fractions import Fraction

frac = Fraction(3, 4)
numerator = frac.numerator # 取出分子
denominator = frac.denominator # 取出分母

arr = [Fraction
(0), Fraction(1, 6), Fraction(1, 2), Fraction(5,
12), Fraction(0), Fraction(-1, 12), Fraction(0)]

# 可以直接做乘除
```

```
def fx(x):
    x = Fraction(x)
    ans = Fraction(0)
    for i in range(1, 7):
        ans += arr[i] * x ** (7 - i)
    return ans
```

## 1.5 bitset

bitset<size> b(a): 長度為size，初始化為a  
 b[i]: 第i位元的值(0 or 1)  
 b.size(): 有幾個位元  
 b.count(): 有幾個1  
 b.set(): 所有位元設為1  
 b.reset(): 所有位元設為0  
 b.flip(): 所有位元反轉

## 2 Math

### 2.1 質數表

```
vector<int> prime_table(int n){
    vector<int> table(n + 1, 0);
    for(int i = 1; i <= n; i++){
        for(int j = i; j <= n; j += i){
            table[j]++;
        }
    }
    return table;
}
```

### 2.2 快速冪

```
#define int long long

// 根據費馬小定
// 理，若 a p 互質，a^(p-2) 為 a 在 mod p 時的乘法逆元
// a ^ (b ^ c
// ) % mod = fast_pow(a, fast_pow(b, c, mod - 1), mod)
typedef unsigned long long ull;
inline int ksc(ull
    x, ull y, int p) { // 0(1)快速乘 (防爆 long long)
    return (x
        * y - (ull)((long double)x / p * y) * p + p) % p;
}

inline int fast_pow(int a, int b, int mod)
{
    // a^b % mod
    int res = 1;
    while(b)
    {
        if(b & 1) res = ksc(res, a, mod);
        a = ksc(a, a, mod);
        b >>= 1;
    }
    return res;
}
```

### 2.3 擴展歐幾里得

```
int gcd(int a, int b)
{
    return b == 0 ? a : gcd(b, a % b);
}

int lcm(int a, int b)
{
    return a * b / gcd(a, b);
}

pair<int, int> ext_gcd
    (int a, int b) //擴展歐幾里德 ax+by = gcd(a,b)
{
    if (b == 0)
        return {1, 0};
    if (a == 0)
        return {0, 1};
    int x, y;
    tie(x, y) = ext_gcd(b % a, a);
    return make_pair(y - (b / a) * x, x);
}
```

## 2.4 矩陣

```
template<typename T>
struct Matrix{
    using rt = std::vector<T>;
    using mt = std::vector<rt>;
    using matrix = Matrix<T>;
    int r, c;
    mt m;
    Matrix(int r, int c): r(r), c(c), m(r, rt(c)){}
    rt& operator[](int i){return m[i];}
    matrix operator+(const matrix &a){
        matrix rev(r, c);
        for(int i=0; i<r; ++i)
            for(int j=0; j<c; ++j)
                rev[i][j]=m[i][j]+a.m[i][j];
        return rev;
    }
    matrix operator-(const matrix &a){
        matrix rev(r, c);
        for(int i=0; i<r; ++i)
            for(int j=0; j<c; ++j)
                rev[i][j]=m[i][j]-a.m[i][j];
        return rev;
    }
    matrix operator*(const matrix &a){
        matrix rev(r, a.c);
        matrix tmp(a.c, a.r);
        for(int i=0; i<a.r; ++i)
            for(int j=0; j<a.c; ++j)
                tmp[j][i]=a.m[i][j];
        for(int i=0; i<r; ++i)
            for(int j=0; j<a.c; ++j)
                for(int k=0; k<c; ++k)
                    rev.m[i][j]+=m[i][k]*tmp[j][k];
        return rev;
    }
    bool inverse(){
        Matrix t(r, r+c);
        for(int y=0; y<r; y++){
            t.m[y][c+y] = 1;
            for(int x=0; x<c; ++x)
                t.m[y][x]=m[y][x];
        }
        if( !t.gas() )
            return false;
        for(int y=0; y<r; y++){
            for(int x=0; x<c; ++x)
                m[y][x]=t.m[y][c+x]/t.m[y][y];
            return true;
        }
    }
    T gas(){
        vector<T> lazy(r, 1);
        bool sign=false;
        for(int i=0; i<r; ++i){
            if( m[i][i]==0 ){
                int j=i+1;
                while(j<r&&!m[j][i])j++;
                if(j==r)continue;
                m[i].swap(m[j]);
                sign=!sign;
            }
            for(int j=0; j<r; ++j){
                if(i==j)continue;
                lazy[j]=lazy[j]*m[i][i];
                T mx=m[j][i];
                for(int k=0; k<c; ++k)
                    m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
            }
        }
        T det=sign?-1:1;
        for(int i=0; i<r; ++i){
            det = det*m[i][i];
            det = det/lazy[i];
            for(auto &j:m[i])j/=lazy[i];
        }
        return det;
    }
};
```

### 2.5 Miller rabin Prime test

```
// fast_pow 去前面抄，需要處理防暴乘法
// 記得 #define int long long 也要放
// long long 範圍內測試過答案正確
// time: O(logn)
```

```

inline bool mr(int x, int p) {
    if (fast_pow(x, p - 1, p) != 1) return 0;
    int y = p - 1, z;
    while (!(y & 1)) {
        y >>= 1;
        z = fast_pow(x, y, p);
        if (z != 1 && z != p - 1) return 0;
        if (z == p - 1) return 1;
    }
    return 1;
}

inline bool prime(int x) {
    if (x < 2) return 0;
    if (x == 2 ||
        x == 3 || x == 5 || x == 7 || x == 43) return 1;
    // 如果把 2
    // 到 37 前 12 個質數都檢查一遍 可以保證 2^78 皆可用
    return mr(2, x)
        && mr(3, x) && mr(5, x) && mr(7, x) && mr(43, x);
}

```

## 2.6 Pollard's Rho

```

// 主函數記得放 srand(time(NULLptr))
// prime 檢測以及快速冪, gcd 等請從前面抄

// 輸入一個數字 p, 隨
// 機回傳一個 非 1 非 p 的因數, 若 p 是質數會無窮迴圈
#define rg register int
inline int rho(int p) {
    int x, y, z, c, g;
    rg i, j;
    while (1) {
        y = x = rand() % p;
        z = 1;
        c = rand() % p;
        i = 0, j = 1;
        while (++i) {
            x = (ksc(x, x, p) + c) % p;
            z = ksc(z, abs(y - x), p);
            if (x == y || !z) break;
            if (!(i % 127) || i == j) {
                g = gcd(z, p);
                if (g > 1) return g;
                if (i == j) y = x, j <= 1;
            }
        }
    }
}

// 回傳隨機一個質因數, 若 input 為質數, 則直接回傳
int prho(int p) {
    if (prime(p)) return p;
    int m = rho(p);
    if (prime(m)) return m;
    return prho(p / m);
}

// 回傳將 n 質因數分解的結果, 由小到大排序
// ex: input: 48, output: 2 2 2 2 3
vector<int> prime_factorization(int n) {
    vector<int> ans;
    while (n != 1) {
        int m = prho(n);
        ans.push_back(m);
        n /= m;
    }
    sort(ans.begin(), ans.end());
    return ans;
}

```

## 2.7 皮薩諾定理

```

// fib(x) % m = fib(x + kn) % m 當 k >= 1, 求 n
// n 為費式數列 % m 會重複的週期
// pisano_period(m) <= 6m
// 通常這都要本地跑

#define int long long

int pisano_period(int m) {
    int pre = 0, cur = 1;
    int temp;

```

```

for (int i = 0; i < m * m; i++) {
    temp = pre;
    pre = cur;
    cur = (temp + cur) % m;
    if (pre == 0 && cur == 1) return i + 1;
}
return 0;
}

```

## 2.8 高斯消去法

```

from fractions import Fraction
def gauss_elimination(matrix, results):
    # 將所有數字轉換為分數
    n = len(matrix)
    augm = [[Fraction(matrix
        [i][j]) for j in range(n)] for i in range(n)]
    augr = [Fraction(results[i]) for i in range(n)]

    # 高斯消去法
    for i in range(n):
        # 尋找主元
        if augm[i][i] == 0:
            for j in range(i + 1, n):
                if augm[j][i] != 0:
                    augm[i], augm[j] = augm[j], augm[i]
                    augr[i], augr[j] = augr[j], augr[i]
                    break

        pivot = augm[i][i]
        if pivot == 0:
            # 如果主元為0, 繼續檢查該行是否全為 0
            if all(augm[i][j] == 0 for j in range(n)):
                if augr[i] != 0:
                    return None # 無解
                else:
                    continue
            # 可能有無限多解, 繼續檢查

        # 將主元行的數字規一化
        for j in range(i, n):
            augm[i][j] /= pivot
            augr[i] /= pivot

        # 將其他行的數字變為0
        for j in range(n):
            if i != j:
                factor = augm[j][i]
                for k in range(i, n):
                    augm[j][k] -= factor * augm[i][k]
                augr[j] -= factor * augr[i]

    # 檢查是否存在無限多解的情況
    for i in range(n):
        if all(augm[i][j]
            [j] == 0 for j in range(n)) and augr[i] != 0:
            return [] # 無限多組解

    return augr

# matrix = [
#     [2, -1, 1],
#     [3, 3, 9],
#     [3, 3, 5]
# ]
# results = [8, -42, 0]
# output = [
#     Fraction(12, 1), Fraction(11, 2), Fraction(-21, 2)]
# Fraction 可以強轉 float

import numpy as np

def gauss_elimination(matrix, ans):
    matrix = np.array(matrix)
    ans = np.array(ans)

    try:
        solution = np.linalg.solve(matrix, ans)
        return [f"{value:.2f}" for value in solution]
    except np.linalg.LinAlgError:
        # 無解或者無限多組解
        return "No Solution"

```

# 有開放 numpy 可以用  
 # 優點：行數短，執行速度快  
 # 缺點：只能用浮點數，無法區分無解及無限多組解

## 2.9 卡特蘭數

```
"""
卡特蘭數 Catalan
公式：H(n) = C(2 * n, n) // (n + 1), n >= 2, n 為正整數
快速計算方式：
1. H(0) = H(1) = 1, H(n)
   = sum(H(i - 1) * H(n - i) for i in range(1, n + 1))
2. H(n) = H(n - 1) * (4 * n - 2) // (n + 1)
3. H(n) = C(2 * n, n) - C(2 * n, n - 1)
"""

"""
可解問題：
有效括號匹配問題：
    給定 n 個左括號與右括號，求有幾種不同的正確括號匹配
二元樹結構問題：給定 n 個節點，求有幾種不同的二元樹結構
將一個凸
    n + 2 邊形劃分成多個三角形，求有幾種不同的劃分方式
狄克路徑：給定 n * n 的網格，
    從左下到右上的路徑中，永不超過對角線的路徑有幾種
一個 stack 在 push 順
    序不變的情況下 (1, 2, 3, ..., n)，有幾種 pop 的方式
在圖上選擇 2 * n 個
    點，將這些點兩兩連接使得 n 條線段不相交的方法有幾種
"""

n = int(input())

catalan = [1 for _ in range(n + 1)]
for i in range(1, n + 1):
    catalan[i] = catalan[i - 1] * (4 * i - 2) // (i + 1)

ans = 0
for i in range(0, n + 1): # 卡特蘭數的平方
    ans += catalan[i] * catalan[n - i]

print(ans)
# 185ms in codeforces, n <= 5000
```

## 2.10 中國剩餘定理

```
// vec[i] = {m_i, x_i}, 求最小非負 x
// 使得 x ≡ x_i (mod m_i) 對所有 i 同時成立；無解回 -1
// 注意 overflow
int CRT(vector<pair<int, int>> &v)
{
    int m = v[0].first, x = (v[0].second % m + m) % m;
    for (int i = 1; i < (int)v.size(); ++i)
    {
        int mi =
            v[i].first, xi = (v[i].second % mi + mi) % mi;
        int g = gcd(m, mi), d = xi - x;
        if (d % g) return -1;
        int m1 = m / g, m2 = mi / g;
        auto ab = ext_gcd((int)m1, (int)m2);
        int inv = ((int)ab.first % m2 + m2) % m2;
        int k = ((d / g) % m2 + m2) % m2;
        k = (k * inv) % m2;
        x = (x + m * k) % (m * m2);
        m *= m2;
        x = (x + m) % m;
    }
    return x;
}
```

## 2.11 Theorem

- Cramer's rule

$$\begin{aligned} ax+by &= e & x &= \frac{ed-bf}{ad-bc} \\ cx+dy &= f & y &= \frac{af-ec}{ad-bc} \end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem
  - Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .
    - The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
    - The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .
- Tutte's Matrix
  - Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .
- Cayley's Formula
  - Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
  - Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .
- Erdős–Gallai theorem
  - A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every  $1 \leq k \leq n$ .
- Gale–Ryser theorem
  - A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .
- Fulkerson–Chen–Anstee theorem
  - A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .
- Pick's theorem
  - For simple polygon, when points are all integer, we have  $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$ .
- Möbius inversion formula
  - $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$
- Spherical cap
  - A portion of a sphere cut off by a plane.
  - $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
  - Volume  $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$ .
  - Area  $= 2\pi rh = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$ .
- Lagrange multiplier
  - Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
  - Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.
- Nearest points of two skew lines
  - Line 1:  $v_1 = p_1 + t_1 d_1$
  - Line 2:  $v_2 = p_2 + t_2 d_2$
  - $n = d_1 \times d_2$
  - $n_1 = d_1 \times n$
  - $n_2 = d_2 \times n$
  - $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
  - $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$
- Derivatives/Integrals
  - Integration by parts:  $\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$
  - $\left| \frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \right| \left| \frac{d}{dx} \cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \right| \left| \frac{d}{dx} \tan^{-1}x = \frac{1}{1+x^2} \right|$
  - $\left| \frac{d}{dx} \tan x = 1 + \tan^2 x \right| \left| \int \tan ax = -\frac{\ln|\cos ax|}{a} \right|$
  - $\left| \int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) \right| \left| \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) \right|$
  - $\int \sqrt{a^2 + x^2} = \frac{1}{2} (x\sqrt{a^2 + x^2} + a^2 \text{asinh}(x/a))$
- Spherical Coordinate
  - $(x, y, z) = (r \sin\theta \cos\phi, r \sin\theta \sin\phi, r \cos\theta)$
  - $(r, \theta, \phi) = (\sqrt{x^2 + y^2 + z^2}, \arccos(z/\sqrt{x^2 + y^2 + z^2}), \arctan2(y, x))$

- Rotation Matrix

$$M(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

## 2.12 Estimation

$n$	2	3	4	5	6	7	8	9	20	30	40	50	100		
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8		
$n$	100	1e3	1e6	1e9	1e12	1e15	1e18								
$d(i)$	12	32	240	1344	6720	26880	103680								
$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8
$n$	2	3	4	5	6	7	8	9	10	11	12	13			
$B_n$	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7			

## 2.13 Euclidean Algorithms

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + 2 \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 2.14 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1 = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$ :  $j$ 's s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$ :  $j$ 's s.t.  $\pi(j) \geq j$ ,  $k$ :  $j$ 's s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 2.15 Tips for Generating Functions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x)' \Rightarrow na_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x) \Rightarrow na_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$

## 3 Graph

### 3.1 DSU

```
class dsu{
public:
    vector<int> parent;
    dsu(int num){
        parent.resize(num);
        for(int i = 0; i < num; i++) parent[i] = i;
    }
    int find(int x){
        if(parent[x] == x) return x;
        return parent[x] = find(parent[x]);
    }
    bool same(int a, int b){
        return find(a) == find(b);
    }
    void Union(int a, int b){
        parent[find(a)] = find(b);
    }
};
```

### 3.2 Dijkstra

```
// 傳入圖的 pair 為 {權重, 點}, 無限大預設 1e9 是情況改
#define pii pair<int, int>
vector<
    int> dijkstra(vector<vector<pii>> &graph, int src){
    int n = graph.size();
    vector<int> dis(n, 1e9);
    vector<bool> vis(n, false);
    priority_queue<pii, vector<pii>, greater<pii>> pq;
    pq.push({0, src});
    dis[src] = 0;
    while(!pq.empty()){
        auto [w, node] = pq.top();
        pq.pop();
        if(vis[node]) continue;
        vis[node] = true;
        for(auto [nw, nn]: graph[node]){
            if(w + nw < dis[nn]){
                dis[nn] = w + nw;
                pq.push({dis[nn], nn});
            }
        }
    }
    return dis;
}
```

### 3.3 SPFA

```
#define pii pair<int, int>
// {在 src 可到達
// 的點中是否存在負環, 最短路徑}, arg 中 n 為點的數量
// arg 中 pair 裡的第一個值為權重, 第二個為點
pair<bool, vector<int>>
SPFA(vector<vector<pii>> &graph, int n, int src){
    vector<int> dis(n + 1, 1e9);
    vector<int> cnt(n + 1, 0);
    vector<bool> vis(n + 1, false);
    queue<int> q;
    vis[src] = true; q.push(src); dis[src] = 0;
    while(!q.empty()){
        auto node = q.front(); vis[node] = false; q.pop();
```

```

for(auto [w, nn]:graph[node]){
    if(w + dis[node] < dis[nn]){
        dis[nn] = w + dis[node];
        if(!vis[nn]){
            if(++cnt[nn] >= n) return {true, {}};
            q.push(nn);
            vis[nn] = true;
        }
    }
}
return {false, dis};
}

```

### 3.4 Floyd Warshell

// 中繼點放外面

```

for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            dis[i][j] = min(dis[i][j], dis[i][k] + dis[k][j]);
        }
    }
}

```

### 3.5 Tarjan SCC

```

class tarjan{
    // 1-base
    int time = 1;
    int id = 1;
    stack<int> s;
    vector<int> low;
    vector<int> dfn;
    vector<bool> in_stack;
    void dfs(int node, vector<vector<int>> &graph){
        in_stack[node] = true;
        s.push(node);
        dfn[node] = low[node] = time++;
        for(auto &j : graph[node]){
            if(dfn[j] == 0){
                dfs(j, graph);
                // 看看往下有沒有辦法回到更上面的點
                low[node] = min(low[node], low[j]);
            }
            else if(in_stack[j]){
                low[node] = min(low[node], low[j]);
            }
        }
        vector<int> t; // 儲存這個強連通分量
        if(dfn[node] == low[node]){
            while(s.top() != node){
                t.push_back(s.top());
                in_stack[s.top()] = false;
                scc_id[s.top()] = id;
                s.pop();
            }
            t.push_back(s.top());
            scc_id[s.top()] = id;
            in_stack[s.top()] = false;
            s.pop();
            id++;
        }
        if(!t.empty()) ans.push_back(t);
    }
public:
    vector<int> scc_id;
    vector<vector<int>> ans;
    // ans ans[i] 代表第 i 個強連通分量裡面包涵的點
    // scc_id[i] 代表第 i 個點屬於第幾個強連通分量
    vector
    <vector<int>> scc(vector<vector<int>> &graph){
        int num = graph.size();
        scc_id.resize(num, -1);
        dfn.resize(num, 0);
        low.resize(num, 0);
        in_stack.resize(num, false);
        for(int i = 1; i < num; i++){
            if(dfn[i] == 0) dfs(i, graph);
        }
        return ans;
    }
};

```

### 3.6 2 SAT

// (a || b) && (c || d) && (e || f) .....

// 用

下面的 tarjan scc 算法來解 2 sat 問題，若事件 a 發生時，事件 b 必然發生，我們須在 a -> b 建立一條有向

// 用

cses 的 Giant Pizza 來舉例子，給定 n 個人 m 個配料表，每個人可以提兩個要求，兩個要求至少要被滿足一個

// 3 5

// + 1 + 2

// - 1 + 3

// + 4 - 2

// 以這

個例子來說，第一個人要求要加 配料1 或者 配料2 其中一項，第二個人要求不要 配料1 或者 要配料3 其中一項

// 試問能不能滿足所有人的要求，我們可以把 要加

配料 i 當作點 i，不加配料 i 當作點 i + m (配料數量)

// 關於第一個人的要求 我們可以看成若不加 配

料1 則必定要 配料2 以及 若不加 配料2 則必定要 配料1

// 關於第二個人要求 可看做加了 配料

1 就必定要加 配料3 以及 不加 配料3 就必定不加 配料1

// 以這些條件建立有向圖，並且

找尋 scc，若 i 以及 i + m 在同一個 scc 中代表無解

// 若要求解，則若 i 的 scc\_id

小於 i + m 的 scc\_id 則 i 為 true，反之為 false

// tarjan 的模板在上面

cin >> n >> m;

vector<vector<int>> graph(m \* 2 + 1);

function<int(int)> tr = [&](int x){

if(x > m) return x - m;

return x + m;

};

for(int i = 0; i < n; i++){

char c1, c2;

int a, b;

cin >> c1 >> a >> c2 >> b;

// a 代表 a 為真，m + a 代表 a 為假

if(c1 == '-') a += m;

if(c2 == '-') b += m;

graph[tr(a)].push\_back(b);

graph[tr(b)].push\_back(a);

}

tarjan t;

auto scc = t.scc(graph);

for(int i = 1; i <= m; i++){

if(t.scc\_id[i] == t.scc\_id[tr(i)]){

cout << "IMPOSSIBLE\n";

return 0;

}

for(int i = 1; i <= m; i++){

if(t.scc\_id[i] < t.scc\_id[tr(i)]){

cout << '+';

}

else cout << '-';

cout << ' ';

}

cout << '\n';

### 3.7 Euler Path

// 1. 無向圖是歐拉圖：

// 非零度頂點是連通的

// 頂點的度數都是偶數

// 2. 無向圖是半歐拉圖(有路沒有環)：

// 非零度頂點是連通的

// 恰有 2 個奇度頂點

// 3. 有向圖是歐拉圖：

// 非零度頂點是強連通的

// 每個頂點的入度和出度相等



```
// 4. 有向圖是半歐拉圖(有路沒有環):
// 非零度頂點是弱連通的
// 至多一個頂點的出度與入度之差為 1
// 至多一個頂點的入度與出度之差為 1
// 其他頂點的入度和出度相等
vector<set<int>> adj;
vector<int> ans;

void dfs(int x) { // Hierholzer's Algorithm
    while (!adj[x].empty()) {
        auto next = *(adj[x].begin());
        adj[x].erase(next);
        adj[next].erase(x);
        dfs(next);
    }
    ans.emplace_back(x);
}

void solve() {
    // 建立雙向邊, set用來防重邊, 點數n, 邊數m
    for (int i = 1; i <= n; i++)
        if (adj[i].size() & 1) return; /* impossible */
    dfs(1);
    if (ans.size() != m + 1) return; /* impossible */
    reverse(ans.begin(), ans.end()); /* then print it */
}
```

### 3.8 Bridge

```
// 橋: 若移除邊會使連通分量變多
// [USAGE] ECC ecc(n); ecc.add_edge(u, v); ecc.solve();
// is_bridge[i]; necc; bln[i];
// 邊是否為橋: 橋連通分量數量; 頂點所屬橋連通分量編號
struct ECC { // 0-base
    int n, dft, ecnt, necc;
    vector<int> low, dfn, bln, is_bridge, stk;
    vector<vector<pii>> G;
    void dfs(int u, int f) {
        dfn[u] = low[u] = ++dft, stk.pb(u);
        for (auto [v, e] : G[u])
            if (!dfn[v])
                dfs(v, e), low[u] = min(low[u], low[v]);
            else if (e != f)
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            if (f != -1) is_bridge[f] = 1;
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = necc;
            bln[u] = necc++, stk.pop_back();
        }
    }
    ECC(int _n): n(_n), dft(),
        , ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
    }
    void solve() {
        is_bridge.resize(ecnt);
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
}; // 8BQube
```

### 3.9 Max flow min cut

```
#define int long long

// dicnic Algorithm Time:  $O(V^2E)$  實際上會快一點
// 記得在 main 裡面 resize graph
// 最小割, 找
// 到最少條的邊切除, 使得從 src 到 end 的 maxflow 為 0
// 枚舉所有邊  $i \rightarrow j$ , src 可
// 以到達 i 但無法到達 j, 那這條邊為最小割裡的邊之一
// 無向圖最大流: 修改 add_edge, 反向邊建為 capacity
// 使用時只要 add_edge 一次

class edge{
public:
    int next;
    int capacity;
    int rev;
    bool is_rev;
    edge(int _n, int _c,
        int _r, int _co, int _ir) : next(_n), capacity
        (_c), rev(_r), cost(_co), is_rev(_ir){};
};
```

```
edge(int _n, int _c, int _r, int _ir) :
    next(_n), capacity(_c), rev(_r), is_rev(_ir){};
};

vector<vector<edge>> graph;
vector<int> level, iter;

void add_edge(int a, int b, int capacity){
    graph[a].push_back
        (edge(b, capacity, graph[b].size(), false));
    graph[b].
        push_back(edge(a, 0, graph[a].size() - 1, true));
}

void bfs(int start) {
    fill(level.begin(), level.end(), -1);
    queue<int> q;
    level[start] = 0;
    q.push(start);
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto& e : graph[v]) {
            if (e.capacity > 0 && level[e.next] < 0) {
                level[e.next] = level[v] + 1;
                q.push(e.next);
            }
        }
    }
}

int dfs(int v, int end, int flow) {
    if (v == end) return flow;
    for (int &i = iter[v]; i < graph[v].size(); i++) {
        edge &e = graph[v][i];
        if (e.capacity > 0 && level[v] < level[e.next]) {
            int d = dfs(e.next, end, min(flow, e.capacity));
            if (d > 0) {
                e.capacity -= d;
                graph[e.next][e.rev].capacity += d;
                return d;
            }
        }
    }
    return 0;
}

int maxflow(int start, int end) {
    int flow = 0;
    level.resize(graph.size() + 1);
    while (true) {
        bfs(start);
        if (level[end] < 0) return flow;
        iter.assign(graph.size() + 1, 0);
        int f;
        while ((f = dfs(start, end, 1e9)) > 0) {
            flow += f;
        }
    }
}
```

### 3.10 Minimum cost maximum flow

```
#define int long long
#define pii pair<int, int>

// Edmonds-Karp Algorithm Time:  $O(VE^2)$  實際上會快一點
// 一條邊的費用為 單位花費 * 流過流量
// 把原本的 BFS 換成 SPFA 而已
// 記得在 main 裡面 resize graph
// MCMF 回傳 {flow, cost}
// 無向圖: add_edge(u,v,C,W), add_edge(v,u,C,W);

class edge{
public:
    int next;
    int capacity;
    int rev;
    int cost;
    bool is_rev;
    edge(int _n, int _c,
        int _r, int _co, int _ir) : next(_n), capacity
        (_c), rev(_r), cost(_co), is_rev(_ir){};
};
```

```

vector<vector<edge>> graph;

void add_edge(int a, int b, int capacity, int cost){
    graph[a].push_back(
        edge(b, capacity, graph[b].size(), cost, false));
    graph[b].push_back(
        (edge(a, 0, graph[a].size() - 1, -cost, true));
}

pii dfs(int now
    , int end, pii data, vector<pii> &path, int idx){
    auto [flow, cost] = data;
    if(now == end) return {flow, 0};
    auto &e = graph[now][path[idx + 1].second];
    if(e.capacity > 0){
        auto [ret, nc] = dfs(e.next, end, {min(flow
            , e.capacity), cost + e.cost}, path, idx + 1);
        if(ret > 0){
            e.capacity -= ret;
            graph[e.next][e.rev].capacity += ret;
            return {ret, nc + ret * e.cost};
        }
    }
    return {0, 0};
}

vector<pii> search_path(int start, int end){
    int n = graph.size() + 1;
    vector<int> dis(n + 1, 1e9);
    vector<bool> vis(n + 1, false);
    vector<pii> ans; queue<int> q;
    vis[start] = true; q.push(start); dis[start] = 0;
    vector<pii> parent(graph.size(), {-1, -1});
    q.push(start);
    while(!q.empty()){
        auto node = q.front(); vis[node] = false; q.pop();
        for(int i = 0; i < graph[node].size(); i++){
            auto &e = graph[node][i];
            if(e.capacity
                > 0 and e.cost + dis[node] < dis[e.next]){
                dis[e.next] = e.cost + dis[node];
                parent[e.next] = {node, i};
                if(!vis[e.next]){
                    q.push(e.next);
                    vis[e.next] = true;
                }
            }
        }
    }
    if(parent[end].first == -1) return ans;
    int now = end;
    while(now != start){
        auto [node, idx] = parent[now];
        ans.emplace_back(node, idx);
        now = node;
    }
    ans.emplace_back(start, -1);
    reverse(ans.begin(), ans.end());
    return ans;
}

pii MCMF(int start, int end){
    int ans = 0, cost = 0;
    while(1){
        vector<bool> visited(graph.size() + 1, false);
        auto tmp = search_path(start, end);
        if(tmp.size() == 0) break;
        auto [flow, c] = dfs(start, end, {1e9, 0}, tmp, 0);
        ans += flow;
        cost += c;
    }
    return {ans, cost};
}

```

### 3.11 二分圖

/\*  
判定二分圖：著色法 dfs 下去，顏色相撞非二分圖

二分圖最大匹配：用 maxflow 去做，一個 src  
點聯通所有左圖，左圖建邊向右圖，右圖再建邊向 end  
點，計算 src 跟 end 的最大流，若要還原，找出左圖  
通往右圖中 capacity 為 0 的邊，他的兩個端點就是答案

最小點覆蓋：選最少的點，保證每條邊  
至少有一個端點被選到， 最小點覆蓋 = 二分圖最大匹配

最大獨立集：選最多的點，滿足這些  
點兩兩間互不相連， 最大獨立集 =  $n$  - 二分圖最大匹配  
\*/

### 3.12 Check cycle

```

vector<int> G[MAXN];
bool visit[MAXN];
/* return if the connected component where u is
contains a cycle*/
bool dfs(int u, int pre) {
    if(visit[u]) return true;
    visit[u] = true;
    for(int v : G[u])
        if(v != pre && dfs(v, u))
            return true;
    return false;
}

```

//check if a graph contains a cycle

```

bool checkCycle(int n) {
    for(int i = 1; i <= n; i++)
        if(!visit[i] && dfs(i, -1))
            return true;
    return false;
}

```

### 3.13 BCC

// [USAGE] bcc.bcc(n); bcc.add\_edge(u, v); bcc.solve();  
// bcc.is\_ap[i]; // i 是否為割點  
// bcc.bcc[j]; // 第 j 個點雙連通分量中包含的所有頂點  
// bcc.nbcc; // 點雙連通分量數量

// [USAGE] bcc.block\_cut\_tree();  
// bcc.ng[i]; // 新圖頂點 i 的所有鄰居  
// bcc  
.bln[i]; // 原圖中的 i 在新圖上的編號(i可以是割點)  
// bcc.cir[j]; // 新圖上的頂點 j 是否為割點

```

struct BCC { // 0-base
    int n, dft, nbcc;
    vector<int> low, dfn, bln, stk, is_ap, cir;
    vector<vector<int>> G, bcc, nG;
    void make_bcc(int u) {
        bcc.emplace_back(1, u);
        for (; stk.back() != u; stk.pop_back())
            bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
        stk.pop_back(), bln[u] = nbcc++;
    }
    void dfs(int u, int f) {
        int child = 0;
        low[u] = dfn[u] = ++dft, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v]) {
                dfs(v, u), ++child;
                low[u] = min(low[u], low[v]);
                if (dfn[u] <= low[v]) {
                    is_ap[u] = 1, bln[u] = nbcc;
                    make_bcc(v), bcc.back().pb(u);
                }
            } else if (dfn[v] < dfn[u] && v != f)
                low[u] = min(low[u], dfn[v]);
        if (f == -1 && child < 2) is_ap[u] = 0;
        if (f == -1 && child == 0) make_bcc(u);
    }
    BCC(int _n): n(_n), dft(),
        nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(v), G[v].pb(u);
    }
    void solve() {
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
    void block_cut_tree() {
        cir.resize(nbcc);
        for (int i = 0; i < n; ++i)
            if (is_ap[i])
                bln[i] = nbcc++;
        cir.resize(nbcc, 1), nG.resize(nbcc);
        for (int i = 0; i < nbcc && !cir[i]; ++i)
            for (int j : bcc[i])

```



```

        if (is_ap[j])
            nG[i].pb(bln[j]), nG[bln[j]].pb(i);
    } // up to 2 * n - 2 nodes!! bln[i] for id
}; // 8BQube

```

## 4 String

### 4.1 trie

```

class trie{
public:
    class node{
    public:
        int count;
        vector<trie::node*> child;
        node(){
            child.resize(26, nullptr);
            count = 0;
        }
        ~node() {
            for (auto c : child)
                if (c) delete c;
        }
    };
    node* root;
    trie(){
        root = new node;
    }
    ~trie() {
        delete root;
    }
    void insert(string s){
        auto temp = root;
        for(int i = 0; i < s.size(); i++){
            if(!temp -> child[s[i] - 'a'])
                temp -> child[s[i] - 'a'] = new node;
            temp = temp -> child[s[i] - 'a'];
        }
        temp -> count++;
    }
    bool search(string &s){
        auto temp = root;
        for(int i = 0; i < s.size(); i++){
            temp = temp -> child[s[i] - 'a'];
            if(!temp) return false;
        }
        if(temp -> count > 0) return true;
        return false;
    }
};

```

### 4.2 KMP

```

vector<int> build(string &s){
    vector<int> next = {0, 0};
    // 匹配失敗跳去哪 (最長共同前後綴)
    int length = s.size(), j = 0;
    for(int i = 1; i < length; i++){
        while(j > 0 and s[j] != s[i]){
            j = next[j];
        }
        if(s[j] == s[i]) j++;
        next.push_back(j);
    }
    return next;
}

int match(string &a, string &b){
    auto next = build(b);
    int length = a.size(), length2 = b.size(), j = 0, count = 0;
    for(int i = 0; i < length; i++){
        while(j > 0 and a[i] != b[j]){
            j = next[j];
        }
        if(a[i] == b[j]) j++;
        if(j == length2){
            count++;
            j = next[j];
        }
    }
    return count;
}

```

### 4.3 Hash

```

vector<int> Pow(int num){
    int p = 1e9 + 7;
    vector<int> ans = {1};
    for(int i = 0; i < num; i++){
        ans.push_back(ans.back() * b % p);
    }
    return ans;
}

vector<int> Hash(string s){
    int p = 1e9 + 7;
    vector<int> ans = {0};
    for(char c:s){
        ans.push_back((ans.back() * b + c) % p);
    }
    return ans;
}

// 閉區間[l, r]
int query
(vector<int> &vec, vector<int> &pow, int l, int r){
    int p = 1e9 + 7;
    int length = r - l + 1;
    return
        (vec[r + 1] - vec[l] * pow[length] % p + p) % p;
}

```

### 4.4 Zvalue

```

vector<int> z_func(string s1){
    int l = 0, r = 0, n = s1.size();
    vector<int> z(n, 0);
    for(int i = 1; i < n; i++){
        if(i
            <= r and z[i - l] < r - i + 1) z[i] = z[i - l];
        else{
            z[i] = max(z[i], r - i + 1);
            while(i + z
                [i] < n and s1[i + z[i]] == s1[z[i]]) z[i]++;
        }
        if(i + z[i] - 1 > r){
            l = i;
            r = i + z[i] - 1;
        }
    }
    return z;
}

```

### 4.5 最長迴文子串

```

// 找到對於每個位置的迴文半徑
vector<int> manacher(string s) {
    string t = "#";
    for (auto c : s) {
        t += c;
        t += '#';
    }
    int n = t.size();
    vector<int> r(n);
    for (int i = 0, j = 0; i
        < n; i++) { // i 是中心, j 是最長回文字串中心
        if (2 * j - i >= 0 && j + r[j] > i) {
            r[i] = min(r[2 * j - i], j + r[j] - i);
        }
        while (i - r[i] >= 0 &&
            i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
            r[i] += 1;
        }
        if (i + r[i] > j + r[j]) {
            j = i;
        }
    }
    return r;
}

// # a # b # a #
// 1 2 1 4 1 2 1
// # a # b # b # a #
// 1 2 1 2 5 2 1 2 1
// 值 -1 代表原回文字串長度
// (id - val + 1) / 2 可得原字串回文開頭

```

## 4.6 Suffix Array

```
struct SuffixArray {
    int n; string s;
    vector<int> sa, rk, lc;
    // 想法：
    // 排序過了，因此前綴長得像的會距離很近在差不多位置
    // n: 字串長度
    // sa: 後綴數組，sa[i] 表示第 i 小的後綴的起始位置
    // rk: 排名數組，rk[i] 表示從位置 i 開始的後綴的排名
    // lc: LCP 數組，
    // lc[i] 表示 sa[i] 和 sa[i + 1] 的最長公共前綴長度
    // 求 sa[i] 跟 sa[j] 的
    // LCP 長度 當 i < j : min(lc[i] ..... lc[j - 1])
    // 求 longest common substring : A +
    // "A" + B 建立 SA，找到 sa 相鄰但不同組中 lc 最大的
    SuffixArray(const string &s_) {
        s = s_; n = s.length();
        sa.resize(n);
        lc.resize(n - 1);
        rk.resize(n);
        iota(sa.begin(), sa.end(), 0);
        sort(sa.begin(), sa.end(),
            [&](int a, int b) { return s[a] < s[b]; });
        rk[sa[0]] = 0;
        for (int i = 1; i < n; ++i)
            rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
        int k = 1;
        vector<int> tmp, cnt(n);
        tmp.reserve(n);
        while (rk[sa[n - 1]] < n - 1) {
            tmp.clear();
            for (int i = 0; i < k; ++i)
                tmp.push_back(n - k + i);
            for (auto i : sa)
                if (i >= k)
                    tmp.push_back(i - k);
            fill(cnt.begin(), cnt.end(), 0);
            for (int i = 0; i < n; ++i)
                ++cnt[rk[i]];
            for (int i = 1; i < n; ++i)
                cnt[i] += cnt[i - 1];
            for (int i = n - 1; i >= 0; --i)
                sa[--cnt[rk[tmp[i]]]] = tmp[i];
            swap(rk, tmp);
            rk[sa[0]] = 0;
            for (int i = 1; i < n; ++i)
                rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] ||
                    sa[i - 1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
            k *= 2;
        }
        for (int i = 0, j = 0; i < n; ++i) {
            if (rk[i] == 0) {
                j = 0;
            } else {
                for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j] == s[sa[rk[i] - 1] + j];)
                    ++j;
                lc[rk[i] - 1] = j;
            }
        }
    }
};
```

## 4.7 AC-Automatan

```
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], ord[len], top;
    int rxn[len][sigma]; // node actually be reached
    int newnode() {
        fill_n(nx[top], sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(string &s) {
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
            X = nx[X][c - 'A'];
        }
        return X; // return the end node of string
    }
    void make_fl() {
```

```
queue<int> q;
q.push(1), fl[1] = 0;
for (int t = 0; !q.empty(); ) {
    int R = q.front();
    q.pop(), ord[t++] = R;
    for (int i = 0; i < sigma; ++i)
        if (~nx[R][i]) {
            int X = rxn[R][i] = nx[R][i], Z = fl[R];
            for (; Z && !~nx[Z][i]; ) Z = fl[Z];
            fl[X] = Z ? nx[Z][i] : 1, q.push(X);
        }
    else rxn[R][i] = R > 1 ? rxn[fl[R]][i] : 1;
}
}
void solve() {
    for (int i = top - 2; i > 0; --i)
        cnt[fl[ord[i]]] += cnt[ord[i]];
}
} ac;
```

## 5 Geometry

### 5.1 Point

```
template<typename T>
class point {
public:
    T x;
    T y;
    point(){}
    point(T _x, T _y){
        x = _x;
        y = _y;
    }
    point<T> operator+(const point<T> &a);
    point<T> operator-(const point<T> &a);
    point<T> operator/(const point<T> &a);
    point<T> operator/(T a);
    point<T> operator*(const T &a);
    bool operator<(const point<T> &a);
};

template<typename T>
point<T> point<T>::operator+(const point<T> &a){
    return point<T>(x + a.x, y + a.y);
}

template<typename T>
point<T> point<T>::operator-(const point<T> &a){
    return point<T>(x - a.x, y - a.y);
}

template<typename T>
point<T> point<T>::operator/(const point<T> &a){
    return point<T>(x / a.x, y / a.y);
}

template<typename T>
point<T> point<T>::operator/(T a){
    return point<T>(x / a, y / a);
}

template<typename T>
point<T> point<T>::operator*(const T &a){
    return point<T>(x * a, y * a);
}

template<typename T>
bool point<T>::operator<(const point<T> &a){
    if (x != a.x) return x < a.x;
    return y < a.y;
}
```

### 5.2 內積, 外積, 距離

```
template<typename T>
T dot(const point<T> &a, const point<T> &b){
    return a.x * b.x + a.y * b.y;
}

template<typename T>
T cross(const point<T> &a, const point<T> &b){
    return a.x * b.y - a.y * b.x;
}

template<typename T>
```

```

T len(point<T> p){
    return sqrt(dot(p, p));
}

template<typename T>
int sign(T x){
    return x == 0 ? 0 : x > 0 ? 1 : -1;
}

template<typename T>
T pointSegDist(point<T> q0, point<T> q1, point<T> p) {
    if (sign(len(q0 - q1)) == 0) return len(q0 - p);
    if (sign(dot(q1 - q0, p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return abs(cross(q1 - q0, p - q0) / len(q0 - q1));
    return min(len(p - q0), len(p - q1));
}

```

### 5.3 向量應用

```

template<typename T>
bool collinearity
(point<T> p1, point<T> p2, point<T> p3){
    //檢查三點是否共線
    return cross(p2 - p1, p2 - p3) == 0;
}

template<typename T>
bool inline(point<T> a, point<T> b, point<T> p){
    //檢查 p 點是否在ab線段
    return collinearity
        (a, b, p) && dot(a - p, b - p) <= 0;
}

template<typename T>
bool intersect
(point<T> a, point<T> b, point<T> c, point<T> d){
    //ab線段跟cd線段是否相交
    return (cross(b - a, c - a) * \
        cross(b - a, d - a) < 0 && \
        cross(d - c, a - c) * \
        cross(d - c, b - c) < 0) \
        || inLine(a, b, c) || \
        inLine(a, b, d) || inLine(c, d, a) \
        || inLine(c, d, b);
}

template<typename T>
point<T> intersection
(point<T> a, point<T> b, point<T> c, point<T> d){
    //ab線段跟cd線段相交的點
    assert(intersect(a, b, c, d));
    return a + (b -
        a) * cross(a - c, d - c) / cross(d - c, b - a);
}

template<typename T>
bool inPolygon(vector<point<T>> polygon, point<T> p){
    //判斷點
    //p是否在多邊形polygon裡，vector裡的點要連續填對
    for(int i = 0; i < polygon.size(); i++){
        if(cross(p - polygon[i], \
            polygon[(i - 1 + polygon.size()) % \
                polygon.size()] - polygon[i]) * \
            cross(p - polygon[i], \
                polygon[(i + 1) % polygon.size()] - polygon[i]) > 0)
            return false;
        return true;
    }
}

template<typename T>
T triangleArea(point<T> a, point<T> b, point<T> c){
    //三角形頂點，求面積
    return abs(cross(b - a, c - a)) / 2;
}

template<typename T, typename F, typename S>
long double triangleArea_Herons_formula(T a, F b, S c){
    //三角形頂點，求面積(給邊長)
    auto p = (a + b + c) / 2;
    return sqrt(p * (p - a) * (p - b) * (p - c));
}

```

```

template<typename T>
T area(vector<point<T>> &p){
    //多邊形頂點，求面積
    T ans = 0;
    for(int i = 0; i < p.size(); i++){
        ans += cross(p[i], p[(i + 1) % p.size()]);
    }
    return ans / 2 > 0 ? ans / 2 : -ans / 2;
}

```

### 5.4 Static Convex Hull

// 需要使  
用前一個向量模板的 *point*，需要 *operator -* 以及 *<*  
// 需要前面向量模板的 *cross*

```

template<typename T>
vector<point<T>> getConvexHull(vector<point<T>>& pnts){
    sort(pnts.begin(), pnts.end());
    auto cmp = [&](point<T> a, point<T> b)
    { return a.x == b.y && a.x == b.y; };
    pnts.erase(unique
        (pnts.begin(), pnts.end(), cmp), pnts.end());
    if(pnts.size() <= 1) return pnts;
    vector<point<T>> hull;
    for(int i = 0; i < 2; i++){
        int t = hull.size();
        for(point<T> pnt : pnts){
            while(hull.size() - t >= 2 &&
                cross(hull.back() - hull[hull.size() - 2], pnt - hull[hull.size() - 2]) < 0)
                // <= 0 或者 < 0 要看點有沒有在邊上
                hull.pop_back();
            hull.push_back(pnt);
        }
        hull.pop_back();
        reverse(pnts.begin(), pnts.end());
    }
    return hull;
}

```

### 5.5 外心, 最小覆蓋圓

```

int sign(double a)
{
    // 小於 eps
    // 回傳 0，否則正回傳 1，負回傳 -1 應付浮點數誤差用
    const double eps = 1e-10;
    return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;
}

// 輸入三個點求外心
template<typename T>
point<T> findCircumcenter(point<T> A, point<T> B, point<T> C, const T eps = 1e-10){
    point<T> AB = B - A;
    point<T> AC = C - A;
    T AB_len_sq = AB.x * AB.x + AB.y * AB.y;
    T AC_len_sq = AC.x * AC.x + AC.y * AC.y;
    T D = AB.x * AC.y - AB.y * AC.x;
    // 若三點接近共線
    assert(fabs(D) < eps);
    // 外心的座標
    T circumcenterX = A.x + (
        AC.y * AB_len_sq - AB.y * AC_len_sq) / (2 * D);
    T circumcenterY = A.y + (
        AB.x * AC_len_sq - AC.x * AB_len_sq) / (2 * D);
    return point<T>(circumcenterX, circumcenterY);
}

template<typename T>
pair<T, point<T>> MinCircleCover(vector<point<T>> &p) {
    // 引入前面的 len 跟 point
    // 回傳最小覆蓋圓{半徑, 中心}
    random_shuffle(p.begin(), p.end());
    int n = p.size();
    point<T> c = p[0]; T r = 0;
    for(int i=1; i<n; i++){
        if(sign(len(c-p[i])-r) > 0) { // 不在圓內
            c = p[i], r = 0;
            for(int j=0; j<i; j++){
                if(sign(len(c-p[j])-r) > 0) {
                    c = (p[i]+p[j])/2.0;
                    r = len(c-p[i]);
                    for(int k=0; k<j; k++) {

```

```

        if(sign(len(c-p[k])-r) > 0) {
            c = findCircumcenter
                (p[i],p[j],p[k]);
            r = len(c-p[i]);
        }
    }
}
}
return make_pair(r, c);
}

```

## 5.6 四邊形旋轉

```

const long double PI = acos(-1);

// 寬 w 高 h 的四邊形，旋轉一個 pi 後在每個角度的寬高
vector
    <pair<double, double>> rotate(double w, double h){
    int freq = 1000; // 自己調整精度
    vector<pair<double, double>> res;
    for (int i = 0; i <= 5; ++i) {
        double theta = (PI * i) / 5;
        double nw
            = c * fabs(cos(theta)) + d * fabs(sin(theta));
        double nh
            = c * fabs(sin(theta)) + d * fabs(cos(theta));
        res.push_back({nw, nh});
    }
    return res;
}

```

## 5.7 旋轉

```

const long double PI = acos(-1);
// 逆時針旋轉
// angle_rad 為弧度
pair<double, double> rotate_point
    (double x, double y, double angle_rad) {
    angle_rad *= PI;
    double
        new_x = x * cos(angle_rad) - y * sin(angle_rad);
    double
        new_y = x * sin(angle_rad) + y * cos(angle_rad);
    return {new_x, new_y};
}

int main() {
    double x = 5, y = 0;
    double angle = 0.5; // 逆時針旋轉 90 度
    auto result = rotate_point(x, y, angle);
    cout << result.first << " " << result.second << endl;
    // 0, 5
    return 0;
}

```

## 5.8 極座標轉直角座標

```

// 極座標轉換為直角座標函數，theta 單位為弧度
const long double PI = acos(-1);
pair<double, double>
    > polar_to_cartesian(double r, double theta) {
    double theta_radians = theta * PI;
    double x = r * cos(theta_radians);
    double y = r * sin(theta_radians);
    return {x, y};
}

int main() {
    double r = 5, theta = 0.5; // 極座標
    auto result = polar_to_cartesian(r, theta);
    cout << result.first << " " << result.second << endl;
    // 0, 5
    return 0;
}

```

## 5.9 直角座標轉極座標

```

// 直角座標轉換為極座標
const long double PI = acos(-1);
std::pair<double, double> cartesian_to_polar(double x, double y) {
    double r = sqrt(x * x + y * y);

```

```

    double theta = atan2(y, x) / PI;
    return {r, theta};
}

int main() {
    double x = 3, y = 4; // 直角座標
    auto result = cartesian_to_polar(x, y);
    cout << "r = " << result
        .first << ", theta = " << result.second << endl;
    // 5, 0.295167
    return 0;
}

```

# 6 Data Structure

## 6.1 Sparse Table

```

class Sparse_Table{
    // 0-base
    // 要改成找最大把min換成max就好
private:
public:
    int spt[500005][22][2];
    Sparse_Table(vector<int> &ar){
        int n = ar.size();
        for (int i = 0; i < n; i++){
            spt[i][0][0] = ar[i];
            // spt[i][0][1] = ar[i];
        }
        for (int j = 1; (1 << j) <= n; j++) {
            for (int i = 0; (i + (1 << j) - 1) < n; i++) {
                spt[i][j][0] = min(spt[i + (1 <<
                    (j - 1))] [j - 1][0], spt[i][j - 1][0]);
                // spt[i][j][1] = max(spt[i + (1 <<
                    (j - 1))] [j - 1][1], spt[i][j - 1][1]);
            }
        }
    }
    int query_min(int l, int r)
    {
        if(l>r) return INT_MAX;
        int j = (int) __lg(r - l + 1);
        ///j = 31 - __builtin_clz(r - l + 1);
        return min
            (spt[l][j][0], spt[r - (1 << j) + 1][j][0]);
    }
    int query_max(int l, int r)
    {
        if(l>r) return INT_MAX;
        int j = (int) __lg(r - l + 1);
        ///j = 31 - __builtin_clz(r - l + 1);
        return max
            (spt[l][j][1], spt[r - (1 << j) + 1][j][1]);
    }
};

```

## 6.2 Segement Tree

```

// 不想要區間加值就把每個函數裡面的 push 都移除
// 最外層呼叫時，每個 id 都傳 1

const int N = 200000 + 9;
int a[N];
int seg[4 * N];
int lazy[4 * N];

inline void pull(
    int id){ seg[id] = seg[id * 2] + seg[id * 2 + 1]; }

inline void apply(int id, int l, int r, int v){
    seg[id] += v * (r - l + 1);
    lazy[id] += v;
}

inline void push(int id, int l, int r){
    if (!lazy[id] || l == r) return;
    int mid = (l + r) / 2;
    apply(id * 2, l, mid, lazy[id]);
    apply(id * 2 + 1, mid + 1, r, lazy[id]);
    lazy[id] = 0;
}

void build(int id, int
    l, int r) { // 編號為 id 的節點，存的區間為 [l, r]
    if (l
        == r) { seg[id] = a[l]; return; } // 葉節點的值

```

```

    int mid =
        (l + r) / 2;                // 將區間切成兩半
    build(id * 2, l, mid);           // 左子節點
    build(id * 2 + 1, mid + 1, r);   // 右子節點
    pull(id);
}

// 區間查詢：回傳 [ql, qr] 的區間和
int query(int id, int l, int r, int ql, int qr) {
    if (r < ql || qr < l) return 0; // 交集為空
    if (ql <= l && r <= qr) return seg[id]; // 完全覆蓋
    push(
        id, l, r);                 // 下傳 lazy
    int mid = (l + r) / 2;
    return query(id * 2, l, mid, ql, qr) // 左
        + query(id * 2 + 1, mid + 1, r, ql, qr); // 右
    // 否則，往左、右進行遞迴
}

// 區間加值：將 [ql, qr] 每個位置都加上 x
void range_add
    (int id, int l, int r, int ql, int qr, int x) {
    if (r < ql || qr < l) return; // 交集為空
    if (ql <= l && r <= qr) { apply(id, l, r, x); return; } // 完全覆蓋
    push(id, l, r)
        ;                          // 下傳 lazy 再往下走
    int mid = (l + r) / 2;
    range_add
        (id * 2, l, mid, ql, qr, x); // 左
    range_add
        (id * 2 + 1, mid + 1, r, ql, qr, x); // 右
    pull(id);
}

// 單點修改 (設值版)：將 a[i] 改成 x
void modify(int id, int l, int r, int i, int x) {
    if (l == r) { seg[id] = x; return; }
    push(id, l, r); // 確保往下的值正確
    int mid = (l + r) / 2;
    if (i
        <= mid) modify(id * 2, l, mid, i, x); // 左
    else modify
        (id * 2 + 1, mid + 1, r, i, x); // 右
    pull(id);
}

```

### 6.3 Link Cut Tree

```

// 通常用於對樹上任兩點間的路徑做加值、修改、查詢等工作
// 與線段樹相同，要修改 LCT 的功能只需更改
// pull、push、fix、query 等函數，再加上需要的懶標即可
// 範例為樹上任兩點 x, y 路徑上的權值 xor
// 和，樹上任意點單點改值
const int N = 300005;
class LinkCutTree {
private:
#define lc(x) node[x].ch[0]
#define rc(x) node[x].ch[1]
#define fa(x) node[x].fa
#define rev(x) node[x].rev
#define val(x) node[x].val
#define sum(x) node[x].sum
    struct Tree {
        int val, sum, fa, rev, ch[2];
    } node[N];
    inline void pull(int x) {
        sum(x) = val(x) ^ sum(lc(x)) ^ sum(rc(x));
    }
    inline void reverse(int x) {
        swap(lc(x), rc(x));
        rev(x) ^= 1;
    }
    inline void push(int x) {
        if (rev(x)) {
            reverse(lc(x));
            reverse(rc(x));
            rev(x) ^= 1;
        }
    }
    inline bool get(int x) { return rc(fa(x)) == x; }
    inline bool isroot(int x) {

```

```

        return (lc(fa(x)) ^ x) && (rc(fa(x)) ^ x);
    }
    inline void update(int x) {
        if (!isroot(x)) update(fa(x));
        push(x);
    }
    void rotate(int x) {
        int y = fa(x), z = fa(y), d = get(x);
        if (!isroot(y))
            node[z].ch[get(y)] = x; // 重要，不能更換順序
        fa(x) = z;
        node[fa(node[x].ch[d ^ 1])].ch[d ^ 1] = y;
        node[x].ch[d ^ 1] = y;
        node[fa(y) = x].ch[d ^ 1] = y;
        pull(y), pull(x); // 先 y 再 x
    }
    void splay(int x) {
        update(x);
        for (int y = fa(x); !isroot(x);
            rotate(x), y = fa(x)) {
            if (!isroot(y)) rotate(get(x) == get(y) ? y : x);
        }
        pull(x);
    }
    int access(int x) {
        int p = 0;
        for (; x; x = fa(p = x)) {
            splay(x), rc(x) = p, pull(x);
        }
        return p;
    }
    inline void makeroot(int x) {
        access(x), splay(x), reverse(x);
    }
    inline int findroot(int x) {
        access(x), splay(x);
        while (lc(x)) { push(x), x = lc(x); }
        return splay(x), x;
    }
    inline void split(int x, int y) {
        makeroot(x), access(y), splay(y);
    }
public:
    inline void init(int len, int *data) {
        for (int i = 1; i <= len; ++i) {
            node[i].val = data[i];
        }
    }
    inline void link(int x, int y) { // 連邊
        makeroot(x);
        if (findroot(y) == x) return;
        fa(x) = y;
    }
    inline void cut(int x, int y) { // 斷邊
        makeroot(x);
        if (findroot(y) != x || fa(y) != x || lc(y))
            return;
        fa(y) = rc(x) = 0;
        pull(x);
    }
    inline void fix(int x, int v) { // 單點改值
        splay(x);
        val(x) = v;
    }
    // 區間查詢
    inline int query(int x, int y) {
        return split(x, y), sum(y);
    }
};

LinkCutTree LCT;
int n, a[N];

signed main() {
    int n, q, op, x, y;
    cin >> n >> q;
    for (int i = 1; i <= n; ++i) { cin >> a[i]; }
    LCT.init(n, a);
    while (q--) {
        cin >> op >> x >> y;
        if (op == 0) {
            cout << LCT.query(x, y) << endl;
        } else if (op == 1) {
            LCT.link(x, y);

```



```

    } else if (op == 2) {
        LCT.cut(x, y);
    } else {
        LCT.fix(x, y);
    }
}
return 0;
}

```

## 6.4 BIT

```

#define lowbit(x) x & -x

void modify(vector<int> &bit, int idx, int val) {
    for(int i = idx; i <= bit.size(); i += lowbit(i)) bit[i] += val;
}

int query(vector<int> &bit, int idx) {
    int ans = 0;
    for(int i = idx; i > 0; i -= lowbit(i)) ans += bit[i];
    return ans;
}

// the first i s.t. a[1]+...+a[i] >= k
int findK(vector<int> &bit, int k) {
    int idx = 0, res = 0;
    int mx = __lg(bit.size()) + 1;
    for(int i = mx; i >= 0; i--) {
        if((idx | (1<<i)) > bit.size()) continue;
        if(res + bit[idx | (1<<i)] < k) {
            idx = (idx | (1<<i));
            res += bit[idx];
        }
    }
    return idx + 1;
}

// O(n)建bit
for (int i = 1; i <= n; ++i) {
    bit[i] += a[i];
    int j = i + lowbit(i);
    if (j <= n) bit[j] += bit[i];
}

```

## 6.5 2D BIT

```

// 2維BIT
#define lowbit(x) (x&-x)

class BIT {
    int n;
    vector<int> bit;

public:
    void init(int _n) {
        n = _n;
        bit.resize(n + 1);
        for(auto &b : bit) b = 0;
    }
    int query(int x) const {
        int sum = 0;
        for(; x; x -= lowbit(x))
            sum += bit[x];
        return sum;
    }
    void modify(int x, int val) {
        for(; x <= n; x += lowbit(x))
            bit[x] += val;
    }
};

class BIT2D {
    int m;
    vector<BIT> bit1D;

public:
    void init(int _m, int _n) {
        m = _m;
        bit1D.resize(m + 1);
        for(auto &b : bit1D) b.init(_n);
    }
    int query(int x, int y) const {
        int sum = 0;
        for(; x; x -= lowbit(x))
            sum += bit1D[x].query(y);
    }
};

```

```

        return sum;
    }
    void modify(int x, int y, int val) {
        for(; x <= m; x += lowbit(x))
            bit1D[x].modify(y, val);
    }
};

```

## 6.6 undo DSU

```

struct dsu_undo{
    vector<int> sz, p;
    int comps;
    dsu_undo(int n){
        sz.assign(n+5, 1);
        p.resize(n+5);
        for(int i = 1; i<=n; ++i) p[i] = i;
        comps = n;
    }
    vector<pair<int, int>> opt;
    int Find(int x){
        return x==p[x]?x:Find(p[x]);
    }
    bool Union(int a, int b){
        int pa = Find(a), pb = Find(b);
        if(pa==pb) return 0;
        if(sz[pa]<sz[pb]) swap(pa, pb);
        sz[pa] += sz[pb];
        p[pb] = pa;
        opt.push_back({pa, pb});
        comps--;
        return 1;
    }
    void undo(){
        auto [pa, pb] = opt.back();
        opt.pop_back();
        p[pb] = pb;
        sz[pa] -= sz[pb];
        comps++;
    }
};

```

## 7 Dynamic Programming

### 7.1 LCS

```

// O(n^2)
int LCS(string t1, string t2) {
    if(t1.size() < t2.size()) swap(t1, t2);
    int len = t1.size();
    vector<vector<int>> dp(2, vector<int>(len + 1, 0));
    for(int j = 1; j <= t2.size(); j++){
        for(int i = 1; i <= len; i++){
            if(t2[j - 1] == t1[i - 1])
                dp[j % 2][i] = dp[(j + 1) % 2][i - 1] + 1;
            else
                dp[j % 2][i] = max(dp[(j + 1) % 2][i], dp[j % 2][i - 1]);
        }
    }
    return dp[t2.size() % 2][t1.size()];
}

// O(nlogn)
// 這裡string 要以 1 base index 所以開頭要補個字元
// d: 記住此數字的前一個數字
// t: 當前LIS位置, num: 根據t2生成出string來找LIS長度
// N: 最大字串長度
#define N 120
int t[N*N], d[N*N], num[N*N];
map<char, vector<int>> dict; // 每個字串出現的index位置
int binarySearch(int l, int r, int v){
    int m;
    while(r>l){
        m = (l+r)/2;
        if(num[v] > num[t[m]]) l = m+1;
        else if(num[v] < num[t[m]]) r = m;
        else return m;
    }
    return r;
}

int LCS(string t1, string t2){
    dict.clear();
    // i = strA.length() -1 才可以逆序
    for(int i = t1.length() - 1; i >= 0; i--) dict[t1[i]].push_back(i);
}

```

```

int k = 0; //生成數列的長度的最長長度
for(int i = 1; i < t2.length
    ) ; i++){ // 依據 strB 的每個字元來生成數列
    for(int j = 0 ; j < dict[t2[i]].size() ; j++){
        //將此字元在 strA 出現的位置放入數列
        num[++k] = dict[t2[i]][j] ;
    }
    if(k==0) return 0;
    d[1] = -1 , t[1] = 1 ; //LIS init
    int len = 1, cur ; // len 由於前面
        已經把 LCS = 0 的機會排除，於是這裡則從 1 開始

    // 標準的 LIS 作法，不斷嘗試將 LCS 生長
    for(int i = 1 ; i <= k ; i++ ){
        if(num[i] > num
            [t[len]]) t[++len] = i , d[i] = t[len-1] ;
        else{
            cur = binarySearch(1,len,i);
            t[cur] = i ;
            d[i] = t[cur-1];
        }
    }
    return len ;
}
}

```

## 7.2 LIS

```

int LIS(vector<int>& save) {
    vector<int> dp;
    int n = save.size();
    for (int i = 0; i < n; i++) {
        auto it = lower_bound(dp.begin(), dp.end(), save[i]);
        if(it == dp.end()) dp.push_back(save[i]);
        else *it = save[i];
    }
    return dp.size();
}

```

## 7.3 Knapsack

```

/**
 * 背包問題：
 * 1. dp[i][j]: 考慮 1~i 個物品，重量為 j 時的最大價格
 * 2. dp[i][j]: 考慮 1~i 個物品，價值為 j 時的最小重量
 */

// 當重量比較輕時 O(nw)
vector<int> dp(sum + 1, 0);
for (int i = 1; i <= n; ++i) {
    for (int j = sum /* bound */; j >= weight[i]; --j) {
        if (dp[j] < dp[j - weight[i]] + price[i]) {
            dp[j] = dp[j - weight[i]] + price[i];
            backtrack[i][j] = 1;
        }
    }
}

// 當重量比較重時 O(nc)
vector<int> dp(sum + 1, 1e9 + 7);
dp[0] = 0;
for (int i = 1; i <= n; ++i) {
    for (int j = sum /* bound */; j >= price[i]; --j) {
        if (dp[j] > dp[j - price[i]] + weight[i]) {
            dp[j] = dp[j - price[i]] + weight[i];
            backtrack[i][j] = 1;
        }
    }
}

// backtrack: 找到當 bound 為 k 時，背包內有哪些東西
// 註：只找到其中一種
int l = n, r = k;
vector<int> ans;
while (l != 0 && r != 0) {
    if (backtrack[l][r]) {
        ans.push_back(l);
        r -= weight[l]; // 當用方法一時，用這行
        r -= price[l]; // 當用方法二時，用這行
    }
    l--;
}
}

```

## 7.4 位元 dp

```

// 檢查第 n 位是否為 1
if(a & (1 << n))

// 強制將第 n 位變成 1
a |= (1 << n)

// 強制將第 n 位變成 0
a &= ~(1 << n)

// 將第 n 位反轉(1變0, 0變1)
a ^= (1 << n)

// 第 0 ~ n - 1 位 全部都是 1
(1 << n) - 1

// 兩個集合的聯集
S = a | b

// 兩個集合的交集
S = a & b

```

## 7.5 經典 dp 轉移式

```

/*
最大區間和：

dp[i] 代表 由第 i 項結尾時的最大區間和
dp[0] = arr[0]
dp[i] = max(dp[i - 1], arr[i])
ans = max_element(dp)
*/

```

## 8 Divide and conquer

### 8.1 逆序數對

```

int merge(
    vector<pair<int, int>>& v, int l, int mid, int r) {
    vector<pair<int, int>> temp(r - l + 1);
    int i = l, j = mid + 1, k = 0, inv_count = 0;
    while (i <= mid && j <= r) {
        if (v[i].second <= v[j].second) {
            temp[k++] = v[i++];
        } else {
            temp[k++] = v[j++];
            inv_count += (mid - i + 1);
        }
    }
    while (i <= mid) temp[k++] = v[i++];
    while (j <= r) temp[k++] = v[j++];
    for (int i = l; i <= r; i++) {
        v[i] = temp[i - l];
    }
    return inv_count;
}

int mergeSort
    (vector<pair<int, int>>& v, int l, int r) {
    int count = 0;
    if (l < r) {
        int mid = l + (r - l) / 2;
        count += mergeSort(v, l, mid);
        count += mergeSort(v, mid + 1, r);
        count += merge(v, l, mid, r);
    }
    return count;
}

signed main()
{
    int n;
    cin >> n;
    vector<pair<int, int>> arr(n);
    for(int i = 0; i < n; i++){
        arr[i].first = i;
        cin >> arr[i].second;
    }
    cout << mergeSort(arr, 0, n - 1) << '\n';
}

```

## 8.2 Mo's algorithm

```
// time complexity:  $n * \sqrt{q} * O(p)$ 
//  $O(p)$  為 add, remove 的時間複雜度
// 若知道  $[l, r]$  的答案 需要快速知道  $[l$ 
//    $- 1, r]$ ,  $[l + 1, r]$ ,  $[l, r - 1]$ ,  $[l, r + 1]$  的答案

int n, q, k, l = 0, r = 0;

array queries = 詢問們;
type ans; //目前答案
void add(type v){/*...*/} //增加一個數字，算新答案
void remove(type v){/*...*/} //移除一個數字，算新答案

vector<tuple<int, int, int, int>> queries(q);
k = sqrt(n);
for(int i = 0; i < q; i++){
    int l, r;
    cin >> l >> r;
    queries[i] = {l / k, r, l, i};
    // 先對 l 的塊，再對 r 排序
}

sort(queries.begin(), queries.end());

add(a[0]);

for(int i = 0; i < q; i++){
    auto [_ , rp, lp, id] = queries[i];
    lp--; rp--;
    while(l > lp) add(a[--l]);
    while(l < lp) remove(a[l++]);
    while(r < rp) add(a[++r]);
    while(r > rp) remove(a[r--]);
    ans_v[id] = ans;
}
```

## 9 Tree

### 9.1 樹直徑

```
int d1[200005], d2[200005], ans;

void dfs(int now, int fa, vector<vector<int>> &graph){
    for(auto i: graph[now]){
        if(i != fa){
            dfs(i, now, graph);
            if(d1[i] + 1 > d1[now]){
                d2[now] = d1[now];
                d1[now] = d1[i] + 1;
            }
            else if(d1[i] + 1 > d2[now]){
                d2[now] = d1[i] + 1;
            }
        }
    }
    ans = max(ans, d1[now] + d2[now]);
}

signed main()
{
    int n;
    cin >> n;
    vector<vector<int>> graph(n + 1);
    for(int i = 0; i < n - 1; i++){
        int a, b;
        cin >> a >> b;
        graph[a].push_back(b);
        graph[b].push_back(a);
    }
    dfs(1, 0, graph);
    cout << ans << '\n';
}
```

### 9.2 LCA

//  $n$  為點數，graph 由子節點往父節點建有向邊  
// graph 要 resize

```
int n, q;
int fa[20][200001];
int dep[200001];

vector<vector<int>> graph;
```

```
void dfs(int now, int lst){
    fa[0][now] = lst;
    for(int &i: graph[now]){
        dep[i] = dep[now] + 1;
        dfs(i, now);
    }
}

void build_lca(int root){
    dep[root] = 1;
    dfs(root, root);
    for(int i = 1; i < 18; i++){
        for(int j = 1; j < n + 1; j++){
            fa[i][j] = fa[i - 1][fa[i - 1][j]];
        }
    }
}

int lca(int a, int b){
    // 預設 a 比 b 淺
    if(dep[a] > dep[b]) return lca(b, a);
    // 讓 a 和 b 跳到同一個地方
    int step = dep[b] - dep[a];
    for(int i = 0; i < 18; i++){
        if(step >> i & 1){
            b = fa[i][b];
        }
    }
    if(a == b) return a;
    for(int i = 17; i >= 0; i--){
        if(fa[i][a] != fa[i][b]){
            a = fa[i][a];
            b = fa[i][b];
        }
    }
    return fa[0][a];
}
```

### 9.3 樹壓平

```
//紀錄 in & out
vector<int> Arr;
vector<int> In, Out;
void dfs(int u) {
    Arr.push_back(u);
    In[u] = Arr.size() - 1;
    for (auto v : Tree[u]) {
        if (v == parent[u])
            continue;
        parent[v] = u;
        dfs(v);
    }
    Out[u] = Arr.size() - 1;
}

//進去出來都紀錄
vector<int> Arr;
void dfs(int u) {
    Arr.push_back(u);
    for (auto v : Tree[u]) {
        if (v == parent[u])
            continue;
        parent[v] = u;
        dfs(v);
    }
    Arr.push_back(u);
}

//用 Treap 紀錄
Treap *root = nullptr;
vector<Treap *> In, Out;
void dfs(int u) {
    In[u] = new Treap(cost[u]);
    root = merge(root, In[u]);
    for (auto v : Tree[u]) {
        if (v == parent[u])
            continue;
        parent[v] = u;
        dfs(v);
    }
    Out[u] = new Treap(0);
    root = merge(root, Out[u]);
}

//Treap 紀錄 Parent
```

```

struct Treap {
    Treap *lc = nullptr, *rc = nullptr;
    Treap *pa = nullptr;
    unsigned pri, size;
    long long Val, Sum;
    Treap(int Val):
        pri(rand()), size(1),
        Val(Val), Sum(Val) {}
    void pull();
};

void Treap::pull() {
    size = 1;
    Sum = Val;
    pa = nullptr;
    if (lc) {
        size += lc->size;
        Sum += lc->Sum;
        lc->pa = this;
    }
    if (rc) {
        size += rc->size;
        Sum += rc->Sum;
        rc->pa = this;
    }
}

//找出節點在中序的編號
size_t getIdx(Treap *x) {
    assert(x);
    size_t Idx = 0;
    for (Treap *child = x->rc; x;) {
        if (child == x->rc)
            Idx += 1 + size(x->lc);
        child = x;
        x = x->pa;
    }
    return Idx;
}

//切出想要的東西
void move(Treap *&root, int a, int b) {
    size_t a_in = getIdx(In[a]), a_out = getIdx(Out[a]);
    auto [L, tmp] = splitK(root, a_in - 1);
    auto [tree_a, R] = splitK(tmp, a_out - a_in + 1);
    root = merge(L, R);
    tie(L, R) = splitK(root, getIdx(In[b]));
    root = merge(L, merge(tree_a, R));
}

```

## 10 Else

### 10.1 Big Number

```

string Add(const string &a, const string &b) {
    int n
        = a.length() - 1, m = b.length() - 1, car = 0;
    string res;
    while (n >= 0 || m >= 0 || car) {
        int x = (n >= 0 ? a[n] - '0' : 0) + (m >= 0 ? b[m] - '0' : 0) + car;
        res += (x % 10) + '0';
        car = x / 10;
        n--, m--;
    }
    while (res.length() > 1 && res.back() == '0') {
        res.pop_back();
    }
    reverse(res.begin(), res.end());
    return res;
}

string Minus(const string &a, const string &b) {
    // Assume a >= b
    int n
        = a.length() - 1, m = b.length() - 1, bor = 0;
    string res;
    while (n >= 0) {
        int x = a[n] - '0' - bor;
        int y = m >= 0 ? b[m] - '0' : 0;
        bor = 0;
        if (x < y) {
            x += 10;
            bor = 1;
        }
        res += x - y + '0';
        n--, m--;
    }
    while (res.length() > 1 && res.back() == '0') {

```

```

        res.pop_back();
    }
    reverse(res.begin(), res.end());
    return res;
}

string Multiple(const string &a, const string &b) {
    string res = "0";
    int n = a.length() - 1, m = b.length() - 1;
    for (int i = m; i >= 0; i--) {
        string add;
        int car = 0;
        for (int j = n; j >= 0 || car; j--) {
            int x = (j >= 0
                ? a[j] - '0' : 0) * (b[i] - '0') + car;
            add += (x % 10) + '0';
            car = x / 10;
        }
        while (add.length() > 1 && add.back() == '0') {
            add.pop_back();
        }
        reverse(add.begin(), add.end());
        res = Add(res, add + string(m - i, '0'));
    }
    return res;
}

```

### 10.2 Ternary Search

```

// return the maximum of $f(x)$ in $[l, r]$
double ternary_search(double l, double r) {
    while (r - l > EPS) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1), f2 = f(m2);
        if (f1 < f2) l = m1;
        else r = m2;
    }
    return f(l);
}

// return the maximum of $f(x)$ in $[l, r]$
int ternary_search(int l, int r) {
    while (r - l > 1) {
        int mid = (l + r) / 2;
        if (f(m) > f(m + 1)) r = m;
        else l = m;
    }
    return r;
}

```

### 10.3 Duipai

```

#include <iostream>
using namespace std;
int main() {
    for (int T=1; T++;) {
        if (system("./random > test.in")) {
            cout << "random RE on " << T << '\n';
            return 0;
        }
        if (system("./sol < test.in > test.out")) {
            cout << "sol RE on " << T << '\n';
            return 0;
        }
        if (system("./bf < test.in > test.ans")) {
            cout << "bf RE on " << T << '\n';
            return 0;
        }
        if (system("./diff -Z test.out test.ans")) {
            cout << "WA on " << T << '\n';
            return 0;
        }
        else {
            cout << "AC on " << T << '\n';
        }
    }
}

```

### 10.4 Random Generator

```

#include <iostream>
#include <random>
int main() {
    std::random_device rd;
    std::mt19937 gen(rd());
    std::uniform_int_distribution<> distrib(1, 100);
    std::cout << "Get Rand: " << distrib(gen) << '\n';
}

```