Contents

1 Basic

1.1 Default Code

```
#include <bits/stdc++.h>
#define int long long
#define endl '\n' // 如果是互動題要把這個註解掉
#define de(x) cout << #x << '=' << x << ", "
#define dd cout << '\n';
// #pragma GCC target("popcnt")
// #pragma GCC optimize("O3")
using namespace std;
int tt = 1;

void pre() {
  cout.tie(nullptr); // 輸出加速
  cin >> tt; // 多筆輸入
}

void solve() {}

signed main() {
  ios_base::sync_with_stdio(false);
  cin.tie(nullptr);
#ifdef LOCAL
  // g++ -DLOCAL -std=c++17 <filename> && ./a.out
  freopen("input.txt", "r", stdin);
  // freopen("output.txt", "w", stdout);
#endif // LOCAL
  pre();
  while (tt--) { solve(); }
  return 0;
}
```

1.2 PBDS

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
template
    <class T> using Tree = tree<T, null_type, less<T
    >, rb_tree_tag, tree_order_statistics_node_update>;

/*
如果有 define int long long 記得拿掉
Tree<int> t 就跟 set<int> t 一樣，有包好 template
rb_tree_tag 使用紅黑樹
第三個參數 less<T> 為由小到大，greater<T> 為由大到小
插入 t.insert(); 刪除 t.erase();
t.order_of_key
    (k); 從前往後數 k 是第幾個 (0-base 且回傳 int 型別)
t.find_by_order(k);
    從前往後數第 k 個元素 (0-base 且回傳 iterator 型別)
t.lower_bound
    (); t.upper_bound(); 用起來一樣 回傳 iterator
可以用 Tree<pair<int, int>> T 來模擬 mutiset
*/
```

1.3 int128 Input Output

```
// 抄 BBuf github 的

#include <bits/stdc++.h>
using namespace std;

void scan(__int128 &x) // 輸入
{
  x = 0;
  int f = 1;
  char ch;
  if((ch = getchar()) == '-') f = -f;
  else x = x*10 + ch-'0';
  while((ch = getchar()) >= '0' && ch <= '9')
    x = x*10 + ch-'0';
  x *= f;
}

void print(__int128 x) // 輸出
{
  if(x < 0)
  {
    x = -x;
    putchar('-');
  }
  if(x > 9) print(x/10);
  putchar(x%10 + '0');
}

int main()
{
  __int128 a, b;
  scan(a);
  scan(b);
  print(a + b);
  puts("");
  print(a*b);
  return 0;
}
```

1.4 Python

```
## Input
# p q 都是整數，中間以空白分開輸入
p, q = map(int, input().split())

# 輸入很多個用空
    白隔開的數字，轉成 float 放進陣列，s 是 input 字串
arr = list(map(float, s.split()))


# 分數用法 Fraction(被除數, 除數)
from fractions import Fraction

frac = Fraction(3, 4)
numerator = frac.numerator # 取出分子
denominator = frac.denominator # 取出分母

arr = [Fraction
    (0), Fraction(1, 6), Fraction(1, 2), Fraction(5,
     12), Fraction(0), Fraction(-1, 12), Fraction(0)]
```

```
# 可以直接做乘除
def fx(x):
    x = Fraction(x)
    ans = Fraction(0)
    for i in range(1, 7):
        ans += arr[i] * x ** (7 - i)
    return ans
```

1.5 bitset

```
bitset<size> b(a):長度為size，初始化為a
b[i]:第i位元的值(0 or 1)
b.size():有幾個位元
b.count():有幾個1
b.set():所有位元設為1
b.reset():所有位元設為0
b.flip():所有位元反轉
```

2 Math

2.1 質數表

```
vector<int> prime_table(int n){
  vector<int> table(n + 1, 0);
  for(int i = 1; i <= n; i++){
    for(int j = i; j <= n; j += i){
      table[j]++;
    }
  }
  return table;
}
```

2.2 快速冪

```
#define int long long

// 根據費馬小定
    理，若 a p 互質，a^(p-2) 為 a 在 mod p 時的乘法逆元
// a ^ (b ^ c
    ) % mod = fast_pow(a, fast_pow(b, c, mod - 1), mod)
typedef unsigned long long ull;
inline int ksc(ull
     x, ull y, int p) {  // O(1)快速乘 (防爆long long)
  return (x
      * y - (ull)((long double)x / p * y) * p + p) % p;
}

inline int fast_pow(int a, int b, int mod)
{
  // a^b % mod
  int res = 1;
  while(b)
  {
    if(b & 1) res =  ksc(res, a, mod);
    a = ksc(a, a, mod);
    b >>= 1;
  }
  return res;
}
```

2.3 擴展歐幾里得

```
int gcd(int a, int b)
{
  return b == 0 ? a : gcd(b, a % b);
}

int lcm(int a, int b)
{
  return a * b / gcd(a, b);
}

pair<int, int> ext_gcd
    (int a, int b) //擴展歐幾里德 ax+by = gcd(a,b)
{
  if (b == 0)
    return {1, 0};
  if (a == 0)
    return {0, 1};
  int x, y;
  tie(x, y) = ext_gcd(b % a, a);
  return make_pair(y - (b / a) * x, x);
}
```

2.4 矩陣

```
template<typename T>
struct Matrix{
  using rt = std::vector<T>;
  using mt = std::vector<rt>;
  using matrix = Matrix<T>;
  int r,c;
  mt m;
  Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}
  rt& operator[](int i){return m[i];}
  matrix operator+(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]+a.m[i][j];
    return rev;
  }
  matrix operator-(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]-a.m[i][j];
    return rev;
  }
  matrix operator*(const matrix &a){
    matrix rev(r,a.c);
    matrix tmp(a.c,a.r);
    for(int i=0;i<a.r;++i)
      for(int j=0;j<a.c;++j)
        tmp[j][i]=a.m[i][j];
    for(int i=0;i<r;++i)
      for(int j=0;j<a.c;++j)
        for(int k=0;k<c;++k)
          rev.m[i][j]+=m[i][k]*tmp[j][k];
    return rev;
  }
  bool inverse(){
    Matrix t(r,r+c);
    for(int y=0;y<r;y++){
      t.m[y][c+y] = 1;
      for(int x=0;x<c;++x)
        t.m[y][x]=m[y][x];
    }
    if( !t.gas() )
      return false;
    for(int y=0;y<r;y++)
      for(int x=0;x<c;++x)
        m[y][x]=t.m[y][c+x]/t.m[y][y];
    return true;
  }
  T gas(){
    vector<T> lazy(r,1);
    bool sign=false;
    for(int i=0;i<r;++i){
      if( m[i][i]==0 ){
        int j=i+1;
        while(j<r&&!m[j][i])j++;
        if(j==r)continue;
        m[i].swap(m[j]);
        sign=!sign;
      }
      for(int j=0;j<r;++j){
        if(i==j)continue;
        lazy[j]=lazy[j]*m[i][i];
        T mx=m[j][i];
        for(int k=0;k<c;++k)
          m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
      }
    }
    T det=sign?-1:1;
    for(int i=0;i<r;++i){
      det = det*m[i][i];
      det = det/lazy[i];
      for(auto &j:m[i])j/=lazy[i];
    }
    return det;
  }
};
```

2.5 Miller rabin Prime test

```
// fast_pow 去前面抄，需要處裡防暴乘法
// 記得 #define int long long 也要放
// long long 範圍內測試過答案正確
// time: O(logn)
```

```cpp
inline bool mr(int x, int p) {
  if (fast_pow(x, p - 1, p) != 1) return 0;
  int y = p - 1, z;
  while (!(y & 1)) {
      y >>= 1;
      z = fast_pow(x, y, p);
      if (z != 1 && z != p - 1) return 0;
      if (z == p - 1) return 1;
  }
  return 1;
}

inline bool prime(int x) {
  if (x < 2) return 0;
  if (x == 2 ||
      x == 3 || x == 5 || x == 7 || x == 43) return 1;
  // 如果把 2
      到 37 前 12 個質數都檢查一遍 可以保證 2^78 皆可用
  return mr(2, x)
      && mr(3, x) && mr(5, x) && mr(7, x) && mr(43, x);
}
```

2.6 Pollard's Rho

```cpp
// 主函數記得放 srand(time(nullptr))
// prime 檢測以及快速冪, gcd 等請從前面抄

// 輸入一個數字 p，隨
    機回傳一個 非 1 非 p 的因數，若 p 是質數會無窮迴圈
#define rg register int
inline int rho(int p) {
  int x, y, z, c, g;
  rg i, j;
  while (1) {
    y = x = rand() % p;
    z = 1;
    c = rand() % p;
    i = 0, j = 1;
    while (++i) {
      x = (ksc(x, x, p) + c) % p;
      z = ksc(z, abs(y - x), p);
      if (x == y || !z) break;
      if (!(i % 127) || i == j) {
        g = gcd(z, p);
        if (g > 1) return g;
        if (i == j) y = x, j <<= 1;
      }
    }
  }
}

// 回傳隨機一個質因數，若 input 為質數，則直接回傳
int prho(int p){
  if(prime(p)) return p;
  int m = rho(p);
  if(prime(m)) return m;
  return prho(p / m);
}

// 回傳將 n 質因數分解的結果，由小到大排序
// ex: input: 48, output: 2 2 2 2 3
vector<int> prime_factorization(int n){
  vector<int> ans;
  while(n != 1){
    int m = prho(n);
    ans.push_back(m);
    n /= m;
  }
  sort(ans.begin(), ans.end());
  return ans;
}
```

2.7 皮薩諾定理

```cpp
// fib(x) % m = fib(x + kn) % m 當 k >= 1，求 n
// n 為費式數列 % m 會重複的週期
// pisano_period(m) <= 6m
// 通常這都要本地跑

#define int long long

int pisano_period(int m) {
  int pre = 0, cur = 1;
  int temp;
  for (int i = 0; i < m * m; i++) {
    temp = pre;
    pre = cur;
    cur = (temp + cur) % m;
    if (pre == 0 && cur == 1) return i + 1;
  }
  return 0;
}
```

2.8 高斯消去法

```python
from fractions import Fraction
def gauss_elimination(matrix, results):
    # 將所有數字轉換為分數
    n = len(matrix)
    augm = [[Fraction(matrix
        [i][j]) for j in range(n)] for i in range(n)]
    augr = [Fraction(results[i]) for i in range(n)]

    # 高斯消去法
    for i in range(n):
        # 尋找主元
        if augm[i][i] == 0:
            for j in range(i + 1, n):
                if augm[j][i] != 0:
                    augm[i], augm[j] = augm[j], augm[i]
                    augr[i], augr[j] = augr[j], augr[i]
                    break

        pivot = augm[i][i]
        if pivot == 0:
            # 如果主元為0，繼續檢查該行是否全為 0
            if all(augm[i][j] == 0 for j in range(n)):
                if augr[i] != 0:
                    return None  # 無解
                else:
                    continue
                        # 可能有無限多解，繼續檢查

        # 將主元行的數字規一化
        for j in range(i, n):
            augm[i][j] /= pivot
        augr[i] /= pivot

        # 將其他行的數字變為0
        for j in range(n):
            if i != j:
                factor = augm[j][i]
                for k in range(i, n):
                    augm[j][k] -= factor * augm[i][k]
                augr[j] -= factor * augr[i]
    # 檢查是否存在無限多解的情況
    for i in range(n):
        if all(augm[i][j
            ] == 0 for j in range(n)) and augr[i] == 0:
            return [] # 無限多組解

    return augr


# matrix = [
#     [2, -1, 1],
#     [3, 3, 9],
#     [3, 3, 5]
# ]
# results = [8, -42, 0]
# output = [
    Fraction(12, 1), Fraction(11, 2), Fraction(-21, 2)]
# Fraction 可以強轉 float


import numpy as np

def gauss_elimination(matrix, ans):
    matrix = np.array(matrix)
    ans = np.array(ans)

    try:
        solution = np.linalg.solve(matrix, ans)
        return [f"{value:.2f}" for value in solution]
    except np.linalg.LinAlgError:
        # 無解或者無限多組解
        return "No Solution"
```

```
# 有開放 numpy 可以用
# 優點：行數短，執行速度快
# 缺點：只能用浮點數，無法區分無解及無限多組解
```

2.9 卡特蘭數

```python
"""
卡特蘭數 Catalan
公式：H(n) = C(2 * n, n) // (n + 1), n >= 2, n 為正整數
快速計算方式：
1. H(0) = H(1) = 1, H(n) = sum(H(i - 1) * H(n - i) for i in range(1, n + 1))
2. H(n) = H(n - 1) * (4 * n - 2) // (n + 1)
3. H(n) = C(2 * n, n) - C(2 * n, n - 1)
"""

"""
可解問題：
有效括號匹配問題：給定 n 個左括號與右括號，求有幾種不同的正確括號匹配
二元樹結構問題：給定 n 個節點，求有幾種不同的二元樹結構
將一個凸 n + 2 邊形劃分成多個三角形，求有幾種不同的劃分方式
狄克路徑：給定 n * n 的網格，從左下到右上的路徑中，永不超過對角線的路徑有幾種
一個 stack 在 push 順序不變的情況下 (1, 2, 3, ..., n)，有幾種 pop 的方式
在圖上選擇 2 * n 個點，將這些點兩兩連接使得 n 條線段不相交的方法有幾種
"""

n = int(input())

catalan = [1 for _ in range(n + 1)]
for i in range(1, n + 1):
    catalan[i] = catalan[i - 1] * (4 * i - 2) // (i + 1)

ans = 0
for i in range(0, n + 1):  # 卡特蘭數的平方
    ans += catalan[i] * catalan[n - i]

print(ans)
# 185ms in codeforces, n <= 5000
```

2.10 中國剩餘定理

```cpp
// vec[i] = {m_i, x_i}，求最小非負 x 使得 x □ x_i (mod m_i) 對所有 i 同時成立；無解回 -1
// 注意 overflow
int CRT(vector<pair<int, int>> &v)
{
  int m = v[0].first, x = (v[0].second % m + m) % m;
  for (int i = 1; i < (int)v.size(); ++i)
  {
    int mi = v[i].first, xi = (v[i].second % mi + mi) % mi;
    int g = gcd(m, mi), d = xi - x;
    if (d % g) return -1;
    int m1 = m / g, m2 = mi / g;
    auto ab = ext_gcd((int)m1, (int)m2);
    int inv = ((int)ab.first % m2 + m2) % m2;
    int k = ((d / g) % m2 + m2) % m2;
    k = (k * inv) % m2;
    x = (x + m * k) % (m * m2);
    m *= m2;
    x = (x + m) % m;
  }
  return x;
}
```

2.11 Theorem

- Cramer's rule

$$\begin{array}{l} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{l} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

- Vandermonde's Identity

$$C(n+m,k)=\sum_{i=0}^{k}C(n,i)C(m,k-i)$$

- Kirchhoff's Theorem
  Denote L be a $n\times n$ matrix as the Laplacian matrix of graph G, where $L_{ii}=d(i)$, $L_{ij}=-c$ where c is the number of edge (i,j) in G.
  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
- Tutte's Matrix
  Let D be a $n\times n$ matrix, where $d_{ij}=x_{ij}$ (x_{ij} is chosen uniformly at random) if $i<j$ and $(i,j)\in E$, otherwise $d_{ij}=-d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.
- Cayley's Formula
  - Given a degree sequence $d_1,d_2,\ldots,d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1,2,\ldots,k$ belong to different components. Then $T_{n,k}=kn^{n-k-1}$.
- Erdős–Gallai theorem
  A sequence of nonnegative integers $d_1\ge\cdots\ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1+\cdots+d_n$ is even and $\displaystyle\sum_{i=1}^{k}d_i\le k(k-1)+\sum_{i=k+1}^{n}$ min(d_i,k) holds for every $1\le k\le n$.
- Gale–Ryser theorem
  A pair of sequences of nonnegative integers $a_1\ge\cdots\ge a_n$ and $b_1,\ldots,b_n$ is bigraphic if and only if $\displaystyle\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\displaystyle\sum_{i=1}^{k}a_i\le\sum_{i=1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- Fulkerson–Chen–Anstee theorem
  A sequence $(a_1,\ b_1),\ \ldots\ ,\ (a_n,\ b_n)$ of nonnegative integer pairs with $a_1\ \ge\ \cdots\ \ge\ a_n$ is digraphic if and only if $\displaystyle\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\displaystyle\sum_{i=1}^{k}a_i\le\sum_{i=1}^{k}\min(b_i,k-1)+\sum_{i=k+1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- Pick's theorem
  For simple polygon, when points are all integer, we have $A=\#\{\text{lattice points in the interior}\}+\frac{\#\{\text{lattice points on the boundary}\}}{2}-1$.
- Möbius inversion formula
  - $f(n)=\sum_{d|n}g(d)\Leftrightarrow g(n)=\sum_{d|n}\mu(d)f(\frac{n}{d})$
  - $f(n)=\sum_{n|d}g(d)\Leftrightarrow g(n)=\sum_{n|d}\mu(\frac{d}{n})f(d)$
- Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: arcsin(a/r).
  - Volume $=\pi h^2(3r-h)/3=\pi h(3a^2+h^2)/6=\pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $=2\pi rh=\pi(a^2+h^2)=2\pi r^2(1-\cos\theta)$.
- Lagrange multiplier
  - Optimize $f(x_1,\ldots,x_n)$ when k constraints $g_i(x_1,\ldots,x_n)=0$.
  - Lagrangian function $\mathcal{L}(x_1,\ldots,x_n,\lambda_1,\ldots,\lambda_k)=f(x_1,\ldots,x_n)-\sum_{i=1}^{k}\lambda_i g_i(x_1,\ldots,x_n)$.
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.
- Nearest points of two skew lines
  - Line 1 : $\boldsymbol{v}_1=\boldsymbol{p}_1+t_1\boldsymbol{d}_1$
  - Line 2 : $\boldsymbol{v}_2=\boldsymbol{p}_2+t_2\boldsymbol{d}_2$
  - $\boldsymbol{n}=\boldsymbol{d}_1\times\boldsymbol{d}_2$
  - $\boldsymbol{n}_1=\boldsymbol{d}_1\times\boldsymbol{n}$
  - $\boldsymbol{n}_2=\boldsymbol{d}_2\times\boldsymbol{n}$
  - $\boldsymbol{c}_1=\boldsymbol{p}_1+\frac{(\boldsymbol{p}_2-\boldsymbol{p}_1)\cdot\boldsymbol{n}_2}{\boldsymbol{d}_1\cdot\boldsymbol{n}_2}\boldsymbol{d}_1$
  - $\boldsymbol{c}_2=\boldsymbol{p}_2+\frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)\cdot\boldsymbol{n}_1}{\boldsymbol{d}_2\cdot\boldsymbol{n}_1}\boldsymbol{d}_2$
- Derivatives/Integrals
  Integration by parts: $\int_a^b f(x)g(x)dx=[F(x)g(x)]_a^b-\int_a^b F(x)g'(x)dx$

$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\cos^{-1}x=-\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$
$\frac{d}{dx}\tan x=1+\tan^2 x$	$\int\tan ax=-\frac{\ln\|\cos ax\|}{a}$	
$\int e^{-x^2}=\frac{\sqrt{\pi}}{2}\mathrm{erf}(x)$	$\int xe^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)$	

$$\int\sqrt{a^2+x^2}=\frac{1}{2}\left(x\sqrt{a^2+x^2}+a^2\mathrm{asinh}(x/a)\right)$$

- Spherical Coordinate

$$(x,y,z)=(r\sin\theta\cos\phi,r\sin\theta\sin\phi,r\cos\theta)$$

$$(r,\theta,\phi)=(\sqrt{x^2+y^2+z^2},\mathrm{acos}(z/\sqrt{x^2+y^2+z^2}),\mathrm{atan2}(y,x))$$

- Rotation Matrix

$$M(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, R_x(\theta_x)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_x & -\sin\theta_x\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$

2.12 Estimation

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8

n	100	1e3	1e6	1e9	1e12	1e15	1e18
$d(i)$	12	32	240	1344	6720	26880	103680

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8

n	2	3	4	5	6	7	8	9	10	11	12	13
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7

2.13 Euclidean Algorithms

- $m=\lfloor\frac{an+b}{c}\rfloor$
- Time complexity: $O(\log n)$

$$f(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)}{2}+\lfloor\frac{b}{c}\rfloor\cdot(n+1) & \\ +f(a \bmod c,b \bmod c,c,n), & a\ge c\vee b\ge c\\ 0, & n<0\vee a=0\\ nm-f(c,c-b-1,a,m-1), & \text{otherwise}\end{cases}$$

$$g(a,b,c,n)=\sum_{i=0}^{n}i\lfloor\frac{ai+b}{c}\rfloor$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor\cdot\frac{n(n+1)}{2} & \\ +g(a \bmod c,b \bmod c,c,n), & a\ge c\vee b\ge c\\ 0, & n<0\vee a=0\\ \frac{1}{2}\cdot(n(n+1)m-f(c,c-b-1,a,m-1) & \\ -h(c,c-b-1,a,m-1)), & \text{otherwise}\end{cases}$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor^2$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor^2\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor^2\cdot(n+1) & \\ +\lfloor\frac{a}{c}\rfloor\cdot\lfloor\frac{b}{c}\rfloor\cdot n(n+1) & \\ +h(a \bmod c,b \bmod c,c,n) & \\ +2\lfloor\frac{a}{c}\rfloor\cdot g(a \bmod c,b \bmod c,c,n) & \\ +2\lfloor\frac{b}{c}\rfloor\cdot f(a \bmod c,b \bmod c,c,n), & a\ge c\vee b\ge c\\ 0, & n<0\vee a=0\\ nm(m+1)-2g(c,c-b-1,a,m-1) & \\ -2f(c,c-b-1,a,m-1)-f(a,b,c,n), & \text{otherwise}\end{cases}$$

2.14 General Purpose Numbers

- Bernoulli numbers
  $B_0-1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$
  $$\sum_{j=0}^{m}\binom{m+1}{j}B_j=0, \text{ EGF is } B(x)=\frac{x}{e^x-1}=\sum_{n=0}^{\infty}B_n\frac{x^n}{n!}.$$
  $$S_m(n)=\sum_{k=1}^{n}k^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k^+n^{m+1-k}$$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.
  $S(n,k)=S(n-1,k-1)+kS(n-1,k), S(n,1)=S(n,n)=1$
  $S(n,k)=\frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$
  $x^n=\sum_{i=0}^{n}S(n,i)(x)_i$
- Pentagonal number theorem
  $$\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2}+x^{k(3k-1)/2}\right)$$
- Catalan numbers
  $$C_n^{(k)}=\frac{1}{(k-1)n+1}\binom{kn}{n}$$
  $$C^{(k)}(x)=1+x[C^{(k)}(x)]^k$$
- Eulerian numbers
  Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\ge j$, k j:s s.t. $\pi(j)>j$.
  $E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$
  $E(n,0)=E(n,n-1)=1$
  $E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

2.15 Tips for Generating Functions

- Ordinary Generating Function $A(x)=\sum_{i\ge 0}a_ix^i$
  - $A(rx)\Rightarrow r^na_n$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)'\Rightarrow na_n$
  - $\frac{A(x)}{1-x}\Rightarrow\sum_{i=0}^{n}a_i$
- Exponential Generating Function $A(x)=\sum_{i\ge 0}\frac{a_i}{i!}x_i$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A^{(k)}(x)\Rightarrow a_{n+k}$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}\binom{n}{i}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1,i_2,\ldots,i_k}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)\Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n=\sum_{i\ge 0}\binom{n}{i}x^i$
  - $\frac{1}{(1-x)^n}=\sum_{i\ge 0}\binom{i}{n-1}x^i$

3 Graph

3.1 DSU

```
class dsu{
 public:
   vector<int> parent;
   dsu(int num){
     parent.resize(num);
     for(int i = 0; i < num; i++) parent[i] = i;
   }
   int find(int x){
     if(parent[x] == x) return x;
     return parent[x] = find(parent[x]);
   }
   bool same(int a, int b){
     return find(a) == find(b);
   }
   void Union(int a, int b){
     parent[find(a)] = find(b);
   }
};
```

3.2 Dijkstra

```
// 傳入圖的 pair 為 {權重, 點}, 無限大預設 1e9 是情況改
#define pii pair<int, int>
vector<
    int> dijkstra(vector<vector<pii>> &graph, int src){
  int n = graph.size();
  vector<int> dis(n, 1e9);
  vector<bool> vis(n, false);
  priority_queue<pii, vector<pii>, greater<pii>> pq;
  pq.push({0, src});
  dis[src] = 0;
  while(!pq.empty()){
    auto [w, node] = pq.top();
    pq.pop();
    if(vis[node]) continue;
    vis[node] = true;
    for(auto [nw, nn]:graph[node]){
      if(w + nw < dis[nn]){
        dis[nn] = w + nw;
        pq.push({dis[nn], nn});
      }
    }
  }
  return dis;
}
```

3.3 SPFA

```
#define pii pair<int, int>
// {在 src 可到達
    的點中是否存在負環, 最短路徑}, arg 中 n 為點的數量
// arg 中 pair 裡的第一個值為權重, 第二個為點
pair<bool, vector<int>>
     SPFA(vector<vector<pii>> &graph, int n, int src){
  vector<int> dis(n + 1, 1e9);
  vector<int> cnt(n + 1, 0);
  vector<bool> vis(n + 1, false);
  queue<int> q;
  vis[src] = true; q.push(src); dis[src] = 0;
  while(!q.empty()){
    auto node = q.front(); vis[node] = false; q.pop();
```

```cpp
    for(auto [w, nn]:graph[node]){
      if(w + dis[node] < dis[nn]){
        dis[nn] = w + dis[node];
        if(!vis[nn]){
          if(++cnt[nn] >= n) return {true, {}};
          q.push(nn);
          vis[nn] = true;
        }
      }
    }
  }
  return {false, dis};
}
```

3.4 Floyd Warshall

```cpp
// 中繼點放外面

for (int k = 0; k < n; k++) {
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      dis[i
          ][j] = min(dis[i][j], dis[i][k] + dis[k][j]);
    }
  }
}
```

3.5 Tarjan SCC

```cpp
class tarjan{
    //  1-base
    int time = 1;
    int id = 1;
    stack<int> s;
    vector<int> low;
    vector<int> dfn;
    vector<bool> in_stack;
    void dfs(int node, vector<vector<int>> &graph){
      in_stack[node] = true;
      s.push(node);
      dfn[node] = low[node] = time++;
      for(auto &j : graph[node]){
        if(dfn[j] == 0){
          dfs(j, graph);
          // 看看往下有沒有辦法回到更上面的點
          low[node] = min(low[node], low[j]);
        }
        else if(in_stack[j]){
          low[node] = min(low[node], low[j]);
        }
      }
      vector<int> t; // 儲存這個強連通分量
      if(dfn[node] == low[node]){
        while(s.top() != node){
          t.push_back(s.top());
          in_stack[s.top()] = false;
          scc_id[s.top()] = id;
          s.pop();
        }
        t.push_back(s.top());
        scc_id[s.top()] = id;
        in_stack[s.top()] = false;
        s.pop();
        id++;
      }
      if(!t.empty()) ans.push_back(t);
    }
  public:
    vector<int> scc_id;
    vector<vector<int>> ans;
    // ans ans[i] 代表第 i 個強連通分量裡面包涵的點
    // scc_id[i] 代表第 i 個點屬於第幾個強連通分量
    vector
        <vector<int>> scc(vector<vector<int>> &graph){
      int num = graph.size();
      scc_id.resize(num, -1);
      dfn.resize(num, 0);
      low.resize(num, 0);
      in_stack.resize(num, false);
      for(int i = 1; i < num; i++){
        if(dfn[i] == 0) dfs(i, graph);
      }
      return ans;
    }
};
```

3.6 2 SAT

```cpp
// (a || b) && (c || d) && (e || f) .....

// 用
     下面的 tarjan scc 算法來解 2 sat 問題，若 事件 a 發
     生時，事件 b 必然發生，我們須在 a -> b 建立一條有向
// 用
      cses 的 Giant Pizza 來舉例子，給定 n 個人 m 個配料
     表，每個人可以提兩個要求，兩個要求至少要被滿足一個

// 3 5
// + 1 + 2
// - 1 + 3
// + 4 - 2

// 以這
     個例子來說，第一個人要求要加 配料1 或者 配料2 其中
     一項，第二個人要求不要 配料1 或者 要配料3 其中一項
// 試問能不能滿足所有人的要求，我們可以把 要加
     配料 i 當作點 i ，不加配料 i 當作點 i + m(配料數量)
// 關於第一個人的要求 我們可以看成若不加 配
     料1 則必定要 配料2 以及 若不加 配料2 則必定要 配料1
// 關於第二個人要求 可看做加了 配料
     1 就必定要加 配料3 以及 不加 配料3 就必定不加 配料1

// 以這些條件建立有向圖，並且
     找尋 scc ，若 i 以及 i + m 在同一個 scc 中代表無解
// 若要求解，則若 i 的 scc_id
      小於 i + m 的 scc_id 則 i 為 true ，反之為 false
// tarjan 的模板在上面
cin >> n >> m;

vector<vector<int>> graph(m * 2 + 1);
function<int(int)> tr = [&](int x){
  if(x > m) return x - m;
  return x + m;
};

for(int i = 0; i < n; i++){
  char c1, c2;
  int a, b;
  cin >> c1 >> a >> c2 >> b;
  // a 代表 a 為真，m + a 代表 a 為假
  if(c1 == '-') a += m;
  if(c2 == '-') b += m;
  graph[tr(a)].push_back(b);
  graph[tr(b)].push_back(a);
}

tarjan t;
auto scc = t.scc(graph);

for(int i = 1; i <= m; i++){
  if(t.scc_id[i] == t.scc_id[tr(i)]){
    cout << "IMPOSSIBLE\n";
    return 0;
  }
}

for(int i = 1; i <= m; i++){
  if(t.scc_id[i] < t.scc_id[tr(i)]){
    cout << '+';
  }
  else cout << '-';
  cout << ' ';
}
cout << '\n';
```

3.7 Euler Path

```cpp
// 1. 無向圖是歐拉圖：
// 非零度頂點是連通的
// 頂點的度數都是偶數

// 2. 無向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是連通的
// 恰有 2 個奇度頂點

// 3. 有向圖是歐拉圖：
// 非零度頂點是強連通的
// 每個頂點的入度和出度相等
```

```cpp
// 4. 有向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是弱連通的
// 至多一個頂點的出度與入度之差為 1
// 至多一個頂點的入度與出度之差為 1
// 其他頂點的入度和出度相等
vector<set<int>> adj;
vector<int> ans;

void dfs(int x) { // Hierholzer's Algorithm
  while (!adj[x].empty()) {
    auto next = *(adj[x].begin());
    adj[x].erase(next);
    adj[next].erase(x);
    dfs(next);
  }
  ans.emplace_back(x);
}

void solve() {
  // 建立雙向邊，set用來防重邊，點數n，邊數m
  for (int i = 1; i <= n; i++)
    if (adj[i].size() & 1) return; /* impossible */
  dfs(1);
  if (ans.size() != m + 1) return; /* impossible */
  reverse(ans.begin(), ans.end()); /* then print it */
}
```

3.8 Bridge

```cpp
// 橋: 若移除邊會使連通分量變多
// [USAGE] ECC ecc(n); ecc.add_edge(u, v); ecc.solve();
// is_bridge[i]; necc; bln[i];
// 邊是否為橋; 橋連通分量數量; 頂點所屬橋連通分量編號
struct ECC { // 0-base
  int n, dft, ecnt, necc;
  vector<int> low, dfn, bln, is_bridge, stk;
  vector<vector<pii>> G;
  void dfs(int u, int f) {
    dfn[u] = low[u] = ++dft, stk.pb(u);
    for (auto [v, e] : G[u])
      if (!dfn[v])
        dfs(v, e), low[u] = min(low[u], low[v]);
      else if (e != f)
        low[u] = min(low[u], dfn[v]);
    if (low[u] == dfn[u]) {
      if (f != -1) is_bridge[f] = 1;
      for (; stk.back() != u; stk.pop_back())
        bln[stk.back()] = necc;
      bln[u] = necc++, stk.pop_back();
    }
  }
  ECC(int _n): n(_n), dft()
    , ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
  void add_edge(int u, int v) {
    G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
  }
  void solve() {
    is_bridge.resize(ecnt);
    for (int i = 0; i < n; ++i)
      if (!dfn[i]) dfs(i, -1);
  }
}; // 8BQube
```

3.9 Max flow min cut

```cpp
#define int long long

// dicnic Algorithm Time: O(V^2E) 實際上會快一點
// 記得在 main 裡面 resize graph
// 最小割，找
   到最少條的邊切除，使得從 src 到 end 的 maxflow 為 0
// 枚舉所有邊 i -> j，src 可
   以到達 i 但無法到達 j ，那這條邊為最小割裡的邊之一
// 無向圖最大流：修改 add_edge，反向邊建為 capacity
   // 使用時只要 add_edge 一次

class edge{
  public:
    int next;
    int capacity;
    int rev;
    bool is_rev;
    edge(int _n, int _c, int _r, int _ir) :
        next(_n), capacity(_c), rev(_r), is_rev(_ir){};
};

vector<vector<edge>> graph;
vector<int> level, iter;

void add_edge(int a, int b, int capacity){
  graph[a].push_back
      (edge(b, capacity, graph[b].size(), false));
  graph[b].
      push_back(edge(a, 0, graph[a].size() - 1, true));
}

void bfs(int start) {
  fill(level.begin(), level.end(), -1);
  queue<int> q;
  level[start] = 0;
  q.push(start);
  while (!q.empty()) {
    int v = q.front();
    q.pop();
    for (auto& e : graph[v]) {
      if (e.capacity > 0 && level[e.next] < 0) {
        level[e.next] = level[v] + 1;
        q.push(e.next);
      }
    }
  }
}

int dfs(int v, int end, int flow) {
  if (v == end) return flow;
  for (int &i = iter[v]; i < graph[v].size(); i++) {
    edge &e = graph[v][i];
    if (e.capacity > 0 && level[v] < level[e.next]) {
      int d = dfs(e.next, end, min(flow, e.capacity));
      if (d > 0) {
        e.capacity -= d;
        graph[e.next][e.rev].capacity += d;
        return d;
      }
    }
  }
  return 0;
}

int maxflow(int start, int end) {
  int flow = 0;
  level.resize(graph.size() + 1);
  while (true) {
    bfs(start);
    if (level[end] < 0) return flow;
    iter.assign(graph.size() + 1, 0);
    int f;
    while ((f = dfs(start, end, 1e9)) > 0) {
      flow += f;
    }
  }
}
```

3.10 Minimum cost maximum flow

```cpp
#define int long long
#define pii pair<int, int>

// Edmonds-Karp Algorithm Time: O(VE^2) 實際上會快一點
// 一條邊的費用為 單位花費 * 流過流量
// 把原本的 BFS 換成 SPFA 而已
// 記得在 main 裡面 resize graph
// MCMF 回傳 {flow, cost}
// 無向圖：add_edge(u,v,C,W), add_edge(v,u,C,W);

class edge{
  public:
    int next;
    int capacity;
    int rev;
    int cost;
    bool is_rev;
    edge(int _n, int _c,
         int _r, int _co, int _ir) : next(_n), capacity
        (_c), rev(_r), cost(_co), is_rev(_ir){};
};
```

```
vector<vector<edge>> graph;

void add_edge(int a, int b, int capacity, int cost){
  graph[a].push_back(
      edge(b, capacity, graph[b].size(), cost, false));
  graph[b].push_back
      (edge(a, 0, graph[a].size() - 1, -cost, true));
}

pii dfs(int now
    , int end, pii data, vector<pii> &path, int idx){
  auto [flow, cost] = data;
  if(now == end) return {flow, 0};
  auto &e = graph[now][path[idx + 1].second];
  if(e.capacity > 0){
    auto [ret, nc] = dfs(e.next, end, {min(flow
        , e.capacity), cost + e.cost}, path, idx + 1);
    if(ret > 0){
      e.capacity -= ret;
      graph[e.next][e.rev].capacity += ret;
      return {ret, nc + ret * e.cost};
    }
  }
  return {0, 0};
}

vector<pii> search_path(int start, int end){
  int n = graph.size() + 1;
  vector<int> dis(n + 1, 1e9);
  vector<bool> vis(n + 1, false);
  vector<pii> ans; queue<int> q;
  vis[start] = true; q.push(start); dis[start] = 0;
  vector<pii> parent(graph.size(), {-1, -1});
  q.push(start);
  while(!q.empty()){
    auto node = q.front(); vis[node] = false; q.pop();
    for(int i = 0; i < graph[node].size(); i++){
      auto &e = graph[node][i];
      if(e.capacity
          > 0 and e.cost + dis[node] < dis[e.next]){
        dis[e.next] = e.cost + dis[node];
        parent[e.next] = {node, i};
        if(!vis[e.next]){
          q.push(e.next);
          vis[e.next] = true;
        }
      }
    }
  }
  if(parent[end].first == -1) return ans;
  int now = end;
  while(now != start){
    auto [node, idx] = parent[now];
    ans.emplace_back(node, idx);
    now = node;
  }
  ans.emplace_back(start, -1);
  reverse(ans.begin(), ans.end());
  return ans;
}

pii MCMF(int start, int end){
  int ans = 0, cost = 0;
  while(1){
    vector<bool> visited(graph.size() + 1, false);
    auto tmp = search_path(start, end);
    if(tmp.size() == 0) break;
    auto [flow, c] = dfs(start, end, {1e9, 0}, tmp, 0);
    ans += flow;
    cost += c;
  }
  return {ans, cost};
}
```

3.11 二分圖

```
/*
判定二分圖: 著色法 dfs 下去，顏色相撞非二分圖

二分圖最大匹配: 用 maxflow 去做，一個 src
     點聯通所有左圖，左圖建邊向右圖，右圖再建邊向 end
     點，計算 src 跟 end 的最大流，若要還原，找出左圖
    通往右圖中 capacity 為 0 的邊，他的兩個端點就是答案

最小點覆蓋: 選最少的點，保證每條邊
    至少有一個端點被選到， 最小點覆蓋 = 二分圖最大匹配

最大獨立集: 選最多的點，滿足這些
    點兩兩間互不相連， 最大獨立集 = n - 二分圖最大匹配
*/
```

3.12 Check cycle

```
vector<int> G[MAXN];
bool visit[MAXN];
/* return if the connected component where u is
    contains a cycle*/
bool dfs(int u, int pre) {
    if(visit[u])    return true;
    visit[u] = true;
    for(int v : G[u])
        if(v != pre && dfs(v, u))
            return true;
    return false;
}

//check if a graph contains a cycle

bool checkCycle(int n) {
    for(int i = 1; i <= n; i++)
        if(!visit[i] && dfs(i, -1))
            return true;
    return false;
}
```

3.13 BCC

```
// [USAGE] BCC bcc(n); bcc.add_edge(u, v); bcc.solve();
// bcc.is_ap[i]; // i 是否為割點
// bcc.bcc[j]; // 第 j 個點雙連通分量中包含的所有頂點
// bcc.nbcc; // 點雙連通分量數量

// [USAGE] bcc.block_cut_tree();
// bcc.nG[i]; // 新圖頂點 i 的所有鄰居
// bcc
    .bln[i]; // 原圖中的 i 在新圖上的編號(i可以是割點)
// bcc.cir[j]; // 新圖上的頂點 j 是否為割點
struct BCC { // 0-base
  int n, dft, nbcc;
  vector<int> low, dfn, bln, stk, is_ap, cir;
  vector<vector<int>> G, bcc, nG;
  void make_bcc(int u) {
    bcc.emplace_back(1, u);
    for (; stk.back() != u; stk.pop_back())
      bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
    stk.pop_back(), bln[u] = nbcc++;
  }
  void dfs(int u, int f) {
    int child = 0;
    low[u] = dfn[u] = ++dft, stk.pb(u);
    for (int v : G[u])
      if (!dfn[v]) {
        dfs(v, u), ++child;
        low[u] = min(low[u], low[v]);
        if (dfn[u] <= low[v]) {
          is_ap[u] = 1, bln[u] = nbcc;
          make_bcc(v), bcc.back().pb(u);
        }
      } else if (dfn[v] < dfn[u] && v != f)
        low[u] = min(low[u], dfn[v]);
    if (f == -1 && child < 2) is_ap[u] = 0;
    if (f == -1 && child == 0) make_bcc(u);
  }
  BCC(int _n): n(_n), dft(),
      nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
  void add_edge(int u, int v) {
    G[u].pb(v), G[v].pb(u);
  }
  void solve() {
    for (int i = 0; i < n; ++i)
      if (!dfn[i]) dfs(i, -1);
  }
  void block_cut_tree() {
    cir.resize(nbcc);
    for (int i = 0; i < n; ++i)
      if (is_ap[i])
        bln[i] = nbcc++;
    cir.resize(nbcc, 1), nG.resize(nbcc);
    for (int i = 0; i < nbcc && !cir[i]; ++i)
      for (int j : bcc[i])
```

```
      if (is_ap[j])
        nG[i].pb(bln[j]), nG[bln[j]].pb(i);
  } // up to 2 * n - 2 nodes!! bln[i] for id
};  // 8BQube
```

4 String

4.1 trie

```
class trie{
  public:
    class node{
      public:
        int count;
        vector<trie::node*> child;
        node(){
          child.resize(26, nullptr);
          count = 0;
        }
        ~node() {
          for (auto c : child)
            if (c) delete c;
        }
    };
    node* root;
    trie(){
      root = new node;
    }
    ~trie() {
      delete root;
    }
    void insert(string s){
      auto temp = root;
      for(int i = 0; i < s.size(); i++){
        if(!temp -> child[s[i] -
            'a']) temp -> child[s[i] - 'a'] = new node;
        temp = temp -> child[s[i] - 'a'];
      }
      temp -> count++;
    }
    bool search(string &s){
      auto temp = root;
      for(int i = 0; i < s.size(); i++){
        temp = temp -> child[s[i] - 'a'];
        if(!temp) return false;
      }
      if(temp -> count > 0) return true;
      return false;
    }
};
```

4.2 KMP

```
vector<int> build(string &s){
  vector<int> next = {0, 0};
  // 匹配失敗跳去哪 (最長共同前後綴)
  int length = s.size(), j = 0;
  for(int i = 1; i < length; i++){
    while(j > 0 and s[j] != s[i]){
      j = next[j];
    }
    if(s[j] == s[i]) j++;
    next.push_back(j);
  }
  return next;
}

int match(string &a, string &b){
  auto next = build(b);
  int length
      = a.size(), length2 = b.size(), j = 0, count = 0;
  for(int i = 0; i < length; i++){
    while(j > 0 and a[i] != b[j]){
      j = next[j];
    }
    if(a[i] == b[j]) j++;
    if(j == length2){
      count++;
      j = next[j];
    }
  }
  return count;
}
```

4.3 Hash

```
vector<int> Pow(int num){
  int p = 1e9 + 7;
  vector<int> ans = {1};
  for(int i = 0; i < num; i++)
    ans.push_back(ans.back() * b % p);
  return ans;
}

vector<int> Hash(string s){
  int p = 1e9 + 7;
  vector<int> ans = {0};
  for(char c:s){
    ans.push_back((ans.back() * b + c) % p);
  }
  return ans;
}

// 閉區間[l, r]
int query
    (vector<int> &vec, vector<int> &pow, int l, int r){
  int p = 1e9 + 7;
  int length = r - l + 1;
  return
       (vec[r + 1] - vec[l] * pow[length] % p + p) % p;
}
```

4.4 Zvalue

```
vector<int> z_func(string s1){
  int l = 0, r = 0, n = s1.size();
  vector<int> z(n, 0);
  for(int i = 1; i < n; i++){
    if(i
        <= r and z[i - l] < r - i + 1) z[i] = z[i - l];
    else{
      z[i] = max(z[i], r - i + 1);
      while(i + z
          [i] < n and s1[i + z[i]] == s1[z[i]]) z[i]++;
    }
    if(i + z[i] - 1 > r){
      l = i;
      r = i + z[i] - 1;
    }
  }
  return z;
}
```

4.5 最長迴文子字串

```
// 找到對於每個位置的迴文半徑
vector<int> manacher(string s) {
  string t = "#";
  for (auto c : s) {
    t += c;
    t += '#';
  }
  int n = t.size();
  vector<int> r(n);
  for (int i = 0, j = 0; i
       < n; i++) {    // i 是中心, j 是最長回文字串中心
    if (2 * j - i >= 0 && j + r[j] > i) {
      r[i] = min(r[2 * j - i], j + r[j] - i);
    }
    while (i - r[i] >= 0 &&
         i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
      r[i] += 1;
    }
    if (i + r[i] > j + r[j]) {
      j = i;
    }
  }
  return r;
  // # a # b # a #
  // 1 2 1 4 1 2 1
  // # a # b # b # a #
  // 1 2 1 2 5 2 1 2 1
  // 值 -1 代表原回文字串長度
  // (id - val + 1) / 2 可得原字串回文開頭
}
```

4.6 Suffix Array

```cpp
struct SuffixArray {
  int n; string s;
  vector<int> sa, rk, lc;
  // 想法 :
       排序過了，因此前綴長得像的會距離很近在差不多位置
  // n: 字串長度
  // sa: 後綴數組，sa[i] 表示第 i 小的後綴的起始位置
  // rk: 排名數組，rk[i] 表示從位置 i 開始的後綴的排名
  // lc: LCP 數組，
      lc[i] 表示 sa[i] 和 sa[i + 1] 的最長公共前綴長度
  // 求 sa[i] 跟 sa[j] 的
       LCP 長度 當 i < j : min(lc[i] ...... lc[j - 1])
  // 求 longest common substring : A +
      "#" + B 建立 SA，找到 sa 相鄰但不同組中 lc 最大的
  SuffixArray(const string &s_) {
    s = s_; n = s.length();
    sa.resize(n);
    lc.resize(n - 1);
    rk.resize(n);
    iota(sa.begin(), sa.end(), 0);
    sort(sa.begin(), sa.end
        (), [&](int a, int b) { return s[a] < s[b]; });
    rk[sa[0]] = 0;
    for (int i = 1; i < n; ++i)
      rk[sa[i]]
          = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
    int k = 1;
    vector<int> tmp, cnt(n);
    tmp.reserve(n);
    while (rk[sa[n - 1]] < n - 1) {
      tmp.clear();
      for (int i = 0; i < k; ++i)
        tmp.push_back(n - k + i);
      for (auto i : sa)
        if (i >= k)
          tmp.push_back(i - k);
      fill(cnt.begin(), cnt.end(), 0);
      for (int i = 0; i < n; ++i)
        ++cnt[rk[i]];
      for (int i = 1; i < n; ++i)
        cnt[i] += cnt[i - 1];
      for (int i = n - 1; i >= 0; --i)
        sa[--cnt[rk[tmp[i]]]] = tmp[i];
      swap(rk, tmp);
      rk[sa[0]] = 0;
      for (int i = 1; i < n; ++i)
        rk[sa[i]] = rk[sa[i - 1]] + (tmp[
            sa[i - 1]] < tmp[sa[i]] || sa[i - 1] + k ==
             n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
      k *= 2;
    }
    for (int i = 0, j = 0; i < n; ++i) {
      if (rk[i] == 0) {
        j = 0;
      } else {
        for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j
             < n && s[i + j] == s[sa[rk[i] - 1] + j]; )
          ++j;
        lc[rk[i] - 1] = j;
      }
    }
  }
};
```

4.7 AC-Automatan

```cpp
struct AC_Automatan {
  int nx[len][sigma], fl[len], cnt[len], ord[len], top;
  int rnx[len][sigma]; // node actually be reached
  int newnode() {
    fill_n(nx[top], sigma, -1);
    return top++;
  }
  void init() { top = 1, newnode(); }
  int input(string &s) {
    int X = 1;
    for (char c : s) {
      if (!~nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
      X = nx[X][c - 'A'];
    }
    return X; // return the end node of string
  }
  void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); ) {
      int R = q.front();
      q.pop(), ord[t++] = R;
      for (int i = 0; i < sigma; ++i)
        if (~nx[R][i]) {
          int X = rnx[R][i] = nx[R][i], Z = fl[R];
          for (; Z && !~nx[Z][i]; ) Z = fl[Z];
          fl[X] = Z ? nx[Z][i] : 1, q.push(X);
        }
        else rnx[R][i] = R > 1 ? rnx[fl[R]][i] : 1;
    }
  }
  void solve() {
    for (int i = top - 2; i > 0; --i)
      cnt[fl[ord[i]]] += cnt[ord[i]];
  }
} ac;
```

5 Geometry

5.1 Point

```cpp
template<typename T>
class point{
    public:
    T x;
    T y;
    point(){}
    point(T _x, T _y){
        x = _x;
        y = _y;
    }
    point<T> operator+(const point<T> &a);
    point<T> operator-(const point<T> &a);
    point<T> operator/(const point<T> &a);
    point<T> operator/(T a);
    point<T> operator*(const T &a);
    bool operator<(const point<T> &a);
};

template<typename T>
point<T> point<T>::operator+(const point<T> &a){
    return point<T>(x + a.x, y + a.y);
}

template<typename T>
point<T> point<T>::operator-(const point<T> &a){
    return point<T>(x - a.x, y - a.y);
}

template<typename T>
point<T> point<T>::operator/(const point<T> &a){
    return point<T>(x / a.x, y / a.y);
}

template<typename T>
point<T> point<T>::operator/(T a){
    return point<T>(x / a, y / a);
}

template<typename T>
point<T> point<T>::operator*(const T &a){
    return point<T>(x * a, y * a);
}

template<typename T>
bool point<T>::operator<(const point<T> &a){
    if(x != a.x) return x < a.x;
    return y < a.y;
}
```

5.2 內積, 外積, 距離

```cpp
template<typename T>
T dot(const point<T> &a,const point<T> &b){
    return a.x * b.x + a.y * b.y;
}

template<typename T>
T cross(const point<T> &a,const point<T> &b){
    return a.x * b.y - a.y * b.x;
}

template<typename T>
```

```cpp
T len(point<T> p){
    return sqrt(dot(p, p));
}

template<typename T>
int sign(T x){
    return x == 0 ? 0 : x > 0 ? 1 : -1;;
}

template<typename T>
T pointSegDist(point<T> q0, point<T> q1, point<T> p) {
    if (sign(len(q0 - q1)) == 0) return len(q0 - p);
    if (sign(dot(q1 - q0, p
        - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return
            abs(cross(q1 - q0, p - q0) / len(q0 - q1));
    return min(len(p - q0), len(p - q1));
}
```

5.3 向量應用

```cpp
template<typename T>
bool collinearity
    (point<T> p1, point<T> p2, point<T> p3){
    //檢查三點是否共線
    return cross(p2 - p1, p2 - p3) == 0;
}

template<typename T>
bool inLine(point<T> a, point<T> b, point<T> p){
    //檢查 p 點是否在ab線段
    return collinearity
        (a, b, p) && dot(a - p, b - p) <= 0;
}

template<typename T>
bool intersect
    (point<T> a, point<T> b, point<T> c, point<T> d){
    //ab線段跟cd線段是否相交
    return (cross(b - a, c - a) * \
        cross(b - a, d - a) < 0 && \
        cross(d - c, a - c) * \
        cross(d - c, b - c) < 0) \
        || inLine(a, b, c) || \
        inLine(a, b, d) || inLine(c, d, a) \
        || inLine(c, d, b);
}

template<typename T>
point<T> intersection
    (point<T> a, point<T> b, point<T> c, point<T> d){
    //ab線段跟cd線段相交的點
    assert(intersect(a, b, c, d));
    return a + (b -
        a) * cross(a - c, d - c) / cross(d - c, b - a);
}

template<typename T>
bool inPolygon(vector<point<T>> polygon, point<T> p){
    //判斷點
        p是否在多邊形polygon裡，vector裡的點要連續填對
    for(int i = 0; i < polygon.size(); i++)
        if(cross(p - polygon[i], \
            polygon[(i - 1 + polygon.size()) % \
            polygon.size()] - polygon[i]) * \
            cross(p - polygon[i], \
            polygon[(i +
              1) % polygon.size()] - polygon[i]) > 0)
            return false;
    return true;
}

template<typename T>
T triangleArea(point<T> a, point<T> b, point<T> c){
    //三角形頂點，求面積
    return abs(cross(b - a, c - a)) / 2;
}

template<typename T,typename F,typename S>
long double triangleArea_Herons_formula(T a, F b, S c){
    //三角形頂點，求面積(給邊長)
    auto p = (a + b + c)/2;
    return sqrt(p * (p - a) * (p - b) * (p - c));
}

template<typename T>
T area(vector<point<T>> &p){
    //多邊形頂點，求面積
    T ans = 0;
    for(int i = 0; i < p.size(); i++)
        ans += cross(p[i], p[(i + 1) % p.size()]);
    return ans / 2 > 0 ? ans / 2 : -ans / 2;
}
```

5.4 Static Convex Hull

```cpp
// 需要使
    用前一個向量模板的 point ， 需要 operator - 以及 <
// 需要前面向量模板的 cross

template<typename T>
vector<point<T>> getConvexHull(vector<point<T>>& pnts){
    sort(pnts.begin(), pnts.end());
    auto cmp = [&](point<T> a, point<T> b)
    { return a.x == b.y && a.x == b.y; };
    pnts.erase(unique
        (pnts.begin(), pnts.end(), cmp), pnts.end());
    if(pnts.size()<=1) return pnts;
    vector<point<T>> hull;
    for(int i = 0; i < 2; i++){
        int t = hull.size();
        for(point<T> pnt : pnts){
            while(hull.size() - t >= 2 &&
                cross(hull.back() - hull[hull.size()
               - 2], pnt - hull[hull.size() - 2]) < 0)
                // <= 0 或者 < 0 要看點有沒有在邊上
                hull.pop_back();
            hull.push_back(pnt);
        }
        hull.pop_back();
        reverse(pnts.begin(), pnts.end());
    }
    return hull;
}
```

5.5 外心, 最小覆蓋圓

```cpp
int sign(double a)
{
  // 小於 eps
      回傳 0，否則正回傳 1 ，負回傳 應付浮點數誤差用
  const double eps = 1e-10;
  return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;
}

// 輸入三個點求外心
template <typename T>
point<T> findCircumcenter(point<
    T> A, point<T> B, point<T> C, const T eps = 1e-10){
    point<T> AB = B - A;
    point<T> AC = C - A;
    T AB_len_sq = AB.x * AB.x + AB.y * AB.y;
    T AC_len_sq = AC.x * AC.x + AC.y * AC.y;
    T D = AB.x * AC.y - AB.y * AC.x;
    // 若三點接近共線
    assert(fabs(D) < eps);
    // 外心的座標
    T circumcenterX = A.x + (
        AC.y * AB_len_sq - AB.y * AC_len_sq) / (2 * D);
    T circumcenterY = A.y + (
        AB.x * AC_len_sq - AC.x * AB_len_sq) / (2 * D);
    return point<T>(circumcenterX, circumcenterY);
}

template<typename T>
pair<T, point<T>> MinCircleCover(vector<point<T>> &p) {
    // 引入前面的 len 跟 point
    // 回傳最小覆蓋圓{半徑,中心}
    random_shuffle(p.begin(), p.end());
    int n = p.size();
    point<T> c = p[0]; T r = 0;
    for(int i=1;i<n;i++) {
        if(sign(len(c-p[i])-r) > 0) { // 不在圓內
            c = p[i], r = 0;
            for(int j=0;j<i;j++) {
                if(sign(len(c-p[j])-r) > 0) {
                    c = (p[i]+p[j])/2.0;
                    r = len(c-p[i]);
                    for(int k=0;k<j;k++) {
```

```
                    if(sign(len(c-p[k])-r) > 0) {
                        c = findCircumcenter
                            (p[i],p[j],p[k]);
                        r = len(c-p[i]);
                    }
                }
            }
        }
    }
    return make_pair(r, c);
}
```

5.6 四邊形旋轉

```
const long double PI = acos(-1);

// 寬 w 高 h 的四邊形，旋轉一個 pi 後在每個角度的寬高

vector
    <pair<double, double>> rotate(double w, double h){
  int freq = 1000; // 自己調整精度
  vector<pair<double, double>> res;
  for (int i = 0; i <= 5; ++i) {
    double theta = (PI * i) / 5;
    double nw
        = c * fabs(cos(theta)) + d * fabs(sin(theta));
    double nh
        = c * fabs(sin(theta)) + d * fabs(cos(theta));
    res.push_back({nw, nh});
  }
  return res;
}
```

5.7 旋轉

```
const long double PI = acos(-1);
// 逆時針旋轉
// angle_red 為弧度
pair<double, double> rotate_point
    (double x, double y, double angle_rad) {
  angle_rad *= PI;
  double
      new_x = x * cos(angle_rad) - y * sin(angle_rad);
  double
      new_y = x * sin(angle_rad) + y * cos(angle_rad);
  return {new_x, new_y};
}

int main() {
  double x = 5, y = 0;
  double angle = 0.5;   // 逆時針旋轉 90 度
  auto result = rotate_point(x, y, angle);
  cout << result.first << " " << result.second << endl;
  // 0, 5
  return 0;
}
```

5.8 極座標轉直角座標

```
// 極座標轉換為直角座標函數，theta 單位為弧度
const long double PI = acos(-1);
pair<double, double
    > polar_to_cartesian(double r, double theta) {
  double theta_radians = theta * PI;
  double x = r * cos(theta_radians);
  double y = r * sin(theta_radians);
  return {x, y};
}

int main() {
  double r = 5, theta = 0.5;  // 極座標
  auto result = polar_to_cartesian(r, theta);
  cout << result.first << " " << result.second << endl;
  // 0, 5
  return 0;
}
```

5.9 直角座標轉極座標

```
// 直角座標轉換為極座標
const long double PI = acos(-1);
std::pair<double
    , double> cartesian_to_polar(double x, double y) {
  double r = sqrt(x * x + y * y);
  double theta = atan2(y, x) / PI;
  return {r, theta};
}

int main() {
  double x = 3, y = 4;  // 直角座標
  auto result = cartesian_to_polar(x, y);
  cout << "r = " << result
      .first << ", theta = " << result.second << endl;
  // 5, 0.295167
  return 0;
}
```

6 Data Structure

6.1 Sparse Table

```
class Sparse_Table{
  // 0-base
  // 要改成找最大把min換成max就好
  private:
  public:
    int spt[500005][22][2];
    Sparse_Table(vector<int> &ar){
      int n = ar.size();
      for (int i = 0; i < n; i++){
          spt[i][0][0] = ar[i];
          // spt[i][0][1] = ar[i];
      }
      for (int j = 1; (1 << j) <= n; j++) {
        for (int i = 0; (i + (1 << j) - 1) < n; i++) {
          spt[i][j][0] = min(spt[i + (1 <<
               (j - 1))][j - 1][0], spt[i][j - 1][0]);
          // spt[i][j][1] = max(spt[i + (1 <<
               (j - 1))][j - 1][1], spt[i][j - 1][1]);
        }
      }
    }
    int query_min(int l, int r)
    {
      if(l>r) return INT_MAX;
      int j = (int)__lg(r - l + 1);
      ///j = 31 - __builtin_clz(r - l+1);
      return min
          (spt[l][j][0], spt[r - (1 << j) + 1][j][0]);
    }
    int query_max(int l, int r)
    {
      if(l>r) return INT_MAX;
      int j = (int)__lg(r - l + 1);
      ///j = 31 - __builtin_clz(r - l+1);
      return max
          (spt[l][j][1], spt[r - (1 << j) + 1][j][1]);
    }
};
```

6.2 Segement Tree

```
// 不想要區間加值就把每個函數裡面的 push 都移除
// 最外層呼叫時，每個 id 都傳 1

const int N = 200000 + 9;
int a[N];
int seg[4 * N];
int lazy[4 * N];

inline void pull(
    int id){ seg[id] = seg[id * 2] + seg[id * 2 + 1]; }

inline void apply(int id, int l, int r, int v){
    seg[id] += v * (r - l + 1);
    lazy[id] += v;
}

inline void push(int id, int l, int r){
    if (!lazy[id] || l == r) return;
    int mid = (l + r) / 2;
    apply(id * 2, l, mid, lazy[id]);
    apply(id * 2 + 1, mid + 1, r, lazy[id]);
    lazy[id] = 0;
}

void build(int id, int
     l, int r) { // 編號為 id 的節點，存的區間為 [l, r]
    if (l
        == r) { seg[id] = a[l]; return; } // 葉節點的值
```

```
    int mid =
        (l + r) / 2;                  // 將區間切成兩半
    build(id * 2, l, mid);                  // 左子節點
    build(id * 2 + 1, mid + 1, r);          // 右子節點
    pull(id);
}

// 區間查詢：回傳 [ql, qr] 的區間和
int query(int id, int l, int r, int ql, int qr) {
    if (r < ql || qr < l) return 0;         // 交集為空
    if (ql <= l && r <= qr) return seg[id]; // 完全覆蓋
    push(
        id, l, r);                         // 下傳 lazy
    int mid = (l + r) / 2;
    return query(id * 2, l, mid, ql, qr)    // 左
         + query(id * 2 + 1, mid + 1, r, ql, qr); // 右
    // 否則，往左、右進行遞迴
}

// 區間加值：將 [ql, qr] 每個位置都加上 x
void range_add
    (int id, int l, int r, int ql, int qr, int x) {
    if (r < ql || qr < l) return;           // 交集為空
    if (ql <= l && r <=
        qr) { apply(id, l, r, x); return; } // 完全覆蓋
    push(id, l, r)
        ;                         // 下傳 lazy 再往下走
    int mid = (l + r) / 2;
    range_add
        (id * 2, l, mid, ql, qr, x);          // 左
    range_add
        (id * 2 + 1, mid + 1, r, ql, qr, x);  // 右
    pull(id);
}

// 單點修改 (設值版)：將 a[i] 改成 x
void modify(int id, int l, int r, int i, int x) {
    if (l == r) { seg[id] = x; return; }
    push(id, l, r); // 確保往下的值正確
    int mid = (l + r) / 2;
    if (i
        <= mid) modify(id * 2, l, mid, i, x);     // 左
    else modify
        (id * 2 + 1, mid + 1, r, i, x);       // 右
    pull(id);
}
```

6.3 Discrete Segement Tree

```
#include <bits/stdc++.h>
#define int long long
#define de(x) cout << #x << '=' << x << ", "
#define dd cout << '\n';

/*
本題給n個長方形 0 <= x1, y1, x2, y2 <= 10^9
求被奇數個長方形覆蓋的面積

想法：掃描線+離散化線段樹
區間xor加值 + 區間sum查詢
*/

/*
先將輸入點離散化 [a1, a2, ... , an] => [1, 2, ... , n]
將每個相鄰離散點區間重新編號 [1, 2] => 編
    號1 [2, 3] => 編號 2 , ... , [n - 1, n] 編號 n - 1
將每個區間的長度都記下 pre[1] = a2 - a1
    , pre[2] = a3 - a2 , ... , pre[n - 1] = an - an-1
若要加值區間[a, b]，則加線段樹上的區間[a, b - 1]
和一般線段樹唯一差別在 void apply
    ()，區間長度從(r - l + 1)變成(pre[r] - pre[l - 1])
*/

/*

testcase 1 :
2
0 0 4 4
1 1 3 3

answer : 12

testcase 2 :
4
0 0 10 10
1 1 11 11
2 2 12 12
3 3 13 13

answer : 72

*/

using namespace std;
int tt = 1;

int a[600009];
int seg[4 * 600009];
int lazy[4 * 600009];
int mp[600009];
int pre[600009];

map<int, int> mp2;

void pull(int id){
    seg[id] = seg[id * 2] + seg[id * 2 + 1];
}
void apply(int id, int l, int r, int v){
    seg[id] = (pre[r] - pre[l - 1]) - seg[id];
    lazy[id] ^= 1;
}

void push(int id, int l, int r){
    if(!lazy[id] || l == r) return;
    int mid = (l + r) / 2;
    apply(id * 2, l, mid, lazy[id]);
    apply(id * 2 + 1, mid + 1, r, lazy[id]);
    lazy[id] = 0;
}

int query(int id, int l, int r, int ql, int qr){
    if(r < ql || qr < l) return 0;
    if(ql <= l && r <= qr) return seg[id];
    push(id, l, r);
    int mid = (l + r) / 2;
    return query(id * 2, l, mid, ql
        , qr) + query(id * 2 + 1, mid + 1, r, ql, qr);
}

void range_add
    (int id, int l, int r, int ql, int qr, int x){
    if(r < ql || qr < l) return ;
    if(ql <= l && r <= qr) {
        apply(id, l, r, x);
        return;
    }
    push(id, l, r);
    int mid = (l + r) / 2;
    range_add(id * 2, l, mid, ql, qr, x);
    range_add(id * 2 + 1, mid + 1, r , ql, qr, x);
    pull(id);
}

void _pre(){
    cout.tie(nullptr);
    //cin >> tt;
}

struct rec{
    int loc, down, top;
};

bool comp(rec a, rec b){
    return a.loc < b.loc;
}

void swap(int &a, int &b){
    int tmp = a;
    a = b;
    b = tmp;
}

void solve(){
    int n, a, b, c, d, i, j, k;
    vector<rec> v;
    cin >> n;
```

```cpp
    set<int> st;
    for(i = 1; i <= n; i ++){
        cin >> a >> b >> c >> d;
        int high = max(b, d);
        int low = min(b, d);
        v.push_back({a, low, high});
        v.push_back({c, low, high});
        st.insert(low);
        st.insert(high);
    }
    int N = 0;
    pre[0] = 0;
    mp[0] = 0;
    vector<int> vv;

    for(auto it = st.begin(); it != st.end(); it ++){
        mp[++ N] = *it;
        mp2[*it]= N;
        vv.push_back(*it);
    }

    for(i = 0; i < (int)vv.size() - 1; i ++){
      pre[i + 1] = pre[i] + (vv[i + 1] - vv[i]);
    }

    for(i = 0; i < (int)v.size(); i ++){
        v[i] = {v[
            i].loc, mp2[v[i].down], mp2[v[i].top] - 1};
    }
    sort(v.begin(), v.end(), comp);
    int ans = 0;

    int lastloc = v[0].loc;
    range_add(1, 1, N - 1, v[0].down, v[0].top, 1);

    for(i = 1; i < (int)v.size(); i ++){
        int down = v[i].down;
        int top = v[i].top;
        int loc = v[i].loc;

        int q = query(1, 1, N - 1, 1, N - 1);

        ans += (loc - lastloc) * q;
        range_add(1, 1, N - 1, down, top, 1);
        lastloc = loc;
    }

    cout << ans << '\n';
}
signed main(){
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
    _pre();
    while(tt--){
        solve();
    }
}
```

6.4 Link Cut Tree

```cpp
// 通常用於對樹上任兩點間的路徑做加值、修改、查詢等工作
// 與線段樹相同，要修改 LCT 的功能只需更改
// pull、push、fix、query 等函數，再加上需要的懶標即可
// 範例為樹上任兩點 x, y 路徑上的權值 xor
// 和，樹上任意點單點改值
const int N = 300005;
class LinkCutTree {
private:
#define lc(x) node[x].ch[0]
#define rc(x) node[x].ch[1]
#define fa(x) node[x].fa
#define rev(x) node[x].rev
#define val(x) node[x].val
#define sum(x) node[x].sum
  struct Tree {
    int val, sum, fa, rev, ch[2];
  } node[N];
  inline void pull(int x) {
    sum(x) = val(x) ^ sum(lc(x)) ^ sum(rc(x));
  }
  inline void reverse(int x) {
    swap(lc(x), rc(x));
    rev(x) ^= 1;
  }
  inline void push(int x) {
    if (rev(x)) {
      reverse(lc(x));
      reverse(rc(x));
      rev(x) ^= 1;
    }
  }
  inline bool get(int x) { return rc(fa(x)) == x; }
  inline bool isroot(int x) {
    return (lc(fa(x)) ^ x) && (rc(fa(x)) ^ x);
  }
  inline void update(int x) {
    if (!isroot(x)) update(fa(x));
    push(x);
  }
  void rotate(int x) {
    int y = fa(x), z = fa(y), d = get(x);
    if (!isroot(y))
      node[z].ch[get(y)] = x; // 重要，不能更換順序
    fa(x) = z;
    node[fa(node[x].ch[d ^ 1]) = y].ch[d] =
      node[x].ch[d ^ 1];
    node[fa(y) = x].ch[d ^ 1] = y;
    pull(y), pull(x); // 先 y 再 x
  }
  void splay(int x) {
    update(x);
    for (int y = fa(x); !isroot(x);
         rotate(x), y = fa(x)) {
      if (!isroot(y)) rotate(get(x) == get(y) ? y : x);
    }
    pull(x);
  }
  int access(int x) {
    int p = 0;
    for (; x; x = fa(p = x)) {
      splay(x), rc(x) = p, pull(x);
    }
    return p;
  }
  inline void makeroot(int x) {
    access(x), splay(x), reverse(x);
  }
  inline int findroot(int x) {
    access(x), splay(x);
    while (lc(x)) { push(x), x = lc(x); }
    return splay(x), x;
  }
  inline void split(int x, int y) {
    makeroot(x), access(y), splay(y);
  }

public:
  inline void init(int len, int *data) {
    for (int i = 1; i <= len; ++i) {
      node[i].val = data[i];
    }
  }
  inline void link(int x, int y) { // 連邊
    makeroot(x);
    if (findroot(y) == x) return;
    fa(x) = y;
  }
  inline void cut(int x, int y) { // 斷邊
    makeroot(x);
    if (findroot(y) != x || fa(y) != x || lc(y))
      return;
    fa(y) = rc(x) = 0;
    pull(x);
  }
  inline void fix(int x, int v) { // 單點改值
    splay(x);
    val(x) = v;
  }
  // 區間查詢
  inline int query(int x, int y) {
    return split(x, y), sum(y);
  }
};

LinkCutTree LCT;
int n, a[N];

signed main() {
  int n, q, op, x, y;
  cin >> n >> q;
  for (int i = 1; i <= n; ++i) { cin >> a[i]; }
```

```
  LCT.init(n, a);
  while (q--) {
    cin >> op >> x >> y;
    if (op == 0) {
      cout << LCT.query(x, y) << endl;
    } else if (op == 1) {
      LCT.link(x, y);
    } else if (op == 2) {
      LCT.cut(x, y);
    } else {
      LCT.fix(x, y);
    }
  }
  return 0;
}
```

6.5 BIT

```
#define lowbit(x) x & -x

void modify(vector<int> &bit, int idx, int val) {
  for(int i = idx
      ; i <= bit.size(); i+= lowbit(i)) bit[i] += val;
}

int query(vector<int> &bit, int idx) {
  int ans = 0;
  for(int i = idx; i > 0; i-= lowbit(i)) ans += bit[i];
  return ans;
}

// the first i s.t. a[1]+...+a[i] >= k
int findK(vector<int> &bit, int k) {
  int idx = 0, res = 0;
  int mx = __lg(bit.size()) + 1;
  for(int i = mx; i >= 0; i--) {
    if((idx | (1<<i)) > bit.size()) continue;
    if(res + bit[idx | (1<<i)] < k) {
      idx = (idx | (1<<i));
      res += bit[idx];
    }
  }
  return idx + 1;
}
//O(n)建bit
for (int i = 1; i <= n; ++i) {
    bit[i] += a[i];
    int j = i + lowbit(i);
    if (j <= n) bit[j] += bit[i];
}
```

6.6 2D BIT

```

//2維BIT
#define lowbit(x) (x&-x)

class BIT {
    int n;
    vector<int> bit;

public:
    void init(int _n) {
        n = _n;
        bit.resize(n + 1);
        for(auto &b : bit) b = 0;
    }
    int query(int x) const {
        int sum = 0;
        for(; x; x -= lowbit(x))
            sum += bit[x];
        return sum;
    }
    void modify(int x, int val) {
        for(; x <= n; x += lowbit(x))
            bit[x] += val;
    }
};

class BIT2D {
    int m;
    vector<BIT> bit1D;

public:
    void init(int _m, int _n) {
        m = _m;
        bit1D.resize(m + 1);
        for(auto &b : bit1D) b.init(_n);
    }
    int query(int x, int y) const {
        int sum = 0;
        for(; x; x-= lowbit(x))
            sum += bit1D[x].query(y);
        return sum;
    }
    void modify(int x, int y, int val) {
        for(; x <= m; x += lowbit(x))
            bit1D[x].modify(y,val);
    }
};
```

6.7 undo DSU

```
struct dsu_undo{
  vector<int>sz,p;
  int comps;
  dsu_undo(int n){
    sz.assign(n+5,1);
    p.resize(n+5);
    for(int i = 1;i<=n;++i)p[i] = i;
    comps = n;
  }
  vector<pair<int,int>>opt;
  int Find(int x){
    return x==p[x]?x:Find(p[x]);
  }
  bool Union(int a,int b){
    int pa = Find(a),pb = Find(b);
    if(pa==pb)return 0;
    if(sz[pa]<sz[pb])swap(pa,pb);
    sz[pa]+=sz[pb];
    p[pb] = pa;
    opt.push_back({pa,pb});
    comps--;
    return 1;
  }
  void undo(){
        auto [pa,pb] = opt.back();
        opt.pop_back();
        p[pb] = pb;
        sz[pa]-=sz[pb];
        comps++;
  }
};
```

7 Dynamic Programing

7.1 LCS

```
// O(n^2)
int LCS(string t1, string t2) {
  if(t1.size() < t2.size()) swap(t1, t2);
  int len = t1.size();
  vector<vector<int>> dp(2, vector<int>(len + 1, 0));
  for(int j = 1; j <= t2.size(); j++){
    for(int i = 1; i <= len; i++){
      if(t2[j - 1] == t1[i - 1])
           dp[j % 2][i] = dp[(j + 1) % 2][i - 1] + 1;
      else dp[j % 2][i]
           = max(dp[(j + 1) % 2][i], dp[j % 2][i - 1]);
    }
  }
  return dp[t2.size() % 2][t1.size()];
}

// O(nlogn)
// 這裡string 要以 1 base index 所以開頭要補個字元
// d:記住此數字的前一個數字
    , t:當前LIS位置, num:根據t2生成出string來找LIS長度
// N: 最大字串長度
#define N 120
int t[N*N], d[N*N], num[N*N];
map<char, vector<int>> dict; // 每個字串出現的index位置
int binarySearch(int l, int r, int v){
    int m;
    while(r>l){
        m = (l+r)/2;
        if(num[v] > num[t[m]])l = m+1;
        else if(num[v] < num[t[m]])r = m;
        else return m;
    }
```

```
    return r;
}
int LCS(string t1, string t2){
    dict.clear();
    //i = strA.length() -1 才可以逆序
    for(int i = t1.length
        ()-1 ; i > 0 ; i--) dict[t1[i]].push_back(i) ;
    int k = 0; //生成數列的長度的最長長度
    for(int i = 1 ; i < t2.length
        () ; i++){ // 依據 strB 的每個字元來生成數列
        for(int j = 0 ; j < dict[t2[i]].size() ; j++)
        //將此字元在 strA 出現的位置放入數列
            num[++k] = dict[t2[i]][j] ;
    }
    if(k==0) return 0;
    d[1] = -1 , t[1] = 1 ; //LIS init
    int len = 1, cur ; // len 由於前面
        已經把 LCS = 0 的機會排除，於是這裡則從 1 開始

    // 標準的 LIS 作法，不斷嘗試將 LCS 生長
    for(int i = 1 ; i <= k ; i++ ){
        if(num[i] > num
            [t[len]]) t[++len] = i , d[i] = t[len-1] ;
        else{
            cur = binarySearch(1,len,i);
            t[cur] = i ;
            d[i] = t[cur-1];
        }
    }
    return len ;

}
```

7.2 LIS

```
int LIS(vector<int>& save) {
  vector<int> dp;
  int n = save.size();
  for (int i = 0; i < n; i++) {
    auto it = lower_bound(dp.begin(),dp.end(),save[i]);
    if(it == dp.end()) dp.push_back(save[i]);
    else *it = save[i];
  }
  return dp.size();
}
```

7.3 Knapsack

```
/**
 * 背包問題：
 * 1. dp[i][j]: 考慮 1~i 個物品，重量為 j 時的最大價格
 * 2. dp[i][j]: 考慮 1~i 個物品，價值為 j 時的最小重量
 */

// 當重量比較輕時 O(nw)
vector<int> dp(sum + 1, 0);
for (int i = 1; i <= n; ++i) {
  for (int j = sum /* bound */; j >= weight[i]; --j) {
    if (dp[j] < dp[j - weight[i]] + price[i]) {
      dp[j] = dp[j - weight[i]] + price[i];
      backtrack[i][j] = 1;
    }
  }
}
// 當重量比較重時 O(nc)
vector<int> dp(sum + 1, 1e9 + 7);
dp[0] = 0;
for (int i = 1; i <= n; ++i) {
  for (int j = sum /* bound */; j >= price[i]; --j) {
    if (dp[j] > dp[j - price[i]] + weight[i]) {
      dp[j] = dp[j - price[i]] + weight[i];
      backtrack[i][j] = 1;
    }
  }
}

// backtrack: 找到當 bound 為 k 時，背包內有哪些東西
// 註：只找到其中一種
int l = n, r = k;
vector<int> ans;
while (l != 0 && r != 0) {
  if (backtrack[l][r]) {
    ans.push_back(l);
    r -= weight[l]; // 當用方法一時，用這行
    r -= price[l]; // 當用方法二時，用這行
  }
  l--;
}
```

7.4 位元 dp

```
// 檢查第 n 位是否為1
if(a & (1 << n))

// 強制將第 n 位變成1
a |= (1 << n)

// 強制將第 n 位變成0
a &= ~(1 << n)

// 將第 n 位反轉(1變0, 0變1)
a ^= (1 << n)

// 第 0 ~ n - 1位 全部都是1
(1 << n) - 1

// 兩個集合的聯集
S = a | b

// 兩個集合的交集
S = a & b
```

7.5 經典 dp 轉移式

```
/*
最大區間和：

dp[i] 代表 由第 i 項結尾時的最大區間和
dp[0] = arr[0]
dp[i] = max(dp[i - 1], arr[i])
ans = max_element(dp)

*/
```

8 Divide and conquer

8.1 逆序數對

```
int merge(
    vector<pair<int, int>>& v, int l, int mid, int r) {
  vector<pair<int, int>> temp(r - l + 1);
  int i = l, j = mid + 1, k = 0, inv_count = 0;
  while (i <= mid && j <= r) {
      if (v[i].second <= v[j].second) {
          temp[k++] = v[i++];
      } else {
          temp[k++] = v[j++];
          inv_count += (mid - i + 1);
      }
  }
  while (i <= mid) temp[k++] = v[i++];
  while (j <= r) temp[k++] = v[j++];
  for (int i = l; i <= r; i++) {
    v[i] = temp[i - l];
  }
  return inv_count;
}

int mergeSort
    (vector<pair<int, int>>& v, int l, int r) {
  int count = 0;
  if (l < r) {
    int mid = l + (r - l) / 2;
    count += mergeSort(v, l, mid);
    count += mergeSort(v, mid + 1, r);
    count += merge(v, l, mid, r);
  }
  return count;
}

signed main()
{
  int n;
  cin >> n;
  vector<pair<int, int>> arr(n);
  for(int i = 0; i < n; i++){
    arr[i].first = i;
    cin >> arr[i].second;
  }
```

```
    cout << mergeSort(arr, 0, n - 1) << '\n';
}
```

8.2 Mo's algorithm

```
// time complexity: n * sqrt(q) * O(p)
// O(p) 為 add, remove 的時間複雜度
// 若知道 [l, r] 的答案 需要快速知道 [l
    - 1, r], [l + 1, r], [l, r - 1], [l, r + 1] 的答案

int n, q, k, l = 0, r = 0;

array queries = 詢問們;
type ans; //目前答案
void add(type v){/*...*/} //增加一個數字，算新答案
void remove(type v){/*...*/} //移除一個數字，算新答案

vector<tuple<int, int, int, int>> queries(q);
k = sqrt(n);
for(int i = 0; i < q; i++){
  int l, r;
  cin >> l >> r;
  queries[i] = {l / k, r, l, i};
  // 先對 l 的塊，再對 r 排序
}

sort(queries.begin(), queries.end());

add(a[0]);

for(int i = 0; i < q; i++){
  auto [_, rp, lp, id] = queries[i];
  lp--; rp--;
  while(l > lp) add(a[--l]);
  while(l < lp) remove(a[l++]);
  while(r < rp) add(a[++r]);
  while(r > rp) remove(a[r--]);
  ans_v[id] = ans;
}
```

9 Tree

9.1 樹直徑

```
int d1[200005], d2[200005], ans;

void dfs(int now, int fa, vector<vector<int>> &graph){
  for(auto i: graph[now]){
    if(i != fa){
      dfs(i, now, graph);
      if(d1[i] + 1 > d1[now]){
        d2[now] = d1[now];
        d1[now] = d1[i] + 1;
      }
      else if(d1[i] + 1 > d2[now]){
        d2[now] = d1[i] + 1;
      }
    }
  }
  ans = max(ans, d1[now] + d2[now]);
}

signed main()
{
  int n;
  cin >> n;
  vector<vector<int>> graph(n + 1);
  for(int i = 0; i < n - 1; i++){
    int a, b;
    cin >> a >> b;
    graph[a].push_back(b);
    graph[b].push_back(a);
  }
  dfs(1, 0, graph);
  cout << ans << '\n';
}
```

9.2 LCA

```
// n 為點數，graph 由子節點往父節點建有向邊
// graph 要 resize

int n, q;
int fa[20][200001];
int dep[200001];

vector<vector<int>> graph;

void dfs(int now, int lst){
  fa[0][now] = lst;
  for(int &i:graph[now]){
    dep[i] = dep[now] + 1;
    dfs(i, now);
  }
}

void build_lca(int root){
  dep[root] = 1;
  dfs(root, root);
  for(int i = 1; i < 18; i++){
    for(int j = 1; j < n + 1; j++){
      fa[i][j] = fa[i - 1][fa[i - 1][j]];
    }
  }
}

int lca(int a, int b){
  // 預設a比b淺
  if(dep[a] > dep[b]) return lca(b, a);
  // 讓a和b跳到同一個地方
  int step = dep[b] - dep[a];
  for (int i = 0; i < 18; i++)
  {
    if(step >> i & 1){
      b = fa[i][b];
    }
  }
  if(a == b) return a;
  for(int i = 17; i >= 0; i--){
    if(fa[i][a] != fa[i][b]){
      a = fa[i][a];
      b = fa[i][b];
    }
  }
  return fa[0][a];
}
```

9.3 樹壓平

```
//紀錄in & out
vector<int> Arr;
vector<int> In, Out;
void dfs(int u) {
  Arr.push_back(u);
  In[u] = Arr.size() - 1;
  for (auto v : Tree[u]) {
    if (v == parent[u])
      continue;
    parent[v] = u;
    dfs(v);
  }
  Out[u] = Arr.size() - 1;
}

//進去出來都紀錄
vector<int> Arr;
void dfs(int u) {
  Arr.push_back(u);
  for (auto v : Tree[u]) {
    if (v == parent[u])
      continue;
    parent[v] = u;
    dfs(v);
  }
  Arr.push_back(u);
}

//用Treap紀錄
Treap *root = nullptr;
vector<Treap *> In, Out;
void dfs(int u) {
  In[u] = new Treap(cost[u]);
  root = merge(root, In[u]);
  for (auto v : Tree[u]) {
    if (v == parent[u])
      continue;
    parent[v] = u;
    dfs(v);
  }
  Out[u] = new Treap(0);
```

```cpp
  root = merge(root, Out[u]);
}
//Treap紀錄Parent
struct Treap {
  Treap *lc = nullptr, *rc = nullptr;
  Treap *pa = nullptr;
  unsigned pri, size;
  long long Val, Sum;
  Treap(int Val):
    pri(rand()), size(1),
    Val(Val), Sum(Val) {}
  void pull();
};

void Treap::pull() {
  size = 1;
  Sum = Val;
  pa = nullptr;
  if (lc) {
    size += lc->size;
    Sum += lc->Sum;
    lc->pa = this;
  }
  if (rc) {
    size += rc->size;
    Sum += rc->Sum;
    rc->pa = this;
  }
}
//找出節點在中序的編號
size_t getIdx(Treap *x) {
  assert(x);
  size_t Idx = 0;
  for (Treap *child = x->rc; x;) {
    if (child == x->rc)
      Idx += 1 + size(x->lc);
    child = x;
    x = x->pa;
  }
  return Idx;
}
//切出想要的東西
void move(Treap *&root, int a, int b) {
  size_t a_in = getIdx(In[a]), a_out = getIdx(Out[a]);
  auto [L, tmp] = splitK(root, a_in - 1);
  auto [tree_a, R] = splitK(tmp, a_out - a_in + 1);
  root = merge(L, R);
  tie(L, R) = splitK(root, getIdx(In[b]));
  root = merge(L, merge(tree_a, R));
}
```

10 Else

10.1 Big Number

```cpp
string Add(const string &a, const string &b) {
    int n
        = a.length() - 1, m = b.length() - 1, car = 0;
    string res;
    while (n >= 0 || m >= 0 || car) {
        int x = (n >= 0 ? a[n] -
          '0' : 0) + (m >= 0 ? b[m] - '0' : 0) + car;
        res += (x % 10) + '0';
        car = x / 10;
        n--, m--;
    }
    while (res.length() > 1 && res.back() == '0') {
        res.pop_back();
    }
    reverse(res.begin(), res.end());
    return res;
}
string Minus(const string &a, const string &b) {
    // Assume a >= b
    int n
        = a.length() - 1, m = b.length() - 1, bor = 0;
    string res;
    while (n >= 0) {
        int x = a[n] - '0' - bor;
        int y = m >= 0 ? b[m] - '0' : 0;
        bor = 0;
        if (x < y) {
            x += 10;
            bor = 1;
        }
        res += x - y + '0';
        n--, m--;
    }
    while (res.length() > 1 && res.back() == '0') {
        res.pop_back();
    }
    reverse(res.begin(), res.end());
    return res;
}
string Multiple(const string &a, const string &b) {
    string res = "0";
    int n = a.length() - 1, m = b.length() - 1;
    for (int i = m; i >= 0; i--) {
        string add;
        int car = 0;
        for (int j = n; j >= 0 || car; j--) {
            int x = (j >= 0
                ? a[j] - '0' : 0) * (b[i] - '0') + car;
            add += (x % 10) + '0';
            car = x / 10;
        }
        while (add.length() > 1 && add.back() == '0') {
            add.pop_back();
        }
        reverse(add.begin(), add.end());
        res = Add(res, add + string(m - i, '0'));
    }
    return res;
}
```

10.2 Tenary Search

```cpp
// return the maximum of $f(x)$ in $[l, r]$
double ternary_search(double l, double r) {
  while(r - l > EPS) {
    double m1 = l + (r - l) / 3;
    double m2 = r - (r - l) / 3;
    double f1 = f(m1), f2 = f(m2);
    if(f1 < f2) l = m1;
    else r = m2;
  }
  return f(l);
}

// return the maximum of $f(x)$ in $(l, r]$
int ternary_search(int l, int r) {
  while(r - l > 1) {
    int mid = (l + r) / 2;
    if(f(m) > f(m + 1)) r = m;
    else l = m;
  }
  return r;
}
```

10.3 Duipai

```cpp
#include <iostream>
using namespace std;
int main() {
    for (int T=1;;T++){
        if (system("./random > test.in")) {
            cout << "random RE on " << T << '\n';
            return 0;
        }
        if (system("./sol < test.in > test.out")) {
            cout << "sol RE on " << T << '\n';
            return 0;
        }
        if (system("./bf < test.in > test.ans")) {
            cout << "bf RE on " << T << '\n';
            return 0;
        }
        if (system("diff -Z test.out test.ans")) {
            cout << "WA on " << T << '\n';
            return 0;
        } else {
            cout << "AC on " << T << '\n';
        }
    }
}
```

10.4 Random Generator

```cpp
#include <iostream>
#include <random>
int main() {
    std::random_device rd;
```

```
    std::mt19937 gen(rd());
    std::uniform_int_distribution<> distrib(1, 100);
    std::cout << "Get Rand: " << distrib(gen) << '\n';
}
```