Lot streaming and scheduling with due-date related objective

BY

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1. ABSTRACT

Consider a Supplier that delivers bread to several fast food companies like McDonalds, Burger king, KFC etc. These industries face a large scale of customers spread over a wide geographical horizon. One of the important challenges that these companies face is delivering the food as fresh as possible. To achieve this objective, they require the delivery of bread in accordance with the consumption rate by the customers. To address this, the production rate of bread by the supplier should deviate less from the consumption rate. This requires flexibility from the supplier to schedule the deliveries considering the customer requirements. Therefore, the problem has to be formulated such that the deviation of production rate from the consumption rate has to be minimized over a specified due-date window.

2. ASSUMPTIONS

Assuming the consumption rate (demand rate) to be linear over a specified season (Due) as shown in figure 1, the production of supplies is split into a pre-determined number of sub-lots depending on the manufacturing cost objective and equipment constraints. The graph in figure 1 is drawn for the case with two-sub lots. The slope of the production function denotes the production rate of the supplies and it is assumed to be considerably larger than the slope of demand rate. The points in the production function indicating 'delivery 1' and 'delivery 2' denotes the completion time of the sub lots one and two respectively. The delivery of these sub-lots are split in such a way that the area under the production function subtracted from the area under the demand function, is minimized. Figure 1 denotes the least deviation of production rate from the demand rate for a specified number of sub-lots and over a given due. This formulation is valid only for permutational flow shop model with consistent sub-lots.

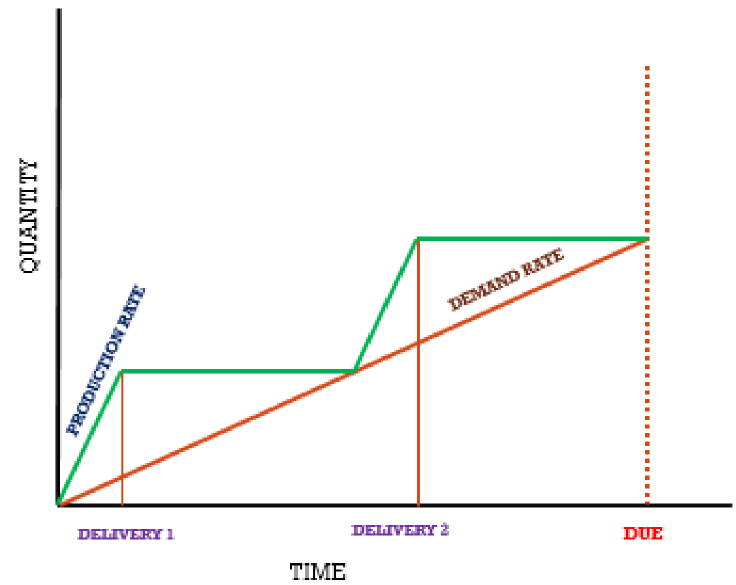


Figure 1

3. MODELLING OF THE PROBLEM (CONSTANT DEMAND RATE FOR ALL THE JOBS)

Consider that the job (The type of bread) requires processing on 'm' machines. Let 'k' be the total number of different jobs (Total number of different types of bread). n(x) is number of sub lots specified for each job. $nk = \sum_{x=1}^{k} n(x)$. is the total number of sub-lots in the sequence for k jobs.

3.1 PARAMETERS:

- The production rate of the x^{th} job - Production(x)
- The demand rate (constant for all the jobs) - demand
- The season over which the demand rate is constant for x^{th} job - due(x)
- Processing time of the lots of x^{th} job in i^{th} machine - p(i,x)
- Binary parameter b(x,j) : 1 if job x occupies the j^{th} slot in sequence; Else 0

3.2 DECISION VARIABLE:

- Completion time on machine i for x^{th} job, j^{th} in sequence: ct(i,x,j)
- Shortage at the j^{th} sequence occupied by job x - shortage(x,j)
- Buffer at the j^{th} sequence occupied by job x - Buffer(x,j)

3.3 COMPONENTS OF OBJECTIVE FUNCTION

The schedule is calculated using the following objective function, for minimum deviation of the production function from the demand function, which is the area under the sections 'a' and 'b' (figure 2) subtracted from area under 'c'(figure 3)

Minimize Deviation = (a + b)*b(x,j) - c + d

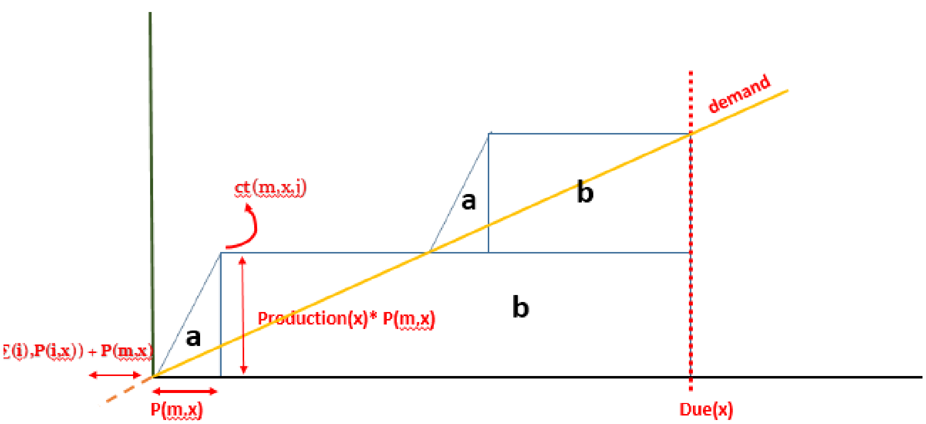


Figure 2

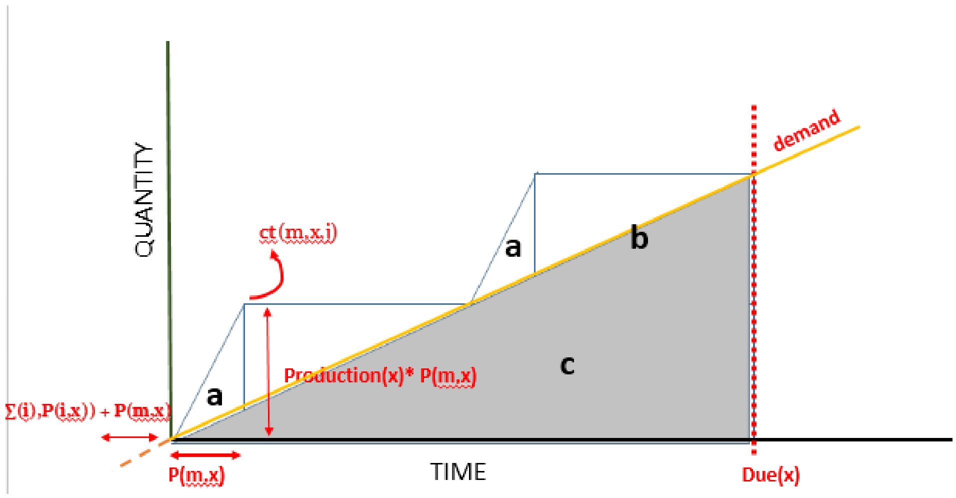


Figure 3

$$\mathbf{a} = \sum_{x=1,j=1}^{x=k,j=nk} \left(\mathbf{production}(\mathbf{x}) * \frac{\mathbf{P}(\mathbf{m},\mathbf{x})^2}{2} \right)$$

This section represents the area under the production line during the time of production. Here, only the processing time at the last machine *p(m,x)* is considered as the delivery is related only to the output from the last machine according to this formulation.

$$\mathbf{b} = \sum_{x=1,j=1}^{x=k,j=nk} \left(\mathbf{production}(\mathbf{x}) * \mathbf{P}(\mathbf{x}) * (\mathbf{Due}(\mathbf{x}) - \ (\textstyle\sum_{i=1}^{m} \left(\mathbf{P}(\mathbf{i},\mathbf{x})\right) + \ \mathbf{P}(\mathbf{m},\mathbf{x}) - \ \mathbf{ct}(\mathbf{m},\mathbf{x},\mathbf{j}) \right)$$

Since the cumulative production rate is considered, the area of this rectangular section is calculated after the production of every sub lot. The length of this rectangle is 'due' subtracted from the completion time of each sub lot. The processing time of all the machines except the last machine is subtracted to synchronise the time scale of production and demand function.

$$\mathbf{c} = \mathbf{demand}^*(\textstyle\sum_{x=1}^{x=k}(\mathbf{Due}(\mathbf{x}) - \ (\sum_{i=1}^{i=m} \mathbf{P}(\mathbf{i},\mathbf{x}) + \mathbf{p}(\mathbf{m},\mathbf{x}))\text{^}\mathbf{2})/\mathbf{2}\ )$$

The area of this triangular section represents the area under the linear demand function.

Other quantities that are required for the calculation of correct schedule are summed in 'd'.

d = ensure_start + Deficit + work_in_process

$$ensure\_start = \sum_{x=1}^{x=k} st(m, x, 1),\ {}^{*}demand$$

$$st(i,x,j) = (ct(i,x,j)-p(i,x))^{*}\ b(x,j)$$

This quantity ensures that start time of the first sub lot in sequence is always from the origin of the graph.

$$Deficit = \sum_{x=1,j=1}^{x=k,j=nk}((shortage(x,j) * wt)\text{^}2) * (demand/2)$$

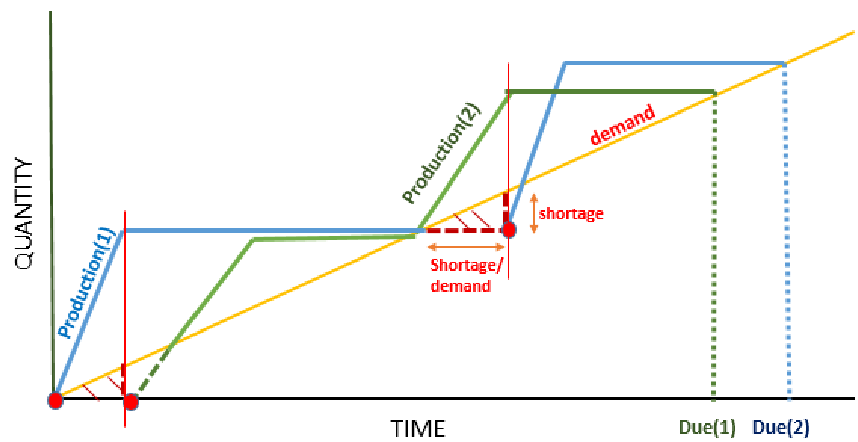
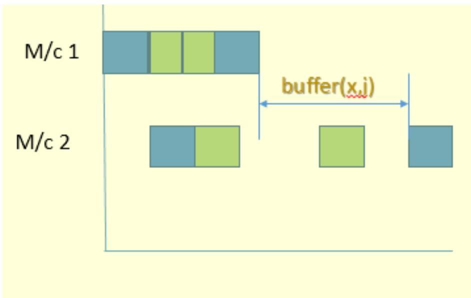


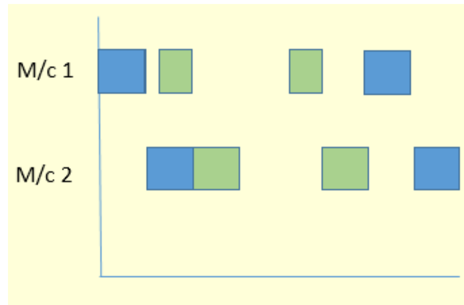
Figure 4

Shortages have to be quantified for two reasons. Firstly, when there are more than one job, the lots of the subsequent jobs except the first face forced shortages as each machine can process only one lot at a time. From figure, it can be seen that during the processing time of the first job, the production of second job is forced to remain *under* the demand region. Secondly, the supplier sometimes wishes to have a small quantity of shortage in order to eliminate a larger deviation. Thus, weightage (wt) can be assigned for the shortages and the area under striped region is calculated. The shortages can be calculated either with reference to start time of each lot, as the effective quantities that have been delivered are quantified only at this point.

$\mathbf{work\_in\_process} = \sum_{x=1,j=1}^{x=k,j=nk}(\mathbf{buffer}(\mathbf{x},\mathbf{j}) * \mathbf{b}(\mathbf{x},\mathbf{j}))$

buffer(x,j) = (st(m,x,j)-ct(1,x,j)) b(x,j)*





Since only the completion time of at the last machine is considered for the calculation of areas, the schedule of the previous machines would shift towards left as shown in figure. This results in work-in-process inventory. Therefore this equation is required to minimize the buffer and ensure that the completion time in the previous machines are scheduled relative to the last machine as shown in figure.

3.4 THE FORMULATION

Thus the objective function is

Min Deviation =

$$\sum_{x=1,j=1}^{x=k,j=nk}\left(production(x) * \frac{P(m,x)^2}{2}\right) + \sum_{x=1,j=1}^{x=k,j=nk}\left(production(x) * P(x) * (Due(x) - \left(\sum_{i=1}^{m}\left(P(i,x)\right) + P(m,x) - ct(m,x,j)\right)\right) \text{ - } demand * \sum_{x=1}^{k}(Due(x) - (\sum_{i=1}^{i=m} P(i,x)))\text{^}2)/2) + \sum_{x=1}^{x=k} st(m,x,1), * demand + \sum_{x=1,j=1}^{x=k,j=nk}((shortage(x,j) * wt)\text{^}2) * demand/2) + \sum_{x=1,j=1}^{x=k,j=nk}(buffer(x,j) * b(x,j))$$

Subject to,

- $\sum_{x=1}^{x=k}(\mathbf{st}(\mathbf{i},\mathbf{x},\mathbf{j})) >= \sum_{x=1}^{x=k}(\mathbf{ct}(\mathbf{i},\mathbf{x},\mathbf{j}-\mathbf{1}) * \mathbf{b}(\mathbf{x},\mathbf{j}))$

This constraint ensures that the start time of every lot in sequence is scheduled only after the completion time of previous lot in the sequence. As each sequence can be occupied by only one of the jobs, the sum over k jobs for every slot in the sequence is included in the constraint.

- $\sum_{x=1}^{x=k}(\mathbf{st}(\mathbf{i},\mathbf{x},\mathbf{j})) >= \sum_{x=1}^{x=k}(\mathbf{ct}(\mathbf{i}-\mathbf{1},\mathbf{x},\mathbf{j}) * \mathbf{b}(\mathbf{x},\mathbf{j}))$

  This constraint ensures that a lot in a machine is processed only after it has been processed in the previous machines.

- **k(x,j)*production(x)*p(m,x) + shortage(x,j) >= demand*(st(m,x,j-(∑(i),P(i,x))+P(m,x))*b(x,j)**

  This constraint ensures that the lots are scheduled in such a way that the production at any point in time within the due-date window does not go below the demand at that point, excluding the calculated shortage for each lot. k(x,j) is a parameter that takes value from 0 to nk – 1 for each job at corresponding slots. For example, for a given sequence as defined by b(x,j), the corresponding values of the parameter k(x,j) would be as follows.

b(x/j)	1	2	3	4	5	6	7	8
1	1	1		1			1	
2			1		1	1		1

k(x/j)	1	2	3	4	5	6	7	8
1	0	1		2			3	
2			0		1	2		3

- **Ct(m,x,j)<=due(x)**

  This constraint ensures that the schedules calculated within the due-date window.

4. RULES FOR SEQUENCE FORMATION (FOR TWO JOBS)

1. Identify the job that has longer due date. Place a sub-lot of that job at the end of the sequence.
2. Identify the job that has highest production rate. Place a sub-lot of that job in the beginning of the sequence. Place a sub-lot of the other job in the immediate succeeding sequence.
3. Alternate the jobs $X_1:X_2$ in the ratio Due(2):Due(1) from the second slot in the sequence.
4. Solve the model for this sequence and generate the Deviation chart. (chart showing the deviation of quantities produced from the demand at the start time of every lot. Ideally the deviations should be zero except the forced negative deviation)
5. Check if there are any lots that have positive deviations.

   If no, the current sequence is the optimal.

   If yes, check if the succeeding lot to the right has lesser deviation. If yes, swap the two lots.
6. Solve the model for the new sequence and check if the value of the objective function is lesser than the previous sequence.

   If no, reject and discard the new sequence. Go to next step.

   If yes, retain the new sequence. Go to next step.
7. Check if it is possible to swap and generate new sequences with the above rule.

   If yes, go to step 5.

   Else, the current sequence is the optimal.

4.1 ILLUSTRATION:

Consider two jobs 1 and 2 with due date window **Due(1) = 23 days**, and **Due(2)=27 days** respectively. The production rate of the job 1 and 2 are **Production(1)=41 units/day,** and **Production(2)=52 units/day** respectively. **Demand rate = 10 units/day.** Each job is specified to have **four** sub lots.

1. Job 2 has longer due date and hence it is placed at the end of the sequence.

							2

2. Job 2 has the highest production rate. Hence it is placed at the beginning of the sequence. Job 1 is placed next in the sequence

2	1						2

3. Due(2)/Due(1) = 27/23 = 1.17 ~ 1
   X1:X2 = 1:1
   Therefore alternate 2 and 1 in the ratio 1:1 from second slot in sequence

2	1	2	1	2	1		2

   If all the lots of a job has been assigned in the sequence, fill the remaining slots with the unassigned sub lots.

2	1	2	1	2	1	1	2

4. Solve the model for the generated sequence and generate the deviation chart.

The calculations that determine the other parameter values are as follows
For x=1
Due(1)=23
Demand rate = 10/day
*Total quantity to be produced = 23*10 = 230*
No. of sub-lots specified = 4
Therefore, size of each sub lot = 230/4 = 57.5
Production rate on first machine = 35 units/day
Therefore, p(1,1) = 57.5/35 = 1.6
Production rate on second machine = 41 units/day

Therefore, $p(2,1) = 57.5/41 = 1.4$

Similarly for $x=2$
Production rate on first machine = 52 units/day
$P(1,2)=1.3$
Production rate on second machine = 52 units/day
$P(2,2)=1.3$

After modelling the problem and the input of parameter values in AIMMS software, the following output is obtained

Value of objective function = 6419.34

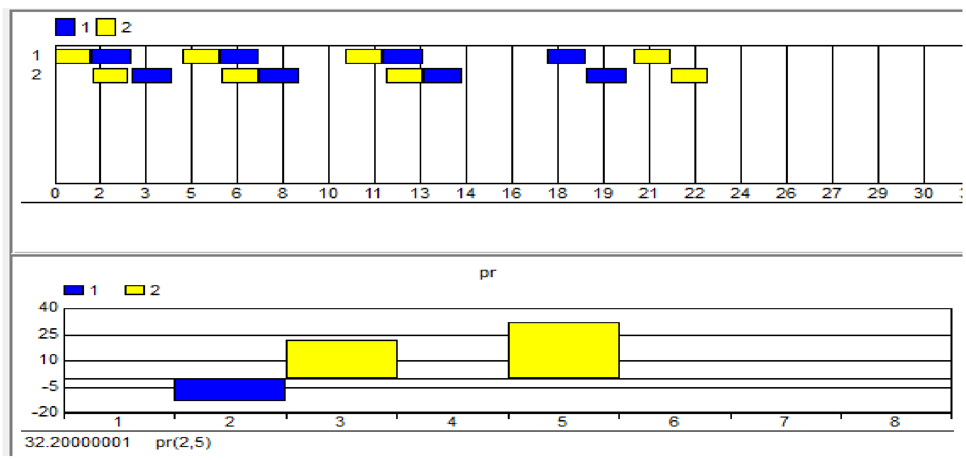


Figure 5- Gantt-chart and deviation chart

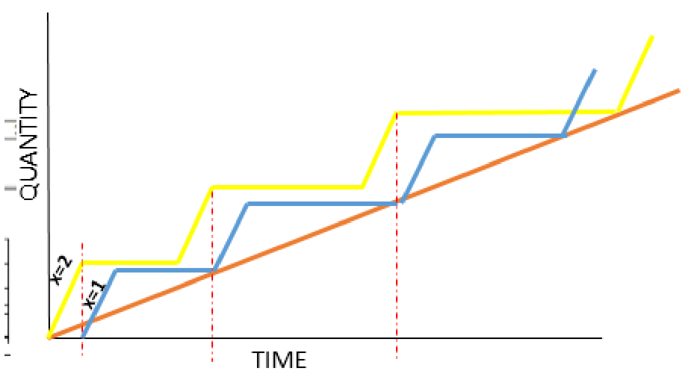


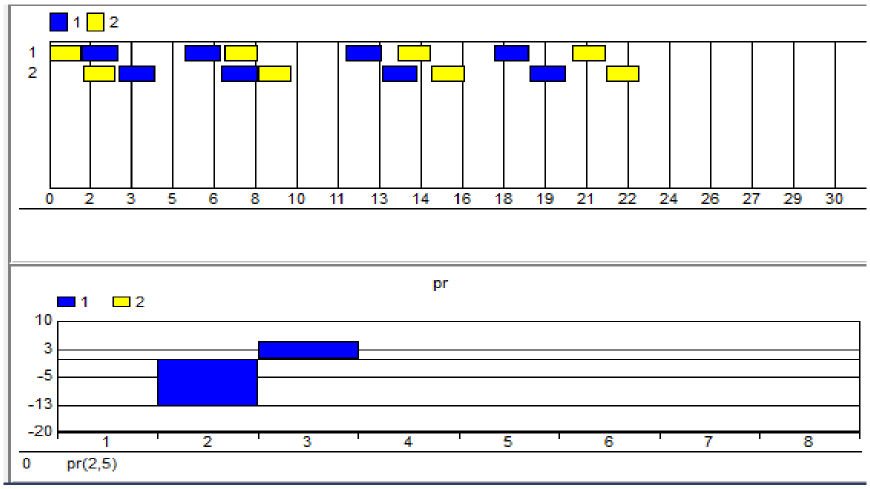
Figure 6

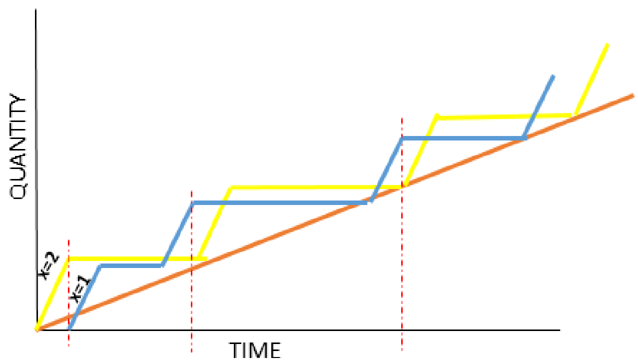
Lots 3 has positive deviation. The succeeding lot 4 has lesser (no) deviation. Therefore swap 3 and 4. Similarly swap 5 and 6.

2	1	1	2	1	2	1	2

5. Solve the model for the new sequence and check if the value of the objective function is lesser than the previous sequence.

Value of objective function = 6214





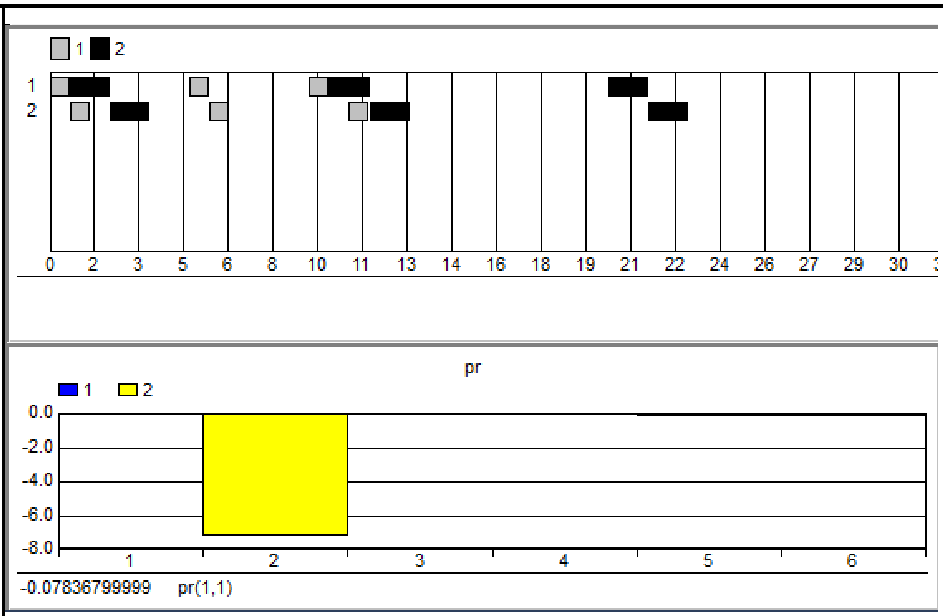
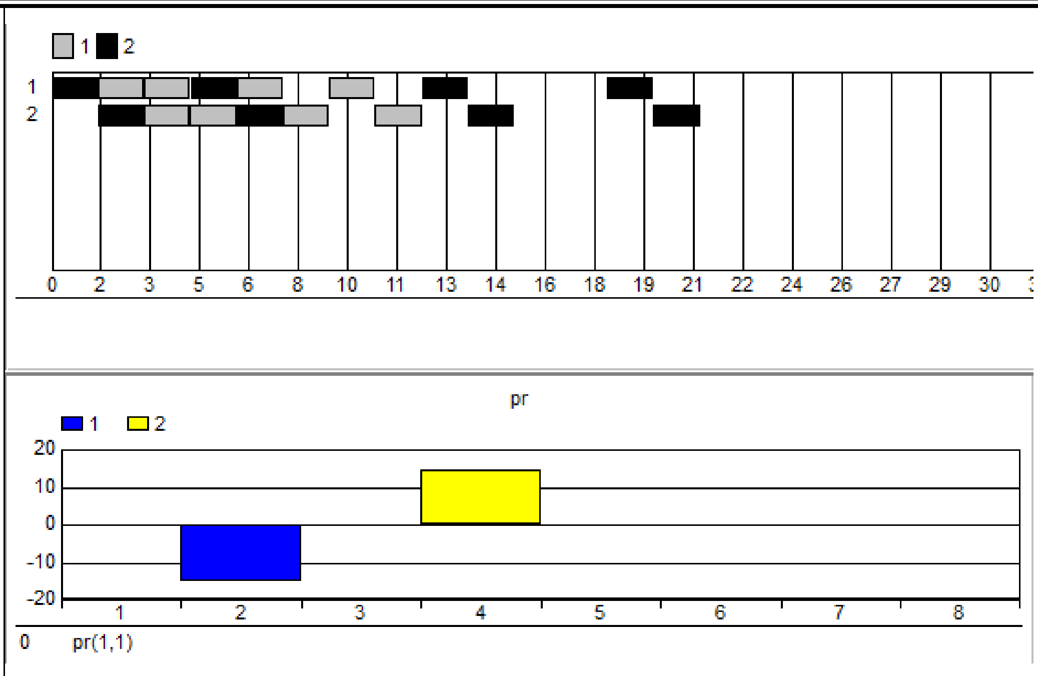
6. Since the value of the objective function is lesser, and no further sequence according to the above rules could be generated, this sequence is the optimal.

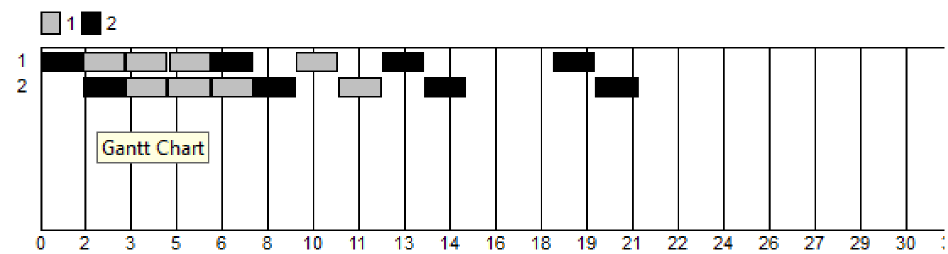
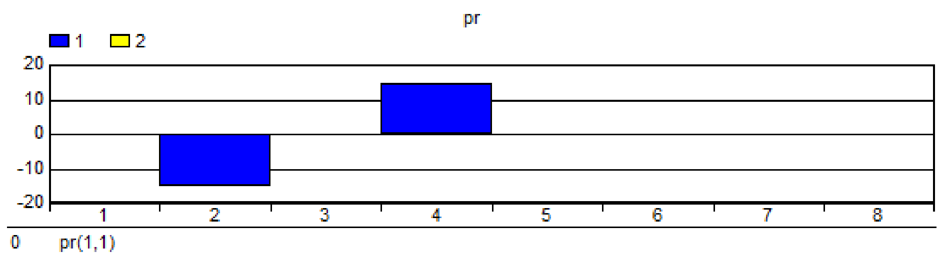
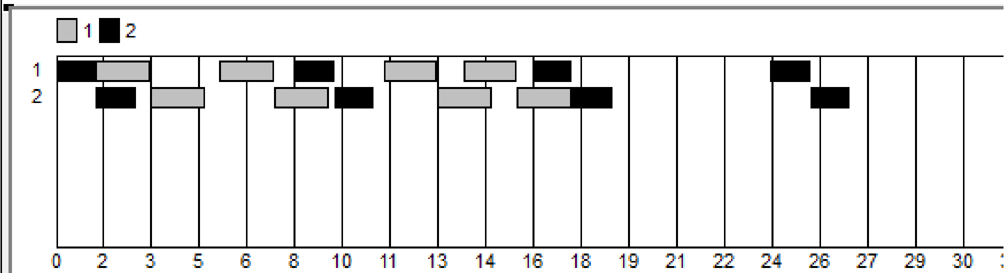
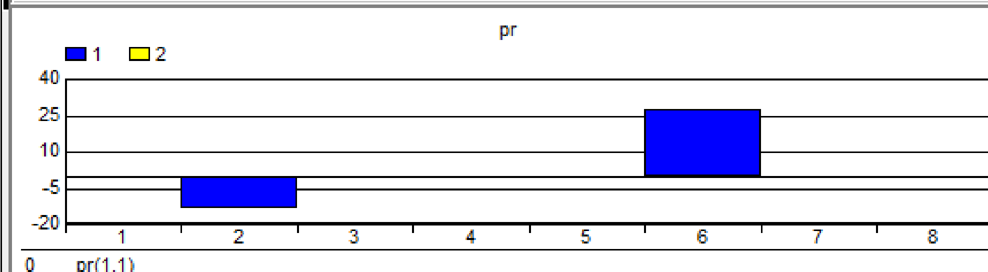
4.2 VALIDATION OF THE RULE AND RESULTS:

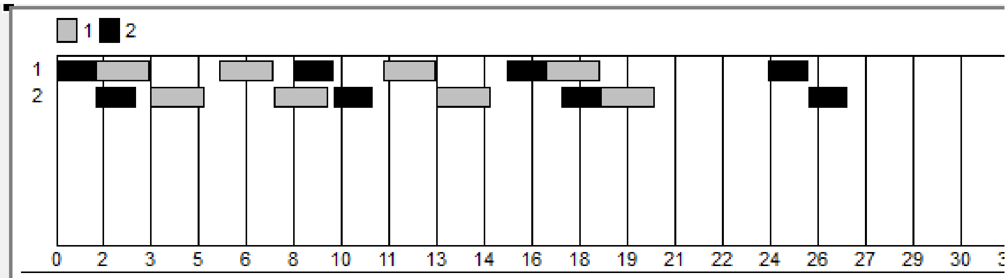
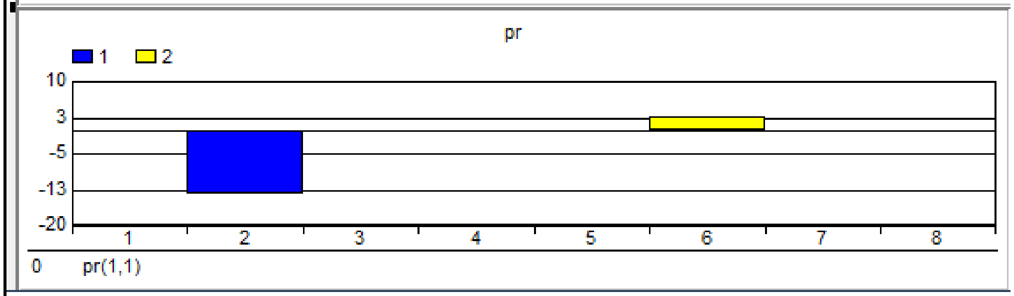
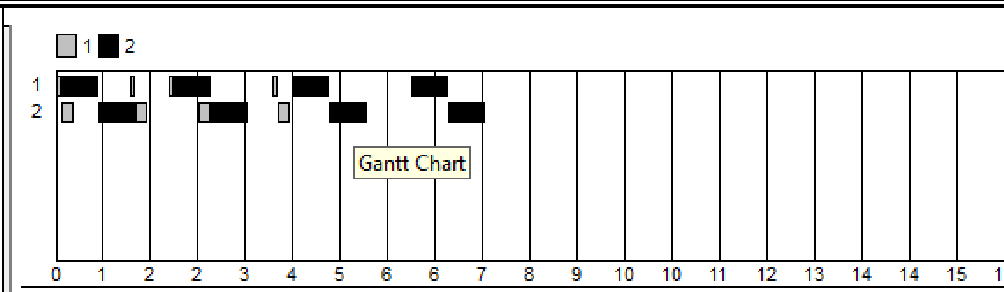
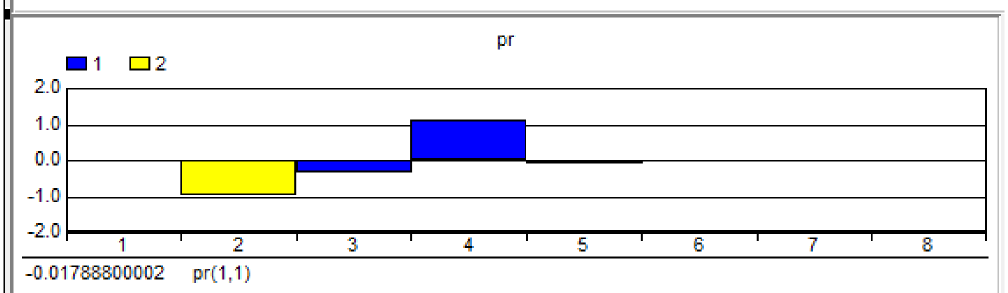
This procedure has been tried out for six different cases with the due dates and production rates varying to its extremes as shown in the following table. The following cases are solved using AIMMS. The iterations marked bold are the optimal.

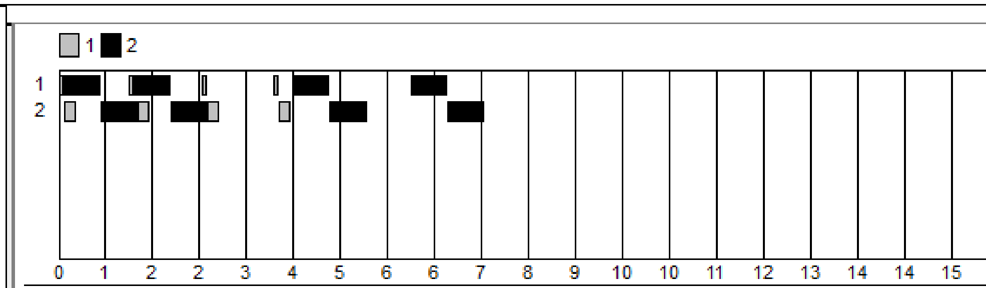
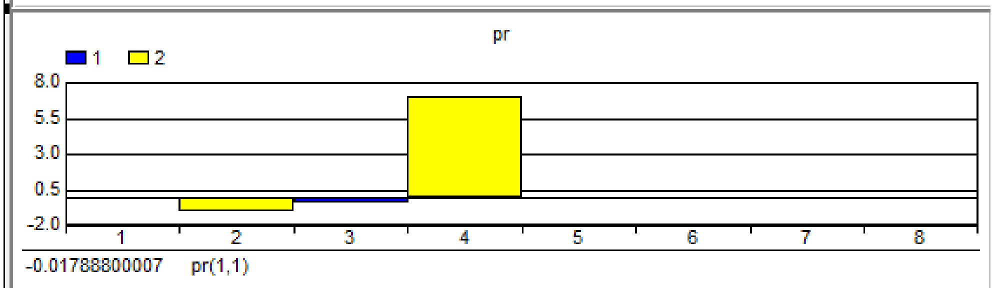
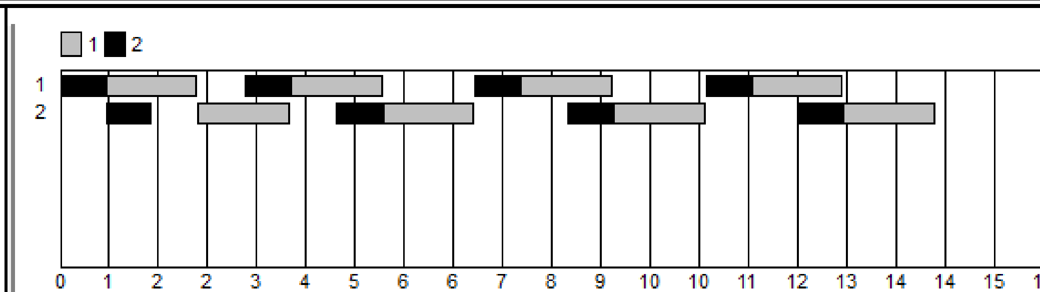
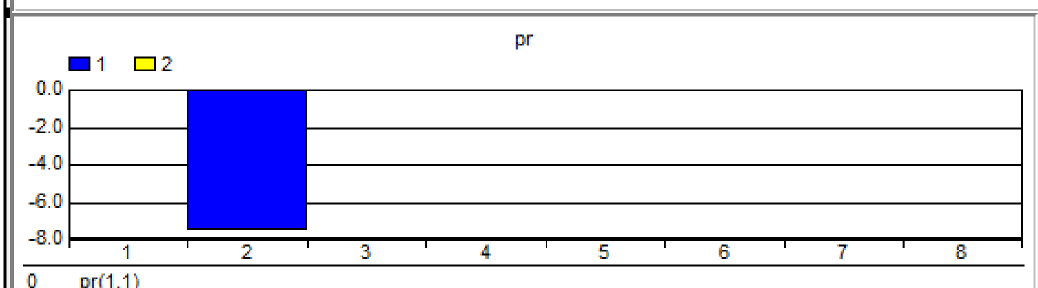
S.No	ANALYSIS OF CASES			ITERATIONS	VALUE OF OBJECTIVE FUNCTION
1	Due(1) = 2*Due(2)	P(1) = P(2)	Due(1) = 15, P(1)=70 Due(2) = 30, P(2)= 70	**1,2,1,1,2,2**	**2891**
2	Due(1) = 2*Due(2)	P(1) = 2*P(2)	Due(1) = 12, P(1)=20 Due(2) = 24, P(2)= 40	2,1,1,2,1,1,2,2	7472
				2,1,1,1,2,1,2,2	**7427**
3	Due(1) <= 2*Due(2)	P(1) = 2*P(2)	Due(1) = 22, P(1)=30 Due(2) = 32, P(2)= 60	2,1,1,2,1,1,2,2	6696
				2,1,1,2,1,2,1,2	**6566**
4	Due(1) <= 2*Due(2)	P(1) <= 2*P(2)	Due(1) = 5, P(1)=58 Due(2) = 8, P(2)= 32	**1,2,1,1,2,1,2,2**	**100**
				1,2,1,2,1,1,2,2	112.5
5	Due(1)=Due(2)	P(1)=2*P(2)	Due(1) = 15, P(1)=25 Due(2) = 15, P(2)= 50	**2,1,2,1,2,1,2,1**	**1987**
6	Due(1) ~ Due(2)	P(1) ~ P(2)	Due(1) = 23, P(1)=41 Due(2) = 27, P(2)= 52	2,1,2,1,2,1,1,2	6508
				2,1,1,2,1,2,1,2	**6167**

The Gantt-chart and the deviation-chart from AIMMS for the cases described above are shown in the following table

CASE	iterations	Sequence	Gantt-chart and deviation chart
Due(1) = 15, P(1)=70 Due(2) = 30, P(2)= 70	1	**1,2,1,1,2,2** **(2891)** (optimal)	
Due(1) = 12, P(1)=20 Due(2) = 24, P(2)= 40	1	2,1,1,2,1,1,2,2 (7472) (swap 4 and 5)	

	2	**2,1,1,1,2,1,2,2** *(7427)* *7427<7472* **(optimal)**	 
Due(1) = 22, P(1)=30 Due(2) = 32, P(2)= 60	1	2,1,1,2,1,1,2,2 (6696) (swap 6 and 7)	 

	2	**2,1,1,2,1,2,1,2** (6566) *6566<6696* **(optimal)**	 
Due(1) = 5, P(1)=58 Due(2) = 8, P(2)= 32	**1**	**1,2,1,1,2,1,2,2** **(100)** (Swap 4 and 5) **(optimal)**	 

	2	1,2,1,2,1,1,2,2 (112.5) *112.5 > 100* *Reject this sequence*	 
Due(1) = 15, P(1)=25 Due(2) = 15, P(2)= 50	1	**2,1,2,1,2,1,2,1** **(1987)** **(optimal)**	 

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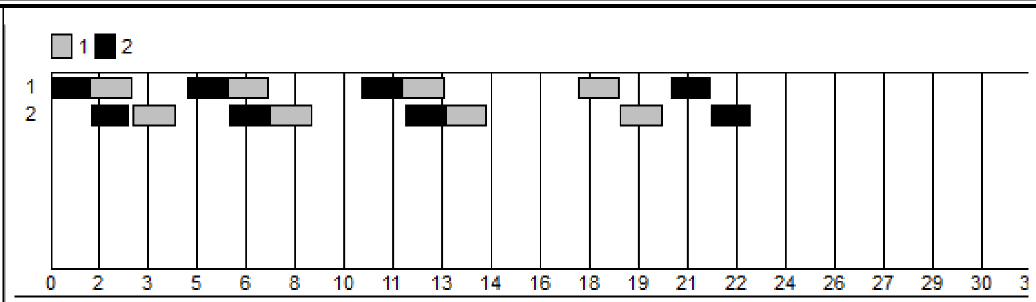
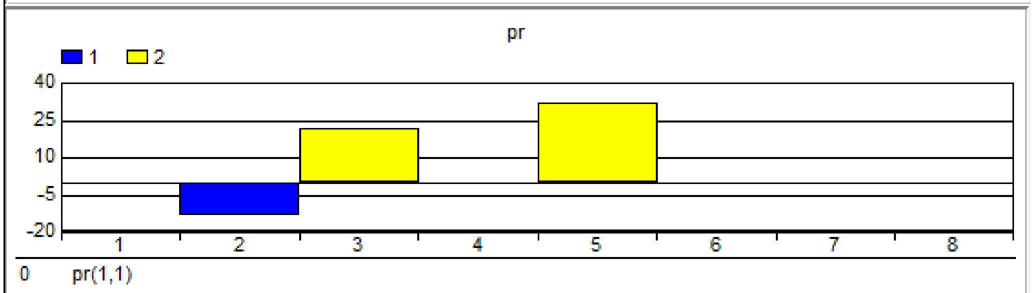
Due(2) = 27, P(2)= 52</td>
<td>1</td>
<td>2,1,2,1,2,1,1,2

(swap 3 and 4)

(swap 5 and 6)

(6508)</td>
<td>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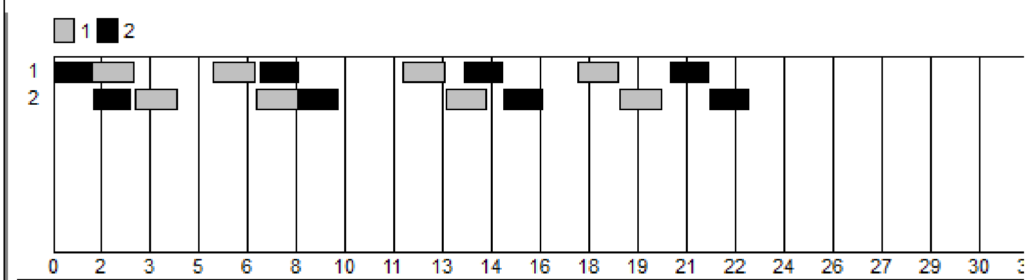
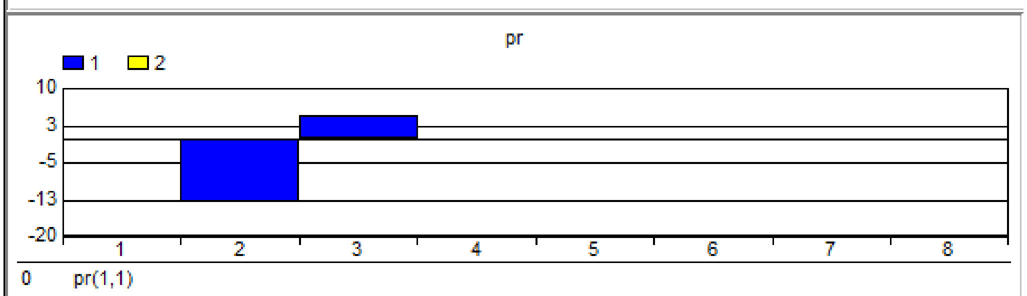
6167<6508

(optimal)</td>
<td>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From the results, it can be seen that, this rules calculates the optimal solution for all the cases. Hence it can be understood that this model works for any cases. Larger difference in the production rate (greater than 2 times) will further shorten the processing time of one of the jobs and hence it would only give a better solution. Thus the optimality is obtained even if the due dates and the production rates are varied over any range.

5. CONCLUSION AND SCOPE OF EXPANSION:

Therefore, with this modelling and the sequencing rule, the *optimal schedule (maximum of 2 jobs)* with the least deviation of the production rate from the consumption rate can be computed for a specified number of deliveries required over a due date window. However, this model is limited to permutational flow-shops. This model can also be used to solve for the cases with more than one job and the optimal sequence could be calculated using heuristic procedures. This model can also be expanded for jobs having different demand rates.

Reference:

Introduction to operations scheduling – Michael Pinedo.