DSP Lab Report

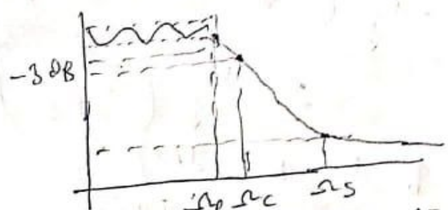
Objective:- By using the Chebyshev Type-I filter and BLT approaches, design a digital filtering system which can process the EEG signal to extract the delta, alpha and gamma waves with specified frequency ranges.

Design & Manual Solutions:-

1 a) (i) Delta — below 4 Hz, to design LPF

$\omega_c = 2 \times \pi \times 4 = 8\pi$ rad/sec, $(= 25.133)$

$f_c = 4\,Hz$, $f_p = 3\,Hz$



$\omega_p = 2\pi \times 3 = 6\pi$ rad/sec $(= 18.85)$

Passband ripple $= 1\,dB = -20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$

$\epsilon = 0.5088$

Stopband attenuation $= 40\,dB = A_s$

Transition bandwidth $= 3\,Hz$

$\Rightarrow \omega_s = 2\pi \times 6 = 12\pi$ rad/s

$F_s = 200\,Hz$

Frequency prewrapping $\Rightarrow$

$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p T}{2}\right) = 18.86$ ~~25.166~~ rad/sec

$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = 37.811$ rad/sec

So we have

$\Omega_s = 37.811$ rad/s

$\Omega_p = 18.86$ rad/s

~~$\Omega_c = 25.166$ rad/s~~

$\epsilon = 0.5088$

$A_s = 40\,dB$

$F_s = 200\,Hz$

parameters of filter design

$$N \geq \frac{\cosh^{-1}\left[\left(\frac{10^{0.1 A_s}-1}{10^{0.1 A_p}-1}\right)^{1/2}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\cosh^{-1}\left[\frac{99.995}{0.5088}\right]}{\cosh^{-1}(2.005)}$$

$N \geq 4.526$

$\boxed{N = 5}$

Ω_c (corrected)

$= 17.497$ rad/s

for

- 30's

for poles $\rightarrow$ $\gamma_K = \frac{\pi(2K-1)}{2N}$, $K = 1, 2, 3, 4, 5$

$\alpha = -\frac{\sinh^{-1}\left(\frac{1}{\varepsilon}\right)}{N} = -0.2856$

$\gamma_K = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$

$S_K = j\Omega_p \left(\cos(\gamma_K + j\alpha) \right)$

$S_K = \Omega_p [\sin\gamma_K \sinh\alpha + j\cos\gamma_K \cosh\alpha]$

~~$S_1 = \Omega_p($~~

$S_1 = -1.687 + j\,18.67$, $S_5 = -1.687 - j\,18.67$

$S_2 = -4.417 + j\,11.54$, $S_4 = -4.417 - j\,11.54$

$S_3 = -5.46$

N is odd

$$H_a(s) = \frac{-P_1 P_2 P_3 P_4 P_5}{(s-P_1)(s-P_2)(s-P_3)(s-P_4)(s-P_5)}$$

$$H_a(s) = \frac{-(351.415)(152.68)(-5.46)}{(s^2+3.374s+351.415)(s^2+8.834s+152.68)(s+5.46)}$$

$$\boxed{H_a(s) = \frac{2.93\times10^5}{(s^2+3.374s+351.415)(s^2+8.834s+152.68)(s+5.46)}}$$

$$s = 400\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

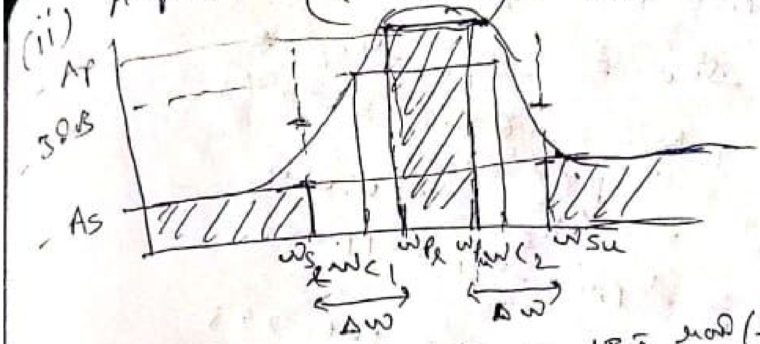
$$H(z) = \frac{2.93\times10^5\,(1+z^{-1})^5}{\left(400^2(1-z^{-1})^2+3.374\times400\times(1-z^{-2})+351.415(1+z^{-1})^2\right)\left(400^2(1-z^{-1})^2+8.834\times400(1-z^{-2})+152.68(1+z^{-1})^2\right)\left(400(1-z^{-1})+5.46(1+z^{-1})\right)}$$

$$H(z) = \frac{2.93\times10^5\,(1+z^{-1})^5}{\left(159001.815z^{-2}-319297.172z^{-1}+161701.015\right)\left(156619.08z^{-2}-319684.64z^{-1}+163686.28\right)\left(394.54z^{-1}+405.46\right)}$$

$$\boxed{H(z) = \frac{273.02\times10^{-10}\,(1+z^{-1})^5}{(1-1.9746z^{-1}+0.9832z^{-2})(1-1.953z^{-1}+0.957z^{-2})(1+0.973z^{-1})}}$$

CS Scanned with CamScanner

(ii) Alpha – (9–13 Hz) BPF



$\omega_{c1} = 2\pi f_{c1} = 18\pi$ rad/s

$\omega_{c2} = 2\pi f_{c2} = 26\pi$ rad/s

$f_{pl} = 10$ Hz, $f_{pu} = 12$ Hz

$f_{sl} = 7$ Hz, $f_{su} = 15$ Hz

$\omega_{pl} = 20\pi$ rad/s, $\omega_{pu} = 24\pi$ rad/s, $\omega_{su} = 30\pi$ rad/s.

$\Rightarrow \omega_{sl} = 14\pi$ rad/s

~~$\varepsilon = 0.5088$~~

prewarping $\Rightarrow$

$\Omega_{pl} = 2/T \tan(\omega_{pl}T/2) = 63.353$ rad/s

$\Omega_{pu} = 2/T \tan(\omega_{pu}T/2) = 76.304$ rad/s

$\Omega_{sl} = 2/T \tan(\omega_{sl}T/2) = 44.160$ rad/s

$\Omega_{su} = 2/T \tan(\omega_{su}T/2) = 96.03$ rad/s

$\Omega_{c1} = 2/T \tan\left(\frac{\omega_{c1}T}{2}\right) = 56.93$ rad/s

$\Omega_{c2} = 2/T \tan\left(\frac{\omega_{c2}T}{2}\right) = 82.84$ rad/s

As, F 40 dB, ~~passb~~

0.5088, ~~bandwidth~~ = 12.951

~~fixing Ω_{pl}~~

centre freq $\Rightarrow \Omega_0^2 = \Omega_{c_1}\Omega_{c_2}$

$\Omega_0 = 68.673$ rad/s.

fixing Ω_{pl}, $\Omega_{pu}' = \Omega_0^2/\Omega_{pl} = 74.44$

$W_p = \Omega_{pu}' - \Omega_{pl} = 11.088$

$= 106.793$

fixing Ω_{sl}, $\Omega_{su}' = \Omega_0^2/\Omega_{sl}$

$W_s = \Omega_{su}' - \Omega_{sl} = 62.633$

fixing Ω_{pu}, $\Omega_{pl}' = \Omega_0^2/\Omega_{pu} =$ ~~61.305~~ ~~49.109~~ 61.805

$W_p = 14.498$

fixing Ω_{su}, $\Omega_{sl}' = \Omega_0^2/\Omega_{su} =$ ~~56.43~~ 49.109

$W_s =$ ~~25.411~~ 46.9205

So we will choose $\Omega_{pl}, \Omega_{pu}, \Omega_{su}, \Omega_{sl}$ such that we have W_p & W_s both are small ~~W_p more~~ & ~~less~~ $\Rightarrow$

$\Omega_{pl} =$ ~~74.44 rad/s~~ 63.353 rad/s

$\Omega_{pu} = 74.44$ rad/s

$\Omega_{sl} = 49.109$ rad/s

$\Omega_{su} = 96.03$ rad/s

$\varepsilon = 0.5088, \quad A_s = 40$ dB

$$\nu_s = \frac{W_s}{W_p} = \frac{46.9205}{11.088} = 4.2316$$

$$N \geq \frac{\cosh^{-1}\left(\left[\frac{10^{0.1A_s} - 1}{\varepsilon}\right]^{1/2}\right)}{\cosh^{-1}(\nu_s)}$$

$N \geq 2.816 \Rightarrow \boxed{N = 3}$

prototype for LPF chebyshev type-I

order = 3 ⇒

$$\gamma_k = \frac{(2k-1)\pi}{2N}, \quad k = 1, 2, 3$$

$$\gamma_k = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

$$\alpha = -0.476$$

$$p_1 = -0.247 + j\,0.966$$

$$p_2 = -0.4942$$

$$p_3 = -0.247 - j\,0.966$$

$$H_a(s) = \frac{-p_1 p_2 p_3}{(s-p_1)(s-p_2)(s-p_3)}$$

$$H_a(s) = \frac{0.4911}{(s^2 + 0.494s + 0.994)(s + 0.4942)}$$

$$\cancel{H_a(s) = \frac{0.4911}{s^3 + 0.912s^2 + 1.23s + 0.4}}$$

3rd order prototype

$$\boxed{H_a(s) = \frac{0.4911}{s^3 + 0.9882s^2 + 1.238s + 0.4912}}$$

for bandpass

$$s \rightarrow \frac{s^2 + \Omega_0^2}{s\,W_p}$$

$$s \rightarrow \frac{s^2 + 4715.98}{11.088\,s}$$

$$H_a(s) = \frac{0.4911}{\left[\left(\frac{s^2 + 4715.98}{11.088s}\right)^2 + 0.494\left(\frac{s^2 + 4715.98}{11.088s}\right) + 0.994\right]\left[\frac{s^2 + 4715.98}{11.088s} + 0.4942\right]}$$

$$\Rightarrow H_a(s) = \frac{0.4911 \times (11.088\,s)^3}{\left[(s^2+4715.98)^2 + 0.494(s^2+4715.98)(11.088\,s)^2 + 0.494(11.088\,s)^2\right]\left[s^2+4715.98 + 11.088(0.4942)s\right]}$$

$$H_a(s) = \frac{669.467\,s^3}{\left(s^4+9431.96\,s^2+22240467.36 + 60.7342\,s^3 + 286421.3\,s^2 + 122.206\,s^2\right)\left(s^2+5.47925\,s + 4715.98\right)}$$

$$H_a(s) = \frac{669.4671\,s^3}{\left(s^4+60.7342\,s^3+295975.466\,s^2+22240467.36\right)\left(s^2+5.485 + 4715.98\right)}$$

$$s \rightarrow 400\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{669.467\,(1-z^{-2})^3}{\left((400)^4(1-z^{-1})^4 + 60.7342(1-z^{-1})^3(1+z^{-1}) \times 400^3 + 295975.5(1-z^{-2})^2 \times 400^2 + 22240467.36(1+z^{-1})^4\right)\left(400^2(1-z^{-1})^2 + 5.48(1-z^{-2}) \times 400 + 4715.98(1+z^{-1})^2\right)}$$

$$\boxed{H(z) = \frac{5.218 \times 10^{-14}(1-z^{-2})^3}{\left(1-1.432z^{-1}+0.768z^{-2}+1.23z^{-3} + 0.89886z^{-4}\right)\left(1-1.86z^{-1}+0.975z^{-2}\right)}}$$

(iii) Gamma - (31 Hz +) , HPF

$f_c = 31\,Hz$ + $f_p = 32\,Hz$, $f_s = 29\,Hz$

$\omega_c = 62\pi$, $\omega_p = 64\pi$, $\omega_s = 58\pi$

$\Omega_c = 2/T \tan\left(\frac{\omega_c T}{2}\right) = 211.79$ rad/s

$\Omega_p = 2/T \tan(\omega_p T/2) = 219.90$ rad/s

$\Omega_s = 2/T \tan(\omega_s T/2) = 195.96$ rad/s

$\varepsilon = 0.5088$

$A_s = 40\,dB$

$$N \geq \frac{\cosh^{-1}\left[\left(\frac{10^{0.1A_s} - 1}{\varepsilon}\right)^{1/2}\right]}{\cosh^{-1}\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$N \geq 12.2066 \Rightarrow \boxed{N = 13}$

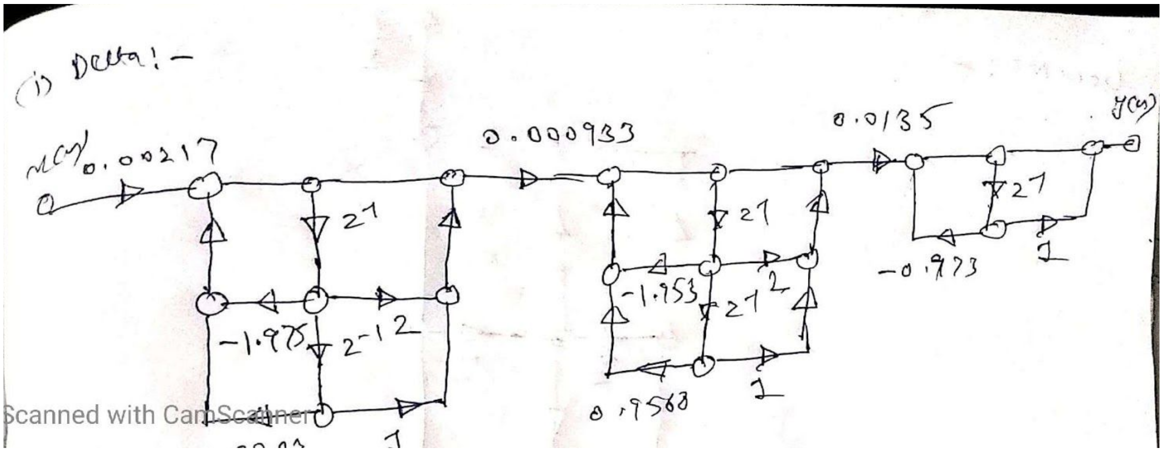
Using Matlab function ⇒

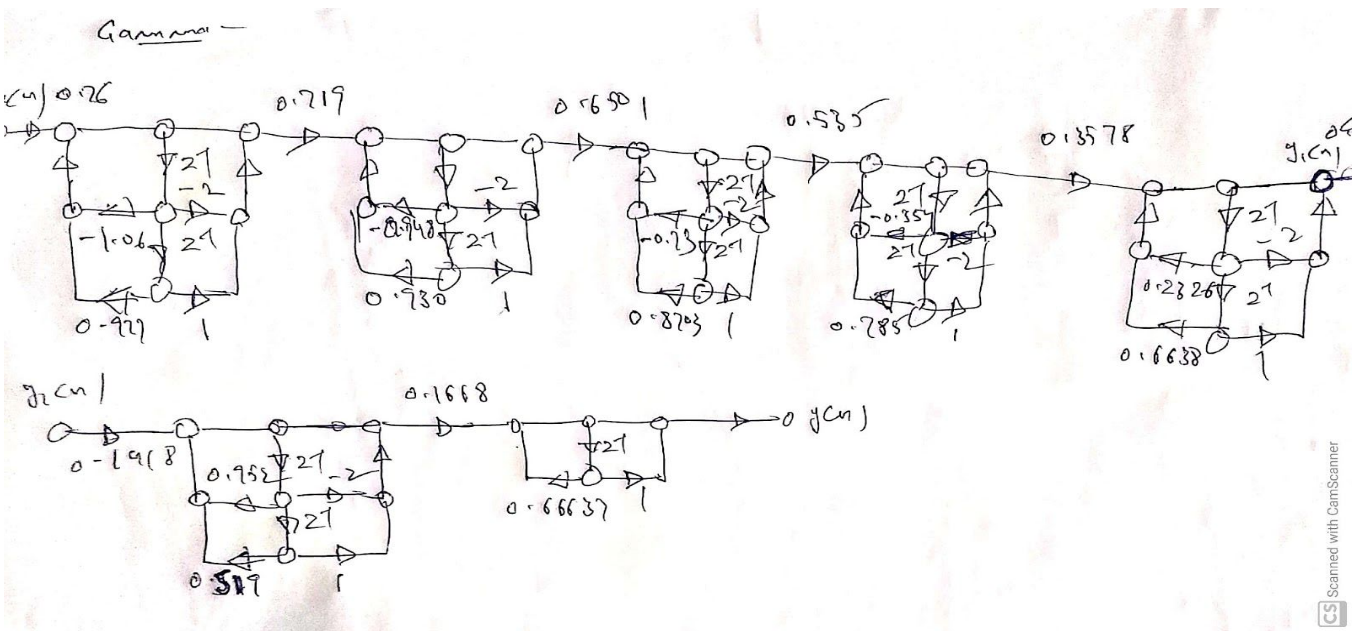
$$H_a(s) = \frac{+\,0.0004298\, s^{13}}{0.0004298\, s^{13} + 1.39\, s^{12} + 1543\, s^{11} + 1.541\times10^{6}\, s^{10} + 8.004\times10^{8}\, s^{9} + 4.895\times10^{11} s^{8} + 1.513\times10^{14}\, s^{7} + 6.676\times10^{16}\, s^{6} + 1.29\times10^{19}\, s^{5} + 4.43\times10^{21}\, s^{4} + 5.061\times10^{23}\, s^{3} + 1.41\times10^{26}\, s^{2} + 7.437\times10^{27} s + 1.725\times10^{30}}$$

$$S = 400\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\boxed{H(z) = \frac{0.002\,(1 + z^{-1})^{13}}{1 - 1.584 z^{-1} + 3.83 z^{-2} + 2.922 z^{-3} + 4.3 z^{-4} - 1.18 z^{-5} + 2.31 z^{-6} + 0.514 z^{-7} + 0.9205 z^{-8} + 0.752 z^{-9} + 0.2423 z^{-10} + 0.475 z^{-11} - 0.0115 z^{-12} + 0.1389 z^{-13}}}$$

Realization Structures:-





Source Code:-

<u>Main File</u>

```
clc;
clear;
close all;
%%  Load The Required Data %%

% Use any of the below three datas as EEG input according to the need.
data = load('signal07_EEG.csv');
% data = load('signal08_EEG.csv');
% data = load('signal09_eegalpha.csv');

%% Required Specifications for filter %%
```

```
% According to the given question the filter parameters calculated for the
% different wave filtering are given below. Please follow to the
% handwritten calculations done to find it.
% To get desired output please put the below specifications for each cases.

% Specifications taken :- Ap = 1 dB, As = 40 dB, Ts = 0.005 sec
% 1. Delta :- Fp = 18.86 rad/s, Fs = 37.811 rad/s
% 2. Alpha :- Fpu = 74.44 rad/s, Fpl = 63.353 rad/s, Fsu = 96.03 rad/s,
% Fsl= 49.109 rad/s.
% 3. Gamma :- Fp = 219.90 rad/s, Fs = 195.96 rad/s

Ap = input('Pass Band Attenuation(in dB): ');
As = input('Stop Band Attenuation(in dB): ');
Ts = input('Sampling Time(s): ');

wave = input('Wave to extract:- Delta/Alpha/Gamma: ','s');

if wave == "Delta"
   Filter_type = "LPF";
elseif wave == "Alpha"
   Filter_type = "BPF";
elseif wave == "Gamma"
   Filter_type = "HPF";
end
if Filter_type == "LPF" || Filter_type == "HPF"
   Fp = input('Pass Band Edge Frequency(in rad/sec): ');
   Fs = input('Stop Band Edge Frequency(in rad/sec): ');
   F_b = [0,0,0,0];
   [N,Pole,a,b] = chebyshevtype1dsp(Ap,As,Fp,Fs,F_b,Filter_type,Ts);
else
   Fpu = input('Passband Upper Edge Frequecy (rad/sec): ');
   Fpl = input('Passband Lower Edge Frequecy (rad/sec): ');
   Fsu = input('Stopband Upper Edge Frequecy (rad/sec): ');
   Fsl = input('Stopband Lower Edge Frequecy (rad/sec): ');
   F_b = [Fpu,Fpl,Fsu,Fsl];
   [N,Pole,a,b] = chebyshevtype1dsp(Ap,As,0,0,F_b,Filter_type,Ts);
end

%% Required Filter and Filtered output %%

filtered_output = filter(real(b),real(a),data);

figure; hold on; grid on;
plot(data,'MarkerSize',5);
plot(filtered_output,'MarkerSize',5);
legend('EEG Signal','Filtered Signal');
```

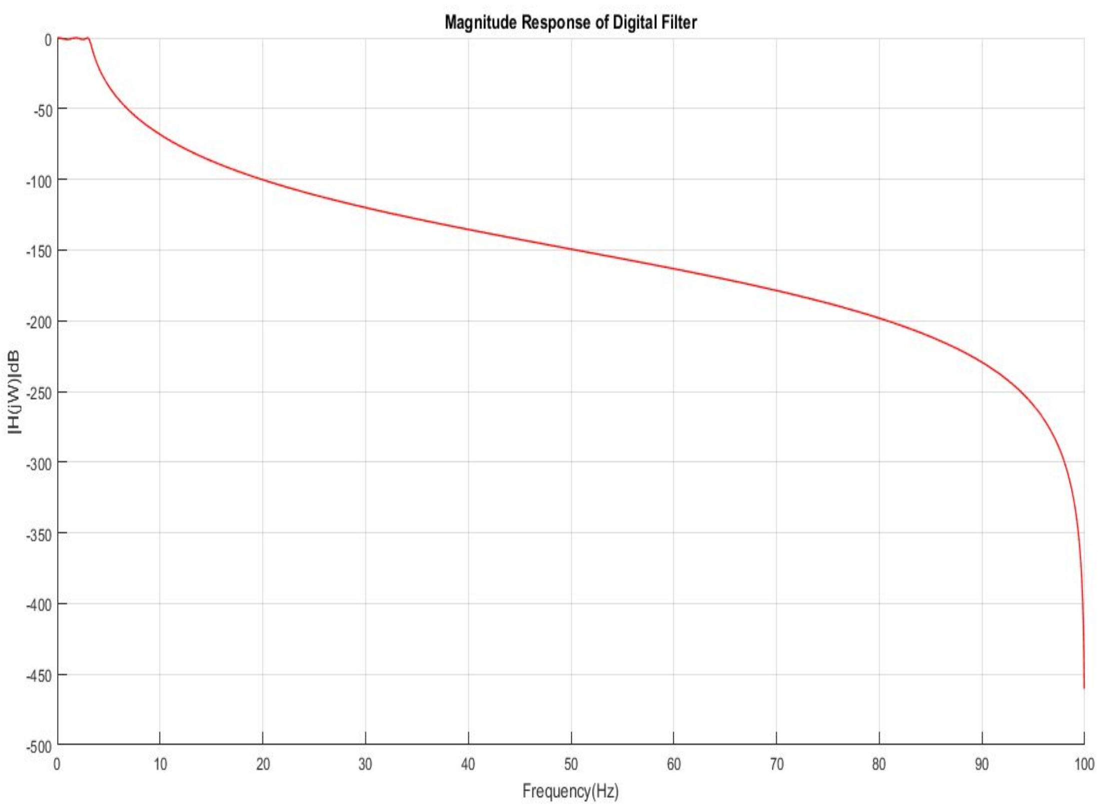
chebyshevtype1dsp_function

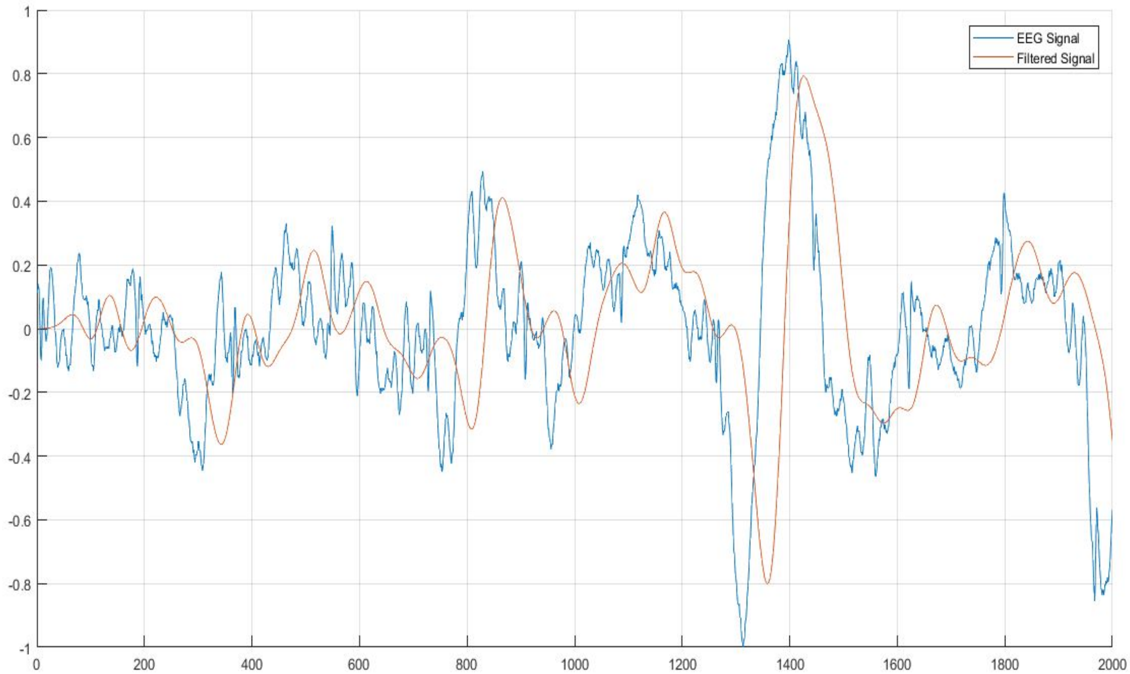
```
function  [N,Pole,a,b]= chebyshevtype1dsp(Ap,As,Fp,Fs,F_b,Filter_type,Ts)
%% Required Filter Design of LPF Prototype %%
  if Filter_type == "LPF"
    gamma = Fs/Fp;
  elseif Filter_type == "HPF"
    gamma = Fp/Fs;
  else
    Wp = F_b(1)-F_b(2);
    Ws = F_b(3)-F_b(4);
    if Filter_type == "BPF"
      gamma = Ws/Wp;
    else
      gamma = Wp/Ws;
    end
  end
  n = acosh(sqrt(10^(0.1*As)-1)/sqrt(10^(0.1*Ap)-1))/acosh(gamma);
  N = ceil(n);
  epsilon = sqrt(10^(0.1*Ap)-1);
  k = 1:1:N;
  xk = (2*k-1)*pi/(2*N);
  y = -asinh(1/epsilon)/N;
  Pole =(1i*cos(xk)*cosh(y)+sin(xk)*sinh(y));

  %% Prototype Conversions from LPF to any of the four filters %%
  s = tf('s');
  if Filter_type == "LPF"
    Fc = Fp*cosh(acosh(1/epsilon)/N);
    s = s/Fc;
  elseif Filter_type == "HPF"
    Fc = Fp*cos(acos(1/epsilon)/N);
    s = Fc/s;
  else
    Wp = F_b(1)-F_b(2);
    Fc = sqrt(F_b(1)*F_b(2));
    if Filter_type == "BPF"
      s = (s^2 + Fc^2)/(Wp*s);
    else
      s = (Wp*s)/(s^2 + Fc^2);
    end
  end
  %% Chebyshev Polynomial Calculation for the final transfer equation %%
  chebyshev_poly=1;
  for i=1:N
    chebyshev_poly = (1-s/Pole(i))*chebyshev_poly;
  end
  G=1;
  if ~mod(N,2)
    G=1/(1+epsilon^2);
```

```
    end
    Ha = G/chebyshev_poly; %analog TF
    Hd = c2d(Ha,Ts,'tustin'); %digital TF
    [b,a] = tfdata(Hd,'v'); %forward and backward coef extraction
    [Hd_mag,w]=freqz(b,a,2048);
    %magintude plot of the desgined digital filter
    figure;
    hold on; grid on;
    plot((w/(Ts*2*pi)),20*log10(abs(Hd_mag)/max(abs(Hd_mag))),'r','LineWidth',1,'MarkerSize',5);
    title('Magnitude Response of Digital Filter');
    xlabel('Frequency(Hz)');
    ylabel('|H(jW)|dB');
end
```

Evaluation Results:- Used the given EEG signal to extract three waves delta, alpha, gamma according to the given specifications.

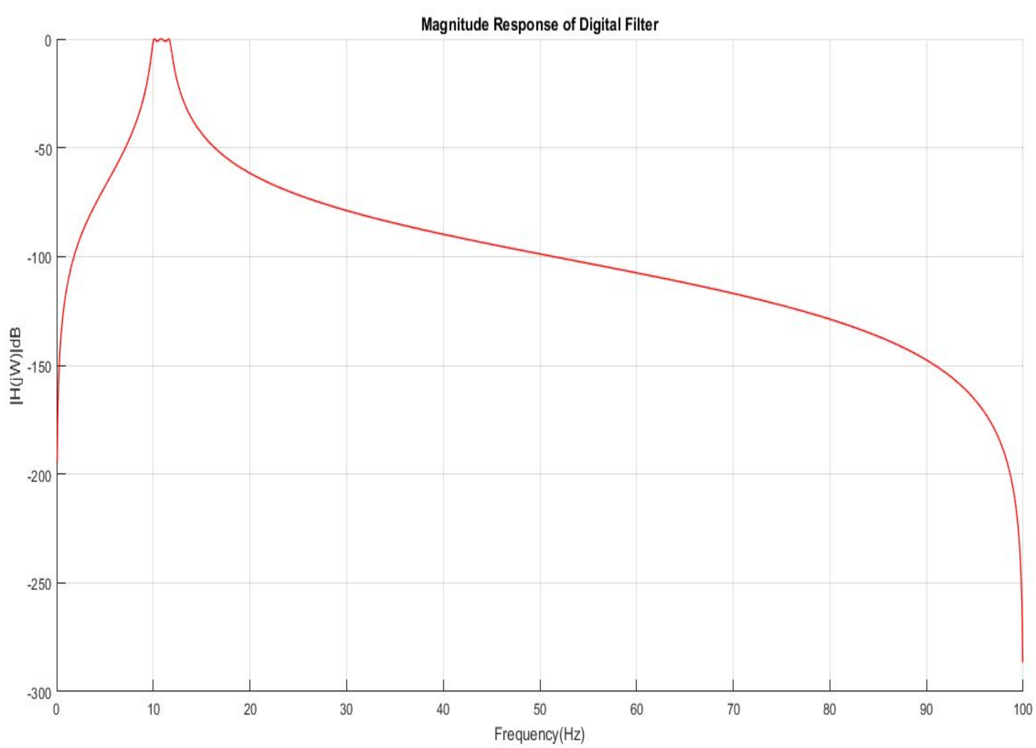
1. Delta wave:-



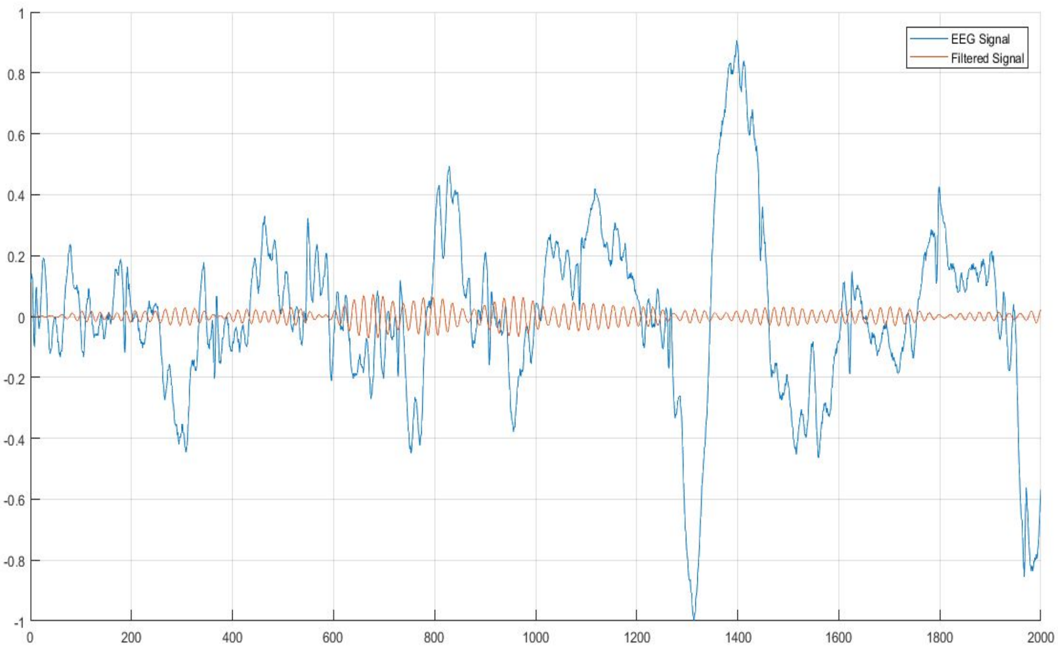


EEG Signal
Filtered Signal
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
200
400
600
800
1000
1200
1400
1600
1800
2000

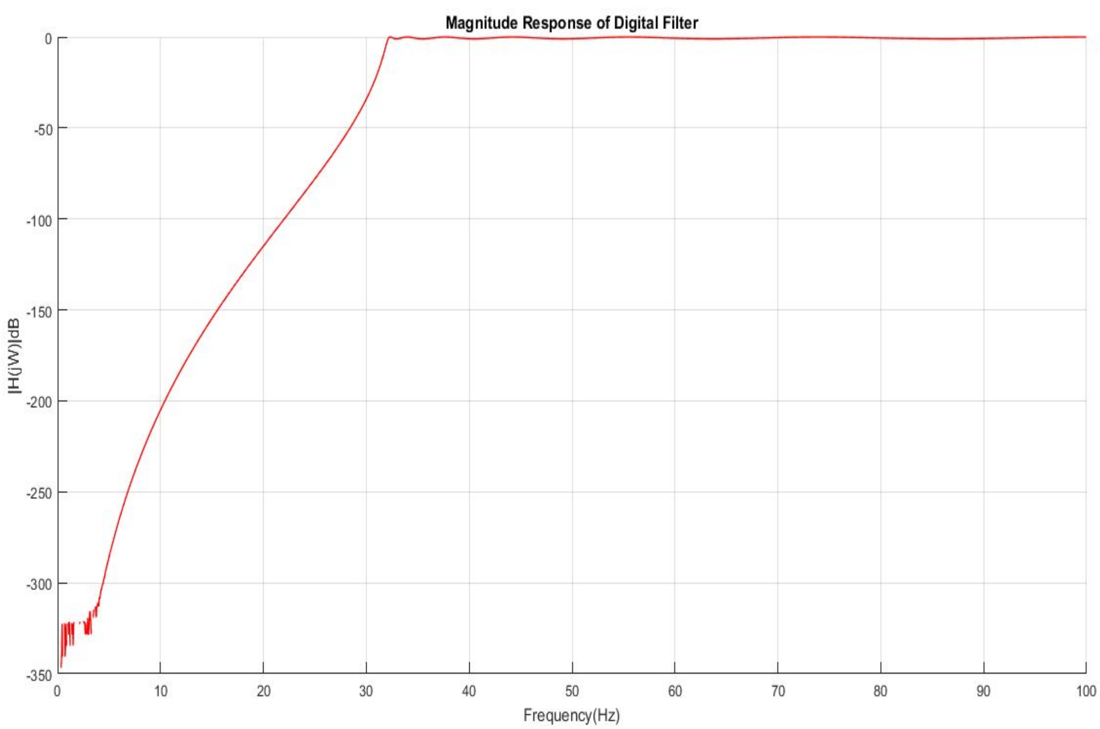
2. Alpha wave:-

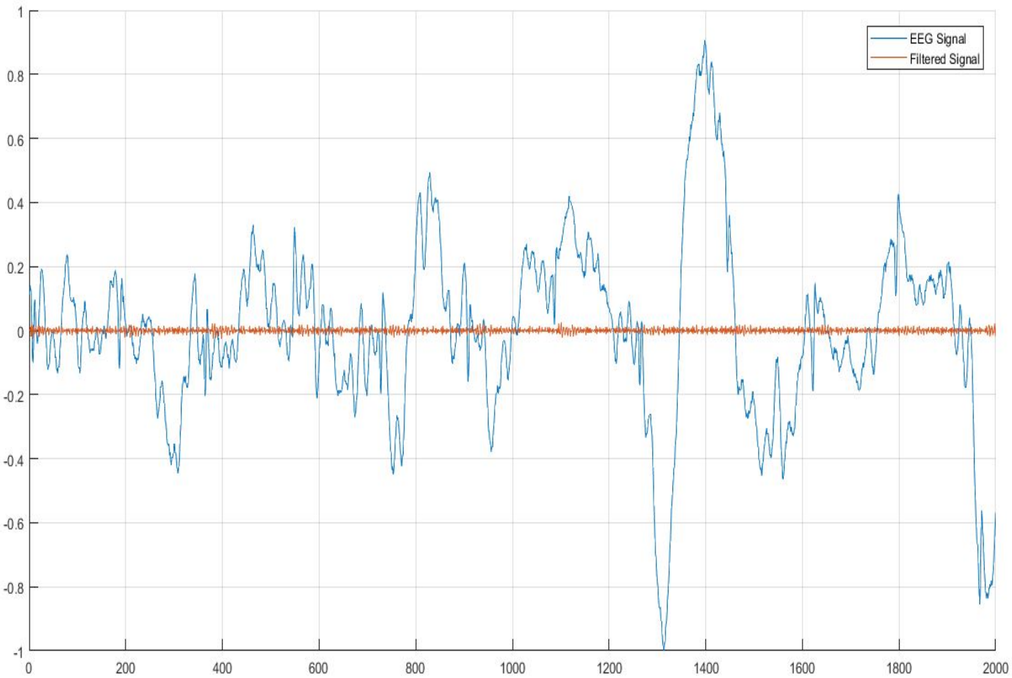


Magnitude Response of Digital Filter
|H(jW)|dB
Frequency(Hz)
0
-50
-100
-150
-200
-250
-300
0
10
20
30
40
50
60
70
80
90
100


EEG Signal
Filtered Signal
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
200
400
600
800
1000
1200
1400
1600
1800
2000

3. Gamma wave:-


Magnitude Response of Digital Filter
|H(jW)|dB
0
-50
-100
-150
-200
-250
-300
-350
0
10
20
30
40
50
60
70
80
90
100
Frequency(Hz)



Discussion & Conclusion:- As, we can see that we got the three waves filtered out from one of the EEG signals given (EEG_signal07).