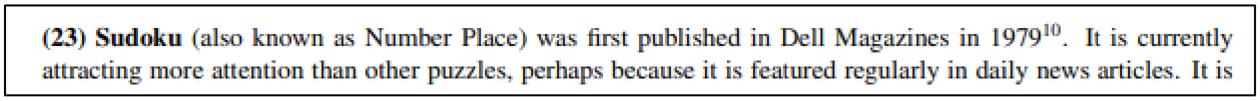
Asa Holland - CS325 - Portfolio Project - March 8, 2021

Step 1) Selection

I chose Puzzle 23 from the Survey of NP-Complete puzzles. This puzzle is Sudoku.

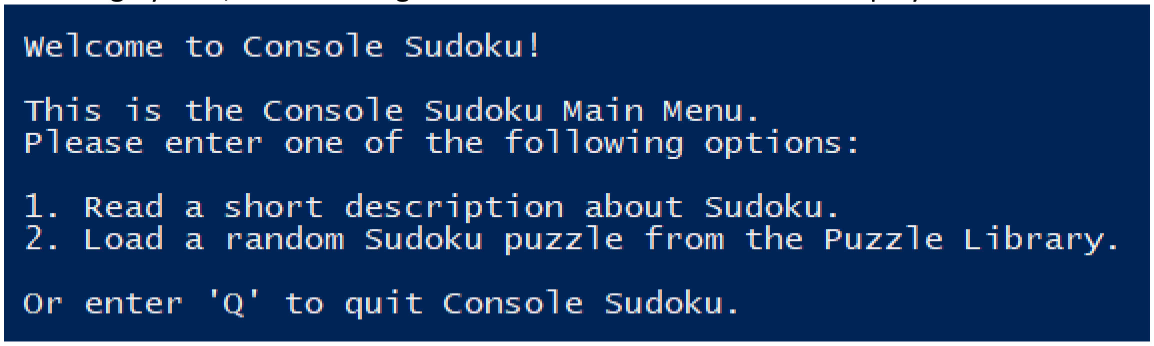

(23) Sudoku (also known as Number Place) was first published in Dell Magazines in 1979[10]. It is currently attracting more attention than other puzzles, perhaps because it is featured regularly in daily news articles. It is

Step 2) Rules of Sudoku

The goal of Sudoku is to completely fill a 9×9 grid with digits. When filled with valid digits, every column, every row, and every 3×3 subgrid that compose the game board contain each digit from 1 to 9. The Sudoku player is provided a grid which is partially complete (some cells are left blank), and the Sudoku player aims to fill in the grid with valid digits so that the puzzle can be completed. If enough blanks are present when the board is first presented, it is possible for multiple valid solutions to exist for a single board, but a good instance of a Sudoku puzzle has only one single valid solution.

Step 3) Implementation Allowing User to Solve the Puzzle

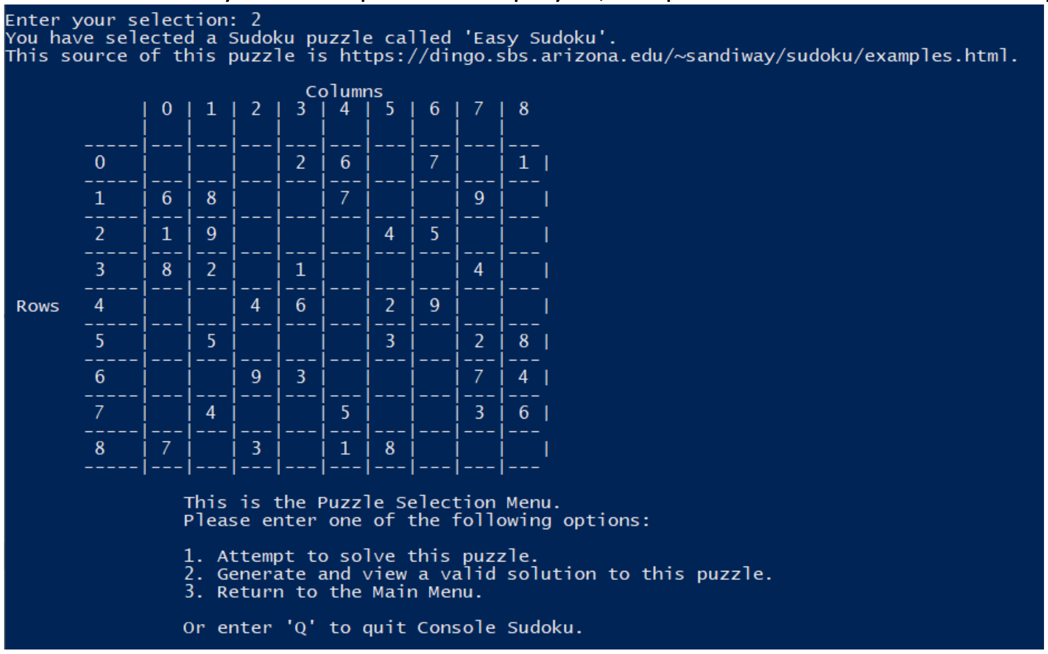
I implemented Sudoku using Python on the command line. When the 'consoleSudoku.py' file is run using Python, the following 'Console Sudoku Main Menu' is displayed in the console.



```
Welcome to Console Sudoku!

This is the Console Sudoku Main Menu.
Please enter one of the following options:

1. Read a short description about Sudoku.
2. Load a random Sudoku puzzle from the Puzzle Library.

Or enter 'Q' to quit Console Sudoku.
```

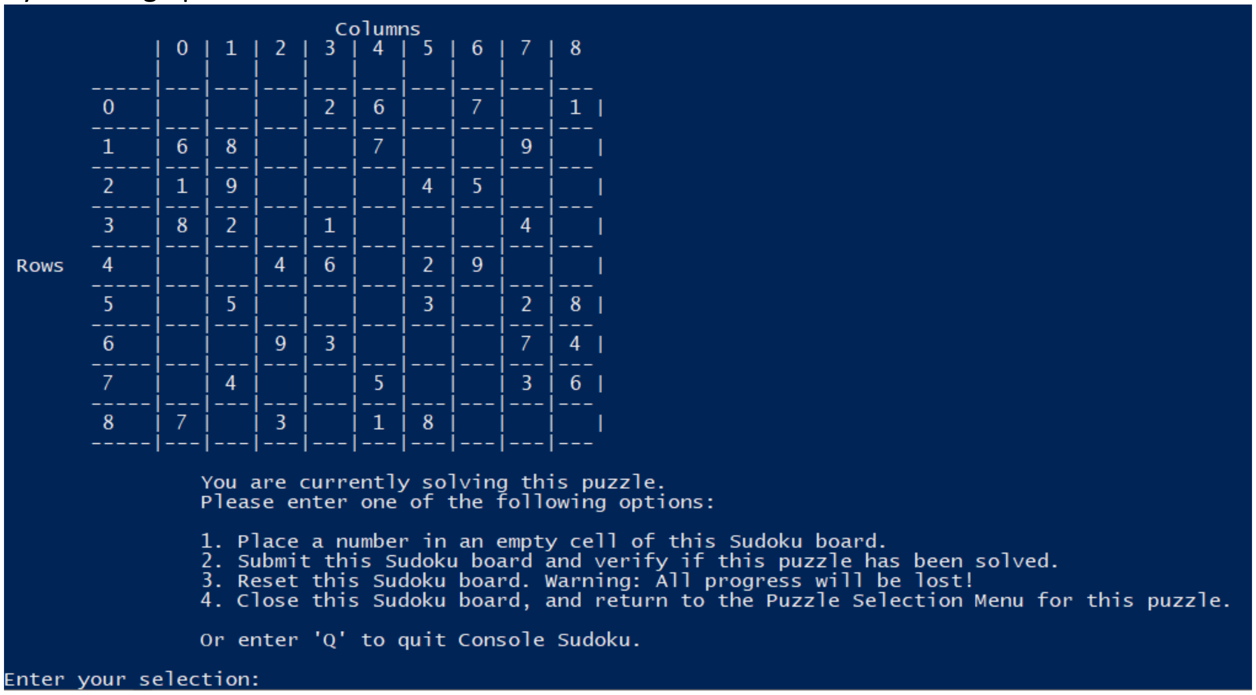
Option 1 displays the same paragraph as rules in Step 2 (above).
Option 2 randomly loads one of three puzzles from a Puzzle library json file and opens the 'Puzzle Selection Menu'.

When a randomly selected puzzle is displayed, the puzzle name & source are displayed as well:



```
Enter your selection: 2
You have selected a Sudoku puzzle called 'Easy Sudoku'.
This source of this puzzle is https://dingo.sbs.arizona.edu/~sandiway/sudoku/examples.html.

                             Columns
            | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
            |   |   |   |   |   |   |   |   |
       -----|---|---|---|---|---|---|---|---|---
        0   |   |   |   | 2 | 6 |   | 7 |   | 1 |
       -----|---|---|---|---|---|---|---|---|---
        1   | 6 | 8 |   |   | 7 |   |   | 9 |   |
       -----|---|---|---|---|---|---|---|---|---
        2   | 1 | 9 |   |   |   | 4 | 5 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        3   | 8 | 2 |   | 1 |   |   |   | 4 |   |
       -----|---|---|---|---|---|---|---|---|---
 Rows   4   |   |   | 4 | 6 |   | 2 | 9 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        5   |   | 5 |   |   |   | 3 |   | 2 | 8 |
       -----|---|---|---|---|---|---|---|---|---
        6   |   |   | 9 | 3 |   |   |   | 7 | 4 |
       -----|---|---|---|---|---|---|---|---|---
        7   |   | 4 |   |   | 5 |   |   | 3 | 6 |
       -----|---|---|---|---|---|---|---|---|---
        8   | 7 |   | 3 |   | 1 | 8 |   |   |   |
       -----|---|---|---|---|---|---|---|---|---

                This is the Puzzle Selection Menu.
                Please enter one of the following options:

                1. Attempt to solve this puzzle.
                2. Generate and view a valid solution to this puzzle.
                3. Return to the Main Menu.

                Or enter 'Q' to quit Console Sudoku.
```

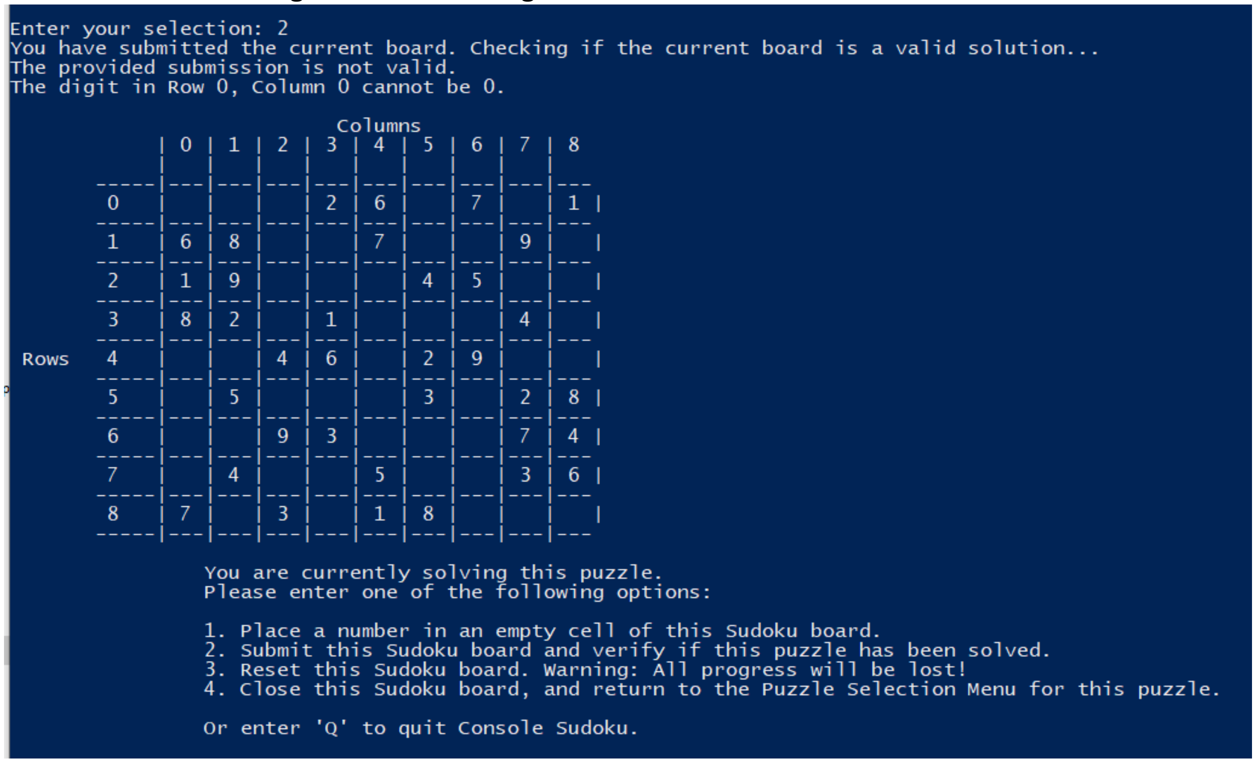
Within the ‘Puzzle Selection Menu’, a user can attempt to solve the randomly selected puzzle by selecting option 1.



```
                          Columns
            | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
            |   |   |   |   |   |   |   |   |
       -----|---|---|---|---|---|---|---|---|---
        0   |   |   |   | 2 | 6 |   | 7 |   | 1 |
       -----|---|---|---|---|---|---|---|---|---
        1   | 6 | 8 |   |   | 7 |   |   | 9 |   |
       -----|---|---|---|---|---|---|---|---|---
        2   | 1 | 9 |   |   |   | 4 | 5 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        3   | 8 | 2 |   | 1 |   |   |   | 4 |   |
       -----|---|---|---|---|---|---|---|---|---
 Rows   4   |   |   | 4 | 6 |   | 2 | 9 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        5   |   | 5 |   |   |   | 3 |   | 2 | 8 |
       -----|---|---|---|---|---|---|---|---|---
        6   |   |   | 9 | 3 |   |   |   | 7 | 4 |
       -----|---|---|---|---|---|---|---|---|---
        7   |   | 4 |   |   | 5 |   |   | 3 | 6 |
       -----|---|---|---|---|---|---|---|---|---
        8   | 7 |   | 3 |   | 1 | 8 |   |   |   |
       -----|---|---|---|---|---|---|---|---|---

                You are currently solving this puzzle.
                Please enter one of the following options:

                1. Place a number in an empty cell of this Sudoku board.
                2. Submit this Sudoku board and verify if this puzzle has been solved.
                3. Reset this Sudoku board. Warning: All progress will be lost!
                4. Close this Sudoku board, and return to the Puzzle Selection Menu for this puzzle.

                Or enter 'Q' to quit Console Sudoku.

Enter your selection:
```

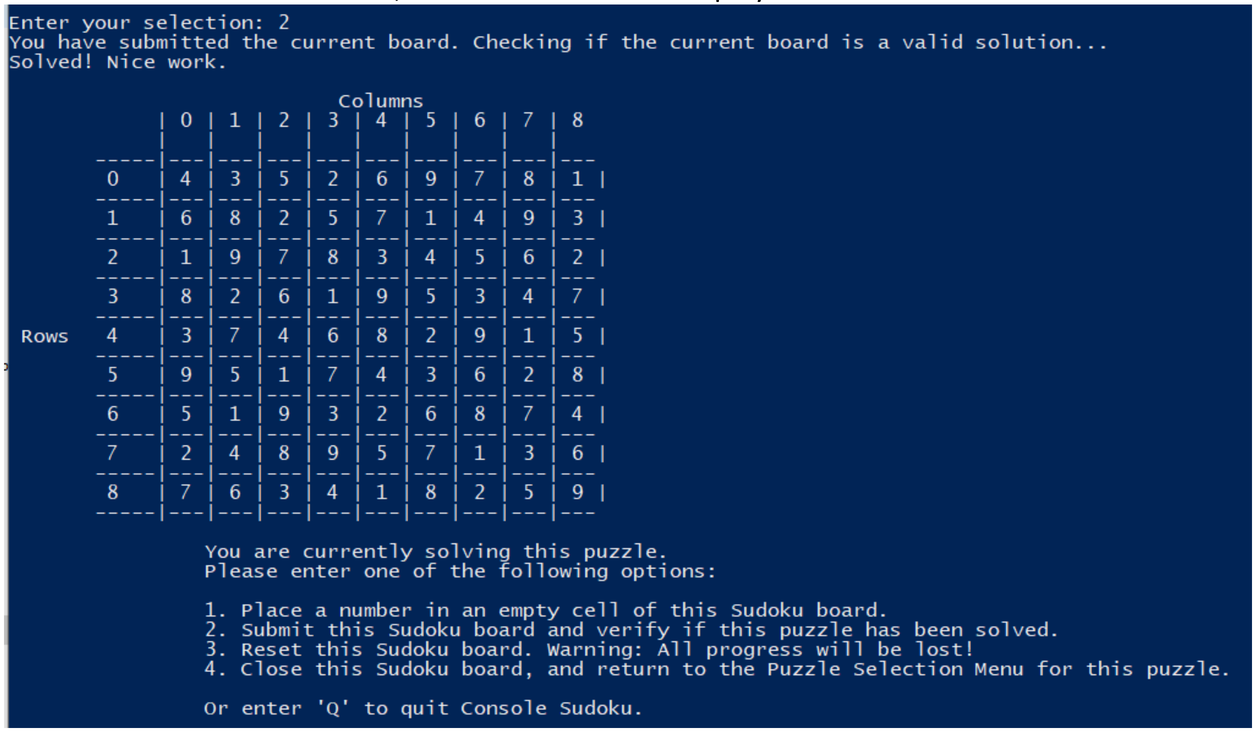
Once option 1 is selected, the console UI will state “You are currently solving this puzzle” and present the user with additional options to enter values into the cells, submit the puzzle, reset the puzzle, or close the puzzle and return to the Puzzle Selection Menu.

Submitting a board will check if the proposed solution is valid. If it is not valid, it will display the first error found during the validation algorithm.



```
Enter your selection: 2
You have submitted the current board. Checking if the current board is a valid solution...
The provided submission is not valid.
The digit in Row 0, Column 0 cannot be 0.

                          Columns
            | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
            |   |   |   |   |   |   |   |   |
       -----|---|---|---|---|---|---|---|---|---
        0   |   |   |   | 2 | 6 |   | 7 |   | 1 |
       -----|---|---|---|---|---|---|---|---|---
        1   | 6 | 8 |   |   | 7 |   |   | 9 |   |
       -----|---|---|---|---|---|---|---|---|---
        2   | 1 | 9 |   |   |   | 4 | 5 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        3   | 8 | 2 |   | 1 |   |   |   | 4 |   |
       -----|---|---|---|---|---|---|---|---|---
 Rows   4   |   |   | 4 | 6 |   | 2 | 9 |   |   |
       -----|---|---|---|---|---|---|---|---|---
        5   |   | 5 |   |   |   | 3 |   | 2 | 8 |
       -----|---|---|---|---|---|---|---|---|---
        6   |   |   | 9 | 3 |   |   |   | 7 | 4 |
       -----|---|---|---|---|---|---|---|---|---
        7   |   | 4 |   |   | 5 |   |   | 3 | 6 |
       -----|---|---|---|---|---|---|---|---|---
        8   | 7 |   | 3 |   | 1 | 8 |   |   |   |
       -----|---|---|---|---|---|---|---|---|---

                You are currently solving this puzzle.
                Please enter one of the following options:

                1. Place a number in an empty cell of this Sudoku board.
                2. Submit this Sudoku board and verify if this puzzle has been solved.
                3. Reset this Sudoku board. Warning: All progress will be lost!
                4. Close this Sudoku board, and return to the Puzzle Selection Menu for this puzzle.

                Or enter 'Q' to quit Console Sudoku.
```

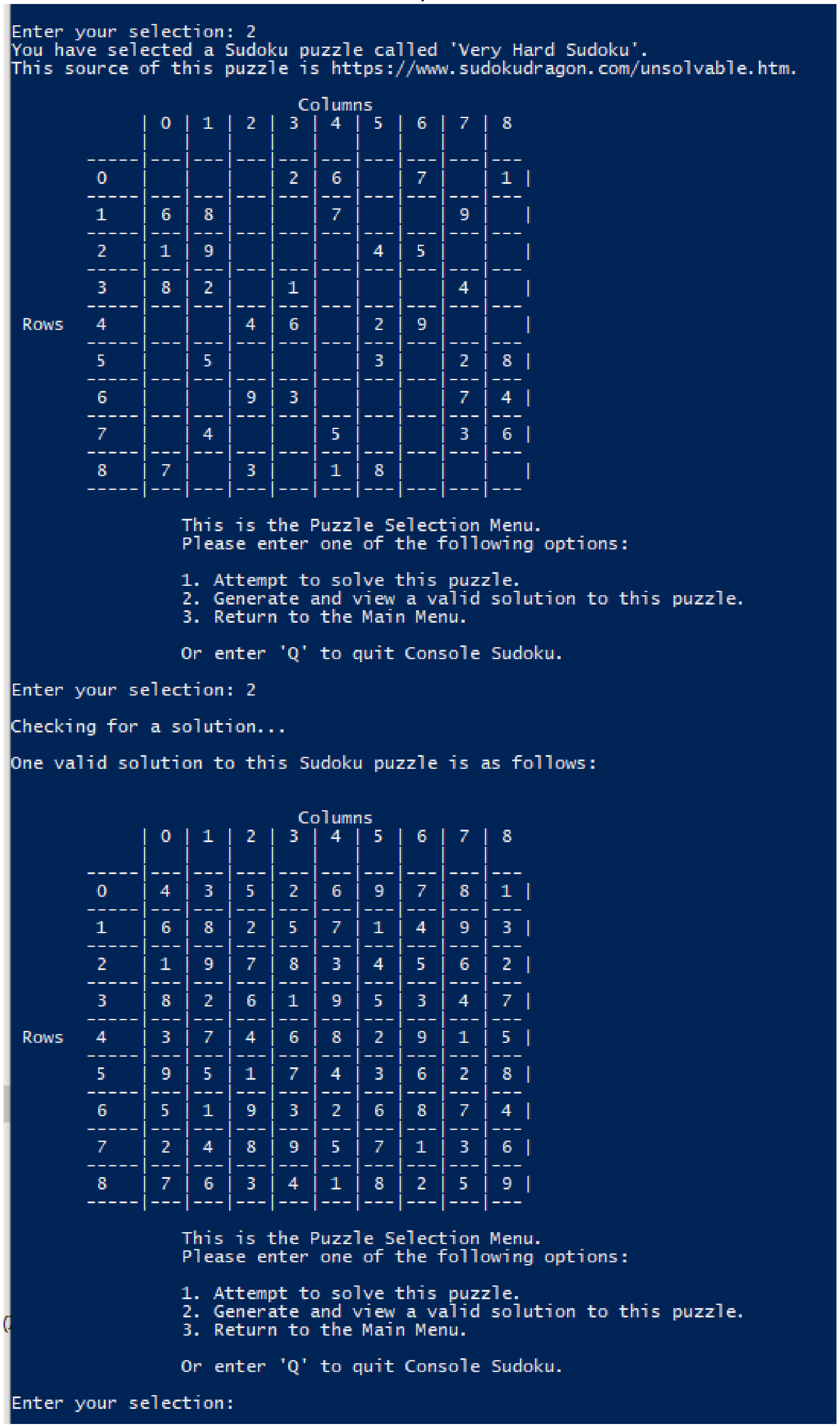
Submitting a valid solution will verify the user’s solution and confirm whether it is actually a correct solution. If it is correct, Console Sudoku will display “Solved! Nice work.” to the user.



```
Enter your selection: 2
You have submitted the current board. Checking if the current board is a valid solution...
Solved! Nice work.

                              Columns
             | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
             |   |   |   |   |   |   |   |   |
        -----|---|---|---|---|---|---|---|---|---
         0   | 4 | 3 | 5 | 2 | 6 | 9 | 7 | 8 | 1 |
        -----|---|---|---|---|---|---|---|---|---
         1   | 6 | 8 | 2 | 5 | 7 | 1 | 4 | 9 | 3 |
        -----|---|---|---|---|---|---|---|---|---
         2   | 1 | 9 | 7 | 8 | 3 | 4 | 5 | 6 | 2 |
        -----|---|---|---|---|---|---|---|---|---
         3   | 8 | 2 | 6 | 1 | 9 | 5 | 3 | 4 | 7 |
        -----|---|---|---|---|---|---|---|---|---
 Rows    4   | 3 | 7 | 4 | 6 | 8 | 2 | 9 | 1 | 5 |
        -----|---|---|---|---|---|---|---|---|---
         5   | 9 | 5 | 1 | 7 | 4 | 3 | 6 | 2 | 8 |
        -----|---|---|---|---|---|---|---|---|---
         6   | 5 | 1 | 9 | 3 | 2 | 6 | 8 | 7 | 4 |
        -----|---|---|---|---|---|---|---|---|---
         7   | 2 | 4 | 8 | 9 | 5 | 7 | 1 | 3 | 6 |
        -----|---|---|---|---|---|---|---|---|---
         8   | 7 | 6 | 3 | 4 | 1 | 8 | 2 | 5 | 9 |
        -----|---|---|---|---|---|---|---|---|---

                You are currently solving this puzzle.
                Please enter one of the following options:

                1. Place a number in an empty cell of this Sudoku board.
                2. Submit this Sudoku board and verify if this puzzle has been solved.
                3. Reset this Sudoku board. Warning: All progress will be lost!
                4. Close this Sudoku board, and return to the Puzzle Selection Menu for this puzzle.

                Or enter 'Q' to quit Console Sudoku.
```

Bonus A: Finding A Solution to the Puzzle

If the user returns to the ‘Puzzle Selection Menu’, the user can specify that they would like to see a solved version of the selected puzzle.



```
Enter your selection: 2
You have selected a Sudoku puzzle called 'Very Hard Sudoku'.
This source of this puzzle is https://www.sudokudragon.com/unsolvable.htm.

                            Columns
              | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
              |   |   |   |   |   |   |   |   |
         -----|---|---|---|---|---|---|---|---|---
          0   |   |   |   | 2 | 6 |   | 7 |   | 1 |
         -----|---|---|---|---|---|---|---|---|---
          1   | 6 | 8 |   |   | 7 |   |   | 9 |   |
         -----|---|---|---|---|---|---|---|---|---
          2   | 1 | 9 |   |   |   | 4 | 5 |   |   |
         -----|---|---|---|---|---|---|---|---|---
          3   | 8 | 2 |   | 1 |   |   |   | 4 |   |
         -----|---|---|---|---|---|---|---|---|---
   Rows   4   |   |   | 4 | 6 |   | 2 | 9 |   |   |
         -----|---|---|---|---|---|---|---|---|---
          5   |   | 5 |   |   |   | 3 |   | 2 | 8 |
         -----|---|---|---|---|---|---|---|---|---
          6   |   |   | 9 | 3 |   |   |   | 7 | 4 |
         -----|---|---|---|---|---|---|---|---|---
          7   |   | 4 |   |   | 5 |   |   | 3 | 6 |
         -----|---|---|---|---|---|---|---|---|---
          8   | 7 |   | 3 |   | 1 | 8 |   |   |   |
         -----|---|---|---|---|---|---|---|---|---

                This is the Puzzle Selection Menu.
                Please enter one of the following options:

                1. Attempt to solve this puzzle.
                2. Generate and view a valid solution to this puzzle.
                3. Return to the Main Menu.

                Or enter 'Q' to quit Console Sudoku.

Enter your selection: 2

Checking for a solution...

One valid solution to this Sudoku puzzle is as follows:


                            Columns
              | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8
              |   |   |   |   |   |   |   |   |
         -----|---|---|---|---|---|---|---|---|---
          0   | 4 | 3 | 5 | 2 | 6 | 9 | 7 | 8 | 1 |
         -----|---|---|---|---|---|---|---|---|---
          1   | 6 | 8 | 2 | 5 | 7 | 1 | 4 | 9 | 3 |
         -----|---|---|---|---|---|---|---|---|---
          2   | 1 | 9 | 7 | 8 | 3 | 4 | 5 | 6 | 2 |
         -----|---|---|---|---|---|---|---|---|---
          3   | 8 | 2 | 6 | 1 | 9 | 5 | 3 | 4 | 7 |
         -----|---|---|---|---|---|---|---|---|---
   Rows   4   | 3 | 7 | 4 | 6 | 8 | 2 | 9 | 1 | 5 |
         -----|---|---|---|---|---|---|---|---|---
          5   | 9 | 5 | 1 | 7 | 4 | 3 | 6 | 2 | 8 |
         -----|---|---|---|---|---|---|---|---|---
          6   | 5 | 1 | 9 | 3 | 2 | 6 | 8 | 7 | 4 |
         -----|---|---|---|---|---|---|---|---|---
          7   | 2 | 4 | 8 | 9 | 5 | 7 | 1 | 3 | 6 |
         -----|---|---|---|---|---|---|---|---|---
          8   | 7 | 6 | 3 | 4 | 1 | 8 | 2 | 5 | 9 |
         -----|---|---|---|---|---|---|---|---|---

                This is the Puzzle Selection Menu.
                Please enter one of the following options:

                1. Attempt to solve this puzzle.
                2. Generate and view a valid solution to this puzzle.
                3. Return to the Main Menu.

                Or enter 'Q' to quit Console Sudoku.

Enter your selection:
```

Note: The process of generating a solution does not compare the provided puzzle with any sort of ‘expected’ solution, and it only generates one possible solution using the solving algorithm. Other solutions may be possible depending on the puzzle selected.

Step 5: Prove the Correctness of the Verification Algorithm

```
def is_valid_sudoku(board_to_test):
        for row in range(9):
                for col in range(9):
                        if temp_board[row][col] == 0:
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
                        temp = temp_board[row][col]
                        temp_board[row][col] = 0
                        if not is_placement_possible(y=row, x=col, n=temp, board=temp_board):
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': temp}
                        temp_board[row][col] = temp
        return {'is_valid': True, 'invalid_row': None, 'invalid_column': None, 'invalid_digit': None}

def is_placement_possible(y, x, n, board):
        for i in range(0, 9):
                if copied_board[y][i] == n:
                        return False
                if copied_board[i][x] == n:
                        return False
        subsquare_x = (x//3)*3
        subsquare_y = (y//3)*3
        for i in range(3):
                for j in range(3):
                        if copied_board[subsquare_y+i][subsquare_x+j] == n:
                                return False
        return True
```

Prove that for all inputs, the algorithm correctly determines whether the input is valid or not.

Prove the algorithm terminates

At maximum, this algorithm will loop over all 81 cells of a provided Sudoku board and check each one to be valid. If it locates an invalid cell prior to completing all 81 cells, it will terminate and return that 'is_valid' for the board is False. If the algorithm does not terminate at any of the prior 81 cells, then it will terminate at the end of the algorithm by retuning that 'is_valid' is True for this particular board. Therefore we have shown that the algorithm will always terminate in either a True or False value for 'is_valid'.

1) If the value of any particular cell is 0, then the algorithm will terminate. This is due to the fact that no valid Sudoku board contains a zero. In the algorithm, it can be seen that if this is the case, the algorithm returns that the 'is_valid' value is False.
2) If the value placed in any particular cell is not a possible input (the value is already found in the row of that cell, the column of that cell, or the subsquare of that cell), then the algorithm will terminate. This is due to the fact that no valid Sudoku board can contain a value that is repeated in the same row, column or subsquare. In the algorithm, it can be seen that if a repeated value is encountered, the algotihm returns that the 'is_valid' value is False
3) If the algorithm has investigated all possible cells on the board, and determined every cell to be valid, then thee entire board has been determined to be valid. This can be seen since the end of the 'is_valid_sudoku' function returns an 'is_valid' value of True, and this return statement is only accessible after all cells of a provided board have been examined and determined to be valid. Thus, the algorithm will always stop after completing x loops over the rows and y loops over the columns.

4) Prove that the algorithm determines a valid solution to each posed board. This is proven using the loop invariant (the property of the loop that should always be true). In this case, the loop invariant is that after looping over x rows and y columns, the 'board' that contains cells with rows <= x and column values <= y will only contain valid inputs (if the algorithm has not terminated into one of the above-mentioned base cases). Put another way, the loop invariant is that if each cell that has been inspected up to the current cell have been valid, then all cells up the current cell are valid.

    a. Prove the base case that if the value of any particular cell is 0, then the algorithm will terminate. Given a provided board where the top left cell is 0 (row 0, column 0), we can walk-through the algorithm to determine the result.

```
def is_valid_sudoku(board_to_test):
    for row in range(9):
        for col in range(9):
            if temp_board[row][col] == 0:
                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
```

In this part of the algorithm, we can see that if the board_to_test contains the topleft cell as zero, then a temporary duplicate of the provided board_to_test array is created. No differences have been incorporated into the duplicate. Then, the outer loop will enter at index 0 for the row range and 0 for the index of the column value. The algorithm with check to see if the value of the cell at the 0th row and 0th column is equal to 0. Since it is in this case, the algorithm will return that the function is not Valid via the "is_valid" key in the result dictionary. Thus, we can see that this base case is addressed by the algorithm.

    b. Prove the 2nd base case. That is, prove that if the algorithm is provided a sudoku board with an impossible placement that the algorithm returns False. The board can be invalid based on a repeated value in the same row, the same column, or the same subsquare. Proving this base case correct requires a proof of each of these three cases. In each of these cases, the first base case (the value being 0) will not be returned. Thus the algorithm will enter the is_placement_possible subroutine to verify that each cell is valid.

```
def is_placement_possible(y, x, n, board):
    for i in range(0, 9):
        if copied_board[y][i] == n:
            return False
        if copied_board[i][x] == n:
            return False
    subsquare_x = (x//3)*3
    subsquare_y = (y//3)*3
    for i in range(3):
        for j in range(3):
            if copied_board[subsquare_y+i][subsquare_x+j] == n:
                return False
    return True
```

1. Provided an array of arrays with the following:
    - Value of the cell in the 0th row and 0th column equals 1
    - Value of the cell in the 0th row and 1st column equals 1

In this case, checking the value of the 0th row and 0th column will return False since 1 will be found within copied_board[i][x].

2. Provided an array of arrays with the following:
    - Value of the cell in the 0th row and 0th column equals 1
    - Value of the cell in the 1st row and 0th column equals 1

In this case, checking the value of the 0th row and 0th column will return False since 1 will be found within copied_board[y][i].

3) Provided an array of arrays with the following:
    - Value of the cell in the 0th row and 0th column equals 1
    - Value of the cell in the 1st row and 1st column equals 1

In this case, the subsquare values of x=0 and y=0 will be determined to be:

subsquare_x = (0//3)*3 = 0
subsquare_y = (0//3)*3 = 0

Then, for each i in 0, 1 and 2 and for each j in 0, 1, and 2,
copied_board[subsquare_y+i][subsquare_x+j] == n will be run.
When i=1 and j=1, copied_board[subsquare_y+i][subsquare_x+j] will equal to copied_board[0+1][0+1], which is equal to copied_board[1][1], which will return the value of 1 which is in that cell. Since 1 is equal to n where n is the provided digit from the initial cell, this will return False.

iii) Finally we need to prove the final base case. This is that if the algorithm is provided a valid Sudoku board, it will return that the algorithm is valid. Given a provided board there all cells meet the conditions of the is_placement_possible subroutine, and no values are zero, we can walk through the algorithm to determine its correctness.

```
def is_valid_sudoku(board_to_test):
    for row in range(9):
        for col in range(9):
            if temp_board[row][col] == 0:
                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
            temp = temp_board[row][col]
            temp_board[row][col] = 0
            if not is_placement_possible(y=row, x=col, n=temp, board=temp_board):
                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': temp}
            temp_board[row][col] = temp
    return {'is_valid': True, 'invalid_row': None, 'invalid_column': None, 'invalid_digit': None}
```

In this case, for every cell that is visited, the value will not be zero so the first base case will not be returned. The value of the cell in the ith row and jth column will be saved as 'temp'. The value of the cell in the ith row and jth column will be assigned to be equal to 0. The result of is_placement_possible() placing temp in the ith row and jth column will return True, so the second base case will not be executed. The value of the cell will be returned to temp (the initial value). This process will proceed for all cells. At the end, the algorithm will return that the solution is valid.

Inductive Step

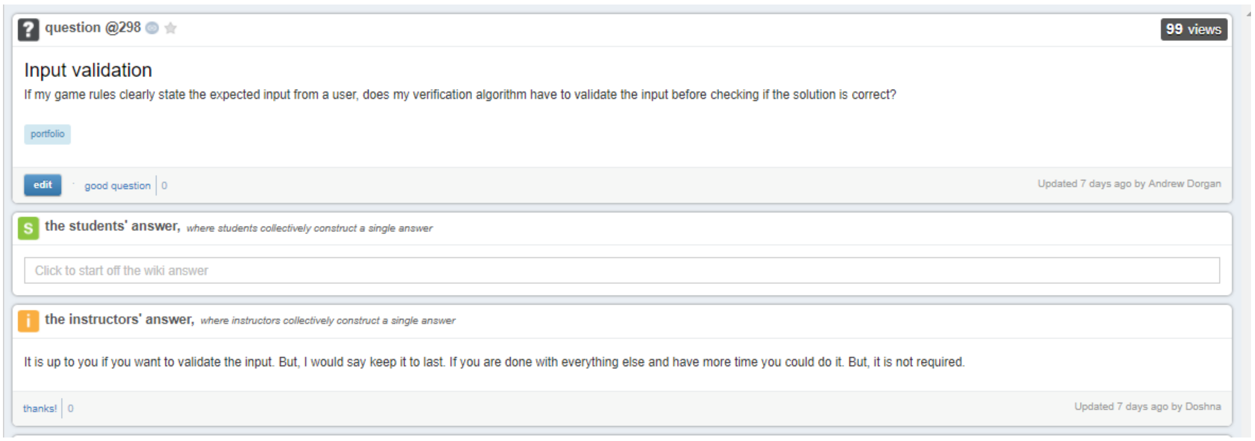
We will assume that the loop invariant (after looping over x rows and y columns, the 'board' that contains cells with rows <= x and column values <= y will only contain valid inputs) is true for some kth cell of an arbitrary Sudoku board. Thus, the value of the ${k_x}^{th}$ row and the ${k_y}^{th}$ column is possible. Using this assumption, we will prove that the loop invariant is also true for the k+1th cell in the Sudoku board (the cell immediately following the kth cell). After looping through k cells of the board, the board should check whether the k+1st cell contains a valid digit. After k repetitions, is_valid_sudoku has not returned on any of the base cases. The inner code will execute on the the ${k_{x+1}}^{th}$ row and the ${k_{y+1}}^{th}$ column (representing the k+1th cell) :

```
if temp_board[row][col] == 0:
        return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
temp = temp_board[row][col]
temp_board[row][col] = 0
if not is_placement_possible(y=row, x=col, n=temp, board=temp_board):
        return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': temp}
temp_board[row][col] = temp
```

If the value in the k+1th cell is not valid (either by containing a zero or containing a repeated value from the same row, column or subsquare, the algorithm will return that the board is not valid. If the value in the k+1th cell is valid, the loop will continue. If the k+1th cell was the last cell in the Sudoku board, then the loop is complete and the function will return that the board is valid. We can see that the loop invariant holds true (after looping over x+1 rows and y+1 columns, the 'board' that contains cells with rows <= x+1 and column values <= y+1 will only contain valid inputs, or will have returned False). We can confidently say that the loop invariant holds for all positive integers of k.

Since we have shown that the loop will always terminate, and that at each termination the is_valid_sudoku() function will return whether the provided board is valid or not base on the current looped cell, we have shown that the algorithm both terminates and produces the correct answer when it terminates. Therefore we have proved the correctness of the verification algorithm.

Note that this algorithm does not verify whether the value of any cell is outside the range of possible sudoku values (1 through 9, or 0 as a placeholder for empty). If somehow the board was provided some character other than 1 through 9 or 0, the algorithm may return false positives. As per piazza post @298, input validation is not required for this portfolio project.



question @298

99 views

Input validation

If my game rules clearly state the expected input from a user, does my verification algorithm have to validate the input before checking if the solution is correct?

portfolio

edit · good question | 0

Updated 7 days ago by Andrew Dorgan

the students' answer, *where students collectively construct a single answer*

Click to start off the wiki answer

the instructors' answer, *where instructors collectively construct a single answer*

It is up to you if you want to validate the input. But, I would say keep it to last. If you are done with everything else and have more time you could do it. But, it is not required.

thanks! | 0

Updated 7 days ago by Doshna

Asa Holland - CS325 - Portfolio Project - March 8, 2021

Step 6) Discuss the Time Complexity of the Verification Algorithm

```
def is_valid_sudoku(board_to_test):
        for row in range(9):
                for col in range(9):
                        if temp_board[row][col] == 0:
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
                        temp = temp_board[row][col]
                        temp_board[row][col] = 0
                        if not is_placement_possible(y=row, x=col, n=temp, board=temp_board):
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': temp}
                        temp_board[row][col] = temp
        return {'is_valid': True, 'invalid_row': None, 'invalid_column': None, 'invalid_digit': None}
```

9 operations
    3 operations
        1 operation

(~27 operations in Best Case)

This algorithm will always run on a board of size 9 rows by 9 columns. Since n (the input size) is not changing, it is not entirely appropriate to use Big O complexity to analyze the time complexity of the verification algorithm. Big O time complexity is meant to assist in determining the time complexity as the input size grows large, which is not relevant in this case.

However, we can discuss the actual complexity due to the number of operations taken by the algorithm while determining a valid solution.

At Best Case, the board is not valid with a zero in the 0th column and 0th row (the top left cell of the Sudoku board). In this case, the Sudoku is_valid_sudoku algorithm will perform the following actions:
Construct a new list. This will take 81 operations as the list will be constructed using 81 values from the provided board.
Enter the 'row' loop at index 0. This will take 1 operation.
    Enter the 'col' loop at index 0. This will take 1 operation.
        Index the temp_board at the [0th] row. This will take 1 operation.
        Index the temp_board row at the [0th] column value. This will take 1 operation.
        Obtain the value at the indexed column at the indexed row of temp_board. 1 operation
        Check to see if this value is equal to 0. This will take 1 operation.
        In this case, the inner return will end the algorithm, retuning 'is_valid' = False
            (retuning takes 1 operation)

Clearly, the process of the Best Case will take a fixed number of operations.

In the Worst Case, the board is entirely valid and will need to check all cells to confirm they are valid. In this case, the Sudoku is_valid_sudoku algorithm will perform the following actions:
    Construct a new list. This will take 81 operations as the list will be constructed using 81 values from the provided board.
    Loop over 'row' loop 9 times. This will take 9 operations.
        Loop over the 'col' loop 9 times. This will take 9 operations.
            Index the value at this cell and check if it equal to 0 (3 operations)
            Save the value of the indexed cell to temp (3 operations: index, index, assignment)
            Assign the value at the indexed cell to 0 (3 operations: index, index, assignment)
            Perform the is_placement_possible subroutine an compare to not
            Assign the value at the cell to temp (3 operations: index, index, assignment)
    Perform the outer return, which will end the algorithm and retuning 'is_valid' = True (1 operation)

```
def is_valid_sudoku(board_to_test):
        for row in range(9):
                for col in range(9):
                        if temp_board[row][col] == 0:
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': 0}
                        temp = temp_board[row][col]
                        temp_board[row][col] = 0
                        if not is_placement_possible(y=row, x=col, n=temp, board=temp_board):
                                return {'is_valid': False, 'invalid_row': row, 'invalid_column': col, 'invalid_digit': temp}
                        temp_board[row][col] = temp
        return {'is_valid': True, 'invalid_row': None, 'invalid_column': None, 'invalid_digit': None}
```

9 operations
    9 operations
        3 operations
        3 operations
        3 operations
        1 op + is_placement_possible
        3 operations
1 operation

(~7,100 operations)

```
def is_placement_possible(y, x, n, board):
        for i in range(0, 9):
                if copied_board[y][i] == n:
                        return False
                if copied_board[i][x] == n:
                        return False
        subsquare_x = (x//3)*3
        subsquare_y = (y//3)*3
        for i in range(3):
                for j in range(3):
                        if copied_board[subsquare_y+i][subsquare_x+j] == n:
                                return False
        return True
```

```
9 operations
   3 operations
   3 operations

1 operation
1 operation
3 operations
   3 operations
      5 operations
1 operation

(~75 operations in worst case)
```

In the worst case, the is_placement_possible subroutine copies the provide board (81 operations), iterates over through each index and checks if the value at each row and column is equal to the same provided number (are cell values duplicated within the same row and column?). Then the subroutine determined the subsquare location and iterates over all 9 cells of the subsquare, checking if those cells contain the provided digit. In the worst case, this subroutine will return True for all provided cells on the provided board. Clearly, in the worst case, the subroutine will perform a fixed number of operations.

Clearly, the process of running the algorithm through the Best Case will take a fixed number of operations.

This means that the actual complexity of the algorithm will vary based on whether the provided board is invalid (and how it is invalid) or if it is valid. The number of operations will range from roughly 100 operations in the Best Case to roughly 7,100 operations in the Worst Case.

Therefore we can conclude that the actual time complexity will depend on the number of values that must be validated. Since the is_valid_sudoku function checks every cell to determine if there is a valid solution, at a maximum, there will be 81 cells checked. To generalize, we can treat this like 9^n where n is the number of cells that must be checked. This is exponential, but is not realistic because that any possible number of cells could be checked, when realistically there can only be a maximum of n <= 81 (if the worst case occurs and all cells are checked, which only happens if the provided sudoku board is valid).

This is still a bounded solution however. The input amount (the overall size of the board) does not change since there are always 81 cells in the board. So technically the time complexity will always be $O(9^{81})$ which is still constant time.

Bonus B) Discuss the Time Complexity of the Solving Algorithm

```
def find_empty_location(board, coordinates):
    for y_i in range(9):                          9 operations
        for x_j in range(9):                          9 operations
            if board[y_i][x_j] == 0:                      3 operations
                coordinates[0]= y_i                           2 operations
                coordinates[1]= x_j                           2 operations
                return True                                   1 operation
    return False                                  1 operation

                                                  Worst Case: ~240 operations
                                                  Best Case: ~10 operations
```

```
def is_placement_possible(y, x, n, board):
    for i in range(0, 9):                                       9 operations
        if copied_board[y][i] == n:                                 3 operations
            return False
        if copied_board[i][x] == n:                                 3 operations
            return False
    subsquare_x = (x//3)*3                                      1 operation
    subsquare_y = (y//3)*3                                      1 operation
    for i in range(3):                                          3 operations
        for j in range(3):                                          3 operations
            if copied_board[subsquare_y+i][subsquare_x+j] == n:         5 operations
                return False
    return True                                                 1 operation

                                                                Worst Case: ~75 operations
                                                                Best Case: ~5 operations
```

```
def solve_sudoku(board_to_solve):
    copied_board = board_to_solve.copy()                                     81 operations
    current_coords = [0, 0]                                                  1 operation
    if not find_empty_location(board=copied_board, coordinates=current_coords):   1 operation + find_empty_location subroutine
        return True                                                              1 operation
    [row, col] = current_coords                                              1 operation
    for digit in range(1, 10):                                               9 operations
        if is_placement_possible(y=row, x=col, n=digit, board=copied_board):     1 operation + is_placement_possible subroutine
            copied_board[row][col] = digit                                           3 operations
            if solve_sudoku(board_to_solve=copied_board):                            1 operation + recursive solve_sudoku call
                return True                                                              1 operation
            copied_board[row][col] = 0                                               3 operations
    return False                                                             1 operation
```

My implementation of the solving algorithm relies on three functions, a solving function as well as two supporting subroutines.

The first subroutine find_empty_location iterates over the provided Sudoku board and searches for empty cells. If it finds an empty cell, it updates a pair of coordinates with the position of that cell and returns True (an empty cell has been found). In the worst case, this will iterate over all cells and determine that none are empty, returning False. In either case, there is standard range of operations that could be performed by this subroutine, depending on how many cells need to be searched to locate an empty cell.

The second subroutine is the same is_placement_possible subroutine used in my verification algorithm. This has the same range of operations as previously discussed, and is dependent on whether or not the value at a particular cell is valid or not, and if not, in which way is it invalid.

The main solve_sudoku function is a recursive, backtracking algorithm which relies on the other two subroutines.

The first base case of solve_sudoku is if no empty locations are found by find_empty_location . Since solve_sudoku is only provided unfinished puzzles, if there are no empty cells, that means that the only way this base case has been reached is if the entire puzzle has been solved with valid input. Therefore we can return True. This will (in the worst case) take approximately 320 operations (~80 operations in solve_sudoku + ~240 operations in find_empty_location).

The second base case of solve_sudoku is if an empty cell is located using 1 instance of find_empty_location (which will befrom a fixed number of operations based on how many cells needed to be examines) and all digits from 1 to 9 are determined to NOT be valid options for that cell by is_placement_possible (the worst case, approximately 75 operations each time). In this case, we know that there either the puzzle is impossible or a mistake was made earlier when assigning digits to cells. Since there will be 9 loops where is_placement_possible is called, this will roughly (9*75) + x operations + ~85 where x is between 10 and 235 based on the operations performed by find_empty_location . Therefore in the worst case (where the last empty location is the last cell in the sudoku board, taking ~235 operations to determine) this base case will perform ~1,000 operations.

The recursive call in solve_sudoku occurs when an empty cell has been identified by find_empty_location and a viable digit option from 1 to 9 has been identified using is_placement_possible. In this case, we will place the value in the cell and then call solve_sudoku to see if the board can still be solved using the placement. If it is valid, we will solve the puzzle. If it is not valid, then we will return False, and backtrack by removing the placed value and placing a new digit in its place. Although not realistic, an upper bound for this recursion would be the hypothetical case in which we would need to recurse on every cell, for every possible input. Therefore, we can generalize the problem to say that a recurrence relation defines the complexity of this recursive call. The recurrence relation is as follows:

$$T(n) = 9\,T(n-1) + O(1)$$

Each Sudoku cell has 9 possible values that must be checked. We can generalize to say there can be n unknown empty cells on a provided Sudoku board for which we are attempting to place a value. Each time we place a value in a cell, the number of unknown values decreases by 1 and we recurse onto that decision. There is a standard amount of operational time complexity associated with each step of the recursion.

We can simplify the recurrence relation to show that it is upper bounded by 9^n possible operations where n is the number of cells that are empty and need assignment. Although this is exponential growth, this is deceptive because it is not a realistic bound based on the types of provided Sudoku boards. Realistically the number of operations performed would be much less than that. A fully blank board with the standard size of a 9x9 grid would have 9^{81} operations plus some constant amount of operations. But that wouldn't make for a particularly fun puzzle.