Application of seasonal ARIMA model for forecasting GDP of United Kingdom

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In this paper, an attempt will be made to forecast the value of Gross Domestic Product for the United Kingdom of Great Britain and Northern Ireland for the fourth quarter of 2021. For this purpose, the country's historical GDP data in the form of a time series will be used, covering the range from the first quarter of 1955 to the third quarter of 2021.

The data is contained in the file `gdp_uk.xlsx` and contains 267 observations. For each of them, the values of two variables were given:

- date – year and quarter,
- gdp – GDP value in million GBP, in current prices, not seasonally adjusted.

The data comes from the resources of Office for National Statistics.

The scope of the data was limited by removing the observations to the 1st quarter of 1980 to eliminate the values disturbing the desired series characteristics.

Initial analysis

Descriptive statistics for the `gdp` variable:

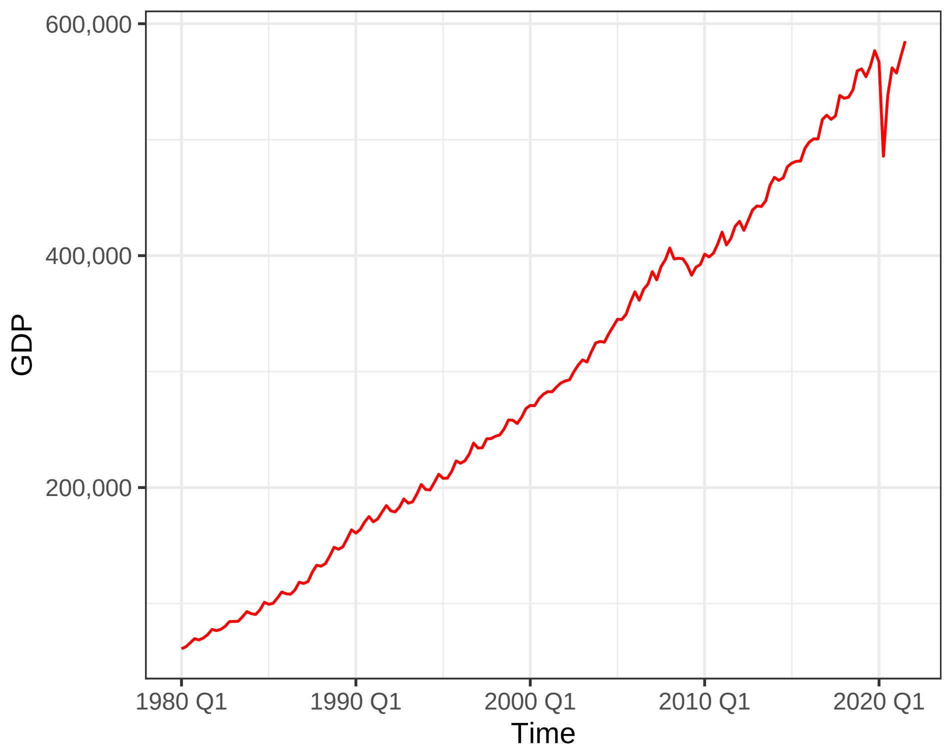
```
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
 61022  167071  280461  294553  412376  584928 
```

```
      SD       CV  Skewness Kurtosis 
153515.8 52.11816 0.1999873 1.835088 
```

The time series average is £ `294,553`m with a standard deviation of £ `153,515`m. The middle value of the series is £ `280,461`m. The smallest value in the series is £ `61,022`m and the highest is £ `584,928`m. The middle 50% of the series is between the first and third quartiles of £ `167,071`m and £ `412,376`m, respectively. The volatility coefficient is `52.12`, which means the average volatility. A skewness of `0.2` means a slightly right-asymmetric distribution. A positive kurtosis of `1.83` indicates a high concentration of the values around the mean.

The chart shows a relatively steady upward trend with two major collapses around 2008 and 2020.

Chart 1. United Kingdom GDP in 1980-2021



Determining time series stationarity

Augmented Dickey Fuller test for integration

$H_0 : \rho = 0$, y_t it is not a stationary process

$H_1 : \rho < 0$, y_t is a stationary process

For unchanged data

```
[1] "ADF without trend and constant"
     statistic  1pct  5pct 10pct
tau1  3.474337 -2.58 -1.95 -1.62
[1] "ADF with constant"
     statistic  1pct  5pct 10pct
tau2   1.61824 -3.46 -2.88 -2.57
phi1  11.65688  6.52  4.63  3.81
[1] "ADF with constant and trend"
     statistic  1pct  5pct 10pct
tau3 -2.012357 -3.99 -3.43 -3.13
phi2  9.526477  6.22  4.75  4.07
phi3  3.684532  8.43  6.49  5.47
```

Without taking into account the intercept and the trend, the test statistic `tau1 = 3.47` is greater than the critical value `-1.95`, which means that at the significance level of 5% there is no reason to reject the null hypothesis of non-stationarity of the series.

Taking into account the intercept, the test statistic `tau2 = 1.61` is greater than the critical value `-2.88`, which means that at the significance level of 5% there are no grounds to reject the null hypothesis of non-stationarity of the series.

Taking into account the intercept and the trend, the test statistic `tau3 = -2.01` is greater than the critical value `-3.43`, which means that at the significance level of 5% there are no grounds to reject the null hypothesis of series non-stationarity.

For first seasonal differences

```
[1] "ADF without trend and constant"
      statistic  1pct  5pct 10pct
tau1 -0.9821618 -2.58 -1.95 -1.62
[1] "ADF with constant"
     statistic  1pct  5pct 10pct
tau2 -3.045680 -3.46 -2.88 -2.57
phi1  4.638144  6.52  4.63  3.81
[1] "ADF with constant and trend"
     statistic  1pct  5pct 10pct
tau3 -5.386365 -3.99 -3.43 -3.13
phi2  9.686629  6.22  4.75  4.07
phi3 14.508674  8.43  6.49  5.47
```

Without taking into account the intercept and the trend, the test statistic `tau1 = -0.98` is greater than the critical value `-1.95`, which means that at the significance level of 5% there is no reason to reject the null hypothesis of non-stationarity of the series.

Taking into account the intercept, the test statistic `tau2 = -3.05` is lower than the critical value `-2.88`, which means that at the significance level of 5% we reject the null hypothesis of non-stationarity of the series.

Taking into account the intercept and the trend, the test statistic `tau3 = -5.39` is lower than the critical value `-3.43`, which means that at the significance level of 5% we reject the null hypothesis of non-stationarity of the series.

From the above tests it appears that the series is stationary after the first differentiation, $X_t \sim \mathsf{I}(1)$.

HEGY test for seasonal integration

$H_A : \pi_1 = 0$, y_t is non-stationary, non-seasonal

$H_B : \pi_2 = 0$, y_t is a non-stationary process with two-period seasonality

$H_C : \pi_3 = \pi_4 = 0$, y_t is a non-stationary process with four-period seasonality

```
Call:
lm(formula = y4[-1:-4] ~ y1_1 + y2_1 + y3_1 + y3_2 + y4_1 + y4_2 +
    y4_3 + y4_4)

Residuals:
   Min     1Q Median     3Q    Max
-79165  -1849    455   2689  19967

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.650e+03  1.602e+03   4.150 5.48e-05 ***
y1_1         2.719e-03  1.084e-03   2.508 0.013175 *
y2_1        -4.263e-01  1.231e-01  -3.463 0.000692 ***
y3_1         3.421e-02  8.478e-02   0.404 0.687117
y3_2         3.805e-02  9.730e-02   0.391 0.696260
y4_1         1.382e-01  9.639e-02   1.434 0.153684
y4_2         4.690e-01  1.805e-01   2.598 0.010287 *
y4_3        -1.457e-01  2.080e-01  -0.700 0.484847
y4_4        -4.995e-01  1.641e-01  -3.043 0.002755 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7853 on 154 degrees of freedom
Multiple R-squared:  0.5654,  Adjusted R-squared:  0.5428
F-statistic: 25.05 on 8 and 154 DF,  p-value: < 2.2e-16

Linear hypothesis test

Hypothesis:
y3_1 = 0
y3_2 = 0

Model 1: restricted model
Model 2: y4[-1:-4] ~ y1_1 + y2_1 + y3_1 + y3_2 + y4_1 + y4_2 + y4_3 +
    y4_4

  Res.Df        RSS Df Sum of Sq    F Pr(>F)
1    156 9516718018
2    154 9496982868  2  19735150 0.16 0.8523
```

Table 1. HEGY test critical values for quarterly data (Hylleberg et al. 1990).

	H_A: t-test $\pi_1 = 0$			H_B: t-test $\pi_2 = 0$			H_C: F-test $\pi_3 = \pi_4 = 0$		
n	1%	5%	10%	1%	5%	10%	1%	5%	10%
48	−3.66	−2.96	−2.62	−2.68	−1.95	−1.60	4.78	3.04	2.32
100	−3.47	−2.88	−2.58	−2.61	−1.95	−1.60	4.77	3.08	2.35
136	−3.51	−2.89	−2.58	−2.60	−1.91	−1.58	4.73	3.00	2.36
200	−3.48	−2.87	−2.57	−2.58	−1.92	−1.59	4.76	3.12	2.37

The t-test statistic for $y_1(-1)$ variable `2.51` is greater than the critical value `-2.89`, which means that at the significance level of 5% there are no grounds to reject the hypothesis of non-seasonal non-stationarity.

The t-test statistic for $y_2(-1)$ variable `-3.46` is lower than the critical value `-1.91`, which means that at the significance level of 5%, we reject the hypothesis of two-period seasonal non-stationarity.

The F-test statistic for the variables $y_3(-1)$ and $y_3(-2)$ `0.16` is lower than the critical value `3.00`, which means that at the significance level of 5% there are no grounds to reject the hypothesis of four-period seasonal non-stationarity.

Table 2. HEGY test hypothesis combinations (Hylleberg et al. 1990).

These hypotheses aren't rejected	These hypotheses are rejected	Stationary variable
H_A, H_B, H_C	–	$\Delta_4 y_t \ (= y_{4t})$
H_A, H_B	H_C	$\Delta_2 y_t \ (= y_{3t})$
H_A, H_C	H_B	$(I-L)(I+L^2)y_t \ (= y_{2t})$
H_B, H_C	H_A	$(I+L)(I+L^2)y_t \ (= y_{1t})$
H_A	H_B, H_C	$\Delta_1 y_t$
H_B	H_A, H_C	$(I+L)y_t$
H_C	H_A, H_B	$(I+L^2)y_t$
–	H_A, H_B, H_C	y_t

Based on the table above, it can be concluded that the series is characterized by a semi-annual stationarity.

Seasonal ARIMA model

Based on the tests performed above, it can be determined that the parameters `d` and `D` in the `SARIMA(p,d,q)(P,D,Q)` model will have the value of `1`. The optimal values of the remaining parameters are not known, therefore several models with parameter values from `0` to `2` will be estimated.

```
   p d q P D Q      AIC
M1 2 1 2 2 1 2 3385.053
M2 2 1 2 1 1 2 3383.471
M3 2 1 2 1 1 1 3381.501
M4 2 1 1 1 1 1 3383.346
M5 1 1 1 1 1 1 3382.138
M6 1 1 1 0 1 1 3380.691
M7 0 1 1 0 1 1 3383.170
```

From the above array it can be concluded that, based on the Akaike criterion, the best model is the `Model 6`.

```
Forecast method: ARIMA(1,1,1)(0,1,1)[4]

Model Information:
Series: dfts
ARIMA(1,1,1)(0,1,1)[4]

Coefficients:
         ar1      ma1     sma1
      0.5358  -0.8339  -0.8453
s.e.  0.2273   0.1716   0.0638

sigma^2 = 63163093:  log likelihood = -1686.35
AIC=3380.69   AICc=3380.95   BIC=3393.04

Error measures:
                   ME     RMSE      MAE       MPE     MAPE      MASE
Training set 677.2466 7754.825 3455.684 0.2782024 1.067464 0.2411356
                    ACF1
Training set -0.03223876

Forecasts:
        Point Forecast    Lo 95    Hi 95
2021 Q4       593756.3 578179.4 609333.1

Standard error:  7947.523
```

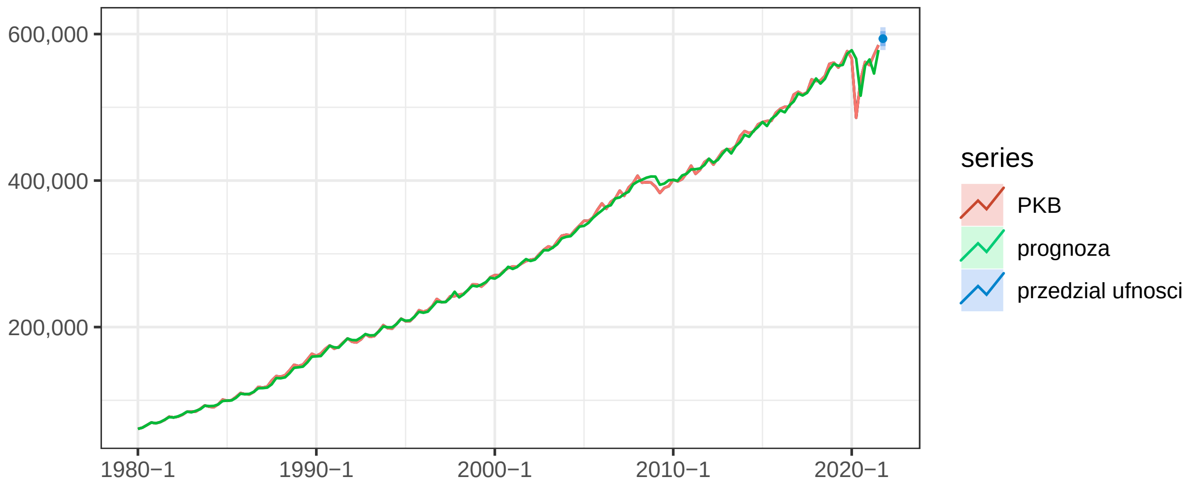
The GDP forecast for the fourth quarter of 2021 for the United Kingdom is £ `593756.3`m with an average estimate error of £ `7947.523`m, which is `1.3%` of the forecast value, which proves that the forecast is acceptable.

The range `<578179.4;609333.1>` has a 95% probability to cover the unknown United Kingdom GDP for the fourth quarter of 2021.

Based on the arithmetic mean of the forecast error, our forecasts were on average £ `677.247`m too low in relation to the real GDP value, which is on average `0.278%` of the forecasted variable, which proves that the forecast is not biased.

The value of the GDP forecast deviates from the true value by an average of £ `3455.684`m, with forecast errors averaging `1.07%` of the true value, which proves that the forecasts are acceptable.

Chart 2. United Kingdom GDP forecast

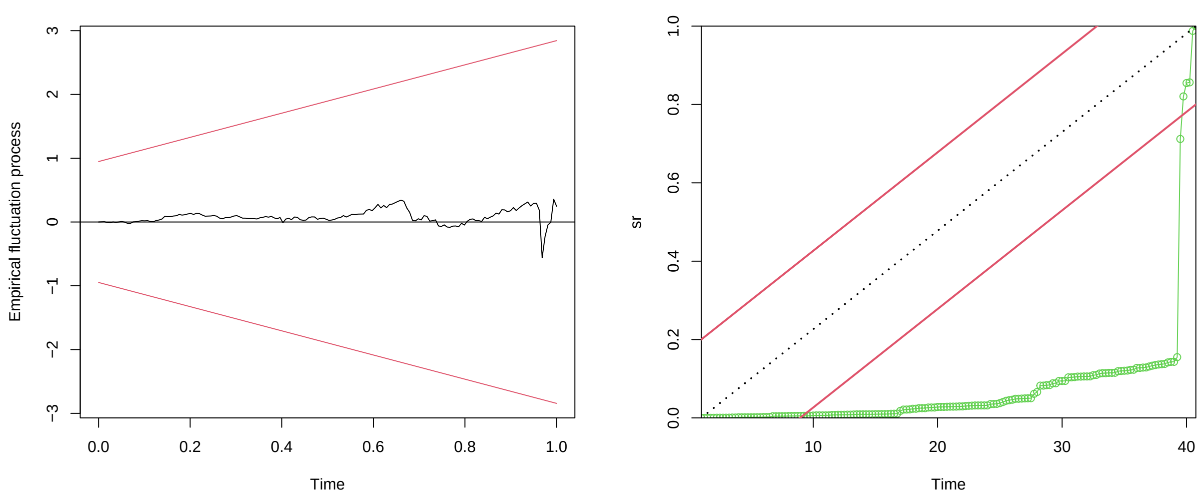


CUSUM stability test

From Chart 3, it can be concluded that the model is suitable for forecasting because the plot of the cumulative sums has not exceeded the confidence interval.

Based on Chart 4, it can be concluded that the model is characterized by an unstable inference precision.

Chart 3. CUSUM and Chart 4. CUSUM-SQ



Chow stability test

$H_0 : \beta_1 = \beta_2$

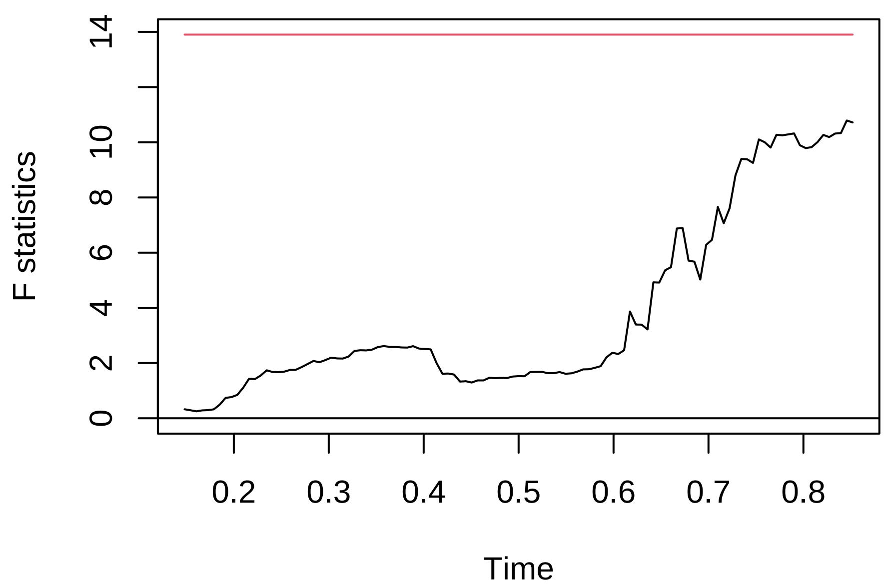
$H_0 : \beta_1 \neq \beta_2$

```
    Chow test

data:  d1s1gdp ~ ma_1 + ma_4
F = 0.50884, p-value = 0.6768
```

The `p-value` is greater than the significance level of 5% which means there is no structural break. The same conclusion can be drawn from the graph of the QLR test.

Chart 5. QLR test for structural break



Bibliography

Hylleberg, S. et al. (1990). "Seasonal integration and cointegration". In: *Journal of Econometrics* 44.1–2, pp. 215–238.