CS1675 Recitation #3
9/14/2018

Linear algebra review problems

The following two example problems are from http://web.pdx.edu/~erdman/LINALG/Linalg_pdf.pdf:

2.2. Exercises

(1) Let $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix}$.

(a) Does the matrix $D = ABC$ exist? ______ If so, then $d_{34} =$ ______ .
(b) Does the matrix $E = BAC$ exist? ______ If so, then $e_{22} =$ ______ .
(c) Does the matrix $F = BCA$ exist? ______ If so, then $f_{43} =$ ______ .
(d) Does the matrix $G = ACB$ exist? ______ If so, then $g_{31} =$ ______ .
(e) Does the matrix $H = CAB$ exist? ______ If so, then $h_{21} =$ ______ .
(f) Does the matrix $J = CBA$ exist? ______ If so, then $j_{13} =$ ______ .

See scanned page on Github.

(3) Let $A = \begin{bmatrix} 1 & 1/3 \\ c & d \end{bmatrix}$. Find numbers c and d such that $A^2 = -I$.

Answer: $c =$ ______ and $d =$ ______ .

See scanned page on Github.

Differentiation review problems

The following two example problems are from http://www.math.mcgill.ca/rags/JAC/dobson/diff.pdf. For all problems, find dy/dx. Note that $dy/dx\ y = ln(x) = 1/x$.

4. $y = \left(e^{x^2+2}\right)^2$

This is a set of three nested functions. The derivative can be expressed with recursive application of the chain rule: $dy/dx = dy/dg \cdot dg/df \cdot dh/dx$
where
$h(x) = x^2 + 2$
$g(h(x)) = e^{h(x)}$
$y(g(h(x))) = (g(h(x)))^2$

Substituting derivatives of the above:
$dy/dx = 2(e^{x^2+2}) \cdot (e^{x^2+2}) \cdot 2x = 4x(e^{x^2+2})^2$

35. $y = \ln \cos x$

This is a set of nested functions, and so chain rule applies again:
$dy/dx = dy/dg \cdot dg/dx$
where
$g(x) = cos(x)$
$y(g(x)) = ln(g(x))$

Substituting derivatives of the above:
$dy/dx = \frac{1}{cos(x)} \cdot -sin(x) = -\frac{sin(x)}{cos(x)} = -tan(x)$

40. $y = \frac{\sqrt{x}+1}{\sqrt{x}-1}$

Here we can use the product rule $uv' + u'v$, where:
$u = (\sqrt{x} + 1)$
$v = (\sqrt{x} - 1)^{-1}$

$u' = \frac{1}{2\sqrt{x}}$ v' by chain rule $= -1(\sqrt{x} - 1)^{-2}) \cdot \frac{1}{2\sqrt{x}}$

Substituting:
$(\sqrt{x} + 1)(-1(\sqrt{x} - 1)^{-2}) \cdot \frac{1}{2\sqrt{x}}) + ((\frac{1}{2\sqrt{x}})(\sqrt{x} - 1)^{-1}) =$
$\frac{-\sqrt{x}-1}{2(\sqrt{x}-1)^2(\sqrt{x})} + \frac{\sqrt{x}-1}{2\sqrt{x}(\sqrt{x}-1)^2} = \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$