

Differentiation problems from right side of slide 102 of 1675 intro slides

(1) $s(y) = 4ye^{2y}$

We can use the product rule: $uv' + u'v$

where $u = 4y$ and $v = e^{2y}$.

$u' = 4$.

For v' , we use the chain rule (derivative of $f(g(x)) = f'(g(x))g'(x)$) where $g(x) = 2y$:
 $v' = 2e^{2y}$

Substituting:

$$uv' + u'v = 4y2e^{2y} + 4e^{2y} = (8y + 4)e^{2y}$$

(2) $p(x) = \frac{\log(x^2)}{x}$

Rearranging: $p(x) = \log(x^2)x^{-1}$

Using product rule, where $u = \log(x^2)$, $v = x^{-1}$, $v' = -x^{-2}$
 u' (by chain rule, where $g(x) = x^2$) : $2x(1/x^2) = 2/x$

Substituting:

$$uv' + u'v = -\log(x^2)x^{-2} + 2/x * 1/x = \frac{-\log(x^2)+2}{x^2}$$

(3) $q(z) = (e^z - z)^3$

By the chain rule, where $g(x) = (e^z - z)$, and $g'(x) = (e^z - 1)$:

$$q'(z) = 3(e^z - z)^2(e^z - 1)$$

Linear algebra review problems

The following two example problems are from MIT Open Courseware, 18.06SC Linear Algebra, Fall 2011 (see ocw.mit.edu):

Problem 1.2: Multiply: $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$.

Solution:
$$\begin{bmatrix} 1 \cdot 3 + 2 \cdot (-2) + 0 \cdot 1 \\ 6 + 0 + 3 \\ 12 - 2 + 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 11 \end{bmatrix}.$$

Problem 1.3: True or false: A 3 by 2 matrix A times a 2 by 3 matrix B equals a 3 by 3 matrix AB . If this is false, write a similar sentence which is correct.

Solution: The statement is true. In order to multiply two matrices, the number of columns of A must equal the number of rows of B . The product AB will have the same number of rows as the first matrix and the same number of columns as the second:

$$A(m \text{ by } n) \text{ times } B(n \text{ by } p) \text{ equals } AB(m \text{ by } p).$$

Matrix Properties question:

The following are possible statements regarding the properties of matrix multiplication in general. Which of the following properties is not guaranteed to hold?

$$A(B * C) = (A * B)C$$

$$A(B + C) = A * B + A * C$$

$$A * B = B * A$$

Solution:

$A * B = B * A$ is false. The commutative property does not hold for matrix multiplication. Even if $B * A$ is defined (i.e., inner dimensions match), the contents of an individual cell in $A * B$ may differ from the same cell in $B * A$ (e.g., $\text{dot}(A(1,:), B(:,1))$ does not necessarily equal $\text{dot}(B(1,:), A(:,1))$).