

Differentiation Formulas

$$\begin{aligned}\frac{d}{dx} k &= 0 \\ \frac{d}{dx} [f(x) \pm g(x)] &= f'(x) \pm g'(x) \\ \frac{d}{dx} [k \cdot f(x)] &= k \cdot f'(x) \\ \frac{d}{dx} [f(x)g(x)] &= f(x)g'(x) + g(x)f'(x) \\ \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) &= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \\ \frac{d}{dx} f(g(x)) &= f'(g(x)) \cdot g'(x) \\ \frac{d}{dx} x^n &= nx^{n-1} \\ \frac{d}{dx} \sin x &= \cos x \\ \frac{d}{dx} \cos x &= -\sin x \\ \frac{d}{dx} \tan x &= \sec^2 x \\ \frac{d}{dx} \cot x &= -\csc^2 x \\ \frac{d}{dx} \sec x &= \sec x \tan x \\ \frac{d}{dx} \csc x &= -\csc x \cot x \\ \frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} a^x &= a^x \ln a \\ \frac{d}{dx} \ln |x| &= \frac{1}{x} \\ \frac{d}{dx} \sin^{-1} x &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \cos^{-1} x &= \frac{-1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan^{-1} x &= \frac{1}{x^2+1} \\ \frac{d}{dx} \cot^{-1} x &= \frac{-1}{x^2+1} \\ \frac{d}{dx} \sec^{-1} x &= \frac{1}{|x|\sqrt{x^2-1}} \\ \frac{d}{dx} \csc^{-1} x &= \frac{-1}{|x|\sqrt{x^2-1}}\end{aligned}$$

Integration Formulas

$$\begin{aligned}\int dx &= x + C \\ \int x^n dx &= \frac{x^{n+1}}{n+1} + C \\ \int \frac{dx}{x} &= \ln |x| + C \\ \int e^x dx &= e^x + C \\ \int a^x dx &= \frac{1}{\ln a} a^x + C \\ \int \ln x dx &= x \ln x - x + C \\ \int \sin x dx &= -\cos x + C \\ \int \cos x dx &= \sin x + C \\ \int \tan x dx &= -\ln |\cos x| + C \\ \int \cot x dx &= \ln |\sin x| + C \\ \int \sec x dx &= \ln |\sec x + \tan x| + C \\ \int \csc x dx &= -\ln |\csc x + \cot x| + C \\ \int \sec^2 x dx &= \tan x + C \\ \int \csc^2 x dx &= -\cot x + C \\ \int \sec x \tan x dx &= \sec x + C \\ \int \csc x \cot x dx &= -\csc x + C \\ \int \frac{dx}{\sqrt{a^2 - x^2}} &= \sin^{-1} \frac{x}{a} + C \\ \int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \tan^{-1} \frac{x}{a} + C \\ \int \frac{dx}{x\sqrt{x^2 - a^2}} &= \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C\end{aligned}$$

**TRIGONOMETRIC IDENTITIES**

• **Reciprocal identities**

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} \\ \tan u &= \frac{1}{\cot u} & \cot u &= \frac{1}{\tan u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u}\end{aligned}$$

• **Pythagorean Identities**

$$\begin{aligned}\sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u \\ 1 + \cot^2 u &= \csc^2 u\end{aligned}$$

• **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

• **Co-Function Identities**

$$\begin{aligned}\sin\left(\frac{\pi}{2} - u\right) &= \cos u & \cos\left(\frac{\pi}{2} - u\right) &= \sin u \\ \tan\left(\frac{\pi}{2} - u\right) &= \cot u & \cot\left(\frac{\pi}{2} - u\right) &= \tan u \\ \csc\left(\frac{\pi}{2} - u\right) &= \sec u & \sec\left(\frac{\pi}{2} - u\right) &= \csc u\end{aligned}$$

• **Parity Identities (Even & Odd)**

$$\begin{aligned}\sin(-u) &= -\sin u & \cos(-u) &= \cos u \\ \tan(-u) &= -\tan u & \cot(-u) &= -\cot u \\ \csc(-u) &= -\csc u & \sec(-u) &= \sec u\end{aligned}$$

• **Sum & Difference Formulas**

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

• **Double Angle Formulas**

$$\begin{aligned}\sin(2u) &= 2 \sin u \cos u \\ \cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u \\ \tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

• **Power-Reducing/Half Angle Formulas**

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos(2u)}{2} \\ \cos^2 u &= \frac{1 + \cos(2u)}{2} \\ \tan^2 u &= \frac{1 - \cos(2u)}{1 + \cos(2u)}\end{aligned}$$

• **Sum-to-Product Formulas**

$$\begin{aligned}\sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)\end{aligned}$$

• **Product-to-Sum Formulas**

$$\begin{aligned}\sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)]\end{aligned}$$

## Hyperbolic and inverse functions

<u><b><i>Identities for Hyperbolic Function</i></b></u>	<u><b><i>The Six Hyperbolic Function</i></b></u>
$\sinh 2x = 2 \sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $\cosh^2 x = \frac{\cosh 2x + 1}{2}$ $\sinh^2 x = \frac{\cosh 2x - 1}{2}$ $\cosh^2 x - \sinh^2 x = 1$ $\tanh^2 x = 1 - \operatorname{sech}^2 x$ $\coth^2 x = 1 + \operatorname{csch}^2 x$	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$ $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$ $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
<u><b><i>Expressing Inverse Hyperbolic Functions As Natural Logarithms</i></b></u>	
$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$ $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$ $\operatorname{sech}^{-1} x = \ln \left( \frac{1 + \sqrt{1 - x^2}}{x} \right)$ $\operatorname{csch}^{-1} x = \ln \left( \frac{1}{x} + \frac{\sqrt{1 + x^2}}{ x } \right)$ $\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$	$(-\infty < x < \infty)$ $(x \geq 1)$ $( x  < 1)$ $(0 < x < 1)$ $(x \neq 0)$ $( x  > 1)$