$$B_{(m,n)} = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx$$

let $x = \frac{y}{y+1}$

$$dx = \frac{dy}{(1+y)^2}$$

at $x = 0 \Rightarrow y = 0$

at $x = 1 \Rightarrow y = \infty$

$$B_{(m,n)} = \int_0^\infty \left(\frac{y}{1+y}\right)^{m-1} * \left[\frac{1+y}{1+y} - \frac{y}{1+y}\right]^{n-1} * \frac{dy}{(1+y)^2}$$

$$= \int_0^\infty \left(\frac{y}{1+y}\right)^{m-1} * \left(\frac{1}{1+y}\right)^{n-1} * \frac{dy}{(1+y)^2}$$

$$= \int_0^\infty y^{m-1} * \left(\frac{1}{1+y}\right)^{m\cancel{-1}} * \left(\frac{1}{1+y}\right)^{n\cancel{-1}} * \cancel{\left(\frac{1}{1+y}\right)^{2}} \, dy$$

$$= \int_0^\infty y^{m-1} * \left(\frac{1}{1+y}\right)^{m+n} dy$$

$$B_{(m,n)} = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}} \, dy$$

$$= \int_0^\infty \frac{y^{m-1}}{(1+y)^{m-1}} * \frac{1}{(1+y)^{n+1}} \, dy$$

$$= \int_0^\infty \left(\frac{y}{1+y}\right)^{m-1} * \frac{1}{(1+y)^{n-1}} * \frac{1}{(1+y)^2} \, dy$$

let $\frac{y}{1+y} = x \Rightarrow y = \frac{x}{1-x}$

$$\frac{(1+y)\,dy - y\,dy}{(1+y)^2} = dx$$

$$\frac{dy}{(1+y)^2} = dx$$

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$\cancel{\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx}$$

$\sin^{2m-1} \times \cos^{2n-1}$

let $X = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\, d\theta$

at $X = 0 \Rightarrow \theta = 0$

$X = 1 \Rightarrow \theta = 90^\circ = \pi/2$

$\because \sin^2\theta + \cos^2\theta = 1$

$X + \cos^2\theta = 1$

$\cos^2\theta = (1-X)$

$$\beta_{(m,n)} = \int_0^{\pi/2} \sin^{2m-2}\theta * \cos^{2n-2}\theta * (2\sin\theta\cos\theta)\,d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta * \cos^{2n-1}\,d\theta$$

$$B_{(m,n)} = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx \qquad \cancel{(a-x) \Rightarrow (1-x/a)}$$

~~let~~ $\cancel{x/a = y \rightarrow dx = a\,dy}$

let $x = y/a \Rightarrow dx = dy/a$

at $x = 0 \qquad y = 0$

at $x = 1 \qquad y = a \qquad 1 - x = 1 - y/a \Rightarrow \frac{a-y}{a}$

$$B(m,n) = \int_0^a \left(y/a\right)^{m-1} * \left(\frac{a-y}{a}\right)^{n-1} dy/a$$

$$= \int_0^a \frac{1}{a^{m-1}} \times \frac{1}{a^{n-1}} * \frac{1}{a} * y^{m-1} * (a-y)^{n-1} \, dy$$

$$= \frac{1}{a^{m+n-1}} * \int_0^a y^{m-1} * (a-y)^{n-1} \, dy$$