DataSci 420

lesson 7: support vector machines

Seth Mottaghinejad

today's agenda

- SVM **pros and cons**
- linear separability and **wide-margin classifiers**
- non-linear separability
- the **kernel trick**
- **soft-margin classifiers**
- **multi-class classification** with SVMs
- **cross validation** for **hyper-parameter tuning**

SVMs in a nutshell

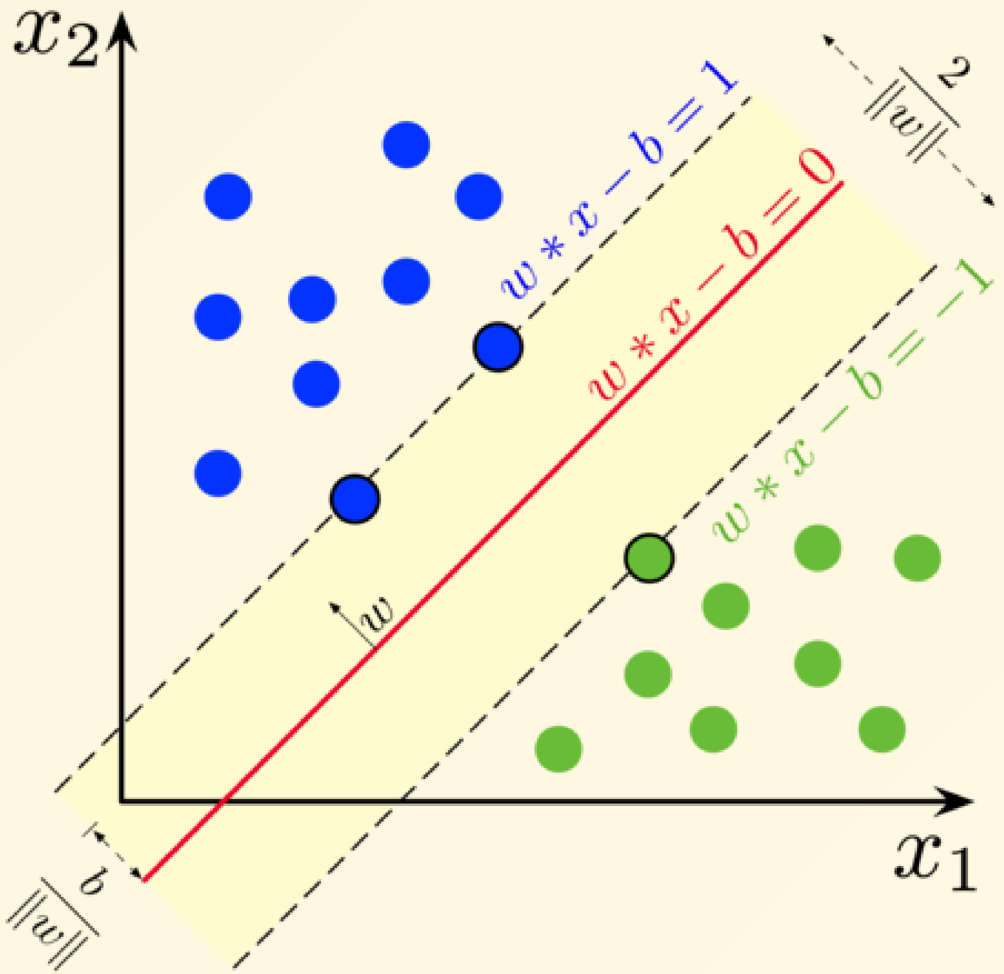
SVMs give us models that capture very **complex relationships** without running the risk of over-fitting:

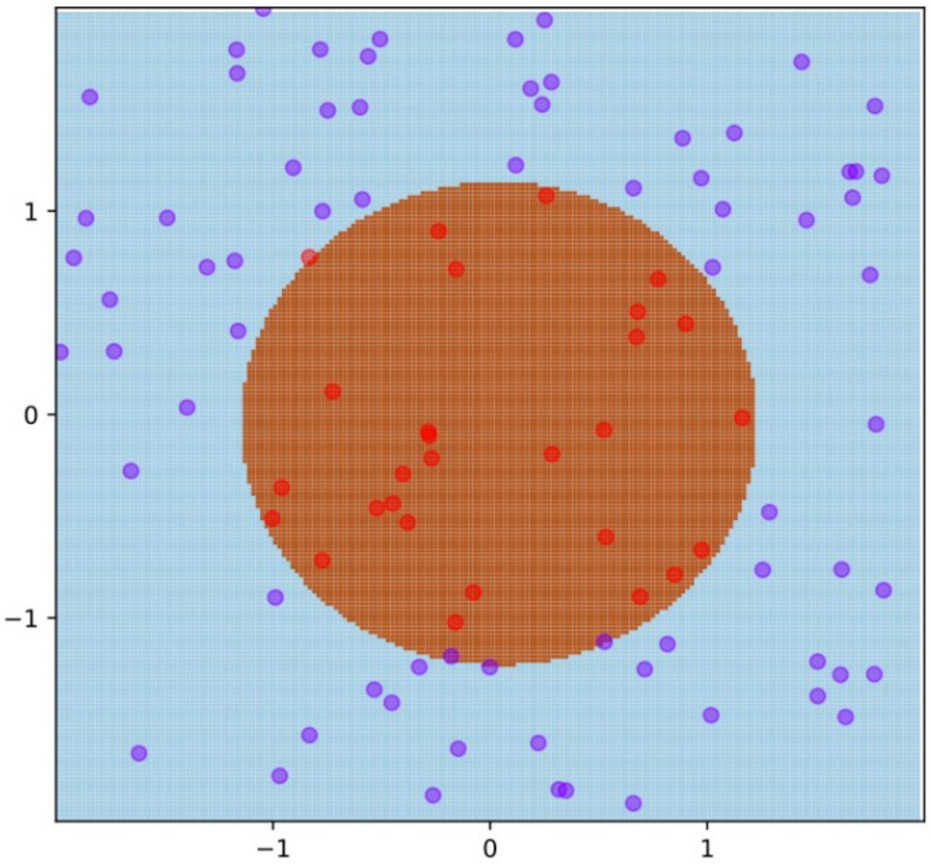
- world-class until the advent of deep learning
- they can run through **a lot of compute**, although the **kernel trick** makes the computation much more efficient
- less affected by outliers (because the separation boundary only depends on the **support vectors**)
- not great for **multi-class** classfication: **one-vs-one**, **one-vs-all**

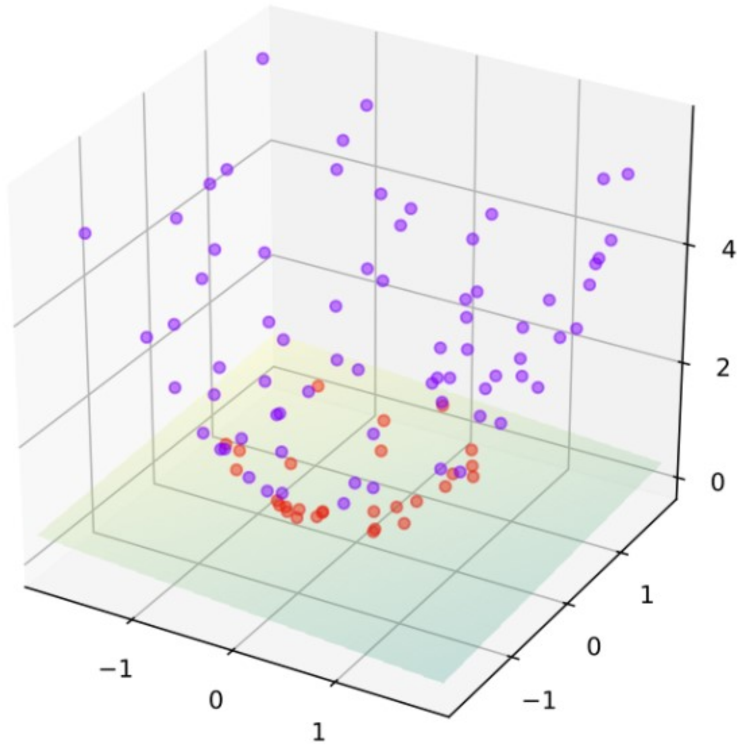
SVM classifier

- there are many lines that offer **linear separability**
- the one that maximizes the **margin** is the best one
- SVM are called **wide-margin classifires**
- the model is explained by its **support vectors**

source: [Wikipedia]()







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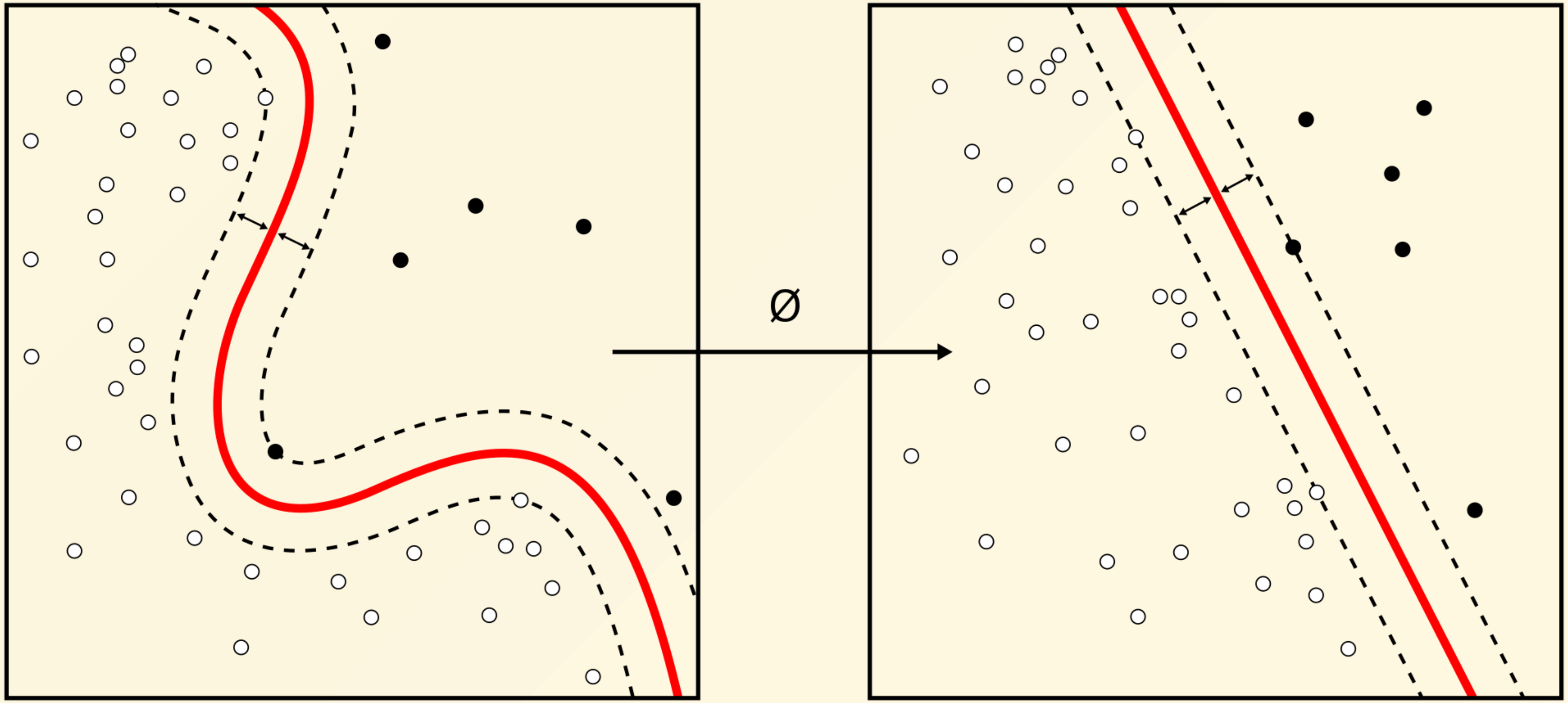


source: [Wikipedia]

SVM motivation

- if data is **linearly separable** (by a hyper-plane), then a **wide-margin classifier** is a better classifier
- when data is not linearly separable, project it to a **higher dimension** ($\phi : X \to Z$) in which the labels are linearly separable
- in Z space, **decision boundary** is linear, pinned down only by a few data points called **support vectors**
- the **pre-image** of decision boundary in X space can look complex, but it's the pre-image of a hyper-plane in Z space

break time

the kernel trick

- the math for SVMs can be challenging: linear algebra including some abstract concepts
- the prediction equation is $g(\mathbf{x}) = \operatorname{sign}(\mathbf{w}^T\mathbf{z} + b)$
- we need to calculate $\mathbf{z}_n^T\mathbf{z}_m$ to find a solution
- with the kernel trick we can do it **without explicitly finding the mappings $\mathbf{x}_i \mapsto \mathbf{z}_i$**
- instead we use the kernel K: $\mathbf{z}_n^T\mathbf{z}_m = K(\mathbf{x}_n, \mathbf{x}_m)$

types of kernels

depending on the choice, kernels introduce new hyper-parameters (such as γ and d)

- **linear** $K(\mathbf{x}_n, \mathbf{x}_m) = \mathbf{x}_n^T \mathbf{x}_m$
- **polynomial** $K(\mathbf{x}_n, \mathbf{x}_m) = (\gamma \mathbf{x}_n^T \mathbf{x}_m + r)^d$ where $\gamma > 0$
- **radial basis function** $K(\mathbf{x}_n, \mathbf{x}_m) = \exp(-\gamma||\mathbf{x}_n - \mathbf{x}_m||^2)$ where $\gamma > 0$ (which corresponds to an infinite dimensional Z space if we look at its Taylor series expansion)

soft-margin classifiers

- **hard-margin classifiers** expect perfect separability, but we can add a **slack variable** and get a **soft-margin classifier**
- when the data is not linearly separable, we can adjust the trade-off between margin width and the classification error using the C **hyper-parameter**
- C is the penalty on data points that are on the wrong side of the decision boundary:
  - higer C means more **tolerance** for misclassification

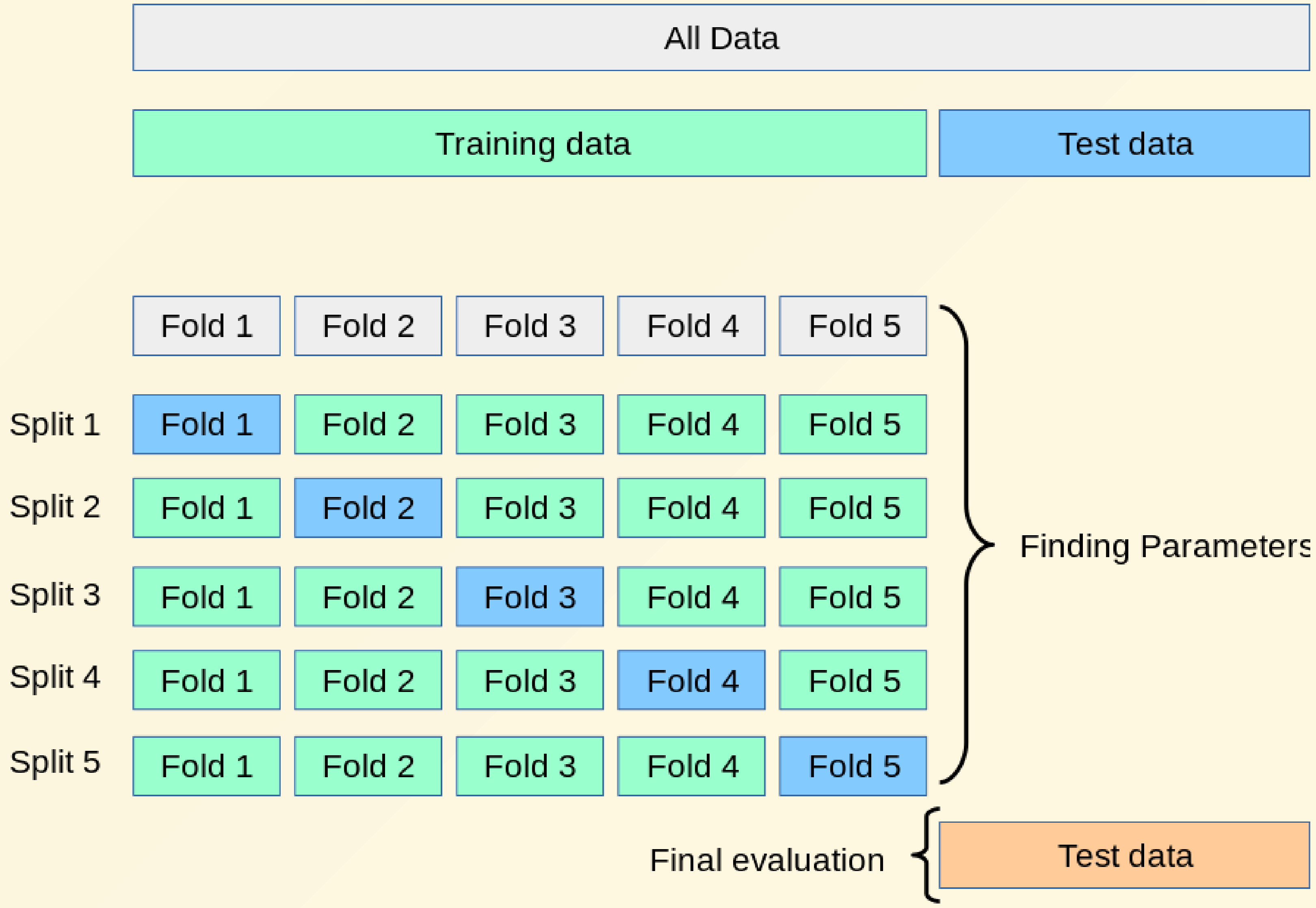
multi-class classification

- let k be the number of classes
- SVMs can only give us binary classifiers but we can still use them to do multi-class classification:
  - **one vs one:** builds $\binom{k}{2}$ classifiers
  - **one vs rest:** (also called. **one vs all**), builds k classifers
- unlike SVMs, neural networks can train multi-class classifiers with a single instance of training
  - logistic regression is like a NN too and can do the same

[break time]()

hyper-parameter tuning

- we can do a **three-way split**:
    - **training data** is for **learning**, **validation data** is for **model selection**, **test data** is for **evaluating final model**
- we can do a **two-way split** and **cross validation**:
    - training data is divided into k folds:
        - $k-1$ folds are for learning, and the kth fold for validation
        - we repeat this k times, one for each fold
    - test data is for **evaluating final model**



source: www.scikit-learn.org

the end