

MARCH 2023



The University of Texas at Austin

GSURE Denoising enables training of higher quality generative priors for accelerated Multi-Coil MRI Reconstruction

Asad Aali¹, Marius Arvinte^{1,2}, Sidharth Kumar¹, Yamin Ishraq Arefeen¹, and Jonathan I. Tamir¹

¹*Chandra Family Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX, United States*, ²*Intel Corporation, Hillsboro, OR, United States*

Asad Aali
PhD Student
UT CSI Lab
ECE, UT Austin



UT Computational Sensing and Imaging Lab

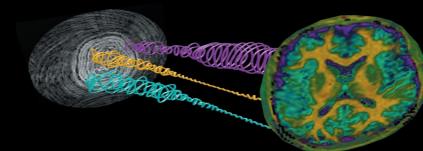
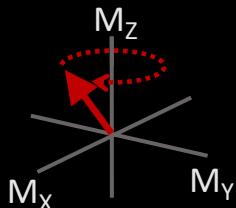
- Joint design of imaging system and software
- Particular focus on application to MRI
- Work with clinicians to translate work to hospital



Jon Tamir, PhD
Assistant Professor, ECE, UT Austin
<http://www.jtsense.com/> <https://github.com/utcsilab>

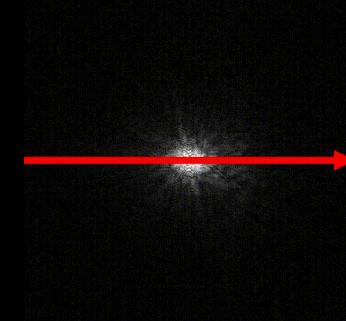
Computational MRI

Imaging physics

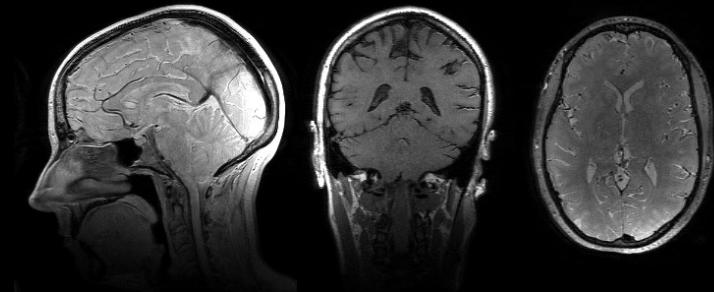


Prior knowledge

Acquisition



Reconstruction



Deep learning inversion for MRI

1. End-to-end supervised training
2. Distribution learning / generative modeling



Generative models are powerful image generators

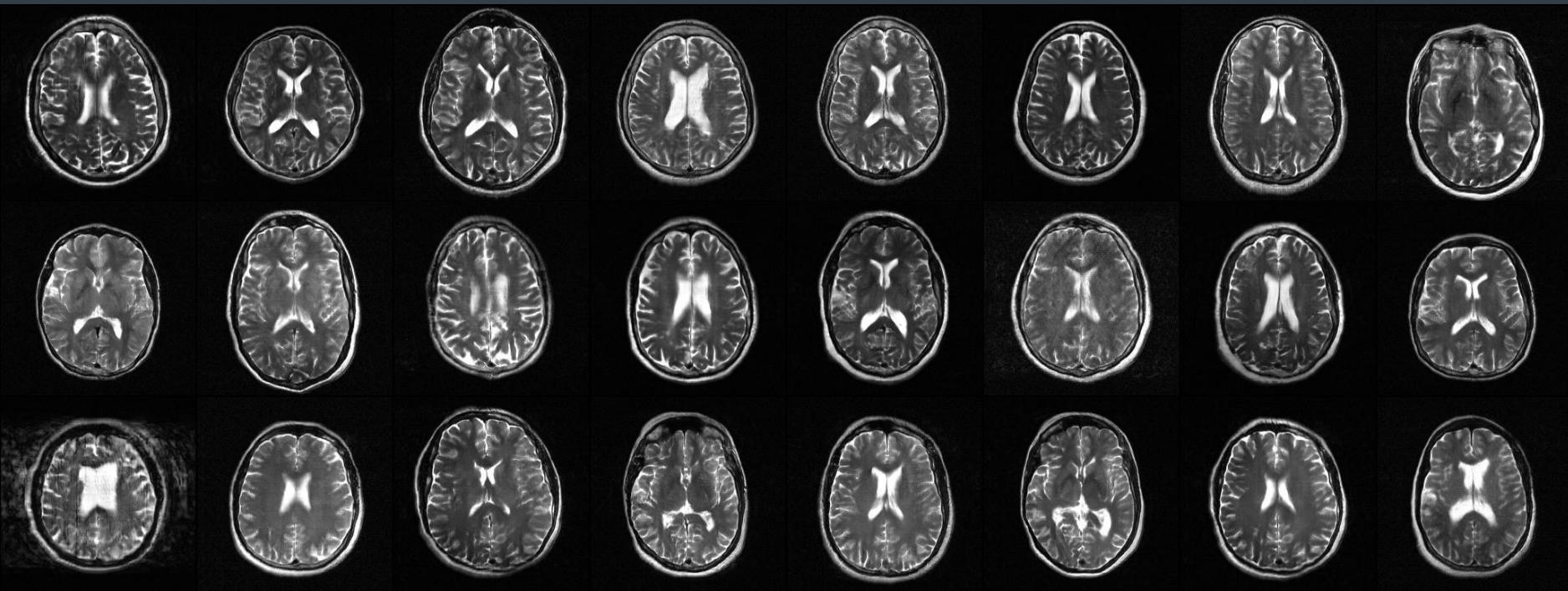


Generative models are powerful image generators



<https://thiscatdoesnotexist.com/>

Generative models are powerful image generators

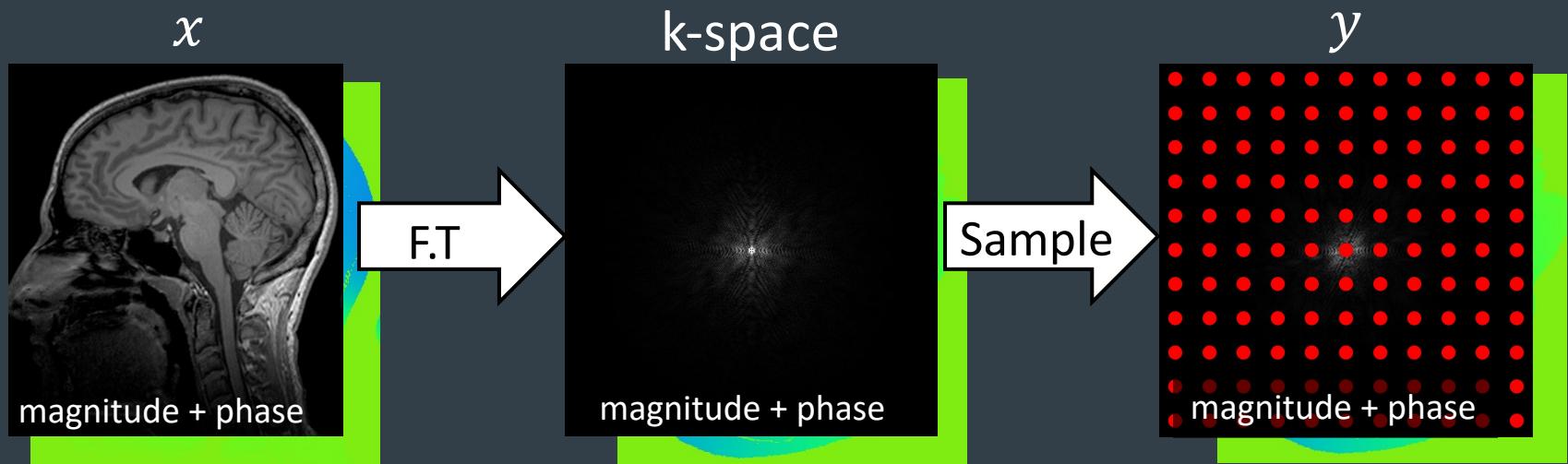


Generative model trained on FastMRI data

MRI: Problem Formulation

Signal is the Fourier transform of the image

$$y = Ax + \text{noise}$$



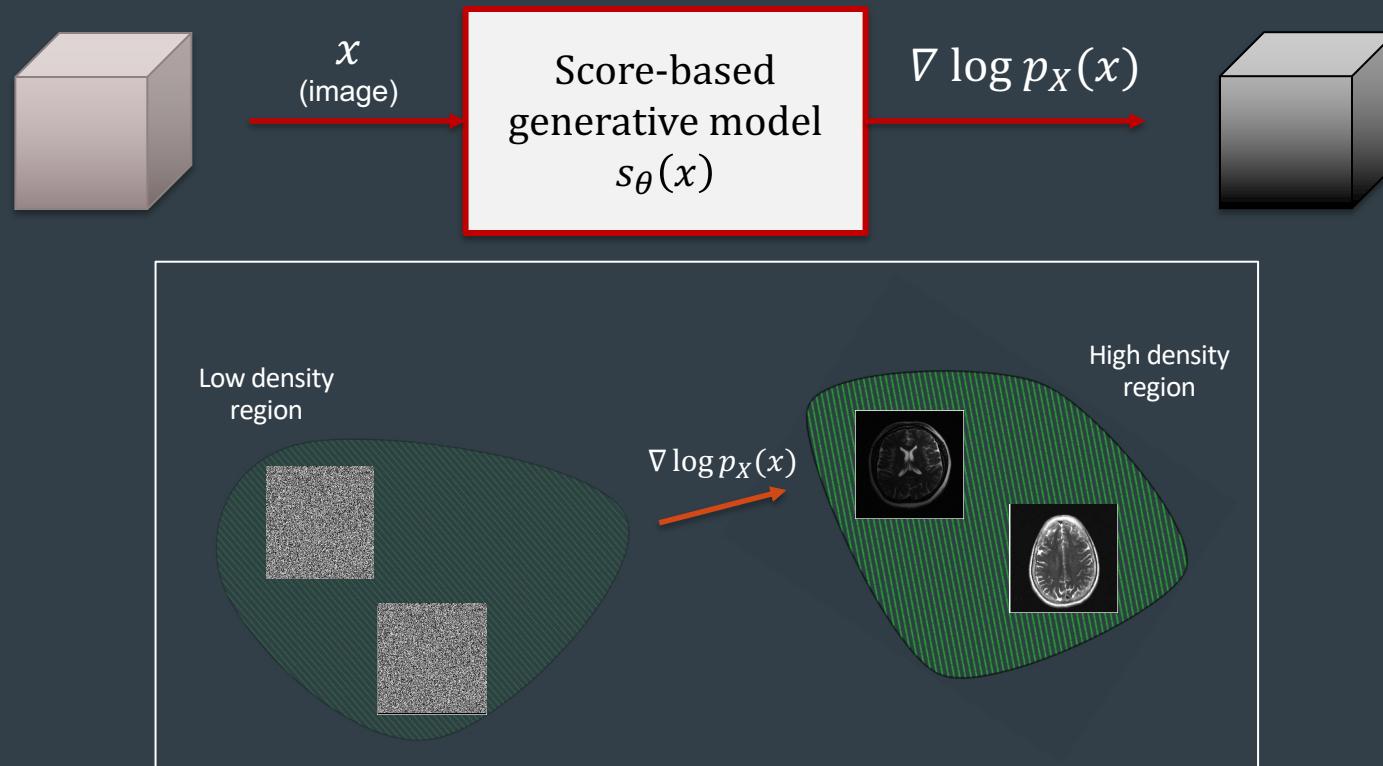
Back to basics: statistical interpretation

$$y = Ax + n$$

Goal: estimate the image from noisy measurements

- **Likelihood function:** described by the imaging physics $p(y|x)$
- **Prior knowledge:** probability distribution for the image: $p(x)$
- MAP estimator: maximize the posterior $\max_x p(x|y)$
- Another option: **sample from the posterior** $x \sim p(x|y)$
- Another option: **conditional expectation** $E [x|y]$

Score-based generative models

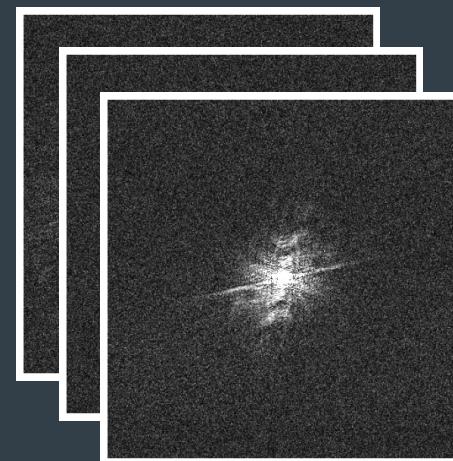


MRI Samples are inherently noisy

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

y

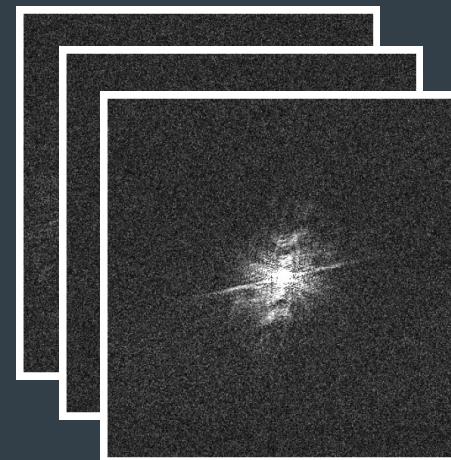


Original K-Space

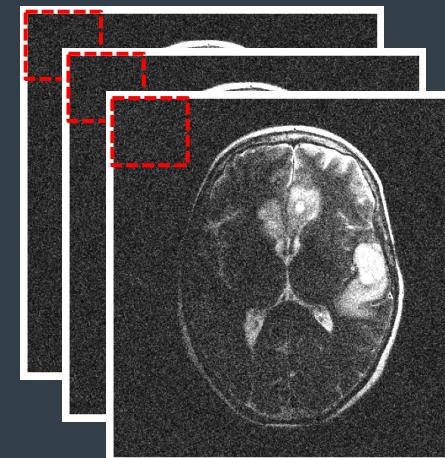
MRI Samples are inherently noisy

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

 y  $F^H y$ 

Original K-Space

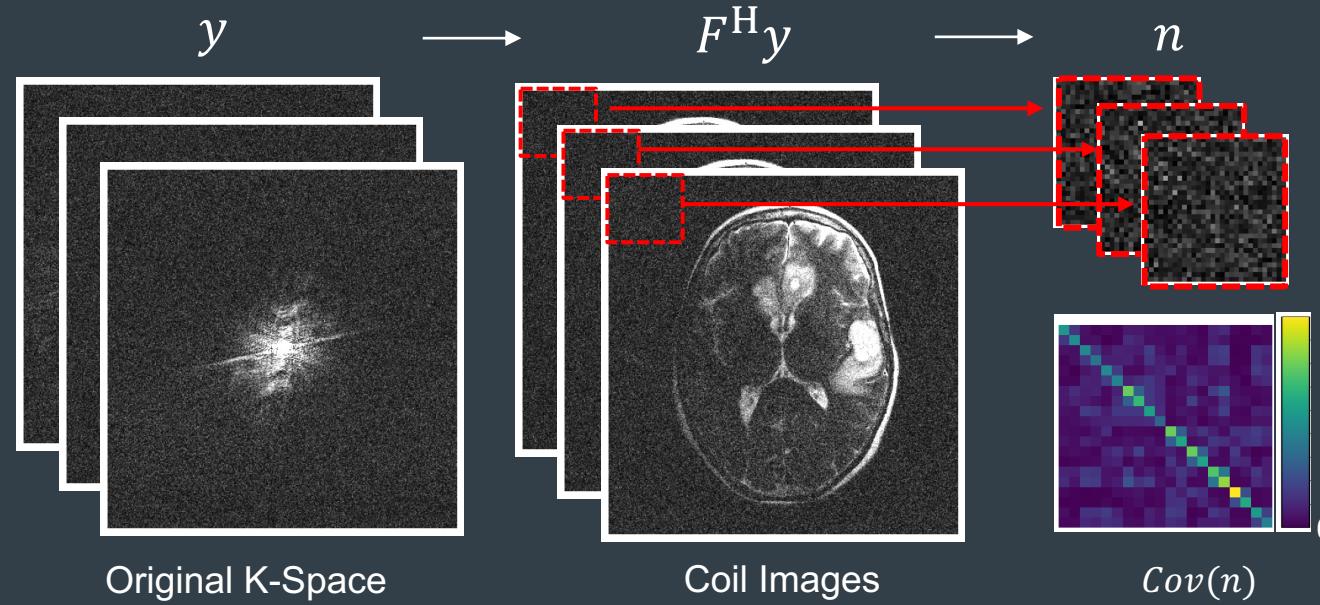


Coil Images

MRI Samples are inherently noisy

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

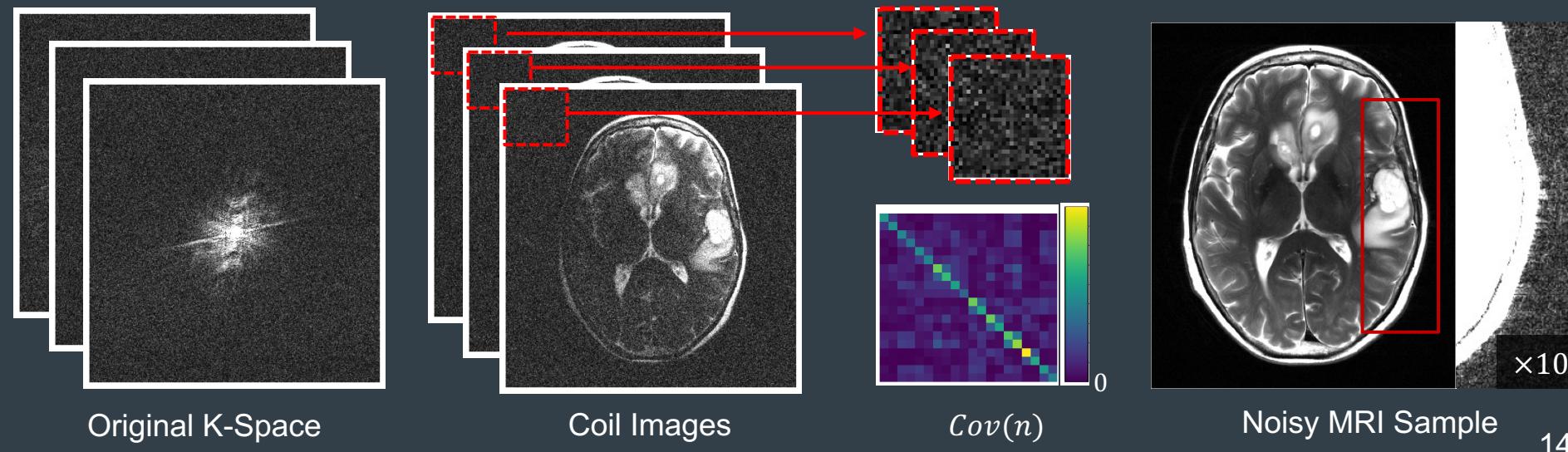


MRI Samples are inherently noisy

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

$$y \longrightarrow F^H y \longrightarrow n \longrightarrow \hat{x}_{\text{noisy}} = A^H y^*$$



Learning from Noisy Data

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

- Case 1: Assume A is Identity -> **SURE-Score**

$$y = x + \text{noise}$$

Learning from Noisy Data

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

- Case 1: Assume A is Identity -> **SURE-Score**

$$y = x + \text{noise}$$

- Case 2: Assume A is a Linear Forward Operator (Fully-Sampled) -> **GSURE-Score**

$$y = FSx + \text{noise}$$

Learning from Noisy Data

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + \text{noise}$$

- Case 1: Assume A is Identity -> **SURE-Score**

$$y = x + \text{noise}$$

- Case 2: Assume A is a Linear Forward Operator (Fully-Sampled) -> **GSURE-Score**

$$y = FSx + \text{noise}$$

- Case 3: Assume A is a Linear Forward Operator (Under-Sampled) -> **GSURE + Ambient Diffusion ???**

$$y = PFSx + \text{noise}$$

*In ambient diffusion, we assume kspace is noise-free.

Case 1: Denoising with SURE $\rightarrow y = x + n$

$$\mathbb{E}_w \left[\frac{1}{n} \|x - f_\theta(y)\|_2^2 \right] = \mathbb{E}_w \left[\frac{1}{n} \|y - f_\theta(y)\|_2^2 \right] - \sigma_w^2 + \frac{2\sigma_w^2}{n} \operatorname{div}_y(f_\theta(y))$$

Does not depend on the clean sample x

- We can train a denoiser that removes Gaussian noise at a single noise level σ_w^2 using the loss [1]:

$$\mathcal{L}(\theta) = \mathbb{E}_{x,w} [\|y - f_\theta(y)\|_2^2 + 2\sigma_w^2 \operatorname{div}_y(f_\theta(y))]$$

Case 2: Denoising with Generalized-SURE

$$y = FSx + n$$

Generalized-SURE Equation

- An unbiased estimate of the MSE

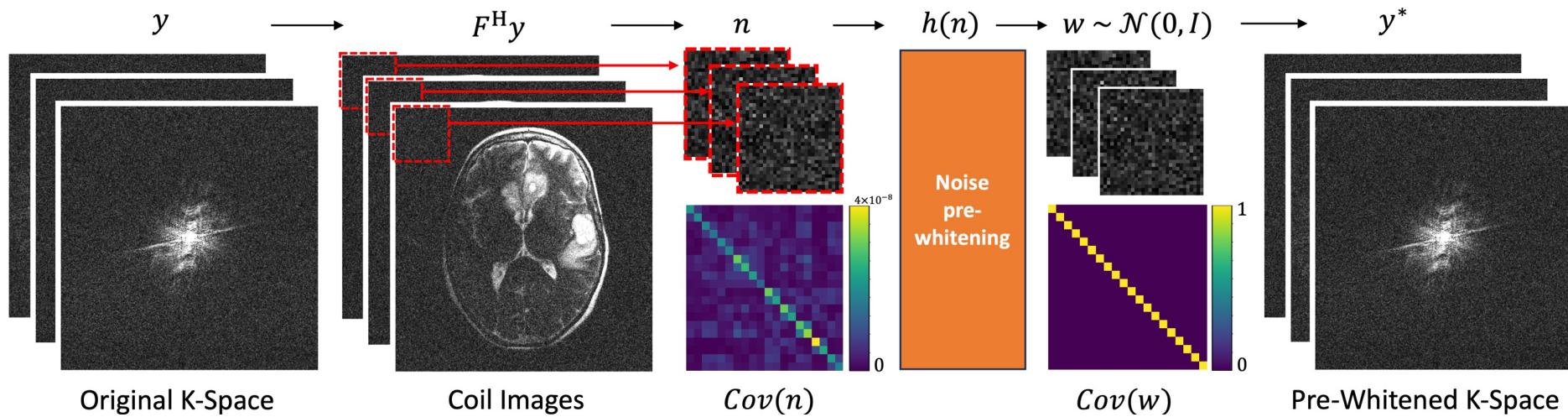
$$s(g) = \|x\|^2 + \|g_\phi(u)\|^2 + 2 \left(\text{Tr} \left(\frac{\partial g_\phi(u)}{\partial u} \right) + g_\phi^T(u) \frac{\partial \ln q(u)}{\partial u} \right)$$

Where:

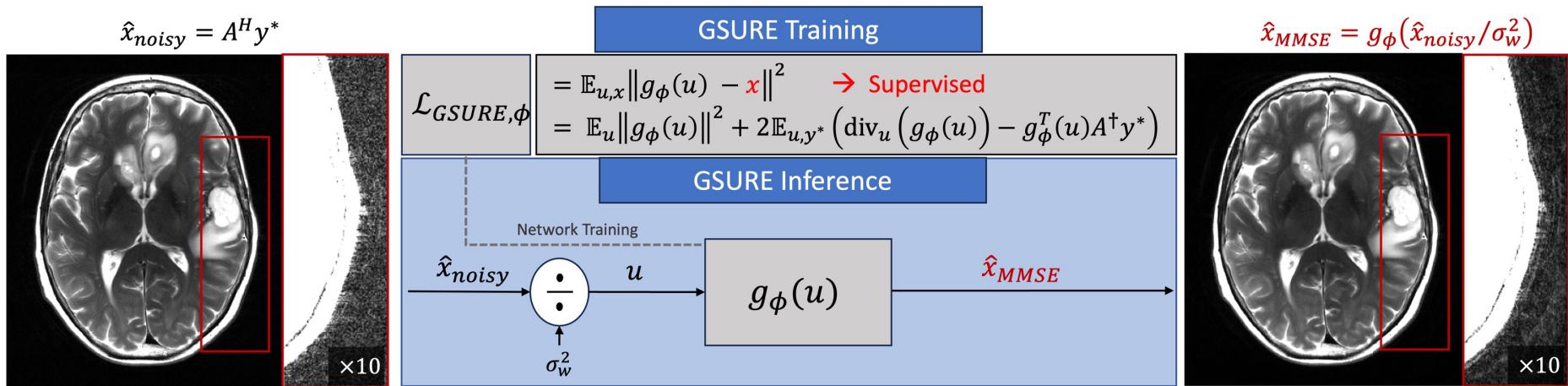
- g_ϕ is the denoiser network
- $u = \frac{A^H y}{\sigma_w^2}$
- $\frac{\partial \ln q(u)}{\partial u} = -A^+ y$
- In the i.i.d. Gaussian case, this equation becomes SURE (previously shown)

$$L_{SURE} = \|x\|^2 + 2\sigma_w^2 \frac{\partial g_\phi(y)}{\partial y} - 2g_\phi^T(y)y$$

FastMRI – Noise Pre-Whitening



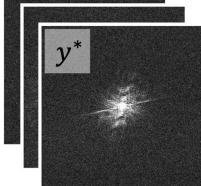
GSURE Training



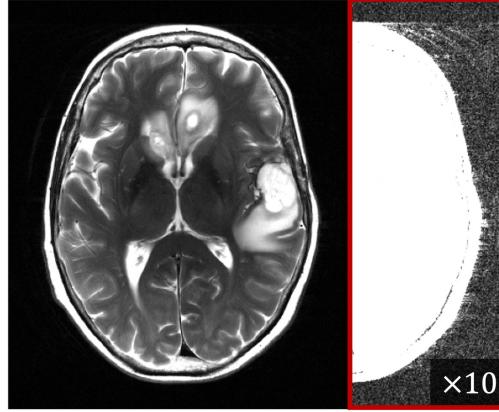
GSURE Denoising – T2 Brain Scans

Original FastMRI

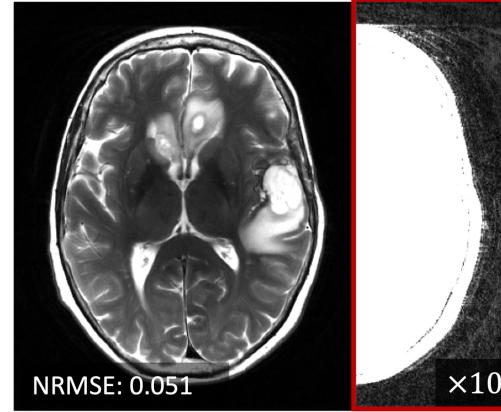
$$SNR_w = 36 \text{ dB}$$



$$\hat{x}_{noisy}$$



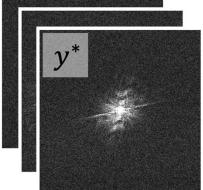
$$\hat{x}_{G \text{ SURE}}$$



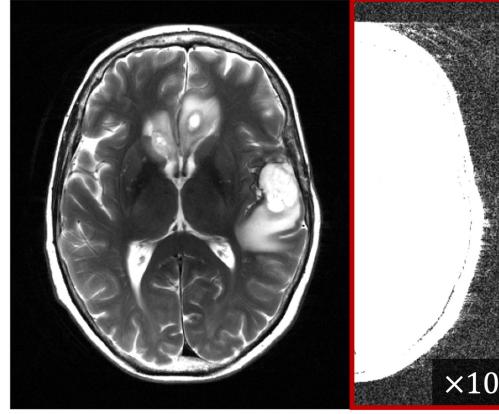
GSURE Denoising – T2 Brain Scans

Original FastMRI

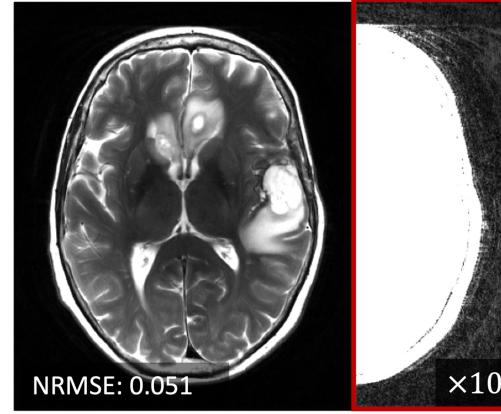
$$SNR_w = 36 \text{ dB}$$



$$\hat{x}_{noisy}$$



$$\hat{x}_{G \text{ SURE}}$$

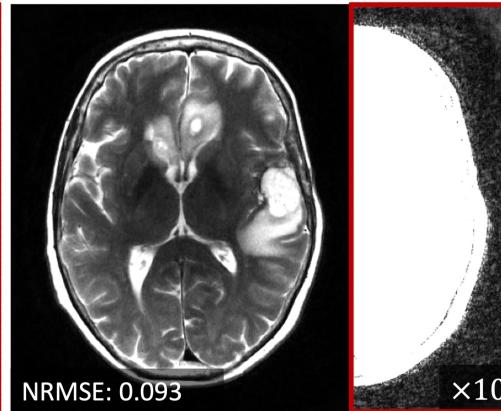
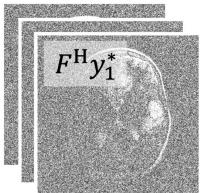
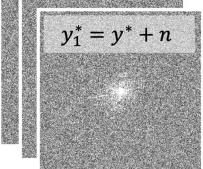


Original FastMRI

+

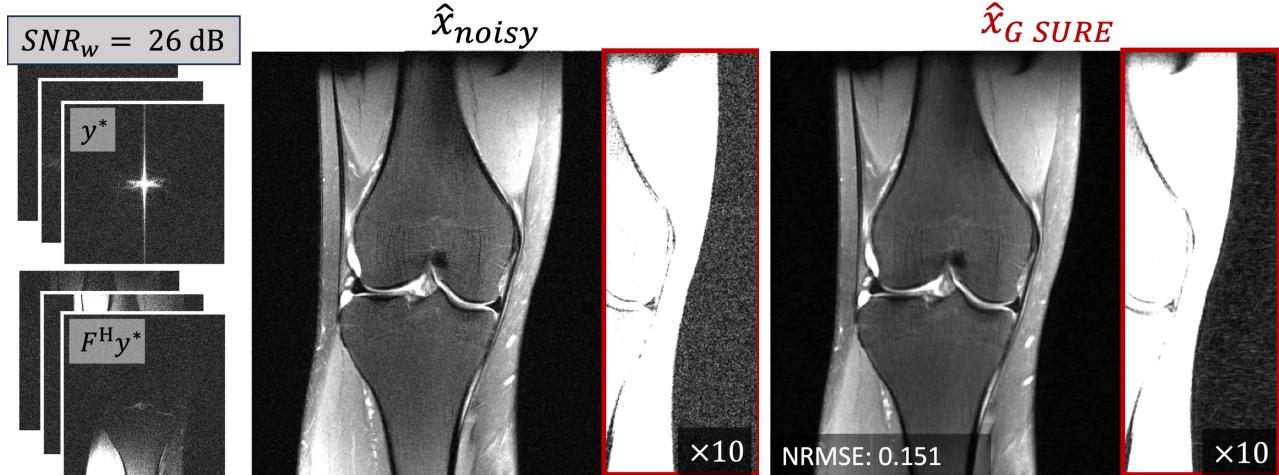
Additive Gaussian Noise

$$SNR_w = 26 \text{ dB}$$



GSURE Denoising – Knee Scans

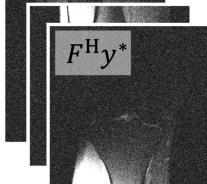
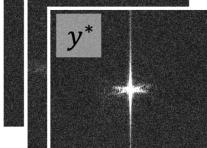
Original FastMRI



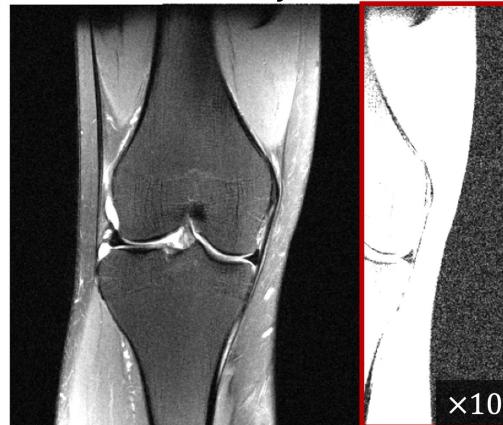
GSURE Denoising – Knee Scans

Original FastMRI

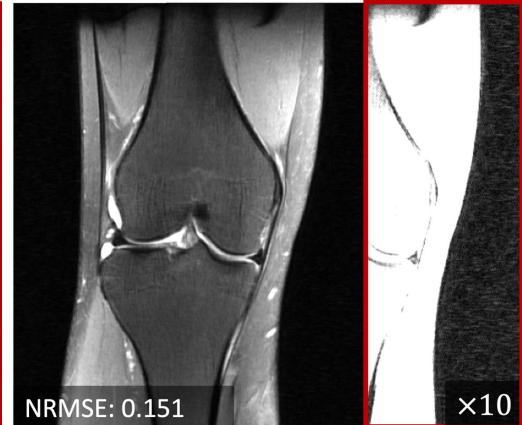
$$SNR_w = 26 \text{ dB}$$



$$\hat{x}_{noisy}$$



$$\hat{x}_{G \text{ SURE}}$$



NRMSE: 0.151

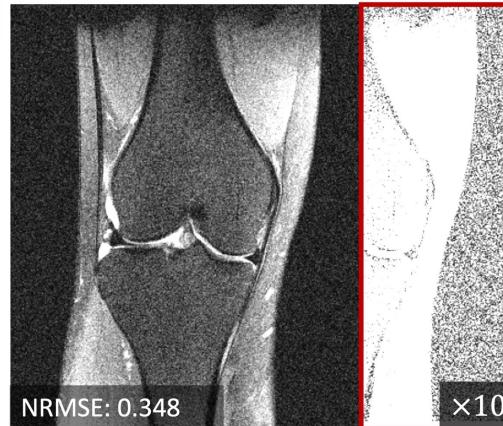
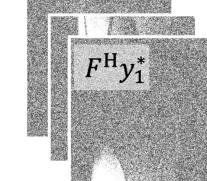
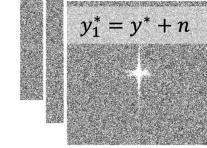
x10

Original FastMRI

+

Additive Gaussian Noise

$$SNR_w = 16 \text{ dB}$$



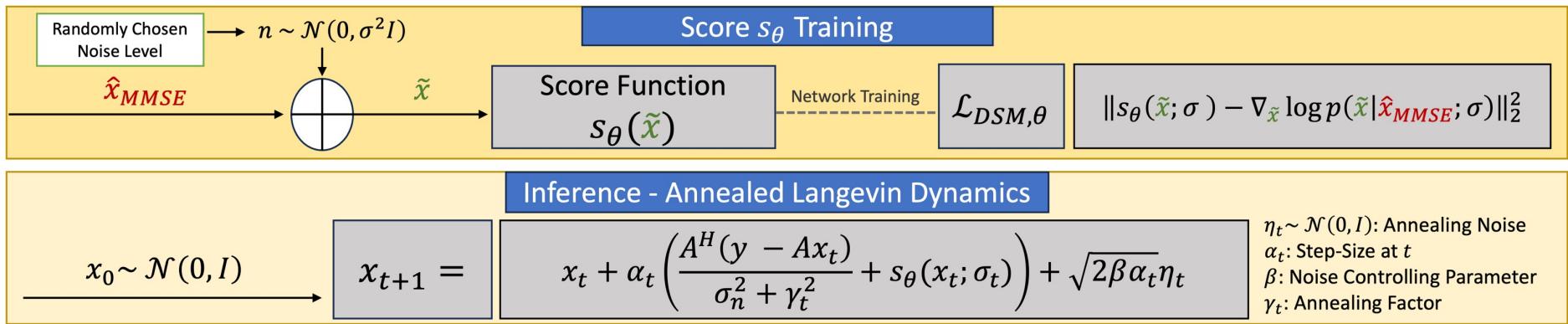
NRMSE: 0.348



NRMSE: 0.197

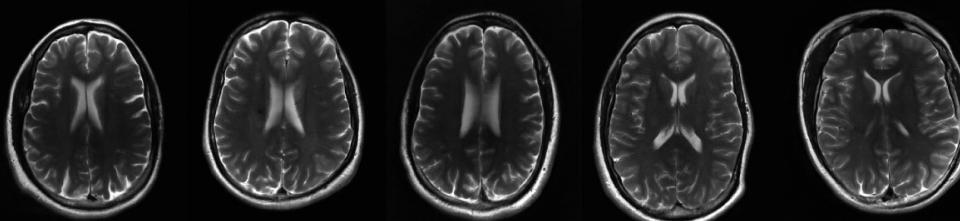
x10

Score Model Training + Inference

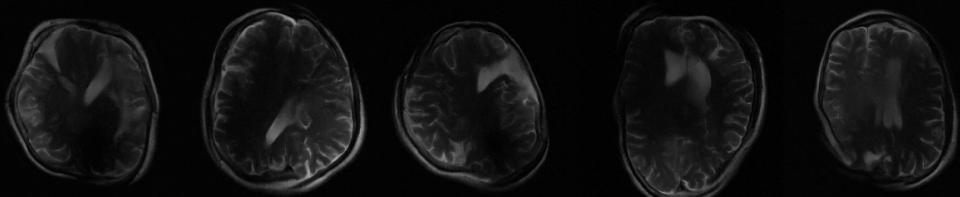


Prior Sampling – $p(x)$

Naive Score
~ 32dB

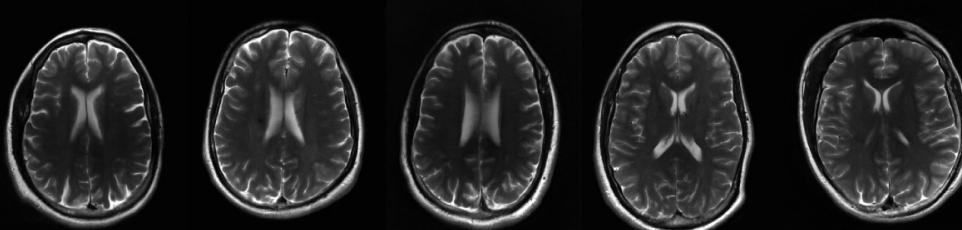


Naive Score
~ 22dB

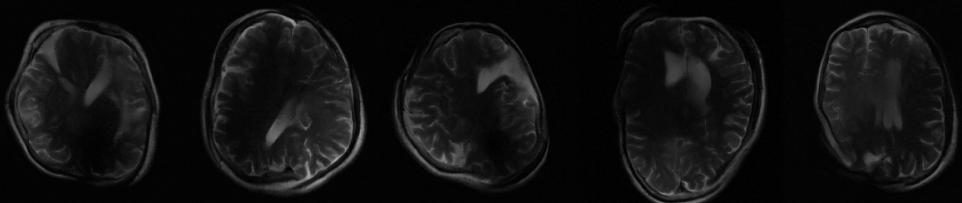


Prior Sampling – $p(x)$

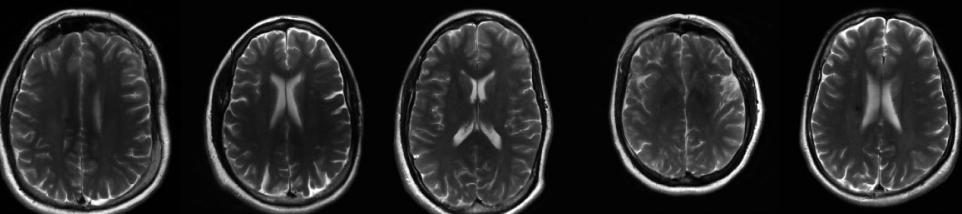
Naive Score
~ 32dB



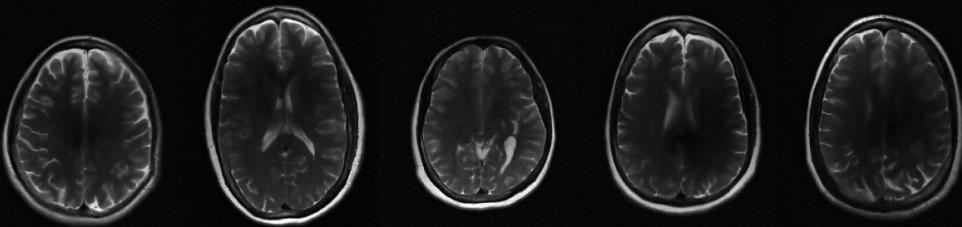
Naive Score
~ 22dB



GSURE-Score
~ 32dB

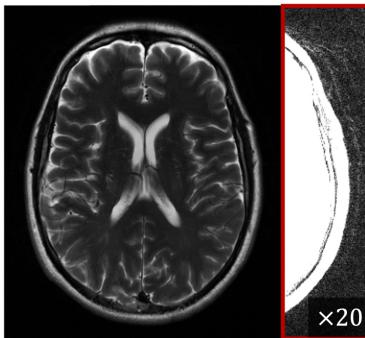


GSURE-Score
~ 22dB

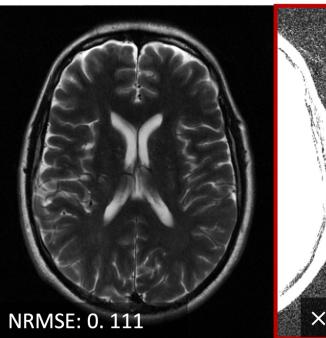


Posterior Sampling - $x \sim p(x|y)$

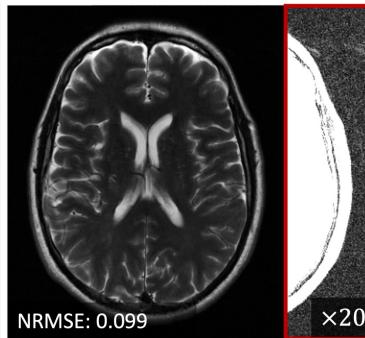
Fully Sampled



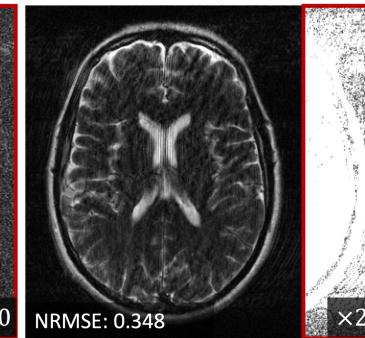
Naive Score @ 32dB



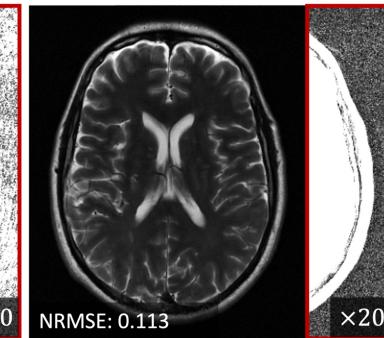
GSURE-Score @ 32dB



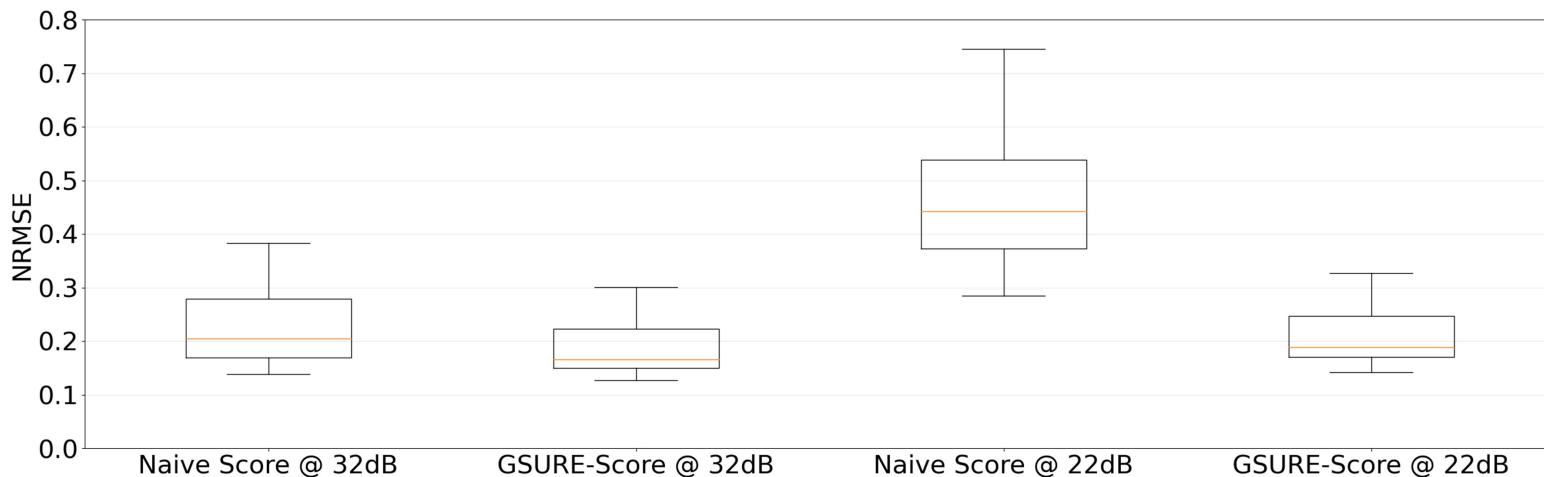
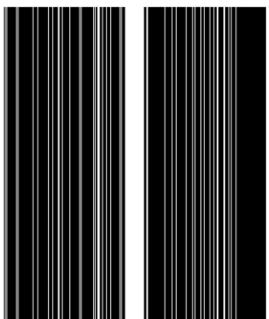
Naive Score @ 22dB



GSURE-Score @ 22dB



R=5



Conclusions

1. MRI samples are inherently noisy
2. Self-supervised techniques like GSURE can successfully remove noise
3. Denoising as a pre-processing step, severely improves the quality of generative priors
4. Priors trained on denoised FastMRI are better inverse problem solvers than naive training

Thank you!

Asad Aali

asad.aali@utexas.edu

<https://www.linkedin.com/in/asadaali/>

PhD Student, UT CSI Lab

