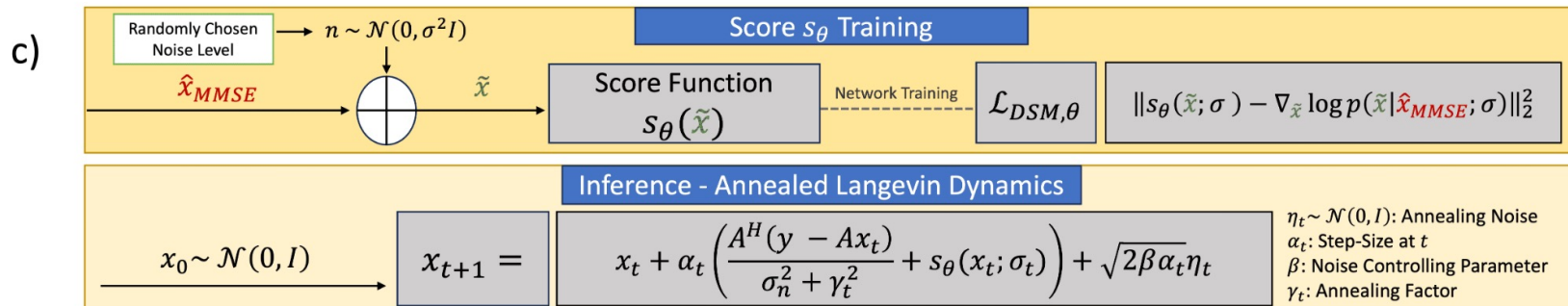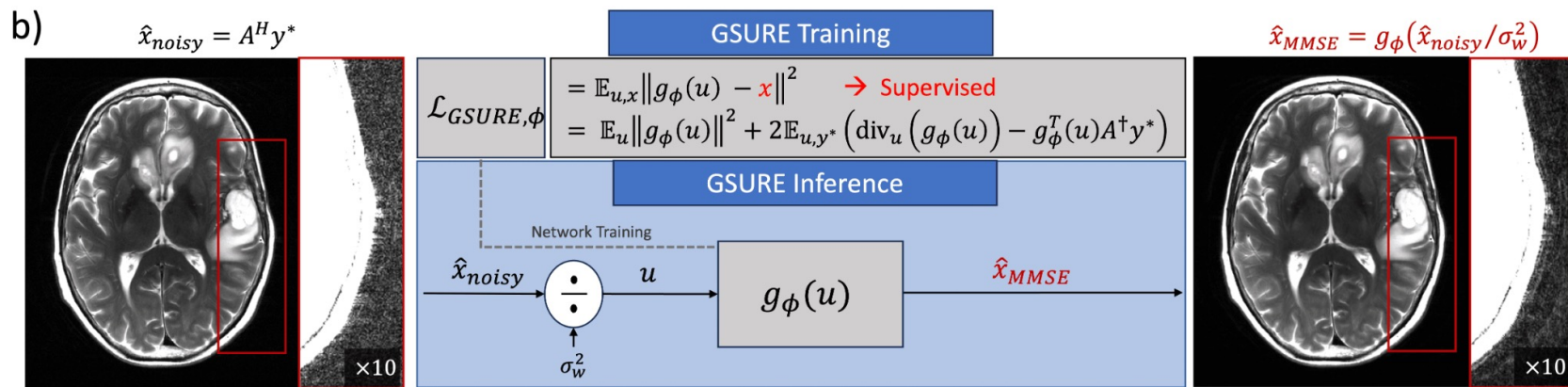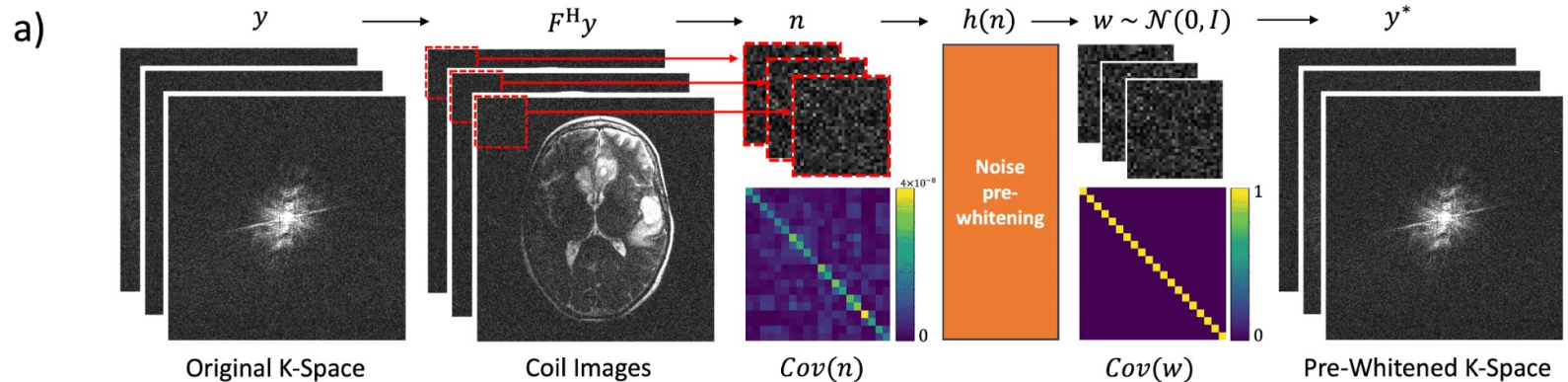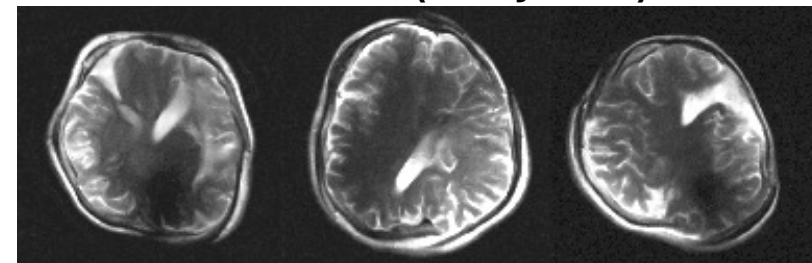# GSURE Denoising enables training higher quality priors for Multi-Coil MRI Reconstruction

Asad Aali, Marius Arvinte, Sidharth Kumar, Yamin Ishraq Arefeen, and Jonathan I. Tamir
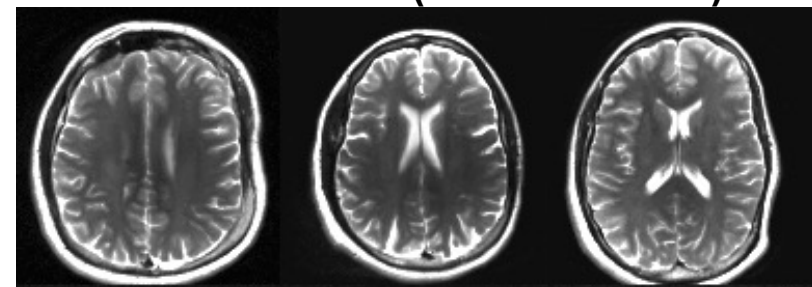
Proposed Pipeline

Naive Score (Noisy Prior)

a)

b)

$\hat{x}_{noisy} = A^H y^*$

$\hat{x}_{MMSE} = g_\phi(\hat{x}_{noisy}/\sigma_w^2)$

GSURE Training

$$\mathcal{L}_{GSURE,\phi} \quad = \mathbb{E}_{u,x}\|g_\phi(u) - x\|^2 \qquad \rightarrow \text{Supervised}$$
$$= \mathbb{E}_u\|g_\phi(u)\|^2 + 2\mathbb{E}_{u,y^*}\left(\text{div}_u\left(g_\phi(u)\right) - g_\phi^T(u)A^\dagger y^*\right)$$

GSURE Inference

Network Training

$\hat{x}_{noisy}$ · u $g_\phi(u)$ $\hat{x}_{MMSE}$

σ_w^2

GSURE-Score (Denoised Prior)

Posterior Evaluation

c)

Randomly Chosen Noise Level → $n \sim \mathcal{N}(0, \sigma^2 I)$

Score s_θ Training

$\hat{x}_{MMSE}$ ⊕ $\tilde{x}$

Score Function $s_\theta(\tilde{x})$

Network Training

$\mathcal{L}_{DSM,\theta}$

$\|s_\theta(\tilde{x}; \sigma) - \nabla_{\tilde{x}} \log p(\tilde{x}|\hat{x}_{MMSE}; \sigma)\|_2^2$

Inference - Annealed Langevin Dynamics

$x_0 \sim \mathcal{N}(0, I)$

$x_{t+1} = \quad x_t + \alpha_t\left(\dfrac{A^H(y - Ax_t)}{\sigma_n^2 + \gamma_t^2} + s_\theta(x_t; \sigma_t)\right) + \sqrt{2\beta\alpha_t}\,\eta_t$

$\eta_t \sim \mathcal{N}(0, I)$: Annealing Noise
α_t: Step-Size at t
β: Noise Controlling Parameter
γ_t: Annealing Factor