MAY 2024

TEXAS
The University of Texas at Austin

GSURE Denoising enables training of higher quality generative priors for accelerated Multi-Coil MRI Reconstruction

Authors: **Asad Aali**[1], Marius Arvinte[2], Sidharth Kumar[1], Yamin Arefeen[1], Jon Tamir[1]

[1]The University of Texas at Austin

[2]Intel Labs

Motivation

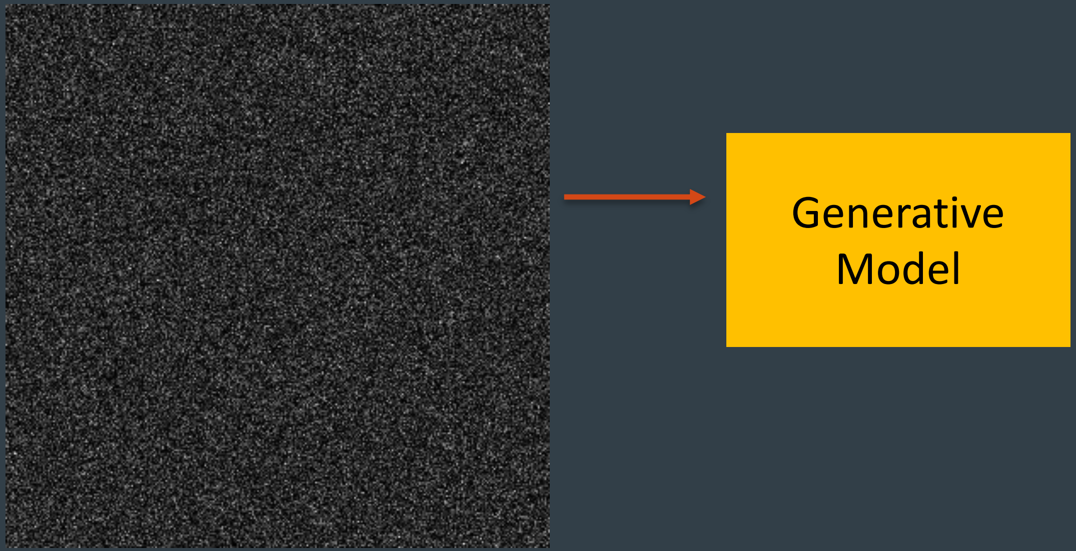
- Deep learning based accelerated MRI reconstruction with Generative Models
  - ✓ Develops a prior with large training databases
  - ✓ Decouples from the forward model

Training Dataset

Motivation

- Generative models are powerful image generators

Sample from Gaussian Distribution



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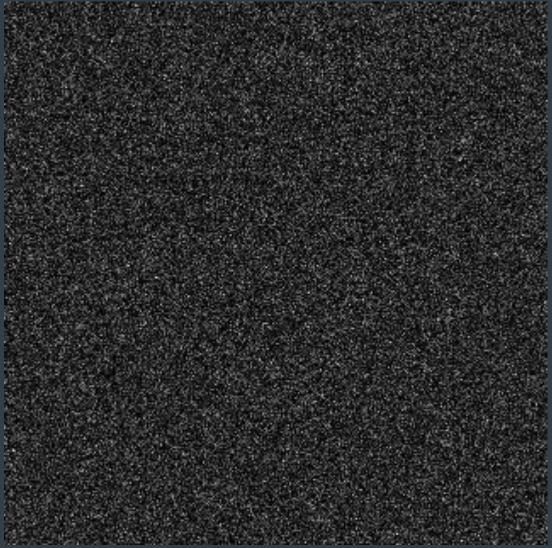
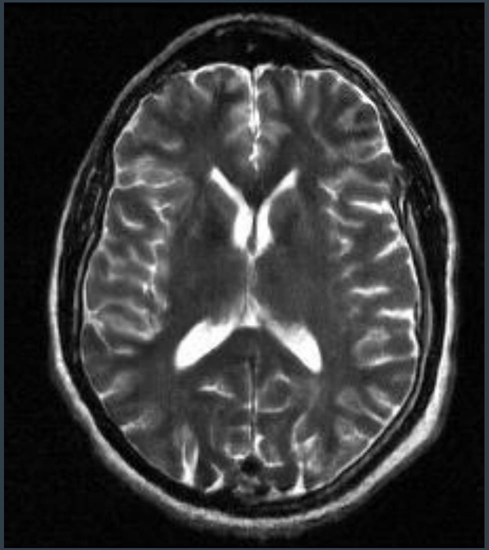


Generative Model

Sample from Image Distribution



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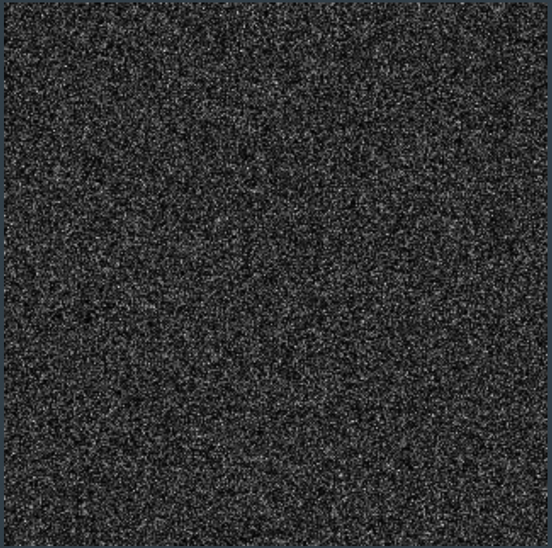

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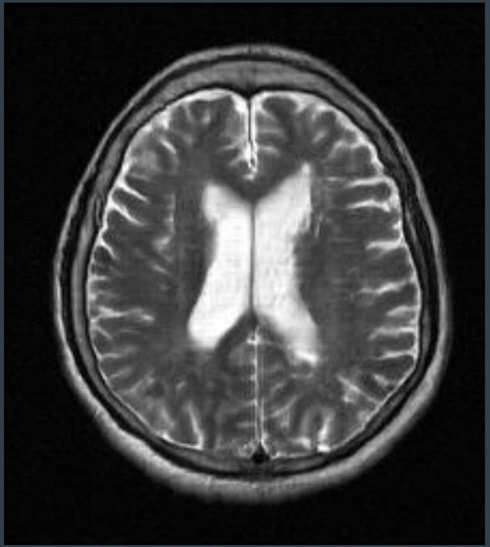

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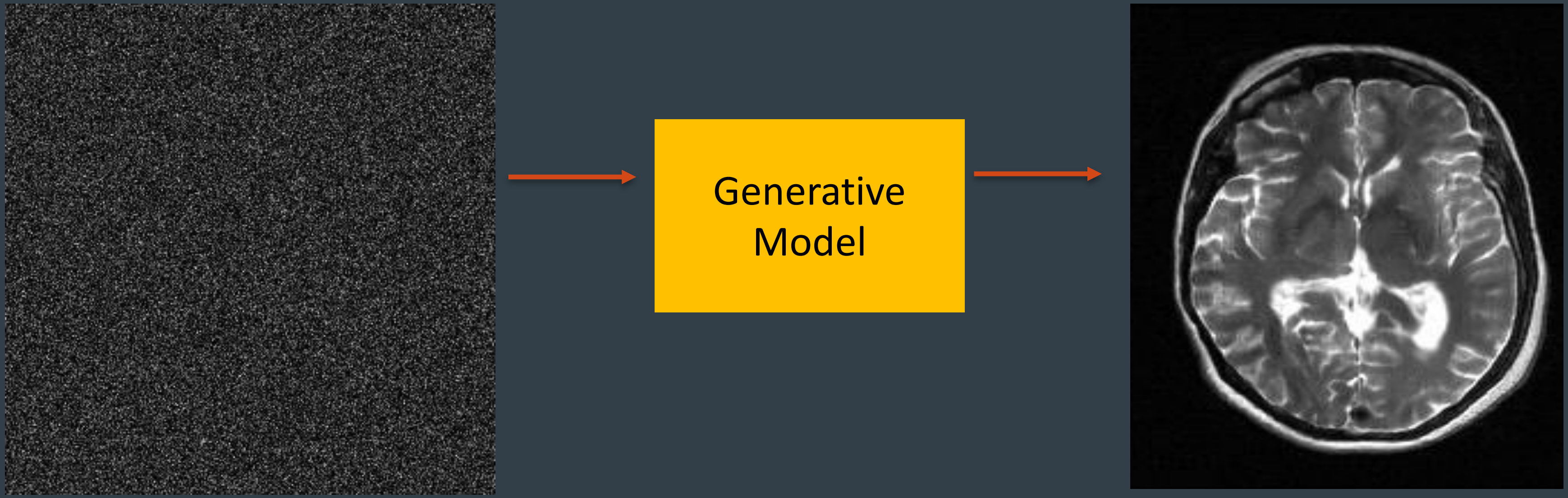

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Sample from Gaussian Distribution

$$\sim \mathcal{N}(z)$$

Image Prior

$\rightarrow$ Generative Model $\rightarrow$

Sample from Image Distribution

$$\sim p(x)$$

Image Prior

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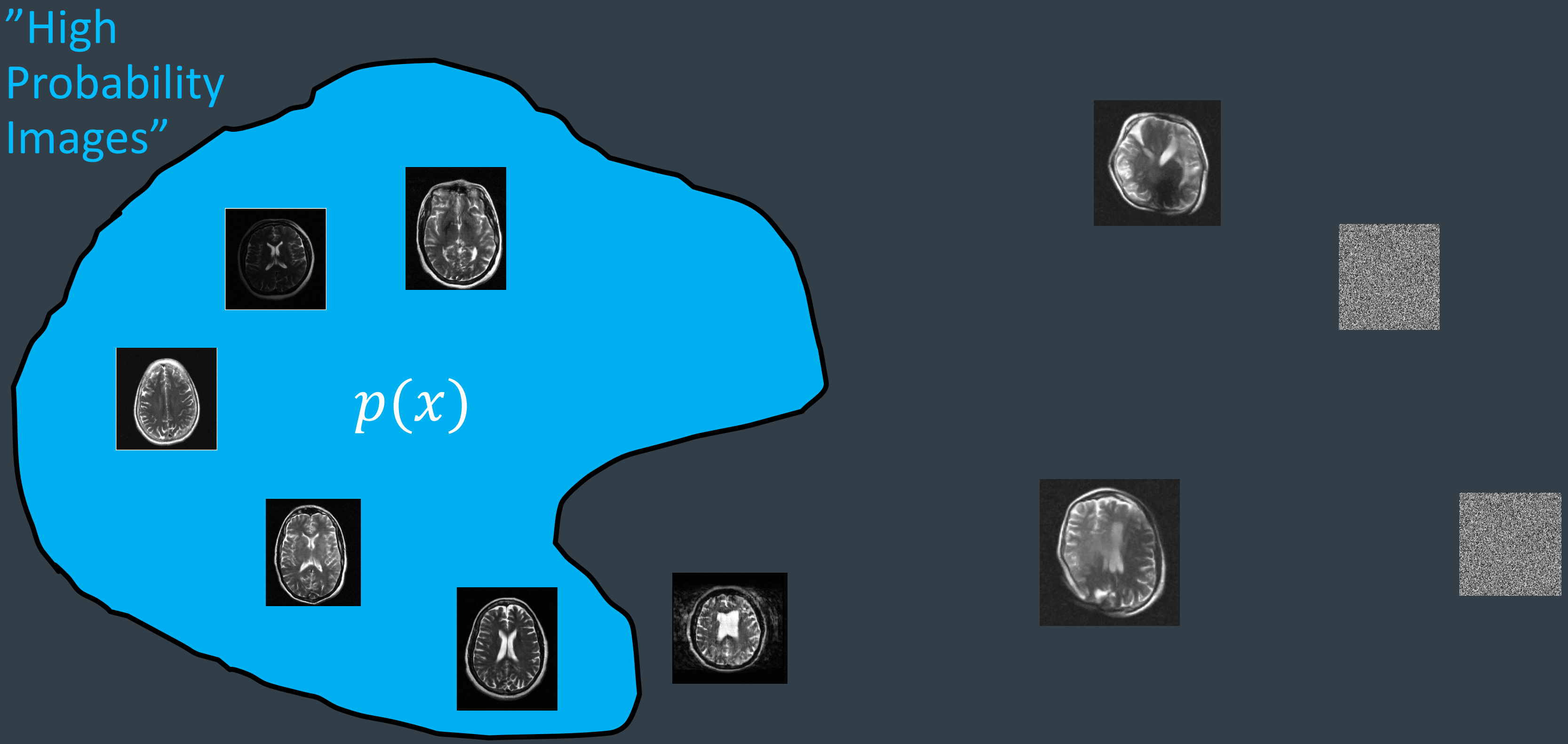
- Using Generative Models to guide accelerated MRI reconstructions.



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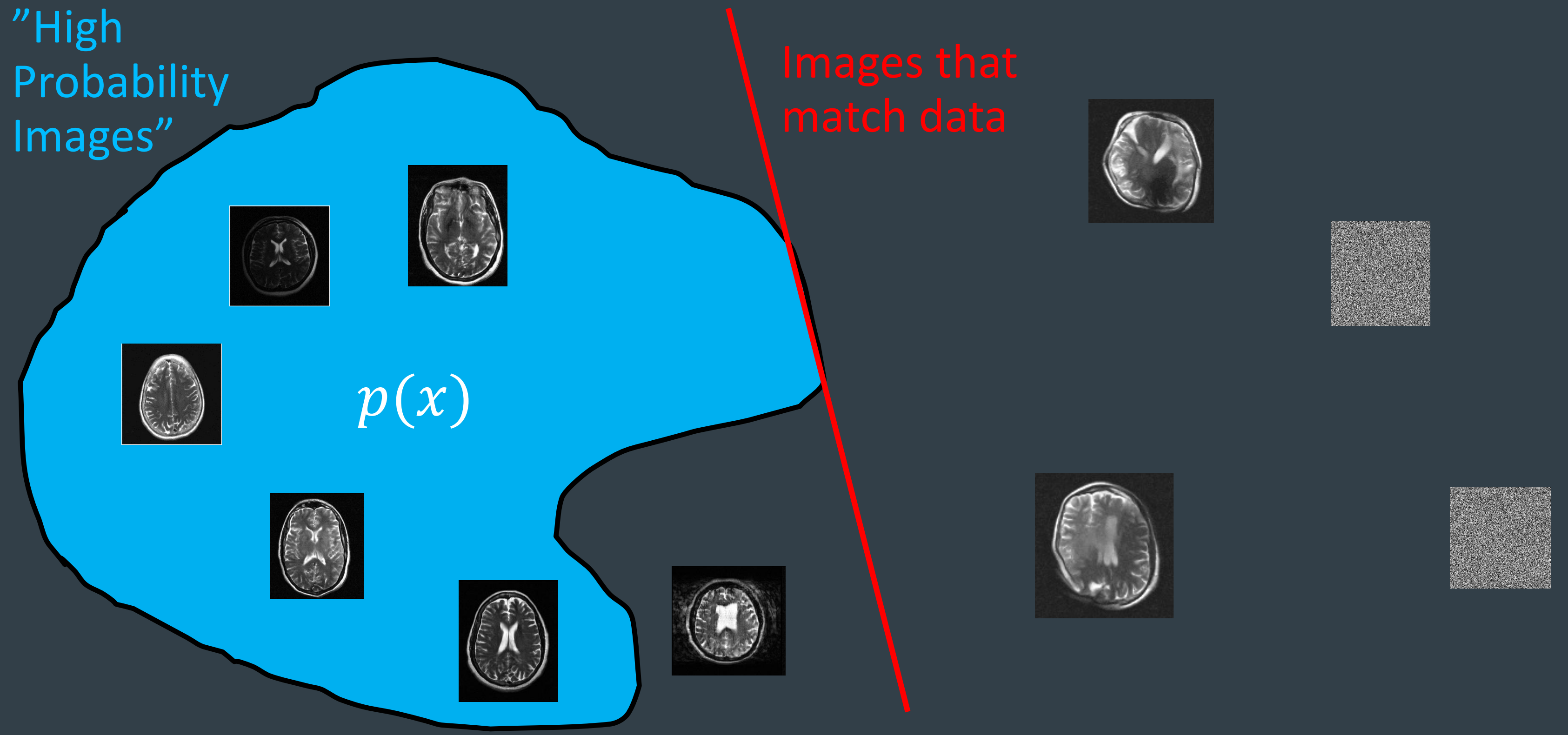

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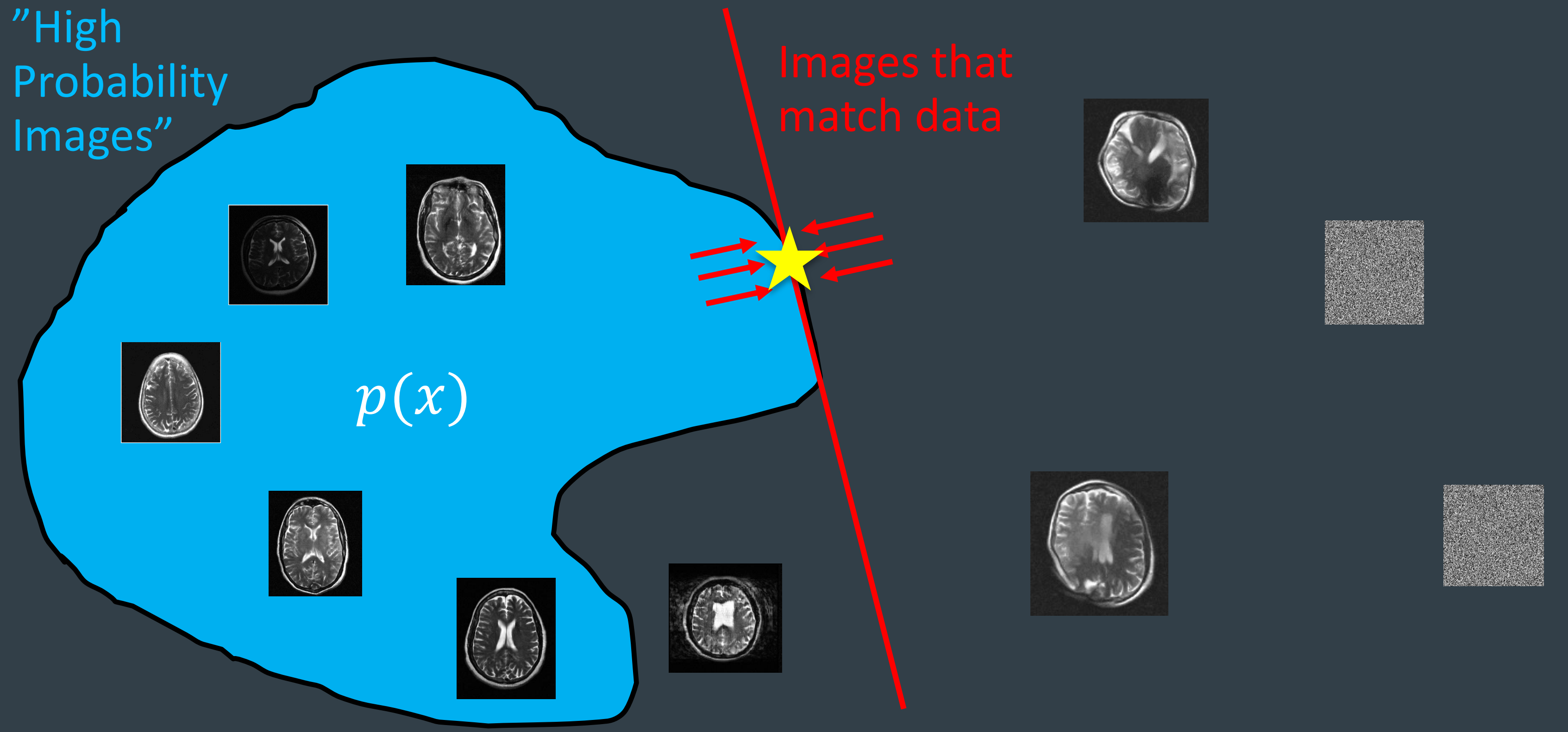

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- Generative Models rely on large amounts of ***high-quality data***.

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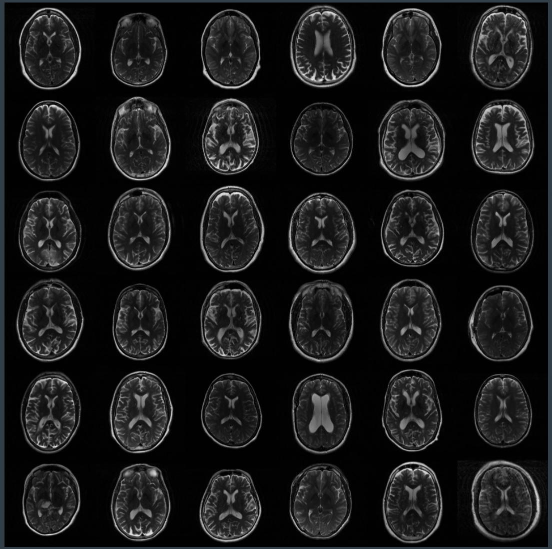

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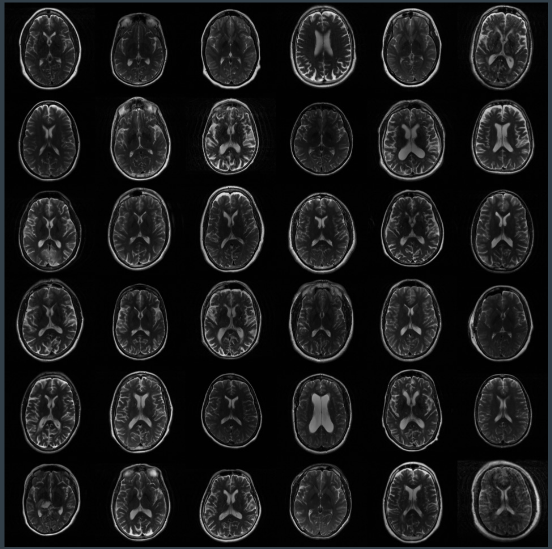

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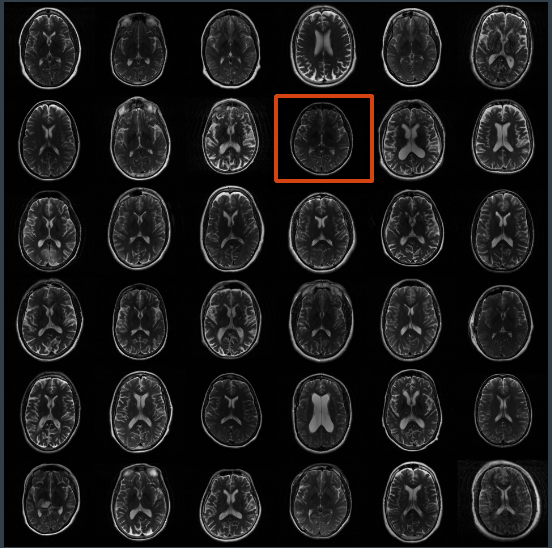

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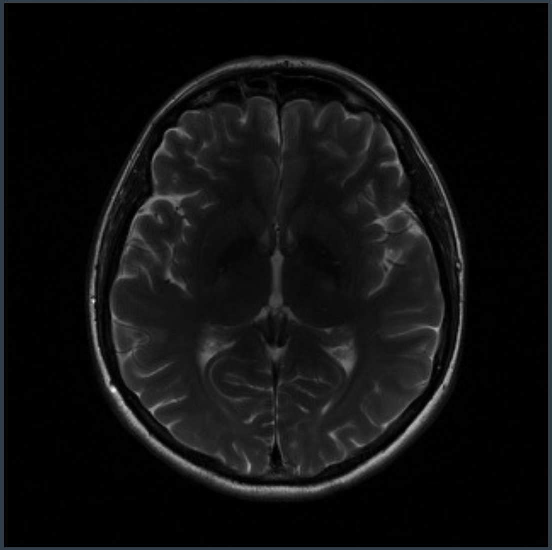

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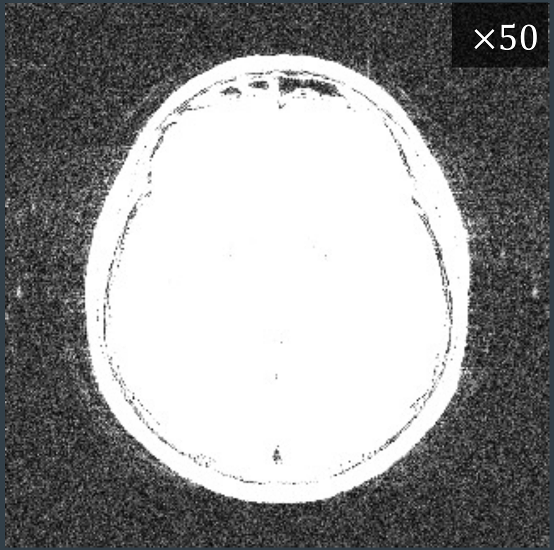

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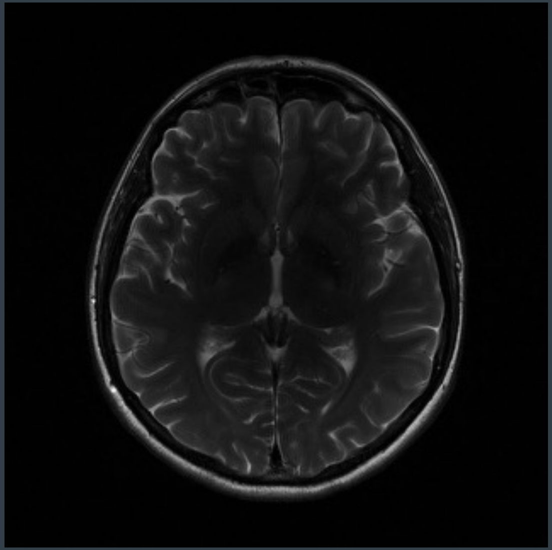

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- Training generative models with noisy datasets leads to a poor prior.

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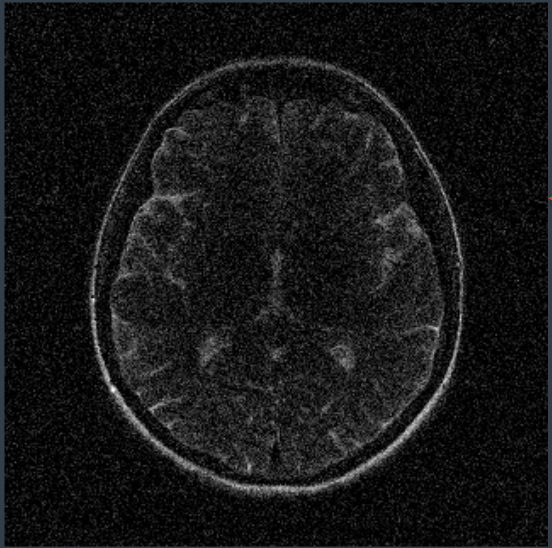

training

Generative Model

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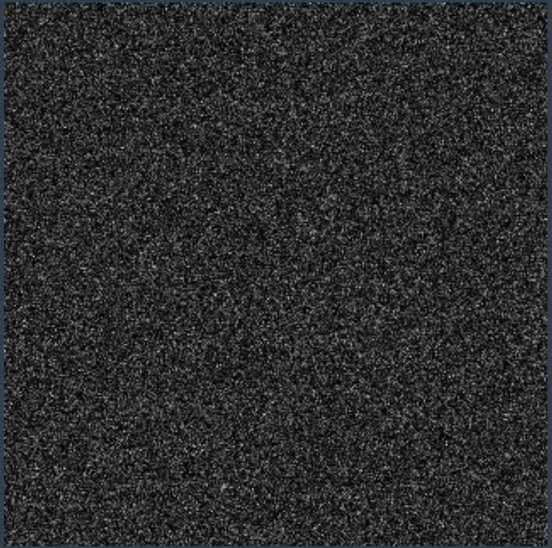
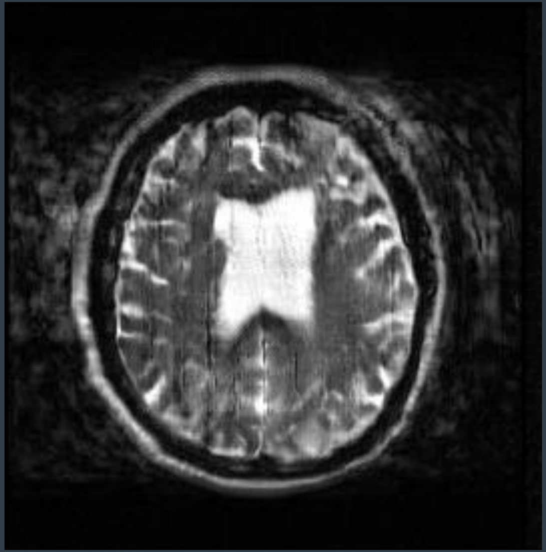


Generative Model

Sample from Image Distribution



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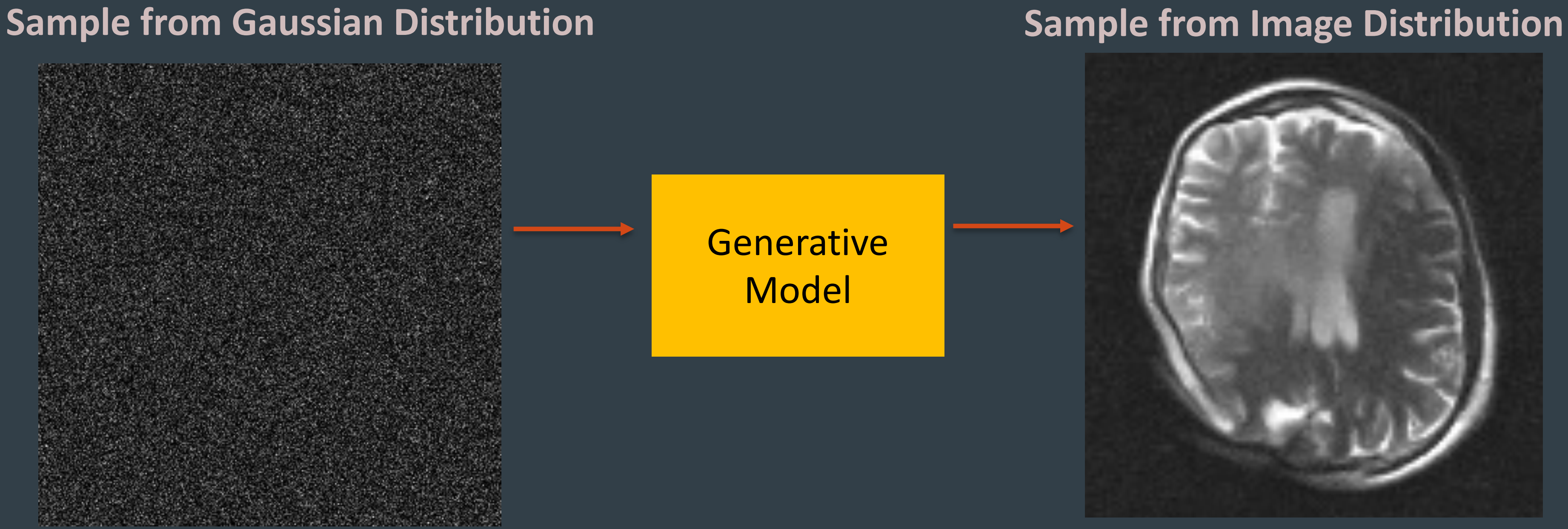

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- Training generative models with noisy datasets leads to a poor prior.
- **Reconstruction performance depends on accuracy of priors**

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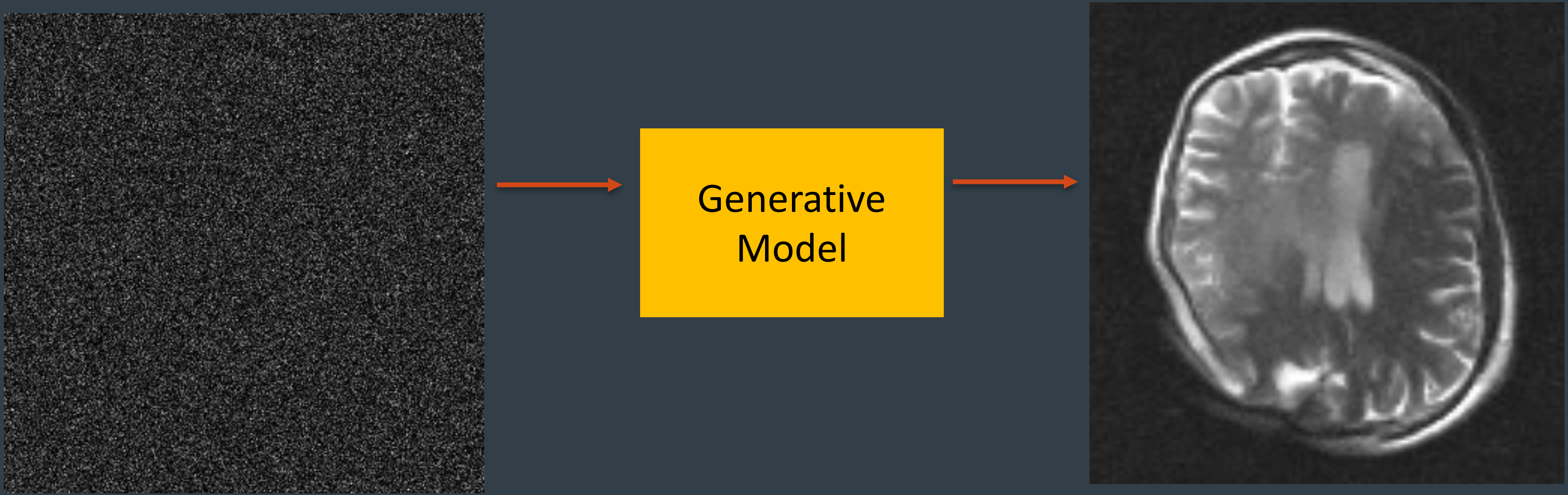

Motivation

Application in real world datasets: low field neo-natal MRI

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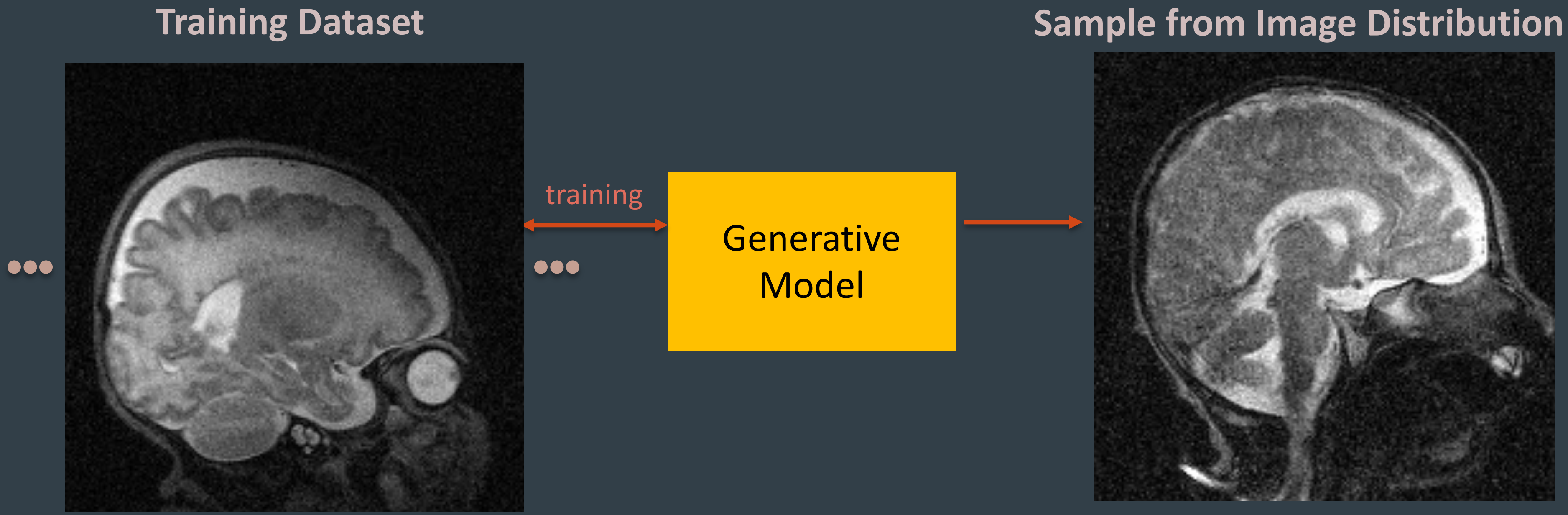
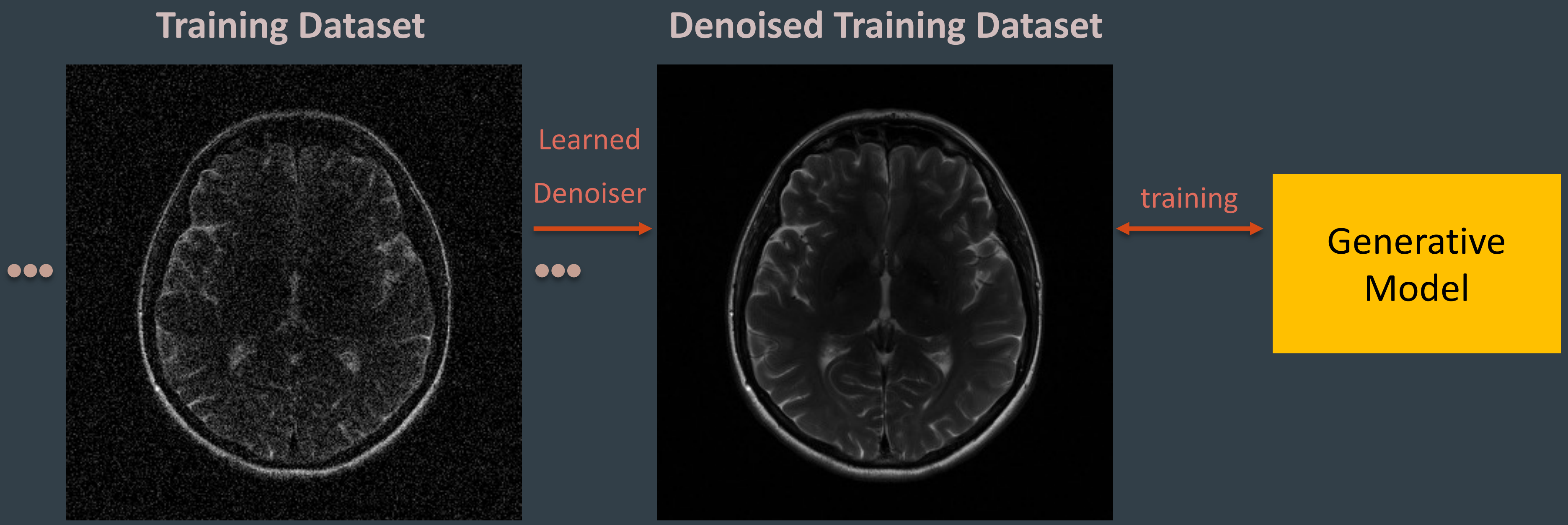
- **Reconstruction performance depends on accuracy of priors**



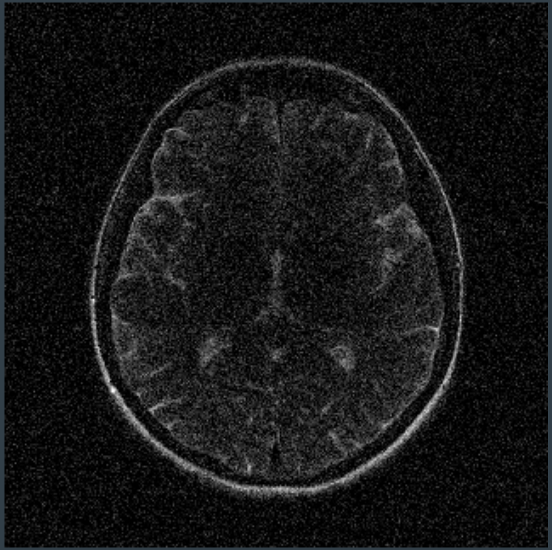
Purpose

- Use a learned denoiser to denoise the training dataset before training the generative model



Purpose

Training a learned denoiser without access clean training samples?



Purpose

Investigate the **feasibility** and **effectiveness** of self-supervised **denoising** MRI samples as a pre-processing step to learning **generative priors** for accelerated MRI reconstruction

Problem Formulation

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Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

$$y = Ax + noise$$

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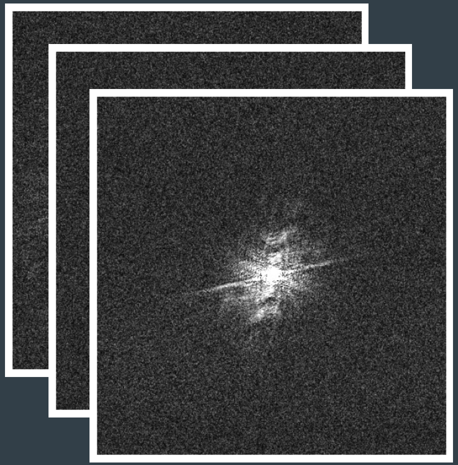

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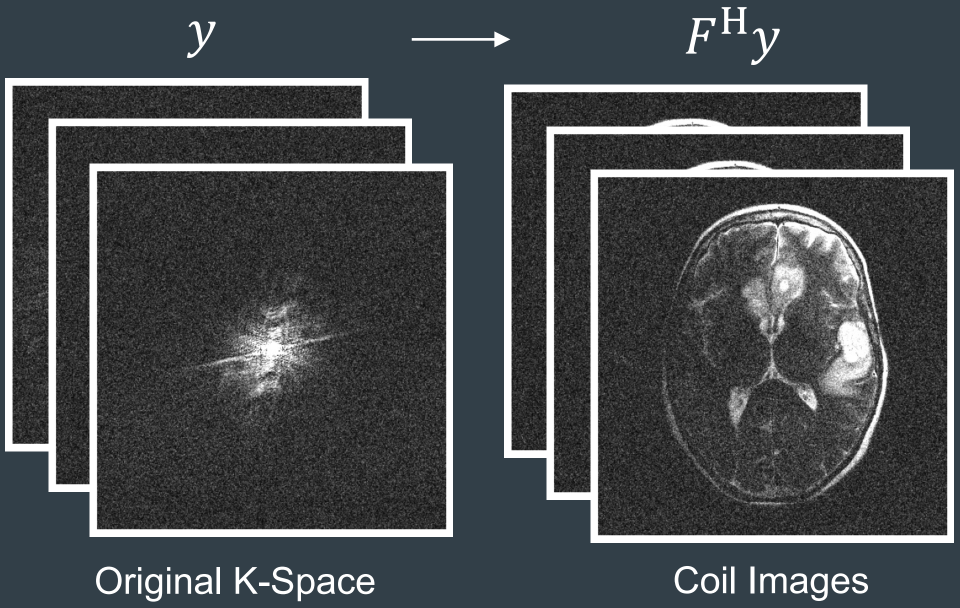

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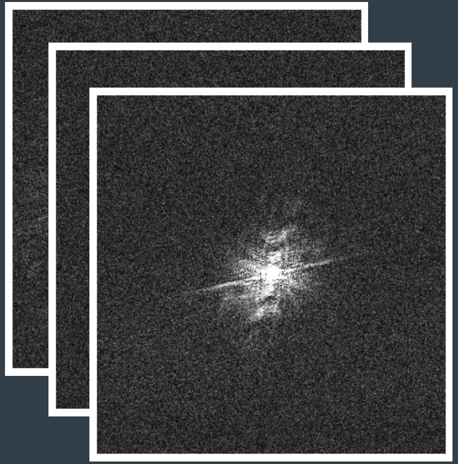
$y \longrightarrow F^{\mathrm{H}}y \longrightarrow \boxed{\hat{x}_{noisy} = A^H y}$



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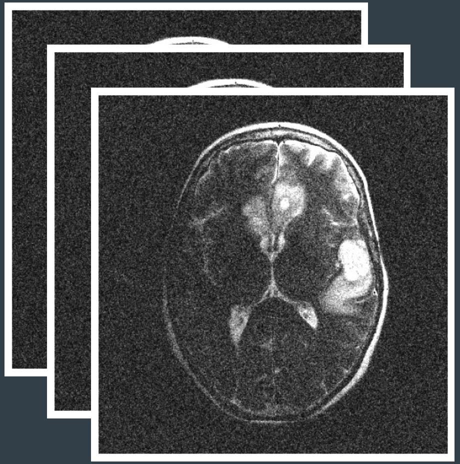
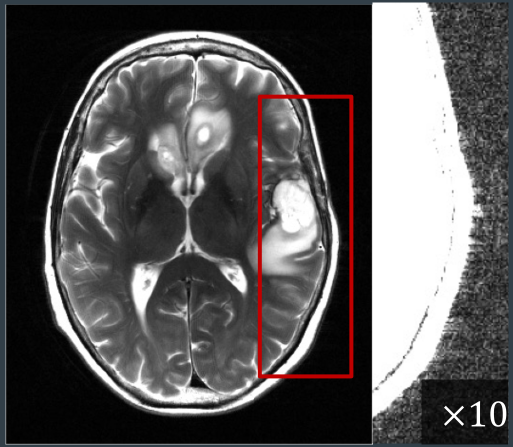


Coil Images



Noisy MRI Sample

Proposed Methods

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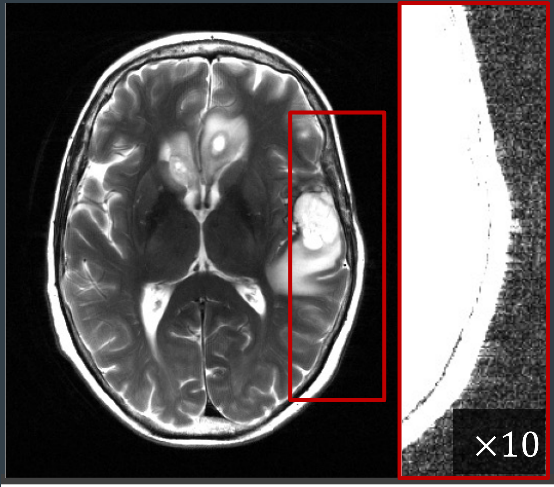
$\hat{x}_{noisy}$



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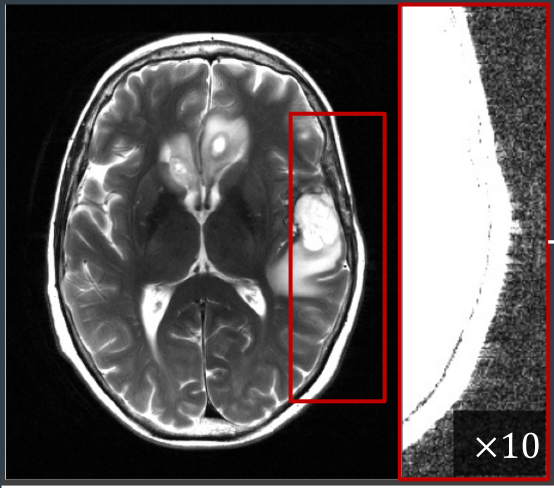

Denoiser Network
g_ϕ

$g_\phi(\hat{x}_{noisy})$

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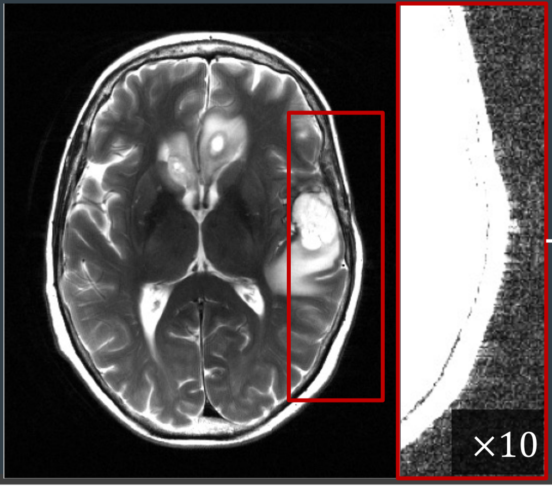

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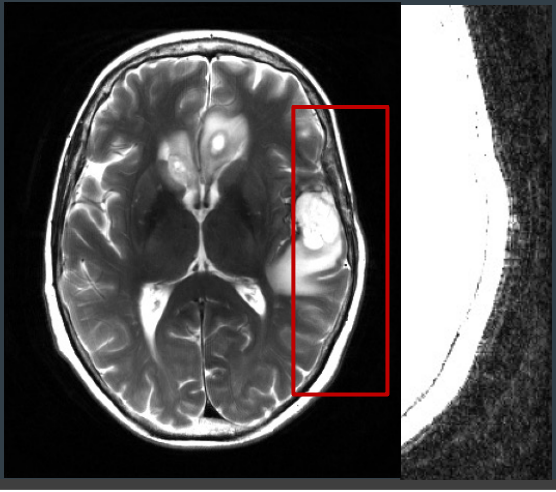
$\hat{x}_{MMSE}$



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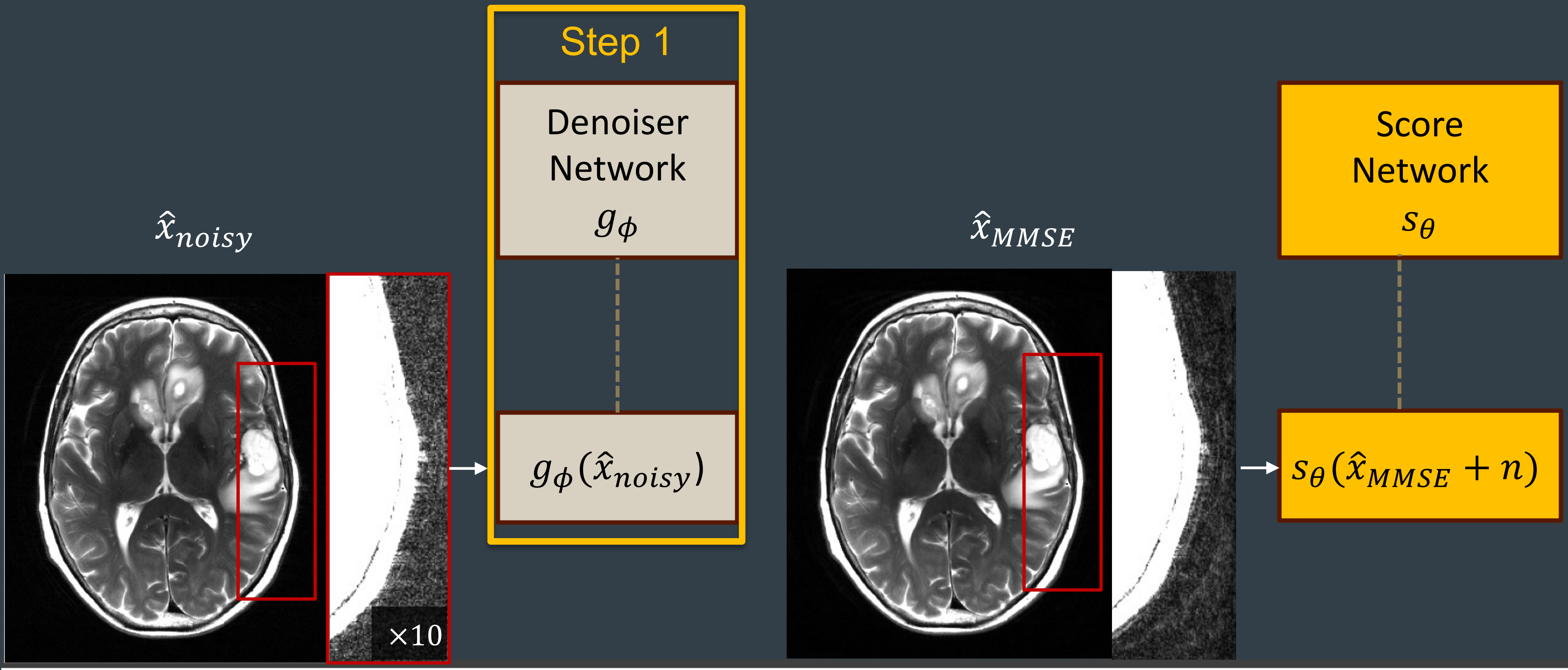

Self-Supervised Denoising

Goal is to learn the **clean distribution** using *noisy* data (i.i.d Gaussian, with known power σ_w^2).

A is a Linear Forward Operator (Fully-Sampled) -> **GSURE**[1,2,3]

$$y = FSx + noise$$

[1]Soltanayev, *NeurIPS,* 2018, [2]Eldar, *IEEE Transactions on Signal Processing,* 2008, [3]Kawar, *TMLR,* 2023

Generalized SURE (GSURE) Basics

- GSURE[1]: Self-supervised denoising technique, only need access to:
  - $\hat{x}_{noisy}$ → **Noisy Samples**
  - Noise **Covariance Matrix**

- An unbiased estimate of the MSE

$$E[\mathrm{L}_{GSURE}] = E\left\|g_\phi(\hat{x}_{noisy}) - x\right\|$$

[1]Yonina, *IEEE Transactions on Signal Processing, 2008*

Pre-Processing Noisy Dataset

- GSURE loss requires the **noise covariance** matrix
- **Pre-whitening** (noise covariance $= I$) makes computation relatively straight-forward

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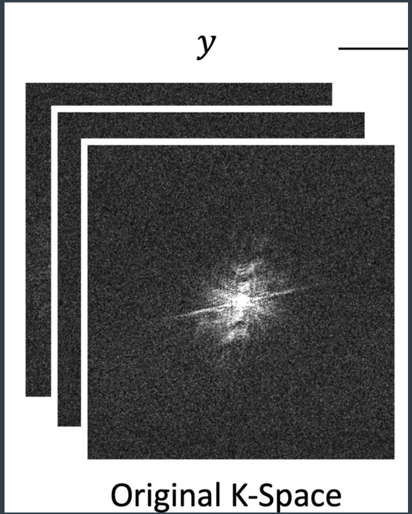

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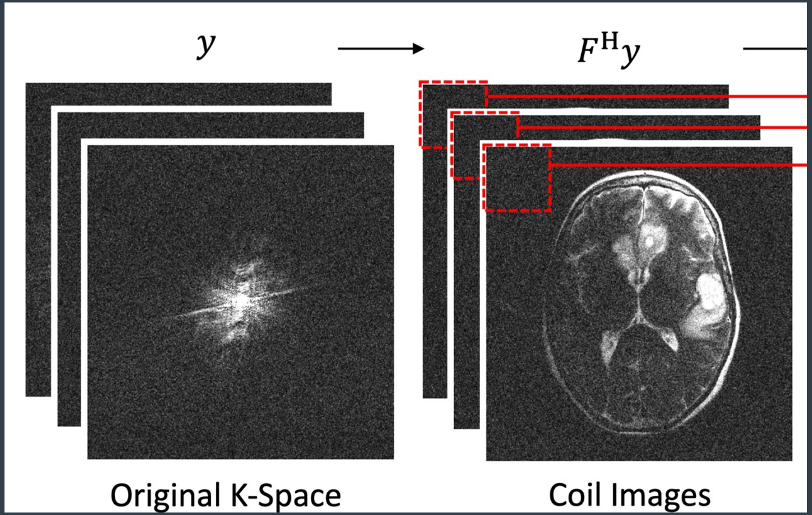

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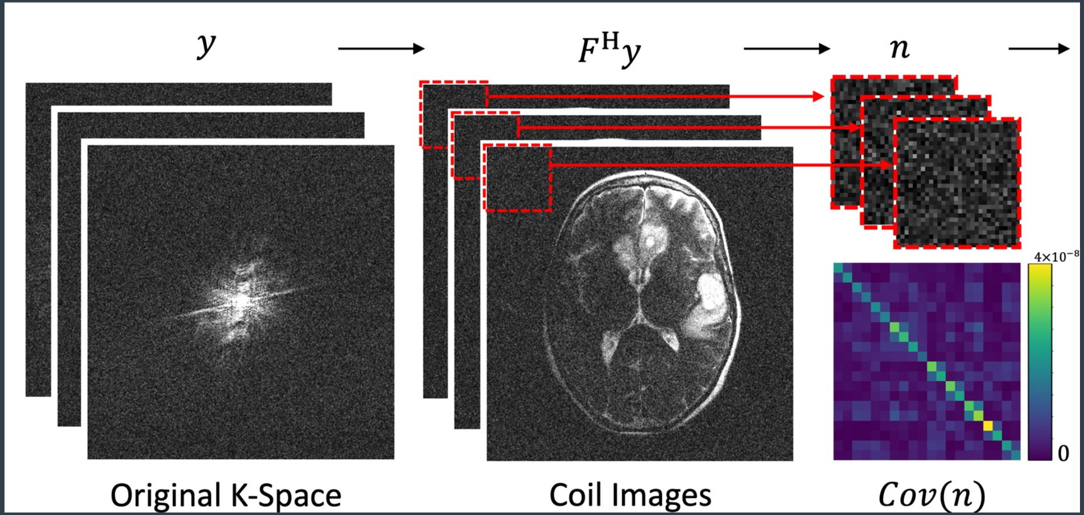

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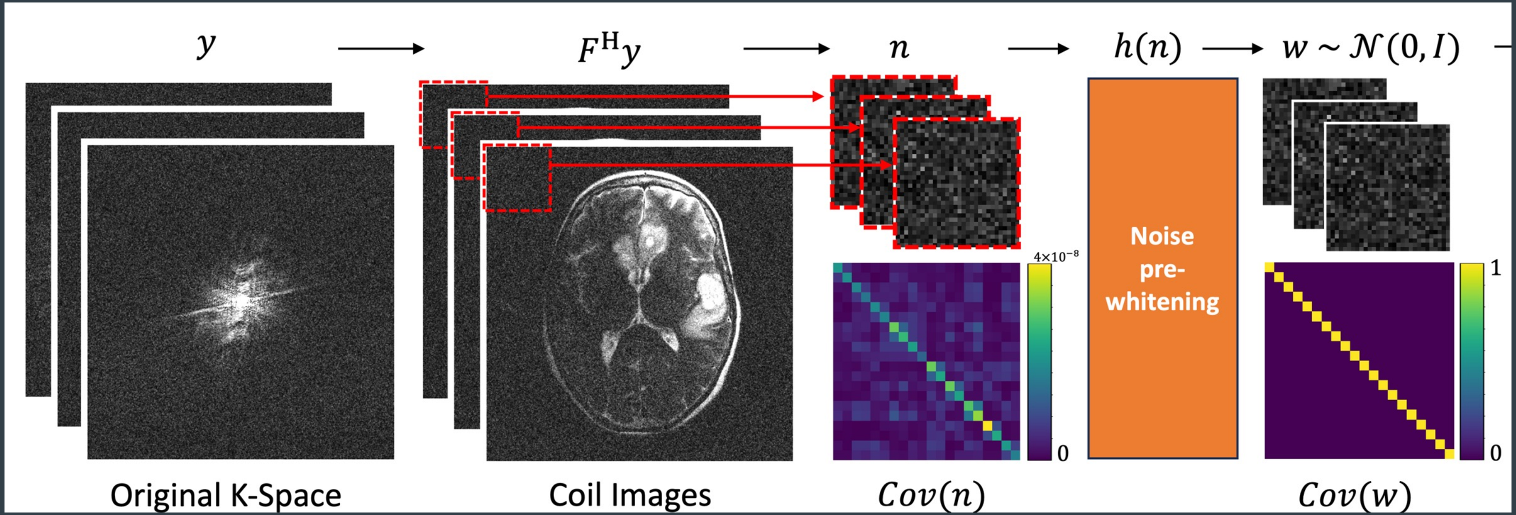

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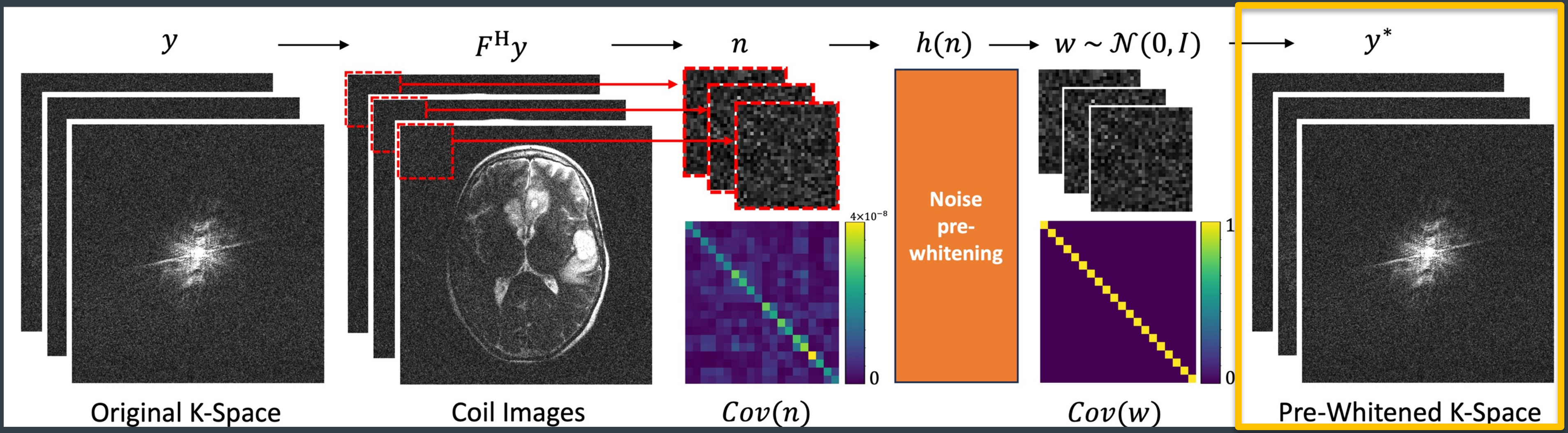
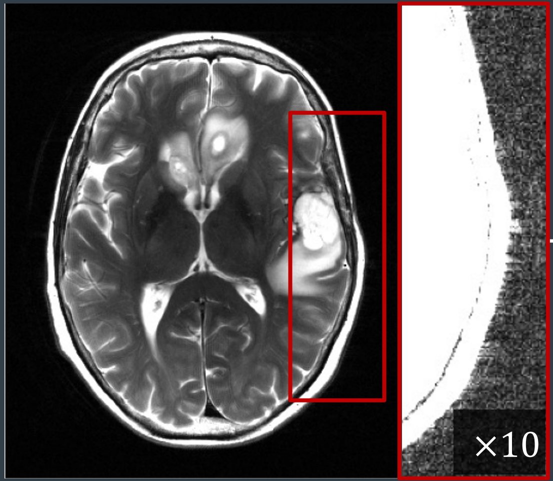


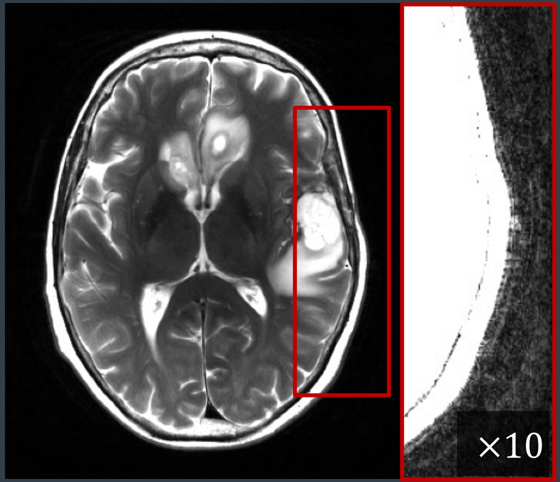
GSURE Denoising - Summary

$\hat{x}_{noisy}$



$g_\phi(\hat{x}_{noisy})$

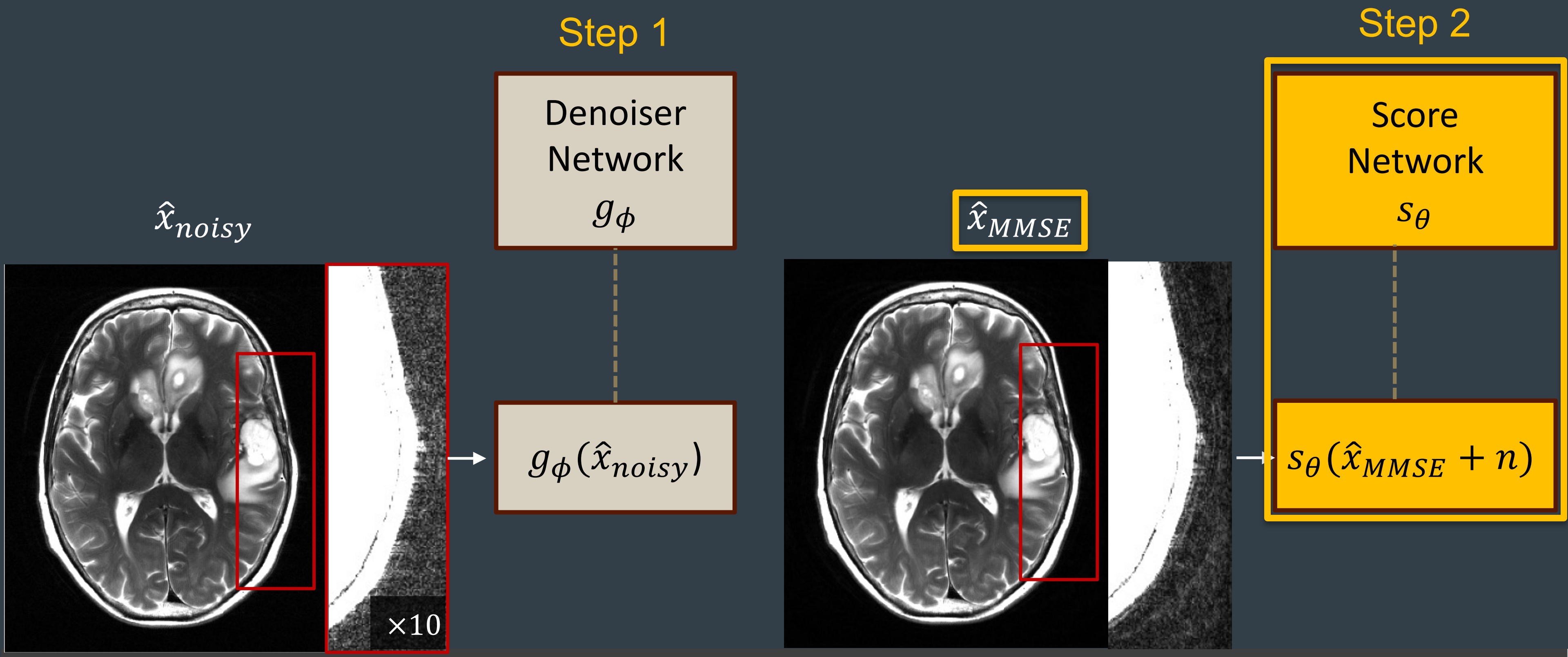
$\hat{x}_{MMSE}$



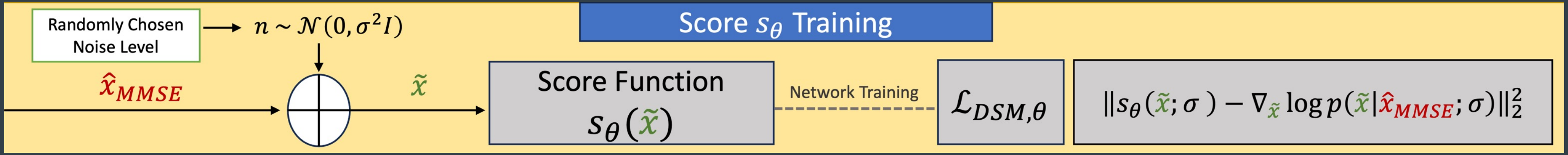
$$L_{GSURE} = \left\|g_\phi(u)\right\|^2 + 2\left(Tr\left(\frac{\partial g_\phi(u)}{\partial u}\right) + g_\phi^T(u)\frac{\partial \ln q(u)}{\partial u}\right)$$

GSURE Training

Proposed Methods



Score-based generative models



Hyvarinen, *JMLR* 2005, Song, *UAI,* 2018, Vincent, *MIT Press* 2011, Song, *NeurIPS* 2019, Dhariwal, *NeurIPS* 2021

Score-based generative models

Score s_θ Training

Randomly Chosen Noise Level → $n \sim \mathcal{N}(0, \sigma^2 I)$

$\hat{x}_{MMSE}$ ⊕ n → $\tilde{x}$ → Score Function $s_\theta(\tilde{x})$

Network Training

$$\mathcal{L}_{DSM,\theta} \quad \| s_\theta(\tilde{x}; \sigma) - \nabla_{\tilde{x}} \log p(\tilde{x} | \hat{x}_{MMSE}; \sigma) \|_2^2$$

Inference - Annealed Langevin Dynamics

$x_0 \sim \mathcal{N}(0, I)$

$$x_{t+1} = x_t + \alpha_t \left(\frac{A^H (y - A x_t)}{\sigma_n^2 + \gamma_t^2} + s_\theta(x_t; \sigma_t) \right) + \sqrt{2\beta\alpha_t}\,\eta_t$$

$\eta_t \sim \mathcal{N}(0, I)$: Annealing Noise
α_t: Step-Size at t
β: Noise Controlling Parameter
γ_t: Annealing Factor

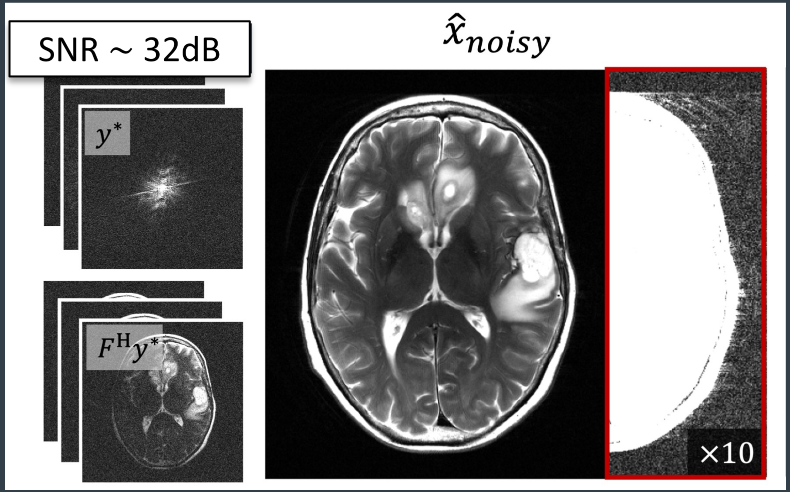
Hyvarinen, *JMLR* 2005, Song, *UAI,* 2018, Vincent, *MIT Press* 2011, Song, *NeurIPS* 2019, Dhariwal, *NeurIPS* 2021

Experiments

1. Evaluation of **Self-Supervised Denoising** (GSURE)
2. **Prior sampling** performance of score models trained on Noisy vs. GSURE Denoised data
3. **Posterior Reconstruction** performance of score models trained on Noisy vs. GSURE Denoised data

T2 Brain Scans

Original FastMRI



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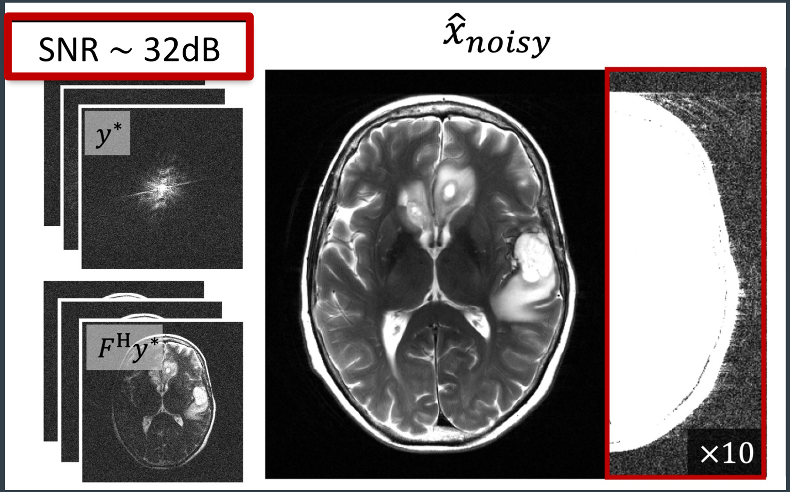

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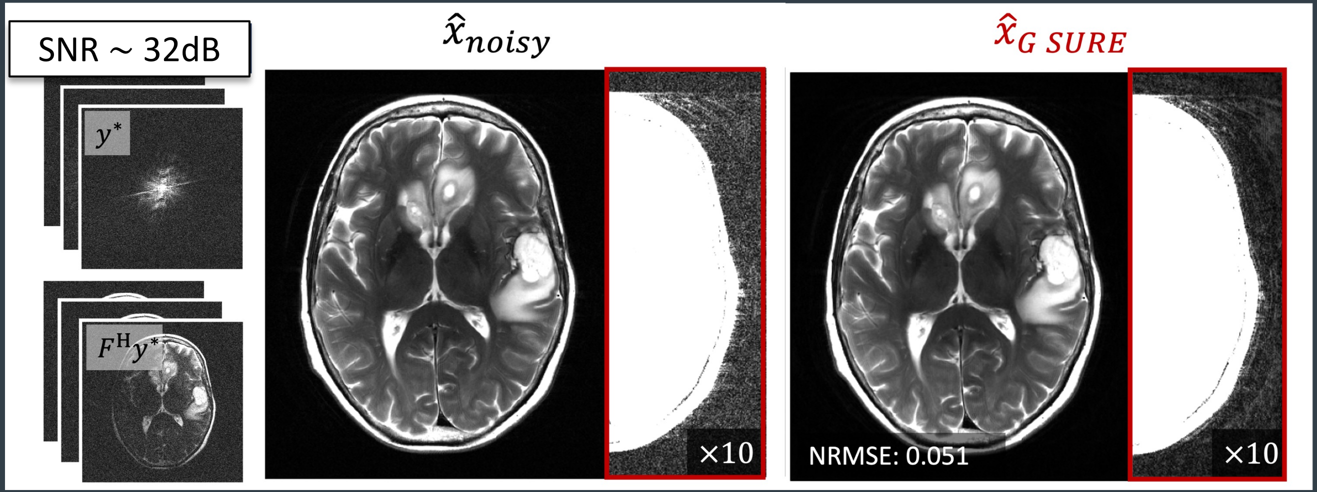

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Original FastMRI

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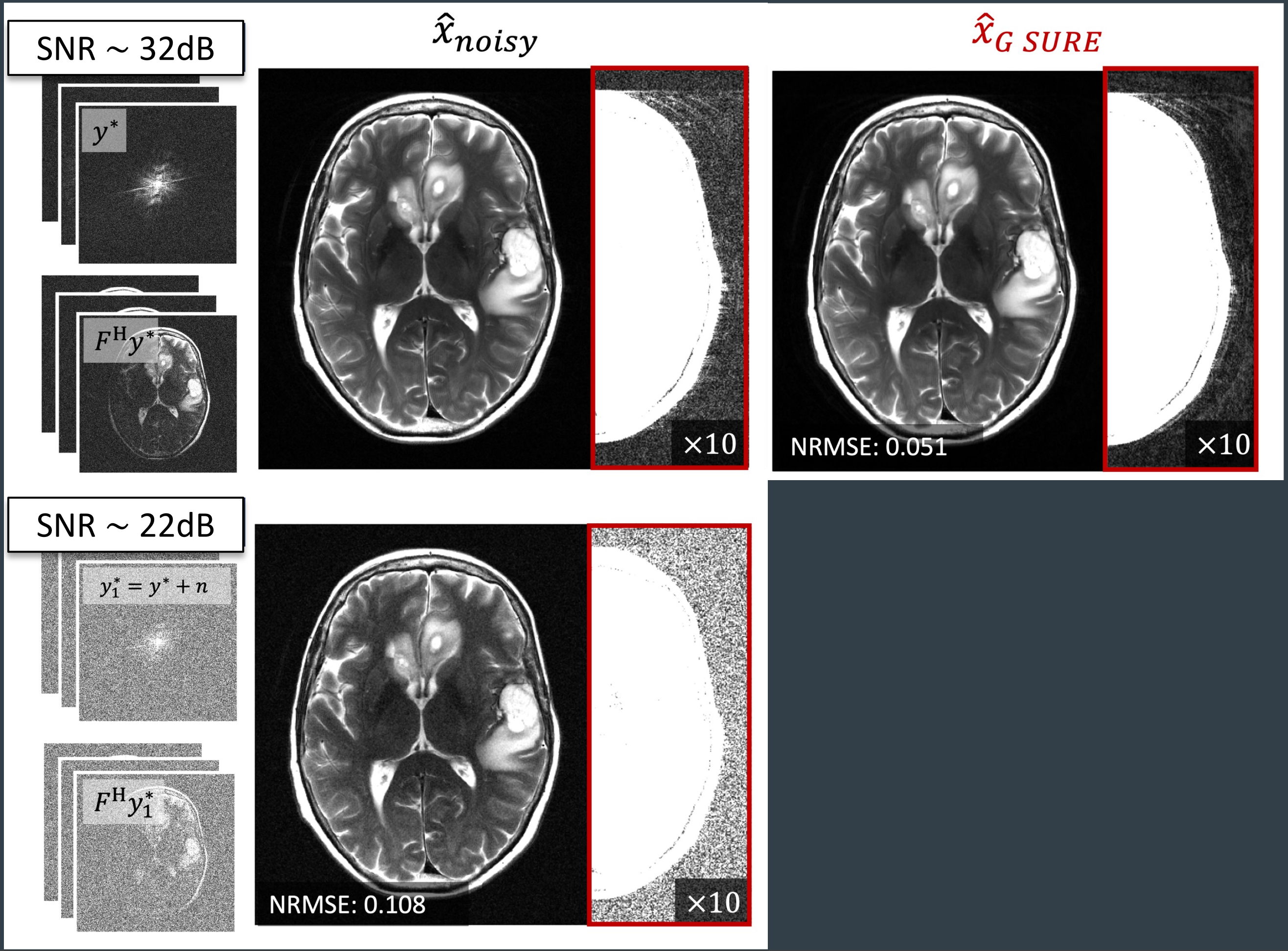
+

Additive Gaussian Noise



T2 Brain Scans

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Original FastMRI

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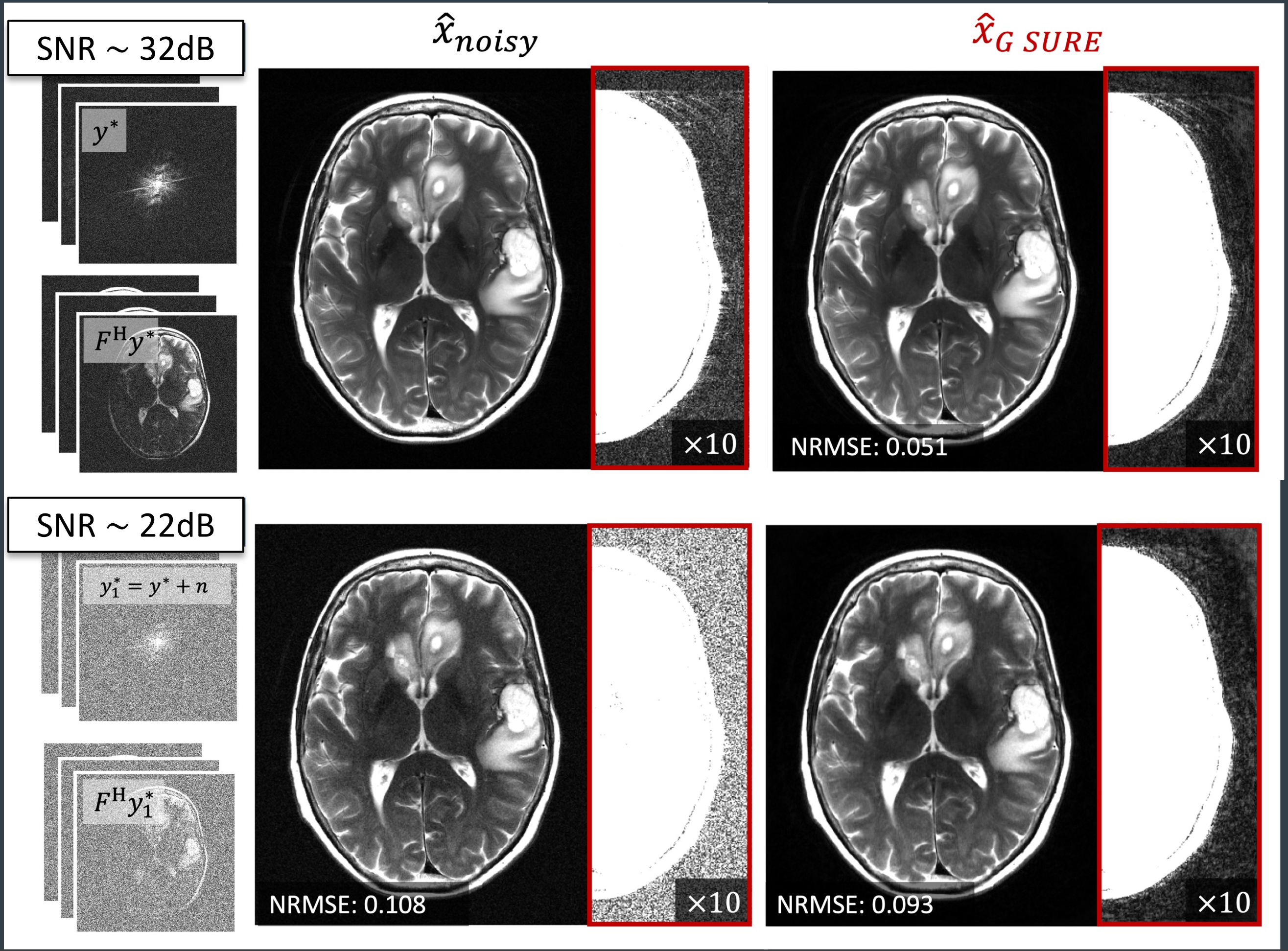


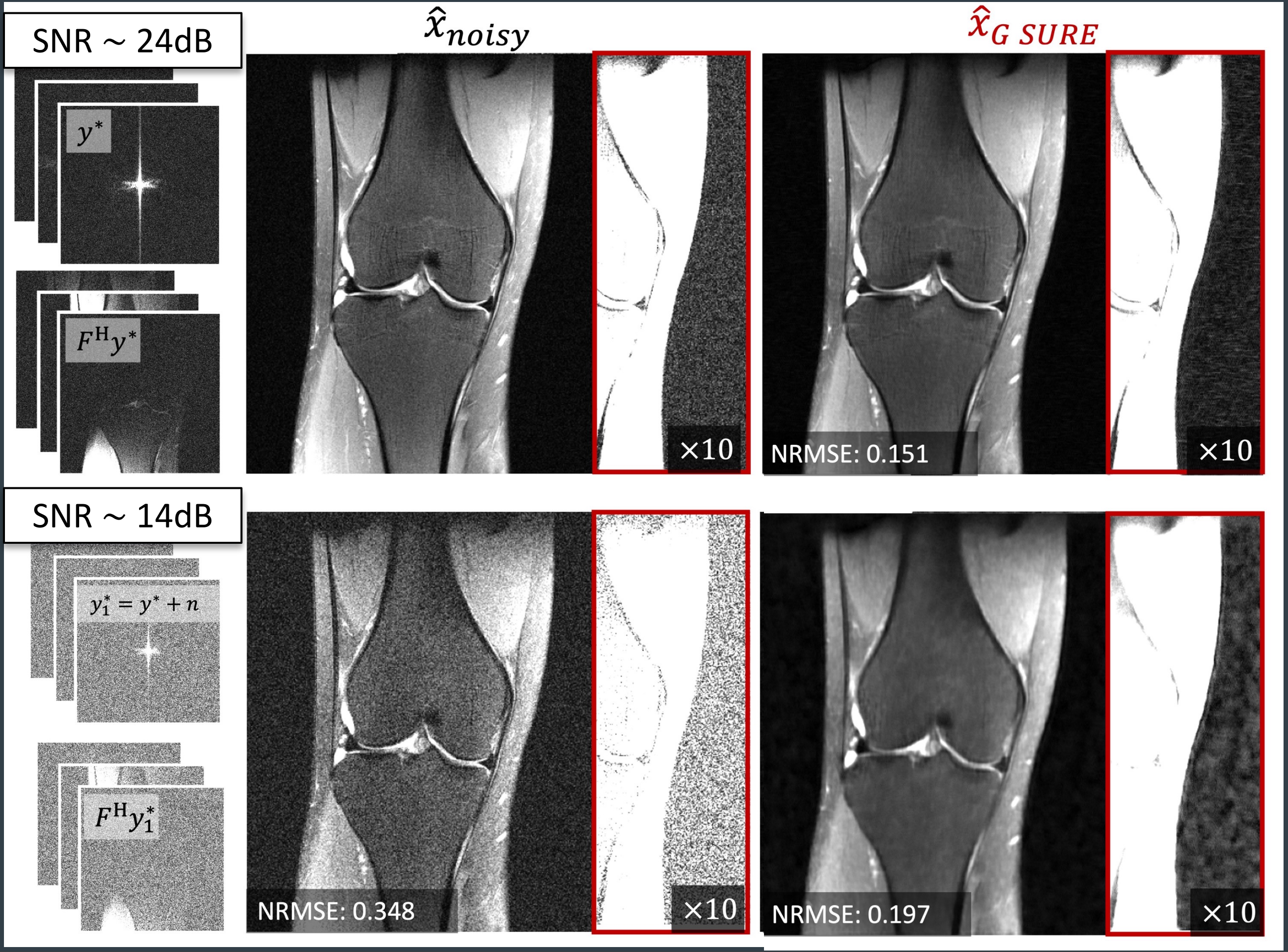
Knee Scans

Original FastMRI

Original FastMRI

+

Additive Gaussian Noise

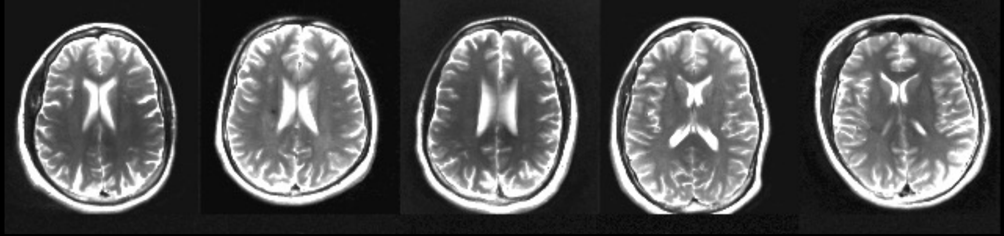


Results

1. Evaluation of **Self-Supervised Denoising** (GSURE)
2. **Prior sampling** performance of score models trained on Noisy vs. GSURE Denoised data
3. **Posterior Reconstruction** performance of score models trained on Noisy vs. GSURE Denoised data

Prior Sampling
$p(x)$

Naive Score
~ 32dB



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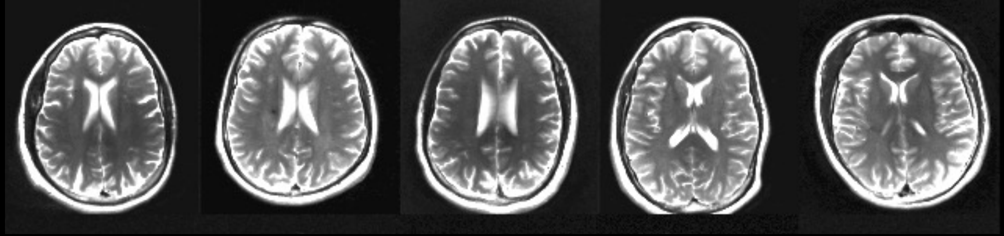
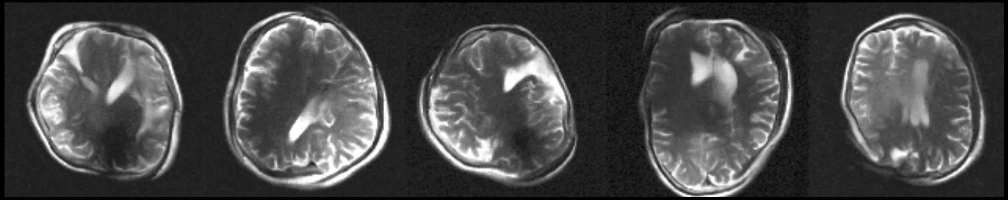


Naive Score
~ 22dB



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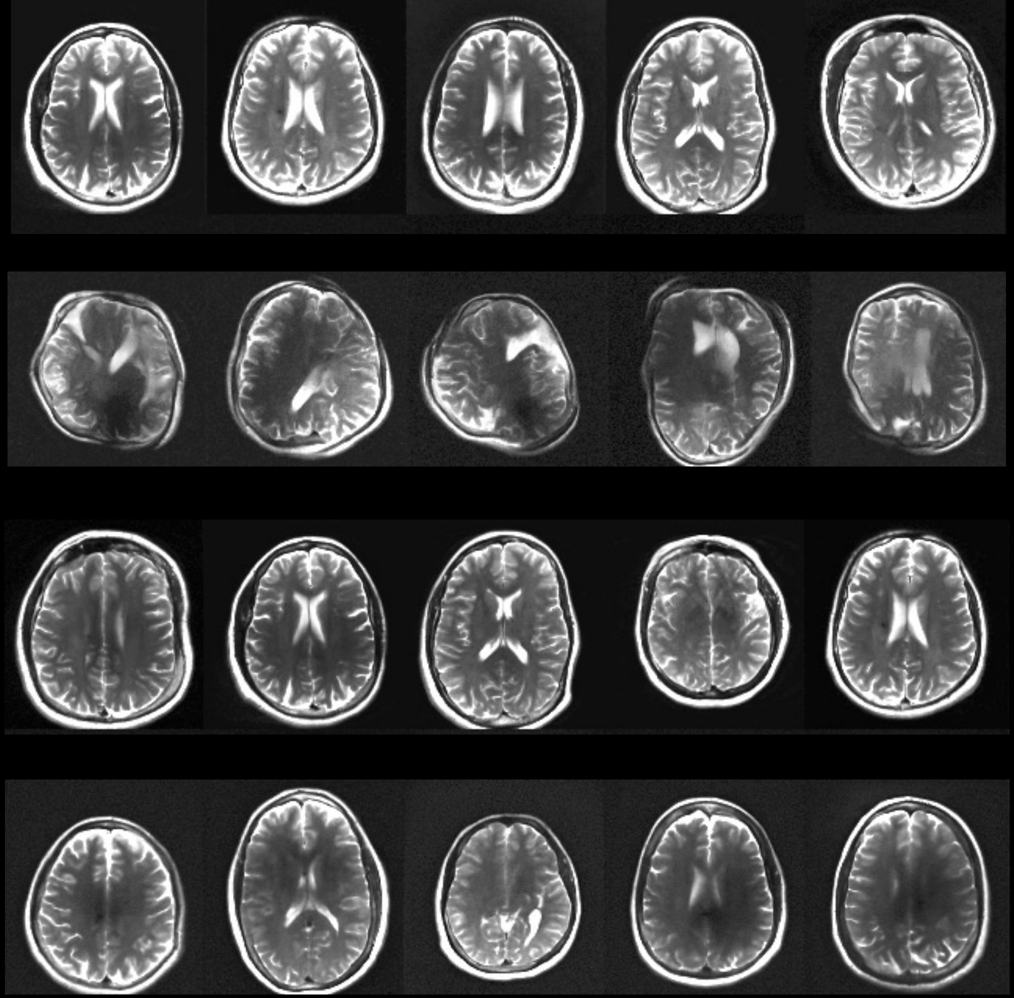
$p(x)$

Naive Score
~ 32dB

Naive Score
~ 22dB

GSURE-Score
~ 32dB

GSURE-Score
~ 22dB

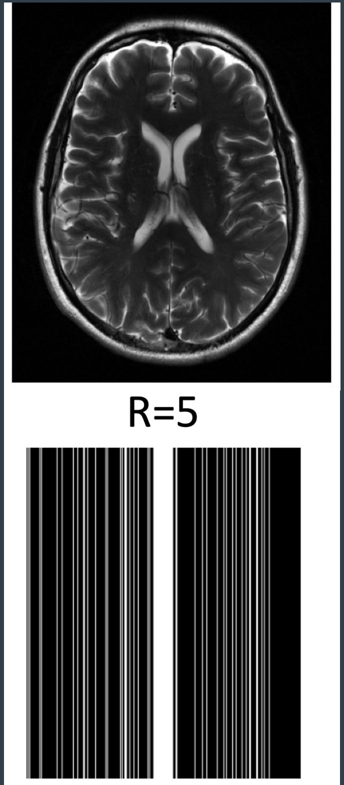


Results

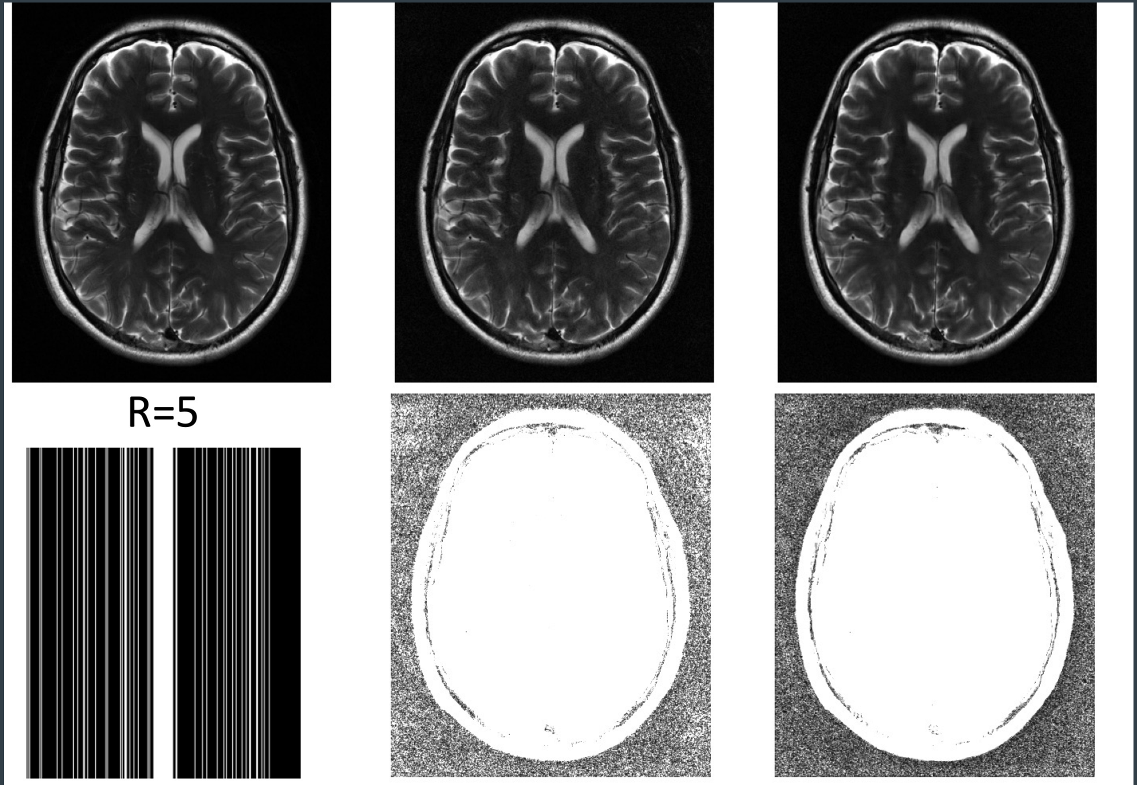
1. Evaluation of **Self-Supervised Denoising** (GSURE)
2. **Prior sampling** performance of score models trained on Noisy vs. GSURE Denoised data
3. **Posterior Reconstruction** performance of score models trained on Noisy vs. GSURE Denoised data

Posterior Sampling $x \sim p(x|y)$

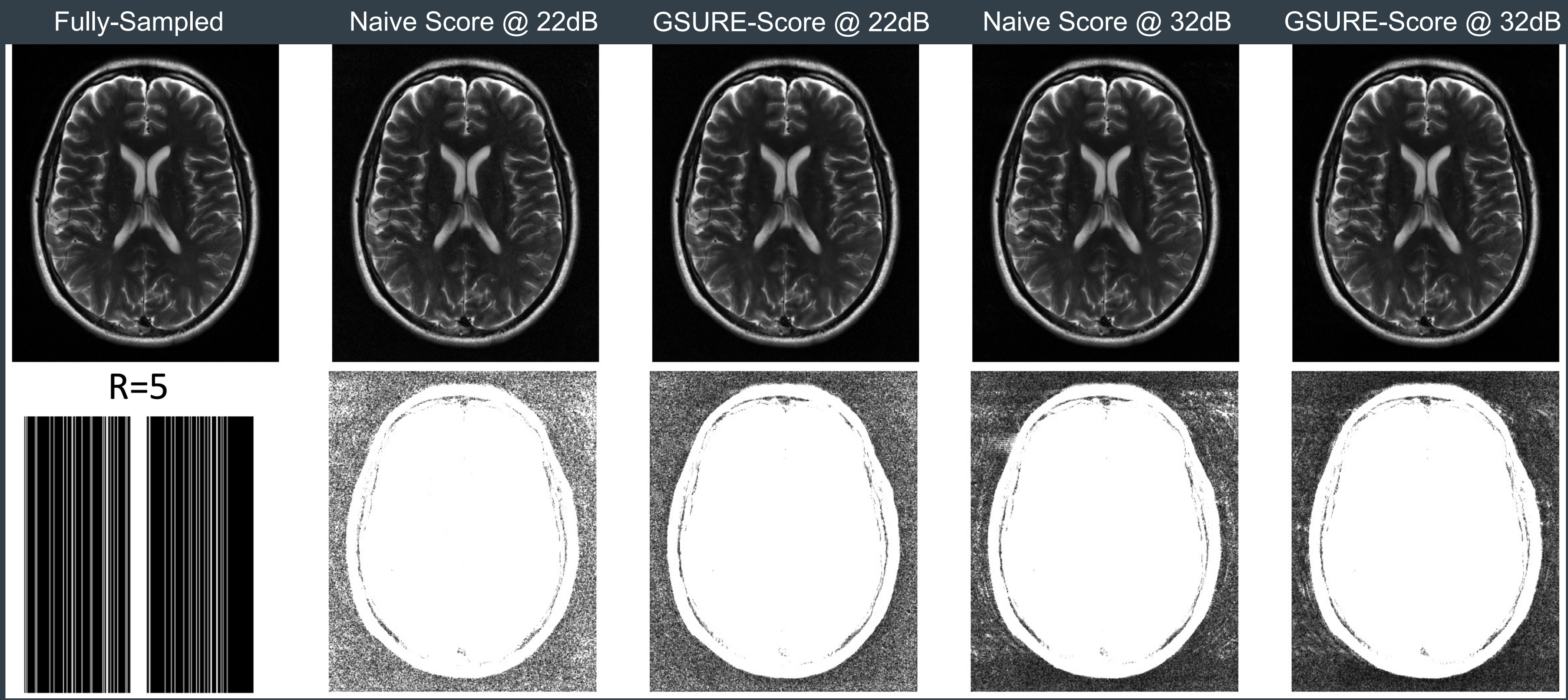
Fully-Sampled | Naive Score @ 22dB | GSURE-Score @ 22dB | Naive Score @ 32dB | GSURE-Score @ 32dB



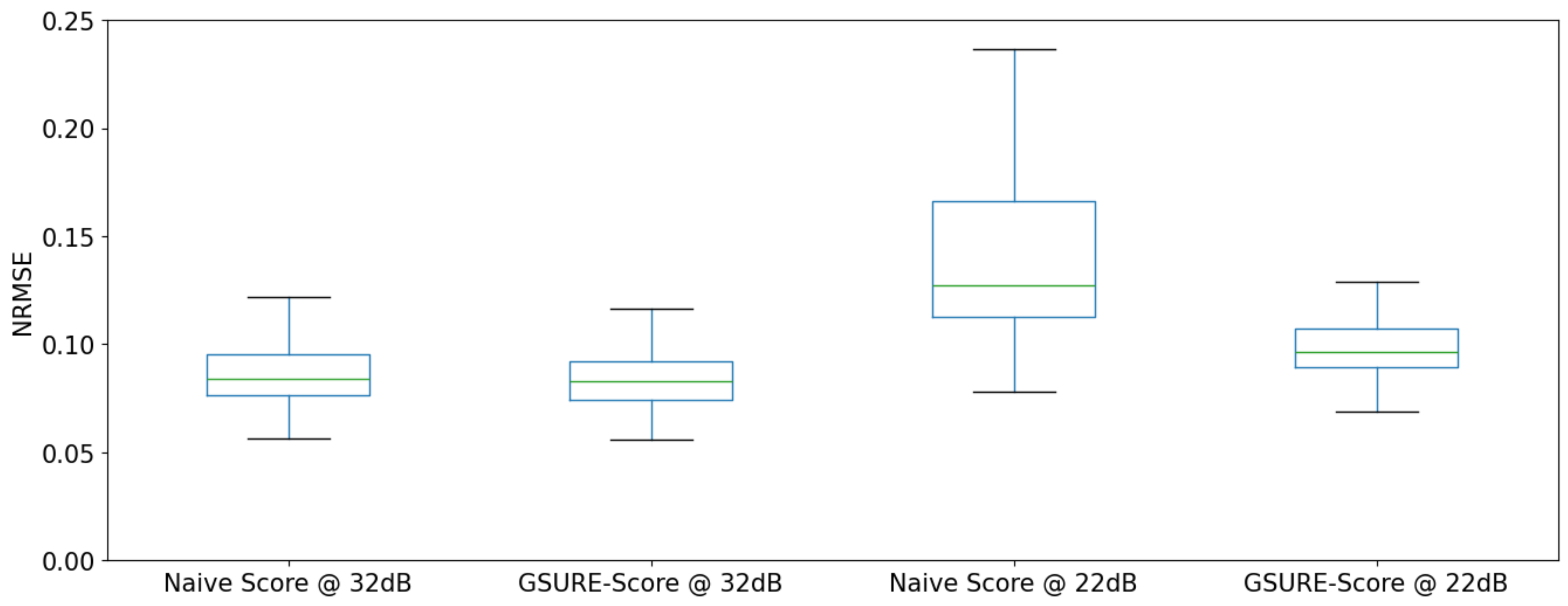
Posterior Sampling $x \sim p(x|y)$



Posterior Sampling $x \sim p(x|y)$



Posterior Sampling $x \sim p(x|y)$



Discussion and Conclusion

1. Self-supervised denoising techniques can successfully **remove noise**
2. Denoising as a pre-processing step helps train more **accurate priors**
3. The benefit of denoising is more visible in **lower SNR** settings
4. Priors trained on denoised FastMRI are **better inverse problem solvers** than naive training
5. We see application in the low-field setting (neo-natal MRI)

Future Works

A is a Linear Forward Operator (**Fully-Sampled**) -> GSURE[1,2,3]

$$y = FSx + noise$$

Assume A is a **Low-Rank** Forward Operator[4]

$$y = \boldsymbol{P}FSx + noise$$

[1]Soltanayev, *NeurIPS,* 2018, [2]Eldar, *IEEE Transactions on Signal Processing,* 2008, [3]Kawar, *TMLR,* 2023, [4]Aali, AmbientDPS, *Arxiv*, 2024

Thank you!

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Source Code: https://github.com/utcsilab/GsureScore-Diffusion.git