Let's wrap this up! Incremental structured decoding with resource constraints

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Main Idea

- Language models have trouble with single-shot constraint satisfaction
- Typically solved via rejection sampling or backtracking style decoders
- We implement an incremental structured decoder for autoregressive LLMs
- Guarantees monotonic progress and preservation of resource constraints
- Ensures all valid words are generable and all generable words are valid

Motivation

Suppose we want to force an autoregressive LLM to generate syntactically valid next tokens $P(x_n \mid x_1, \ldots, x_{n-1})$, under certain resource constraints. Here is a concrete example: "Generate an arithmetic expression with two or more variables in ten or fewer tokens.". If we sample the partial trajectory,

`( x + ( y * (`

then we will spend quite a long time rejecting invalid completions, because this trajectory has passed the point of no return. Even though `(` is a locally valid continuation, we need to avoid this scenario, because we would like a linear sampling delay and to guarantee this, we must avoid backtracking.

Semiring Parsing

Given a CFG, $G : \mathcal{G} = \langle V, \Sigma, P, S\rangle$, in Chomsky Normal Form (CNF), we may construct a recognizer $R_\mathcal{G} : \Sigma^n \to \mathbb{B}$ for strings $\sigma : \Sigma^n$ as follows. Let 2^V be our domain, where 0 is $\varnothing$, $\oplus$ is $\cup$, and $\otimes$ be defined as:

$$s_1 \otimes s_2 = \{C \mid \langle A, B\rangle \in s_1 \times s_2, (C \to AB) \in P\}$$

If we define $\hat\sigma_r = \{w \mid (w \to \sigma_r) \in P\}$, then construct a matrix with unit nonterminals on the superdiagonal, $M_0[r+1=c](G,\sigma) = \hat\sigma_r$ the fixpoint $M_{i+1} = M_i + M_i^2$ is fully determined by the first diagonal:

$$M_0 = \begin{pmatrix} \varnothing & \hat\sigma_1 & \varnothing & \cdots & \varnothing \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \varnothing \\ & & & & \hat\sigma_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix} \Rightarrow \begin{pmatrix} \varnothing & \hat\sigma_1 & \Lambda & \cdots & \varnothing \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \Lambda \\ & & & & \hat\sigma_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix} \Rightarrow \ldots \Rightarrow M_\infty = \begin{pmatrix} \varnothing & \hat\sigma_1 & \Lambda & \cdots & \Lambda_\sigma^* \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \Lambda \\ & & & & \hat\sigma_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix}$$

CFL membership is recognized by $R(G,\sigma) = [S \in \Lambda_\sigma^*] \Leftrightarrow [\sigma \in \mathcal{L}(G)]$.

Porous Completion

Let us consider an example with two holes, $\sigma = 1\ \_\ \_$, with the grammar being $G = \{S \to NON, O \to + \mid \times, N \to 0 \mid 1\}$. This can be rewritten into CNF as $G' = \{S \to NL, N \to 0 \mid 1, O \to \times \mid +, L \to ON\}$.

	2^V	$\mathbb{B}^{\lvert V\rvert}$	$\mathbb{B}^{\lvert V\rvert} \to \mathbb{B}^{\lvert V\rvert}$
M_0	$\begin{pmatrix} \{N\} & & \\ & \{N,O\} & \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & & \\ & \square\blacksquare\blacksquare\square & \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & & \\ & V_{1,2} & \\ & & V_{2,3} \end{pmatrix}$
M_1	$\begin{pmatrix} \{N\} & \varnothing & \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & V_{0,2} & \\ & V_{1,2} & V_{1,3} \\ & & V_{2,3} \end{pmatrix}$
M_∞	$\begin{pmatrix} \{N\} & \varnothing & \{S\} \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \square\square\square\blacksquare \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & V_{0,2} & V_{0,3} \\ & V_{1,2} & V_{1,3} \\ & & V_{2,3} \end{pmatrix}$

This procedure decides if $\exists \sigma' \in \mathcal{L}(G) \mid \sigma' \sqsubseteq \sigma$ but forgets provenance.

Regular Expression Propagation

Regular expressions that permit union, intersection and concatenation are called generalized regular expressions (GREs). These can be constructed as follows:

$$\begin{aligned} &\mathcal{L}(\ \varnothing\ ) = \varnothing && \mathcal{L}(\ R^*\ ) = \{\varepsilon\} \cup \mathcal{L}(R \cdot R^*) \\ &\mathcal{L}(\ \varepsilon\ ) = \{\varepsilon\} && \mathcal{L}(\ R \vee S\ ) = \mathcal{L}(R) \cup \mathcal{L}(S) \\ &\mathcal{L}(\ a\ ) = \{a\} && \mathcal{L}(\ R \wedge S\ ) = \mathcal{L}(R) \cap \mathcal{L}(S) \\ &\mathcal{L}(\ R \cdot S\ ) = \mathcal{L}(R) \times \mathcal{L}(S) && \mathcal{L}(\ \neg R\ ) = \Sigma^* \setminus \mathcal{L}(R) \end{aligned}$$

Finite slices of a CFL are finite and therefore regular a fortiori. Just like sets, bitvectors and other datatypes, we can propagate GREs through a parse chart. Here, the algebra will carry GRE$^{|V|}$, where $0 = [\varepsilon]_{v\in V}$, and $\oplus, \otimes$ are defined:

$$s_1 \otimes s_2 = \Big[\bigvee_{(v \to AB) \in P} s_1[A] \cdot s_2[B]\Big]_{v \in V} \qquad s_1 \oplus s_2 = \big[s_1[v] \vee s_2[v]\big]_{v\in V}$$

Initially, we have $M_0[r+1=c](G,\sigma) = \Sigma$. Now after computing the fixpoint, when we unpack $\Lambda_\sigma^*[S]$, this will be a GRE recognizing the finite CFL slice.

Brzozowski Differentiation

Janusz Brzozowski (1964) introduced a derivative operator $\partial_a : \textsc{Reg} \to \textsc{Reg}$, which slices a given prefix off a language: $\partial_a L = \{b \in \Sigma^* \mid ab \in L\}$. The Brzozowski derivative over a GRE is effectively a normalizing rewrite system:

$$\begin{aligned} &\partial_a(\ \varnothing\ ) = \varnothing && \delta(\ \varnothing\ ) = \varnothing \\ &\partial_a(\ \varepsilon\ ) = \varnothing && \delta(\ \varepsilon\ ) = \varepsilon \\ &\partial_a(\ a\ ) = \varepsilon && \delta(\ a\ ) = \varnothing \\ &\partial_a(\ b\ ) = \varnothing \text{ for each } a \neq b && \delta(\ R^*\ ) = \varepsilon \\ &\partial_a(\ R^*\ ) = (\partial_x R) \cdot R^* && \delta(\ \neg R\ ) = \varepsilon \text{ if } \delta(R) = \varnothing \\ &\partial_a(\ \neg R\ ) = \neg \partial_a R && \delta(\ \neg R\ ) = \varnothing \text{ if } \delta(R) = \varepsilon \\ &\partial_a(\ R \cdot S\ ) = (\partial_a R) \cdot S \vee \delta(R) \cdot \partial_a S && \delta(\ R \cdot S\ ) = \delta(R) \wedge \delta(S) \\ &\partial_a(\ R \vee S\ ) = \partial_a R \vee \partial_a S && \delta(\ R \vee S\ ) = \delta(R) \vee \delta(S) \\ &\partial_a(\ R \wedge S\ ) = \partial_a R \wedge \partial_a S && \delta(\ R \wedge S\ ) = \delta(R) \wedge \delta(S) \end{aligned}$$

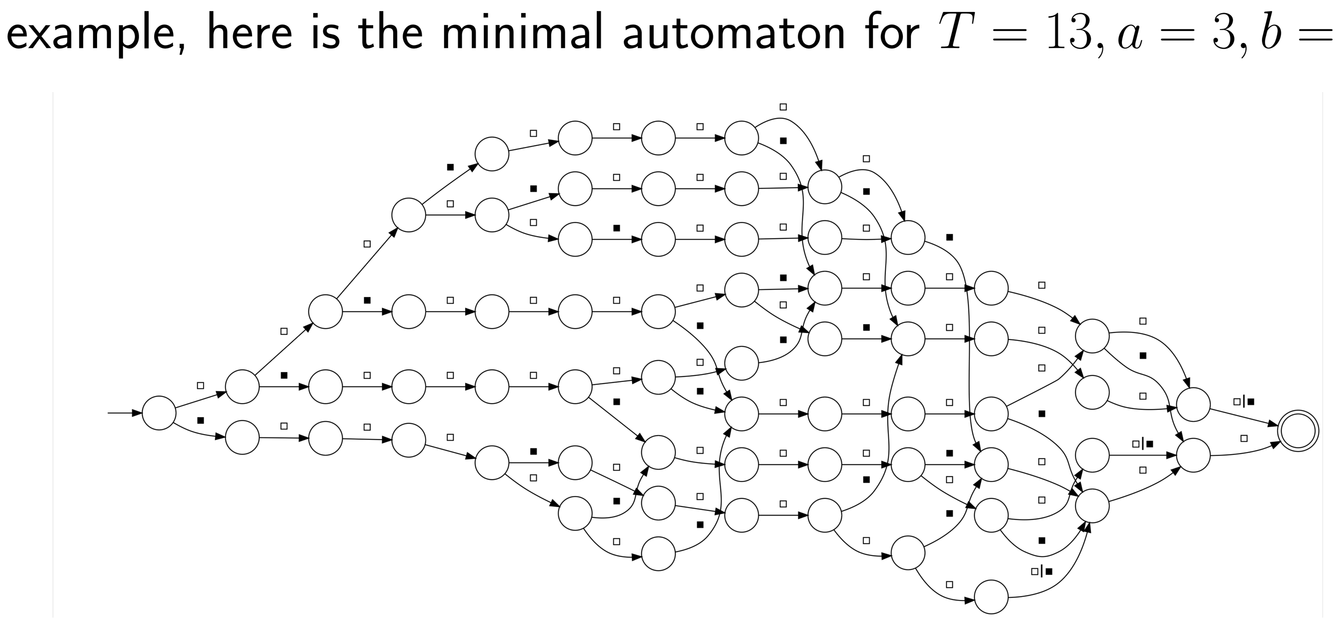
The key property we care about is, this formulation allows us to sample lazily from language intersections, without first materializing the product automaton.

Example: Determinantal Point Processes

Consider a time series, A, whose points which are not too close nor far apart, and $n \leq \sum_{i=1}^{|A|} \mathbf{1}[A_i = \blacksquare]$. We want to sample the typical set using an LLM.

- The words are bitvectors of some length, T, i.e., $A = \{\square, \blacksquare\}^T$
- Consecutive $\blacksquare$ separated by $\square^{[a,b]}$, i.e., $B = \square^*(\blacksquare\square^{[a,b]})^{[n,\infty)}\{\blacksquare, \epsilon\}\square^*$

The DPP language is regular. Let C be an FSA such that $\mathcal{L}(C) = \mathcal{L}(A) \cap \mathcal{L}(B)$. For example, here is the minimal automaton for $T = 13, a = 3, b = 5, n = 2$.



This automaton for $\mathcal{L}(C)$ can grow very large, and we may only need to sample a small sublanguage with distributional support. Question: Can we incrementally subsample $\mathcal{L}(C)$ while ensuring partial trajectories always lead to acceptance?