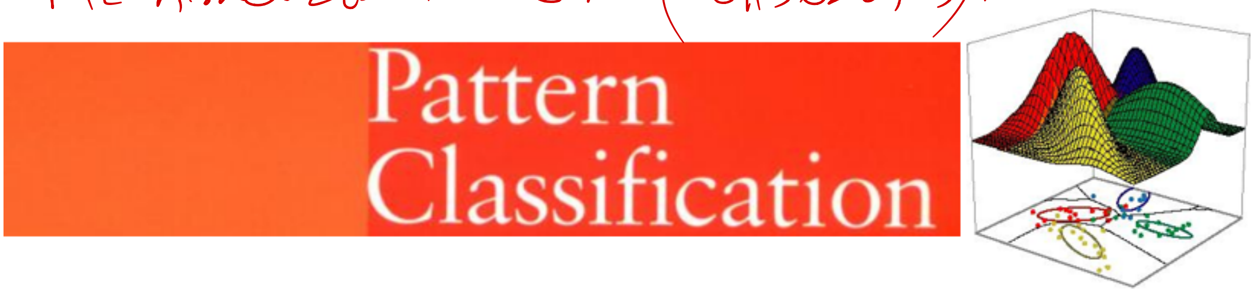
Md Asaduzzaman Jabin (811562873), ECE, UGA



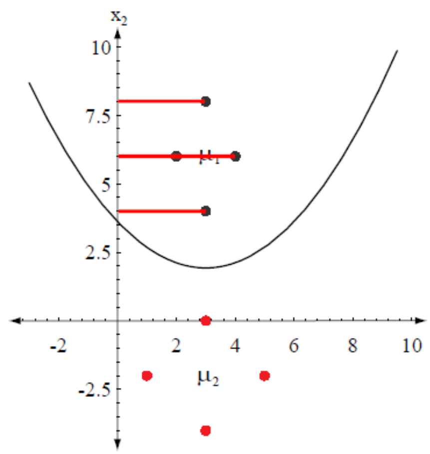
Due 10/06/2022

Instructor: Thirimachos Bourlai

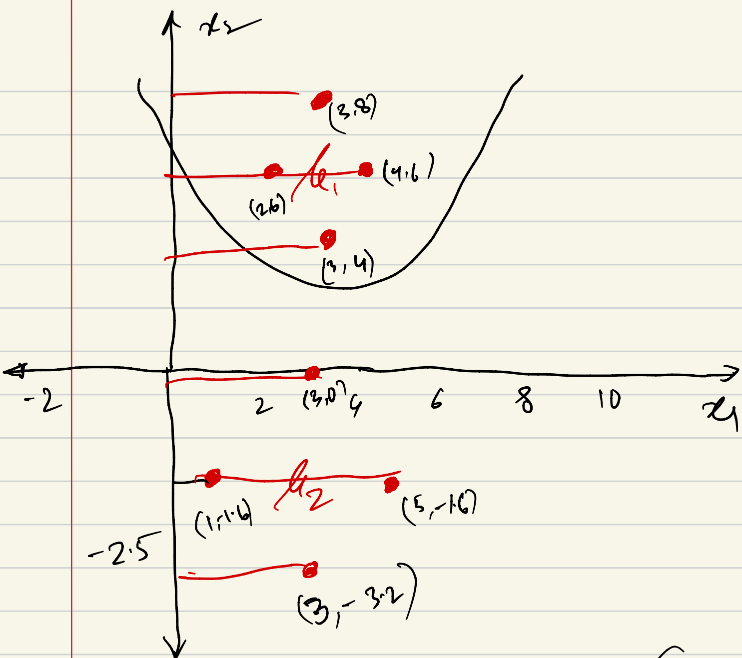
Homework 3

Calculate the decision boundary for the two-category two-dimensional data in the Example figure.

Use Duda's book example and solution (Section 2.6, after figure 2.16) and work on it, providing ALL details until you determine the boundary: setting the observations (samples), showing on paper step by step how you calculate the means, covariance matrices, determinants, inverse matrices, how do you compute each part of the Eqs. 64 – 67, before in the end having the same equation (decision boundary) as the one provided by Duda on the same page.



The computed Bayes decision boundary for two Gaussian distributions, each based on four data points.



We know,

$$\mu = \mathcal{E}[x] = \int x \, p(x) \, dx \quad \cdots \text{(1)}$$

$$\Sigma = \mathcal{E}\left[(x-\mu)(x-\mu)^T\right] = \int (x-\mu)(x-\mu)^T P(x) \, dx \quad \cdots \text{(2)}$$

$$g_i(x) = x^T W_i x + w_i^T x + w_{i0} \quad \cdots \text{(3)}$$

$$W_i = -\frac{1}{2} \Sigma_i^{-1} \quad \cdots \text{(4)}$$

$$w_i = \Sigma_i^{-1} \mu_i \quad \cdots \text{(5)}$$

$$w_{i0} = -\frac{1}{2} \mu_i^T \Sigma_i^{-1} \mu_i - \frac{1}{2} \ln |\Sigma_i| + \ln P(\omega_i) \quad \cdots \text{(6)}$$

So, Total mean $\mu_{1x} = \frac{3+3+2+4}{4}$

$= 3$

$\mu_{1y} = \frac{6+6+8+4}{4}$

$= 6$

So, $\boxed{\mu_1 = \begin{bmatrix}3\\6\end{bmatrix}}$

So, Total mean $\mu_{2x} = \frac{3+3+1+5}{4}$

$= 3$

$\mu_{1y} = \frac{0 + -1.6 + -1.6 + -3.2}{3}$

$= -2$

$\boxed{\text{So, } \mu_2 = \begin{bmatrix}3\\-2\end{bmatrix}}$

So, calculate variance, $\Sigma = \frac{\sum_{i=1}^{n}(x_i - x_{avg})(y_i - y_{avg})}{n-1}$

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From 7 $\Rightarrow$

$$\text{So,}\quad \Sigma_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \therefore \Sigma_1^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \therefore \Sigma_2^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

Now, from eq 3 - 6 $\Rightarrow$

$$P(W_1) = P(W_2) = 0.5$$

$$\therefore g_1(x) = g_2(x)$$

by calculating eq 6, 5, 4 $\Rightarrow$

$$g_1(x) = g_2(x) \Rightarrow$$

$$x_2 = 3.514 - 1.125x_1 + 0.1875x_1^2$$

here, it's a parabola equation with vertex $\begin{pmatrix} 3 \\ 1.83 \end{pmatrix}$

So, this would be the 2D decision boundary (Ans)