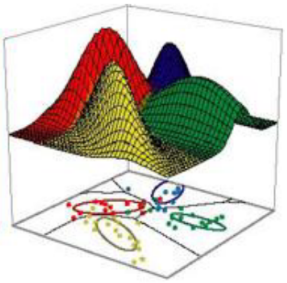
Pattern Classification



Instructor: Thirimachos Bourlai

Homework 1

Due 08/29/22, 12.30 p.m.

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***Instructions**: Solve the problems using a pen and submit right before next Wednesday's class.*

- **Problem 1**: A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34. The probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?
- **Problem 2**: A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which can detect any part that is obviously defective and discard it. What is the probability that a part is good given that it passed the inspection machine?
- **Problem 3**: Your neighbor has 2 children. You learn that he has a son, Joe. What is the probability that Joe's sibling is a brother? Joe's sibling is equally likely to have been born male or female suggests that the probability the other child is a boy is 1/2. Is this correct?
- **Problem 4**: Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both defective fuses in the first two tests?
- **Problem 5**: Six cards are selected at random (without replacement) from a standard deck of 52 cards. What is the probability there will be no pairs? (Two cards of the same denomination)
- **Problem 6**: Consider the parts problem again, but now assume that a one-year warranty is given for the parts that are shipped to customers. Suppose that a good part fails within the first year with probability 0.01, while a slightly defective part fails within the

first year with probability 0.10. What is the probability that a customer receives a part that fails within the first year and therefore is entitled to a warranty replacement?

- **Problem 7**:
  - Urn 1 contains 5 white balls and 7 black balls.
  - Urn 2 contains 3 whites and 12 black.
  - A fair coin is flipped; if it is Heads, a ball is drawn from Urn 1, and if it is Tails, a ball is drawn from Urn 2.
  - Suppose that this experiment is done, and you learn that a white ball was selected. What is the probability that this ball was in fact taken from Urn 2? (i.e., that the coin flip was Tails)

Problem 1: A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34. The probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Solve:

$$P(\text{white} \mid \text{black}) = \frac{P(\text{black} \cap \text{white})}{P(\text{black})}$$

$$= \frac{0.34}{0.47} = 0.723 \quad (\text{Ans})$$

Problem 2: A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which can detect any part that is obviously defective and discard it. What is the probability that a part is good given that it passed the inspection machine?

∴ Pass inspection probability $= P(O\bar{d})$

here $P(g)$ & $P(\bar{O}d)$ are independent probability

$= 1 - P(Od)$

$= 1 - 0.08$

$= 0.92$

$P(g) = 0.9$

$P(\bar{g}d) = 0.02$

$P(Od) = 0.08$

$\therefore P(g \mid O\bar{d}) = \frac{P(g \cap \bar{O}\bar{d})}{P(\bar{O}\bar{d})} = \frac{P(g)}{P(\bar{O}\bar{d})}$ (g & $\bar{O}\bar{d}$ independent) $= \frac{0.9}{0.92}$

$= 0.978$

(Ans)

Problem 3: Your neighbor has 2 children. You learn that he has a son, Joe. What is the probability that Joe's sibling is a brother? Joe's sibling is equally likely to have been born male or female suggests that the probability the other child is a boy is 1/2. Is this correct?

∴ here, Sample space $n(S) = 2$

event of son $n(A) = 1$

∴ probability $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2} = 0.5$

So, yes, probability of child is a boy is $1/2$. (Ans)

Problem 4: Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both defective fuses in the first two tests?

here, first test & 2nd test are totally independent. So we can use multiplication rule.

$$P(T_2|T_1) = \frac{P(T_2 \cap T_1)}{P(T_1)}$$

$$\Rightarrow P(T_2 \cap T_1) = P(T_1) \cdot P(T_2|T_1)$$

$$= \frac{2}{7} \cdot \frac{1}{6}$$

$$= \frac{2}{42} = 0.04761$$

$n(S) = 7$
$n(T_1) = 2$
$n(T_2) = 1$

(Ans)

Problem 5: Six cards are selected at random (without replacement) from a standard deck of 52 cards. What is the probability there will be no pairs? (Two cards of the same denomination)

All test events are independent, so we could apply multiplication rule.

4 color cards have same value

$$P(T_1 \cap T_2 \cap T_3 \cap T_3 \cdots) = P(T_1)\, P(T_2|T_1)\, P(T_3|(T_2 \cap T_1)) \cdots$$

4 color card

$$= \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48} \cdot \frac{32}{47}$$

$$= 0.34$$

Problem 6: Consider the parts problem again, but now assume that a one-year warranty is given for the parts that are shipped to customers. Suppose that a good part fails within the first year with probability 0.01, while a slightly defective part fails within the
first year with probability 0.10. What is the probability that a customer receives a part
that fails within the first year and therefore is entitled to a warranty replacement?

From problem 2

probability of parts good & passed machine

$$P(g|\text{passed}) = 0.978 = P(g)$$

probability of part failed & passed machine

$$P(\bar{g}) = 1 - 0.978 = 0.022$$

Total probability of failed

$$P = P(g)\, P(f|g) + P(sd)\, P(f|sd)$$

$$= 0.9 \times 0.01 + 0.02 \times 0.1$$

$$= 0.011$$

$P(g) = 0.9$

$P(f|g) = 0.01$

$P(f|sd) = 0.10$

$P(sd) = 0.02$

Problem 7:

- Urn 1 contains 5 white balls and 7 black balls.
- Urn 2 contains 3 whites and 12 black.
- A fair coin is flipped; if it is Heads, a ball is drawn from Urn 1, and if it is Tails, a ball is drawn from Urn 2.
- Suppose that this experiment is done, and you learn that a white ball was selected. What is the probability that this ball was in fact taken from Urn 2? (i.e., that the coin flip was Tails)

$$P_T(\text{white} \mid T) = \frac{P(\text{white})\,P(T \mid \text{white})}{\sum P(\text{white})\,P(T_i \mid \text{white})}$$

$$= \frac{\frac{3}{15} \cdot \frac{1}{2}}{\underbrace{3/15 \cdot 1/2}_{\text{Tail}} + \underbrace{5/12 \times 1/2}_{\text{head}}}$$

$$= 0.3243$$

Ans