# Artificial Neural Network : Training

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# Learning of neural networks: Topics

- Concept of learning
- Learning in
    - Single layer feed forward neural network
    - multilayer feed forward neural network
    - recurrent neural network
- Types of learning in neural networks

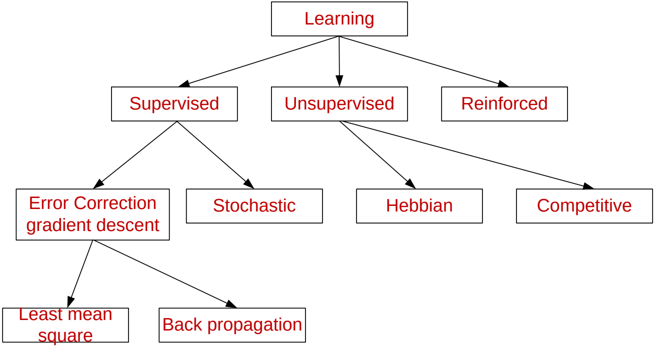
# Concept of Learning

# The concept of learning

- The learning is an important feature of human computational ability.
- Learning may be viewed as the change in behavior acquired due to practice or experience, and it lasts for relatively long time.
- As it occurs, the effective coupling between the neuron is modified.
- In case of artificial neural networks, it is a process of modifying neural network by updating its weights, biases and other parameters, if any.
- During the learning, the parameters of the networks are optimized and as a result process of curve fitting.
- It is then said that the network has passed through a learning phase.

# Types of learning

- There are several learning techniques.
- A taxonomy of well known learning techniques are shown in the following.



In the following, we discuss in brief about these learning techniques.

# Different learning techniques: Supervised learning

- **Supervised learning**

  In this learning, every input pattern that is used to train the network is associated with an output pattern.

- This is called ”training set of data”. Thus, in this form of learning, the input-output relationship of the training scenarios are available.
- Here, the output of a network is compared with the corresponding target value and the error is determined.
- It is then feed back to the network for updating the same. This results in an improvement.
- This type of training is called learning with the help of teacher.

# Different learning techniques: Unsupervised learning

- **Unsupervised learning**

  If the target output is not available, then the error in prediction can not be determined and in such a situation, the system learns of its own by discovering and adapting to structural features in the input patterns.

- This type of training is called learning without a teacher.

# Different learning techniques: Reinforced learning

- **Reinforced learning**

  In this techniques, although a teacher is available, it does not tell the expected answer, but only tells if the computed output is correct or incorrect. A reward is given for a correct answer computed and a penalty for a wrong answer. This information helps the network in its learning process.

- **Note :** Supervised and unsupervised learnings are the most popular forms of learning. Unsupervised learning is very common in biological systems.

  It is also important for artificial neural networks : training data are not always available for the intended application of the neural network.

# Different learning techniques : Gradient descent learning

- **Gradient Descent learning :**
  This learning technique is based on the minimization of error $E$ defined in terms of weights and the activation function of the network.
- Also, it is required that the activation function employed by the network is differentiable, as the weight update is dependent on the gradient of the error $E$.
- Thus, if $\Delta W_{ij}$ denoted the weight update of the link connecting the $i$-th and $j$-th neuron of the two neighboring layers then

  $$\Delta W_{ij} = \eta \frac{\partial E}{\partial W_{ij}}$$

  where $\eta$ is the **learning rate parameter** and $\frac{\partial E}{\partial W_{ij}}$ is the **error gradient** with reference to the weight $W_{ij}$
- The least mean square and back propagation are two variations of this learning technique.

# Different learning techniques : Stochastic learning

- **Stochastic learning**

  In this method, weights are adjusted in a probabilistic fashion. Simulated annealing is an example of such learning (proposed by Boltzmann and Cauch)

# Different learning techniques: Hebbian learning

## Hebbian learning

- This learning is based on correlative weight adjustment. This is, in fact, the learning technique inspired by biology.
- Here, the input-output pattern pairs $(x_i, y_i)$ are associated with the weight matrix $W$. $W$ is also known as the correlation matrix.
- This matrix is computed as follows.

  $$W = \sum_{i=1}^{n} X_i Y_i^T$$

  where $Y_i^T$ is the transpose of the associated vector $y_i$

# Different learning techniques : Competitive learning

- **Competitive learning**

  In this learning method, those neurons which responds strongly to input stimuli have their weights updated.

- When an input pattern is presented, all neurons in the layer complete and the winning neuron undergoes weight adjustment.
- This is why it is called a Winner-takes-all strategy.

In this course, we discuss a generalized approach of supervised learning to train different type of neural network architectures.

# Training SLFFNNs

# Single layer feed forward NN training

- We know that, several neurons are arranged in one layer with inputs and weights connect to every neuron.
- Learning in such a network occurs by adjusting the weights associated with the inputs so that the network can classify the input patterns.
- A single neuron in such a neural network is called perceptron.
- The algorithm to **train a perceptron** is stated below.
- Let there is a perceptron with $(n + 1)$ inputs $x_0, x_1, x_2, \cdots, x_n$ where $x_0 = 1$ is the bias input.
- Let $f$ denotes the transfer function of the neuron. Suppose, $\bar{X}$ and $\bar{Y}$ denotes the input-output vectors as a training data set. $\bar{W}$ denotes the weight matrix.

With this input-output relationship pattern and configuration of a perceptron, the algorithm **Training Perceptron** to train the perceptron is stated in the following slide.

# Single layer feed forward NN training

1. Initialize $\bar{W} = w_0, w_1, \cdots, w_n$ to some random weights.
2. For each input pattern $x \in \bar{X}$ do Here, $x = \{x_0, x_1, ... x_n\}$
   - Compute $I = \sum_{i=0}^{n} w_i x_i$
   - Compute observed output $y$

$$y = f(I) = \begin{cases} 1 & , \text{if } I > 0 \\ 0 & , \text{if } I \leq 0 \end{cases}$$

$\bar{Y'} = \bar{Y'} + y$ Add $y$ to $\bar{Y'}$, which is initially empty

3. If the desired output $\bar{Y}$ matches the observed output $\bar{Y'}$ then output $\bar{W}$ and exit.
4. Otherwise, update the weight matrix $\bar{W}$ as follows :
   - For each output $y \in \bar{Y'}$ do
   - If the observed out $y$ is 1 instead of 0, then $w_i = w_i - \alpha x_i$, $(i = 0, 1, 2, \cdots n)$
   - Else, if the observed out $y$ is 0 instead of 1, then $w_i = w_i + \alpha x_i$, $(i = 0, 1, 2, \cdots n)$
5. Go to step 2.

# Single layer feed forward NN training

In the above algorithm, $\alpha$ is the learning parameter and is a constant decided by some empirical studies.

**Note :**

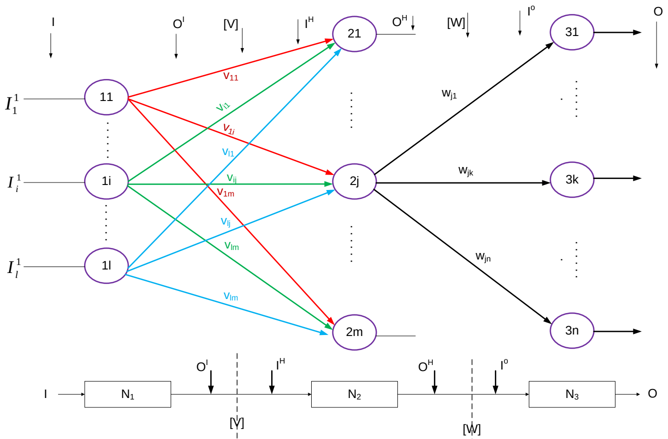
- The algorithm Training Perceptron is based on the supervised learning technique
- ADALINE : Adaptive Linear Network Element is also an alternative term to perceptron
- If there are 10 number of neutrons in the single layer feed forward neural network to be trained, then we have to iterate the algorithm for each perceptron in the network.

# Training MLFFNNs

# Training multilayer feed forward neural network

- Like single layer feed forward neural network, supervisory training methodology is followed to train a multilayer feed forward neural network.
- Before going to understand the training of such a neural network, we redefine some terms involved in it.
- A block digram and its configuration for a three layer multilayer FF NN of type $l - m - n$ is shown in the next slide.

# Specifying a MLFFNN



Input layer

$|N_1|=l$

Hidden Layer

$|N_2|=m$

Output Layer

$|N_3|=n$

Linear transfer function, ɵ

$$f_i^l = (I_i, \theta_i)$$

Log-Sigmoid transfer function

$$f_j^m = (I_j^H, \alpha_j) = \frac{1}{1+e^{-\alpha_j I_j^H}}$$

Tan-Sigmoid transfer function

$$f_k^n = (I_k^o, \alpha_k) = \frac{e^{+\alpha_o I_o} - e^{-\alpha_o I_o}}{e^{+\alpha_o I_o} + e^{-\alpha_o I_o}}$$

# Specifying a MLFFNN

- For simplicity, we assume that all neurons in a particular layer follow same transfer function and different layers follows their respective transfer functions as shown in the configuration.
- Let us consider a specific neuron in each layer say *i*-th, *j*-th and *k*-th neurons in the input, hidden and output layer, respectively.
- Also, let us denote the weight between i-th neuron ($i = 1, 2, \cdots, l$) in input layer to j-th neuron ($j = 1, 2, \cdots, m$) in the hidden layer is denoted by $v_{ij}$.
- The weight matrix between the input to hidden layer say $V$ is denoted as follows.

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1j} & \cdots & v_{1m} \\ v_{21} & v_{22} & \cdots & v_{2j} & \cdots & v_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{i1} & v_{i2} & \cdots & v_{ij} & \cdots & v_{im} \\ v_{l1} & v_{l2} & \cdots & v_{lj} & \cdots & v_{lm} \end{bmatrix}$$

# Specifying a MLFFNN

- Similarly, $w_{jk}$ represents the connecting weights between $j - th$ neuron($j = 1, 2, \cdots, m$) in the hidden layer and $k$-th neuron ($k = 1, 2, \cdots n$) in the output layer as follows.

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1k} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2k} & \cdots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{j1} & w_{j2} & \cdots & w_{jk} & \cdots & w_{jn} \\ w_{m1} & w_{m2} & \cdots & w_{mk} & \cdots & w_{mn} \end{bmatrix}$$

# Learning a MLFFNN

Whole learning method consists of the following three computations:

1. **Input layer computation**
2. **Hidden layer computation**
3. **Output layer computation**

In our computation, we assume that $< T_0, T_I >$ be the training set of size $|T|$.

# Input layer computation

- Let us consider an input training data at any instant be $I^I = [I_1^1, I_2^1, \cdots, I_i^1, I_l^1]$ where $I^I \in T_I$
- Consider the outputs of the neurons lying on input layer are the same with the corresponding inputs to neurons in hidden layer. That is,

$$O^I = I^I$$

$$[l \times 1] = [l \times 1] \qquad \text{[Output of the input layer]}$$

- The input of the j-th neuron in the hidden layer can be calculated as follows.

$$I_j^H = v_{1j}o_1^I + v_{2j}o_2^I +, \cdots, +v_{ij}o_j^I + \cdots + v_{lj}o_l^I$$

where $j = 1, 2, \cdots m$.

[Calculation of input of each node in the hidden layer]

- In the matrix representation form, we can write

$$I^H = V^T \cdot O^I$$

$$[m \times 1] = [m \times l]\ [l \times 1]$$

# Hidden layer computation

- Let us consider any j-th neuron in the hidden layer.
- Since the output of the input layer's neurons are the input to the j-th neuron and the j-th neuron follows the log-sigmoid transfer function, we have

  $$O_j^H = \frac{1}{1+e^{-\alpha_H \cdot I_j^H}}$$

  where $j = 1, 2, \cdots, m$ and $\alpha_H$ is the constant co-efficient of the transfer function.

# Hidden layer computation

Note that all output of the nodes in the hidden layer can be expressed as a one-dimensional column matrix.

$$O^H = \begin{bmatrix} \cdots \\ \cdots \\ \vdots \\ \frac{1}{1+e^{-\alpha_H \cdot I_j^H}} \\ \vdots \\ \cdots \\ \cdots \end{bmatrix}_{m\times 1}$$

# Output layer computation

Let us calculate the input to any k-th node in the output layer. Since, output of all nodes in the hidden layer go to the k-th layer with weights $w_{1k}, w_{2k}, \cdots, w_{mk}$, we have

$$I_k^O = w_{1k} \cdot o_1^H + w_{2k} \cdot o_2^H + \cdots + w_{mk} \cdot o_m^H$$

where $k = 1, 2, \cdots, n$

In the matrix representation, we have

$$I^O = W^T \cdot O^H$$
$$[n \times 1] = [n \times m]\ [m \times 1]$$

# Output layer computation

Now, we estimate the output of the k-th neuron in the output layer. We consider the tan-sigmoid transfer function.

$$O_k = \frac{e^{\alpha_o \cdot I_k^o} - e^{-\alpha_o \cdot I_k^o}}{e^{\alpha_o \cdot I_k^o} + e^{-\alpha_o \cdot I_k^o}}$$

for $k = 1, 2, \cdots, n$

Hence, the output of output layer's neurons can be represented as

$$O = \begin{bmatrix} \cdots \\ \cdots \\ \vdots \\ \frac{e^{\alpha_o \cdot I_k^o} - e^{-\alpha_o \cdot I_k^o}}{e^{\alpha_o \cdot I_k^o} + e^{-\alpha_o \cdot I_k^o}} \\ \vdots \\ \cdots \\ \cdots \end{bmatrix}_{n \times 1}$$

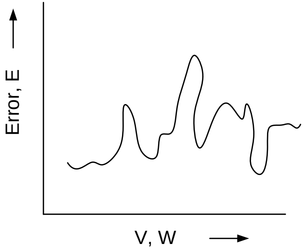
# Back Propagation Algorithm

- The above discussion comprises how to calculate values of different parameters in $l - m - n$ multiple layer feed forward neural network.
- Next, we will discuss how to train such a neural network.
- We consider the most popular algorithm called Back-Propagation algorithm, which is a supervised learning.
- The principle of the **Back-Propagation algorithm** is based on the error-correction with **Steepest-descent method**.
- We first discuss the method of steepest descent followed by its use in the training algorithm.

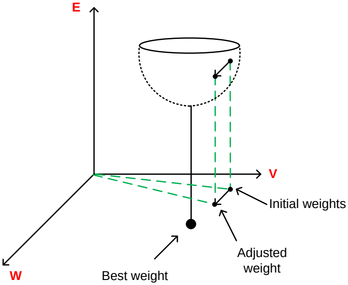
# Method of Steepest Descent

- Supervised learning is, in fact, error-based learning.
- In other words, with reference to an external (teacher) signal (i.e. target output) it calculates error by comparing the target output and computed output.
- Based on the error signal, the neural network should modify its configuration, which includes synaptic connections, that is , the weight matrices.
- It should try to reach to a state, which yields minimum error.
- In other words, its searches for a suitable values of parameters minimizing error, given a training set.
- Note that, this problem turns out to be an optimization problem.

# Method of Steepest Descent



(a) Searching for a minimum error



(b) Error surface with two parameters V and W

# Method of Steepest Descent

- For simplicity, let us consider the connecting weights are the only design parameter.
- Suppose, $V$ and $W$ are the wights parameters to hidden and output layers, respectively.
- Thus, given a training set of size $N$, the error surface, $E$ can be represented as

  $$E = \sum_{i=1}^{N} e^{i}\left(V, W, I_i\right)$$

  where $I_i$ is the i-th input pattern in the training set and $e^{i}(...)$ denotes the error computation of the i-th input.
- Now, we will discuss the steepest descent method of computing error, given a changes in $V$ and $W$ matrices.

# Method of Steepest Descent

- Suppose, A and B are two points on the error surface (see figure in Slide 30). The vector $\vec{AB}$ can be written as

$$\vec{AB} = (V_{i+1} - V_i) \cdot \bar{x} + (W_{i+1} - W_i) \cdot \bar{y} = \Delta V \cdot \bar{x} + \Delta W \cdot \bar{y}$$

The gradient of $\vec{AB}$ can be obtained as

$$e_{\vec{AB}} = \frac{\partial E}{\partial V} \cdot \bar{x} + \frac{\partial E}{\partial W} \cdot \bar{y}$$

Hence, the unit vector in the direction of gradient is

$$\bar{e}_{\vec{AB}} = \frac{1}{|e_{\vec{AB}}|} \left[ \frac{\partial E}{\partial V} \cdot \bar{x} + \frac{\partial E}{\partial W} \cdot \bar{y} \right]$$

# Method of Steepest Descent

- With this, we can alternatively represent the distance vector AB as

$$\vec{AB} = \eta \left[ \frac{\partial E}{\partial V} \cdot \bar{x} + \frac{\partial E}{\partial W} \cdot \bar{y} \right]$$

where $\eta = \frac{k}{|e_{\vec{AB}}|}$ and $k$ is a constant

- So, comparing both, we have

$$\Delta V = \eta \frac{\partial E}{\partial V}$$
$$\Delta W = \eta \frac{\partial E}{\partial W}$$

This is also called as **delta rule** and $\eta$ is called **learning rate**.

# Calculation of error in a neural network

- Let us consider any k-th neuron at the output layer. For an input pattern $I_i \in T_I$ (input in training) the target output $T_{Ok}$ of the k-th neuron be $T_{Ok}$.
- Then, the error $e_k$ of the k-th neuron is defined corresponding to the input $I_i$ as

$$e_k = \frac{1}{2}\left(T_{Ok} - O_{Ok}\right)^2$$

where $O_{Ok}$ denotes the observed output of the k-th neuron.

# Calculation of error in a neural network

- For a training session with $I_i \in T_I$, the error in prediction considering all output neurons can be given as

$$e = \sum_{k=1}^{n} e_k = \frac{1}{2}\sum_{k=1}^{n} (T_{Ok} - O_{Ok})^2$$

  where $n$ denotes the number of neurons at the output layer.

- The total error in prediction for all output neurons can be determined considering all training session $< T_I, T_O >$ as

$$E = \sum_{\forall I_i \in T_I} e = \frac{1}{2}\sum_{\forall t \in <T_I, T_O>} \sum_{k=1}^{n} (T_{Ok} - O_{Ok})^2$$

# Supervised learning : Back-propagation algorithm

- The back-propagation algorithm can be followed to train a neural network to set its topology, connecting weights, bias values and many other parameters.
- In this present discussion, we will only consider updating weights.
- Thus, we can write the error $E$ corresponding to a particular training scenario $T$ as a function of the variable $V$ and $W$. That is

$$E = f(V, W, T)$$

- In BP algorithm, this error $E$ is to be minimized using the gradient descent method. We know that according to the gradient descent method, the changes in weight value can be given as

$$\Delta V = -\eta \frac{\partial E}{\partial V} \tag{1}$$

and

$$\Delta W = -\eta \frac{\partial E}{\partial W} \tag{2}$$

# Supervised learning : Back-propagation algorithm

- Note that $-ve$ sign is used to signify the fact that if $\frac{\partial E}{\partial V}$ (or $\frac{\partial E}{\partial W}$) $> 0$, then we have to decrease $V$ and vice-versa.

- Let $v_{ij}$ (and $w_{jk}$) denotes the weights connecting i-th neuron (at the input layer) to j-th neuron(at the hidden layer) and connecting j-th neuron (at the hidden layer) to k-th neuron (at the output layer).

- Also, let $e_k$ denotes the error at the k-th neuron with observed output as $O_{O_k^o}$ and target output $T_{O_k^o}$ as per a sample intput $I \in T_I$.

# Supervised learning : Back-propagation algorithm

- It follows logically therefore,

  $$e_k = \frac{1}{2}(T_{O_k^o} - O_{O_k^o})^2$$

  and the weight components should be updated according to equation (1) and (2) as follows,

  $$\bar{w_{jk}} = w_{jk} + \Delta w_{jk} \tag{3}$$

  where $\Delta w_{jk} = -\eta \frac{\partial e_k}{\partial w_{jk}}$
  and

  $$\bar{v_{ij}} = v_{ij} + \Delta v_{ij} \tag{4}$$

  where $\Delta v_{ij} = -\eta \frac{\partial e_k}{\partial v_{ij}}$
- Here, $v_{ij}$ and $w_{ij}$ denotes the previous weights and $\bar{v_{ij}}$ and $\bar{w_{ij}}$ denote the updated weights.
- Now we will learn the calculation $\bar{w_{ij}}$ and $\bar{v_{ij}}$, which is as follows.

# Calculation of $\bar{w_{jk}}$

We can calculate $\frac{\partial e_k}{\partial w_{jk}}$ using the chain rule of differentiation as stated below.

$$\frac{\partial e_k}{\partial w_{jk}} = \frac{\partial e_k}{\partial O_{O_k^o}} \cdot \frac{\partial O_{O_k^o}}{\partial I_k^o} \cdot \frac{\partial I_k^o}{\partial w_{jk}} \tag{5}$$

Now, we have

$$e_k = \frac{1}{2}(T_{O_k^o} - O_{O_k^o})^2 \tag{6}$$

$$O_{O_k^o} = \frac{e^{\theta_o I_k^o} - e^{-\theta_o I_k^o}}{e^{\theta_o I_k^o} + e^{-\theta_o I_k^o}} \tag{7}$$

$$I_k^o = w_{1k} \cdot O_1^H + w_{2k} \cdot O_2^H + \cdots + w_{jk} \cdot O_j^H + \cdots w_{mk} \cdot O_m^H \tag{8}$$

# Calculation of $\bar{w}_{jk}$

Thus,

$$\frac{\partial e_k}{\partial O_{O_k^o}} = -(T_{O_k^o} - O_{O_k^o}) \tag{9}$$

$$\frac{\partial O_{o_k^o}}{\partial I_k^o} = \theta_o(1 + O_{O_k^o})(1 - O_{O_k^o}) \tag{10}$$

and

$$\frac{\partial I_k^o}{\partial w_{ij}} = O_j^H \tag{11}$$

# Calculation of $\bar{w_{jk}}$

Substituting the value of $\frac{\partial e_k}{\partial O_{O_k^o}}$, $\frac{\partial O_{O_k^o}}{\partial I_k^o}$ and $\frac{\partial I_k^o}{\partial w_{jk}}$ we have

$$\frac{\partial e_k}{\partial w_{jk}} = -(T_{O_k^o} - O_{O_k^o}) \cdot \theta_o(1 + O_{O_k^o})(1 - O_{O_k^o}) \cdot O_j^H \tag{12}$$

Again, substituting the value of $\frac{\partial E_k}{\partial w_{jk}}$ from Eq. (12) in Eq.(3), we have

$$\Delta w_{jk} = \eta \cdot \theta_o(T_{O_k^o} - O_{O_k^o}) \cdot (1 + O_{O_k^o})(1 - O_{O_k^o}) \cdot O_j^H \tag{13}$$

Therefore, the updated value of $w_{jk}$ can be obtained using Eq. (3)

$$\bar{w_{jk}} = w_{jk} + \Delta w_{jk} = \eta \cdot \theta_o(T_{O_k^o} - O_{O_k^o}) \cdot (1 + O_{O_k^o})(1 - O_{O_k^o}) \cdot O_j^H + w_{jk} \tag{14}$$

# Calculation of $\bar{v}_{ij}$

Like, $\frac{\partial e_k}{\partial w_{jk}}$, we can calculate $\frac{\partial e_k}{\partial V_{ij}}$ using the chain rule of differentiation as follows,

$$\frac{\partial e_k}{\partial v_{ij}} = \frac{\partial e_k}{\partial O_{O_k^o}} \cdot \frac{\partial O_{O_k^o}}{\partial I_k^o} \cdot \frac{\partial I_k^o}{\partial O_j^H} \cdot \frac{\partial O_j^H}{\partial I_j^H} \cdot \frac{\partial I_j^H}{\partial v_{ij}} \tag{15}$$

Now,

$$e_k = \frac{1}{2}(T_{O_k^o} - O_{O_k^o})^2 \tag{16}$$

$$O_k^o = \frac{e^{\theta_o I_k^o} - e^{-\theta_o I_k^o}}{e^{\theta_o I_k^o} + e^{-\theta_o I_k^o}} \tag{17}$$

$$I_k^o = w_{1k} \cdot O_1^H + w_{2k} \cdot O_2^H + \cdots + w_{jk} \cdot O_j^H + \cdots w_{mk} \cdot O_m^H \tag{18}$$

$$O_j^H = \frac{1}{1 + e^{-\theta_H I_j^H}} \tag{19}$$

# Calculation of $\bar{v}_{ij}$

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$$I_j^H = v_{ij} \cdot O_1^H + v_{2j} \cdot O_2^H + \cdots + v_{ij} \cdot O_j^I + \cdots v_{ij} \cdot O_l^I \tag{20}$$

Thus

$$\frac{\partial e_k}{\partial O_{O_k^o}} = -(T_{O_k^o} - O_{O_k^o}) \tag{21}$$

$$\frac{\partial O_k^o}{\partial I_k^o} = \theta_o(1 + O_{O_k^o})(1 - O_{O_k^o}) \tag{22}$$

$$\frac{\partial I_k^o}{\partial O_j^H} = w_{ik} \tag{23}$$

$$\frac{\partial O_j^H}{\partial I_j^H} = \theta_H \cdot (1 - O_j^H) \cdot O_j^H \tag{24}$$

$$\frac{\partial I_j^H}{\partial v_j^i} = O_i^I = I_i^I \tag{25}$$

# Calculation of $\bar{v}_{ij}$

From the above equations, we get

$$\frac{\partial e_k}{\partial v_{ij}} = -\theta_o \cdot \theta_H (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \cdot O_j^H \cdot I_i^H \cdot w_{jk} \tag{26}$$

Substituting the value of $\frac{\partial e_k}{\partial v_{ij}}$ using Eq. (4), we have

$$\Delta v_{ij} = \eta \cdot \theta_o \cdot \theta_H (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \cdot O_j^H \cdot I_i^H \cdot w_{jk} \tag{27}$$

Therefore, the updated value of $v_{ij}$ can be obtained using Eq.(4)

$$\bar{v}_{ij} = v_{ij} + \eta \cdot \theta_o \cdot \theta_H (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \cdot O_j^H \cdot I_i^H \cdot w_{jk} \tag{28}$$

# Writing in matrix form for the calculation of $\bar{V}$ and $\bar{W}$

we have

$$\Delta w_{jk} = \eta \left| \theta_o \cdot (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \right| \cdot O_j^H \tag{29}$$

is the update for $k$-th neuron receiving signal from $j$-th neuron at hidden layer.

$$\Delta v_{ij} = \eta \cdot \theta_o \cdot \theta_H (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \cdot (1 - O_j^H) \cdot O_j^H \cdot I_i^I \cdot w_{jk} \tag{30}$$

is the update for $j$-th neuron at the hidden layer for the $i$-th input at the $i$-th neuron at input level.

# Calculation of $\bar{W}$

Hence,

$$[\Delta W]_{m \times n} = \eta \cdot \left[O^H\right]_{m \times 1} \cdot [N]_{1 \times n} \tag{31}$$

where

$$[N]_{1 \times n} = \left\{\theta_o(T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o})\right\} \tag{32}$$

where $k = 1, 2, \cdots n$
Thus, the updated weight matrix for a sample input can be written as

$$\left[\bar{W}\right]_{m \times n} = [W]_{m \times n} + [\Delta W]_{m \times n} \tag{33}$$

# Calculation of $\bar{V}$

Similarly, for $[\bar{V}]$ matrix, we can write

$$\Delta v_{ij} = \eta \cdot \left| \theta_o (T_{O_k^o} - O_{O_k^o}) \cdot (1 - O^2_{O_k^o}) \cdot w_{jk} \right| \cdot \left| \theta_H (1 - O_j^H) \cdot O_j^H \right| \cdot \left| I_i^I \right| \tag{34}$$

$$= \eta \cdot w_j \cdot \theta^H \cdot (1 - O_j^H) \cdot O_j^H \tag{35}$$

Thus,

$$\Delta V = \left[ I^I \right]_{l \times 1} \times \left[ M^T \right]_{1 \times m} \tag{36}$$

or

$$\left[ \bar{V} \right]_{l \times m} = [V]_{l \times m} + \left[ I^I \right]_{l \times 1} \times \left[ M^T \right]_{1 \times m} \tag{37}$$

This calculation of Eq. (32) and (36) for one training data $t \in < T_O, T_I >$. We can apply it in incremental mode (i.e. one sample after another) and after each training data, we update the networks $V$ and $W$ matrix.

# Batch mode of training

A batch mode of training is generally implemented through the minimization of mean square error (MSE) in error calculation. The MSE for k-th neuron at output level is given by

$$\bar{E} = \frac{1}{2} \cdot \frac{1}{|T|} \sum_{t=1}^{|T|} \left( {T^t}_{O_k^o} - {O^t}_{O_k^o} \right)^2$$

where $|T|$ denotes the total number of training scenariso and $t$ denotes a training scenario, i.e. $t \in < T_O, T_I >$
In this case, $\Delta w_{jk}$ and $\Delta v_{ij}$ can be calculated as follows

$$\Delta w_{jk} = \frac{1}{|T|} \sum_{\forall t \in T} \frac{\partial \bar{E}}{\partial W}$$

and

$$\Delta v_{ij} = \frac{1}{|T|} \sum_{\forall t \in T} \frac{\partial \bar{E}}{\partial V}$$

Once $\Delta w_{jk}$ and $\Delta v_{ij}$ are calculated, we will be able to obtain $\bar{w_{jk}}$ and $\bar{v_{ij}}$

# Any questions??