

Practice Problem - 3.5

1. int $a=0, b=0;$
 int $\text{matrix}[N][M];$
 for($i=0; i < N; i++$) {
 $a = a + i;$
 $\text{matrix}[i][i] = a;$ }
 for($j=0; j < M; j++$) {
 $b = b + 2*j;$ }

Hence, in first loop matrix $[i][i] = a$, for matrix must have the capability to access matrix $[N-1][N-1]$.

So, $M \geq N$

$$\text{a) Time complexity} = O(N) + O(M) \\ = O(N+M)$$

$$\text{b) Space complexity} = O(1) + O(N \cdot M) \\ = O(N \cdot M) \\ (\text{Ans})$$

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3. int $a=0, i=N;$
 vector<int> vec;
 while ($i > 0$) {
 $a += i;$
 $i /= 2;$
 vec.push_back(a);
 }

if $N=10$,
 $i=10, i=5, i=2, i=1, i=0$
 $\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \times$

Suppose, loop will run k times,

$$\therefore 2^k = n \\ k = \log_2 n$$

size of vector is also k

$$\therefore \text{Time complexity} = O(\log_2 n) \\ \therefore \text{Space} \quad n = O(\log_2 n)$$

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2. int $a=0;$
 for ($i=0; i < N; i++$) {
 for ($j=N; j > i; j--$) {
 $a = a + i + j;$ }
 }

1st loop will run N times.

2nd " " " $\frac{N}{i}$ times.

if $N=6$;

$$\begin{array}{ll} i=0 \rightarrow 6=N & \text{Sum} \\ i=1 \rightarrow 5=(N-1) & = 6N - 15 \\ i=2 \rightarrow 4=(N-2) & \\ i=3 \rightarrow 3=(N-3) & \text{F.C.} = O(N \cdot N) \\ i=4 \rightarrow 2=(N-4) & \\ i=5 \rightarrow 1=(N-5) & \end{array}$$

∴ Total

$$= N \times N - \frac{N(N-1)}{2}$$

$$= N^2 - \frac{N^2}{2} - \frac{N}{2}$$

$$= \frac{N^2}{2} - \frac{N}{2}$$

$$\approx N^2 - N$$

$$\approx N^2$$

∴ Time complexity = $O(N^2)$

∴ Space complexity = $O(1)$

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