A Complete Mathematical Walkthrough of AdaBoost Using a Small Dataset

Dataset Definition

We consider a binary classification dataset

$$D = \{(x_i, y_i)\}_{i=1}^{5}$$

x_i: feature vectors

y_i: True class label

$y_i \in \{+1, -1\}$

* Training sample:

Sample	True label y_i
x_1	$+1$
x_2	$+1$
x_3	-1
x_4	-1
x_5	$+1$

* Initialization of sample weights:

$$w_i^{(1)} = \frac{1}{N} = \frac{1}{5} = 0.2$$

Thus,

$$w_1 = w_2 = w_3 = w_4 = w_5 = 0.2$$

* Training the first weak learner

First weak learner: $h_1(x)$

Sample	True label y_i	Prediction $h_1(x)$
x_1	$+1$	$+1$
x_2	$+1$	-1 →
		-1

x_1	+1	+1 →
x_2	+1	−1
x_3	−1	−1 →
x_4	−1	+1
x_5	+1	

Misclassified samples: x_2, x_4

* Computing the weighted Error:

$$\epsilon_1 = \sum_{i=1}^{N} w_i \cdot \mathbb{I}(y_i \neq h_1(x_i))$$

$$\epsilon_1 = w_2 + w_4$$
$$= 0.2 + 0.2 = 0.4$$

* Compute model weight (alpha α)

$$\alpha = \frac{1}{2} \ln\left(\frac{1-\epsilon}{\epsilon}\right)$$

$$\alpha_1 = \frac{1}{2} \ln\left(\frac{1-\epsilon_1}{\epsilon_1}\right)$$
$$= \frac{1}{2} \ln\left(\frac{1-0.4}{0.4}\right)$$
$$= \frac{1}{2} \ln(1.5) = 0.202$$

* Update sample weights

Rule:

✓ Correctly Classified samples:

$$w_i^{new} = w_i^{(1)} \times e^{-\alpha}$$

Misclassified samples:

$$w_i^{new} = w_i^{(1)} \times e^{\alpha}$$

$$w_i^{new} = w_i^{(1)} \cdot e^{-\alpha y_i h_i(x_i)}$$

$-\alpha$

$y_i h_i(x_i)$

$w_i^{(1)} \cdot e^{+\alpha}$

$w_i^{(1)} \cdot e^{-\alpha}$

$(-1, +1)$

$h_1(x_i)$

$(-1, +1)$

y_i	$h_1(x_i)$	
−1	−1	+1
+1	+1	+1
−1	+1	−1
+1	−1	−1

$\Rightarrow$ $y_i h_1(x_i)$ equals +1 for correct prediction

$y_i h_1(x_i)$ equal −1 for misclassification

$w_i^{(1)} \cdot e$

$w_i^{(1)} \cdot e^{-\alpha}$

New weights!

$e^{\alpha} = e^{0.202}$

$= 1.224$

↓ misclassification

$e^{-\alpha} = e^{-0.202}$

$= 0.818$

↓ proper classification

Correct prediction (x_1, x_3, x_5):

$0.20 \times 0.818 = 0.1636$

Wrong prediction (x_2, x_4):

$0.20 \times 1.224 = 0.2448$

* Normalize weights:

Total weights $= 3 \times 0.1636 + 2 \times 0.2448$

$= 0.9804$

Normalize:

Sample	New weight
x_1	$0.1636 / 0.9804 = 0.167$
x_2	$0.2448 / 0.9804 = 0.25$
x_3	0.167
x_4	0.25
x_5	0.167

x_5

Final Strong Classifier:

$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \downarrow \alpha_t \downarrow h_t(x)\right)$$

sign +1 or -1