Project Report:
Stereo Matching using Graphcut-based Optimization

Seyed Abbas Sadat
Simon Fraser University
sas21@sfu.ca

I. INTRODUCTION

In this project we developed a stereo matching system based on graph-cut optimization framework [1]. In stereo matching problem, 2 images taken from 2 cameras that are on the same horizontal line are to be matched pixel by pixel. Therefore, for each pixel on the left image, we should find its corresponding pixel on the right image. According to the assumptions, we know that if (i, j) is the coordinate of the pixel in the left image, the coordinate of the corresponding pixel in the right image will be $(i, j - d)$. The unknown variable d is called disparity and is defined in the image space. Disparity is inversely proportional to the depth of the pixel, i.e., the higher the disparity the closer the object. The disparity is not always observable using only 2 images because part of the scene can be occluded in either of the left or right images and hence no matched pixel can be found.

The stereo matching problem is formulated as: given a left and right image, find the most probable disparity image. This *maximum a posteriori* problem is often solved by finding a disparity image that minimizes some energy function. The energy function includes some terms that promote data fidelity (data terms) as well as other terms that support prior assumptions about the solution, e.g., smoothness (smoothness term). These energy functions are non-convex and finding a global optimum it very difficult. Instead, approaches that result in approximation solutions are preferred. One way to reach a local optimum in the minimization is to perform gradient descent. Graph-cuts offer a framework that enables us to generate a movement (step) of current solution that is steepest in terms of minimizing the energy function. By generating and performing these steps iteratively, we reach a local optimum.

In section II we briefly explain how to generate gradient descent moves for a general pixel labelling problem.

II. GRAPHCUT-BASED OPTIMIZATION

The energy function that we use for stereo matching has the following form:

$$E(f) = \sum_{\{p,q\} \in \mathcal{N}} V_{p,q}(f_p, f_q) + \sum_{p \in \mathcal{P}} D_p(f_p)$$

Here $D_p(f_p)$ is the data term, i.e. cost of assigning label f_p to pixel p. This cost is calculated based on the input data (left and right images in stereo matching). The $V_{p,q}(f_p, f_q)$ term is the pairwise cost of two neighbours p, q having labels f_p, f_q respectively.

Assuming that we are at some arbitrary solution we generate an $\alpha\beta - swap$ move or $\alpha - expansion$ move. Next, if performing the new move reduces the energy function, we apply the move on the current solution and generate a new move again iteratively.

For computing the best move at each solution, i.e. a move with the largest reduction of energy function, we construct a graph network based on the current solution and the input images. The weights of the edges are carefully defined so that the cost of any *cut* in the created graph equals the cost of the specific move in the labelling problem (plus a constant). Therefore if we find the minimum cut of the graph, we can convert it to a move that reduces the energy function the most. The following sections describe $\alpha\beta - swap$ and $\alpha - expansion$ moves.

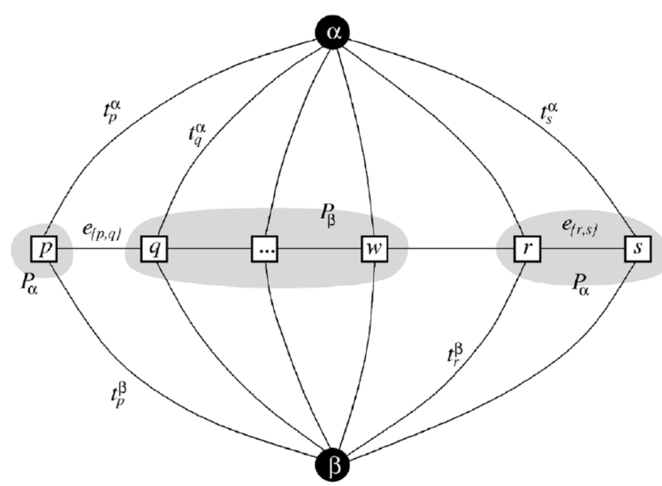


Fig. 1: The graph corresponding to a $\alpha\beta - swap$.

A. $\alpha\beta - swap$

In $\alpha\beta - swap$ move, subset of the pixel having label α can be changed to label β and vice versa. Therefore only the union of the pixel with α or β will change labels and other pixel remain the same. Therefore in each cycle of the algorithm this move is computed for each possible label pair and then is applied on the current solution if it reduces the energy function. In order to find the best possible label change in an $\alpha\beta - swap$, the minimum graph-cut problem shown in figure 1 is solved:

The terminals are the labels α and β and the middle nodes are all the pixels with either α or β label. The weight of the terminal edges are the data terms $D_p(\alpha)$ or $D_p(\beta)$, i.e. the cost of assigning the terminal label to the pixel. The weight of the edges between middle nodes are the pairwise smoothness term $V_{p,q}(\alpha, \beta)$. Please see [1] for details.

B. $\alpha - expansion$

In $\alpha - expansion$ move, a subset of the pixel that do not have label α can be changed to label α and labels of the rest of the pixels remain unchanged. Therefore in each cycle of the algorithm this move is computed for each possible label and then is applied on the current solution if it reduces the energy function. In order to find the best possible label change in an $\alpha - expansion$, the minimum graph-cut problem shown in figure 2 is solved:

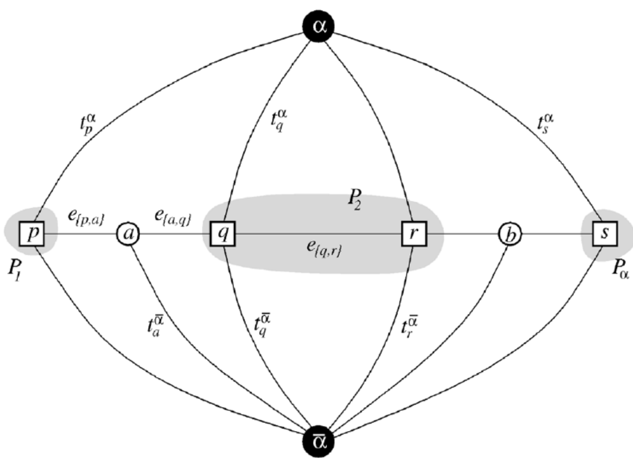


Fig. 2: The graph corresponding to a $\alpha - expansion$.

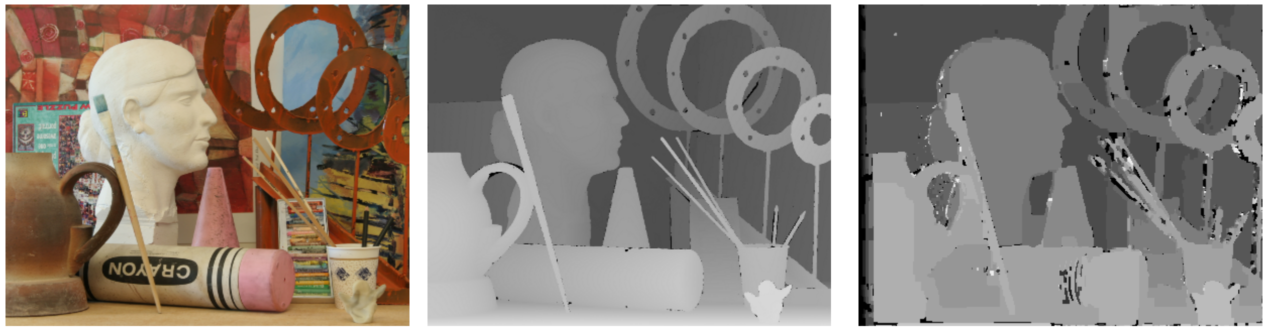
The terminals are the labels α and $\overline{\alpha}$. Selecting $\overline{\alpha}$ simply means that the pixel will keep its current label. The middle nodes are all the pixels with labels other than α. The weight of the terminal edges are the data terms $D_p(\alpha)$ or $D_p(f_p)$, i.e. the cost of assigning the terminal label to the pixel. Notice that $D_p(f_p)$ is used when $\overline{\alpha}$ is selected for pixel p and f_p is the current label of pixel p. The weight of the edges between middle nodes that with the same labels are the pairwise smoothness term $V_{p,q}(\alpha, f_p)$ in which f_p is the current label of the pair pixels. If the labels of the two neighbour pixels are different we add another axillary node with edges to both corresponding neighbour nodes with weights $V(\alpha, f_p)$ and $V(f_p, \alpha)$ and an edge to the terminal $\overline{\alpha}$ with weight $V(f_q, f_p)$. Please see [1] for details.

III. Stereo Matching

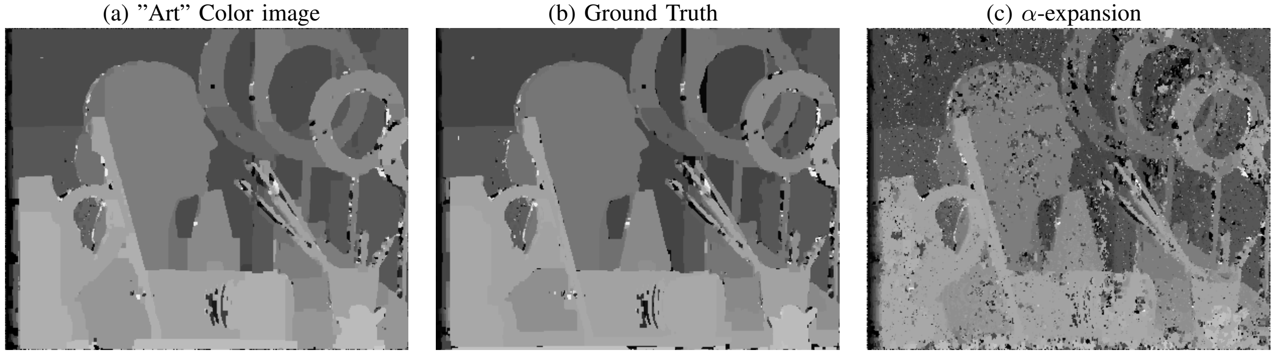
IV. Experiments and Results

References

[1] Y. Boykov, O. Veksler, and R. Zabih, "Fast approximate energy minimization via graph cuts," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 23, no. 11, pp. 1222–1239, 2001.

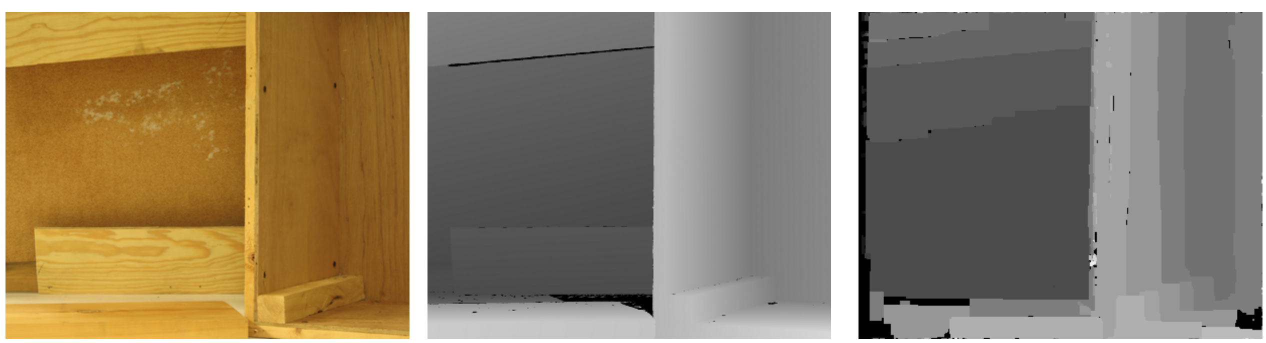


(a) "Art" Color image (b) Ground Truth (c) α-expansion

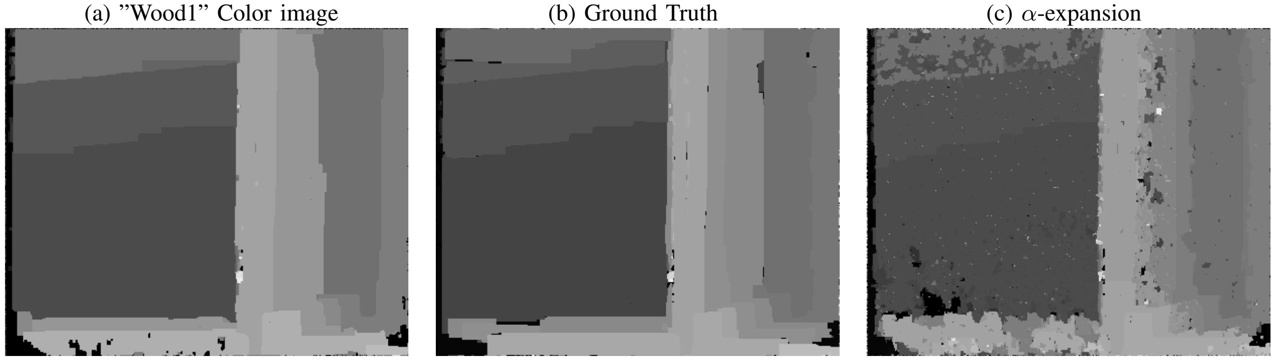


(d) $\alpha\beta$-swap (e) α-expansion - subpixel accuracy (f) $\alpha\beta$-swap - subpixel accuracy

Fig. 3: Disparity image for the "Art" dataset using the both $\alpha\beta$-swap and α-expansion with pixel and subpixel accuracies.



(a) "Wood1" Color image (b) Ground Truth (c) α-expansion



(d) $\alpha\beta$-swap (e) α-expansion - subpixel accuracy (f) $\alpha\beta$-swap - subpixel accuracy

Fig. 4: Disparity image for the "Wood1" dataset using the both $\alpha\beta$-swap and α-expansion with pixel and subpixel accuracies.