3-§. Bir noma'lumli birinchi darajali taqqoslamalar sistemasini yechish.

Ushbu birinchi darajali taqqoslamalar sistemasi

$$\grave{A}_1\tilde{o} \equiv \hat{A}_1 (\mathrm{mod}\ m_1), \quad A_2 x \equiv B_2 (\mathrm{mod}\, m_2), \quad \cdots \ , A_k x \equiv B_k (\mathrm{mod}\, m_k) \tag{1}$$

berilgan bo’lsin. Bu sistema yechimga ega bo’lishligi uchun avvalo (1) dagi har bir taqqoslama yechimga ega bo’lishi kerak. Bu taqqoslamalarning har birini yechib, (1) ni quyidagicha yozib olish mumkin.

$$x \equiv b_1 (\mathrm{mod}\, m_1, \quad x \equiv b_2 (\mathrm{mod}\, m_2), \cdots, x \equiv b_k (\mathrm{mod}\, m_k). \tag{2}$$

(2) sistemani yechaylik. (2) ning birinchi taqqoslamasidan

$$x = b_1 + m_1 t_1, \qquad t_1 \in Z. \tag{3}$$

Bulardan (2) dagi ikkinchi taqqoslamani qanoatlantiruvchilarini ajratib olamiz:

$$b_1 + m_1 t_1 \equiv b_2 (\mathrm{mod}\, m_2). \qquad (4)$$

Bundan $m_1 t_1 \equiv b_2 - b_1 (mod m_2)$. Faraz etaylik, $(m_1,m_2)=d$ bo'lsin. U holda agarda b_2-b_1 ayirma d ga bo'linmasa, (4) taqqoslama yechimga ega emas. Agarda $d|b_2$-b_1 bo'lsa, (4) d ta yechimga ega va

$$\frac{m_1}{d} t_1 \equiv \frac{b_2 - b_1}{d} \left(\mathrm{mod}\, \frac{m_2}{d} \right), \qquad \left(\frac{m_1}{d}, \frac{m_2}{d} \right) = 1 \qquad (5)$$

taqqoslama yagona $t_1 \equiv t' \left(mod \frac{m_2}{d} \right)$ yoki $t_1 = t' + \frac{m_2}{d} t_2,\ \ t_2 \in Z$ yechimga ega. $t_1$ning bu qiymatini (3) ga olib borib qo'yib (2) dagi birinchi 2 ta taqqoslamani qanoatlantiruvchi

$$x = b_1 + m_1 \left(t' + \frac{m_2}{d} t_2 \right) = b_1 + m_1 t' + \frac{m_1 m_2}{d} t_2 = b_1 + m_1 t' + [m_1, m_2] t_2$$

ni topamiz. Agarda $x_2 = b_1 + m_1 t'$ deb olsak, u holda

$$x = x_2 + [m_1, m_2] t_2 \quad \text{yoki} \quad x \equiv x_2 \left(\mathrm{mod} [m_1, m_2] \right)$$

ni hosil qilamiz. Shu usulni davom ettirib, $x \equiv x_k (mod\ [m_1, m_2, \ldots\ , m_k])$ ni, ya'ni (2) ning yechimini hosil qilamiz. (2)- sistemada $(m_1, m_j) = 1, \quad i \neq j$,

$M = m_1 \cdot m_2 \ldots m_k, M_i = \dfrac{M}{m_i}$ bo'lsin.U holda (2) -sistemaning yechimi $x \equiv x_0 (\mathrm{mod}\, M)$ bo'ladi. Bu yerda

$$x_0 = M_1 \cdot M_1' b + M_2 \cdot M_2' b_2 + \ldots + M_k M_k' \cdot b_k \qquad (6)$$

va $M_1', M_2', \ldots, M_k'$ lar ushbu taqqoslamalar sistemasidan aniqlanadi:

$$M_1 M_1' \equiv 1\ (\ \mathrm{mod}\ m_1), M_2 \cdot M_2' \equiv 1 (\mathrm{mod}\, m_2), \ \cdots, \quad M_k M_k' \equiv 1 (\mathrm{mod}\, m_k). \qquad (7)$$

(2)-sistemani yechish qadimgi xitoy masalasi deb ataluvchi $m_1$ga bo'lganda b_1, m_2 ga bo'lganda $b_2, \ldots, m_k$ ga bo'lganda $b_k$qoldiq qoluvchi x sonini toping degan masalaning o'zginasidir.

267. Taqqoslamalar sistemasini yeching:

1) $\begin{cases} x \equiv 6 (\mathrm{mod}\, 15) \\ x \equiv 18 (\mathrm{mod}\, 21), \\ x \equiv 3 (\mathrm{mod}\, 12) \end{cases}$ 2) $\begin{cases} x \equiv 13 (\mathrm{mod}\, 14) \\ x \equiv 6 (\mathrm{mod}\, 35) \\ x \equiv 26 (\mathrm{mod}\, 45) \end{cases}$ 3) $\begin{cases} x \equiv 19 (\mathrm{mod}\, 56) \\ x \equiv 3 (\mathrm{mod}\, 24) \\ x \equiv 7 (\mathrm{mod}\, 20) \end{cases}$

4) $\begin{cases} x \equiv 4 (\mathrm{mod}\, 5) \\ x \equiv 1 (\mathrm{mod}\, 12) \\ x \equiv 7 (\mathrm{mod}\, 14) \end{cases}$

5) $\begin{cases} x \equiv 13 \pmod{16} \\ x \equiv 3 \pmod{10} \\ x \equiv 9 \pmod{14} \end{cases}$ 6) $\begin{cases} x \equiv 9 \pmod{10} \\ x \equiv 10 \pmod{15} \\ x \equiv 11 \pmod{12} \end{cases}$ 7) $\begin{cases} x \equiv 7 \pmod{9} \\ x \equiv 2 \pmod{7} \\ x \equiv 3 \pmod{12} \end{cases}$ 8) $\begin{cases} x \equiv 5 \pmod{12} \\ x \equiv 2 \pmod{8} \\ x \equiv 2 \pmod{11} \end{cases}$

9) $\begin{cases} x \equiv 7 \pmod{10} \\ x \equiv 2 \pmod{5} \\ x \equiv 8 \pmod{9} \end{cases}$ 10) $\begin{cases} x \equiv 8 \pmod{7} \\ x \equiv 3 \pmod{11} \\ x \equiv 9 \pmod{13} \end{cases}$ 11) $\begin{cases} x \equiv 2 \pmod{5} \\ x \equiv 8 \pmod{11} \\ x \equiv 12 \pmod{15}. \end{cases}$

268. Modullari juft-jufti bilan o'zaro tub bo'lgan taqqoslamalar sistemasini yeching.

1) $\begin{cases} x \equiv 1 \pmod{6} \\ x \equiv 2 \pmod{7} \\ x \equiv 3 \pmod{11}, \end{cases}$ 2) $\begin{cases} 2x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \\ 3x \equiv 4 \pmod{11}, \end{cases}$ 3) $\begin{cases} 3x \equiv 1 \pmod{17} \\ 4x \equiv 3 \pmod{5} \\ 2x \equiv 5 \pmod{9}, \end{cases}$ 4) $\begin{cases} 5x \equiv 2 \pmod{9} \\ 3x \equiv -1 \pmod{13} \\ x \equiv 6 \pmod{11}, \end{cases}$

5) $\begin{cases} 6x \equiv 1 \pmod{35} \\ 3x \equiv 4 \pmod{17} \\ 10x \equiv 7 \pmod{13}, \end{cases}$ 6) $\begin{cases} 8x \equiv 7 \pmod{17} \\ 5x \equiv 11 \pmod{6} \\ x \equiv -1 \pmod{19}, \end{cases}$ 7) $\begin{cases} 11x \equiv -4 \pmod{18} \\ 7x \equiv 1 \pmod{11} \\ 3x \equiv 5 \pmod{7}, \end{cases}$ 8) $\begin{cases} 21x \equiv -2 \pmod{23} \\ 12x \equiv 3 \pmod{9} \\ x \equiv 6 \pmod{11}, \end{cases}$

9) $\begin{cases} x \equiv 3 \pmod{29} \\ x \equiv -5 \pmod{12} \\ 2x \equiv 7 \pmod{11}, \end{cases}$ 10) $\begin{cases} 6x \equiv 5 \pmod{31} \\ x \equiv -2 \pmod{29} \\ 5x \equiv 3 \pmod{27}, \end{cases}$ 11) $\begin{cases} x \equiv 1 \pmod{7} \\ x \equiv 3 \pmod{9} \\ x \equiv 5 \pmod{11}. \end{cases}$

269. m_1, m_2, m_3 sonlariga bo'lganda mos ravishda r_1, r_2, r_3 qoldiq qoluvchi eng kichik natural sonni toping.

№	m_1	m_2	m_3	r_1	r_2	r_3	№	m_1	m_2	m_3	r_1	r_2	r_3
1	7	8	9	1	2	3	**7**	13	21	23	9	1	13
2	3	4	5	1	2	3	**8**	3	5	8	2	4	1
3	9	10	13	3	5	6	**9**	3	5	8	2	4	1
4	4	5	7	2	3	4	**10**	5	7	9	4	6	1
5	3	7	8	2	4	5	**11**	7	13	17	6	12	16
6	7	13	17	4	9	1							

270. a ning qanday qiymatida berilgan taqqoslamalar sistemasi yechimga ega?

1) $\begin{cases} x \equiv 5(\bmod 18) \\ x \equiv 8(\bmod 21) \\ x \equiv a(\bmod 35), \end{cases}$ 2) $\begin{cases} x \equiv a(\bmod 7) \\ x \equiv 2(\bmod 9) \\ x \equiv 7(\bmod 11), \end{cases}$ 3) $\begin{cases} x \equiv 5(\bmod 12) \\ x \equiv a(\bmod 11) \\ x \equiv 3(\bmod 15), \end{cases}$ 4) $\begin{cases} x \equiv 11(\bmod 20) \\ x \equiv 1(\bmod 15) \\ x \equiv a(\bmod 18), \end{cases}$

5) $\begin{cases} x \equiv 19(\bmod 24) \\ x \equiv 10(\bmod 21) \\ x \equiv a(\bmod 9), \end{cases}$ 6) $\begin{cases} x \equiv 6(\bmod 15) \\ x \equiv 18(\bmod 21) \\ x \equiv a(\bmod 11), \end{cases}$ 7) $\begin{cases} x \equiv 19(\bmod 56) \\ x \equiv 3(\bmod 24) \\ x \equiv a(\bmod 20), \end{cases}$ 8) $\begin{cases} x \equiv 3(\bmod 5) \\ x \equiv 2(\bmod 7) \\ x \equiv a(\bmod 9), \end{cases}$

9) $\begin{cases} x \equiv 1(\bmod 3) \\ x \equiv 5(\bmod 7) \\ x \equiv a(\bmod 11), \end{cases}$ 10) $\begin{cases} x \equiv 14(\bmod 19) \\ x \equiv 5(\bmod 25) \\ x \equiv a(\bmod 10) \end{cases}$ 11) $\begin{cases} x \equiv 5(\bmod 11) \\ x \equiv 4(\bmod 7) \\ x \equiv a(\bmod 9). \end{cases}$

271. Absissalar o'qining qaysi butun nuqtalarida shu nuqtalardan o'tkazilgan perpendikulyar berilgan to'g'ri chiziqlarning barchasini bir vaqtda butun koordinatali nuqtalarda kesadi.
1) $x = 2 + 5y$, $x = 1 + 8y$, $x = 3 + 11y$;
2) $4x - 7y = 9$, $2x + 9y = 15$, $5x - 13y = 12$;
3) $3x - 5y = 1$, $2x + 3y = 3$, $5x - 7y = 7$;
4) $x + 7y = 2$, $x - 5y = 3$, $2x + 7y = 6$;
5) $2x - 3y = 1$, $x - 5y = 3$, $x - 11y = 2$;
6) $11x + 5y = 6$, $10x + 11y = 9$, $12x + 13y = -1$;
7) $3x - 7y = 5$, $5x - 8y = 4$, $11x + 13y = -2$;
8) $10x - 9y = 1$, $x - 7y = 3$, $x + 5y = 2$;
9) $11x + 17y = 5$, $19x - 37y = 1$, $11x - 7y = 4$;
10) $x - 19y = 2$, $5x - 13y = 1$, $10x + 13y = -3$;
11) $x - 7y = 5$, $3x + 8y = 7$, $x = 11 + 3y$.

272. a). Agar o'nlik sanoq sistemasidagi $N = 4x87y6$ sonining 56 ga bo'linishi ma'lum bo'lsa, uni toping.

b). Agar o'nlik sanoq sistemasidagi $N = xyz138$ sonining 7 ga bo'linishi, $138xyz$ sonini 13 ga bo'lganda qoldiq 6, $x1y3z8$ sonini 11 ga bo'lganda 5 qoldiq qolishi ma'lum bo'lsa, N ni toping.

c). Agar o'nlik sanoq sistemasidagi $N = 13xy45z$ sonining 792 ga bo'linishi ma'lum bo'lsa, x, y, z larni toping.

273. Taqqoslamalar sistemasini yeching:

$a)\begin{cases} x + 3y \equiv 5(mod7) \\ 4x \equiv 5(mod7), \end{cases}$ $b)\begin{cases} 9y \equiv 15(mod12) \\ 7x - 3y \equiv 1(mod12), \end{cases}$ $c)\begin{cases} x \equiv 2(mod4) \\ x - 2y \equiv 1(mod4), \end{cases}$

$$d)\begin{cases} 9y \equiv 15(mod12) \\ 3x - 7y \equiv 1(mod12), \end{cases} e)\begin{cases} 3x - 5y \equiv 1(mod12) \\ 9y \equiv 15(mod12). \end{cases}$$

274. Taqqoslamalar sistemasini yeching:

$$a)\begin{cases} x + 2y \equiv 3(mod5) \\ 4x + y \equiv 2(mod5), \end{cases} \qquad b)\begin{cases} x + 2y \equiv 0(mod5) \\ 3x + 2y \equiv 21(mod5), \end{cases}$$

$$c)\begin{cases} 3x + 4y \equiv 29(mod143) \\ 2x - 9y \equiv -84(mod143), \end{cases} \qquad d)\begin{cases} x + 2y \equiv 4(mod5) \\ 3x + y \equiv 2(mod5), \end{cases}$$

$$e)\begin{cases} x + 5y \equiv 5(mod6) \\ 5x + 3y \equiv 1(mod6), \end{cases} \qquad f)\begin{cases} 5x - y \equiv 3(mod6) \\ 2x + 2y \equiv -1(mod6). \end{cases}$$

$$g)\begin{cases} x - y \equiv 2(mod6) \\ 4x + 2y \equiv 2(mod6), \end{cases} \qquad h)\begin{cases} 4x - y \equiv 2(mod6) \\ 2x + 2y \equiv 0(mod6). \end{cases}$$

275. a)

$$\begin{cases} a_1x + b_1y \equiv c_1 \\ a_2x + b_2y \equiv c_2 \end{cases}(modm) \qquad (1)$$

taqqoslamalar sistemasida $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ bilan m o’zaro tub bo’lsa, u yagona yechimga ega ekanligini isbotlang.

b) (1)-taqqoslamalar sistemasida $(D, m) = d > 1$ bo’lsa, uning yechimga ega bo’lmaslik shartini toping.

c) (1)-taqqoslamalar sistemasida $D \equiv D_1 \equiv D_2 \equiv 0(modm)$ bo’lsa, uning yechimlari to’plami (1) dagi 1-taqqoslamaning yechimlari to’plami bilan bir xil bo’lishini isbotlang. Bunda

$$D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}.$$