VI.1 -§.

348. 1).Berilgan kasr $\frac{127}{52}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $127 = 52 \cdot \boxed{2} + 23;\ 52 = 23 \cdot \boxed{2} + 6;\ \ 23 = 6\ \cdot \boxed{3} + 5;\ 6 = 5\ \cdot \boxed{1} + 1;\ \ 5 = 1 \cdot \boxed{5}$. Bundan $\frac{127}{52} = (2,2,3,1,5)$. **Javob:**$(2,2,3,1,5)$.

2). Berilgan kasr $\frac{24}{35}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $24 = 35 \cdot \boxed{0} + 35;\ 35 = 24 \cdot \boxed{1} + 11;\ \ 24 = 11\ \cdot \boxed{2} + 2;\ 11 = 2\ \cdot \boxed{5} + 1;\ \ 2 = 1 \cdot \boxed{2}$. Bundan $\frac{127}{52} = (0,1,2,5,2)$. **Javob:**$(0,1,2,5,2)$.

3). Berilgan kasr $1{,}23 = \frac{123}{100}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $123 = 100 \cdot \boxed{1} + 23;\ 100 = 23 \cdot \boxed{4} + 8;\ \ 23 = 8\ \cdot \boxed{2} + 7;\ 8 = 7\ \cdot \boxed{1} + 1;\ \ 7 = 1 \cdot \boxed{7}$. Bundan $1{,}23 = (1,4,2,1,7)$. **Javob:**$(1,4,2,1,7)$.

4).Berilgan kasr $\frac{29}{37}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $29 = 37 \cdot \boxed{0} + 29;\ 37 = 29 \cdot \boxed{1} + 8;\ \ 29 = 8\ \cdot \boxed{3} + 5;\ 8 = 5\ \cdot \boxed{1} + 3;\ \ 5 = 3 \cdot \boxed{1} + 2,\ \ 3 = 2 \cdot \boxed{1} + 1,\ \ 2 = 1 \cdot \boxed{2}$. Bundan $\frac{29}{37} = (0,1,3,1,1,1,2)$. **Javob:**$(0,1,3,1,1,1,2)$.

349. Berilgan chekli uzluksiz kasrlarga mos qisqarmas oddiy kasr $\frac{a}{b}$ ni topish uchun munosib kasrlar $\frac{P_k}{Q_k}$ dan foydalanamiz. Bunda $(P_k, Q_k) = 1$ va $\frac{P_n}{Q_n} = \frac{a}{b}$.

1). $(1,1,2,1,2,1,2) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	2	1	2	1	2
P_i	$P_0 = 1$	1	2	5	7	19	26	71
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	11	15	41

Demak,$(1,1,2,1,2,1,2) = \frac{71}{41}$. **Javob:** $\frac{71}{41}$.

2). $(0,1,2,3,4,5) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	1	2	3	4	5
P_i	$P_0 = 1$	0	1	2	7	30	157
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	10	43	225

Demak,$(0,1,2,3,4,5) = \frac{157}{225}$. **Javob:** $\frac{157}{225}$.

3). $(5,4,3,2,1) = (5,4,3,3) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		5	4	3	3
P_i	$P_0 = 1$	5	21	68	225
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	13	43

Demak,$(5,4,3,2,1) = \frac{225}{43}$. **Javob:**$\frac{225}{43}$.

4). $(a, a, a, a, a) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		a	a	a	a	a
P_i	$P_0 = 1$	a	$a^2 + 1$	$a^3 + 2a$	$a^4 + 3a^2 + 1$	$a^5 + 4a^3 + 3a$
Q_i	$Q_0 = 0$	$Q_1 = 1$	a	$a^2 + 1$	$a^3 + 2a$	$a^4 + 3a^2 + 1$

Demak,$(a, a, a, a, a) = \frac{a^5+4a^3+3a}{a^4+3a^2+1}$. **Javob:** $\frac{a^5+4a^3+3a}{a^4+3a^2+1}$.

5). $(a,b,a,b,a)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		a	b	a	b	a
P_i	$P_0=1$	a	$ab+1$	a^2b+2a	$a^2b^2+3ab+1$	$a^3b^2+4a^2b+3a$
Q_i	$Q_0=0$	$Q_1=1$	b	$ab+1$	ab^2+2b	$a^2b^2+3ab+1$

Demak,$(a,a,a,a,a)=\frac{a^3b^2+4a^2b+3a}{a^2b^2+3ab+1}$. **Javob:** $\frac{a^3b^2+4a^2b+3a}{a^2b^2+3ab+1}$.

6). $(2,1,1,3,1,2)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		2	1	1	3	1	2
P_i	$P_0=1$	2	3	5	18	23	64
Q_i	$Q_0=0$	$Q_1=1$	1	2	7	9	25

Demak,$(2,1,1,3,1,2)=\frac{64}{25}$. **Javob:** $\frac{64}{25}$.

7). $(1,1,2,3,4)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	2	3	4
P_i	$P_0=1$	1	2	5	17	73
Q_i	$Q_0=0$	$Q_1=1$	1	3	10	43

Demak,$(1,1,2,3,4)=\frac{73}{43}$. **Javob:** $\frac{73}{43}$.

8). $(2,5,3,2,1,4,2,3)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		2	5	3	2	1	4	2	3
P_i	$P_0=1$	2	11	35	81	116	545	1206	4163
Q_i	$Q_0=0$	$Q_1=1$	5	16	37	53	249	551	1902

Demak,$(2,5,3,2,1,4,2,3)=\frac{4163}{1902}$. **Javob:** $\frac{4163}{1902}$.

350. Berilgan $\frac{a}{b}$ kasrni uzluksiz kasrlarga yoyishdan foydalanib qisqartirish uchun uni chekli uzluksiz kasrlarga yoyib, munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz hamda bunda $(P_k, Q_k) = 1$ va $\frac{P_n}{Q_n} = \frac{a}{b}$ ekanliklaridan foydalanamiz.

1). $\frac{3587}{2743}$ ni uzluksiz kasrlarga yoyamiz. U holda $3587 = 2743 \cdot \boxed{1} + 844$; $2743 = 844 \cdot \boxed{3} + 211$; $844 = 211 \cdot \boxed{4}$ dan $\frac{3587}{2743} = (1, 3, 4)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	3	4
P_i	$P_0 = 1$	1	4	17
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	13

Demak, $\frac{3587}{2743} = (1, 3, 4) = \frac{17}{13}$. Tekshirish $\frac{3587}{2743} = \frac{17 \cdot 211}{13 \cdot 211} = \frac{17}{13}$.

Javob: $\frac{17}{13}$.

2). $\frac{1043}{3427}$ ni uzluksiz kasrlarga yoyamiz. U holda $1043 = 3427 \cdot \boxed{0} + 1043$; $3427 = 1043 \cdot \boxed{3} + 298$; $1043 = 298 \cdot \boxed{3} + 149$; $298 = 149 \cdot \boxed{2}$ dan $\frac{1043}{3427} = (0, 3, 3, 2)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		0	3	3	2
P_i	$P_0 = 1$	0	1	3	7
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	10	23

Demak, $\frac{1043}{3427} = (0, 1, 3, 3) = \frac{7}{23}$. Tekshirish $\frac{1043}{3427} = \frac{7 \cdot 149}{23 \cdot 149} = \frac{7}{23}$.

Javob: $\frac{7}{23}$.

3). $\frac{3653}{3107}$ ni uzluksiz kasrlarga yoyamiz. U holda $3653 = 3107 \cdot \boxed{1} + 546$; $3107 = 546 \cdot \boxed{5} + 377$; $546 = 377 \cdot \boxed{1} + 169$; $377 = 169 \cdot \boxed{2} + 39$; $169 = 39 \cdot \boxed{4} + 13$; $39 = 13 \cdot \boxed{3}$ dan $\frac{3653}{3107} = (1, 5, 1, 2, 4, 3)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	5	1	2	4	3
P_i	$P_0 = 1$	1	6	7	20	87	281
Q_i	$Q_0 = 0$	$Q_1 = 1$	5	6	17	74	239

Demak, $\frac{3653}{3107} = (1, 5, 1, 2, 4, 3) = \frac{281}{239}$. Tekshirish: $\frac{3653}{3107} = \frac{281 \cdot 13}{239 \cdot 13} = \frac{281}{239}$. **Javob:** $\frac{281}{239}$.

4). $\frac{11281}{6583}$ ni uzluksiz kasrlarga yoyamiz. U holda $11281 = 6583 \cdot \boxed{1} + 4698$; $6583 = 4698 \cdot \boxed{1} + 1885$; $4698 = 1885 \cdot \boxed{2} + 928$; $1885 = 928 \cdot \boxed{2} + 29$; $928 = 29 \cdot \boxed{32}$ dan $\frac{11281}{6583} = (1,1,2,2,32)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	1	2	2	32
P_i	$P_0 = 1$	1	2	5	12	389
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	7	227

Demak, $\frac{11281}{6583} = (1,1,2,2,32) = \frac{389}{227}$. Tekshirish: $\frac{11281}{6583} = \frac{389 \cdot 29}{227 \cdot 29} = \frac{389}{227}$. **Javob:** $\frac{389}{227}$.

5). $\frac{1491}{2247}$ ni uzluksiz kasrlarga yoyamiz. U holda $1491 = 2247 \cdot \boxed{0} + 1491$; $2247 = 1491 \cdot \boxed{1} + 756$; $1491 = 756 \cdot \boxed{1} + 735$; $756 = 735 \cdot \boxed{1} + 21$; $735 = 21 \cdot \boxed{35}$ dan $\frac{1491}{2247} = (0,1,1,1,35)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		0	1	1	1	35
P_i	$P_0 = 1$	0	1	1	2	71
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	3	107

Demak, $\frac{1491}{2247} = (0,1,1,1,35) = \frac{71}{107}$. Tekshirish: $\frac{1491}{2247} = \frac{71 \cdot 21}{107 \cdot 21} = \frac{71}{107}$. **Javob:** $\frac{71}{107}$.

351. 1). $(x, 2,3,4) = \frac{73}{30}$ tenglamalarni yechish uchun uning chap tomoni orqali ifodalanuvchi qisqarmas kasrni topib olamiz. Buning uchun $\frac{P_n}{Q_n}$ −munosib kasrni hisoblaymiz:

q_i		x	2	3	4
P_i	$P_0 = 1$	x	$2x + 1$	$7x + 3$	$30x + 13$
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	30

Bundan $\frac{30x+13}{30} = \frac{73}{30} \rightarrow 30x + 13 = 73 \rightarrow 30x = 60 \rightarrow x = 2$. **Javob:** $x = 2$.

2). $7(xyz + x + z) = 10(yz + 1)$ tenglamalarni yechish uchun uni

$$\frac{xyz + x + z}{yz + 1} = \frac{10}{7}$$

ko'rinishida yozib olib, uning chap va o'ng tomonlarini uzluksiz kasrlarga yoyamiz. $xyz + x + z = (yz + 1) \cdot \boxed{x} + z$; $yz + 1 = z \cdot \boxed{y} + 1$; $z = 1 \cdot \boxed{z}$ bundan $\frac{xyz+x+z}{yz+1} = (x, y, z)$. Shuningdek $10 = 7 \cdot \boxed{1} + 3$; $7 = 3 \cdot \boxed{2} + 1$; $3 = 1 \cdot \boxed{3}$ dan

$\frac{10}{7} = (1,2,3)$. Hosil bo'lgan yoyilmalarni yuqoridagi tenglamaga olib borib qo'ysak, $(x, y, z) = (1,2,3)$ bundan esa $x = 1, y = 2, z = 3$ kelib chiqadi.

Javob: $x = 1, y = 2, z = 3$.

352. Berilgan kasrlarni uzluksiz kasrga yoyib uni $\frac{P_4}{Q_4} -$ munosib kasr bilan almashtirib, xatoligini aniqlash hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozish uchun berilgan kasrlarni uzluksiz kasrga yoymiz va $\frac{P_4}{Q_4} -$ munosib kasrni aniqlaymiz. Bundagi xatolik $\frac{1}{Q_4 Q_5}$ dan oshmaydi.

1). $\frac{29}{37} -$ kasrni uzluksiz kasrlarga yoyamiz. U holda $29 = 37 \cdot \boxed{0} + 29$; $37 = 29 \cdot \boxed{1} + 8$; $29 = 8 \cdot \boxed{3} + 5$; $8 = 5 \cdot \boxed{1} + 3$; $5 = 3 \cdot \boxed{1} + 2$; $3 = 2 \cdot \boxed{1} + 1$; $2 = 1 \cdot \boxed{2}$. Bundan $\frac{29}{37} = (0, 1, 3, 1, 1, 1, 2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		0	1	3	1	1	1	2
P_i	$P_0 = 1$	0	1	3	4	7	11	29
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	4	5	9	14	37

Bu yerdan $\frac{P_4}{Q_4} = \frac{4}{5} = 0{,}8$. Bundagi xatolik $\frac{1}{Q_4 Q_5} = \frac{1}{5 \cdot 9} = \frac{1}{45} \approx 0{,}02$ ga teng. Bulardan foydalanib, berilgan kasrni quyidagicha yozishimoz mumkin:

$\frac{29}{37} \approx \frac{4}{5}(-0{,}02) = 0{,}78$. **Javob:** $\frac{29}{37} \approx \frac{4}{5}(-0{,}02)$.

2). $\frac{163}{159} -$ kasrni uzluksiz kasrlarga yoyamiz. U holda $163 = 159 \cdot \boxed{1} + 4$; $159 = 4 \cdot \boxed{39} + 3$; $4 = 3 \cdot \boxed{1} + 1$; $3 = 1 \cdot \boxed{3}$. Bundan $\frac{163}{159} = (1, 39, 1, 3)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	39	1	3
P_i	$P_0 = 1$	1	40	41	163
Q_i	$Q_0 = 0$	$Q_1 = 1$	39	40	79

Bu yerdan $\frac{P_4}{Q_4} = \frac{163}{79}$. Bundan ko'rinaduki, $\frac{P_4}{Q_4} -$ munosib kasr berilgan kasrning o'ziga teng. Shuning uchun ham bu yerda xatolik no'lga teng bo'ladi.

Javob: $\frac{163}{159} = \frac{163}{159}(\pm 0)$.

3). $\frac{648}{385} -$ kasrni uzluksiz kasrlarga yoyamiz. U holda $648 = 385 \cdot \boxed{1} + 263$; $385 = 263 \cdot \boxed{1} + 122$; $263 = 122 \cdot \boxed{2} + 19$; $122 = 19 \cdot \boxed{6} + 8$; $19 = 8 \cdot$

$\boxed{2} + 3; 8 = 3 \cdot \boxed{2} + 2;\ \ 3 = 2 \cdot \boxed{1} + 1;\ 2 = 1 \cdot \boxed{2}$. Bundan $\frac{648}{385} = (1, 1, 2, 6, 2, 2, 1, 2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	1	2	6	2	2	1	2
P_i	$P_0 = 1$	1	2	5	32	69	170	239	648
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	19	41	101	142	385

Bu yerdan $\frac{P_4}{Q_4} = \frac{32}{19} = 0{,}6842.$ Bundagi xatolik $\frac{1}{Q_4 Q_5} = \frac{1}{19 \cdot 41} = \frac{1}{779} \approx 0{,}0013$ ga teng. Bulardan foydalanib berilgan kasrni quyidagicha yozishimoz mumkin:

$\frac{648}{385} \approx \frac{32}{19}(-0{,}0013) = 1{,}6831.$

Javob: $\frac{648}{385} \approx \frac{32}{19}(-0{,}0013).$

4). $\frac{1882}{1651}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $1882 = 1651 \cdot \boxed{1} + 231;\ \ 1651 = 231 \cdot \boxed{7} + 34;\ \ 231 = 34 \cdot \boxed{6} + 27;\ 34 = 27 \cdot \boxed{1} + 7;\ \ 27 = 7 \cdot \boxed{3} + 6; 7 = 6 \cdot \boxed{1} + 1;\ \ 6 = 1 \cdot \boxed{6}$. Bundan $\frac{1882}{1651} = (1, 7, 6, 1, 3, 1, 6)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	7	6	1	3	1	6
P_i	$P_0 = 1$	1	8	49	57	220	277	1882
Q_i	$Q_0 = 0$	$Q_1 = 1$	7	43	50	193	207	1651

Bu yerdan $\frac{P_4}{Q_4} = \frac{57}{50} = 1{,}14.$ Bundagi xatolik $\frac{1}{Q_4 Q_5} = \frac{1}{50 \cdot 193} = \frac{1}{9650} \approx 0{,}000103$ ga teng. Bulardan foydalanib, berilgan kasrni quyidagicha yozishimiz mumkin:

$$\frac{1882}{1651} \approx \frac{57}{50}(-0{,}000103) = 1{,}139897.$$

Javob: $\frac{1882}{1651} \approx \frac{57}{50}(-0{,}000103).$

353. Berilgan kasrlarni uzluksiz kasrga yoying va uni $\frac{P_5}{Q_5}$ – munosib kasr bilan almashtirib, xatoliogini aniqlang hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozing. Bu misolda ham 352-misoldagi singari mulohaza yuritamiz.

1). $\frac{571}{359}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $571 = 359 \cdot \boxed{1} + 212;\ \ 359 = 212 \cdot \boxed{1} + 147;\ \ 212 = 147 \cdot \boxed{1} + 65;\ 147 = 65 \cdot \boxed{2} + 17;\ \ 65 = 17 \cdot \boxed{3} + 14;\ 17 = 14 \cdot \boxed{1} + 3;\ \ 14 = 3 \cdot \boxed{4} + 2,\ 3 = 2 \cdot \boxed{1} + 1,\ 2 = 1 \cdot \boxed{2}$. Bundan $\frac{571}{359} = (1, 1, 1, 2, 3, 1, 4, 1{,}2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	1	1	2	3	1	4	1	2
P_i	$P_0 = 1$	1	2	3	8	27	35	167	202	571
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	5	17	22	105	127	359

Bu yerdan $\frac{P_5}{Q_5} = \frac{27}{17} = 1{,}14.$ Bundagi xatolik $\frac{1}{Q_5Q_6} = \frac{1}{17\cdot 22} = \frac{1}{374} \approx 0{,}0027$ ga teng. Bulardan foydalanib berilgan kasrni quyidagicha yozishimiz mumkin:

$\frac{571}{359} \approx \frac{27}{17}(+0{,}0027)$. **Javob:**$\frac{571}{359} \approx \frac{27}{17}(+0{,}0027)$.

2).$\frac{2341}{1721}$ –kasrni uzluksiz kasrlarga yoyamiz. U holda $2341 = 1721 \cdot \boxed{1} + 620$; $1721 = 620 \cdot \boxed{2} + 481$; $620 = 481 \cdot \boxed{1} + 139$; $481 = 139 \cdot \boxed{3} + 64$; $139 = 64 \cdot \boxed{2} + 11$; $64 = 11 \cdot \boxed{5} + 9$; $11 = 9 \cdot \boxed{1} + 2$, $9 = 2 \cdot \boxed{4} + 1$, $2 = 1 \cdot \boxed{2}$. Bundan $\frac{2341}{1721} = (1, 2, 1, 3, 2, 5, 1, 4{,}2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	2	1	3	2	5	1	4	2
P_i	$P_0 = 1$	1	3	4	15	34	185	219	1061	2341
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	3	11	25	136	161	780	1721

Bu yerdan $\frac{P_5}{Q_5} = \frac{34}{25} = 1{,}14.$ Bundagi xatolik $\frac{1}{Q_5Q_6} = \frac{1}{25\cdot 136} = \frac{1}{3400} \approx 0{,}00029$ ga teng. Bulardan foydalanib, berilgan kasrni quyidagicha yozishimiz mumkin:

$\frac{2341}{1721} \approx \frac{34}{25}(+0{,}00029)$.

Javob: $\frac{2341}{1721} \approx \frac{34}{25}(+0{,}00029)$.

354. Qo'yilgan masala berilgan kasrni shunday munosib kasr bilan almashtirish kerakki, bunda xatolik 0,001 dan kichik bo'lsin degan masalani yechishga keltiriladi. $\frac{571}{359}$ – kasrni uzluksiz kasrga 353.1)- misolda yoygan edik. Unga ko'ra $\frac{571}{359} = (1, 1, 1, 2, 3, 1, 4, 1{,}2)$ va munosib kasrlar $\frac{P_1}{Q_1} = \frac{1}{1}$, $\frac{P_2}{Q_2} = \frac{2}{1}$, $\frac{P_3}{Q_3} = \frac{3}{5}$, $\frac{P_4}{Q_4} = \frac{8}{5}$, $\frac{P_5}{Q_5} = \frac{27}{17}$, $\frac{P_6}{Q_6} = \frac{35}{22}$, $\frac{P_7}{Q_7} = \frac{167}{105}$, $\frac{P_8}{Q_8} = \frac{202}{127}$, $\frac{P_9}{Q_9} = \frac{571}{359}$ dan iborat. O'sha misolda biz berilgan kasrni $\frac{P_5}{Q_5} = \frac{27}{17}$ kasr bilan almashtirib, xatoligini hisoblagan edik. Bunda xatolik $\approx 0{,}0027$ ga teng chiqqan edi. Bu bizdan talab etilayotgan aniqlikdan katta. Shuning uchun ham $\frac{P_6}{Q_6} = \frac{35}{22}$ ni olib, xatoligini hisoblab ko'ramiz. Bunda $\frac{1}{Q_6Q_7} = \frac{1}{22\cdot 105} = \frac{1}{2310} < 0{,}00044 < 0{,}001$ bo'lgani uchun bu aniqlik bizni qanoatlantiradi. Shunday qilib tishlari soni nisbatan kam bo'lgan uzatmani texnik jihatidan qurish mumkin. Bu uzatmada tishlari soni kam bo'lgani uchun qulay va tishlari katta bo'lgani uchun mustahkam bo'ladi. **Javob:** mumkin.

355. Berilgan uzluksiz kasrni $\alpha_n = (\underbrace{2,\ 2, 2, \dots\ , 2}_{n\ ta})$ deb belgilab olamiz.

Agar $n = 2k -$ juft son bo'lsa, u holda

$$\alpha_2 = (2, 2) = 2 + \frac{1}{2} \to \frac{\alpha_2}{2} = 1 + \frac{1}{4} = (1,4);$$

$$\alpha_4 = (2, 2, \alpha_2) = 2 + \frac{1}{2+\frac{1}{\alpha_2}} \to \frac{\alpha_4}{2} = 1 + \frac{1}{4+(\alpha_2:2)} = 1 + \frac{1}{4+\frac{1}{1+\frac{1}{4}}} = (1,4, 1, 4)\ \text{bo'ladi.}$$

Endi faraz qilaylik $\frac{\alpha_{2k}}{2} = (\underbrace{1, 4, 1,4,\ \dots\ , 1,4}_{2k\ ta})$ o'ronli bo'lsin. U holda

$\frac{\alpha_{2(k+1)}}{2} = \frac{(2,2,\alpha_{2k})}{2} = 1 + \frac{1}{4+(\alpha_{2k}:2)} = (\underbrace{1, 4, 1,4,\ \dots\ , 1,4}_{2(k+1)\ ta})$bajariladi. Demak,

matematik induksiya metodiga asosan agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1, 4, 1,4,\ \dots\ , 1,4}_{2(k+1)\ ta})$ dan iborat bo'lar ekan.

Endi $n = 2k + 1 -$ toq son bo'lsin. U holda

$$\alpha_1 = (2), \qquad \alpha_3 = (2, 2,2) = 2 + \frac{1}{2+\frac{1}{2}} \to \frac{\alpha_3}{2} = 1 + \frac{1}{4+\frac{1}{1}} = (1,5).$$

Bundan umumiy holda $\frac{\alpha_{2k+1}}{2} = (\underbrace{1, 4, 1,4,\ \dots\ , 1,4,1,5}_{2k+1\ ta})$bo'ladi.

Javob: agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1, 4, 1,4,\ \dots\ , 1,4}_{2k\ ta})$ dan iborat va $n = 2k + 1 -$ toq son bo'lsa $(\underbrace{1, 4, 1,4,\ \dots\ , 1,4,1,5}_{2k+1\ ta})$bo'ladi.

356. Berilgan uzluksiz kasrni $\alpha_n = (\underbrace{a, a, a, a,\ \dots\ , a, a}_{n\ ta}), a > 1$ deb belgilab olamiz. Agar $n = 2k -$ juft son bo'lsa, u holda

$$\alpha_2 = (a, a) = a + \frac{1}{a} \to \frac{\alpha_2}{a} = 1 + \frac{1}{a^2} = (1, a^2);$$

$$\alpha_4 = (a, a, \alpha_2) = a + \frac{1}{a+\frac{1}{\alpha_2}} \to \frac{\alpha_4}{2} = 1 + \frac{1}{a^2+(\alpha_2:2)} = 1 + \frac{1}{a^2+\frac{1}{1+\frac{1}{a^2}}} = (1, a^2, 1, a^2)$$

bo'ladi. Endi faraz qilaylik $\frac{\alpha_{2k}}{a} = (\underbrace{1, a^2, 1, a^2,\ \dots\ , 1, a^2}_{2k\ ta})$ o'ronli bo'lsin. U holda

$$\frac{\alpha_{2(k+1)}}{2} = \frac{(a, a, \alpha_{2k})}{2} = 1 + \frac{1}{a^2 + (\alpha_{2k}: 2)} = (\underbrace{1, a^2, 1, a^2,\ \dots\ , 1, a^2}_{2(k+1)\ ta})$$

bajariladi. Demak, matematik induksiya metodiga asosan agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1, a^2, 1, a^2,\ \dots\ , 1, a^2}_{2k\ ta})$ dan iborat bo'lar ekan.

Endi $n = 2k + 1 -$ toq son bo'lsin. U holda

$\alpha_1 = (a), \qquad \alpha_3 = (a,a,a) = a + \dfrac{1}{a + \frac{1}{a}} \rightarrow \dfrac{\alpha_3}{a} = 1 + \dfrac{1}{a^2 + \frac{1}{1}} = (1, a^2 + 1).$

Bundan umumiy holda $\frac{\alpha_{2k+1}}{a} = (\underbrace{1, a^2, 1, a^2, \ \dots \ , 1, a^2, 1, a^2 + 1}_{2k+1\ ta})$bo'ladi.

Javob: agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1, a^2, 1, a^2, \ \dots \ , 1, a^2}_{2k\ ta})$ dan iborat va $n = 2k + 1 -$ toq son bo'lsa $(\underbrace{1, a^2, 1, a^2, \ \dots \ , 1, a^2 1, a^2 + 1}_{2k+1\ ta})$bo'ladi.

357. $\left(\frac{P_{n+2}}{P_n} - 1\right) \cdot \left(1 - \frac{P_{n-1}}{P_{n+1}}\right) = \left(\frac{Q_{n+2}}{Q_n} - 1\right)\left(1 - \frac{Q_{n-1}}{Q_{n+1}}\right)$ tenglikni isbotlash uchun rekurent formulalar

$$P_k = P_{k-1}q_k + P_{k-2} \text{ va } Q_k = Q_{k-1}q_k + Q_{k-2} \qquad (*)$$

lardan foydalanamiz. Avvalo (*) da $k = n + 2$ deb olamiz. U holda $P_{n+2} = P_{n+1}q_{n+2} + P_n$ va $Q_{n+2} = Q_{n+1}q_{n+2} + Q_n$ larga ega bo'lamiz. Bulardan $q_{n+2} = \frac{P_{n+2} - P_n}{P_{n+1}}$ va $q_{n+2} = \frac{Q_{n+2} - Q_n}{Q_{n+1}}$ bo'lgani uchun

$$\frac{P_{n+2} - P_n}{P_{n+1}} = \frac{Q_{n+2} - Q_n}{Q_{n+1}} \rightarrow \left(\frac{P_{n+2}}{P_n} - 1\right)\frac{P_n}{P_{n+1}} = \left(\frac{Q_{n+2}}{Q_n} - 1\right)\frac{Q_n}{Q_{n+1}}. \quad (*_1)$$

Endi (*) da $k = n + 1$ deb olamiz. U holda $P_{n+1} = P_n q_{n+1} + P_{n-1}$ va $Q_{n+1} = Q_n q_{n+1} + Q_{n-1}$ larga ega bo'lamiz. Bundan $P_n q_{n+1} = P_{n+1} - P_{n-1}$ va $Q_n q_{n+1} = Q_{n+1} - Q_{n-1}$ hosil bo'ladi. $(*_1)$ tenglikning ikkala tomonini q_{n+1}ko'paytirib oxirgi ikki tenglikdan foydalansak $\left(\frac{P_{n+2}}{P_n} - 1\right)\frac{P_n q_{n+1}}{P_{n+1}} = \left(\frac{Q_{n+2}}{Q_n} - 1\right)\frac{Q_n q_{n+1}}{Q_{n+1}} \rightarrow \left(\frac{P_{n+2}}{P_n} - 1\right)\frac{P_{n+1} - P_{n-1}}{P_{n+1}} = \left(\frac{Q_{n+2}}{Q_n} - 1\right)\frac{Q_{n+1} - Q_{n-1}}{Q_{n+1}}$ kelib chiqadi. Bundan esa isbotlanishi talab etilgan tenglik

$$\left(\frac{P_{n+2}}{P_n} - 1\right) \cdot \left(1 - \frac{P_{n-1}}{P_{n+1}}\right) = \left(\frac{Q_{n+2}}{Q_n} - 1\right)\left(1 - \frac{Q_{n-1}}{Q_{n+1}}\right)$$

kelib chiqadi.

358. 357-misoldagi(*) formulaning birinchisiga asosan $P_n = P_{n-1}q_n + P_{n-2} \rightarrow \frac{P_n}{P_{n-1}} = q_n + \frac{P_{n-2}}{P_{n-1}}$. Bu yerda $\frac{P_{n-1}}{P_{n-2}} = q_{n-1} + \frac{P_{n-3}}{P_{n-2}}$, $\frac{P_{n-2}}{P_{n-3}} = q_{n-2} + \frac{P_{n-4}}{P_{n-3}}$, $\dots$, $\frac{P_2}{P_1} = q_2 + \frac{P_0}{P_1} = q_2 + \frac{1}{q_1}$ bo'lgani uchun $\frac{P_n}{P_{n-1}} = q_n + \cfrac{1}{q_{n-1} + \cfrac{1}{q_{n-2} + \ddots + \cfrac{1}{q_1}}} = (q_n, q_{n-1}, \dots, q_1)$.

357-misoldagi(*) formulaning birinchisiga asosan $Q_n = Q_{n-1}q_n + Q_{n-2} \rightarrow \frac{Q_n}{Q_{n-1}} = q_n + \frac{Q_{n-2}}{Q_{n-1}}$. Bu yerda $\frac{Q_{n-1}}{Q_{n-2}} = q_{n-1} + \frac{Q_{n-3}}{Q_{n-2}}$, $\frac{Q_{n-2}}{Q_{n-3}} = q_{n-2} + \frac{Q_{n-4}}{Q_{n-3}}$, $\dots$, $\frac{Q_2}{Q_1} = q_2 + \frac{Q_0}{Q_1} = q_2$ bo'lgani uchun

$$\frac{Q_n}{Q_{n-1}} = q_n + \cfrac{1}{q_{n-1} + \cfrac{1}{q_{n-2} + \ddots + \cfrac{1}{q_2}}} = (q_n, q_{n-1}, \dots q_2).$$

359. $(P_n, P_{n-1}) = d$ bo'lsin. U holda $P_n = P_{n-1}q_n + P_{n-2}$ dan $(P_{n-1}, P_{n-2}) = d$. Shu mulohazani takrorlab, $n = 3$ da $d = (P_2, P_1) = (q_2q_1 + 1, q_1) = 1$. Demak, $d = (P_n, P_{n-1}) = (P_{n-1}, P_{n-2}) = \ . \ . \ . = (P_2, P_1) = 1.$ $(Q_n, Q_{n-1}) = 1$ ham aynan yuqoridagi singari mulohazalar yordamida isbotlanadi.

360. Tenglikni matematik induksiya meatodidan foydalanib isbotlaymiz.

$n = 1$ bo'lsa, berilgan tenglik to'g'ri

$$2 = \frac{\left(1+\sqrt{2}\right)^2 - \left(1-\sqrt{2}\right)^2}{\left(1+\sqrt{2}\right) - \left(1-\sqrt{2}\right)} = \left(1+\sqrt{2}\right) + \left(1-\sqrt{2}\right) = 2$$

tenglikga aylanadi. Endi faraz qilaylik isbotlanayotgan tenglik n uchun o'rinli bo'lsin, ya'ni

$$\alpha_n = (\underbrace{2,2,2,\dots,2}_{nta}) = \frac{\left(1+\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n+1}}{\left(1+\sqrt{2}\right)^{n} - \left(1-\sqrt{2}\right)^{n}}$$

u holda

$$\alpha_{n+1} = (2, \alpha_n) = 2 + \frac{1}{\alpha_n} = 2 + \frac{\left(1+\sqrt{2}\right)^{n} - \left(1-\sqrt{2}\right)^{n}}{\left(1+\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n+1}} =$$

$$\frac{2\left(1+\sqrt{2}\right)^{n+1} + \left(1+\sqrt{2}\right)^{n} - 2\left(1-\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n}}{\left(1+\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n+1}} =$$

$$\frac{\left(1+\sqrt{2}\right)^{n}(2 + 2\sqrt{2} + 1) - \left(1-\sqrt{2}\right)^{n}(2 - 2\sqrt{2} + 1)}{\left(1+\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n+1}} =$$

$$\frac{\left(1+\sqrt{2}\right)^{n+2} - \left(1-\sqrt{2}\right)^{n+2}}{\left(1+\sqrt{2}\right)^{n+1} - \left(1-\sqrt{2}\right)^{n+1}}$$

bajariladi, ya'ni isbotlanayotgan tenglik $n + 1$ uchun o'rinli. Shuning uchun ham matematik induksiya prinsipiga asosan isbotlanayotgan tenglik ixtiyoriy n natural soni uchun o'rinli.

361. $ax + by = c, a > 0, b > 0, (a, b) = 1$ tenglamani qaraymiz. $\frac{a}{b}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $P_n = a$, $Q_n = b$ bo'lgani uchun berilgan munosabat $P_nQ_{n-1} - Q_nP_{n-1} = (-1)^n$dan

$aQ_{n-1} - bP_{n-1} = (-1)^n \to a((-1)^n cQ_{n-1}) + b((-1)^{n-1}cP_{n-1}) = c$ ga ega bo'lamiz. Bundan $x_0 = (-1)^n cQ_{n-1}$ va $y_0 = (-1)^{n-1}cP_{n-1}$. Demak, biz munosib kasrlardan foydalanib, ikki noma'lumli birinchidarajalianiqmas tenglamaning birta yechimini topishimiz mumkin bo'lar ekan. U holda umumiy yechim $x = x_0 + bt, y = y_0 - at, t \in Z$ bo'ladi.

362.1).$38x + 117y = 209$ tenglamani qaraymiz. $\frac{38}{117}$ kasrni uzluksiz kasrga yoyib, munosib kasrlarini topamiz. U holda $38 = 117 \cdot \boxed{0} + 38;\ \ 117 = 38 \cdot \boxed{3} + 3;\ \ 38 = 3 \cdot \boxed{12} + 2;\ 3 = 2 \cdot \boxed{1} + 1;\ \ 2 = 1 \cdot \boxed{2}$. Bundan $\frac{38}{117} = (0, 3, 12, 1, 2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		0	3	12	1	2
P_i	$P_0 = 1$	0	1	12	$\boxed{13}$	38
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	37	$\boxed{40}$	117

Bu yerdan $n = 5, P_{n-1} = P_4 = 13,\ Q_{n-1} = Q_4 = 40$ bo'lgani uchun $x_0 = -209 \cdot 40 = -8360$ va $y_0 = 209 \cdot 13 = 2717$. Bundan berilgan tenglamaning umumiy yechimi $x = -8360 + 117t,\ y = 2717 - 38t, t \in Z$ ni hosil qilamiz.

Javob:$x = -8360 + 117t,\ y = 2717 - 38t, t \in Z$.

2). $122x + 129y = 2$ tenglamani qaraymiz. $\frac{122}{129}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $122 = 129 \cdot \boxed{0} + 122;\ \ 129 = 122 \cdot \boxed{1} + 7;\ \ 122 = 7 \cdot \boxed{17} + 3;\ 7 = 3 \cdot \boxed{2} + 1;\ \ 3 = 1 \cdot \boxed{3}$. Bundan $\frac{122}{129} = (0, 1, 17, 2, 3)$. Endi munosib kasrlarini aniqlaymiz.

q_i		0	1	17	2	3
P_i	$P_0 = 1$	0	1	17	$\boxed{35}$	122
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	18	$\boxed{37}$	129

Bu yerdan $n = 5, P_{n-1} = P_4 = 35,\ Q_{n-1} = Q_4 = 37$ bo'lgani uchun $x_0 = -2 \cdot 37 = -74$ va $y_0 = 2 \cdot 35 = 70$. Bundan berilgan tenglamaning umumiy yechimi $x = -74 + 129t,\ y = 70 - 122t, t \in Z$ ni hosil qilamiz.

Javob: $x = -74 + 129t,\ y = 70 - 122t, t \in Z$.

3). $119x - 68y = 34 \ \rightarrow 119x + 68(-y) = 34$ tenglamani qaraymiz. $\frac{119}{68}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $119 = 68 \cdot \boxed{1} + 51;\ 68 = 51 \cdot \boxed{1} + 17;\ \ 51 = 17 \cdot \boxed{3} + 0$. Bundan $\frac{119}{68} = (1, 1, 3)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	1	3
P_i	$P_0 = 1$	1	$\boxed{2}$	7
Q_i	$Q_0 = 0$	$Q_1 = 1$	$\boxed{1}$	4

Bu yerdan berilgan tenglama $7x - 4y = 2$ tenglamaga teng kuchli ekanligi kelib chiqadi va demak, $n = 3, P_{n-1} = P_2 = 2,\ Q_{n-1} = Q_2 = 1$ bo'lgani uchun $x_0 = -2 \cdot 1 = -2$ va $y_0 = -2 \cdot 2 - 4$. Bundan berilgan tenglamaning umumiy yechimi $x = -2 - 4t,\ y = -4 - 7t, t \in Z$ ni hosil qilamiz.

Javob: $x = -2 - 4t,\ y = -4 - 7t, t \in Z.$

4). $258x - 175y = 113\ \rightarrow 258x + 175(-y) = 113$ tenglamani qaraymiz. $\frac{258}{175}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $258 = 175 \cdot \boxed{1} + 83;\ 175 = 83 \cdot \boxed{2} + 9;\ \ 83 = 9 \cdot \boxed{9} + 2;\ 9 = 2 \cdot \boxed{4} + 1;$

$2 = 1 \cdot \boxed{2} + 0.$ Bundan $\frac{258}{175} = (1, 2, 9, 4, 2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	2	9	4	2
P_i	$P_0 = 1$	1	3	28	$\boxed{115}$	258
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	19	$\boxed{78}$	175

Bu yerdan $n = 5,\ P_{n-1} = P_4 = 115,\ Q_{n-1} = Q_4 = 78$ bo'lgani uchun $x_0 = -78 \cdot 113 = -8814$ va $y_0 = -115 \cdot 113 = -12995$. Bundan berilgan tenglamaning umumiy yechimi $x = -8814 + 175t,\ y = -12995 - 258t, t \in Z$ ni hosil qilamiz.

Javob: $x = -8814 + 175t,\ y = -12995 - 258t, t \in Z.$

5).$41x + 114y = 5$ tenglamani qaraymiz. $\frac{41}{114}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $41 = 114 \cdot \boxed{0} + 41;\ 114 = 41 \cdot \boxed{2} + 32;\ \ 41 = 32 \cdot \boxed{1} + 9;\ 32 = 9 \cdot \boxed{3} + 5;\ 9 = 5 \cdot \boxed{1} + 4;\ 5 = 4 \cdot \boxed{1} + 1; 4 = 1 \cdot \boxed{4} + 0.$ Bundan $\frac{41}{114} = (0, 2, 1, 3, 1,1,4)$. Endi munosib kasrlarini aniqlaymiz.

q_i		0	2	1	3	1	1	4
P_i	$P_0 = 1$	0	1	1	4	5	$\boxed{9}$	41
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	3	11	14	$\boxed{25}$	114

Bu yerdan $n = 7,\ P_{n-1} = P_6 = 9,\ Q_{n-1} = Q_5 = 25$ bo'lgani uchun $x_0 = -25 \cdot 5 = -125$ va $y_0 = 9 \cdot 5 = 45$. Bundan berilgan tenglamaning umumiy yechimi $x = -125 + 114t,\ y = 45 - 41t,\ t \in Z$ ni hosil qilamiz.

Javob: $x = -125 + 114t,\ y = 45 - 41t,\ t \in Z.$

6). $70x + 33y = 1$ tenglamani qaraymiz. $\frac{70}{33}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $70 = 33 \cdot \boxed{2} + 4;\ 33 = 4 \cdot \boxed{8} + 1;\ \ 4 = 1 \cdot \boxed{4} + 0.$ Bundan $\frac{70}{33} = (2, 8, 4,)$. Endi munosib kasrlarini aniqlaymiz.

q_i		2	8	4
P_i	$P_0 = 1$	2	$\boxed{17}$	70
Q_i	$Q_0 = 0$	$Q_1 = 1$	$\boxed{8}$	333

Bu yerdan $n = 3,\ P_{n-1} = P_2 = 17,\ Q_{n-1} = Q_2 = 8$ bo'lgani uchun $x_0 = -8 \cdot 1 = -8$ va $y_0 = 17 \cdot 1 = 17$. Bundan berilgan tenglamaning umumiy yechimi $x = -8 + 33t,\ y = 17 - 70t,\ t \in Z$ ni hosil qilamiz.

Javob: $x = -8 + 33t,\ y = 17 - 70t,\ t \in Z.$

363. Buning uchun berilgan $\frac{A}{B} = \frac{a^4+3a^2+1}{a^3+2a}$ kasrni uzluksiz kasrlarga yoyib, munosib kasrlarini aniqlaymiz. Agar bunda $\frac{P_n}{Q_n} = \frac{A}{B}$ tenglik o'rinli bo'lsa, berilgan kasr qisqarmas kasr bo'ladi.

$\frac{a^4+3a^2+1}{a^3+2a}$ kasrni uzluksiz kasrga yoyib munosib kasrlarini topamiz. U holda $a^4 + 3a^2 + 1 = (a^3 + 2a) \cdot \boxed{a} + (a^2 + 1);\ a^3 + 2a = (a^2 + 1) \cdot \boxed{a} + a;\ (a^2 + 1) = a \cdot \boxed{a} + 1; a = 1 \cdot \boxed{a} + 0.$

Bundan $\frac{a^4+3a^2+1}{a^3+2a} = (a, a, a, a)$. Endi munosib kasrlarini aniqlaymiz.

q_i		a	a	a	a
P_i	$P_0 = 1$	a	$a^2 + 1$	$a^3 + 2a$	$a^4 + 3a^2 + 1$
Q_i	$Q_0 = 0$	$Q_1 = 1$	a	$a^2 + 1$	$a^3 + 2a$

Bu yerdan $\frac{P_n}{Q_n} = \frac{a^4+3a^2+1}{a^3+2a}$ ni hosil qilamiz. Demak, berilgan kasr qisqarmas kasr ekan.

364. 358-misolga asosan $\frac{P_n}{P_{n-1}} = (q_n, q_{n-1}, \dots, q_1)$. Bundan masalaning shartini e'tiborga olib, $\frac{P_n}{P_{n-1}} = (q_n, q_{n-1}, \dots, q_1) = (q_1, q_2, \dots, q_n) = \frac{P_n}{Q_n}$ ekanligini hosil qilamiz. $\frac{P_n}{P_{n-1}}$ kasr (359-misol) qisqarmas kasr, shuningdek, $\frac{P_n}{Q_n}$ ham qisqarmas kasr bo'lgani uchun oxirgi tenglikdan $P_{n-1} = Q_n$ ga ega bo'lamiz.

365. Tengsizlikni $Q_n = Q_{n-1}q_n + Q_{n-2} \geq 2Q_{n-2}$ munosabatdan foydalanib, matematik induksiya metodi yordamida isbotlaymiz. $n = 2$ da yuqoridagi munosabatdan $Q_2 = Q_1q_2 + Q_0 \geq 2Q_0$ kelib chiqadi. Bunda $Q_1 = 1,\ Q_0 = 0$

ekanligini e`tiborga olsak, doimo bajariladigan $Q_2 = q_2 \geq 0$ munosabatga ega bo'lamiz. $n = 3$ da yuqoridagi munosabatdan $Q_3 = Q_2q_3 + Q_1 \geq 2Q_1$ kelib chiqadi. Bunda $Q_2 = q_2, Q_1 = 1,\ Q_0 = 0$ ekanligini e`tiborga olsak doimo bajariladigan $Q_3 = q_2q_3 + 1 \geq 2$ munosabatga ega bo'lamiz.

Endi faraz qilaylik, isbotlanishi talab etilayotgan tengsizlik n=k uchun o'rinli bo'lsin, ya'ni $Q_k \geq 2^{\frac{k-1}{2}}$ bajarilsin. U holda n=k+1 da $Q_{k+1} \geq 2^{\frac{k}{2}}$ ning bajarilishini ko'rsatamiz. Haqiqatan ham, induktivlik farazimizga asosan $Q_{k-1} \geq 2^{\frac{k-2}{2}}$bo'lgani uchun $Q_{k+1} = Q_kq_{k+1} + Q_{k-1} \geq 2Q_{k-1} \geq 2 \cdot 2^{\frac{k-2}{2}} = 2^{\frac{k}{2}}$ bajariladi. Demak, isbotlanayotgan munosabat barcha n≥2 natural sonlar uchun o'rinli.

366. $P_nQ_{n-1} - P_{n-1}Q_n = (-1)^n$ tenglikdan foydalanib, $ax \equiv b(modm)$, $(a, m) = 1$ taqqoslamani yechich uchun $\frac{P_n}{Q_n} = \frac{m}{a}$ ekanligidan foydalanamiz. Bunda $(P_n, Q_n) = 1$ bo'lganidan $P_n = m,\ Q_n = a$ kelib chiqadi. Bundan foydalanib, yuqoridagi tenglikni quyidagicha yozish mumkin:

$$mQ_{n-1} - aP_{n-1} = (-1)^n \to aP_{n-1} = (-1)^{n-1} + mQ_{n-1} \to a(P_{n-1}b)$$
$$= (-1)^{n-1}b + m(Q_{n-1}b) \ \to a((-1)^{n-1}P_{n-1}b)$$
$$= b + m((-1)^{n-1}Q_{n-1}b).$$

Oxirgi tenglikdan taqqoslamaga o'tsak, $a((-1)^{n-1}P_{n-1}b) \equiv b(modm)$ hosil bo'ladi. Bundan va berilgan taqqoslamadan $x \equiv (-1)^{n-1}P_{n-1}b(modm)$ kelib chiqadi. **Javob:** $x \equiv (-1)^{n-1}P_{n-1}b(modm)$.

367. 1). $\frac{308}{95}$ kasrni uzluksiz kasrlarga yoyib $(n-1)-$ munosib kasrning surati P_{n-1} ni topamiz. U holda $308 = 95 \cdot \boxed{3} + 23;\ 95 = 23 \cdot \boxed{4} + 3;\ \ 23 = 3 \cdot \boxed{7} + 2,\ \ 3 = 2 \cdot \boxed{1} + 1,\ \ 2 = 1 \cdot \boxed{2} + 0.$

Bundan $\frac{308}{95} = (3, 4, 7, 1,\ 2).$ Endi munosib kasrlarini aniqlaymiz.

q_i		3	4	7	1	2
P_i	$P_0 = 1$	3	13	94	$\boxed{107}$	308

Bu yerdan $n = 5,\ P_{n-1} = P_4 = 107$ bo'lgani 366-misoldagi $x \equiv (-1)^{n-1}P_{n-1}b(modm)$ formuladan $x \equiv 107 \cdot 59(mod308) \to x \equiv 6313(mod308) \to \square \equiv 308 \cdot 20 + 153(mod308) \to x \equiv 153(mod308)$ ni hosil qilamiz.

Javob: $x \equiv 153(mod308)$.

2). $\frac{132}{91}$ kasrni uzluksiz kasrlarga yoyib $(n-1)-$ munosib kasrning surati P_{n-1} ni topamiz. U holda $132 = 91 \cdot \boxed{1} + 41;\ 91 = 41 \cdot \boxed{2} + 93;\ \ 41 = 9 \cdot \boxed{4} + 5,\ \ 9 = 5 \cdot \boxed{1} + 4,\ \ 5 = 4 \cdot \boxed{1} + 1,\ \ 4 = 1 \cdot \boxed{4} + 0$.

Bundan $\frac{132}{91} = (1, 2, 4, 1, 1, 4)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	2	4	1	1	4
P_i	$P_0 = 1$	1	3	13	16	$\boxed{29}$	132

Bu yerdan $n = 6,\ P_{n-1} = P_5 = 29$ bo’lgani 366-misoldagi $x \equiv (-1)^{n-1} P_{n-1} b(mod m)$ formuladan $x \equiv -29 \cdot 1(mod 132) \to x \equiv -29(mod 132) \to x \equiv 103(mod 132)$ni hosil qilamiz.

Javob: $x \equiv 103(mod 132)$.

VI.2 -§.

368. 1). $\alpha = \frac{587}{103}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo’ladigan xatolikni baholashimiz talab etilayapti. Buning uchun, avvalo $\frac{587}{103}$ ni uzluksiz kasrlarga yoyib munosib kasrlarini topamiz: $587 = 103 \cdot \boxed{5} + 72$; $103 = 72 \cdot \boxed{1} + 31$; $72 = 31 \cdot \boxed{2} + 10$; $31 = 10 \cdot \boxed{3} + 1$; $10 = 1 \cdot \boxed{10} + 0$. Demak $\frac{587}{103} = (5,1,2,3,10)$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz: $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		5	1	2	3	10
P_i	$P_0 = 1$	5	6	17	$\boxed{57}$	587
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	$\boxed{10}$	103

Demak, $\frac{P_4}{Q_4} = \frac{57}{10}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{587}{103} - \frac{57}{10}\right| = \left|\frac{5870-5871}{1030}\right| = \frac{1}{1030} \approx 0{,}00097 < 0{,}001$ va shuning uchun ham $\frac{587}{103} \approx \frac{57}{10}(-0{,}001) = 5{,}699$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$. Taqqoslash uchun $\frac{587}{103} = 5{,}69902912621$ ekanligini ta’kidlab o’tamiz. **Javob:** $\frac{57}{10},\ \Delta\alpha = \frac{1}{1030}$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4 Q_5}\left(= \frac{1}{10 \cdot 103} = \frac{1}{1030}\right)$ ko’rinishda baholash ham mumkin.

2). $\alpha = 3{,}14159$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo’ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $3{,}14159$ ni uzluksiz kasrlarga yoyib munosiob kasrlarini topamiz: $3{,}14159 = 3 + \frac{14159}{100000}$, bunda $14159 = 100000 \cdot \boxed{0} + 14159$; $100000 = 14159 \cdot \boxed{7} + 887$; $14159 = 887 \cdot \boxed{15} + 854$; $887 = 854 \cdot \boxed{1} + 33$; $854 = 33 \cdot \boxed{25} + 29$,

$33 = 29 \cdot \boxed{1} + 4,\ 29 = 4 \cdot \boxed{7} + 1,\ 4 = 1 \cdot \boxed{4} + 0$. Demak, $3{,}14159 = (3{,}7{,}15{,}1, 25, 7, 4)$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz:

$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		3	7	15	1	25	7	4
P_i	$P_0 = 1$	3	22	333	$\boxed{355}$	9208		
Q_i	$Q_0 = 0$	$Q_1 = 1$	7	106	$\boxed{113}$	2931		

Demak, $\frac{P_4}{Q_4} = \frac{355}{113}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|3{,}14159 - \frac{355}{113}\right| = |3{,}14159 - 3{,}14159292| < 0{,}000003 < 0{,}00001$ va shuning uchun ham $3{,}14159 \approx \frac{355}{113}(-0{,}00001) = 3{,}1415829$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{355}{113},\ \Delta\alpha = 0.00001$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4 Q_5}\left(= \frac{1}{113 \cdot 2931} = \frac{1}{331203} < 0{,}00001\right)$ ko'rinishda baholash ham mumkin.

3). $\alpha = \frac{-1+\sqrt{5}}{2}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun, avvalo $\frac{-1+\sqrt{5}}{2}$ ni uzluksiz kasrlarga yoyib munosib kasrlarini topamiz:

$\alpha = \frac{-1+\sqrt{5}}{2} = 0 + \frac{1}{\frac{2}{-1+\sqrt{5}}} = 0 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{2}{-1+\sqrt{5}} = \frac{2(1+\sqrt{5})}{4} = \frac{1+\sqrt{5}}{2} = 1 + \left(\frac{1+\sqrt{5}}{2} - 1\right) = 1 + \frac{\sqrt{5}-1}{2} = 1 + \frac{2}{\sqrt{5}+1} = 1 + \frac{1}{\frac{1+\sqrt{5}}{2}} = 1 + \frac{1}{\alpha_1}$. Demak, $\alpha = \frac{-1+\sqrt{5}}{2} = (0{,}1{,}1{,}1, 1, 1, \dots)$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz:

$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	1	1	1	1	1	…
P_i	$P_0 = 1$	0	1	1	$\boxed{2}$	3	5	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	$\boxed{3}$	5	8	…

Demak, $\frac{P_4}{Q_4} = \frac{2}{3}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{-1+\sqrt{5}}{2} - \frac{2}{3}\right| = |0{,}61803399 - 0{,}66666667| < 0{,}049 < 0{,}05$ va shuning uchun ham $\frac{-1+\sqrt{5}}{2} \approx \frac{2}{3}(-0{,}05) = 0{,}61666667$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{2}{3}$, $\Delta\alpha = 0{,}05$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4 Q_5}\left(= \frac{1}{3\cdot 5} = \frac{1}{15} < 0{,}067 < 0{,}1\right)$ ko'rinishda baholash ham mumkin.

4). $\alpha = \frac{2-\sqrt{3}}{5}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{2-\sqrt{3}}{5}$ ni uzluksiz kasrlarga yoyib munosib kasrlarini topamiz: ($\sqrt{3} = 1{,}73050807$)

$\alpha = \frac{2-\sqrt{3}}{5} = 0 + \frac{1}{\frac{5}{2-\sqrt{3}}} = 0 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{5}{2-\sqrt{3}} = 5\left(2+\sqrt{3}\right) = 10 + 5\sqrt{3} = 18 + \left(5\sqrt{3} - 8\right) = 18 + \frac{75-64}{5\sqrt{3}+8} = 18 + \frac{11}{5\sqrt{3}+8} = 18 + \frac{1}{\frac{8+5\sqrt{3}}{11}} = 18 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{8+5\sqrt{3}}{11} = 1 + \left(\frac{8+5\sqrt{3}}{11} - 1\right) = 1 + \frac{5\sqrt{3}-3}{11} = 1 + \frac{1}{\alpha_3};$$

$\alpha_3 = \frac{11}{5\sqrt{3}-3} = \frac{11\left(5\sqrt{3}+3\right)}{66} = 1 + \left(\frac{5\sqrt{3}+3}{6} - 1\right) = 1 + \frac{5\sqrt{3}-3}{6} = 1 + \frac{1}{\alpha_4}$;

$$\alpha_4 = \frac{6}{5\sqrt{3}-3} = \frac{6(5\sqrt{3}+3)}{66} = \frac{5\sqrt{3}+3}{11} = 1 + \left(\frac{5\sqrt{3}+3}{11} - 1\right) = 1 + \frac{5\sqrt{3}-8}{11}$$

$$= 1 + \frac{1}{\frac{11}{5\sqrt{3}-8}} = 1 + \frac{1}{\alpha_5}; \alpha_5 = \frac{11}{5\sqrt{3}-8} = 5\sqrt{3} + 8, \ldots .$$

Demak, $\alpha = \frac{2-\sqrt{3}}{5} = (0{,}18{,}1{,}1, 1, 16, \ldots)$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz. Uni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	18	1	1	1	16	…
P_i	$P_0 = 1$	0	1	1	$\boxed{2}$	3	50	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	18	19	$\boxed{37}$	56	933	…

Demak, $\frac{P_4}{Q_4} = \frac{2}{37}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{2-\sqrt{3}}{4} - \frac{2}{37}\right| = |0{,}06737299 - 0{,}05405405| < 0{,}014 < 0{,}02$ va shuning uchun ham $\frac{2-\sqrt{3}}{4} \approx \frac{2}{37}(+0{,}014) = 0{,}06805405$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{2}{3}$, $\Delta\alpha = 0{,}05$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4 Q_5}\left(= \frac{1}{37\cdot 56} = \frac{1}{2072} < 0{,}0005 < 0{,}001\right)$ ko'rinishda baholash ham mumkin.

5). $\alpha = \frac{1+\sqrt{5}}{2}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{1+\sqrt{5}}{2}$ ni uzluksiz kasrlarga yoyib munosib kasrlarini topamiz: ($\sqrt{5} = 2{,}236067975$)

$\alpha = \frac{1+\sqrt{5}}{2} = 1 + \left(\frac{1+\sqrt{5}}{2} - 1\right) = 1 + \frac{\sqrt{5}-1}{2} = 1 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{2}{\sqrt{5}-1} = \frac{2(\sqrt{5}+1)}{4} = \frac{\sqrt{5}+1}{2} = \alpha$. Demak, $\alpha = \frac{1+\sqrt{5}}{2} = ((1))$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	1	1	1	1	…
P_i	$P_0 = 1$	1	2	3	$\boxed{5}$	8	13	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	$\boxed{3}$	5	8	…

Demak, $\frac{P_4}{Q_4} = \frac{5}{3}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{1+\sqrt{5}}{2} - \frac{5}{3}\right| = |1{,}618033989 - 1{,}66666666| < 0{,}04864 < 0{,}05$ va shuning uchun ham $\frac{1+\sqrt{5}}{2} \approx \frac{5}{3}(-0{,}04864) = 1{,}61802666$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{5}{3}$, $\Delta\alpha = 0{,}05$.

Eslatma: Bu yerdagi $\frac{P_i}{Q_i}\left(\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \dots\right)$ sonlariga Fibonachchi ketma-ketligi deyiladi.

6). $\alpha = \frac{-1+\sqrt{2}}{2}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib, uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{-1+\sqrt{2}}{2}$ ni uzluksiz kasrlarga yoyib munosib kasrlarini topamiz: ($\sqrt{2} = 2{,}236067975$)

$\alpha = \frac{-1+\sqrt{2}}{2} = 0 + \frac{-1+\sqrt{2}}{2} = 1 + \frac{1}{\frac{2}{-1+\sqrt{2}}} = 1 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{2}{\sqrt{2}-1} = 2(\sqrt{2}+1) = 4 + (2\sqrt{2} - 2) = 4 + \frac{(2\sqrt{2}-2)(2\sqrt{2}+2)}{(2\sqrt{2}+2)} = 4 + \frac{2}{\sqrt{2}+1} = 4 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{\sqrt{2}+1}{2} = 1 + \frac{\sqrt{2}-1}{2} = 1 + \frac{1}{\alpha_3}; \alpha_3 = \frac{2}{\sqrt{2}-1} = \alpha_1.$$

Demak, $\alpha = \frac{-1+\sqrt{2}}{2} = (0, (4, 1))$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	4	1	4	1	4	…
P_i	$P_0 = 1$	0	1	1	$\boxed{5}$	6	29	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	5	$\boxed{24}$	29	140	…

Demak, $\frac{P_4}{Q_4} = \frac{5}{24}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{-1+\sqrt{2}}{2} - \frac{5}{24}\right| = |0{,}2071067812 - 0{,}2083333333| < 0{,}001227 < 0{,}002$ va shuning uchun ham$\frac{-1+\sqrt{2}}{2} \approx \frac{5}{24}(-0{,}001227) = 0{,}20710633333$. Bu yerda xatolikni "– " ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{5}{24}$, $\Delta\alpha = 0{,}002$.

369. Buning uchunuchun berilgan $\frac{1261}{881}$ kasrni uzluksiz kasrga yoyamiz. Berilgan aniqlikni ta’minlash uchun k ni $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}0001}} = 100$ bajariladigan tanlash kifoya. Avvalo $\frac{1261}{881}$ kasrni uzluksiz kasrga yoyamiz: $1261 = 881 \cdot \boxed{1} + 380$; $881 = 380 \cdot \boxed{2} + 121$; $380 = 121 \cdot \boxed{3} + 17$; $121 = 17 \cdot \boxed{7} + 2$; $17 = 2 \cdot \boxed{8} + 1$, $2 = 1 \cdot \boxed{2} + 0$. Demak, $\frac{1261}{881} = (1{,}2{,}3{,}7{,}8{,}2)$ ekan. Endi munosib kasrni aniqlaymiz:$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	2	3	7	8	2
P_i	$P_0 = 1$	1	3	10	73	594	1261
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	51	$\boxed{415}$	881

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 5$ va $Q_5 = 415$. Shuning uchun ham $\frac{1261}{881} \approx \frac{594}{415}(-0.0001)$ deb yoza olamiz. Lekinda $\left|\frac{1261}{881} - \frac{P_k}{Q_k}\right| < 0{,}0001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_4}{Q_4}$ tekshirib ko’ramiz. Bu holda

$\left|\frac{1261}{881} - \frac{P_4}{Q_4}\right| = \left|\frac{1261}{881} - \frac{73}{51}\right| = |1{,}43132803632 - 1{,}43137254901| = 0{,}00004451269 < 0{,}00005 < 0{,}0001$ bajariladi va shu uchun $\frac{1261}{881} \approx \frac{73}{51}(-0.0001)$ deb yozish mumkin. Lekinda $\left|\frac{1261}{881} - \frac{P_3}{Q_3}\right| = \left|\frac{1261}{881} - \frac{10}{7}\right| = |1{,}43132803632 -$

$1{,}42857142857| = 0{,}00275660775 > 0{,}0001$. Berilgan shartlarni qanoatlantiruvchi $\frac{1261}{881}$ kasrga eng yaxshi yaqinlashish sifatida $\frac{P_4}{Q_4} = \frac{73}{51}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{73}{51}$.

370. 1).$\sqrt{2} = 1 + (\sqrt{2} - 1) = 1 + \frac{1}{\sqrt{2}+1} = 1 + \frac{1}{2+(\sqrt{2}-1)} = 1 + \frac{1}{2+\frac{1}{(\sqrt{2}+1)}} = 1 + \frac{1}{2+\frac{1}{2+\ddots}}$=(1,(2)) bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}001}} > 31$, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		1	2	2	2	2	2	…
P_i	$P_0 = 1$	1	3	7	17	41	99	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	5	12	29	$\boxed{70}$	…

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 6$ va $Q_6 = 70$. Shuning uchun ham $\frac{P_4}{Q_4} = \frac{99}{70}$, ya'ni $\sqrt{2} \approx \frac{99}{70}(-0{,}001)$ deb yoza olamiz. Lekin $\left|\sqrt{2} - \frac{P_k}{Q_k}\right| < 0{,}001$ shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_5}{Q_5}$ tekshirib ko'ramiz. Bu holda $\left|\sqrt{2} - \frac{P_5}{Q_5}\right| = \left|\sqrt{2} - \frac{41}{29}\right| = |1{,}4142135624 - 1{,}41379310344\,| = 0{,}000421 < 0{,}001$ bajariladi va shu uchun $\sqrt{2} \approx \frac{41}{29}(+0{,}001)$ deb yozish mumkin. Lekin$\left|\sqrt{2} - \frac{P_4}{Q_4}\right| = \left|\sqrt{2} - \frac{17}{12}\right| = |1{,}4142135624 - 1{,}4166666667| = 0{,}00245310426 > 0{,}001$. Berilgan shartlarni qanoatlantiruvchi $\sqrt{2}$ ga eng yaxshi yaqinlashish sifatida $\frac{P_5}{Q_5} = \frac{41}{29}$ munosib kasrni olsak bo'ladi.

Javob: $\frac{41}{29}$.

2).$\sqrt{3} = 1 + (\sqrt{3} - 1) = 1 + \frac{2}{\sqrt{3}+1} = 1 + \frac{1}{\frac{\sqrt{3}+1}{2}} = 1 + \frac{1}{1+(\sqrt{3}-1)} = 1 + \frac{1}{1+\frac{2}{\sqrt{3}+1}} = 1 + \frac{1}{1+\frac{1}{\frac{\sqrt{3}+1}{2}}} = (1,(1,2))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}001}} > 31$, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		1	1	2	1	2	1	2	…
P_i	$P_0 = 1$	1	2	5	7	19	26	71	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	11	15	41	…

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 7$ va $Q_7 = 41$. Shuning uchun ham $\frac{P_7}{Q_7} = \frac{71}{41}$, ya'ni $\sqrt{3} \approx \frac{71}{41}(+0{,}001)$ deb yoza olamiz. Lekinda $\left|\sqrt{3} - \frac{P_k}{Q_k}\right| < 0{,}001$ shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_6}{Q_6}$ tekshirib ko'ramiz. Bu holda

$$\left|\sqrt{3} - \frac{P_6}{Q_6}\right| = \left|\sqrt{3} - \frac{26}{15}\right| = |1{,}73050807 - 1{,}73333333| = 0{,}031 > 0{,}001$$

bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{3}$ga eng yaxshi yaqinlashish sifatida $\frac{P_7}{Q_7} = \frac{71}{41}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{71}{41}$.

3).$\sqrt{7} = 2 + \left(\sqrt{7} - 2\right) = 2 + \frac{3}{\sqrt{7}+2} = 2 + \frac{1}{\frac{\sqrt{7}+2}{3}} = 2 + \frac{1}{\alpha_1}$,

bu yerda $\alpha_1 = \frac{\sqrt{7}+2}{3} = 1 + \left(\frac{\sqrt{7}+2}{3} - 1\right) = 1 + \frac{\sqrt{7}-1}{3} = 1 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{3}{\sqrt{7}-1} = \frac{3(\sqrt{7}+1)}{6} = \frac{\sqrt{7}+1}{2} = 1 + \frac{\sqrt{7}-1}{2} = 1 + \frac{1}{\frac{2}{\sqrt{7}-1}} = 1 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{2}{\sqrt{7}-1} = \frac{2(\sqrt{7}+1)}{6} = \frac{\sqrt{7}+1}{3} = 1 + \frac{\sqrt{7}-2}{3} = 1 + \frac{1}{\frac{3}{\sqrt{7}-2}} = 1 + \frac{1}{\alpha_4};$$

$$\alpha_4 = \frac{3}{\sqrt{7}-2} = \frac{3}{\sqrt{7}-2} = \sqrt{7} + 2 = 4 + \left(\sqrt{7} - 2\right) = 4 + \frac{3}{\sqrt{7}+2} = 4 + \frac{1}{\frac{\sqrt{7}+2}{3}} = 4 + \frac{1}{\alpha_4};$$

$\alpha_4 = \frac{\sqrt{7}+2}{3} = \alpha_1$. Demak, $\sqrt{7} = (2, (1,1,1,4))$

bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}001}} > 31$, ya'ni $31 < Q_k$ shartni

qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		2	1	1	1	4	1	1	1	4	…
P_i	$P_0 = 1$	2	3	5	8	37	45	82	127	590	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	3	14	17	31	$\boxed{48}$	223	…

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 48$. Shuning uchun ham $\frac{P_8}{Q_8} = \frac{127}{48}$, ya'ni $\sqrt{7} \approx \frac{127}{48}(+0{,}001)$ deb yoza olamiz. Lekin $\left|\sqrt{7} - \frac{P_k}{Q_k}\right| < 0{,}001$shartni qanoatlantiruvchi eng kichik maxrajli munosib

kasr bilan almashtirish talab etilgani uchun $\frac{P_7}{Q_7}$ tekshirib ko'ramiz. Bu holda

$\left|\sqrt{7} - \frac{P_7}{Q_7}\right| = \left|\sqrt{7} - \frac{82}{31}\right| = |2{,}645751311 - 2{,}64583333333| = 0{,}00008 <$ $0{,}001$ bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{7}$ga eng yaxshi yaqinlashish sifatida $\frac{P_7}{Q_7} = \frac{82}{31}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{82}{31}$.

4).$\sqrt{11} = 3 + \left(\sqrt{11} - 3\right) = 3 + \frac{2}{\sqrt{11}+3} = 3 + \frac{1}{\frac{\sqrt{11}+3}{2}} = 3 + \frac{1}{\alpha_1}$,

bu yerda $\alpha_1 = \frac{\sqrt{11}+3}{2} = 3 + \left(\frac{\sqrt{11}+3}{2} - 3\right) = 3 + \frac{\sqrt{11}-3}{2} = 3 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{2}{\sqrt{11}-3} = \sqrt{11} + 3 = 6 + (\sqrt{11} - 3) = 6 + \frac{2}{\sqrt{11}+3} = 6 + \frac{1}{\frac{\sqrt{11}+3}{2}}$$

$$= 6 + \frac{1}{\alpha_3};$$

$\alpha_3 = \frac{\sqrt{11}+3}{2} = \alpha_1$. Demak, $\sqrt{11} = (3, (3{,}6))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}001}} >$ 31, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		3	3	6	3	6	…
P_i	$P_0 = 1$	3	10	63	199	125 7	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	19	$\boxed{60}$	379	…

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 4$ va $Q_4 = 60$. Shuning uchun ham $\frac{P_4}{Q_4} = \frac{199}{60} = 3{,}31(6)$, ya'ni $\sqrt{11} \approx \frac{199}{60}(-0{,}001)$ deb yoza olamiz. Lekinda $\left|\sqrt{11} - \frac{P_k}{Q_k}\right| < 0{,}001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_3}{Q_3}$ tekshirib ko'ramiz. Bu holda $\left|\sqrt{11} - \frac{P_3}{Q_3}\right| = \left|\sqrt{11} - \frac{63}{19}\right| = |3{,}3166247903 - 3{,}31578947368| < 0{,}00084 <$ $0{,}001$ bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{11}$ ga eng yaxshi yaqinlashish sifatida $\frac{P_3}{Q_3} = \frac{63}{19}$ munosib kasrni olsak bo'ladi.

Javob: $\frac{63}{19}$.

371. 1).$x^2-5x+2=0$ tenglamaning ildizlarini topamiz. $x_{1,2}=\frac{5\pm\sqrt{25-4\cdot1\cdot2}}{2}=\frac{5\pm\sqrt{17}}{2}$; $x_1=\frac{5+\sqrt{17}}{2}$, $x_2=\frac{5-\sqrt{17}}{2}$. Avvalo birinchi ildiz $x_1=\frac{5+\sqrt{17}}{2}$ ni qaraymiz. $x_1=\frac{5+\sqrt{17}}{2}=4+\frac{\sqrt{17}-3}{2}=4+\frac{1}{\alpha_1}$, bu yerda $\alpha_1=\frac{2}{\sqrt{17}-3}=\frac{2(\sqrt{17}+3)}{8}=\frac{\sqrt{17}+3}{4}=1+\frac{\sqrt{17}-1}{4}=1+\frac{1}{\frac{4}{\sqrt{17}-1}}=1+\frac{1}{\alpha_2}$;

$$\alpha_2=\frac{4}{\sqrt{17}-1}=\frac{\sqrt{17}+1}{4}=1+\frac{\sqrt{17}-3}{4}=1+\frac{1}{\frac{4}{\sqrt{17}-3}}=1+\frac{1}{\alpha_3};$$

$\alpha_3=\frac{4}{\sqrt{17}-3}=\frac{\sqrt{17}+3}{2}=3+\frac{\sqrt{17}-3}{2}=3+\frac{1}{\frac{2}{\sqrt{17}-3}}=3+\frac{1}{\alpha_1}$. Demak, $x_1=\frac{5+\sqrt{17}}{2}=\left(4,(1,1,3)\right)$ bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0,0001}}=100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		4	1	1	3	1	1	3	1	1	3	…
P_i	$P_0=1$	4	5	9	32	41	73	260	333	593	2112	…
Q_i	$Q_0=0$	$Q_1=1$	1	2	7	9	16	57	73	$\boxed{130}$	463	…

Jadvaldan $Q_k>100$ shartni qanoatlantiruvchi eng kichik k bu $k=8$ va $Q_8=130$. Shuning uchun ham $\frac{P_8}{Q_8}=\frac{593}{130}=4,56153846153$, ya'ni $\frac{5+\sqrt{17}}{2}\approx\frac{593}{130}(-0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_8Q_9}=\frac{1}{130\cdot463}=\frac{1}{60190}<0,000017<0,0001$ bo'ladi.

Endi ikkinchi $x_2=\frac{5-\sqrt{17}}{2}$ ildizni qaraymiz. $x_2=\frac{5-\sqrt{17}}{2}=0+\frac{1}{\frac{2}{5-\sqrt{17}}}=0+\frac{1}{\alpha_1}$, bu yerda $\alpha_1=\frac{2}{5-\sqrt{17}}=\frac{2(\sqrt{17}+5)}{8}=\frac{\sqrt{17}+5}{4}=2+\frac{\sqrt{17}-3}{4}=2+\frac{1}{\frac{4}{\sqrt{17}-3}}=2+\frac{1}{\alpha_2}$;

$$\alpha_2=\frac{4}{\sqrt{17}-3}=\frac{\sqrt{17}+3}{2}=3+\frac{\sqrt{17}-3}{2}=3+\frac{1}{\frac{2}{\sqrt{17}-3}}=3+\frac{1}{\alpha_3};$$

$$\alpha_3=\frac{2}{\sqrt{17}-3}=\frac{\sqrt{17}+3}{4}=1+\frac{\sqrt{17}-1}{4}=1+\frac{1}{\frac{4}{\sqrt{17}-1}}=1+\frac{1}{\alpha_4};$$

$\alpha_4=\frac{4}{\sqrt{17}-1}=\frac{\sqrt{17}+1}{4}=1+\frac{\sqrt{17}-3}{4}=1+\frac{1}{\frac{4}{\sqrt{17}-3}}=1+\frac{1}{\alpha_2}$.

Demak, $x_2 = \frac{5-\sqrt{17}}{2} = (0,2,(3,1,1))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} =$ 100 shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		0	2	3	1	1	3	1	1	3	1	1	..
P_i	$P_0 = 1$	0	1	3	4	7	25	32	57	203	260	463	..
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	9	16	57	73	130	463	593	1056	..

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 130$. Shuning uchun ham $\frac{P_8}{Q_8} = \frac{57}{130} = 0,43846153846$, ya'ni $\frac{5-\sqrt{17}}{2} \approx \frac{57}{130}(+0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_8 Q_9} = \frac{1}{130 \cdot 463} = \frac{1}{60190}<0,000017<$ $0,0001$ bo'ladi.

Javob: $x_1 = \frac{5+\sqrt{17}}{2} \approx \frac{593}{130}(-0,0001); x_2 = \frac{5-\sqrt{17}}{2} \approx \frac{57}{130}(+0,0001)$.

$4x^2 + 20x + 23 = 0$ tenglamaning ildizlarini topamiz.

$x_{1,2} = \frac{-10\pm\sqrt{100-92}}{4} = \frac{-10\pm2\sqrt{2}}{4} = \frac{-5\pm\sqrt{2}}{2};\ x_1 = \frac{-5+\sqrt{2}}{2},\ x_2 = \frac{-5-\sqrt{2}}{2}$.

Avvalo birinchi ildiz $x_1 = \frac{-5+\sqrt{2}}{2}$ ni qaraymiz.

$x_1 = \frac{-5+\sqrt{2}}{2} = -2 + \frac{\sqrt{2}-5}{2} + 2 = -2 + \frac{\sqrt{2}-1}{2} = -2 + \frac{1}{\alpha_1}$,

bu yerda

$\alpha_1 = \frac{2}{\sqrt{2}-1} = 2\left(\sqrt{2}+1\right) = 4 + 2\sqrt{2} - 2 = 4 + \frac{2}{\sqrt{2}+1} = 4 + \frac{1}{\frac{\sqrt{2}+1}{2}} = 4 + \frac{1}{\alpha_2}; \alpha_2 =$ $\frac{\sqrt{2}+1}{2} = 1 + \frac{\sqrt{2}-1}{2} = 1 + \frac{1}{\frac{2}{\sqrt{2}-1}} = 1 + \frac{1}{\alpha_1}$.

Demak, $x_1 = \frac{-5+\sqrt{2}}{2} = \left(-2,(4,1)\right)$bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} =$ 100 shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-2	4	1	4	1	4	1	…
P_i	$P_0 = 1$	-2	-7	-9	-43	-52	-251	-303	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	5	24	29	$\boxed{140}$	169	…

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 6$ va $Q_6 = 140$. Shuning uchun ham $\frac{P_6}{Q_6} = -\frac{251}{140} = -1,79285714285$, bunda $\frac{-5+\sqrt{2}}{2} \approx$

$-1{,}792893238$, ya'ni $\frac{-5+\sqrt{2}}{2} \approx -\frac{251}{140}(-0{,}0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_8Q_9} = \frac{1}{140 \cdot 169} = \frac{1}{23660}$ <0,000043<0,0001 bo'ladi.

Endi ikkinchi $x_2 = \frac{-5-\sqrt{2}}{2}$ ildizni qaraymiz. $x_2 = \frac{-5-\sqrt{2}}{2} = -4 + \frac{3-\sqrt{2}}{2} = -4 + \frac{1}{\frac{2}{3-\sqrt{2}}} = -4 + \frac{1}{\alpha_1}$, bu yerda

$$\alpha_1 = \frac{2}{3-\sqrt{2}} = \frac{2(\sqrt{2}+3)}{7} = 1 + \frac{2\sqrt{2}-1}{7} = 1 + \frac{1}{\frac{7}{2\sqrt{2}-1}} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{7}{2\sqrt{2}-1} = 2\sqrt{2}+1 = 3 + (2\sqrt{2}-2) = 3 + \frac{1}{\frac{\sqrt{2}+1}{2}} = 3 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{\sqrt{2}+1}{2} = ((1{,}4)).$$

Demak, $x_2 = \frac{-5-\sqrt{2}}{2} = (-4,1,3,(1,4))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-4	1	3	1	4	1	4	1	…
P_i	$P_0 = 1$	-4	-3	-13	-16	-77	-93	-449	-547	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	4	5	24	29	$\boxed{140}$	169	…

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 7$ va $Q_7 = 140$. Shuning uchun ham

$\frac{P_7}{Q_7} = -\frac{449}{140} = -3{,}20714285714$, $\frac{-5-\sqrt{2}}{2} = -3{,}207106812$, ya'ni $-\frac{449}{140}(-0{,}0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8Q_9} = \frac{1}{140 \cdot 169} = \frac{1}{23660} < 0{,}0001$ bo'ladi.

Javob: $x_1 = \frac{-5+\sqrt{2}}{2} \approx -\frac{251}{140}(-0{,}0001)$; $x_2 = \frac{-5-\sqrt{2}}{2} \approx -\frac{449}{140}(-0{,}0001)$.

3). $x^2 + 9x + 6 = 0$ tenglamaning ildizlarini topamiz.

$$x_{1,2} = \frac{-9 \pm \sqrt{81-24}}{2} = \frac{-9 \pm \sqrt{57}}{2}; \ x_1 = \frac{-9+\sqrt{57}}{2}, \qquad x_2 = \frac{-5-\sqrt{57}}{2}.$$

Avvalo birinchi ildiz $x_1 = \frac{-9+\sqrt{57}}{2}$ ni qaraymiz.

$$x_1 = \frac{-9+\sqrt{57}}{2} = -1 + \frac{\sqrt{57}-9}{2} + 1 = -1 + \frac{\sqrt{57}-7}{2} = -1 + \frac{1}{\alpha_1},$$

bu yerda

$$\alpha_1=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{\sqrt{57}-5}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_2};$$

$$\alpha_2=\frac{4}{\sqrt{57}-5}=\frac{\sqrt{57}+5}{8}=1+\frac{\sqrt{57}-3}{8}=1+\frac{1}{\frac{8}{\sqrt{57}-3}}=1+\frac{1}{\alpha_3};$$

$$\alpha_3=\frac{8}{\sqrt{57}-3}=\frac{\sqrt{57}+3}{6}=1+\left(\frac{\sqrt{57}+3}{6}-1\right)=1+\frac{\sqrt{57}-3}{6}=1+\frac{1}{\alpha_4};$$

$$\alpha_4=\frac{6}{\sqrt{57}-3}=\frac{\sqrt{57}+3}{8}=1+\left(\frac{\sqrt{57}+3}{8}-1\right)=1+\frac{\sqrt{57}-5}{8}=1+\frac{1}{\alpha_5};$$

$$\alpha_5=\frac{8}{\sqrt{57}-5}=\frac{\sqrt{57}+5}{4}=3+\frac{\sqrt{57}-7}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-7}}=3+\frac{1}{\alpha_6};$$

$$\alpha_6=\frac{4}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{2}=7+\frac{\sqrt{57}-7}{2}=7+\frac{1}{\frac{2}{\sqrt{57}-7}}=7+\frac{1}{\alpha_7};$$

$$\alpha_7=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_8}; \quad \alpha_8=\alpha_2$$

Demak, $x_1=\frac{-9+\sqrt{57}}{2}=(-1,3,(1,1,1,3,7,3))$ bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0{,}0001}}=100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-1	3	1	1	1	3	7	3	…
P_i	$P_0=1$	-1	-2	-3	-5	-8	-29	-211	-662	…
Q_i	$Q_0=0$	$Q_1=1$	3	4	7	11	40	$\boxed{291}$	913	…

Jadvaldan $Q_k>100$ shartni qanoatlantiruvchi eng kichik k bu $k=7$ va $Q_7=291$. Shuning uchun ham $\frac{P_7}{Q_7}=-\frac{211}{291}=-0{,}72508591065$, bunda $\frac{-9+\sqrt{57}}{2}\approx-0{,}725082783$, ya'ni $x_1=\frac{-9+\sqrt{57}}{2}\approx-\frac{211}{291}(+0{,}0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_7Q_8}=\frac{1}{40\cdot291}=\frac{1}{11640}$ <0,000086<0,0001 bo'ladi.

Endi ikkinchi $x_2=\frac{-9-\sqrt{57}}{2}$ ildizni qaraymiz.

$x_2=\frac{-9-\sqrt{57}}{2}=-9+\frac{9-\sqrt{57}}{2}=-9+\frac{1}{\frac{2}{9-\sqrt{57}}}=-9+\frac{1}{\alpha_1}$, bu yerda

$$\alpha_1 = \frac{2}{9-\sqrt{57}} = \frac{\sqrt{57}+9}{12} = 1 + \frac{\sqrt{57}-3}{12} = 1 + \frac{1}{\frac{12}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_2};$$

$\alpha_2 = \frac{12}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{4} = 2 + \frac{\sqrt{57}-5}{4} = 2 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 2 + \frac{1}{\alpha_3}$;bunda $x_1$ni hisoblaganmizdagi singari $\alpha_3 = \frac{4}{\sqrt{57}-5} = ((1,1,1,3,7,3))$.

Demak, $x_2 = \frac{-9-\sqrt{57}}{2} = (-9,1,2,(1,1,1,3,7,3))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		-9	1	2	1	1	1	3	7	3	…
P_i	$P_0 = 1$	-9	-8	-25	-33	-58	-91	-331	-2408	-7555	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	7	11	40	$\boxed{291}$	913	…

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 291$. Shuning uchun ham $\frac{P_8}{Q_8} = -\frac{2408}{291} = -8,27491408934$,$\frac{-9-\sqrt{57}}{2} = -8,2749172175$, ya'ni $x_2 = \frac{-9-\sqrt{57}}{2} \approx -\frac{2408}{291}(-0,0001)$ deb yoza olamiz. Bunda xatolik

$$< \frac{1}{Q_8 Q_9} = \frac{1}{291 \cdot 913} = \frac{1}{265683} < 0,000004 < 0,0001 \text{ bo'ladi.}$$

Javob: $x_1 = \frac{-9+\sqrt{57}}{2} \approx -\frac{211}{291}(+0,0001)$; $x_2 = \frac{-9-\sqrt{57}}{2} \approx -\frac{2408}{291}(-0,0001)$.

4). $2x^2 - 3x - 6 = 0$ tenglamaning ildizlarini topamiz. $x_{1,2} = \frac{3 \pm \sqrt{9+48}}{4} = \frac{3 \pm \sqrt{57}}{4}$; $x_1 = \frac{3+\sqrt{57}}{4}$, $x_2 = \frac{3-\sqrt{57}}{4}$. Avvalo birinchi ildiz $x_1 = \frac{3+\sqrt{57}}{4}$ ni qaraymiz.

$x_1 = \frac{3+\sqrt{57}}{4} = 2 + \frac{\sqrt{57}-5}{4} = 2 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 2 + \frac{1}{\alpha_1}$, bu yerda

$$\alpha_1 = \frac{4}{\sqrt{57}-5} = \frac{\sqrt{57}+5}{8} = 1 + \frac{\sqrt{57}-3}{8} = 1 + \frac{1}{\frac{8}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{8}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{6} = 1 + \frac{\sqrt{57}-3}{6} = 1 + \frac{1}{\frac{6}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{6}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{8} = 1 + \frac{\sqrt{57}-5}{8} = 1 + \frac{1}{\alpha_4};$$

$$\alpha_4 = \frac{8}{\sqrt{57}-5} = \frac{\sqrt{57}+5}{4} = 3 + \frac{\sqrt{57}-7}{4} = 3 + \frac{1}{\alpha_5};$$

$$\alpha_5=\frac{4}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{2}=7+\frac{\sqrt{57}-7}{2}=7+\frac{1}{\frac{2}{\sqrt{57}-7}}=7+\frac{1}{\alpha_6};$$

$$\alpha_6=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{\sqrt{57}-5}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_7};$$

$$\alpha_7=\frac{4}{\sqrt{57}-5}=\alpha_1$$

Demak, $x_1=\frac{3+\sqrt{57}}{4}=\left(2,(1,1,1,3,7,3)\right)$ bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0,0001}}=100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		2	1	1	1	3	7	3	1	…
P_i	$P_0=1$	2	3	5	8	29	211	662	873	…
Q_i	$Q_0=0$	$Q_1=1$	1	2	3	11	80	$\boxed{251}$	331	…

Jadvaldan $Q_k>100$ shartni qanoatlantiruvchi eng kichik k bu $k=7$ va $Q_7=251$. Shuning uchun ham $\frac{P_7}{Q_7}=\frac{662}{251}=2,6374501992,$ bunda $x_1=\frac{3+\sqrt{57}}{4}\approx 2,63745860875$, ya'ni $x_1=\frac{3+\sqrt{57}}{4}\approx\frac{662}{251}(+0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_7Q_8}=\frac{1}{251\cdot331}=\frac{1}{83081}<$0,000013<0,0001 bo'ladi.

Endi ikkinchi $x_2=\frac{3-\sqrt{57}}{4}$ ildizni qaraymiz. $x_2=\frac{3-\sqrt{57}}{4}=-2+\frac{11-\sqrt{57}}{4}=-2+\frac{1}{\frac{4}{11-\sqrt{57}}}=-2+\frac{1}{\alpha_1},$ bu yerda

$$\alpha_1=\frac{4}{11-\sqrt{57}}=\frac{\sqrt{57}+11}{16}=1+\frac{\sqrt{57}-5}{16}=1+\frac{1}{\frac{16}{\sqrt{57}-5}}=1+\frac{1}{\alpha_2};$$

$$\alpha_2=\frac{16}{\sqrt{57}-5}=\frac{\sqrt{57}+5}{2}=6+\frac{\sqrt{57}-7}{2}=6+\frac{1}{\frac{2}{\sqrt{57}-7}}=6+\frac{1}{\alpha_3};$$

$$\alpha_3=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{\sqrt{57}-5}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_4};$$

$$\alpha_4=\frac{4}{\sqrt{57}-5}$$

bunda x_1 ni hisoblaganmizdagi singari $\alpha_4=((1,1,1,3,7,3))$.

Demak, $x_2 = \frac{3-\sqrt{57}}{4} = (-2,1,6,3,(1,1,1,3,7,3))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-2	1	6	3	1	1	1	3	7	3	…
P_i	$P_0 = 1$	-2	-1	-8	-25	-33	-58	-91	-331	-2408	-7555	…
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	7	22	29	51	80	$\boxed{291}$	2117	6642	…

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 291$.

Shuning uchun ham $\frac{P_8}{Q_8} = -\frac{331}{291} = -1,13745704467$, $x_2 = \frac{3-\sqrt{57}}{4} = -1,13745860875$, ya'ni $x_2 = \frac{3-\sqrt{57}}{4} \approx -\frac{331}{291}(-0,0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8 Q_9} = \frac{1}{291 \cdot 2117} = \frac{1}{616047} < 0,000002 < 0,00001$ bo'ladi.

Javob: $x_1 = \frac{3+\sqrt{57}}{4} \approx \frac{662}{251}(+0,0001)$; $x_2 = \frac{3-\sqrt{57}}{4} \approx -\frac{331}{291}(-0,0001)$.

372. $A = \alpha - \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$ ayirmani qaraymiz. Bu yerda $\alpha = \frac{P_{n+1} q_{n+2} + P_n}{Q_{n+1} q_{n+2} + Q_n}$ bo'lgani uchun

$$A = \frac{P_{n+1} q_{n+2} + P_n}{Q_{n+1} q_{n+2} + Q_n} - \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$$
$$= \frac{P_{n+1} Q_n q_{n+2} + P_n Q_n + P_{n+1} Q_{n+1} q_{n+2} + P_n Q_{n+1} - P_n Q_{n+1} q_{n+2} - P_n Q_n}{(Q_{n+1} q_{n+2} + Q_n)\,(Q_n + Q_{n+1})}$$
$$- \frac{P_{n+1} Q_{n+1} q_{n+2} + P_{n+1} Q_n}{(Q_{n+1} q_{n+2} + Q_n)(Q_n + Q_{n+1})} =$$
$$\frac{(P_{n+1} Q_n q_{n+2} - P_n Q_{n+1} q_{n+2}) + P_n Q_{n+1} - P_{n+1} Q_n}{(Q_{n+1} q_{n+2} + Q_n)\,(Q_n + Q_{n+1})} =$$
$$\frac{(P_{n+1} Q_n - P_n Q_{n+1}) q_{n+2} - (P_{n+1} Q_n - P_n Q_{n+1})}{(Q_{n+1} q_{n+2} + Q_n)\,(Q_n + Q_{n+1})} = \frac{(P_{n+1} Q_n - P_n Q_{n+1})(q_{n+2} - 1)}{(Q_{n+1} q_{n+2} + Q_n)\,(Q_n + Q_{n+1})}$$
$$= \frac{(-1)^n (q_{n+2} - 1)}{(Q_{n+1} q_{n+2} + Q_n)\,(Q_n + Q_{n+1})}$$

ayirmaning ishorasi n ning juft toqligiga bog'liq bo'lib, agar $n = 2k -$ juft son bo'lsa, $\alpha > \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$; agar $n = 2k + 1 -$ toq son bo'lsa, $\alpha < \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$ bajariladi. Tushunarliki, $\frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$ kasr $\frac{P_n}{Q_n}$ va α sonlari orasida yotadi. Shuning uchun ham

$$\left|\alpha-\frac{P_n}{Q_n}\right|>\left|\frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}-\frac{P_n}{Q_n}\right|=\frac{1}{Q_n(Q_n+Q_{n+1})}$$

bajariladi.

Eslatma. Isbotlangan tengsizlik $\left|\alpha-\frac{P_n}{Q_n}\right|$ uchub quyi chegarani beradi va shuning uchun ham u bizga ma'lum bo'lgan $\left|\alpha-\frac{P_n}{Q_n}\right|<\frac{1}{Q_nQ_{n+1}}$ tengsizlikni to'ldiradi.

373. Bu yerda $\frac{P_n}{Q_n}=\frac{P_{n-1}q_n+P_{n-2}}{Q_{n-1}q_n+Q_{n-2}}$ bo'lgani uchun

$$\frac{P_{n-1}(q_n+m)+P_{n-2}}{Q_{n-1}(q_n+m)+Q_{n-2}}-\frac{P_{n-1}q_n+P_{n-2}}{Q_{n-1}q_n+Q_{n-2}}$$

$$=\frac{(P_{n-1}Q_{n-2}-P_{n-2}Q_{n-1})m}{(Q_{n-1}(q_n+m)+Q_{n-2})\,(Q_{n-1}q_n+Q_{n-2})}=$$

$\frac{(-1)^{n-2}\cdot m}{(Q_{n-1}(q_n+m)+Q_{n-2})\,(Q_{n-1}q_n+Q_{n-2})}$ juft tartibli munosib kasrlar ortadi, toq tartiblilari esa kamayadi.

374. Bu yerda

$$\left|\alpha-\frac{P_n}{Q_n}\right|+\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|=\left|\frac{P_n}{Q_n}-\frac{P_{n-1}}{Q_{n-1}}\right|=\frac{1}{Q_{n-1}Q_n}<\frac{1}{2Q_n^2}+\frac{1}{2Q_{n-1}^2}$$

munosabat o'rinli bo'lgani uchun $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|$ ifoda aynan $\frac{1}{2Q_{n-1}^2}$ dan kichik bo'lishi mumkin. Chunki 372- masalaga ko'ra $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|>\frac{1}{Q_{n-1}(Q_{n-1}+Q_n)}$ bo'lgani uchun, albatta $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|>\frac{1}{\frac{1}{2Q_n^2}}$ bajariladi.

VI.3-§.

375. 1).$(\overline{2,3})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x=(2,3,x)$ ko'rinishda yozib olib, uning munosib kasrlarini topamiz:

q_i		2	3	x
P_i	$P_0=1$	2	7	$7x+2$
Q_i	$Q_0=0$	$Q_1=1$	3	$3x+1$

Bundan $\frac{7x+2}{3x+1}=x\to 3x^2-6x-2=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda

$$x_{1,2}=\frac{3\pm\sqrt{9+6}}{3}=\frac{3\pm\sqrt{15}}{3}=1\pm\frac{\sqrt{15}}{3}=1\pm\sqrt{\frac{5}{3}}=1\pm\sqrt{1,(6)}$$

hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $1+\sqrt{1,(6)}$ dan iborat bo'ladi. **Javob:**$1+\sqrt{\frac{5}{3}}$.

2).$(\overline{1,1,2,2})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x=(1,1,2,2,x)$ ko'rinishda yozib olib, uning munosib kasrlarini topamiz:

q_i		1	1	2	2	x
P_i	$P_0=1$	1	2	5	12	$12x+5$
Q_i	$Q_0=0$	$Q_1=1$	1	3	7	$7x+3$

Bundan $\frac{12x+5}{7x+3}=x\rightarrow 7x^2-9x-5=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $x_{1,2}=\frac{9\pm\sqrt{81+28\cdot5}}{14}=\frac{9\pm\sqrt{201}}{14}$ hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $\frac{9+\sqrt{201}}{14}$ dan iborat bo'ladi. **Javob:** $\frac{9+\sqrt{201}}{14}$.

3). $(\overline{5,4,3})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x=(5,4,3,x)$ ko'rinishda yozib olib uning munosib kasrlarini topamiz:

		5	4	3	x
P_i	$P_0=1$	5	21	68	$68x+21$
Q_i	$Q_0=0$	$Q_1=1$	4	13	$13x+4$

Bundan $\frac{68x+21}{13x+4}=x\rightarrow q_i 13x^2-64x-21=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda

$$x_{1,2}=\frac{32\pm\sqrt{1024+13\cdot21}}{13}=\frac{32\pm\sqrt{1297}}{13}$$

hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $\frac{32+\sqrt{1297}}{13}$ dan iborat bo'ladi. **Javob:** $\frac{32+\sqrt{1297}}{13}$.

4). $\alpha=(1,2,3,\overline{4})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(1,2,3,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega=(\overline{4})=$

$4+\frac{1}{\omega}$. Avvalo ω ni aniqlaymiz. $\omega=4+\frac{1}{\omega}$ dan $\omega^2-4\omega-1=0$. Bu tenglamaning yechimi $\omega_{1,2}=2\pm\sqrt{5}$ dan iborat bo'lib, $\omega>0$ bo'lgani uchun $\omega=2+\sqrt{5}$.

Endi $\alpha=(1,2,3,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz.

q_i		1	2	3	ω
P_i	$P_0=1$	1	3	10	$10\omega+3$
Q_i	$Q_0=0$	$Q_1=1$	2	7	$7\omega+2$

Bundan

$$\frac{10\omega+3}{7\omega+2}=\alpha\rightarrow\alpha=\frac{23+10\sqrt{5}}{16+7\sqrt{5}}=\frac{(23+10\sqrt{5})(16-7\sqrt{5})}{(16+7\sqrt{5})(16-7\sqrt{5})}=\frac{18-\sqrt{5}}{11}$$

hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\frac{18-\sqrt{5}}{11}$ dan iborat bo'ladi. **Javob:** $\frac{18-\sqrt{5}}{11}$.

5). $\alpha=(0,1,1,1,1,\overline{2,2,2})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(0,1,1,1,1,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega=(\overline{2,2,2})$. Avvalo ω ni aniqlaymiz. $\omega=(2,2,2,\omega)$

q_i		2	2	2	ω
P_i	$P_0=1$	2	5	12	$12\omega+5$
Q_i	$Q_0=0$	$Q_1=1$	2	5	$5\omega+2$

dan $\frac{12\omega+5}{5\omega+2}=\omega\rightarrow 5\omega^2-10\omega-5=0\rightarrow\omega^2-2\omega-1=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2}=\frac{1\pm\sqrt{1+1}}{1}=1\pm\sqrt{2}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega=1+\sqrt{2}$. Endi $\alpha=(0,1,1,1,1,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz.

q_i		0	1	1	1	1	ω
P_i	$P_0=1$	0	1	1	2	3	$3\omega+2$
Q_i	$Q_0=0$	$Q_1=1$	1	2	3	5	$5\omega+3$

Bundan

$$\frac{3\omega+2}{5\omega+3}=\alpha \to \alpha=\frac{5+3\sqrt{2}}{8+5\sqrt{2}}=\frac{(5+3\sqrt{2})(8-5\sqrt{2})}{(8+5\sqrt{2})(8-5\sqrt{2})}=\frac{10-\sqrt{2}}{14}$$

hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\frac{10-\sqrt{2}}{14}$ dan iborat bo'ladi. **Javob:** $\frac{10-\sqrt{2}}{14}$.

6). $\alpha=(a,\overline{a,2a,})=(a,\omega)=a+\frac{1}{\omega}$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(a,\overline{a,2a,})=(a,\omega)=a+\frac{1}{\omega}$ ko'rinishda yozib olamiz. Bunda $\omega=(\overline{a,2a})=(a,2a,\omega)$. Avvalo ω ni aniqlaymiz.

q_i		a	$2a$	ω
P_i	$P_0=1$	a	$2a^2+1$	$(2a^2+1)\omega+a$
Q_i	$Q_0=0$	$Q_1=1$	$2a$	$2a\omega+1$

dan $\frac{(2a^2+1)\omega+a}{2a\omega+1}=\omega \to 2\omega^2-2a\omega-1=0, (a\neq 0)$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2}=\frac{a\pm\sqrt{a^2+2}}{2}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega=\frac{a+\sqrt{a^2+2}}{2}$. Endi $\alpha=(a,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz. $\alpha=a+\frac{1}{\omega}=a+\frac{2}{a+\sqrt{a^2+2}}=\frac{(a+\sqrt{a^2+2})\cdot\sqrt{a^2+2}}{a+\sqrt{a^2+2}}=\sqrt{a^2+2}$ hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\sqrt{a^2+2}$ dan iborat bo'ladi. **Javob:** $\sqrt{a^2+2}$.

7). $\alpha=(\overline{2,2,1,1})=(2,2,1,1,\omega)$uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(2,2,1,1,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega=(2,2,1,1,\omega)$. ω ni aniqlaymiz. Buning uchun esa munosib kasrlardan foydalanamiz.

q_i		2	2	1	1	ω
P_i	$P_0=1$	2	5	7	12	$12\omega+7$
Q_i	$Q_0=0$	$Q_1=1$	2	3	5	$5\omega+3$

dan $\frac{12\omega+7}{5\omega+3}=\omega \to 5\omega^2-9\omega-7=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2} = \frac{-9\pm\sqrt{221}}{10}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega = \frac{-9+\sqrt{221}}{10}$. Shunday qilib izlanayotgan kvadrat irratsionallik $\frac{-9+\sqrt{221}}{10}$ dan iborat bo'ladi. **Javob:** $\frac{-9+\sqrt{221}}{10}$.

376. Bir xil chala bo'linmali cheksiz davriy uzluksiz kasrni $\alpha = (a, a, a, \dots) = (a, \alpha) = a + \frac{1}{\alpha}$ ko'rinishida yozib olish mumkin. Bundan $\alpha^2 - a\alpha - 1 = 0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz.

U holda $\alpha = \frac{a+\sqrt{a^2+4}}{2}$ hosil bo'ladi. Shunday qilib, izlanayotgan kvadrat irratsionallik $\frac{a+\sqrt{a^2+4}}{2}$ dan iborat bo'ladi. Misol uchun: $a = 2$ bo'lsa, $\alpha = (2,2,\dots) = (\overline{2}) = \frac{2+2\sqrt{2}}{2} = 1 + \sqrt{2}$; $a = 3$ bo'lsa, $\alpha = (3,3,\dots) = (\overline{3}) = \frac{3+\sqrt{13}}{2}$ va hokazo. **Javob:** $\frac{a+\sqrt{a^2+4}}{2}$.

377.1). $\frac{P_k}{Q_k} = \frac{10}{3}$, $\alpha_{k+1} = \sqrt{2}$ bo'lsa, α ni topish kerak. $\frac{P_k}{Q_k} = \frac{10}{3}$ da $(P_k, Q_k) = 1$ bo'lgani uchun $P_k = 10, Q_k = 3$ ni hosil qilamiz. Ikkinchi tomondan $\frac{P_k}{Q_k} = \frac{10}{3} = 3 + \frac{1}{3}$ bo'lgani uchun $P_{k-1} = 3$, $Q_{k-1} = 1$ kelib chiqadi. Bu qiymatlarni $\alpha = \frac{P_k\alpha_{k+1}+P_{k-1}}{Q_k\alpha_{k+1}+Q_{k-1}}$ da foydalansak $\alpha = \frac{10\sqrt{2}+3}{3\sqrt{2}+1} = \frac{57-\sqrt{2}}{17}$ ekanligi kelib hiqadi.

Javob: $\alpha = \frac{57-\sqrt{2}}{17}$.

2). $\frac{P_k}{Q_k} = \frac{37}{13}$, $\alpha_{k+1} = \frac{1+\sqrt{3}}{2}$ bo'lsa, α ni topish kerak. $\frac{P_k}{Q_k} = \frac{37}{13}$ da $(P_k, Q_k) = 1$ bo'lgani uchun $P_k = 37, Q_k = 13$ ni hosil qilamiz. Ikkinchi tomondan

$$\frac{P_k}{Q_k} = \frac{37}{13} = 2 + \frac{11}{13} = 2 + \frac{1}{\frac{13}{11}} = 2 + \frac{1}{1+\frac{2}{11}} = 2 + \frac{1}{1+\frac{1}{\frac{11}{2}}} = 2 + \frac{1}{1+\frac{1}{5+\frac{1}{2}}} = (2,1,5,2)$$

bo'lgani uchun

q_i		2	1	5	2
P_i	$P_0 = 1$	2	3	17	37
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	6	13

dan $P_{k-1} = 17$, $Q_{k-1} = 6$ kelib chiqadi. Bu qiymatlarni $\alpha = \frac{P_k\alpha_{k+1}+P_{k-1}}{Q_k\alpha_{k+1}+Q_{k-1}}$ da foydalansak

$$\alpha = \frac{37\left(\frac{1+\sqrt{3}}{2}\right)+17}{13\left(\frac{1+\sqrt{3}}{2}\right)+6} = \frac{71+37\sqrt{3}}{25+13\sqrt{3}} = \frac{(71+37\sqrt{3})(25-13\sqrt{3})}{(25+13\sqrt{3})(25-13\sqrt{3})} = \frac{166+\sqrt{3}}{59}$$

ekanligi kelib hiqadi. **Javob:** $\alpha = \frac{166+\sqrt{3}}{59}$.

378.1).$\alpha=\sqrt{x^2+1}=x+\left(\sqrt{x^2+1}-x\right)=x+\frac{1}{\sqrt{x^2+1}+x}=x+\frac{1}{\alpha_1}$, bunda $\alpha_1=\sqrt{x^2+1}+x=2x+\left(\sqrt{x^2+1}-x\right)=2x+\frac{1}{\sqrt{x^2+1}+x}=2x+\frac{1}{\alpha_1}$. Demak, $\alpha=(x,\overline{2x})$. Misol uchun $x=1$ da $\sqrt{2}=(1,\overline{2})$; $x=2$ da $\sqrt{5}=(2,\overline{4})$; $x=3$ da $\sqrt{10}=(3,\overline{6})$ va hakozo. Endi $\frac{P_3}{Q_3}$ aniqlaymiz.

q_i		x	$2x$	$2x$	...
P_i	$P_0=1$	x	$2x^2+1$	$4x^3+3x$	…
Q_i	$Q_0=0$	$Q_1=1$	$2x$	$4x^2+1$	…

Bundan $\frac{P_3}{Q_3}=\frac{4x^3+3x}{4x^2+1}$. **Javob:** $\alpha=(x,\overline{2x})$ va $\frac{P_3}{Q_3}=\frac{4x^3+3x}{4x^2+1}$.

2).$\alpha=\sqrt{a^4+2a}=a^2+\left(\sqrt{a^4+2a}-a^2\right)=a^2+\frac{2a}{\sqrt{a^4+2a}+a^2}=a^2+\frac{1}{\frac{\sqrt{a^4+2a}+a^2}{2a}}=a^2+\frac{1}{\alpha_1}$, bunda $\alpha_1=\frac{\sqrt{a^4+2a}+a^2}{2a}=a+\left(\frac{\sqrt{a^4+2a}+a^2}{2a}-a\right)=a+\frac{\sqrt{a^4+2a}-a^2}{2a}=a+\frac{1}{\frac{2a}{\sqrt{a^4+2a}-a^2}}=a+\frac{1}{\alpha_2}$. Bu yerda $\alpha_2=\frac{2a}{\sqrt{a^4+2a}-a^2}=\sqrt{a^4+2a}+a^2=2a^2+(\sqrt{a^4+2a}-a^2)=2a^2+\frac{2a}{\sqrt{a^4+2a}-a^2}=2a^2+\frac{1}{\alpha_1}$. Demak, $\alpha=\left(a^2,\overline{a,2a^2}\right)$. Endi $\frac{P_3}{Q_3}$ aniqlaymiz.

q_i		a^2	a	$2a^2$	...
P_i	$P_0=1$	a^2	a^3+1	$2a^5+3a^2$	…
Q_i	$Q_0=0$	$Q_1=1$	a	$2a^3+1$	…

Bundan $\frac{P_3}{Q_3}=\frac{2a^5+3a^2}{2a^3+1}$. **Javob:** $\alpha=\left(a^2,\overline{a,2a^2}\right)$ va $\frac{P_3}{Q_3}=\frac{2a^5+3a^2}{2a^3+1}$.

379. $\alpha=\sqrt{a^2+a+1}$ ni uzliksiz kasrga yoyamiz. U holda quyidagiga ega bo'lamiz:

$$\alpha=a+\left(\sqrt{a^2+a+1}-a\right)=a+\frac{a+1}{\sqrt{a^2+a+1}+a}=a+\frac{1}{\alpha_1},\ \text{bunda}$$

$\alpha_1=\frac{\sqrt{a^2+a+1}+a}{a+1}=\frac{(a+1)+(\sqrt{a^2+a+1}-1)}{a+1}=1+\frac{(\sqrt{a^2+a+1}-1)\cdot(\sqrt{a^2+a+1}+1)}{(a+1)\cdot(\sqrt{a^2+a+1}+1)}=1+\frac{a^2+a+1-1}{(a+1)\cdot(\sqrt{a^2+a+1}+1)}=1+\frac{a}{\sqrt{a^2+a+1}+1}=1+\frac{1}{\frac{\sqrt{a^2+a+1}+1}{a}}=1+\frac{1}{\alpha_2}$ bo'lib

$\alpha_2=\frac{\sqrt{a^2+a+1}+1}{a}=\frac{a+(\sqrt{a^2+a+1}+1-a)}{a}=1+\frac{\sqrt{a^2+a+1}+1-a}{a}=1+\frac{1}{\frac{a}{\sqrt{a^2+a+1}-(a-1)}}=$
$1+\frac{1}{\alpha_3}$ bo'ladi. Bulardan foydalanib $\frac{P_3}{Q_3}$ ni aniqlaymiz.

q_i		a	1	1	...
P_i	$P_0=1$	a	$a+1$	$2a+1$	...
Q_i	$Q_0=0$	$Q_1=1$	1	2	...

Bundan $\frac{P_3}{Q_3}=\frac{2a+1}{2}$ ekanligi kelib chiqadi.

380. Avvalo berilgan kvadrat uchhadning musbat ildizini aniqlaymiz.
$bx^2-abx-a=0\to x=\frac{ab+\sqrt{a^2b^2+4ab}}{2b}=a+\left(\frac{ab+\sqrt{a^2b^2+4ab}}{2b}-a\right)=a+$
$\frac{\sqrt{a^2b^2+4ab}-ab}{2b}=a+\frac{1}{\frac{2b}{\sqrt{a^2b^2+4ab}-ab}}=a+\frac{1}{\alpha_1}$, bunda $\alpha_1=\frac{2b}{\sqrt{a^2b^2+4ab}-ab}$

$$\alpha_1=\frac{2b}{\sqrt{a^2b^2+4ab}-ab}=\frac{\sqrt{a^2b^2+4ab}+ab}{2a}=b+\frac{\sqrt{a^2b^2+4ab}-ab}{2a}$$
$$=b+\frac{1}{\frac{2a}{\sqrt{a^2b^2+4ab}-ab}}=b+\frac{1}{\alpha_2}\text{ bo'lib}\alpha_2=\frac{2a}{\sqrt{a^2b^2+4ab}-ab}$$
$$=\frac{\sqrt{a^2b^2+4ab}+ab}{2b}=x.$$

Demak, $x=\frac{ab+\sqrt{a^2b^2+4ab}}{2b}=(\overline{a,b})$, ya'ni berilgan tenglamaning musbat ildizi davr uzunligi 2 ga teng bo'lgan sof davriy uzluksiz kasrga yoyilar ekan.

381. 380-misolda $x_1=(\overline{a,b})$ ning $bx^2-abx-a=0$ tenglamaning musbat ildizi ekanligini ko'rsatgan edik. Berilgan tenglamani $x^2-ax-\frac{a}{b}=0$ ko'rinishda yozish mumkin. Bundan, Viyet teoremasiga asosan $x_1+x_2=a\to$

$$x_2=a-x_1=a-(\overline{a,b})=a-\left(a+\frac{1}{b+\frac{1}{a+\frac{1}{b+\ddots}}}\right)=-\left(\frac{1}{b+\frac{1}{a+\frac{1}{b+\ddots}}}\right)=-\frac{1}{(\overline{b,a})}$$

bo'lishi kerak ekanligi kelib chiqadi. Shunday qilib $x_2=-\frac{1}{(\overline{b,a})}$.

382. Bu holda $\alpha=(\overline{a_1,a_2,\dots,a_n})$ soni $x=\frac{P_{n-1}x+P_{n-2}}{Q_{n-1}x+Q_{n-2}}$ tenglamani qanoatlantiradi, ya'ni $f(x)=Q_{n-1}x^2+(Q_{n-2}-P_{n-1})x-P_{n-2}$ ko'phadning musbat ildizi bo'lishi kerak. Bu ko'phadning ikkinchi ildizi α ga qo'shma $\overline{\alpha}$ bo'lib, $f(0)=-P_{n-2}<0$ va $f(-1)=(Q_{n-1}-Q_{n-2})+(P_{n-1}-P_{n-2})>0$ bo'ladi, chunki n ning o'sishi bilan cheksiz uzluksiz kasrning maxraji o'sadi. Shuningdek, cheksiz uzluksiz kasrning surati P_n monoton o'suvchi bo'ladi. Bu holda $\alpha > 1$ bo'lgani uchun $\overline{\alpha} \in (-1; 0)$ bo'lishi kerak.

383. Bu yerda $x = (a, \overline{b, c}) = a + \frac{1}{(\overline{b,c})}$ bo'lgani chun $x - a = \frac{1}{(\overline{b,c})} \rightarrow (\overline{b, c}) = \frac{1}{x-a}$ bo'ladi. Bunda $(\overline{b, c})$ soni (380-misol) soni $cx^2 - bcx - b = 0$ tenglamaning ildizi. U holda bu tenglamaning ikkinchi ildizi 381-misolga asosan $-\frac{1}{(\overline{c,b})} = \frac{1}{x-a}$ tenglikdan topish mumkin. Bundan $(\overline{c, b}) = -x + a \rightarrow x = a - (\overline{c, b})$ kelib chiqadi.

384. 381-misolga asosan $x_1 = (\overline{a, b})$ soni $bx^2 - abx - a = 0$ tenglamaning musbat ildizi ekanligini ko'rgan edik. Uning ikkinchi ildizi $x_2 = -\frac{1}{(\overline{b,a})} = -\left(0, (\overline{b, a})\right)$ dan iborat bo'ladi. Berilgan tenglamani $x^2 - ax - \frac{a}{b} = 0$ ko'rinishda yozish mumkin. Bundan, Viyet teoremasiga asosan $x_1 \cdot x_2 = -\frac{a}{b} \rightarrow x_1 \cdot x_2 = (\overline{a, b}) \cdot \left(0, (\overline{b, a})\right) = \frac{a}{b}$. **Javob:**$(\overline{a, b}) \cdot \left(0, (\overline{b, a})\right) = \frac{a}{b}$.

385. Bu yerda $\alpha = a + \frac{1}{b+\frac{1}{c}} = a + \frac{c}{bc+1} = \frac{abc+a+c}{bc+1}$ va

$\beta = c + \frac{1}{b+\frac{1}{a}} = c + \frac{a}{ab+1} = \frac{abc+a+c}{ab+1}$ bo'lgani uchun $\frac{\alpha}{\beta} = \frac{ab+1}{bc+1}$ ekanligi kelib chiqadi. $x = (\overline{a, b, c})$ va $y = (\overline{c, b, a})$ lar mos ravishda quyidagi tenglamalarni qanoatlantiradi:

$$x = a + \frac{1}{b + \frac{1}{c+\frac{1}{x}}} = a + \frac{1}{b + \frac{x}{cx+1}} = a + \frac{cx + 1}{bcx + b + x} = \frac{abcx + (a + c)x + ab + 1}{bcx + b + x}$$

$$\rightarrow \frac{(bc + 1)x^2 + bx - [abcx + (a + c)x + \square b + 1]}{bcx + b + x} = 0 \rightarrow$$

$$(bc + 1)x^2 - (abc + a + c - b)x - (ab + 1) = 0.$$

Shunga o'xshash $(ab + 1)y^2 - (abc + a + c - b)y - (bc + 1) = 0.$

Bu tenglamalarni yechib

$$x = \frac{(abc + a + c - b) + \sqrt{(abc + a + c - b)^2 + 4(bc + 1)(ab + 1)}}{2(bc + 1)};$$

$$y = \frac{(abc + a + c - b) + \sqrt{(abc + a + c - b)^2 + 4(bc + 1)(ab + 1)}}{2(ab + 1)}$$

larga ega bo'lamiz. Bulardan

$$\frac{x}{y} = \frac{ab + 1}{bc + 1} = \frac{\alpha}{\beta}$$

kelib chiqadi.

386. Agar n natural soni uchun $\sqrt{n} = (q_1,\ q_2,\ \dots)$ bo'lsa, u holda $\sqrt{n} + q_1 = (2q_1,\ q_2,\ \dots) > 1$ va $-1 < q_1 - \sqrt{n} < 0$ bajariladi. Shuning uchun ham $\sqrt{n} + q_1$ ifoda sof uzluksiz kasrga yoyiladi, ya'ni $\sqrt{n} + q_1 = (\overline{2q_1,\ q_2,\ \dots,\ q_n})$. Bundan $\sqrt{n} = (q_1, \overline{q_2,\ \dots,\ q_n, 2q_1})$. Bu esa isbotlanishi talab etilgan tasdiq. Misol uchun $\sqrt{2} = (1, \overline{2})$; $\sqrt{8} = (1, \overline{2,\ 4})$.

VI.4 -§.

387. Malumki, agar α (haqiqiy yoki kompleks soni) biror ratsional koeffisitsiyentli darajasi $n \geq 1$ bo'lgan $f(x)$ ko'phadning ildizi bo'lsa, α ga algebraik son deyiladi. Shu ta'rifning bajarilishini tekshiramiz.

1). $\alpha = \frac{3}{5}$ soni algebraik son, chunki u $x - \frac{3}{5} = 0$ tenglamaning, ya'ni $5x - 3 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa birinchi darajali butun koeffitsiyentli ko'phad.

2). $\alpha = \sqrt{3}$ soni algebraik son, chunki u $x^2 - 3 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa ikkinchi darajali butun koeffitsiyentli ko'phad.

3). $\alpha = \sqrt[3]{3}$ soni algebraik son, chunki u $x = \sqrt[3]{3} \to x^3 = 3 \to x^3 - 3 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa uchinchi darajali butun koeffitsiyentli ko'phad.

4). $\alpha = 1 + \sqrt{2}$ soni algebraik son, chunki u $x = 1 + \sqrt{2} \to x - 1 = \sqrt{2} \to (x-1)^2 - 2 = 0 \to x^2 - 2x - 1 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa ikkinchi darajali butun koeffitsiyentli ko'phad.

5). $\alpha = 2 - \sqrt{2}$ soni algebraik son, chunki u $x = 2 - \sqrt{2} \to 2 - x = \sqrt{2} \to (2 - x)^2 - 2 = 0 \to x^2 - 4x + 2 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa ikkkinchi darajali butun koeffitsiyentli ko'phad.

6). $\alpha = 1 + i$ soni algebraik son, chunki u $x = 1 + i \to x - 1 = i \to (x-1)^2 = i^2 \to x^2 - 2x + 2 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa ikkinchi darajali butun koeffitsiyentli ko'phad.

7). $\alpha = \sqrt{3} + \sqrt{5}$ soni algebraik son, chunki u $x = \sqrt{3} + \sqrt{5} \to x^2 = 8 + 2\sqrt{15} \to (x^2 - 8)^2 = 60 \to x^4 - 16x^2 + 4 = 0$tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa to'rtinchi darajali butun koeffitsiyentli ko'phad.

8). $\alpha = \sqrt[4]{4 - \sqrt[3]{2}}$ soni algebraik son, chunki u $x = \sqrt[4]{4 - \sqrt[3]{2}} \to x^4 = 4 - \sqrt[3]{2} \to (x^4 - 4)^3 = -2 \to x^{12} - 12x^8 + 48x^4 - 62 = 0$tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa 12- darajali butun koeffitsiyentli ko'phad.

9). $\alpha = a + \sqrt[n]{b}$ (a,b lar ratsional sonlar) soni algebraik son, chunki u $x = a + \sqrt[n]{b} \rightarrow (x-a)^n = b \rightarrow (x-a)^n - b = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa $n -$ darajali ratsional koeffitsiyentli ko'phad.

10). $\alpha = a + i\sqrt{b}$ (a,b lar ratsional sonlar) soni algebraik son, chunki u $x = a + i\sqrt{b} \rightarrow (x-a)^2 = -b \rightarrow x^2 - 2ax + a^2 + b = 0$ tenglama-ning ildizi. Oxirgi tenglamaning chap tomoni esa ikkinchi darajali ratsional koeffitsiyentli ko'phad.

11). $\alpha = cos\frac{\pi}{n} + isin\frac{\pi}{n}$ soni algebraik son, chunki u $z = cos\frac{\pi}{n} + isin\frac{\pi}{n} \rightarrow z^n = cos\pi + isin\pi \rightarrow z^n + 1 = 0$tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa *n*- darajali butun koeffitsiyentli ko'phad.

12). $\alpha = sin10^0$ soni algebraik son, chunki $(cos\varphi + isin\varphi)^3 = cos3\varphi + isin3\varphi$. Ikkinchi tomondan esa $(cos\varphi + isin\varphi)^3 = cos^3\varphi + 3icos^2\varphi\, sin\varphi + 3i^2cos\varphi\, sin^2\varphi + i^3si\square^3\varphi$ dan $sin3\varphi = 3cos^2\varphi\, sin\varphi - sin^3\varphi = 3(1 - sin^2\varphi)\, sin\varphi - sin^3\varphi = 3sin\varphi - 4sin^3\varphi$ kelib chiqadi. Oxirgi tenglikda $\varphi = 10^0$ deb olsak $sin30^0 = 3 = 3sin10^0 - 4sin^3 10^0 \rightarrow \frac{1}{2} = 3sin10^0 - 4sin^3 10^0 \rightarrow 6sin10^0 - 8sin^3 10^0 - 1 = 0$tenglikka kelamiz. Demak $\alpha = sin10^0$ soni $8x^3 - 6x^2 + 1 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa 3- darajali butun koeffitsiyentli ko'phad.

388. Algebraik sonning tartibi deb u ildizi bo'lgan ratsional koeffitsientli eng kichik darajali ko'phadning daraja ko'rsatkichiga aytiladi.

1).$\alpha = a + bi, (a, b \in Z, b \neq 0) \rightarrow (x - a) = bi \rightarrow (x-a)^2 = -b^2 \rightarrow$ $(x-a)^2 + b^2 = 0 \rightarrow x^2 - 2ax + (a^2 + b^2) = 0$. Bu yerdan $\alpha = a + bi$ ning 2-darajali butun koeffitsiyentli ko'phadning ildizi ekanligi kelib chiqadi. Lekin $\alpha = a + bi$ soni darajasi 2 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi. Agar $x + c = 0$ tenglamaning ildizi desak, $a + bi + c = 0 \rightarrow$ $\begin{cases} a + c = 0 \\ b = 0 \end{cases}$ ni hosil qilamiz. Shart bo'yicha $b \neq 0$. Shuning uchun ham qaralayotgan son ikkinchi tartibli algebrai sondir.

2). $\alpha = \sqrt[3]{3}$ sonining $x^3 - 3 = 0$ tenglamaning ildizi ekanligini 387. 3-misolda ko'rgan edik. Oxirgi tenglamaning chap tomoni esa uchinchi darajali butun koeffitsiyentli ko'phad.

Lekin $\alpha = \sqrt[3]{3}$ soni darajasi 2 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi. Agar $\sqrt[3]{3}$ ni $x^2 + px + q = 0$ tenglamaning ildizi desak, $\sqrt[3]{9} + p\sqrt[3]{3} + q = 0 \rightarrow \sqrt[3]{9} + p\sqrt[3]{3} = -q \rightarrow (\sqrt[3]{9} + p\sqrt[3]{3})^2 = q^2 \rightarrow$

$3\sqrt[3]{3} + 6p + p^2\sqrt[3]{9} = q^2$ ni hosil qilamiz. Bu yerda $\sqrt[3]{9} = -p\sqrt[3]{3} - q$ bo'lgani uchun $3\sqrt[3]{3} + 6p + p^2\left(-q - p\sqrt[3]{3}\right) = q^2$ kelib chiqadi. Bundan $(3 - p^3)\sqrt[3]{3} = (-6p + p^2q + q^2) \rightarrow \sqrt[3]{3} = \frac{-6p + p^2q + q^2}{3 - p^3} -$ ratsional son bo'lishi kerak. $3 - p^3 \neq 0$

bo'lgani uchun bunday bo'lishi mumkin emas. Shuning uchun ham qaralayotgan son uchinchi tartibli algebraik sondir.

3). $\alpha = \sqrt[3]{2} - 1$ sonining $(x+1)^3 - 2 = 0 \rightarrow x^3 + 3x^2 + 3x - 1 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa uchinchi darajali butun koeffitsiyentli ko'phad.

Lekin $\alpha = \sqrt[3]{2} - 1$ soni darajasi 3 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi. Agar $\alpha = \sqrt[3]{2} - 1$ ni $x^2 + px + q = 0$ tenglamaning ildizi desak, $\left(\sqrt[3]{2} - 1\right)^2 + p\left(\sqrt[3]{2} - 1\right) + q = 0 \rightarrow \sqrt[3]{4} + (p-2)\sqrt[3]{2} = p - q - 1 \rightarrow \left(\sqrt[3]{4} + (p-2)\sqrt[3]{2}\right)^2 = (p-q-1)^2 \rightarrow 2\sqrt[3]{2} + 4(p-2) +$
$(p-2)^2\sqrt[3]{4} = (p-q-1)^2 \rightarrow 2\sqrt[3]{2} + +(p-2)^2\sqrt[3]{4} = (p-q-1)^2 - 4(p-2)$ ni hosil qilamiz. Bu yerda $\sqrt[3]{4} = p - q - 1 - (p-2)\sqrt[3]{2}$ bo'lgani uchun $2\sqrt[3]{2} + +(p-2)^2(p-q-1-(p-2)\sqrt[3]{2}) = (p-q-1)^2 - 4(p-2)$ kelib chiqadi. Bundan $[(p-2)^3 - 2]\sqrt[3]{2} = (p-2)^2(p-q-1) - (p-q-1)^2 - 4(p-2) \rightarrow$ $\sqrt[3]{2} = \frac{(p-2)^2(p-q-1)-(p-q-1)^2-4(p-2)}{(p-2)^3-2} -$ ratsional son bo'lishi kerak. $(p-2)^3 - 2 \neq 0$ bo'lgani uchun bunday bo'lishi mumkin emas. Shuning uchun ham qaralayotgan son uchinchi tartibli algebraik sondir.

4). $\alpha = \sqrt{2} - \sqrt{3}$ soni $x^2 = 5 - 2\sqrt{6} \rightarrow (5 - x^2)^2 = 24 \rightarrow x^4 - 10x^2 + 1 = 0$ tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa to'rtinchi darajali butun koeffitsiyentli ko'phad.

Lekin $\alpha = \sqrt{2} - \sqrt{3}$ soni darajasi 4 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi. Agar $\alpha = \sqrt{2} - \sqrt{3}$ni $x^3 + ax^2 + bx + c = 0$ tenglamaning ildizi desak $\left(\sqrt{2} - \sqrt{3}\right)^3 + a\left(\sqrt{2} - \sqrt{3}\right)^2 + b\left(\sqrt{2} - \sqrt{3}\right) + c = 0 \rightarrow$
$2\sqrt{2} - 6\sqrt{3} + 9\sqrt{2} - 3\sqrt{3} + 2a - 2\sqrt{6}a + 3a + \sqrt{2}b - \sqrt{3}b + c = 0 \rightarrow$
$(b+11)\sqrt{2} - (b+9)\sqrt{3} - 2\sqrt{6}a = -(5a + c)$ ni hosil qilamiz. Bu yerda a, b, c ratsional sonlar bo'lgani uchun
oxirgi tenglikning o′ng tomoni ratsional son chap tomoni esa irratsional Shuning uchun ham bu tenglik o'rinli emas. Demak, qaralayotgan son to'rtinchi tartibli algebraik sondir.

5). $\alpha = \sqrt{3} + \sqrt{5}$ soni$x^2 = 8 + 2\sqrt{15} \rightarrow (x^2 - 8)^2 = 60 \rightarrow x^4 - 16x^2 + 4 = 0$tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa to'rtinchi darajali butun koeffitsiyentli ko'phad.

Lekin $\alpha = \sqrt{3} + \sqrt{5}$ soni darajasi 4 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi. Agar $\alpha = \sqrt{3} + \sqrt{5}$ni $x^3 + ax^2 + bx + c = 0$ tenglamaning ildizi desak $\left(\sqrt{3} + \sqrt{5}\right)^3 + a\left(\sqrt{3} + \sqrt{5}\right)^2 + b\left(\sqrt{3} + \sqrt{5}\right) + c = 0 \rightarrow$
$3\sqrt{3} + 9\sqrt{5} + 15\sqrt{3} + 5\sqrt{5} + 3a + 2\sqrt{15}a + 5a + \sqrt{3}b + \sqrt{5}b + c = 0 \rightarrow$

$(b+18)\sqrt{3}+(b+14)\sqrt{5}+2\sqrt{15}a=-(8a+c)$ ni hosil qilamiz. Bu yerda a,b,c ratsional sonlar bo'lgani uchun oxirgi tenglikning o'ng tomoni ratsional son chap tomoni esa irratsional. Shuning uchun ham bu tenglik o'rinli emas. Demak, qaralayotgan son to'rtinchi tartibli algebraik sondir.

6). $\alpha=2+i$ soni$(x-2)^2=-1\to x^2-4x+5=0$tenglamaning ildizi. Oxirgi tenglamaning chap tomoni esa ikkinchi darajali butun koeffitsiyentli ko'phad. Lekin $\alpha=2+i$soni darajasi 2 dan kichik bo'lgan ratsional koeffitsiyentli ko'phadning ildizi bo'la olmaydi.

Agar $\alpha=2+i$ni $x+a=0$ tenglamaning ildizi desak $2+i+a=0\to 2+a=0$ va $1=0$ mumkin bo'lmagan tenglikka ega bo'lamiz. Demak, qaralayotgan son 2-tartibli algebraik sondir.

389. 1). Chunki berilgan $x^3+2\sqrt{2}x^2+2=0$ tenglamaning ildizlari

$(x^3+2\sqrt{2}x^2+2)\left(x^3-2\sqrt{2}x^2+2\right)=(x^3+2)^2-8x^4=x^6+4x^3+4-8x^4=x^6-8x^4+4x^3+4=0$ tenglamaning ildizi bo'ladi. Oxirgi tenglama esa butun koeffitsientli tenglamadir.

2). Chunki berilgan $x^2+2ix+10=0$ tenglamaning ildizlari

$(x^2+2ix+10)(x^2-2ix+10)=(x^2+10)^2+4x^2=x^4+20x^2+100+4x^2=0\to x^4+24x^2+100=0$ tenglamaning ildizi bo'ladi. Oxirgi tenglama esa butun koeffitsiyentli tenglamadir.

390. Buning uchun berilgan tenglamaning chap tomonidagi ko'phadning ratsional sonlar maydonida keltirilmaydigan ko'phad ekanligini ko'rsatish yetarli. Buni ko'rsatish uchun Eyzenshteyin alomatidan foydalanamiz. Unga ko'ra butun koeffitsiyentli $f(x)=a_nx^n+a_{n-1}x^{n-1}+\dots+a_1x+a_0$ ko'phadning ratsional sonlar maydonida keltirilmaydigan bo'lishi uchun ko'phadning bosh hadining koeffitsiyentidan boshqa barcha hadlarning koeffitsiyentlari biror p tub soniga bo'linib, ozod hadi a_0 esa p ga bo'lingani holda p^2ga bo'linmasligi kerak.

1). $x^3+2x^2-4x+2=0$ tenglamaning chap tomonidagi $f(x)=x^3+2x^2-4x+2$ ko'phad ratsional sonlar maydonida keltirilmaydigan ko'phad, chunki bu ko'phadning bosh hadining koeffitsiyentidan boshqa barcha hadlarning koeffitsiyentlari $2,-4,2$ lar 2 tub soniga bo'linib, ozod hadi 2 esa 2 ga bo'lingani holda $2^2=4$ ga bo'linmaydi.

2). $2x^5+6x^3-9x^2-15=0$ tenglamaning chap tomonidagi $f(x)=2x^5+6x^3-9x^2-15$ ko'phad ratsional sonlar maydonida keltirilmaydigan ko'phad, chunki bu ko'phadning bosh hadining koeffitsiyentidan boshqa barcha hadlarning koeffitsiyentlari $0,6,-9,0,-15$ lar 3 tub soniga bo'linib, ozod hadi -15 esa 3 ga bo'lingani holda $3^2=9$ ga bo'linmaydi.

3). $x^4-5x^2+10x+20=0$ tenglamaning chap tomonidagi $f(x)=x^4-5x^2+10x+20$ ko'phad ratsional sonlar maydonida keltirilmaydigan ko'phad, chunki bu ko'phadning bosh hadining koeffitsiyentidan boshqa barcha hadlarning koeffitsiyentlari $0,-5,10,20,$ lar 5 tub soniga bo'linib, ozod hadi 20 esa 5 ga bo'lingani holda $5^2=25$ ga bo'linmaydi.

4). $x^5-3x^2+12x-6=0$ tenglamaning chap tomonidagi $f(x)=x^5-3x^2+12x-6$ ko'phad ratsional sonlar maydonida keltirilmaydigan ko'phad, chunki bu ko'phadning bosh hadining koeffitsiyentidan boshqa barcha hadlarning koeffitsiyentlari $0,0,3,12,-6,$ lar 3 tub soniga bo'linib, ozod hadi -6 esa 3 ga bo'lingani holda $3^2=9$ ga bo'linmaydi.

391. Nazariy qismda keltilgan Liuvill teoremasining natijasidan foydalanamiz. Unga ko'ra quyidagi shartlarni qanoatlantiruvchi har bir cheksiz uzluksiz kasr $\alpha=(q_1,q_2,\ q_3,\ldots)$ transendent son bo'ladi. Bu shartlar $q_i>(Q_{i-1})^{i-1},\ i=k+1,$ $k+2,..$ va q_i lar $(i=1,2,3,..k)$ ixtiyoriy sonlar bo'lishi kerak. Agar biz $k=2$ va $q_1=1,\ q_2=2$ deb olsak $q_3>(Q_2)^2=(Q_1q_2+Q_0)^2=4$ dan $q_3=5$ ni hosil qilamiz. $q_4>(Q_3)^3=(Q_2q_3+Q_1)^3=21^3=9261$ dan $q_4=9262$ ni hosil qilamiz. Shunday mulohazani davom ettirib

$\alpha=(1,2,5,9262\ \ldots)$ sonni hosil qilamiz. Bu son Liuvill teoremasining natijasiga asosan trantsendent son bo'ladi.

392. Berilgan α Liuvil sonining trantsendent ekanligini ko'rsatish uchun Liuvill teoremasining natijasining shartlarining bajarilishini ko'rsatamiz. Liuvill teoremasining natijasiga ko'ra agar α haqiqiy son bo'lib, ixtiyoriy, natural son $n\geq 1$ va ixtiyoriy haqiqiy son $c>0$ uchun hech bo'lmasa birorta ratsional kasr $\frac{a}{b}$, $\left(\frac{a}{b}\neq\alpha\right)$ mavjud bo'lib $\left|\alpha-\frac{a}{b}\right|<\frac{c}{b^n}$ shart bajarilsa, α trantsendent son bo'ladi.

Ushbu $r=\frac{1}{10^{1!}}+\frac{1}{10^{2!}}+\cdots+\frac{1}{10^{k!}}=\frac{c}{10^{k!}}$ ratsional sonini qaraymiz. U holda

$\alpha-r=\frac{1}{10^{(k+1)!}}+\frac{1}{10^{(k+2)!}}+\ \ldots<\frac{1}{10^{(k+1)!}}\left(1+\frac{1}{2}+\frac{1}{2^2}+\cdots\right)=\frac{2}{10^{(k+1)!}}$ bo'ladi va $k\to\infty$ da $\alpha-r\to 0$. Shunday qilib, $\frac{2}{10^{(k+1)!}}<\frac{\varepsilon}{(10^{k!})^n}$ bo'lganda $\alpha-r<\frac{\varepsilon}{(10^{k!})^n}$ bajariladi.

393. A.O.Gelfond teoremasiga asosanagar α soni 0 va 1ga teng bo'lmagan algebraik son, β esa darajasi 2 dan kichik bo'lmagan algebraik (irratsional) son bo'lsa, α^β $-$soni trantsendent son bo'ladi.

1). Agar $lg2-$ algebraik irratsional son bo'lsa, Gelfond teoremasiga asosan $10^{lg2}=2$ soni trantsendent son bo'lishi kerak edi. Lekin bu son algebraik son. Demak, $lg2-$ trantsendent son.

2). Agar $log_2 10 -$ algebraik irratsional son bo’lsa, Gelfond teoremasiga asosan $2^{log_2 10} = 10$ soni trantsendent son bo’lishi kerak edi. Lekin bu son algebraik son. Demak, $log_2 10 -$ trantsendent son.

3). Agar $ln5 -$ algebraik irratsional son bo’lsa, Lindeman teoremasiga ko’ra teoremasiga asosan $y = e^x$ tenglamada $x = 0,\ y = 1$ dan boshqa hollarda x va y sonlari bir vaqtda algebraic son bo’la olmaydi. Bizning misolimizda $e^{ln5} = 5$, ya’ni $y = 5$ algebraik son. Demak, $x = ln5$ $-$trantsendent son.

4). Agar $3^{\sqrt{2}}$ sonining trantsendent son ekanligini ko’rsatish uchun Gelfond teoremasida $\alpha = 3$ va $\beta = \sqrt{2}$ deb olish olish kifoya.

5). Agar $5^{\sqrt{3}}$ sonining trantsendent son ekanligini ko’rsatish uchun Gelfond teoremasida $\alpha = 5$ va $\beta = \sqrt{3}$ deb olish olish kifoya.

6). Agar $2^{i\sqrt{3}}$ sonining trantsendent son ekanligini ko’rsatish uchun Gelfond teoremasida $\alpha = 2$ va $\beta = i\sqrt{3}$ deb olish olish kifoya. Chunki $\beta^2 = -3 \rightarrow \beta^2 + 3 = 0$, ya’ni β ikkinchi darajali algebraik irratsionalikdan iborat.

7). Agar 3^{1-i} sonining trantsendent son ekanligini ko’rsatish uchun Gelfond teoremasida $\alpha = 3$ va $\beta = 1 - i$ deb olish olish kifoya. Chunki $(1-\beta)^2 = -1 \rightarrow 1 - 2\beta + \beta^2 = -1 \rightarrow \beta^2 - 2\beta + 2 = 0$, ya’ni β ikkinchi darajali $x^2 - 2x + 2 = 0$ tenglamning ildizi .

8). Agar $5^{2-i\sqrt{2}}$ sonining trantsendent son ekanligini ko’rsatish uchun Gelfond teoremasida $\alpha = 5$ va $\beta = 2 - i\sqrt{2}$ deb olish olish kifoya. Chunki $\alpha = 5$ algebraik son, $\beta = 2 - i\sqrt{2}$ esa $(2-\beta)^2 = -2 \rightarrow 4 - 4\beta + \beta^2 = -2 \rightarrow \beta^2 - 4\beta + 6 = 0$ tenglamaning, ya’ni β ikkinchi darajali $x^2 - 4x + 6 = 0$ tenglamning ildizi.