III-qism. Misollarning yechimlari.

I.1-§.

1. Qoldiqli bo'lish haqidagi teoremaga asosan: $m = 13 \cdot 17 + r, 0 \le r < 13$ bo'lgani uchun $r = 12$ da eng katta m soni hosil bo'ladi va bu holda $m = 13 \cdot 17 + 12 = 170 + 51 + 12 = 233.$

2. 1). Qoldiqli bo'lish haqidagi teoremadan foydalanamiz: $a = b \cdot q + r, 0 \le r < b$ bizda $a = 25$ $q = 3$ va $b, r = ?$ Shuning uchun ham $r = 25 - 3b \rightarrow 0 \le 25 - 3b < b \rightarrow 3b \le 25 < 4b \rightarrow b = 7; 8$ va $r = 4{,}1.$

2). Qoldiqli bo'lish haqidagi teoremadan foydalanamiz: $a = b \cdot q + r, 0 \le r < b$ bizda $a = -30$ $q = -4$ va $b, r = ?$ Shuning uchun ham $r = -30 + 4b \rightarrow 0 \le -30 + 4b < b \rightarrow -4b \le -30 < -3b \rightarrow b = 8, 9$ va $r = 2, 6.$

3. *a)* $N = 2n + 1,\ N^2 = (2n + 1)^2 = 4n^2 + 4n + 1 = 4n(n + 1) + 1.$ Bu yerda $n(n + 1) \,\vdots\, 2$ bo'linganligi uchun $N^2 = 8q + 1.$

$b)$ $x = n^2 + (n + 1)^2 = 2n(n + 1) + 1.$ Bu yerda $n(n + 1) \,\vdots\, 2$ bo'linganligi uchun $x = 4q + 1.$

4. Bizda $p \ge 5$ –tub son. Ma'lumki, N natural sonni 6 ga bo'lganda $N = 6q + r,$ $r = 0{,}1{,}2{,}3{,}4{,}5$ bo'ladi. $r = 0{,}2{,}3{,}4$ bo'lganda N tub son bo'lmaydi yoki 5 dan kichik tub son bo'ladi. Demak, $p \ge 5$ –tub son $p = 6q + 1$ yoki$p = 6q + 5$ ko'rinishida bo'lishi mumkin.

5. 4-misolga asosan $p \ge 5$ tub son $p = 6q + 1$ yoki $p = 6q + 5$ ko'rinishida bo'lishi mumkun. Agar $p = 6q + 1$ ko'rinishda bo'lsa, u holda $p^2 = 36q^2 + 12q + 1 = 12q(3q + 1) + 1 = 12q(q + 1 + 2q) + 1 = 12q(q + 1) + 24q^2 + 1 = 24Q + 1.$

Agar$p = 6q + 5$ bo`lsa, u holda $p^2 = 36q^2 + 60q + 25 = 12q(3q + 5) + 25 = 12q(q + 1 + 2q + 4) + 25 = 12q(q + 1) + 24q(q + 2) + 24 + 1 = 24Q + 1.$

6. Misolning shartiga asosan

$$\begin{cases} a = mq_1 + 1 \\ b = mq_2 + 1 \end{cases}$$

bo'lgani uchun $ab = (mq_1 + 1)(mq_2 + 1) = m^2 q_1 q_2 + +m(q_1 + q_2) + 1 = m(mq_1 q_2 + q_1 + q_2) + 1 = mQ + 1.$

7. $3m + 2 = x^2$ tenglamaning natural sonlarda yechimga ega emasligini isbotlashimiz kerak. Buning uchun $x = 3q$, $x = 3q + 1,$ $x = 3q + 2$ larni tenglamaga qo'yib tekshirib ko'ramiz.

$x = 3q$ bo'lsa, $3m + 2 = 9q^2$ bajarilmaydi;

$x = 3q + 1$ bo'lsa, $3m + 2 = 9q2 + 6q + 1 = 3q + 1$ bajarilmaydi;

$x = 3q + 2$ bo'lsa, $3m + 2 = 9q2 + 12q + 4 = 3T + 1$ bajarilmaydi.

Demak, tenglama natural sonlarda yechimga ega emas.

8. $15^n = 7q_n + 1$ ekanligini matematik induksiya usuli orqali isbotlaymiz. $n = 1$ uchun $15 = 7 \cdot 2 + 1$ tasdiq to'g'ri. Endi faraz qilaylik nk daisbotlangan tasdiq o'rinli bo'lsin, ya'ni $15^k = 7q_k + 1$tenglik bajarilsin u holda$15^{k+1} = 15^k \cdot 15 = (7q_k + 1) \cdot 15 = (7q_k + 1)(7 \cdot 2 + 1) = 98q_k + 7 \cdot q_k + 7 \cdot 2 + 1 = 7(15q_k + 2) + 1 = 7Q + 1.$ Demak, matematik induksiya metodiga ko'ra isbotlangan tasdiq ixtiyoriy n natural son uchun o'rinli.

9. $2^{2^n} + 1 = x_n$ni qaraymiz, matematik induksiya usulidan foidalanib, $n = 2$ da $x_2 = 2^{2^2} + 1 = 2^4 + 1 = 17$ tasdiq o'rinli.Endi faraz qilaylik $n = k$ da tasdiq o`rinli bo'lsin, ya'ni $x_k = 2^{2^k} + 1 = 10q + 7$ soni 7 raqami bilan tugasin. U holda

$$x_{k+1} = 2^{2^{k+1}} + 1 = 2^{2^{k}\cdot 2} + 1 = (2^{2^k})^2 + 1 = (10q + 6)^2 + 1 =$$
$$= 100q^2 + 120q + 36 + 1 = 10(10q^2 + 12q + 3) + 7 =$$
$$= 10l + 7.$$

Endi $y_n = 2^{4^n} - 5$ $(n = 1,2,3 \dots)$ ni qaraymiz. $n = 1$ da $y = 11 = 10 + 1.$ Faraz qilaylik, $y_k = 2^{4^k} - 5 = 10q + 1$ bo'lsin. U holda

$$y_{k+1} = 2^{4^{k+1}} - 5 = 2^{4^{k}\cdot 4} - 5 = (2^{4^k})^4 - 5 = (10q + 6)^4 - 5 = 10t + 36 - 5$$
$$= 10t_1 + 1.$$

Demak matematik induksiya prinsipiga asosan isbotlanayotgan tasdiq ixtiyoriy n natural soni uchun o'rinli.

10. $l = 2n + 1$va $m = 2s + 1$ lar toq sonlar berilgan bo'lsin. $l^2 + m^2$ yig'indini qaraymiz: $l^2 + m^2$=$(2n + 1)^2 + (2s + 1)^2 = 4n^2 + 4n + 1 + 4s^2 + 4s + 1 = 4(n^2 + n + s^2 + s) + 2 = 4M + 2 = 2(2M + 1)$, bunda $M = n^2 + n + s^2 + s$ va $(2M + 1)$-toq son biror butun sonning kvadrati bo'lsa ham 2 soni esa butun sonning kvadratiga teng bo'la olmaydi. Shuning uchun ham $l^2 + m^2 = 2(2M + 1)$ soni butun sonning kvadratiga teng bo'lmaydi.

11. Qaralayotgan uchburchakning katetlari x, y va gipotenuzasini z bilan belgilaylik. Ikkala katet ham 3 ga bo'linmasa ularning har biri $3q + 1$ yoki

$3q + 2$ ko'rinishda bo'ladi. Bundan agar $x = 3q + 1,\ y = 3q + 1$ bo'lsin, u holda $x^2 + y^2 = (3q + 1)^2 + (3q + 1)^2 = 3Q_1 + 1 + 3Q_2 + 1 = 3Q + 2.$

Agar $x = 3q + 1,\ y = 3q + 2$ bo'lsa, u holda $x^2 + y^2 = (3q + 1)^2 + (3q + 2)^2 = 3Q_1 + 1 + 3Q_2 + 4 = 3(Q_1 + Q_2 + 1) + 2 = 3Q + 2.$

Agar $x = 3q + 2,\ y = 3q + 2$ bo'lsa , u holda

$$x^2 + y^2 = (3q + 2)^2 + (3q + 2)^2 = 3Q_1 + 4 + 3Q_2 + 4 = 3(Q_1 + Q_2 + 2) + 2$$
$$= 3Q + 2.$$

$x^2 + y^2 = z^2$ bo'lgani uchun gipotenuzaning kvadrati z^2 ham va 2 ning o'zi ham 3 ga bo'linmaydi, ya'ni $z^2 = 3Q + 2$. Lekin bu holda z^2 ni 3 ga bo'lsak 2 emas *1* qoldiq qolish, kerak (7- masalaga qarang) shuning uchun ham $x \vdots 3$ yoki $y \vdots 3.$

12. Agar x katet 5 ga bo'linmasa, uni $x = 5q + r, 1 \leq r \leq 4$ deb yoza olamiz. Bundan $x^2 = 5Q_1 + r^2$, $r = 1,2,3,4$;

$$\begin{cases} r = 1 \ da \ x^2 = 5Q_1 + 1 \\ r = 2 \ da \ \ x^2 = 5Q_1 + 4 \\ r = 3 \ da \ \ x^2 = 5Q_2 + 4 \\ r = 4 \ da \ \ x^2 = 5Q_3 + 1 \end{cases}$$

ya'ni$x^2 = 5Q + 1$ yoki $x^2 = 5\,Q + 4$ bo'lar ekan. Endi agar y katet ham 5 ga bo'linmasa, u holda uni ham $y^2 = 5T + 1$ yoki$y^2 = 5T + 4$ ko'rinishda ifodalash mumkin bo'ladi va bulardan

$$x^2 + y^2 = 5K + r, \ \ r = 0, 2, 3 \qquad (*)$$

ni hosil qilamiz. Agar z gipotenuza 5 ga bo'linmasa, uning kvadrati z^2ni 5 ga bo'lishdan quidagicha qoldiqlar hosil bo'ladi:

$z = 5q + 1$, $z = 5q + 2$, $z = 5q + 3$,$z = 5q + 4$, $z^2 = 5l_1 + 1$, $z^2 = 5l_2 + 4$, $z^2 = 5l_3 + 4$, $z^2 = 5l_3 + 1$ ya'ni z^2 ni 5 ga bo'lishdan 1 yoki 4 qoldiq qoladi. Shuning uchun, $(*)$ da r uchun faqat bizda $r = 0$ imkoniyat mavjud.U holda $x = 5q$, ya'ni x katet 5 ga bo'linadi. Birinchi x katet va $z$gipotenuza 5 ga bo'linmasligidan ikkinchi y katetning 5 ga bo'linishi ham shunga o'xshash isbotlanadi.

13. $S_n = 1 + 2 + 3 + \cdots + n = \frac{(1+n)}{2} \cdot n$ dan foydalanamiz. Agar $n = 5q$ ko'rinishda bo'lsa, u holda $S_n = \frac{(5q+1)5\cdot q}{2} = 5Q$ bo'ladi.

Agarda $n = 5q + 1$ ko'rinishda bo'lsa, u holda

$$S_n = \frac{(5q + 2)(5q + 1)}{2} = \frac{25q^2 + 15q + 2}{2}$$
$$= \frac{5q(5q + 3) + 2}{2} \frac{5q\big((q + 1) + (4q + 2)\big) + 2}{2}$$
$$= \frac{5q(q + 1)}{2} + 5q(2q + 1) + 1 = 5\left(\frac{q(q + 1)}{2} + q(2q + 1)\right) + 1$$
$$= 5Q + 1$$

bo'ladi.

Agarda $n = 5q + 2$ ko'rinishda bo'lsa, u holda

$$S_n = \frac{(5q + 3)(5q + 2)}{2} = \frac{25q^2 + 25q + 6}{2} = \frac{25q(q + 1)}{2} + 3 = 5Q + 3$$

bo'ladi. Agar $n = 5q + 3$ bo`lsa, u holda

$$S_n=\frac{(5q+4)(5q+3)}{2}=\frac{25q^2+35q+12}{2}=\frac{5q(5q+7)}{2}+6$$
$$=\frac{5q\big((q+1)+4q+6\big)}{2}+6=\frac{5q(q+1)}{2}+\frac{5q(2q+3)}{1}+5+1$$
$$=5\left(\frac{q(q+1)}{2}+q(2q+3)+1\right)+1=\ 5Q+1$$

bo’ladi. $n=5q+4$ bo`lsa, u holda

$$S_n=\frac{(5q+5)(5q+4)}{2}=\frac{5(q+1)\big(q+4(q+1)\big)}{2}=\frac{5q(q+1)}{2}+10(q+1)^2$$
$$=5\left(\frac{q(q+1)}{2}+2(q+1)^2\right)=5Q$$

Demak $n=\ 5q+1$ va $n=5q+3,\ q=0,1,2,...,$ ko`rinishidagi $n$ lar uchun $S_n$ yigindini 5 ga bo`lsak, 1 qoldiq chiqar ekan.

14. $ax-\ by$ ifodaga bx qo`shib va ayirib quyidagicha yozib olamiz:

$ax-\ by\ =\ ax-\ by+bx-bx\ =(a-b)x+b(x-y)$. Shartga ko`ra bu tenglikning chap tomoni $m$ ga bo`linadi, ya’ni $ax-\ by\ =m\cdot k;$ shuning uchun o`ng tomoni ham $m$ ga bo`linadi. Ya`ni $a-b=m\cdot l$va $(b,m)=1$ bo`lganda $b(x-y)=(ax-by)+(b-a)x=mk-ml=\ m(k-l)$va demak $(x-y)\ \vdots\ m$ kelib chiqadi.

15. Berilgan ifodalarda quyidagicha shakl o`zgartirish qilamiz:

$4^n+15n-1=(1+3)^n+15n-1=1+n\cdot 3+\ 3^2\cdot\frac{n(n-1)}{2!}+\frac{n(n-1)(n-2)}{3!}\cdot 3^3+\ \cdots+\ 3^n+15n-1=18n+9Q=9\cdot(2n+Q)$

Demak, bu tenglikning o`ng tomoni 9 ga bo`linadi, demak chap tomoni ham 9 ga bo`linishi kerak.

16. 1) $f(n)=10^n+18n-1$ ifodaning 27 ga bo`linishini ko`rsatamiz. Buning uchun matematik induksiya metodidan foydalanamiz. $f(1)=27\ \vdots\ 2.$

Endi n=k uchun $f(k)\ \vdots\ 27,$ ya`ni $f(k)=27q$ bo`lsin. $f(k+1)$ ni qaraymiz: $f(k+1)=10^{k+1}+18(k+1)-1=10\cdot 10^k+18k+17=(10^k+18k-1)+9\cdot 10^k+18=27q+9(10^k+2),$ bu yerda 10^k+2 ifoda k ning natural qiymatlarida 3 ga bo’linadi. Shuning uchun ham oxirgi tenglikning o’ng tomoni 27 ga va demak chap tomoni ham 27 ga bo’linadi. Shunday qilib matematik induksiya prinsipiga ko’ra istalgan natural n uchun $f(x)\ \vdots\ 2.$

2). Endi $F(n)=3^{2n+3}+40n-27$ ning 64 ga bo’linishini isbotlaymiz. $f(1)=243+40-27=256\ \vdots\ 64.$ Faraz qilaylik, $f(k)\ \vdots\ 64,$ ya`ni $f(k)=64q$ bo’lsin. U holda

$$f(k+1)=3^{2(k+1)+3}+40(k+1)-27=3^{2k+3}\cdot 3^2+40k+13$$
$$=(3^{2k+3}+40k-27)+8\cdot 3^{2k+3}+40=64q+8(3^{2k+3}+5)$$

tenglik o'rinli. Endi $g(k) = 3^{2k+3} + 5$ ning 8 ga bo'linishini ko'rsatamiz. $g(1) = 3^{2\cdot 1+3} + 5 = 248$ bo'lib, bu son 8 ga bo'linadi, ya`ni $g(1) \vdots 8$.

$g(k) \vdots 8$ bo'lsin deb faraz qilaylik, ya`ni $g(k) = 8l$ bo'lsin, u holda $g(k+1) = 3^{2(k+1)+3} + 5 = 3^{2k+3} \cdot 3^2 + 5 = (3^{2k+3} + 5) + 8 \cdot 3^{2k+3} = 8l + 8 \cdot 3^{2k+3} = 8(l + 3^{2k+3}) = 8s$, demak ixtiyoriy $k \in N$ uchun $g(k) = 3^{2k+3} + 5$ ifoda 8 ga bo'linadi. Shuning uchun ham $F(k+1) \vdots 64$. Shunday qilib ixtiyoriy $k \in N$ uchun $F(k) \vdots 64$.

17. 1) $f(n) = \frac{n}{2n^2+1}$ kasrni qaraymiz. Bu kasr qisqarmas kasr, chunki $(n;\ 2n^2 + 1\ ) = 1$. Kasr sof davriy kasrga yoyilishi uchun uning maxrajida 2 va 5 sonlarning ko`paytuvchi sifatida qatnashmasligi kerak. Shuni tekshiramiz: $2n^2\ + 1$ ifoda 2 ga bo`linmaydi (2 ga bo`lsa, 1 qoldiq qoladi).

Endi maxrajda 5 ko`paytuvchi sifatida qatnashmasligini ko`rsatamiz. $n = 5q + r$, $r = 0,1,2,3,4$ deb olamiz, u holda $g(n) = 2n^2 + 1$ dan

$g(5q + r) = 2(5q + r)^2 + 1 = 2(25q^2 + 10qr + r^2) + 1 = 5Q + g(r),\ (*)$

bunda $r = 0,1,2,3,4$. r ning bu qiymatlarda $g(r)$ ning 5 ga bo`linmasligini ko`rsatamiz. Buni bevosita tekshirish orqali amalga oshirish mumkin.

$$g(0) = 1, \qquad g(1) = 3, \qquad g(2) = 9, \qquad g(3) = 19, \qquad g(4) = 33$$

larnig birortasi ham 5 ga bo'linmaydi. (*) dan $g(n)$ ning 5 ga bo'linmasligi kelib chiqadi.

2). Endi $f(n) = \frac{n}{n^2+n+1}$ ni qaraymiz. Bu yerda ham $(n;\ n^2\ + n + 1) = 1$ va $g(n) = n^2 + n + 1 = n(n+1) + 1 = 2q + 1$, ya'ni maxraj 2 ga bo`linmaydi. Bu yerda $g(5q + r) = (5q + r)^2 + 5q + r + 1 = 25q^2 + 10qr + r^2 + 5q + r + 1 = 5Q + r^2 + r + 1 =\ 5Q + g(r)$. $(*)$

Bunda $g(r)$ ifoda $(r = 0,1,2,2,3,4\ )$ 5 ga bo`linmaydi. Haqiqatan ham, $g(0) = 1,\ g(1) = 3,\ g(2) = 7,\ g(3) = 13, g(4) = 21$ sonlarning birortasi ham 5ga karrali emas.$(*)$ dan berilgan kasrning maxrajida 5 soni ko`paytuvchi sifatida qatnashmaydi degan xulosa kelib chiqadi. Demak $f(n)$ kasr son davriy kasrga yoyiladi.

18. $N_1 = \overline{a_1a_2a_3}$, $N_2 = \overline{b_1b_2b_3}$ lar uch xonali sonlar bo'lsin, u holda $M = \overline{a_1a_2a_3b_1b_2b_3} = \overline{a_1a_2a_3} \cdot\cdot 10^3 + b_1b_2b_3 = N_1 \cdot 10^3 + N_2 = (N_1 + N_2) + 999N_1 = (N_1 + N_2) + 37 \cdot 27N_1$ va masalaning sharti bo`yicha $(N_1 + N_2) \vdots 37$, ya'ni $N_1 + N_2 = 37q$. Shuning uchun ham $M = 37q + 37 \cdot 27N_1 = 37(q + 27N_1)$ va demak $M \vdots 37$.

19. *a)*.Birinchi usul. $m^5 - m = m(m^4 - 1) = m(m^2 + 1)(m^2 - 1) = m(m^2 + 1)(m - 1)(m + 1) = (m - 1)m(m + 1)(m^2 - 4 + 5) = (m - 2)(m - 1)m(m + 1)(m + 2) + 5(m - 1)m(m + 1) = 5! \cdot \frac{(m-2)(m-1)m(m+1)(m+2)}{5!} + 5(m - 1)m(m + 1) = 5\left(4! \cdot C_{m+1}^5 + (m - 1)m(m + 1)\right),$

Bu yerda C_{m+1}^5 butun son bo'lgani uchun $(m^5-m) \vdots 5$.

Ikkinchi usul. $m=5x+y, 0 \le y \le 4$ deb olsak, $m^5-m=(5x+y)^5-5x-y=5Q+(y^5-y)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonidagi birinchi qo'shiluvchi 5 ga bo'linadi. Ikkinchi qo'shiluvchi $g(y)=(y^5-y)$ ning 5 ga bo'linishini esa $y=0,1,2,3,4$ qiymatlarda bevosita tekshirib ko'rish mumkin: $g(0)=0,\ g(1)=0,\ g(2)=30,\ g(3)=240,\ g(4)=1020$ bularning barchasi 5 ga bo'linadi. Demak, $(m^5-m) \vdots 5$.

b). $m=6x+y, 0 \le y \le 5$ deb olsak, $m(m^2+5)=(6x+y)((6x+y)^2+5)=(6x+y)(36x^2+12xy+y^2+5)=6Q+(y^3+5y)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonidagi birinchi qo'shiluvchi 6 ga bo'linadi. Ikkinchi qo'shiluvchi $g(y)=(y^3+5y)$ ning 5 ga bo'linishini esa $y=0,1,2,3,4,5$ qiymatlarda bevosita tekshirib ko'rish mumkin: $g(0)=0,\ g(1)=6,\ g(2)=18,$ $g(3)=42,\ g(4)=84$ bularning barchasi 6 ga bo'linadi. Demak, $m(m^2+5) \vdots 6$.

c). $f(m)=m(m+1)(2m+1)=m(m+1)((m+2)+m-1=m(m+1)(m+2)+m(m-1)(m+1)=6(C_{m+2}^3+C_{m+1}^3)$.

Bu yerda ikkala had ham 3 ta ketma–ket natural sonlar ko`paytmasidan iborat.

C_{m+2}^3, C_{m+1}^3 lar mos ravishda $m+2$ elementdan 3 tadan, $m+1$ elementdan 3 tadan tuzilgan gruppalashlar sonini bildirgani uchun ular natural sonlar.

Demak $f(m) \vdots 6$.

20. $S=l+(l+1)+\cdots+(l+2n)$ yig`indini qaraymiz. Arifmetik progressiya hadlari yig`indisi topish formulasiga asosan

$$S=\frac{l+l+2n}{2}(2n+1)=(l+n)(2n+1)$$

Bundan $S \vdots (2n+1)$ ekanligi kelib chiqadi.

21. *a)* $N=1000q+r$ sonini qaraymiz. Bundan $N=100q+r-q=7\cdot 11\cdot 13q+(r-q)$. Demak, berilgan N sonning 7,11 yoki 13 ga bo`linishi uchun uning mingliklar soni $q$ va $N$ ni 1000 ga bo`lishidan chiqqan qoldiq r ning ayirmasi $r-q$ ning 7,11 13 ga bo`linishi zarur va yetarlidir.

b) $N=368312=368\cdot 1000+312;\ \ 368-312=56$ .56 soni 7 ga bo`linadi. Demak, berilgan 368312 soni ham 7 ga bo`linadi. 56 soni 11 ga ham 13 ga ham bo`linmaydi shuning uhun ham $N=368312$ soni 11ga ham13 ga ham bo`linmaydi.

22. $N_1=\overline{a_n a_{n-1} \dots a_1 a_0},\ N_2=\overline{b_m b_{m-1} \dots b_1 b_0}$ sonlarni qaraymiz. Shart bo'yicha

$$\sum_{i=0}^{n} a_i=\sum_{j=0}^{m} b_j.$$

Bundan

$N_1=a_n\cdot 10^n+a_{n-1}\cdot 10^{n-1}+\cdots+a_1\cdot 10^1+a_0=a_n\cdot(9+1)^n+$

$+a_{n-1}\cdot(9+1)^{n-1}+\cdots+a_1\cdot 10^1+a_0=9Q+a_n+a_{n-1}+\cdots$
$\ldots+a_1+a_0=9Q+\sum_{i=0}^{n}a_i.$

Shuningdek,

$$N_2=9T+\sum_{j=0}^{m}b_j.$$

U holda $N_1-N_2=9Q-9T=9(Q-T)$, ya'ni $(N_1-N_2)\,\vdots\,9$.

23. $7+77+777+\cdots+\underbrace{777\ldots77}_{n\ ta}=7\cdot(1+11+111+\cdots+\underbrace{111\ldots11)}_{n\ ta}=7\cdot$
$\left(\frac{10-1}{9}+\frac{10^2-1}{9}+\frac{10^3-1}{9}+\cdots+\frac{10^n-1}{9}\right)=\frac{7}{9}\cdot(10+10^2+10^3+$

$$+\cdots+10^n-n)=\frac{7}{9}\cdot\left(\frac{10(1-10^n)}{1-10}-n\right)=\frac{7}{81}\cdot(10^{n+1}+9n-10).$$

Bu yerda biz geometrik progressiya hadlari yig'indisini topish formulasi

$$S_n=\frac{b_1(1-q^n)}{1-q}$$

dan foydalandik.

24. $N=\underbrace{444\ldots44}_{n\ ta}\cdot\underbrace{888\ldots88}_{n\ ta}$ sonini qaraymiz.

$N=\underbrace{444\ldots44}_{n\ ta}\cdot 10^n+\underbrace{888\ldots88}_{n\ ta}=(4\cdot10^{n-1}+4\cdot10^{n-2}+\cdots+4\cdot10+4)\cdot$

$$\cdot10^n+(8\cdot10^{n-1}+8\cdot10^{n-2}+\cdots+8\cdot10+8)=4\cdot\frac{10^n-1}{9}\cdot10^n+$$

$$+8\cdot\frac{10^n-1}{9}=\frac{4}{9}\cdot\frac{10^n-1}{9}\cdot(10^n+2)=\left[\frac{2}{3}\cdot(10^n-1)\right]\cdot\left[\frac{2}{3}\cdot(10^n-1+3)\right]\cdot$$

Bu yerda

$$10^n-1=9\cdot(10^{n-1}+10^{n-2}+\cdots+10+1)$$

bo'lgani uchun

$\frac{2}{3}\cdot(10^n-1)=\underbrace{666\ldots66}_{n\ ta}$ va $\frac{2}{3}(10^n-1+3)=\underbrace{666\ldots68}_{n\ ta}$

Demak, $N=\underbrace{666\ldots66}_{n\ ta}\cdot\underbrace{666\ldots68}_{n\ ta}$.

25. $N=\underbrace{111\ldots1}_{n\ ta}\underbrace{55\ldots5}_{n\ ta}6=\frac{10^{n+1}-1}{9}\cdot10^{n+1}+5\cdot10\cdot\frac{10^n-1}{9}+6=$

$$\frac{(10^{n+1})^2+40\cdot10^n-50}{9}+6=\frac{(10^{n+1}+2)^2}{3^2}=\left(\frac{(10^{n+1}-1)}{3}+1\right)^2=$$

$$\left(\underbrace{333...3}_{n\ ta}+1\right)^2.$$

26. $S_n = (n+1)(n+2)\,...\,(n+n)$ ifodani $n!$ ga kopaytirib bo’lamiz:

$$S_n = \frac{1\cdot 2\cdot 3\cdots n(n+1)(n+2)\cdots 2n}{1\cdot 2\cdot 3\cdots n} = \frac{(2\cdot 4\cdot\cdots\cdot 2n)\cdot\big(1\cdot 3\cdot 5\cdot\cdots(2n-1)\big)}{1\cdot 2\cdot 3\cdots n}$$
$$= 2^n\cdot(2n-1)!!\,.$$

Demak, $S_n \vdots\, 2^n$.

I. 2-§.

27. Berilgan a va b sonlaridan foydalanib Evklid algoritmini tuzib olamiz:

1) $a = 546$ va $b = 231$

$$546 = 231\cdot 2 + 84$$
$$231 = 84\cdot 2 + 63$$
$$84 = 63\cdot 1 + 21$$
$$63 = 21\cdot 3$$
$$(546; 231) = 21;\quad J: 21.$$

2) $a = 6253$ va $b = 1001$

$$6253 = 1001\cdot 6 + 247$$
$$1001 = 247\cdot 4 + 13$$
$$247 = 13\cdot 19$$
$$(6253; 1001) = 13;\quad j: 13.$$

2) $2257 = 1517\cdot 1 + 740, 1517 = 740\cdot 2 + 37, 740 = 37\cdot 20.\ \ J: 37.$

28. Berilgan sonlarni tub ko’paytuvchilarga ajratib yozib olamiz:

a)

$$\begin{array}{r|l} 420 & 2 \\ 210 & 2 \\ 105 & 3 \\ 35 & 5 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 126 & 2 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 525 & 3 \\ 175 & 5 \\ 35 & 5 \\ 7 & 7 \\ 1 & 1 \end{array}$$

bo’lgani uchun $420 = 2\cdot 2\cdot 5\cdot 3\cdot 7$; $126 = 2\cdot 3\cdot 3\cdot 7$; $525 = 3\cdot 5\cdot 5\cdot 7$ va bulardan $(420; 126; 525) = 3\cdot 7 = 21$;

$$[420; 126; 525] = 2\cdot 2\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7 = 6300.\ J: 21\ va\ 6300.$$

b)

$$\begin{array}{r|l} 529 & 23 \\ 23 & 23 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1541 & 23 \\ 67 & 67 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1817 & 23 \\ 79 & 79 \\ 1 & \end{array}$$

bo’lgani uchun $529 = 23 \cdot 23$; $1541 = 23 \cdot 67$; $1817 = 23 \cdot 79$

va $(529; 1541; 1817) = 23$; $[529; 1541; 1817] = 23 \cdot 23 \cdot 67 \cdot 79 = 529 \cdot 67 \cdot 79 = 2799997$. *j*: 23 va 2799997

29. *a*). Bunda$(6; 35) = (6; 143) = (35,143) = 1$, ya’ni bu sonlar $6, 35,143$ juft-juft bilan o’zaro tub. Shuning uchun ham ularning EKUKi berilgan sonlarning ko’paytmasiga teng, ya’ni $[6; 35; 143] = 6 \cdot 35 \cdot 14$.

b) n va *n+1* sonlari o’zaro tub $(n; n+1) = 1$ shuning uchun ham $[n, n+1] = n(n+1)$.

30. $2n$ va $2n+2$ lar ikkita ketma-ket keluvchi juft sonlar bo’lsin. U holda

$$(2n; 2n+2) = 2(n; n+1) = 2.$$

Endi $2n+1$va $2n+3$ lar ikkita ketma-ket keluvchi toq sonlar bo’lsin. U holda

$$2n+3 = (2n+1) \cdot 1 + 2, \qquad 2n+1 = 2n+1$$

lardan Evklid algoritmiga asosan $(2n+1, 2n+3) = 1$ ekanligi kelib chiqadi.

31.$(cb; bc; ca) = c(b; b; a) = c(a; b)$ tenglik o’rinli. Ikkinchi tomondan esa$(a; b; c) = \big((a; b); c\big) = d$ => $(a; b) = dx$ va $c = dy$. Shuning uchun ham $(cb; bc; ca) = d^2xy$ => $(cb; bc; ca) \vdots (a; b; c)^2$.

32. $(a+b; a-b) = d$ bo’lsin. U holda $a+b = dx$ $a-b = dy$ deb yoza olamiz. Bundan $2a = d(x+y), 2b = d(x-y)$, ya`ni d soni $2a$ va $2b$ larning umumiy bo’luvchisi. Shartga asosan $(a; b)$=1 bo’lgani uchun $(2a; 2b) = 2$. Shuning uchun ham $2 \vdots d$ $d = 1$ yoki $d = 2$.

33. Aytaylik $(a, a+b) = d$ bo’lsin, u holda $a = dxa + b = dy$ yoki $dx + b = dy$ bundan esa $b \vdots d$ bo’lishini va $d = UB\ (a, b)$, (UB-umumiy bo’luvchi) ekanligini topamiz $\frac{a}{b}$ qisqarmas kasr bo’lganidan *d=1*. Demak $\frac{a}{a+b}$ kasr qisqarmas kasr.

34. a va b lar toq sonlar va $a - b = 2^n$ bo’lsin. U holda $(a; b) = d$ −toq son bo’ladi. $a = dx$, $b = dy$, $(x; y) = 1$ deb olsak, $a - b = d(x-y) = 2^n$ => $2^n \vdots d$ $\Rightarrow d = 1$

35. *a)*$(d, m) = \big(d; [dx : dy]\big) = d(1; [x; y]) = d$.

b)$(ab, m) = (dm, m) = m(d; 1) = m$. Bunda $d = (a, b)$ $m = [a, b]$.

c) $(a+b; ab) = x$ $(a; b) = 1$. Faraz etaylik, p soni $a+b$ va ab ning umumiy bo’luvchisi bo’lsin. U holda $a \vdots p$ yoki $b \vdots p$. U holda $(a+b) \vdots p$ bajarilgani uchun p soni a va b sonining umumiy bo`luvchisi bo’ladi. $(a, b) = 1$ bo’lgani uchun $p = 1$ va demak $(a+b; ab) = x = 1$.

d) $(a+b; m) = ?$. $m = [a; b]$va$(a, b) = d$ bo’lsin, u holda $a = dx$, $b = dy$ va $(x; y) = 1$. Bulardan $(a+b; m) = (d(x+y); [dx; dy]) = (d(x+y); d[x; y]) = d(x+y; xy)$.

b) misolga asosan $(x+y; xy) = 1$ va demak $(a+b; m) = d$, ya`ni $(a+b; [a; b]) = (a; b)$.

36. $a)\quad (n;2n+1)=\big(n;n+(n+1)\big),\quad d=(n;2n+1),\qquad n=dx$

$$d=(n;2n+1)=(dx;2dx+1)\quad\Longrightarrow\quad d=1.$$

$b)\quad (10n+9;n+1)=d,\quad 10n+9=dx,\quad n+1=dy,\ \ (x;y)=1.$

$$10(dy-1)+9=dx\Rightarrow\ 10dy-1=dx\Longrightarrow\ 10dy-dx=1\Rightarrow\ d(10y-x)=1$$

$$\Rightarrow\ d=1.$$

$c)$ $(3n+1;10n+3)=d$ bo'lsin, u holda

$$\begin{cases}3n+1=dx\\10n+3=dy\end{cases}\Rightarrow 10dx-3dy=1\Rightarrow d(10x-3y)=1\Rightarrow d=1.$$

Eslatma: a) va c) misollarni Evklid algoritmidan foydalanib ham ishlash mumkin.

37. Agar $x=[a;b]=\dfrac{ab}{(a;b)}$ bo'lsa, u holda $\left(\frac{x}{a};\frac{x}{b}\right)=\left(\frac{b}{(a,b)};\frac{a}{(a,b)}\right)$, bunda

$(a;b)=d$ va $b=db_1$ $a=da_1$va demak, $\left(\dfrac{x}{a};\dfrac{x}{b}\right)=1$. Aksincha, agar $\left(\dfrac{x}{a};\dfrac{x}{b}\right)=1$

bo'lsa $x=[a;b]y$ deb olib

$$1=\left(\frac{x}{a};\frac{x}{b}\right)=\left(\frac{[a;b]y}{a};\frac{[a;b]y}{b}\right)=y\left(\frac{[a;b]}{a};\frac{[a;b]}{b}\right)=\left(\frac{b}{(a,b)};\frac{a}{(a,b)}\right)=y,$$

ya'ni $y=1$. Bundan $x=[a,b]$ni hosil qilamiz.

38. a,b,c - toq sonlar bo'lib $(a;b;c)=D$ bo'lsin.U holda $a=Dx,\ b=Dy, c=Dz$ va x,y,z lar toq sonlar bo'ladi. U holda $\frac{a+b}{2}=D\cdot\frac{x+y}{2}$; $\frac{a+c}{2}=D\cdot\frac{x+z}{2}$; $\frac{b+c}{2}=D\cdot\frac{y+z}{2}$ bo'lib $\frac{x+y}{2}$; $\frac{x+z}{2}$; $\frac{y+z}{2}$ lar butun sonlar bo'ladilar.

Bu yerdan D ning $\frac{a+b}{2},\frac{a+c}{2},\frac{b+c}{2}$ sonlarining umumiy bo'luvchisi ekanligi kelib chiqadi. Endi agar $\left(\frac{a+b}{2},\frac{a+c}{2},\frac{b+c}{2}\right)=d$ desak $d\ \vdots\ D$. Ikkinchi tomondan esa d soni a,b,c larning umumiy bo'luvchisi, ya`ni $D\ \vdots\ d$. Bulardan $D=d$. Haqiqatan ham $\frac{a+b}{2}=dx\Rightarrow a+b=2dx$. Shuningdek $a+c=2dy$ va $b+c=2dz$. Oxirgi tenglikdan $c=2dz-b$ ga egamiz. Buni $a+c=2dy$ tenglikga olib borib qo'ysak, $a+2dz-b=2dy\Rightarrow a-b=2dy-2dz$ hosil bo'ladi. Y holda bundan va birinchi tenglikdan

$$\begin{cases}a+b=2dx\\a-b=2dy-2dz\end{cases}\Rightarrow a=d(x+y-z)\Rightarrow a\ \vdots\ d$$

va $b=d(x-y+z)\Rightarrow b\ \vdots\ d$. Bulardan $c=2dz-b_1d=d(2z-b_1)\Rightarrow c\ \vdots\ d$. d soni a,b,c larning umumiy bo'luvchisi.

39.1) agar

$$a=cq+r;\quad b=cq+r_1\qquad\qquad (1)$$

bo'lsa $(a,b,c)=(c;r;r_1)$ ekanligini isbotlashimiz kerak. $(a;b;c)=d$ deb olaylik. U holda (1) dan $r\ \vdots\ d$ va $r_1\ \vdots\ d$ ekanligi kelib chiqadi. Biz $d=(c;r;r_1)$

ekanligini ko`rsatamiz. $(c;r;r_1)=D$ bo`lsin. U holda (1) dan $a \vdots D$ va $b \vdots D$ kelib chiqadi. Shuningdek, $c \vdots D$. Demak, $D=(a;b;c)$ va $D=d$.

2) a) $667=299\cdot 2+69$ va $391=299\cdot 1+92$ bo’lgani uchun 1)-misolga asosan $(299,391,667)=(299,69,92)=(23\cdot 13;\ 23\cdot 3;\ 23\cdot 4)=23\cdot(13;3;4)=23$.

b). $(588;2058;2849)=(588;497;294)=(2^2\cdot 3\cdot 7^2;\ 7\cdot 71;\ 2\cdot 3\cdot 7^2)=7$, bunda 2889$=588\cdot 4+497$ va $2058=588\cdot 3+294$; $497=71\cdot 7$ ekanligidan foydalandik.

40. Faraz qilaylik $(a,b)=d$ bo’lsin, unda $a=dx,\ b=dy$ bo’lib, bu yerda $(x,y)=1$ bo’ladi. U holda $5a+3b=d(5x+3y)$ va $13a+8b=d(13x+8y)$ bo’lib, bundan esa $UB(5a+3b,13a+8b)=d$ ekanligini topamiz. Endi $(5\mathrm{x}+3\mathrm{y},13\mathrm{x}+8\mathrm{y})=1$ ekanligini isbotlash kerak. Aytaylik

$(5\mathrm{x}+3\mathrm{y},13\mathrm{x}+8\mathrm{y})=\delta$ va $5\mathrm{x}+3\mathrm{y}=\ \delta u, 13\mathrm{x}+8\mathrm{y}=\delta v$ bo’lsin, u holda

$x=\delta(8u-3v)\ \ vay=\delta(5v-13u)$ bo’ladi, ammo $(\mathrm{x},\mathrm{y})=1$ edi, shuning uchun $\delta=1$. Shunday qilib $(5a+3b,13a+8b)=d$.

41.$(n,n+1,n+2)$va $[n,n+1,n+2]$ larni topish kerak.

$$(n,n+1,n+2)=\big((n,n+1),n+2\big)=(1,n+2)=1$$

$$[n,n+1,n+2]=\big[[n,n+1],n+2\big]=[n(n+1),n+2]=$$

$$\frac{n(n+1)(n+2)}{\big(n(n+1);(n+2)\big)}=\frac{n(n+1)(n+2)}{\big(n;(n+2)\big)},$$

chunki $(n+1;n+2)=1$. Bu yerda

$$(n;n+2)=\begin{cases}1, \text{agar } n \text{ toq son bo'lsa;}\\ 2, \text{agar } n \text{ juft son bo'lsa.}\end{cases}$$

Haqiqatan ham, $n=2k-$juft son bo’lsin. U holda $(n;n+2)=(2k;2k+2)=2(k;k+1)=2$; Endi agar $n=2k+1-$ toq son bo’lsa, u holda $(n;n+2)=(2k+1;2k+3)=1$, chunki Evklid algoritmiga asosan $2k+3=(2k+1)\cdot 1+2$, $\ 2k+1=2\cdot k+\boxed{1}$, $\ 2=1\cdot 2+0$.

Shunday qilib,

$$[n,n+1,n+2]=\begin{cases}n(n+1)(n+2), & \text{agar } n \text{ toq son bo'lsa;}\\ \frac{1}{2}n(n+1)(n+2), & \text{agar } n \text{ juft son bo'lsa.}\end{cases}$$

42. $\quad nab=ax+by,\ \ (a,b)=1 \qquad\qquad (*)$

bo'lsa, $x=bk$ $(k=1,2,3,4.\dots)$ deb yoza olamiz. U holda (*) dan $nab=abk+by$. Bundan $y=na-ak=a(n-k)$. Bu yerda a,b,x,y lar natural sonlar bo'lgani uchun $k=1,2,3,4,\dots.n-1(n>1)$, shunday qilib $x=bk, y=a(n-k)$, $k=1,2,3,4,\dots.n-1$. Demak, nab ni $n-1$ ta ko’rsatilgan ko’rinishda ifodalash mumkin ekan.

43.Evklid algoritmidan foydalanamiz. Y holda quyidagilarga ega bo’lamiz:

$899 = 493 \cdot 1 + 406,\ \ 493 = 406 \cdot 1 + 87,\ \ 406 = 87 \cdot 4 + 58,\ \ 87 = 58 \cdot 1 + \boxed{29},\ \ 58 = 29 \cdot 2$, bundan $(899{,}493) = 29$. Shuning uchun ham

$$29 = 87 - 58 \cdot 1 = 87 - (406 - 87 \cdot 4) \cdot 1 = 5 \cdot 87 - 406 =$$
$$= 5(493 - 406 \cdot 1) - 406 = 5 \cdot 493 - 6 \cdot 406$$
$$= 5 \cdot 493 - 6(899 - 493) = 11 \cdot 493 - 6 \cdot 899.$$

Shunday qilib,$29 = 899(-6) + 11 \cdot 493$ $\qquad x = -6, y = 11.$

44. $a = cq + r$ bo'lsin. U holda $(a, c) = (c, r) = 1$ bo'ladi. Shuningdek $b = c \cdot q_1 + r_1$, bo'lsa, $(b, c) = (c, r_1)$ bo'ladi. Bulardan $ab = cQ + rr_1$ ga ega bo'lamiz. Oxirgi tenglikdan $(ab; c) = (c; rr_1) = 1$.

45.$m > n$ va$d = (m, n)$ bo'lsin. U holda $m = dm_1, n = dn_1$ va $m_1 - n_1 > 0$ bo'ladi. Agar $d > m - n = d(m_1 - n_1)$ yoki $0 < m_1 - n_1 < 1$bo'lishi kerak $m_1 - n_1$ −butun son bo'lgani uchun ham bunday bo'lisih mumkin emas. Demak, $(m, n) \leq m - n\ \ (m > n)$.

46. Tushunarliki, $ab \vdots (ab, c)$ va $bc \vdots (ab, c)$ bajariladi. Shuning uchun ham $(ab, bc) \vdots (ab, c)$.Lekinda shartga ko'ra $(a, c) = 1$ bo'lgani uchun $(ab; bc) = b(a, c) = b$ va bundan $b \vdots (ab; c)$.

47. 20-masaladan $c \vdots (ac; b)$ va $(a; b) = 1$ dan$(c, b) \vdots \vdots (ac; b)$. Ikkinchi tomondan $(ac; b) \vdots (c; b)$. Shunday qilib $(ac; b) = (c; b)$ bo'ladi.

48. Faraz qilaylik, $(mn, mk, nk) = d$ bo'lsin. U holda

$$mnk = dx. \qquad (1)$$

Bundan

$$x = \frac{mnk}{d} = m \cdot \frac{nk}{d} = n \cdot \frac{mk}{d} = k \cdot \frac{mn}{d}.$$

Demak, x soni m, n, k larning umumiy karralisi va $x = [m, n, k] \cdot q, q \geq 1$ deb yoza olamiz.$(*)$dan

$$\frac{x}{m} = \frac{nk}{d}, \qquad \frac{[m, n, k]q}{m} = \frac{nk}{d}, \qquad \frac{[m; n; k]q}{n} = \frac{mk}{d}, \qquad \frac{[m; n; k]q}{k} = \frac{mn}{d}.$$

Bulardan q soni $\frac{nk}{d}, \frac{mk}{d}, \frac{mn}{d}$ sonlarning umumiy bo'luvchisi. Farazimizga ko'ra $\left(\frac{nk}{d}, \frac{mk}{d}, \frac{mn}{d}\right) = 1$, shuning uchun ham $q = 1$ va $x = [m, n, k]$. Endi (1) dan isbotlanish talab etilgan tenglik kelib chiqadi.

49. a) berilgan sistemadagi ikkinchi tenglama

$$\begin{cases} x = 30u \\ y = 30v \\ (u, v) = 1 \end{cases}$$

sistemaga teng kuchli, shu sababli sistemaning birinchi tenglamasi $u + v = 5$ ko'rinishda bo'ladi, bundan esa $u = 1{,}2{,}3{,}4$ (yoki $v = 1{,}2{,}3{,}4$) bo'lishi mumkinligini ko'ramiz. $u\,(v)$ning topilgan bu qiymatlari bo'yicha $\mathrm{x} = 30{,}60{,}90{,}120$ (yoki $\mathrm{y} = 30, 60, 90, 120$) bo'lishini topamiz. y ning x ra mos qiymatlarini $\mathrm{y} = 150 - x$ tenglikdan topamiz.

Shunday qilib, sistemaning yechimlari $(30,120), (60,90), (90,60), (120,30)$ juftliklardan iborat ekan.

b) berilgan sistema $\begin{cases}(x,y)=45\\ \frac{x}{y}=\frac{11}{7}\end{cases}$ dagi birinchi tenglama

$\begin{cases}x=45u\\ y=45v\\ (u,v)=1\end{cases}$ sistemaga teng kuchli, shu sababli sistemaning ikkinchi

tenglamasidan $u=11$ va $v=7$ bo'lishini topamiz. Demak, $\mathrm{x}=495$, $\mathrm{y}=315$ bo'lar ekan.

$c)$ $\begin{cases}xy=8400\\ (x,y)=20\end{cases} \Rightarrow \begin{cases}x=20x_1\\ y=20y_1\\ (x_1,y_1)=1\\ 400x_1y_1=8400\end{cases} \Rightarrow \begin{cases}x_1\cdot y_1=21\\ (x_1,y_1)=1.\end{cases}$

Bundan $x_1=1,3,7,21$ va $x=20,60,140,420$ $\quad y_1=21,7,3,1;\quad y=420,140,60,20.$

$d)$ $\begin{cases}\frac{x}{y}=\frac{5}{9}\\ (x.y)=28\end{cases} \left|\begin{matrix}x=28x_1\\ y=28y_1\\ (x_1;y_1)=1\end{matrix}\right| \Rightarrow \begin{cases}\frac{x_1}{y_1}=\frac{5}{9}\\ (x_1;y_1)=1\end{cases} \Rightarrow \begin{cases}x_1=\frac{5}{9}y_1\\ (x_1;y_1)=1\end{cases}$

$$y_1=9,\quad y=29\cdot 9=252 \text{ va } x_1=5,\quad x=28\cdot 5=140.$$

e)

$$\begin{cases}xy=20\\ [x,y]=10\end{cases} \Rightarrow \left|[x,y]=\frac{xy}{(x,y)}\right| \Rightarrow 10=\frac{20}{(x,y)} \Rightarrow (x,y)=2$$

$$\left|\begin{matrix}x=2x_1\\ y=2y_1\\ (x_{1,},y_1)=1\end{matrix}\right| \to \begin{cases}x_1y_1=5\\ (x_1,y_1)=1\end{cases} \to \left|\begin{matrix}x_1=5;1\\ y_1=1;5\end{matrix}\right| \to \left|\begin{matrix}x=10;2\\ y=2;10\end{matrix}\right|.$$

50. $m=10q+1$ dan $(m,q)=1$ kelib chiqadi. $N=10a+b$ ning ikkala tomonini q ga ko'paytiramiz. U holda

$$Nq=10aq+bq=(m-1)a+bq=am-(a-bq)$$

hosil bo'ladi. Agar $(a-bq)\,\vdots\,m$ bo'lsa, $(m,q)=1$ bo'gani uchun N sonining m bo'linishi kelib chiqadi.

51. $N=10a+b$ ning ikkala tomonini $q+1$ ga ko'paytiramiz va $m=10q+9$ ekanligidan foydalanamiz. U holda $(q+1)N=10a(q+1)+b(q+1)=(10q)a+10a+bq+b=(m-9)a+10a+bq+b=ma+a+b(q+1)$. 10.b)-misolga asosan $(q+1;m)=(q+1{,}10q+9)=1$ bo'lgani uchun $a+b(q+1)$ soni m ga bo'linsa N soni m ga bo'linadi.

52. $10N_1$ sonini qaraymiz. $10N_1 = a_n a_{n-1} \cdots a_2 a_1 \cdot 10 + 20a_0 = N + 19a_0$, $(19,10) = 1$. Endi agar $N_1 \vdots 19$ bo'lsa,$N \vdots 19$ va aksincha $N \vdots 19$ bo'lsa, $N_1 \vdots 19$ bo'ladi.

53.26-misoldagi qoidani N=3086379 soniga tadbiq etamiz.

$$N_1 = 308637 + 18 = 308655, \qquad N_2 = 30865 + 10 = 30875$$

$$N_3 = 3087 + 10 = 3097, N_4 = 309 + 14 = 323,$$

$$N_5 = 32 + 6 = 38 \; va \; 38:19 = 2.$$

Demak, 3086379 soni ham 19 ga bo'linadi.

I.3-§.

54. $6n + 1 = p_1 - p_2$ bo'lsin, u holda: *a)*agar $p_2 = 2$ bo'lsa $p_1 = 6n + 3 = 3(2n + 1)$ bo'lishi kerak. $p_1$tub son bo'lgani uchun bunday bo'lishi mumkin emas.

b) p_2 toq tub son bo'lsin, ya`ni $p_2 = 2k + 1$, bu holda $p_1 = 6n + 2k + 2 = 2(3n + k + 1)$bo'ladi. Bunday bo'lishi ham mumkin emas. $6n + 1$ ni 2 ta tub sonning ayirmasi ko'rinishida ifodalab bo'lmaydi.

55. $N = 2k + 1 = p_1 - p_2$dan tub sonlarning bitta juft son bo'lishi kerak ekanligi kelib chiqadi, $p_2 = 2$ desak $N = p_1 - 2$, bunda p_1 tub son.

56. Faraz qilaylik, $N^2 = n^2 + p$ bo'lsin, u holda $N^2 - n^2 = p$ bo'lib, bundan haqli ravishda $(N - n)(N + n) = p$ tenglikni yoza olamiz va bundan esa $N - n = 1, N + n = \mathrm{p}$ kelib chiqadi. Demak $2N = \mathrm{p} + 1$ yoki $\mathrm{p} = 2N - 1 = 6m + 3$ bo'lib bu esa p tub son deb qilingan farazimizga ziddir, demak farazimiz noto'g`ri va $N = 3m + 2$ $(m = 1,2, \dots)$ sonning kvadratini natural son kvadrati va tub sonning yig'indisi ko'rinishida ifodalash mumkin emas ekan.

57. 1-usul. $a = p \cdot a_1$va $p < a_1$ bo'lsin. Agar $p > \sqrt{a}$bo'lsa, $a_1 > \sqrt{a}$ bo'ladi va bu tengsizliklarni hadlarni ko'paytirsak$p \cdot a_1 > a$ bo'lar edi. Demak, $p \leq \sqrt{a}$.

2-usul. $a = p \cdot a_1$va $p < a_1$ bo'lsin. U holda $p^2 < p \cdot a_1$ yoki $p^2 < a$. Bundan $p < \sqrt{a}$.

Agar a tub son bo'lsa, bu teorema o'rinli emas, chunki bu holda a ning eng kichik tub bo'luvchisi ham $a = p$ bo'ladi.

58. 1)$11 < \sqrt{127} < 12$bo'lgani uchun 127 ni ketma-ket 2,3,5,7,11 tub sonlariga bo'lib ko'ramiz. Agar shularning birortasi ham bo'linmasa, 127 soni tub son bo'ladi, aks holda tub son bo'lmaydi. 127 soni 2,3,5,7,11 larning birortasiga ham bo'linmaydi. Demak 127 tub son.

3) $30 < \sqrt{919} < 31$bo'lgani uchun 919 ni ketma-ket $2, 3, 5, 7, 11,13,$

$17,19, 23, 29$ tub sonlariga bo'lib ko'ramiz. Agar shularning birortasi ham bo'linmasa, 919soni tub son bo'ladi, aks holda tub son bo'lmaydi. 919 soni bu sonlarning birortasiga ham bo'linmaydi. Demak, 919 tub son.

3) $86 < \sqrt{7429} < 87$ va 7429 soni $2,3,5,7,11,13$ larga bo'linmaydi, lekin 17 ga bo'linadi, ya'ni $7429 = 17 \cdot 437$. Shuning uchun ham u murakkab son.

59. 1) 100 va 110, bu yerda $10 < \sqrt{110} < 11$. Shuning uchun ham berilgan sonlar orasidagi $2,3,5,7$ ga karralilarni o'chirib chiqamiz. 2 ga karralilarini tushirib qoldirsak: $101,103,105\ ,107,109$ lar qoladi.Bular orasidan 3 ga bo'linadiganlarini o'chirsak:$101,103,107,109$ lar qoladi. Bular orasida 5 ga va 7ga karralisi yo'q. Shuning uchun ham $101,103,107,109$ lar qaralayotgan oraliqdagi tub sonlar.

2) 190 va 200, bu yerda $14 < \sqrt{200} < 15$ bo'lgani uchun 1)- misoldagi singari ish tutib berilgan sonlar orasidagi $2,3,5,7,11,13$ ga karralilarni o'chirib chiqamiz. 2 ga karrali sonlarni tushirib qoldirsak, $191, 193, 195,197, 199$ lar qoladi. Bular orasidan 3, 5 ga bo'linadiganlarini tushirib qoldirsak, $191, 193, 197, 199$ lar qoladi. Bu sonlar orasida 7 ga, 11 ga yoki 13 ga bo'linadiganlari yo'q. Shuning uchun ham bu sonlar tub sonlardir.

3)200 va 220, $14 < \sqrt{220} < 15$ bo'lgani uchun 2- misoldagi singari ish tutib berilgan sonlar orasidagi $2,3,5,7,11,13$ ga karralilarni o'chirib chiqamiz. 2 ga karrali sonlarni tushirib qoldirsak, $201,203, 205, 207,209,211, 213, 215, 217, 219$ lar qoladi. Bular orasidan $3, 5$ ga bo'linadiganlarini tushirib qoldirsak, 203, $209, 211, 217$ lar qoladi. Bu sonlar orasida 7 ga bo'linadiganlarini tushirib qoldirsak $209, 211$ lar qoladi. Bulardan 209 soni 11 ga karrali. 211 esa 11 ga ham 13 ga ham bo'linmaydi. Shuning uchun ham 211 qaralayotgan oraliqdagi yagona tub sondir.

4)2640 va 2680. Bu yerda $51 < \sqrt{2680} < 52$. Bo'lgani uchun berilgan oraliqdagi sonlar orasidan 2 dan 47 gacha bo'lgan tub sonlarga bo`linadiganlarini tushirib qoldiramiz. U holda $2647, 2657, 2659, 2663, 2671, 2677$ sonlarining berilgan oraliqdagi tub sonlar ekanligiga ishonch hosil qilamiz.

60. Faraz qilaylik, p soni $n! - 1$ sonining tub bo'luvchisi bo'lsin. U holda $p \leq n! - 1$ bajariladi. Bundan $p < n!$. Ikkinchi tomondan $n!$ soni p ga bo'linmaydi, shuning uchun ham $p > n$. Bulardan $n < p < n!$ Isbotdan tub sonlar sonining cheksiz ko'p ekanligi kelib chiqadi.

61. $21! + 2, 21! + 3, \dots, 21! + 20, 21! + 21$ larning barchasi murakkab sonlar.

62. $n = 1$da1,11,15 bo'lib faqat 11 tub son. $n = 2$ da 2,12,16 bo'lib, bularning birortasi ham tub son emas. $n \geq 3$ bo'lsa, uni $n = 3k + r, r = 0,1,2$ deb yozish mumkin. $r = 0$ bo'lsa, $3k$, $3k + 10, 3k + 14$. Bularning uchalasi tub bo'ladigan faqat 1 ta k=1 qiymati mavjud. Bu holda $n = 3$ va 3,13,17 tub sonlari hosil bo'ladi. $r = 1$ bo'lsa, $3k + 11$, $3k + 15$ va $3k + 15$ tub emas. $r = 2$ da $3k + 2, 3k + 12, 3k + 16$ va $3k + 12$ soni tub son emas. Demak, $n, n + 10, n + 14$ sonlar bir vaqtda tub bo'ladigan n ning faqat 1 ta qiymati $n = 3$ mavjud ekan.

63. Tub sonlarni 3 ta sinfga $p = 3q, p = 3q + 1, p = 3q + 2$ bo'lamiz. 1-sinfda faqat 1 ta $p = 3$ tub soni($q = 1$)mavjud. Bu holda $2p^2 + 1 = 2 \cdot 9 + 1 = 19$-tub son bo'ladi.

Agar $p = 3q + 1$ ko'rinishda tub son bo'lsa, (2 va 3- sinflar cheksiz ko'p tub sonlar mavjud), $2p^2 + 1 = 2(9q^2 + 6q + 1) + 1 = 18q^2 + 12q + 3 = 3(6q^2 + 4q + 1) -$ murakkab son bo'ladi.

Endi, agar $p = 3q + 2$ ko`rinishdagi tub son bo'lsa, u holda $2p^2 + 1 = 2(9q^2 + 12q + 4) + 1 = 18q^2 + 24q + 9 = 3(6q^2 + 8q + 3)$, ya`ni bu holda ham murakkab son bo'ldi. Shunday qilib p ning faqat bitta $p = 3$ qiymatida $2p^2 + 1 = 19$ tub son bo`lar ekan.

64. Barcha natural sonlarni 5 ga bo'lib qoldiqlari bo'yicha $N = 5n + r, 0 \leq r \leq 4$ deb yoza olamiz. Agar $r = 0$ bo'lsa $N = 5n$ bo'lib faqat $n = 1$ da tub son $p = 5$ bo'ladi va bu holda $4p^2 + 1 = 4 \cdot 25 + 1 = 101$, $6p^2 + 1 = 6 \cdot 25 + 1 = 151$ lar ham tub son bo'ladi. Qolgan hollarda $p = 5n + r$ n=1,2,3,4 tub son bo'lsa, u holda $4p^2 + 1 = 4(25n^2 + 10nr + r^2) + 1 = 5(20n^2 + 8nr) + 4r^2 + 1$ va $6p^2 + 1 = 6(25n^2 + 10nr + r^2) + 1 = 5(30n^2 + 12nr + 6r^2 + 1)$, bu yerda $4r^2 + 1$ ifoda $r = 1$ da 5, $r = 4$ da 65, $r = 2$ da $6r^2 + 1 = 25$, $r = 3$ da 55 ga teng qiymat qabul qiladi, ya`ni $p = 5n \pm 1$ ko'rinishda bo'lsa $4p^2 + 1$ ifoda 5 ga bo'linadi, agarda $p = 5n \pm 2$ ko'rinishidagi tub son bo'lsa, u holda $6p^2 + 1$ ifoda 5 ga bo'linadi. Demak izlanayotgan qiymat bitta $p = 5$.

65. 1) $p + 5$ va $p + 10$ lar uchun $p = 2$ da $p + 10 = 12$ murakkab son. Agar $p = 2k + 1$ toq tub son bo'lsa, $p + 5 = 2k + 6 = 2(k + 3)$ murakkab son bo'ladi.

2) $p, p + 2, p + 5$ uchun $p = 2$ da $p + 2 = 4$ murakkab son. $p = 2k + 1$ bo'lsa, $p + 5 = 2k + 6 = 2(k + 1)$ murakkab son.

3) $2^n - 1$; $2^n + 1, (n > 2)$ uchun sonlarning $3q + r$; $r = 0,1,2$ ko'rinishidagi yozuvidan foydalanamiz. Bunda quyidagi uchta holni qaraymiz:

$$1)2^n = 3q \qquad 2)2^n = 3q + 1 \qquad 3)2^n = 3q + 2.$$

Birinchi holda $2^n = 3q$ bo'lishi mumkin emas. Ikkinchi hol $2^n = 3q + 1$ bo'lsa, $2^n - 1 = 3q$ bo'ladi. $q = 1$ da $2^n - 1 = 3$ tub son bo'ladi va $n = 2$ da $2n + 1 = 5$ ham tub son, lekin misolning shartida $n > 2$. Uchinchi holda $2^n = 3q + 2$ bo'lsa, $2^n + 1 = 3(q + 1)$bo'ladi. Shunday qilib $2^n - 1$ va $2^n + 1$, $n > 2$ bo'lganda bir vaqtda tub son bo'lmas ekan.

66. Agar $p = 3$ bo'lsa, p va $8p^2 + 1 = 73$ lar tub sonlar hamda $8p^2 + 2p + 1 = 8 \cdot 9 + 6 + 1 = 79$ tub son bo'ladi, $p = 3k + 1$ bo'lsa, $8p^2 + 1 =8(9k^2 + 6k + 1) + 1 = 72k^2 + 48k + 9 = 3(24k^2 + 16k + 3)$ murakkab son bo'ladi. Shuningdek, agarda $p = 3k + 2$ bo'lsa, $8p^2 + 1 =8(9k^2 + 12k + 4) + 1 = 72k^2 + 96k + 33 = 3(24k^2 + 32k + 11)$ murakkab son bo'ladi.

67. $2^{18}+3^{18}=(2^6)^3+(3^6)^3=(2^6+3^6)(2^{12}-2^6\cdot 3^6+3^{12})=(2^2+3^2)(2^4-2^2\cdot 3^2+3^4)(2^{12}-2^6\cdot 3^6+3^{12})=13\cdot 61\cdot 488881=13\cdot 61\cdot 37\cdot 73\cdot 181.$

68. $a>3;\, m=3q_1+1;\ \ n=3q_2+2$ bo'lsin.

a)$a=3q; q>1$ bo'lsin, u holda a murakkab son ekanligi ma'lum.

b) $a=3q+1$ bo'lsin, u holda $a+m=3q+1+3q_1+1=3(q+q_1)+2$ va $a+n=3q+3q_2+2+1=3(q+q_2+1)$ bo'ladi. Bunda $a+n$ murakkab son;

c) $a=3q+2$ ko'rinishda bo'lsin, u holda $a+m=3(q+q_1+1)$ murakkab son. Demak, berilgan shartlarda $a>3, a+m;\ a+n$ lar bir vaqtda tub sonlar bo'la olmas ekan.

69. 1) $n^4+4=n^4-4n^2+4n^2+4=(n^2+2)^2-4n^2=(n^2+2+2n)\,(n^2+2-2n)=((n+1)^2+1)((n-1)^2+1)$ bo'lib, bu esa $n>1$ da murakkab son bo'lishligini anglatadi.

2) $n^4+n^2+1=(n^2+1)^2-n^2=(n^2-n+1)\cdot(n^2+n+1), n>1$ da murakkab son bo'ladi.

70. $p, p+2, p+4, (\,p>3)$ sonlarni qaraymiz. $p>3$ tub sonlar $3q+1; 3q+2$ ko'rinishlarida bo'ladi. $p=3q+1\ \ (q=2,4,..)$ deb olsak, u holda $p+2=3(q+1)$ murakkab son bo'ladi, agarda $p=3q+2, (q=1,3,5\,...)$ bo'lsa, u holda $p+4=3(q+2)$ murakkab son bo'ladi. $p=3q$ bo'lsa, $q=1$ da tub son $p=3;\ p+2=5;\ p+4=7,$ yagona egizak tub sonlar uchligini hosil bo'ladi.

71. $p=3n+2$ ko'rinishidagi tub son bo'lsin.U holda $N=3\cdot 5\cdot 7\cdots p+2$ sonini qaraymiz.N soni ham $3n+2$ ko'rinishidagi son, chunki $N=3(5\cdot 7\cdot\ldots\cdot p)+2$ deb yoza olamiz. N sonining kanonik yoyilmasida p dan katta murakkab son qatnashadi va ularning orasida albatta $3n+2$ ko'rinishidagi tub son mavjud, agar N ning barcha bo'luvchilari $3n+1$ ko'rininishida bo'lsa, N ham shunday ko'rinishda bo'lishi kerak bo'lar edi. Demak, p qanday bo'lishidan qat'iy nazar p dan katta $3n\ +\ 2$ ko'rinishidagi tub son mavjud ekan.

72. $\prod_{i=1}^{n} p_i-1=p_k\cdot q(k>n, q\geq 1)$ bo'lib, bundan $p_k\leq\prod_{i=1}^{n} p_i-1$ va $p_k\leq\prod_{i=1}^{n} p_i$. Shunday ekan $p_{n+1}\leq\prod_{i=1}^{n} p_i$.

73. Ma'lumki $p_5=11$, ya'ni beshinchi tub son 11 ga teng va 2·5=10 bo'lib, 11>10 bo'ladi. Agar $p_n>2n$, (n=5,6,7…) bo'lsa, u holda $p_{n+1}-p_n\geq 2$ ekanligidan $p_{n+1}-2n>2$ yoki $p_{n+1}>2(n+1)$ kelib chiqadi, bu esa isbotlanishi talab qilinayotgan tengsizlikni beradi.

74. $p_n\leq 2^{2^{n-1}}$dan $n=1$ da $p_1\leq 2. (p_1=2$ bajariladi). $n=2$ da $\square_2=3<4;\ n=3$ da $p_3=5<16$ bajariladi. Endi faraz qilaylik, $n=k$ da $(k=2,3,\dots n)\ p_k<2^{2^{k-1}}$ tengsizlik o'rinli bo'lsin. U holda

$$p_{n-1} \leq \prod_{k=1}^{n} p_k + 1 \leq 2 \cdot 2^2 \cdot 2^{2^2} \cdots 2^{2^{n-1}} + 1 = 2^{2^n} \cdot 2^{-1} + 1 < 2^{2^n}.$$

Demak, berilgan munosabat ixtiyoriy n natural soni uchun o'rinli.

75. Faraz qilaylik $2^n - 1$ tub son bo'lib n murakkab son bo'lsin, u holda $n = n_1 \cdot n_2$ $(n_1 > 1, n_2 > 1)$ deb yoza olamiz. Bundan $2^n - 1 = 2^{n_1 n_2} - 1 = (2^{n_1})^{n_2} - 1$ murakkab son bo'ladi. $2^n - 1$ tub son degan teskari tasdiq hamma vaqt ham o'rinli emas. Masalan: $2^{11} - 1 = 2048 - 1 = 2047 = 23 \cdot 89$.