4 -§. Algebraik va transtsendent sonlar

Ushbu

$$a_0x^n + a_1x^{n-1} + \dots + a_n = 0, \quad (a_0 \neq 0) \qquad (1)$$

ratsional koeffitsientli n-darajali tenglamaning ildizi α ga algebraik son deyladi. Aks holda α ga transtsendent son deyladi. Boshqacha qilib aytganda algebraik bo'lmagan sonlarga transtsendent sonlar deyiladi.

Ta'rifdan umuman olgandaα algebraik son, bu kompleks son bo'lishi kelib chiqadi. Ma'lumki, ratsional koeffitsientli tenglamani hamma vaqt butun koeffitsientli tenglamaga keltirish mumkin.

Agar α

$$x^n + a_1x^{n-1} + \dots + a_n = 0, \qquad (2)$$

ratsional koeffitsientli n-darajali bosh hadining koeffitsienti 1 ga teng bo'lgan tenglamaning ildizi bo'lsa, α ga butun algebraik son deyladi. Agar α (1) tenglamaning ildizi bo'lib darajasi undan kichik bo'lgan algebraik tenglamaning ildizi bo'lmasa, α ga n-tartibli algebraik son deyladi.

Agar α va β lar algebraik sonlar bo'lsa, u holda $\alpha + \beta, \alpha - \beta,\ \alpha \cdot \beta$ lar va agar $\beta \neq 0$ bo'lsa $\frac{\alpha}{\beta}$ ham algebraik son bo'ladi. Bundan tashqari quyidagi teoremalar o'rinli.

Liuvil teoremasi. Har bir haqiqiy n-tartibli α algebraik son uchun shunday c>0 soni mavjudki, α dan farqli barcha $\frac{a}{b}-$ ratsional sonlar uchun $\left|\alpha - \frac{a}{b}\right| > \frac{c}{b^n}$ munosabat o'rinli bo'ladi.

Natija. Agar $q_n > (Q_{n-1})^{n-1}, n = 1,2, \dots$ bo'lsa, $\alpha = (q_0,\ q_1, q_2, \dots)$ irratsional son transtsendent son bo'ladi.

Gelfond teoremasi. Agar α soni 0 va 1 dan farqli algebraik son, β esa tartibi 2 dan kichik bo'lmagan algebraik son bo'lsa, u holda $\alpha^\beta -$ transtsendent son bo'ladi.

Lindeman teoremasi. $x = 0$ va $y = 1$ dan boshqa hollarda $y = e^x$ tenglamada *x* va *y* sonlari bir vaqtda algebraik son bo'la olmaydi.

387. Quyidagi sonlarning algebraik sonlar ekanligini ko'rsating:

1). $\frac{3}{5}$; 2) $\sqrt{3}$; 3). $\sqrt[3]{3}$, 4). $1+\sqrt{2}$; 5). $2-\sqrt{2}$; 6). $1+i$;

7). $\sqrt{3}+\sqrt{5}$; 8). $\sqrt[4]{4-\sqrt[3]{2}}$; 9). $a+\sqrt[n]{b}$; 10). $a+i\sqrt{b}$

(bunda a va b lar ratsional sonlar); 11). $\cos\frac{\pi}{n}i+\sin\frac{\pi}{n}$; 12). $sin10^0$.

388. Quyidagi algebraik sonlarning tartibini aniqlang: 1). $a+bi$ (a va $b\neq 0$ lar ratsional sonlar); 2). $\sqrt[3]{3}$, 3). $\sqrt[3]{2}-1$; 4). $\sqrt{2}-\sqrt{3}$; 5). $\sqrt{3}+\sqrt{5}$; 6). $2+i$.

389. Berilgan tenglamalarning ildizlarining algebraik sonlar ekanligini isbotlang: 1). $x^3+2\sqrt{2}x^2+2=0$; 2). $x^2+2ix+10=0$.

390. Quyidagi berilgan tenglamalarning ildizlarining tartibi berilgan tenglamaning tartibiga teng bo'lgan algebraik sonlar ekanligini isbotlang:

1). $x^3+2x^2-4x+2=0$; 2). $2x^5+6x^3-9x^2-15=0$,

3). $x^4-5x^2+10x+20=0$, 4). $x^5-3x^2+12x-6=0$.

391. Liuvil metodidan foydalanib, birorta transtsendent sonni quring.

392. Liuvil soni $\alpha=\frac{1}{10^{1!}}+\frac{1}{10^{2!}}+\frac{1}{10^{3!}}+$ ... ning transtsendent ekanligini isbotlang.

393. Gelfond teoremasidan foydalanib quyidagi sonlarning transtsendent ekanligini isbotlang:

1). $lg2$; 2) $log_2 10$; 3). $ln5$; 4). $3^{\sqrt{2}}$; 5). $5^{\sqrt{3}}$;

6). $2^{i\sqrt{3}}$; 7). 3^{1-i}; 8). $5^{2-i\sqrt{2}}$.