2-§. Butun qism va kasr qism funksiyalari.

$y = [x]$ – funksiyasi x ning barcha haqiqiy qiymatlarida aniqlangan bo'lib, x dan katta bo'lmagan va unga eng yaqin turgan butun sonni ifodalaydi. Bu funksiyaga x ning butun qismi deyiladi.

Tushunarliki, $[x] \le x < [x] + 1$ qo'sh tengsizlik o'rinli. x ni hamma vaqt $x = [x] + \alpha$, (bunda $0 \le \alpha < 1$) ko'pinishda yozish mumkin. Bundan $\alpha = \{x\} = x - [x]$. Bu tenglik yordamida aniqlanuvchi $y = \{x\}$ – funksiyaga kasr qism funksiyasi yoki x ning kasr qismi deyiladi.

Agar x_1 va x_2 sonlardan hech bo'lmaganda bittasi butun son bo'lsa, u holda

$$[x_1 + x_2] = [x_1] + [x_2]$$

tenglik o'rinli bo'ladi.

Sonning butun qismi uchun $\left[\frac{x}{m}\right] = \left[\frac{[x]}{m}\right]$ ayniyat o'rinli. $n!$ sonning kanonik yoyilmasida p tub son

$$\left[\frac{m}{p}\right]+\left[\frac{m}{p^2}\right]+\cdots+\left[\frac{m}{p^s}\right]$$

daraja ko'rsatgich bilan qatnashadi, bu yerda $s,\ p^s \le m < p^{s+1}$ tengsizlikdan aniqlanadi.

81. Sonlarning butun qismini toping: a) -2,7; b) $2+\sqrt[3]{987}$; c) $\frac{7-\sqrt{21}}{2}$; d) $\frac{10}{3+\sqrt{3}}$

;

e) $1,(3)+2tg\frac{\pi}{4}$; i) $3+\sin\frac{13\pi}{7}$; j) $3-2\cos\frac{90\pi}{181}$; f) $2-\lg 2512$;

$l=2-\lg\overline{abcd}$; k) $\sqrt{30}+\sqrt[3]{10}$.

82. $[\pi]^{[e]}+[e]=[e]^{[\pi]}+[\pi]$ tenglikni isbotlang. Bu yerda $\pi=3,14...-$ aylana uzunligining uning diametriga nisbati va

$$e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=2,7....$$

83. $\left[\frac{p}{4}\right]$ ning $\frac{p-1}{4}$ yoki $\frac{p-3}{4}$ ga tengligini isbotlang. Bu yerda $p>2$ tub son.

84. $\left[\frac{a}{m}\right]=\frac{a-r}{m}$ tenglikni isbotlang , bu yerda *r* soni $a$ni m bo'lgandagi qoldiq.

85. $\frac{[nx]}{n}\le x<\frac{[nx]}{n}+\frac{1}{n}$, $n=1,2,...$tengsizlikni isbotlang.

86. $\left[\frac{x+y}{n}\right]$ning $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]$ ga, yoki $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+1$ ga teng ekanligini isbotlang.

87. Arap $m-$toq son bo'lsa, u holda $\left[\frac{m}{2}\right]=\frac{m-1}{2}$ ekanligini isbotlang.

88. Funksiya grafigini chizing:

$a)\ y=[x];\quad b)\ y=\{x\};\ c)\ y=\left[-\frac{x}{2}\right];$

$d)\ y=\left[\frac{x^2}{2}-1\right];\quad e)\ y=[sinx].$

89. Tenglamani yeching.

a) $\left[x^2\right]=2$; b) $\left[3x^2-x\right]=x+1$; c) $[x]=\frac{3}{4}x$ d) $\left[x^2\right]=x.$

90. $[12,4m]=87$ tenglamani qanoatlantiruvchi $m$natural sonning mavjud emasligini isbotlang.

91. $[-x]$ va $[x]$ funksiyalar orasidagi bog'lanishni aniqlang.

92. $[x_1+x_2+\cdots+x_n]\ge[x_1]+[x_2]+\cdots+[x_n]$ tengsizlikni isbotlang.

93. $[nx]\ge n[x]$ tengsizlikni isbotlang, bunda $n=1,2,3,...$

94. 10^6 va 10^7 sonlarning orasida 786 ga karrali nechta natural son bor.

95. 1000 dan kichik nechta natural son 5 ga ham 7 ga ham bo'linmaydi.

96. 36 soni bilan o'zaro tub, 100 dan katta bo'lmagan natural sonlar sonini toping.

97. 2017! soni nechta no'l bilan tugaydi.

98. $p^n!=1\cdot 2\cdot 3\cdots p^n$ning kanonik yoyilmasida p tub soni qanday daraja ko'rsatkich bilan ishtirok etadi.

99. 100! ko'paytmada 6 soni qanday daraja ko'rsatkich bilan ishtirok etadi.

100. 11! sonining kanonik yoyilmasini toping.

101. $N=\dfrac{101\cdot 102\cdots 1000}{7^{\alpha}}$ son butun son bo'ladigan eng katta natural sonni toping.

102. $(2m)!!$ sonining kanonik yoyilmasida p tup soni qanday daraja ko'rsatkich bilan qatnashishini toping.

103. x ning $[x]-2\left[\dfrac{x}{2}\right]=1$ tenglama to'g'ri tenglikka aylanadigan qiymatlavrini toping.

104. $\left[ax^2+bx+c\right]=d$ (bu yerda $a\neq 0, d$ −butun son) ko'rinishdagi tenglama yechimining mavjudlik shartini toping .

105. a va b lar natural sonlar, $f(x)$ berilgan kesmada manfiy bo`lmagan uzluksiz funksiya bo`lsa, $a\leq x\leq b,\ 0\leq y\leq f(x)$ egri chiziqli trapetsiyada nechta butun koordinatali nuqtalar bo`ladi.

106. $x^2+y^2=\ 6{,}5^2$ doirada nechta butun koordinatali nuqta bor.

107. 12317 dan katta bo'lmagan va 1575 bilan o`zaro tub bo`lgan butun musbat sonlarning sonini aniqlang.