3-§. Eyler va Ferma teoremalari

Eyler teoremasi. Agar $m > 1$ va $(a, m)=1$ bo'lsa, $a^{\varphi(m)} \equiv 1(mod m)$ bo'ladi. Xususiy holda, agar $m= p$ tub songa teng bo'lsa, Eyler teoremasidan quyidagi Ferma teoremasi kelib chiqadi.

Ferma teoremasi. Agar p tub son va $(a, p)=1$ bo'lsa, u holda $a^{p-1} \equiv 1(mod\ m)$ bo'ladi. Ferma teoremasidan ixtiyoriy a butun musbat soni uchun $a^p - a \equiv 0(modp)$ ning bajarilishi kelib chiqadi.

220. a) agar $(a, 7) = 1$ bo'lsa, $(a^{12} - 1) \vdots 7$; b) agar $(a, 65) = (b, 65) = 1$ bo'lsa, $(a^{12} - b^{12}) \vdots 65$ ekanligini isbotlang.

221. Kanonik yoyilmasiga 2 va 5 kirmaydigan n natural sonining $12-$darajasining birliklar xonasidagi raqami 1 ga teng ekanligini isbotlang.

222. $a^{p-1} + p - 1$ ko'rinishdagi son murakkab ekanligini isbotlang, bu yerda $a \not\equiv 0(modp)$.

223. $2^{11\cdot31} \equiv 2(mod 11 \cdot 31)$ekanligini isbotlang .

224. 2^{30} sonni 13 ga bo'lgandagi qoldiqni toping.

225. 3^{59} sonini 17 ga bo'lgandagi qoldiqni toping.

226. $a^{n(p-1)+1} \equiv a(modp)$ ekanligini isbotlang.

227. 317^{259}sonini 15 ga bo'lgandagi qoldiqni toping.

228. $3^{80} + 7^{80}$ sonini *11* ga bo'lgandagi qoldiqni toping.

229. $3^{100} + 4^{100}$ sonini *7* ga bo'lgandagi qoldiqni toping.

230. 197^{157} sonini 35 ga bo'lgandagi qoldiqni toping.

231. $n = 73 \cdot 37$ uchun $2^{n-1} \equiv 1(mod\ n)$ ekanligini ko'rsating.

232. $1^{30} + 2^{30} + \ ... \ + 10^{30} \equiv -1(mod 11)$ekanligini isbotlang.

233. Ixtiyoriy x butun soni uchun 1) $x^7 \equiv x(mod 42)$; 2) $x^{13} \equiv x(mod 2730)$ ekanligini isbotlang.

234. Agar p va q lar har xil tub sonlar bo'lsa, $p^{q-1} + q^{p-1} \equiv 1(modpq)$ ekanligini isbotlang.

235. 2^{100} sonining oxirgi ikkita raqamini toping.

236. 3^{100} sonining oxirgi raqamini toping.

237. 243^{402} sonining oxirgi uchta raqamini toping.

238. Agar $(n, 6) = 1$ bo'lsa, $n^2 \equiv 1(mod 24)$ ekanligini isbotlang.

239. Agar p tub son bo'lsa, $\sum_{i=1}^{p-1} i^{k(p-1)} + 1 \equiv 0(modp)$ taqqoslamaning o'rinli ekanligini ko'rsating.

240. Agar p tub son bo'lsa, $(\sum_{i=1}^{n} a_i)^p \equiv \sum_{i=1}^{n} a_i^p (modp)$ taqqoslamaning o'rinli ekanligini ko'rsating.

241. Agar $(a, m) = 1$ bo'lsa $a^x \equiv 1(modm)$ taqqoslamaning eng kichik natural yechimi φ(m) ning bo'luvchisi ekanligini isbotlang.

242. Agar $N = \sum_{i=1}^{n} a_i$ soni 30ga bo'linsa, u holda $M = \sum_{i=1}^{n} a_i^5$ sonining ham 30 ga bo'linishi isbotlang.

243. Ixtiyoriy butun sonning $100-$darajasi 125 ga bo'linadi yoki 125 ga bo'lganda 1 qoldiq qolishini isbotlang.

244. Agar $(a, 10) = 1$ bo'lsa, $a^{100n+1} \equiv a(mod1000)$ ning bajarilishini ko'psating. Bunda n natural son.

245. m va n lar natural sonlar bo'lsalar, $a^{6m} + a^{6n} \equiv 0(mod7)$ taqqoslamaning faqat a soni 7 ga karrali bo'lgandagina o'rinli ekanligini isbotlang.

246. $5^{p^2} + 1 \equiv 0\ (modp^2)$ taqqoslamani qanoatlantiruvchi p tub sonini toping.

247. $p > 3$ tub son bo'lsa, p va $2p + 1$ lar tub sonlar bo'lsalar, u holda $4p + 1$ ning murakkab son ekanligini ko'rsating.