

4-§. Eyler funksiyasi

Eyler funksiyasi – m dan katta bo'lmagan va m bilan o'zaro tub sonlar sonini bildiradi va $\varphi(m)$ orqali belgilanadi. Agar $m=p-1$ tub son bo'lsa, u holda ta'rifdan $\varphi(p)=p-1$ ekanligi va agar $m=p^\alpha$ bo'lsa, $\varphi(p^\alpha) = p^\alpha - p^{\alpha-1} = p^\alpha \left(1 - \frac{1}{p}\right)$; umuman agar $m = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$ bo'lsa, u holda

$$\begin{aligned}\varphi(m) &= p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n} \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_n}\right) \\ &= m \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_n}\right)\end{aligned}$$

ekanligi kelib chiqadi. Eyler funksiyasi multiplikativ funksiyadir, ya'ni u aynan nolga teng emas hamda $(m, n) = 1$ shartni qanoatlantiruvchi m, n lar uchun $\varphi(mn) = \varphi(m)\varphi(n)$ bajariladi.

131. $y = \varphi(x)$ funksiya`ning o`zgarishini grafik shaklda tasvirlang. Bu yerda x -natural son, $\varphi(x)$ – Eyler funksiyasi .

132. Hisoblang: 1) $\varphi(125)$, 2) $\varphi(1000)$, 3) $\varphi(180)$, 4) $\varphi(360)$, 5) $\varphi(1440)$, 6) $\varphi(1890)$, 7) $\varphi(11^3)$, 8) $\varphi(23^2)$, 8) $\varphi(12 \cdot 19)$, 10) $\varphi(24 \cdot 28 \cdot 45)$.

133. Maxraji m ga teng qisqarmas musbat to`g`ri kasrlarning soni nechta.

134. 1 dan 120 gacha natural sonlar orasida 30 bilan o`zaro tub bo`lmagan sonlar soni nechta.

135. Quyidagi formulalarning o`rinli ekanligini ko`rsating: a) $\varphi(2^\alpha) = 2^{\alpha-1}$; b) $\varphi(p^\alpha) = p^{\alpha-1}\varphi(p)$; c) $\varphi(m^\alpha) = m^{\alpha-1}\varphi(m)$ (m, α lar natural sonlar, p esa tub son).

136. $\varphi(2m)$ ning qiymati $\varphi(m)$ yoki $2\varphi(m)$ bo`lishi mumkinligini isbotlang. Bu hollarning har biri uchun o`rinli kriteriya`ni toping.

137. Quyidagi tengliklarni o`rinli ekanligini isbotlang:

$$a) \varphi(4n+2) = \varphi(2n+1); \quad b) \varphi(4n) = \begin{cases} 2\varphi(n), & \text{agar } (n, 2) = 1 \text{ bo'lsa;} \\ 2\varphi(2n), & \text{agar } (n, 2) = 2 \text{ bo'lsa.} \end{cases}$$

138. Tenglamani yeching: a) $\varphi(5^x) = 100$; b) $\varphi(7^x) = 294$; c) $\varphi(p^x) = p^{x-1}$; d) $\varphi(3^x \cdot 5^x) = 600$, bunda x va y natural sonlar.

139. Agar $m \geq 3$ bo`lsa $\varphi(m)$ ning qiymati juft son ekanligini isbotlang.

140. Agar $\varphi(x) = a$ tenglamaning $x = m$ ildizi bo`lsa, u holda $x = 2m$ ham ildiz bo`lishini isbotlang. Bu yerda $(m, 2) = 1$.

141. Agar $(m, n) > 1$ bo`lsa, $\varphi(m \cdot n)$ va $\varphi(m) \cdot \varphi(n)$ sonlarini taqqoslang.

142. $\varphi(m \cdot n) = \varphi(m) \cdot \varphi(n) \cdot \frac{d}{\varphi(d)}$ ekanligini isbotlang. Bu yerda $(m, n) = d$.

143. Agar $\delta = (m, n)$ va $\mu = [m, n]$ bo`lsa, $\varphi(m \cdot n) = \varphi(\delta) \cdot \varphi(\mu)$ ekanligini isbotlang.

144. $\varphi(1) + \varphi(p) + \varphi(p^2) + \dots + \varphi(p^\alpha)$ yig`indini toping. Bunda α -natural son.

145. Gauss ayniyatini isbotlang: $\varphi(d_1) + \varphi(d_2) + \dots + \varphi(d_k) = m$, $(\sum_{d \in \mathcal{D}} \varphi(d) = m)$, bunda d_i – m ning natural bo`luvchilari.

146. m bilan o`zaro tub va m dan kichik natural sonlar yig`indisi $\left(S = \sum_{\substack{x \leq m, \\ (x, m) = 1}} 1 \right)$ uchun formula chiqaring.

147. p bilan o`zaro tub va p dan katta bo`lmagan natural sonlar yig`indisi p^2 bilan o`zaro tub va p^2 dan katta bo`lmagan natural sonlar sonidan ikki marta kam bo`lishini isbotlang.

148. Tenglamani yeching:

$$1) \varphi(x) = p - 1, \quad 2) \varphi(x) = 14, \quad 3) \varphi(x) = 8, \quad 4) \varphi(x) = 12.$$

149. Tenglamani yeching: a) $\varphi(x) = 2^\alpha$; b) $\varphi(p^x) = 6 \cdot p^{x-2}$.

150. Tenglamani yeching: $\varphi(m) = 3600$, bu yerda $m = 3^\alpha \cdot 5^\beta \cdot 7^\gamma$.

151. Tenglamani yeching: $\varphi(x) = 120$, bu yerda $x = p_1 \cdot p_2$ va $p_1 - p_2 = 2$.

152. Tenglamani yeching: $\varphi(m) = 11424$, bu yerda $m = p_1^2 \cdot p_2^2$.

153. Tenglamani tekshiring: a) $\varphi(x) = \varphi(px)$; b) $\varphi(px) = p\varphi(x)$;

c) $\varphi(p_1x) = \varphi(p_2x)$; p_1, p_2 turli tub sonlar.

154. Tenglamani yeching:

$$a) \varphi(x) = \frac{x}{2}; \quad b) \varphi(x) = \frac{x}{3}; \quad c) \varphi(x) = \frac{x}{4}.$$

155. Tenglamani tekshiring: $\varphi(p^x) = a$.

156. Eyler funksiyasi xossaligidan foydalanib, barcha tub sonlar to'plami cheksiz ekanligini isbotlang.

157. Maxraji 2 dan n gacha bo'lgan barcha musbat to'g'ri, qisqarmas kasrlar sonini aniqlang.

158. 300 dan kichik va u bilan EKUBi 20 ga teng bo'lgan natural sonlarning sonini aniqlang.