II.1-§.

76. 1) $\pi(5) = 3$; 2)$\pi(10) = 4$; 3) $\pi(25) = 9$; 4) $\pi(37) = 12$; 5) $\pi(200) = 46$; 6) $\pi(1000) = 168$.

77. 1) $\pi(100) = \frac{100}{ln100} = \frac{100}{ln4+ln25} = \frac{100}{2ln10} = \frac{50}{2,3026} \approx 22.$

Nisbiy xatolikni hisoblaymiz:

$$\omega = \frac{\Delta\pi(x)}{\pi(x)} = \frac{25-22}{25} = \frac{3}{25} = 0,12 = 12\%.$$

2)

$$\pi(500) = \frac{500}{ln500} = \frac{5\cdot 100}{ln5 + ln100} = \frac{500}{1,6094 + 4,6052} = \frac{500}{6,2146} \approx 80;$$

$$\omega = \frac{95-80}{95} = \frac{15}{95} = \frac{3}{16} \approx 0,16 = 16\%.$$

3)

$$\pi(1000) = \frac{1000}{ln1000} = \frac{1000}{3ln10} = \frac{1000}{3\cdot 2,3026} = \frac{1000}{6,9078} \approx 145;$$

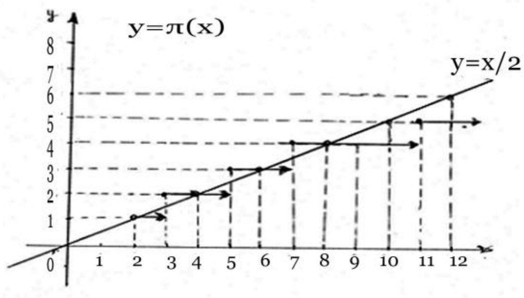
$$\omega = \frac{168-145}{168} = \frac{23}{168} \approx 0,14 = 14\%.$$

4)

$$\pi(3000) = \frac{3000}{ln3000} = \frac{3000}{ln3 + ln1000} = \frac{3000}{8,0064} \approx 3,75;$$

$$\omega = \frac{427-375}{427} = \frac{52}{427} \approx 0,12 = 12\%.$$

78.



1-shakl

79. Chebishyev tengsizligidan

$$\frac{a}{Inx} < \frac{\pi(x)}{x} < \frac{6}{Inx}$$

Bu tengsizlikning ikkala tomonidan $x \to +\infty$ limitga o`tsak:

$$\lim_{x\to\infty} \frac{a}{lnx} = a \cdot \lim_{x\to\infty} \frac{1}{lnx} = a \cdot 0 = 0$$

va

$$\lim_{x\to\infty} \frac{b}{lnx} = 0$$

larga ega bo'lamiz. Bulardan

$$\lim_{x\to\infty} \frac{\pi(x)}{x} = 0$$

ekanligi kelib chiqadi .

Isbotlanganidan xulosa qilish mumkinmi, $\pi(x)$f unksiya x gaqaragandasekin o’sadi. $\frac{\pi(x)}{x}$ nisbatni L.Eyler $[1, x]$ kesmadagi tub sonlarning o'rtacha zichligi deb atagan.

80. Tushunarliki $\pi(p) < p$. Bunda $-p < -\pi(p)$ oxirgi tengsizlikning ikkala tomoniga $p\pi(p)$ ni qo’shamiz. U holda

$$p\pi(p) - p < (p-1)\pi(p)$$

hosil bo’ladi. Buni

$$\frac{\pi(p)-1}{p-1}<\frac{\pi(p)}{p}$$

ko’rinishida yozish mumkin. $\pi(p)-1$=$\pi(p-1)$ bo'lgani uchun

$$\frac{\pi(p-1)}{p-1}<\frac{\pi(p)}{p}$$

ni hosil qilamiz. m murakkab son bo’lsa, $\pi(m-1)=\pi(m)$ bo’lgani uchun

$$\frac{\pi(m-1)}{m}=\frac{\pi(m)}{m}$$

bo’ladi. Bundan

$$\frac{\pi(m)}{m}<\frac{\pi(m-1)}{m-1}$$

kelib chiqadi.

II.2-§.

81. *a)* $[-2{,}7]=-2{,}7-\{-2{,}7\}=-2{,}7-0{,}3=-3.$

b) $\left[2+\sqrt[3]{987}\right]$ hisoblang. Bu yerda $9<\sqrt[3]{987}<10$ bo'lgani uchun $\left[\sqrt[3]{987}\right]=9$ va demak, $\left[2+\sqrt[3]{987}\right]=2+\left[\sqrt[3]{987}\right]=2+9=11.$

c) $\sqrt{21}=4+\alpha, 0<\propto<1$ bo'lgani uchun $\left[\frac{7-\sqrt{21}}{2}\right]=\left[\frac{7-(4+\alpha)}{2}\right]=\left[\frac{3-\propto}{2}\right]=1$ bo’ladi.

d) $\frac{10}{3+\sqrt{3}}=\left[\frac{10(3-\sqrt{3})}{9-3}\right]=\frac{30-10\sqrt{3}}{6}=\frac{30-(17+\propto)}{6}=\frac{13-\propto}{6}=2.$

e)$\left[1,(3)+2\operatorname{tg}\frac{\pi}{4}\right]=[1,(3)+2]=[1,(3)]+2=1+2=3.$

i)$\left[3+\sin\frac{13\pi}{7}\right]=\left[3+\sin\left(2\pi-\frac{\pi}{7}\right)\right]=\left[3-\sin\frac{\pi}{7}\right]=3+\left[-\sin\frac{\pi}{7}\right]=3-1=2.$

j)$\left[3-2cos\frac{90\pi}{181}\right]=[3-\alpha]=2,$ chunki $0<cos\frac{90\pi}{181}<\frac{1}{2}.$

f). Bu yerda $\lg 2512=x\Rightarrow 2512=10^x$ $3<x<4$ ya’ni $x=3+\alpha, 0<\alpha<1$ bo’lgani uchun $[2-\log_{10}2512]=[2-(3+\alpha)]=[-1-\alpha]=-2.$

l).$log_{10}\overline{abcd}=x\Rightarrow\overline{abcd}=10^x\Rightarrow 3<x<4$ bo’lgani uchun agar $\overline{abcd}>1000$ bo'lsa, $[2-log_{10}\overline{abcd}]=2-[(3+\propto)]=2-4=-2$ va agar $\overline{abcd}=1000$ bo'lsa, u holda $[2-log_{10}\overline{abcd}]=[2-3]=-1;$

k) $\sqrt{30}+\sqrt[3]{10}=(5+\propto)+(2+\beta);$ $0<\propto<0{,}5; 0<\beta<0{,}2.$ $\left[\sqrt{30}+\sqrt[3]{10}\right]=[7+\propto+\beta]=7;$ chunki $0<\propto+\beta<1.$

82. Berilgan tenglikning chap tomoni$[\pi]^{[e]} + [e] = 3^2 + 2 = 11$ o'ng tomoni $[e]^{[\pi]} + [\pi] = 2^3 + 3 = 8 + 3 = 11$. Bu tengliklarning o'ng tomonlari teng, Shuning uchun ham chap tomonlari ham teng bo'lishi kerak.

83. $p = 4k + 1$ yoki $p = 4k + 3$ ko'rinishida deb olishimiz mumkin. $p = 4k + 1$ ko'rinishda bo'lsa, $\left[\frac{p}{4}\right] = \left[\frac{4k+1}{4}\right] = \left[k + \frac{1}{4}\right] = kva\frac{p-1}{4} = \frac{4k+1-1}{4} = k$; ya`ni $\frac{p}{4} = \frac{p-1}{4}$; agar $p = 4k + 3$ ko'rinishida bo'lsa, $\left[\frac{p}{4}\right] = \left[k + \frac{3}{4}\right] = k = \frac{p-3}{4}$.

84. $a = mq + r,\ 0 \le r < m$ deb yozib olsak, $\left[\frac{a}{m}\right] = q + \frac{r}{m}; 0 \le \frac{r}{m} < 1$ bo'ladi. Bundan $\left[\frac{a}{m}\right] = q = \frac{a-r}{m}$.

85. Berilgan munosabat $[nx] \le nx < [nx] + 1, n = 1,2,\ldots$ munosabatga teng kuchli. Buning to'g'ri ekanligi esa butun qism funksiyasi ta'rifidan bevosita kelib chiqadi.

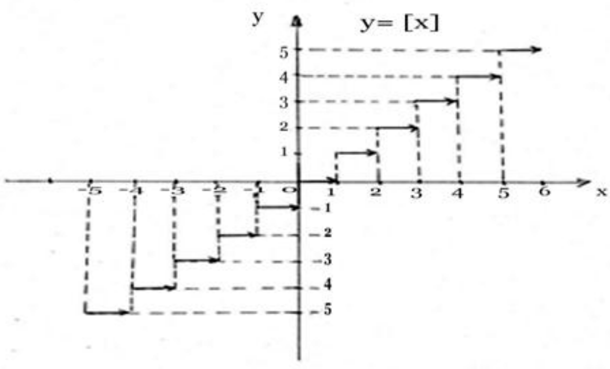
86. $\frac{x+y}{n} = \frac{x}{n} + \frac{y}{n} = \left[\frac{x}{n}\right] + \propto + \left[\frac{y}{n}\right] + \beta;\ 0 \le \propto < 1$ va $0 \le \beta < 1$. Бундан $\left[\frac{x+y}{n}\right] = \left[\frac{x}{n}\right] + \left[\frac{y}{n}\right] + [\propto + \beta]$; бунда $0 \le \propto + \beta < 2$. Shuning uchun ham $[\propto + \beta] = 0$ yoki 1. Birinchi holda $\left[\frac{x+y}{n}\right] = \left[\frac{x}{n}\right] + \left[\frac{y}{n}\right]$ bo'ladi. Ikkinchi holda esa $\left[\frac{x+y}{n}\right] = \left[\frac{x}{n}\right] + \left[\frac{y}{n}\right] + 1$.

87.*1-usul* . m toq son bo'lsa, $m = 2q + 1$ deb yoza olamiz va

$$\left[\frac{m}{2}\right] = \left[\frac{2q+1}{2}\right] = \left[q + \frac{1}{2}\right] = q = \frac{m-1}{2}.$$

2-usul. $\left[\frac{m}{2}\right] = \frac{m-1}{2}$ tenglik $\frac{m-1}{2} \le \frac{m}{2} < \frac{m-1}{2} + 1$ ga, ya`ni $\frac{m-1}{2} \le \frac{m}{2} < \frac{m+1}{2}$ ga teng kuchli. Bundan$\frac{m-1}{2} - \frac{m}{2} \le 0 < \frac{m+1}{2} - \frac{m}{2}$ yoki $-\frac{1}{2} \le 0 < \frac{1}{2}$, ya`ni $-1 \le 0 \le 1$, doimo bajariladigan munosabat kelib chiqadi.

88. a) $y = [x]$ ning grafigini (2-shakl) chizamiz $(0 \le x < 1;\quad y = 0)$; $(1 \le x < 2\,; y = 1)$; $(2 \le x < 3;\ y = 2)$ va hokazo $(n \le x < n + 1; y = n)$. Bularni Dekart koordinatalar sistemasida tasvirlaymiz:

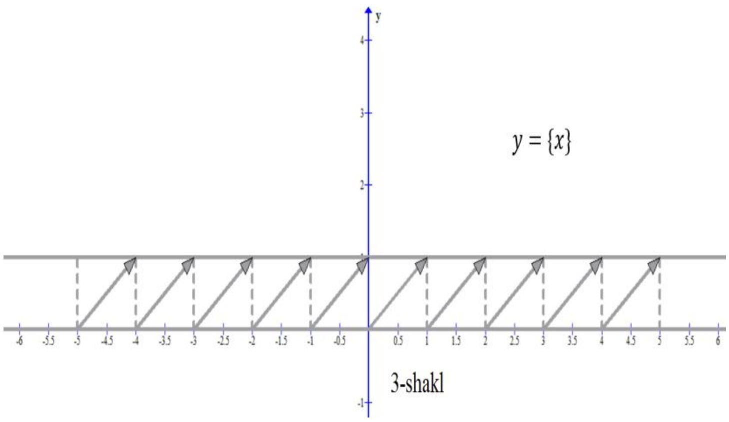


2-shakl

b) $y = \{x\}$ ning grafigini chizmiz.

$$\begin{pmatrix} 0 \le x < 1 \\ 0 \le y < 1 \end{pmatrix};\ \begin{pmatrix} 1 \le x < 2 \\ 0 \le y < 1 \end{pmatrix};\begin{pmatrix} 2 \le x < 3 \\ 0 \le y < 1 \end{pmatrix};\ \cdots;\begin{pmatrix} n-1 \le x < n \\ 0 \le y < 1 \end{pmatrix}$$

Bularni Dekart koordinatalar sistemasida tasvirlab berilgan funksiyaning grafigiga



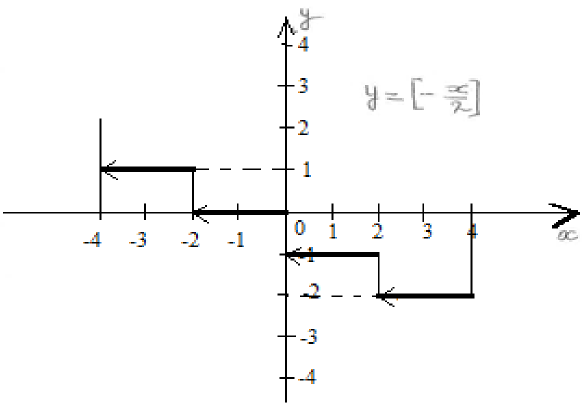
3-shakl

ega bo'lamiz (3-shakl).

c) $y = \left[-\frac{m}{2}\right]$ ning grafigini chizamiz.

$$\begin{pmatrix} 0 < x \le 2 \\ y = -1 \end{pmatrix};\ \begin{pmatrix} -2 < x \le 0 \\ y = 0 \end{pmatrix};\begin{pmatrix} -4 < x \le -2 \\ y = 1 \end{pmatrix};\ \cdots;\begin{pmatrix} -2n < x \le -2(n-1) \\ y = n-1 \end{pmatrix}$$

Bularni Dekart koordinatalar sistemasida tasvirlab berilgan funksiya`ning grafigiga ega bo'lamiz (4-shakl).

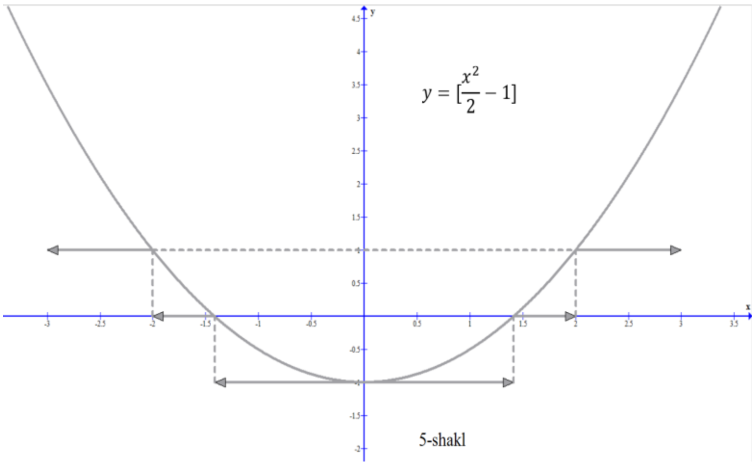


4-shakl

$d)\ y=\left[\frac{x^2}{2}-1\right]$ ning grafigini chizmiz. Bu yerda $\frac{x^2}{2}-1=0 \Rightarrow x^2=2 \Rightarrow$ $x_1=-\sqrt{2},\ x_2=+\sqrt{2}.$

Bundan $\binom{-\sqrt{2}<x<\sqrt{2}}{y=-1}$; $\binom{\sqrt{2}\le x<2}{y=0}$; $\binom{2\le x<\sqrt{6}}{y=1}$; $\cdots$

x_1=-$\sqrt{2}$, x_2=+$\sqrt{2}$.

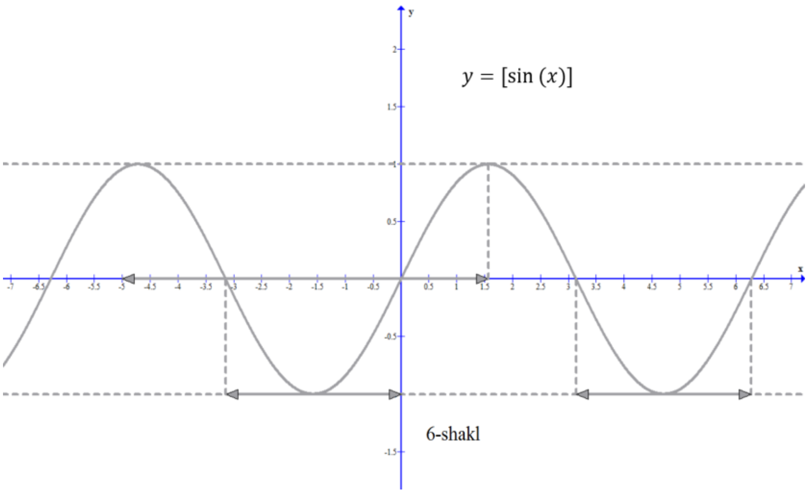


5-shakl

e) $y=[\sin x]$. Bu yerda

$$y=[\sin x]=\begin{cases} 1,\ \text{agar}\ x=\dfrac{\pi}{2}+2\pi k, k\in Z\ \text{bo'lsa;} \\ 0,\ \text{agar}\ 2\pi k\le x<\pi+2\pi k, k\in Z\ va\ x\ne\dfrac{\pi}{2}+2\pi k, k\in Z\ \text{bo'lsa;} \\ -1,\ \text{agar}\ \pi+2\pi k\le x<2\pi k, k\in Z\ va\ x\ne\dfrac{\pi}{2}+2\pi k, k\in Z\ \text{bo'lsa} \end{cases}$$

ekanligini e'tiborga olsak quyidagi grafikni hosil qilamiz (6-shakl).



6-shakl

89. *a*) $[x^2] = 2 \Rightarrow 2 \leq x^2 < 3 \Rightarrow \sqrt{2} \leq |x| < \sqrt{3}$. Avvalo $\sqrt{2} \leq |x|$ dan $x \leq -\sqrt{2}, x \geq \sqrt{2}$ va $|x| < \sqrt{3}$ dan $-\sqrt{3} < x < \sqrt{3}$ ga ega bo'lamiz. Bulardan $-\sqrt{3} < x \leq -\sqrt{2}$ va $\sqrt{2} \leq x < \sqrt{3}$.

b) $[3x^2 - x] = x + 1$ dan $x + 1$ butun son bo'lishi kerak. Buning uchun x butun bo'lishi kerak. x butun son bo'lsa, $3x^2 - x$ ham butun son bo'ladi. U holda $3x^2 - x = x + 1$ tenglamaga ega bo'lamiz. Bundan $3x^2 - 2x - 1 = 0.$ Bu tenglamani yechib $x_{1,2} = \frac{1 \pm \sqrt{1+3}}{3} = \frac{1 \pm 2}{3}$ ni ya'ni $x_1 = 1$, $x_2 = -\frac{1}{3}$ larga ega bo'lamiz. Bu yerda $x_2 = -\frac{1}{3}$ kasr son bo'lgani uchun tenglamani qanoatlantirmaydi. Javob $x = 1$.

c) $[x] = \frac{3}{4}x \Rightarrow \frac{3}{4}x \leq x < \frac{3}{4}x + 1$ va $\frac{3}{4}x$ butun son bo'lishi kerak. Bulardan $3x \leq 4x < 3x + 4 \;\Rightarrow\; 0 \leq x < 4 \;\; x = 0, \frac{4}{3}, \frac{8}{3}$. Bundan $x = 0, \frac{4}{3}, \frac{8}{3}$ ekanligi kelib chiqadi. Demak 3ta yechimi bor.

d) $[x^2] = x \Rightarrow x \leq x^2 < x + 1$ va x butun son bo'lishi kerak ekanligi kelib chiqadi. Bulardan$0 \leq x^2 - x < 1.$ Bu qo'sh tengsizlikni yechamiz.

A. $x^2 - x - 1 < 0$ tengsizlikni yechamiz. Uning o'ng tomoni shoxlari yuqoriga qaragan parabola bo'lgani uchun ham tengsizlikning yechimlari $x^2 - x - 1 = 0$ tenglamaning ikkala yechimlari orasidagi sonlardan iborat bo'ladi. $x^2 - x - 1 = 0$ tenglamaning yechimlari

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

dan iborat. Shuning uchun ham $x^2 - x - 1 < 0$ tengsizlikning yechimi $\left(\frac{1-\sqrt{5}}{2}; \frac{1+\sqrt{5}}{2}\right)$ oraliqdan iborat.

B. Endi $x^2 - x \geq 0$ tengsizlikni yechamiz. Uning o'ng tomoni shoxlari yuqoriga qaragan parabola bo'lgani uchun ham tengsizlikning yechimlari $]-\infty, x_1] \cup [x_2, +\infty[$ dan iborat bo'ladi. $x^2 - x = 0$ tenglamaning yechimlari$x_1 = 0$ va $x_2 = 1$ lardan iborat. Shuning uchun ham $x^2 - x \geq 0$ tengsizlikning yechimi $]-\infty, 0] \cup [1, +\infty[$ dan iborat.

Endi qarab chiqilgan A va B hollarni birlashtirib, $0 \leq x^2 - x < 1$ qo'sh tengsizlikning yechimini topamiz. U holda $x \in \left]\frac{1-\sqrt{5}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{5}}{2}\right[$ va x butun son bo'lishi kerak. Demak, qaralayotgan tengsizlikning butun son klardagi yechimlari $x = 0, 1$ dan iborat.

90. $[12{,}4m] = 87 \Rightarrow 87 \leq 12{,}4m < 88 \;\Rightarrow \frac{870}{124} \leq m < \frac{880}{124} \Rightarrow \frac{435}{62} \leq m < \frac{440}{62} \Rightarrow 7\frac{1}{62} \leq m < 7\frac{3}{31} \Rightarrow m \notin N.$

91. Agar x butun son bo'lsa, u holda $[-x] = -[x]$.Agar x kasr son bo'lsa,$[-x] = y$ deb olsak, $y < -x < y + 1$ bajarilishi kerak. Bundan $-y - 1 < x < -y$ yoki $[x] = -y - 1 = -[-x] - 1$.

Shunday qilib

$$[-x] = \begin{cases} -[x]\text{ga; agar } x \text{ butun son bo'lsa;} \\ -[x] - 1 \text{ ga; agar } x \text{ kasir son bo'lsa.} \end{cases}$$

92. $x_i = [x_i] + \propto_i \quad 0 \leq \propto_i < 1$ deb olsak,

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} [x_i] + \sum_{i=1}^{n} \propto_i$$

bo'ladi. Bundan

$$\left[\sum_{i=1}^{n} x_i\right] = \sum_{i=1}^{n} [x_i] + \left[\sum_{i=0}^{n} \propto_i\right].$$

Bu yerda

$$\left[\sum_{i=0}^{n} \propto_i\right] \geq 0$$

bo'lgani uchun

$$\left[\sum_{i=1}^{n} x_i\right] \geq \sum_{i=1}^{n} [x_i] \qquad (*)$$

bajariladi.

93. 12-masalada $x_1 = x_2 = \cdots = x_n = x$ deb olamiz. U holda (*) munosabat $[nx] \geq n[x]$ ko'rinishni oladi.

94. $[1, N]$ kesimda m soniga karrali sonlarning soni $\left[\frac{N}{m}\right]$ ga teng. Shuning uchun ham 10^6 va 10^7sonlari orasidagi 786 ga karrali natural sonlarning soni

$$\left[\frac{10^7}{786}\right] - \left[\frac{10^6}{786}\right] = \left[\frac{10000000}{786}\right] - \left[\frac{1000000}{786}\right] = 12722 - 1272 = 11450.$$

95. 1000 dan kichik natural sonlarning soni 999 ta ularning orasida 5 ga karralilari soni $\left[\frac{999}{5}\right]$ ga, 7 ga karralilari soni $\left[\frac{999}{7}\right]$ ga teng. Bu sonlar orasida 5 va7ga karralilari ham bor. Shuning uchun ham 1000 dan kichik 5ga ham 7 ga ham bo'linmaydigan natural sonlar soni $999 - \left[\frac{999}{5}\right] - \left[\frac{999}{7}\right] + \left[\frac{\left[\frac{999}{5}\right]}{7}\right] = 999 - 199 - 142 + 28 = 686$ ga teng.

96. $36 = 2^2 \cdot 3^2$ bo'lgani uchun n soni 36 bilan o'zaro tub bo'lishi uchun $(n, 2) = (n, 3) = 1$ bo'lishi kerak. Shuning uchun ham 36 soni bilan o'zaro tub 100 dan katta bo’lmagan natural sonlarning soni $100-\left[\frac{100}{2}\right]-\left[\frac{100}{3}\right]+\left[\frac{100}{6}\right]=100-50-33+16=116-83=33.$

97. Agar ko’paytmada 2 va 5 birgalikda ko’paytuvchi sifatida necha marta qatnashsa, ko’paytma shuncha nol bilan tugaydi. Albatta $2017!$ da 2 soni 5 ga qaraganda ko’proq ko’paytuvchi sifatida qatnashadi. Shuning uchun ham masalani yechish uchun 5 ning $2017!$ da nechanchi daraja bilan qatnashishini aniqlash kifoya.

$$\alpha=\left[\frac{2017}{5}\right]+\left[\frac{2017}{5^2}\right]+\left[\frac{2017}{5^3}\right]+\left[\frac{2017}{5^4}\right]=403+80+16+3=502.$$

Demak, $2017!$ ko’paytma 502 ta nol bilan tugaydi.

98. $N!$ ning tub ko’paytuvchilarga yoyilmasida p tub soni

$$\alpha=\left[\frac{N}{p}\right]+\left[\frac{N}{p^2}\right]+\cdots+\left[\frac{N}{p^k}\right],\quad p^k\leq N$$

daraja bilan qatnashadi $p^n=N$ deb olsak,

$$\propto=\left[\frac{p^n}{p}\right]+\left[\frac{p^n}{p^2}\right]+\cdots+\left[\frac{p^n}{p^n}\right]=p^{n-1}+p^{n-2}+\cdots+p+1=\frac{(1-p^n)}{1-p}=\frac{p^n-1}{p-1}$$

hosil boladi.

99. $6=3\cdot 2$ bo’lgani uchun $100!$ ko’paytmada 6 ning qaysi daraja bilan qatnashishini aniqlash uchun 3 ning qaysi daraja bilan qatnashishini aniqlash kifoya.

$$\alpha=\left[\frac{100}{3}\right]+\left[\frac{100}{9}\right]+\left[\frac{100}{27}\right]+\left[\frac{100}{81}\right]=33+11+3+1=48.$$

Demak,$100!$ ko’paytmada 6 soni 48-daraja bilan qatnashadi.

100. Ma’lumki, $n!$ sonining kanonik yoyilmasi $n!=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdots p_k^{\alpha_k}$

ko’rinishida bo’lib, bu yerda p_i lar tub sonlar, α_i lar esa p_i tub sonining $n!$ sonida qanday daraja qatnashishini bildiradi va

$$\alpha=\left[\frac{N}{p}\right]+\left[\frac{N}{p^2}\right]+\cdots+\left[\frac{N}{p^n}\right]$$

ko’rinishda topiladi. Demak ,

$$\alpha_1=\left[\frac{11}{2}\right]+\left[\frac{11}{2^2}\right]+\left[\frac{11}{2^3}\right]=5+2+1=8;$$

$$\alpha_2=\left[\frac{11}{3}\right]+\left[\frac{11}{3^2}\right]=3+1=4;$$

$$\alpha_3=\left[\frac{11}{5}\right]=2;\quad \alpha_4=\left[\frac{11}{7}\right]=1;\quad \alpha_5=\left[\frac{11}{11}\right]=1$$

bo’lgani uchun $11!=2^8\cdot 3^4\cdot 5^2\cdot 7\cdot 11.$

101. Avvalo berilgan Nsonining ko’rinishini $N=\frac{1000!}{100!\cdot 7^{\alpha}}$ shaklda yozib olamiz va bu yerda N soni butun son bo’lishligi uchun $1000!$ ning kanonik yoyilmasi tarkibida 7 tub soni qanday daraja k bilan qatnashishini aniqlashimiz kerak: (N ning suratida)

$$k=\left[\frac{1000}{7}\right]+\left[\frac{1000}{7^2}\right]+\left[\frac{1000}{7^3}\right]=142+20+2=164;$$

(N ning maxrajida)

$$l=\left[\frac{100}{7}\right]+\left[\frac{100}{7^2}\right]+\alpha=14+2+\alpha=16+\alpha.$$

Bularga asosan $N=\frac{7^{164}\cdot Q}{7^{16+\alpha}}=7^{148-\alpha}\cdot Q$. Bu yerda Q natural son va $(Q,7)=1$. Bundan $148-\alpha\geq 0\Rightarrow 0\leq\alpha\leq 148$. Demak, α ning eng katta qiymati 148 ga teng.

102. Ma'lumki, $(2m)!!=m!\cdot 2^m$. Bundan, agar $p=2$ bo'lsa, u holda $2^k\leq m<2^{k+1}$ bo'lgani uchun, izlangan daraja ko'rsatkich $m+\sum_{i=1}^{k}\left[\frac{m}{2^i}\right]$ ga teng bo'ladi. Agar $p>2$ bo'lsa, u holda izlangan daraja ko'rsatkich $\sum_{i=1}^{s}\left[\frac{m}{p^i}\right]$ bu yerda $p^s\leq m<p^{s+1}$.

103. Berilgan tenglama avvalo ko'rinishida $[x]=1+2\left[\frac{x}{2}\right]$ yozib olamiz, agar bu tenglamaning chap tomomnini y belgilasak, u holda quyidagiga ega bo'lamiz:

$$\begin{cases} y=[x] \\ y=1+2\left[\frac{x}{2}\right]. \end{cases}$$

Bundan esa

$$\begin{cases} y=[x] \\ \frac{y-1}{2}=1+2\left[\frac{x}{2}\right] \end{cases}$$

sistemani hosil qilamiz. $\frac{y-1}{2}$ ning butun qiymatlarini m belgilab

$$\begin{cases} 2m+1=[x] \\ m=\left[\frac{x}{2}\right] \end{cases} \text{ yoki } \begin{cases} 2m+1\leq x<2m+2 \\ 2m\leq x<2m+2 \end{cases} \text{ ni topamiz.}$$

Bu yerdan $2m+1\leq x<2m+2,\quad m=0,\pm 1,\ \pm 2,...$ ni hosil qilamiz.

104. $y=ax^2+bx+c$ funksiya va demak $y=\left[ax^2+bx+c\right]$ funksiya $a>0$ bo'lganda quyidan va $a<0$ da yuqorida chegaralangan. Ikkala holda ham $y=\left[ax^2+bx+c\right]=\left[a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4a\tilde{n}}{4a^2}\right)\right]$ funksiya`ning qiymatlarining aniq chegarasi $\left[-\frac{b^2-4ac}{4a}\right]$ sondan iborat bo'ladi. Shuning учун $a>0$ bo'lganda berilgan tenglama $\left[-\frac{b^2-4ac}{4a}\right]\le d$ bo'lganda va faqat shu holda yechimga ega, agarda $a<0$ bo'lsa, u holda $\left[-\frac{b^2-4ac}{4a}\right]\le d$ bo'lsa yechim mavjud bo'ladi.

105. Har bir $x=k$ $(a\le k\le b)$ butun absissaga egri chiziqli trapetsiya ichidagi va chegarasidagi $[f(x)]+1$ ta butun ordinata mos keladi. Shuning uchun ham izlanayotgan nuqtalar soni

$$\sum_{k=a}^{b}([f(k)]+1) \text{ ga teng.}$$

106. Buning uchun avvalo 1-chorakdagi shu aylana ichidagi butun nuqtalar sonini aniqlaymiz. Aylana tenglamasini y ga nisbatan yechib, 1-chorakga mos qismi $y=\sqrt{6{,}5^2-k^2}$ ni olib 25-misolni tadbiq etamiz. U holda $\sum_{k=0}^{6}([\sqrt{6{,}5^2-k^2}\,]+1)=7+7+7+6+6+5+3=41$ hosil boladi. Demak, izlnayotgan nuqtalar soni $N=4\cdot 41-4\cdot 7=164-28=136$ ta.

107. n dan katta bo'lmagan va $p_1,p_2,...,p_k$ tub sonlarning har biri bilan o'zaro tub bo'lgan sonlarning soni $B(n;p_1;p_2;...;p_k)=[n]-\left[\frac{n}{p_1}\right]-\left[\frac{n}{p_2}\right]-\cdots-\left[\frac{n}{p_k}\right]+\left[\frac{n}{p_1p_2}\right]+\cdots+\left[\frac{n}{p_{k1}p_k}\right]-\left[\frac{n}{p_1p_2p_3}\right]-\cdots-\left[\frac{n}{p_{k-2}p_{k-1}p_k}\right]+\cdots+(-1)^k\left[\frac{n}{p_1p_2\cdots p_k}\right]$ formula bilan topiladi. Shunga asosan $1575=3^2\cdot 5^2\cdot 7$ bo'lgani uchun $12317-\left[\frac{12317}{3}\right]-\left[\frac{12317}{5}\right]-\left[\frac{12317}{7}\right]+\left[\frac{12317}{15}\right]+\left[\frac{12317}{21}\right]+\left[\frac{12317}{35}\right]-\left[\frac{12317}{105}\right]=12317-4105-2463-1759+821+586+351-117=5631.$

II.3-§.

108.1). Bu yerda $375=3\cdot 5^3$ bo'lgani uchun (1) va (2)- formulalardan

$$\tau(375)=\tau(3\cdot 5^3)=(1+1)(3+1)=8;$$

$$\sigma(375)=\frac{3^2-1}{3-1}\cdot\frac{5^4-1}{5-1}=4\cdot\frac{624}{4}=624$$

larga ega bo'lamiz.

2).$720=2^4\cdot 3^2\cdot 5$ bo'lgani uchun(1) va (2)- formulalardan

$$\tau(720)=5\cdot 3\cdot 2=30;$$

$$\sigma(720)=\frac{2^5-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{5^2-1}{5-1}=31\cdot 13\cdot 6=31\cdot 78=2418$$

lar kelib chiqadi.

3). Bu yerda $957=3\cdot 11\cdot 29$ bo'lgani uchun (1) va (2)- formulalardan

$$\tau(957) = (1+1)(1+1)(1+1) = 8;$$
$$\sigma(957) = \frac{3^2-1}{3-1} \cdot \frac{11^2-1}{11-1} \cdot \frac{29^2-1}{29-1} = 4 \cdot 12 \cdot 30 = 48 \cdot 30 = 1440$$
lar kelib chiqadi.

4).$988 = 2^2 \cdot 13 \cdot 19$ bo'lgani uchun(1) va (2)- formulalardan
$$\tau(988) = 3 \cdot 2 \cdot 2 = 12;$$
$$\sigma(988) = \frac{2^3-1}{2-1} \cdot \frac{13^3-1}{13-1} \cdot \frac{19^2-1}{19-1} = 7 \cdot 14 \cdot 20 = 1960.$$
5).$990 = 2 \cdot 3^2 \cdot 5 \cdot 11; \quad \tau(990) = 2 \cdot 3 \cdot 2 \cdot 2 = 24,$
$$\sigma(990) = \frac{2^2-1}{2-1} \cdot \frac{3^3-1}{3-1} \cdot \frac{5^2-1}{5-1} = 3 \cdot 13 \cdot 6 \cdot 12 = 2808.$$
6).$1200 = 2^4 \cdot 3 \cdot 5^2;\ \tau(1200) = 5 \cdot 2 \cdot 3 = 30,$
$$\sigma(1200) = \frac{2^5-1}{2-1} \cdot \frac{3^2-1}{3-1} \cdot \frac{5^3-1}{5-1} = 31 \cdot 4 \cdot 31 = 3844.$$
7). $1440 = 2^5 \cdot 3^2 \cdot 5;\ \tau(1440) = 6 \cdot 3 \cdot 2 = 36,$
$$\sigma(1440) = \frac{2^6-1}{2-1} \cdot \frac{3^3-1}{3-1} \cdot \frac{5^2-1}{5-1} = 63 \cdot 13 \cdot 6 = 4914.$$
8). $1500 = 2^2 \cdot 3 \cdot 5^3;\ \tau(1500) = 3 \cdot 2 \cdot 4 = 24,$
$$\sigma(1500) = \frac{2^3-1}{2-1} \cdot \frac{3^2-1}{3-1} \cdot \frac{5^4-1}{5-1} = 7 \cdot 4 \cdot 156 = 4368.$$

9). $1890 = 2 \cdot 3^3 \cdot 5 \cdot 7; \tau(1890) = 2 \cdot 4 \cdot 2 \cdot 2 = 32,$
$$\sigma(1890) = \frac{2^2-1}{2-1} \cdot \frac{3^4-1}{3-1} \cdot \frac{5^2-1}{5-1} \cdot \frac{7^2-1}{7-1} = 3 \cdot 40 \cdot 6 \cdot 8 = 5760.$$
10). $4320 = 2^5 \cdot 3^3 \cdot 5;\ \tau(4320) = 6 \cdot 4 \cdot 2 = 48,$
$$\sigma(4320) = \frac{2^6-1}{2-1} \cdot \frac{3^4-1}{3-1} \cdot \frac{5^2-1}{5-1} = 63 \cdot 40 \cdot 6 = 15120.$$
109. 1). $360 = 2^3 \cdot 3^2 \cdot 5,\ \ d = 2^\alpha \cdot 3^\beta \cdot 5^\gamma,\ \ 0 \le \alpha \le 3,\ \ 0 \le \beta \le 2, 0 \le \gamma \le 1.$ Shuning uchun ham

$(1+2+4+8)(1+3+9)(1+5) = (1+2+4+8)(1+3+9+5+15+45) = 1+3+9+5+15+45+2+6+18+10+30+90+4+12+36+20+60+180+8+24+72+40+120+360;$

Bo'luvchilar:
$1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360$. Ularning jami soni $24\ ta.$

2). $720 = 2^4 \cdot 3^2 \cdot 5, \quad (1+2+4+8+16)(1+3+9)(1+5)$
$= (1+2+4+8+16)(1+3+9+5+15+45)$
$= 1+2+4+8+16+3+6+12+24+48+9+18+36+72$
$+ 144+5+10+20+40+80+15+30+60+120+240+360$
$+ 720.$

Bo'luchilar $1,2,3,4,5,6,8,9,10,12,15,16,18,20,24,30,$
$36,40,45,48,60,72,80,90,120,144,180,240,360,720. Jami: 30ta.$

3). $954 = 2 \cdot 3^2 \cdot 53, \quad (1+2)(1+3+9)(1+53)$
$= (1+2)(1+3+9+53+159+447)$
$= 1+3+9+53+159+447+2+6+18+106+318+954.$

Bo'luchilar: $1,2,3,6,9,18,53,106,159,318,477,954.$ $Jami: 12\ ta$.

4). $988 = 2^2 \cdot 13 \cdot 19, \quad (1+2+4)(1+13)(1+19)$
$= (1+2+4)(1+13+19+247)$
$= 1+13+19+247+2+26+38+494+4+52+76+988$

Bo'luvchilar: $1,2,4,13,19,26,38,52,76,247,494,988.$ $Jami: 12\ ta.$

5). $600 = 2^3 \cdot 3 \cdot 5^2, \quad (1+2+4+8)(1+3)(1+5+25) = (1+3+5+15+25+75)(1+2+4+8) = 1+3+5+15+25+75+2+6+10+30+50+150+4+12+20+60+100+300+8+24+40+120+200+600$

Bo'luvchilar: $1,2,3,4,5,6,8,10,12,15,20,24,25,30,40,50,60,75,100,$
$120,150,200,300,600.$ $Jami: 24\ ta.$

110. $\tau(x) = 6,\ \sigma(x) = 28,\ x = p_1{}^{\alpha} \cdot p_2{}^{\beta} \quad \alpha \geq 1,\ \beta \geq 1$ bo'lgani uchun $\tau(x) = (\alpha+1)(\beta+1) = 6,$ bundan $\alpha = 1,$ □ $= 2$ va $x = p_1 \cdot p_2{}^2$.

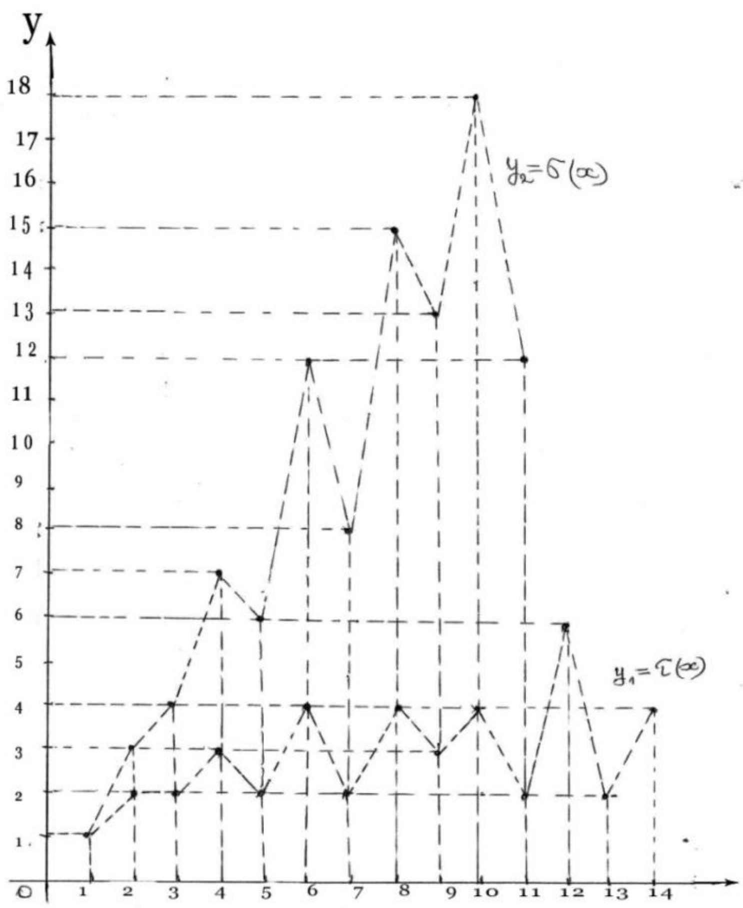
Bu holda $\sigma(x) = (p_1+1)\frac{p_2{}^3-1}{p_2-1} = (p_1+1)(p_2{}^2+p_2+1) = (p_1+1)(p_2(p_2+1)+1) = 28,$ bu yerda $p_2(p_2+1)$ juft son bo'lgani uchun $p_2(p_2+1)+1$ toq son, shuning uchun ham $p_1+1=4,\ p_2(p_2+1)+1=7,\ p_1=3,$ $p_2(p_2+1)=6,\ p_2=2$ demak, $x = p_1 p_2{}^2 = 3 \cdot 4 = 12.$

111. $N = p^{\alpha} \cdot q^{\beta}, \quad N^2 = p^{2\alpha} \cdot q^{2\beta}, \quad N^3 = p^{3\alpha} \cdot q^{3\beta}$. Bulardan $\tau(N^2) = (2\alpha+1)(2\beta+1) = 15 = 3 \cdot 5,$ bundan $\alpha = 1,\ \beta = 2$ (yoki $\alpha = 2;\ \beta = 1$). $\tau(N^3) = (3\alpha+1)(3\beta+1) = 4 \cdot 7 = 28$ ta.

112. $\tau(x)$ va $\sigma(x)$ larning grafigni sxematik tasvirlang. Buning uchun berilgan funksiyalarning qiymatlari jadvalini tuzib olamiz:

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\tau(x)$	1	2	2	3	2	4	2	4	3	4	2	6	3	4	4	5
$\sigma(x)$	1	3	4	7	6	12	8	15	13	18	12	28	14	24	24	31

Bu qiymatlarni Dekart koordinatalar sistemasida belgilab quyidagi grafiklarga



7-shakl

ega bo'lamiz (7-shakl).

113. $p_2 - p_1 = 2,\ \ p_1 = p_2 - 2,\ \sigma(p_1) = \frac{{p_1}^2 - 1}{p_1 - 1} = p_1 + 1 = 1 + (p_2 - 2) = p_2 - 1 = \varphi(p_2)$.

114. $m = 2^\alpha$ bo'lsin. U holda $\sigma(m) = \frac{2^{\alpha+1}-1}{2-1}$ bo'lgani uchun tenglama $2^{\alpha+1} - 1$=$2 \cdot 2^\alpha - 1 = 2^{\alpha+1} - 1$, ya'ni $m = 2^\alpha$ $(\alpha = 0,1,2,3,\ldots)$ ko'rinishdagi sonlar berilgan tenglamaning yechimi,

$\alpha = 0,1,2,3,\ldots$ qiymatlar bersak, m ning cheksiz ko'p natural qiymatlari hosil bo'ladi.

115. 1). Agar p tub soni m yoki n ning kanonik yoyilmasiga biror α daraja ko'rsatkichi bilan kirsa, u holda $\tau(mn)$ da ham, shuningdek, $\tau(m)\tau(n)$ da ham $(\alpha+1)$ ko'paytuvchi qatnashadi. Agarda m va n larning kanonik yoyilmasida mos ravishda p^αva p^β lar qatnashsa, u holda mn ning kanonik yoyilmasida $p^{\alpha+\beta}$ishtirok etadi. Bu holda $\tau(mn)$da $\alpha+\beta+1$ ko'paytuvchi qatnashadi. $\alpha+\beta+$

$1 < (\alpha+1)(\beta+1)$bo'lgani uchun $\tau(m)\tau(n) > \tau(mn)$ bo'ladi, ya'ni agar $(m,n) > 1$ bo'lsa, $\tau(m)\tau(n) > \tau(mn)$ bo'lar ekan.

2). Agar p tub soni m yoki n ning kanonik yoyilmasiga biror α daraja ko'rsatkichi bilan kirsa, u holda $\sigma(mn)$ da ham, shuningdek, $\sigma(m)\sigma(n)$ da ham $\frac{p^{\alpha+1}-1}{p-1}$ ko'paytuvchi qatnashadi.

Agar m va n larning kanonik yoyilmasiga mos ravishda p^{α} va p^{β} lar tegishli bo`lsa, u holda $mn$ ning kanonik yoyilmasida $p^{\alpha+\beta}$ qatnashadi. Bu holda $\sigma(mn)$ ning tarkibida qatnashuvchi $\frac{p^{\alpha+\beta+1}-1}{p-1}$ ko`paytuvchiga, $\sigma(m)\cdot\sigma(n)$ ning tarkibidagi

$$\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}=\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1}{(p-1)^2}$$

ko`paytma mos keladi. Bu yerda

$$\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1}{(p-1)}-\left(p^{\alpha+\beta+1}-1\right)$$
$$=\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1-p^{\alpha+\beta+2}+p^{\alpha+\beta+1}-1}{(p-1)}=$$
$$\frac{p(1-p^{\alpha})-p^{\beta}+p^{\alpha+\beta}}{p-1}=\frac{p(p^{\alpha}-1)(p^{\beta}-1)}{p-1}>0$$

ya`ni

$$\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}>\frac{p^{\alpha+\beta+1}-1}{p-1}.$$

Demak, agar $(m,n) > 1$bo`lsa, u holda $\sigma(m)\sigma(n) > \sigma(mn)$ bo`ladi.

116. m ning barcha natural bo`luvchilari $d_1, d_2, \ldots, d_{\tau(m)}$ bo`lsin, u holda biz

$$\delta(m)=\prod_{i=1}^{\tau(m)} d_i$$

uchun formula chiqarishimiz kerak. Bunda $\frac{m}{d_1}, \frac{m}{d_2}, \ldots \frac{m}{d_{\tau(m)}}$ lar ham m ning barcha bo`luvchilari bo`lgani uchun

$$\delta(m)=\prod_{i=1}^{\tau(m)}\frac{m}{d_i}=m^{\tau(m)}\prod_{i=1}^{\tau(m)}\frac{1}{d_i}=m^{\tau(m)}\cdot\frac{1}{\prod_{i=1}^{\tau(m)} d_i}=\frac{m^{\tau(m)}}{\delta(m)}.$$

Bunda $\delta^2(m)=m^{\tau(m)}$, yoki $\delta(m)=\sqrt{m^{\tau(m)}}$. Xususiy holda $\delta(10)=\sqrt{10^{\tau(10)}}=\sqrt{10^4}=10^2=100.$

117. Masalaning shartiga asosan $m=\sqrt{m^{\tau(m)}}$, bundan $\tau(m)=2$, ya`ni $m$ natural soni faqat 2ta bo`luvchiga ega bo`lishi kerak, demak, u tub son bo`lishi kerak. Shunday qilib o`zining barcha natural bo`luvchilari ko`paytmasiga teng bo`lgan sonlar natural sonlar to`plami tub sonlar to`plami bilan ustma-ust tushadi.

118. n ning kanonik yoyilmasi $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots p_k^{\alpha_k}$ bo`lsin. U holda $\sigma_k(n) = (1 + p_1^k + p_1^{2k} + \cdots + p_1^{\alpha_1 k})(1 + p_2^k + p_2^{2k} + \cdots + p_2^{\alpha_2 k}) \dots (1 + p_s^k + p_s^{2k} + \cdots + p_s^{\alpha_s k}) = \frac{p_1^{k(\alpha_1+1)}-1}{p_1^k-1} \cdot \frac{p_2^{k(\alpha_2+1)}-1}{p_2^k-1} \dots \frac{p_s^{k(\alpha_s+1)}-1}{p_s^k-1}$, ya`ni

$$\sigma_k(n) = \prod_{i=1}^{s} \frac{p^{k(\alpha_i+1)} - 1}{p_i^k - 1}.$$

Tushunarliki, $\sigma_0(n) = \tau(n)$, $\sigma_1(n) = \sigma(n)$.

119. 1).$\sigma_2(12) = \sigma_2(2^2 \cdot 3) = \frac{2^{2\cdot 3}-1}{2^2-1} \cdot \frac{3^{2\cdot 2}-1}{3^2-1} = \frac{63}{3} \cdot \frac{80}{8} = 210.$

2).$\sigma_2(18) = \sigma_2(2 \cdot 3^2) = \frac{2^{2\cdot 2}-1}{2^2-1} \cdot \frac{3^{2\cdot 3}-1}{3^2-1} = \frac{15\cdot 728}{3\cdot 8} = 5 \cdot 91 = 455.$

3).$\sigma_3(36) = \sigma_3(2^2 \cdot 3^2) = \frac{2^{3\cdot 3}}{2^3-1} \cdot \frac{3^{3\cdot 3}-1}{3^2-1} = \frac{511}{7} \cdot 91 = 73 \cdot 91 = 6643.$

4).$\sigma_2(16) = \sigma_2(2^4) = \frac{2^{2\cdot 5}-1}{2^2-1} = \frac{1023}{3} = 341.$

5).$\sigma_3(8) = \sigma_3(2^3) = \frac{2^{3\cdot 4}-1}{2^3-1} = \frac{4095}{7} = 585.$

120. 1).$\sigma(28) = \sigma(2^2 \cdot 7) = \frac{2^3-1}{2-1} \cdot \frac{4^2-1}{7-1} = 7 \cdot \frac{48}{6} = 7 \cdot 8 = 56 = 2 \cdot 28.$

Ya`ni $n = 28$ da $\sigma(n) = 2n$ tenglik o`rinli. Shuning uchin ham $n = 28$ – mukammal son.

2).$\sigma(469) = \sigma(2^4 \cdot 31) = \frac{2^5-1}{2-1} \cdot \frac{31^2-1}{31-1} = 31 \cdot 32 = 992 = 2 \cdot 496.$

3).$\sigma(8128) = \sigma(2^6 \cdot 127) = \frac{2^7-1}{2-1} \cdot \frac{127^2-1}{127-1} = 127 \cdot 128 = 16256 = 2 \cdot 8128.$

121. $\sigma(N) = \sigma(p^n) = \frac{p^{n+1}-1}{p-1} = p^n + (p^{n-1} + p^{n-2} + \cdots + p + 1) = p^n + \frac{(1-p^n)}{1-p} = p^n + \frac{p^n-1}{p-1} < 2p^n = 2 \cdot N$, ya`ni $\sigma(n) < 2N$.

122. $\sigma(N) = \sigma\left(p^\alpha \cdot q^\beta\right) = \frac{p^{\alpha+1}-1}{p-1} \cdot \frac{q^{\beta+1}-1}{q-1} < \frac{p^{\sigma+1}}{p-1} \cdot \frac{q^{\beta+1}}{q-1} = N \cdot \frac{p}{p-1} \cdot \frac{q}{q-1}$

Shart bo`yicha $p \geq 3, q \geq 5$. Shuning uchun ham $\sigma(N) < \frac{3}{2} \cdot \frac{5}{4} N = \frac{15}{8} N < 2N$. Demak $\sigma(N) < 2N$.

123. 1). Shartga ko`ra $\delta(n) = 5832$, 9-masalada istalgan formulaga asosan $\delta(n) = \sqrt{n^{\tau(n)}} = 5832 = 2^3 \cdot 3^6$. Demak, $n = 2^\alpha \cdot 3^\beta$ ko`rinishda bo`lishi kerak. Bulardan

$\sqrt{(2^\alpha \cdot 3^\beta)^{\tau(2^\alpha \cdot 3^\beta)}} = (2^\alpha \cdot 3^\beta)^{\frac{(\alpha+1)(\beta+1)}{2}} = 2^3 \cdot 3^6$, ya`ni

$$2^{\frac{\alpha(\alpha+1)(\beta+1)}{2}} = 2^3, 3^{\frac{\beta(\alpha+1)(\beta+1)}{2}} = 3^6$$

ga ega bo`lamiz. Bularga asosan $\alpha(\alpha + 1)(\beta + 1) = 6, \beta(\alpha + 1)(\beta + 1) = 12$ bundan

$\begin{cases}\alpha(\alpha+1)(\beta+1)=1\cdot3\cdot3\\(\alpha+1)\beta(\beta+1)=2\cdot2\cdot3\end{cases}\Rightarrow \alpha=1,\ \beta=2$ ekanligi kelib chiqadi va $n=2\cdot 3^2=18$ hosil bo`ladi.

2). Shartga ko`ra $\sqrt{n^{\tau(n)}}=3^{30}\cdot5^{40}$ , n ni $n=3^\alpha\cdot5^\beta$ ko`rinishda izlaymiz. U holda $(3^\alpha\cdot5^\beta)^{\frac{(\alpha+1)(\beta+1)}{2}}=3^{30}\cdot5^{40}$

ga ega bo`lamiz. Bundan $\alpha(\alpha+1)(\beta+1)=60;\ (\alpha+1)\beta(\beta+1)=80.$

Buni quyidagicha yozib olish mumkin:

$$\begin{cases}\alpha(\alpha+1)(\beta+1)=3\cdot4\cdot5\\\beta(\alpha+1)(\beta+1)=4\cdot4\cdot5\end{cases}\Rightarrow \alpha=3,\qquad \beta=4\,.$$

Demak $n=3^3\cdot5^4=27\cdot625=16875.$

124. N sonining barcha bo`luvchilarini o`sib borish tartibida joylashtirib chiqamiz: $1,d_1,d_2,\dots,\frac{N}{d_1},\frac{N}{d_1},\frac{N}{1}$. Bularning soni $(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)$ ta. Bularni 2 tadan olib, $1\cdot\frac{N}{1},d_1\cdot\frac{N}{d_1},d_2\cdot\frac{N}{d_2},\dots$ N ning barcha 2 ta ko`paytuvchi ko`rinishida ifodalanishlariga ega bo`lamiz. Ularning son $\frac{(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)}{2}$ ga teng, agar $N$ to`liq kvadrat bo`lmasa va $\frac{(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)+1}{2}$ ga teng, agar $N$ to`liq kvadrat bo`lsa. Bularni birlashtirsak, $N$ ni 2 ta ko`paytuvchi ko'rinishda ifodalashlar soni $\left[\frac{1+(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)}{2}\right]$ ga teng degan xulosaga kelamiz.

125. Bizda $N=2^\alpha\cdot3^\beta\cdot7^\gamma$. Bundan $\tau(N)=(\alpha+1)(\beta+1)(\gamma+1);$

$$\tau(5N)=(\alpha+1)(\beta+2)(\gamma+1)=(\alpha+1)(\beta+1)(\gamma+1)+8;$$
$$\tau(7N)=(\alpha+1)(\beta+1)(\gamma+2)=(\alpha+1)(\beta+1)(\gamma+1)+12;$$
$$\tau(8N)=(\alpha+4)(\beta+1)(\gamma+1)=(\alpha+1)(\beta+1)(\gamma+1)+18.$$

Bulardan $(\alpha+1)(\gamma+1)(\beta+2-\beta-1)=8$, ya`ni

$\begin{cases}(\alpha+1)(\gamma+1)=8\\(\alpha+1)(\beta+1)=12\\(\beta+1)(\gamma+1)=6\end{cases}$ ga ega bo`lamiz.

$$(\alpha+1)(\beta+1)(\gamma+1)=\sqrt{8\cdot12\cdot6}=\sqrt{16\cdot36}=4\cdot6=4\cdot3\cdot2.$$

Bulardan $(\alpha+1)=4,\ \ \alpha=3;(\beta+1)=3,\ \ \beta=2;(\gamma+1)=2,\ \gamma=1$

va $N=2^3\cdot5^2\cdot7=1400.$

126. Masalaning sharti bo`yicha $N=2^x\cdot3^y\cdot5^z$va$\frac{N}{2}=2^{x-1}\cdot3^y\cdot5^z$, $\frac{N}{3}=2^x\cdot3^{y-1}\cdot5^z$, $\frac{N}{5}=2^x\cdot3^y\cdot5^{z-y}$. Bulardan

$$\begin{cases}\tau\left(\dfrac{N}{2}\right)=\tau(N)-30\\ \tau\left(\dfrac{N}{3}\right)=\tau(N)-35\\ \tau\left(\dfrac{N}{5}\right)=\tau(N)-42\end{cases} \Rightarrow \begin{cases}x(y+1)(z+1)=(x+1)(y+1)(z+1)-30\\ y(x+1)(z+1)=(x+1)(y+1)(z+1)-35\\ z(x+1)(y+1)=(x+1)(y+1)(z+1)-42.\end{cases}$$

Oxirgi sistemani quyidagicha yozib olish mumkin.

$$\begin{cases}(y+1)(z+1)=30\\ (x+1)(z+1)=35\\ (x+1)(y+1)=42.\end{cases}$$

Buni tanlash usuli bilan yechamiz:$(x+1)^2(y+1)^2(z+1)^2=2^2\cdot3^2\cdot5^2\cdot7^2\Rightarrow$ $(x+1)(y+1)(z+1)=2\cdot3\cdot5\cdot7$,bu yerda $(x+1)(y+1)=42$ bo`lishi kerak, shuning uchun $x+1=7\ \ y+1=6$, u holda $(x+1)(z+1)=35$ dan $(z+1)=5$ kelib chiqadi va bu yechimlar $(y+1)(z+1)=30$ tenglamani qanoatlantiradi. Shunday qilib $x=6,\ y=5,\ z=4$va$N=2^6\cdot3^5\cdot5^4=64\cdot243\cdot625=9720000$.

127. $2^{\alpha+1}-1$ tub son bo`lsin, u holda $N=2^{\alpha}(2^{\alpha+1}-1)$ ning mukammal son ekanligini ko`rsatamiz. $N=2^{\alpha+1}-1=p$ deb olsak,

$\sigma(N)=\sigma(2^{\alpha}\cdot p)=\frac{2^{\alpha+1}-1}{2-1}\cdot\frac{p^2-1}{p-1}=(2^{\alpha+1}-1)(p+1)=(2^{\alpha+1}-1)(2^{\alpha+1})=2N$, ya`ni N −mukammal son.

128. Buni isbotlash uchun har qanday juft mukammal sonning $2^{\alpha}(2^{\alpha+1}-1)$ ko`rinishida ifodalanishini ko`rsatish yetarli. Bunda $2^{\alpha+1}-1$ tub son. Faraz qilaylik, $N=2^{\alpha}\cdot q, (q;2)=1$ juft son mukammal son bo`lsin, ya`ni u uchun

$\sigma(N)=2N$ tenglik bajarilsin. Bundan $\sigma(2^{\alpha}q)=2^{\alpha+1}q$ yoki

$$\frac{2^{\alpha+1}-1}{2-1}\sigma(q)=2^{\alpha+1}q.$$

Bu yerdan $\sigma(q)=\frac{2^{\alpha+1}}{2^{\alpha+1}-1}q$ va q soni $2^{\alpha+1}-1$ ga bo'linishi kerak. U holda $q=(2^{\alpha+1}-1)k$ va $\sigma(q)=2^{\alpha+1}k$ bo'ladi. Bu yerdan k va $(2^{\alpha+1}-1)k$ lar q ning bo'luvchilari bo'lib, ularning yig'indisi uchun $2^{\alpha+1}k=\sigma(q)$ bajariladi. U holda q ning boshqa bo'luvchilari yo'q bo'lishi kerak. Demak, $q=(2^{\alpha+1}-1)k$ soni tub son ekan, ya'ni $k=1$ va $2^{\alpha+1}-1$ tub son.

129. $N=2^{\alpha}\cdot p_1p_2$ deb olsak, masalaning shartiga ko'ra $\sigma(N)=\sigma(2^{\alpha}p_1p_2)=\frac{2^{\alpha+1}-1}{2-1}\cdot\frac{{p_1}^2-1}{p_1-1}\cdot\frac{{p_2}^2-1}{p_2-1}=$

$$(2^{\alpha+1}-1)(p_1+1)(p_2+1)=3N=3\cdot2^{\alpha}p_1p_2, \qquad (1)$$

bu yerda $p_1>p_2$ toq sonlar.

Agar $\alpha=0$ bo'lsa, $(p_1+1)(p_2+1)=3p_1p_2$ yoki $p_1+p_2+1=2p_1p_2$. Bu oxirgi tenglik o'rinli emas, chunki chap toq son o'ng tomoni esa juft son. Demak,

$\alpha \neq 0$ bo'lsa. $\alpha = 1$ bo'lsin. U holda $3(p_1+1)(p_2+1) = 6p_1p_2$ yoki $p_1+p_2+1 = p_1p_2$, ya'ni $p_1+1 = p_2(p_1-1)$. Bunda $p_1 - 1$ juft son, ya'ni $p_1 - 1 = 2n$, u holda $2n+2 = 2np_2$ bundan $n+1 = np_2 \rightarrow n(p_2-1) = 1 \rightarrow n = 1,\ p_2 = 2$. Bunday bo'lishi uchun ham mumkin emas chunki masalaning shartida p_2 – toq tub son. Demak, $\alpha \neq 1$. $\alpha = 2$

$\alpha = 2$ bo'lsin. Bu holda (1) dan $7(p_1+1)(p_2+1) = 12p_1p_2 \rightarrow 7p_1 + 7p_2 + 7 = 5p_1p_2 \rightarrow 7(p_1+p_2+1) = 5p_1p_2$. Bundan $p_1 = 7$ (yoki $p_2 = 7$) va $8 + p_1 = 5p_2 \rightarrow p_2 = 2$ yoki ($p_1 = 2$). Bunday bo'lishi ham mumkin emas.

Demak, $\alpha \neq 2, \alpha = 3$ bo'lsin. Bu holda (1) dan $15(p_1+1)\,(p_2+1) = 24p_1p_2 \rightarrow 5(p_1+1)(p_2+1) = 8p_1p_2$. Bundan $5(p_1+p_2+1) = 3p_1p_2$va$p_1 = 5$ (yoki $p_2 = 5$) hamda $6 + p_2 = 3p_2 \rightarrow p_2 = 3$ $(p_1 = 3)$. Shunday qilib berilgan masalaning shartini qanoatlantiruvchi eng kichik natural son $N = 2^3 \cdot 5 \cdot 3 = 120$ ekan.

130. Faraz qilaylik , $N = {p_1}^{\alpha_1}{p_2}^{\alpha_2} \cdots {p_k}^{\alpha_k}$ bo'lsin. U holda

$$\tau(N) = (\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)$$

a). Agar $\tau(N)$ toq son bo'lsa, $(1+\alpha_i)$ $(i = 1,2,\ldots,k)$ ko'paytuvchilarning har biri toq son bo'lishi kerak, ya'ni $\alpha_i(i = 1,2,\ldots,k)$ lar juft bo'lishi kerak. Bu esa N butun sonning to'la kvadratiga teng degani.

b). Aksincha, agar N biror sonning kvadratiga teng bo'lsa, α_i $(i = 1,2,\ldots,k)$ lar juft sonlar $\alpha_i + 1$ lar esa toq natural sonlar bo'lishi kerak. U holda $\tau(N) = \prod_{i=1}^{k}(1+\alpha_i)$ ham toq son bo'ladi.

II.4-§.

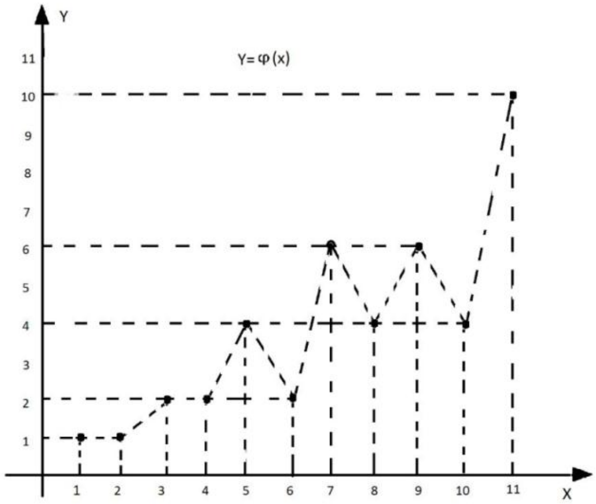
131. Eyler funksiyasi$y = \varphi(x)$ ning qiymatlari jadvalini tuzamiz.

x	1	2	3	4	5	6	7	8	9	10	11
$\varphi(x)$	1	1	2	2	4	2	6	4	6	4	10

Bu qiymatlarni (nuqtalarni) Dekart koordinatalar sistemasida belgilab chiqib uzlukli chiziq bilan belgilab chiqsak, $y = \varphi(x)$ funksiya`ning o'zgarishini xarakterlovchi chiziqqa ega bo'lamiz.

132. 1). $\varphi(125) = \varphi(5^3) = 5^3 - 5^2 = 100;$

2). 1000 ni tub ko'paytuvchilarga ajratib $\varphi(x)$ ning multiplikativligidan foydalanamiz.$\varphi(1000) = \varphi(2^3 \cdot 5^3) = \varphi(2^3) \cdot \varphi(5^3) = (2^3 - 2^2)(5^3 - 5^2) = 4 \cdot 100 = 400;$



8-shakl

3). $\varphi(180) = \varphi(18 \cdot 10) = \varphi(2^2 \cdot 3^2 \cdot 5) = \varphi(2^2) \cdot \varphi(3^2) \cdot \varphi(5) = (2^2 - 2) \cdot (3^2 - 3)(5 - 1) = 2 \cdot 6 \cdot 4 = 48;$

4). $\varphi(360) = \varphi(2^3 \cdot 3^2 \cdot 5) = (2^3 - 2^2)(3^2 - 3)(5 - 1) = 4 \cdot 6 \cdot 4 = 64;$

5). $\varphi(1440) = \varphi(12^2 \cdot 10) = 4(2^5 \cdot 3^2 \cdot 5) = (2^5 - 2^4)(3^2 - 3)(5 - 1) = 16 \cdot 6 \cdot 4 = 384;$

6). $\varphi(1890) = \varphi(2)\varphi(3^3)\varphi(5)\varphi(7) = (2 - 1)(3^3 - 3^2) \cdot 4 \cdot 6 = 18 \cdot 24 = 432;$

7). $\varphi(11^3) = 11^3 - 11^2 = 121 \cdot 11 = 1331;$

8). $\varphi(23^2) = 23^2 - 23 = 506;$

9). $\varphi(12 \cdot 17) = \varphi(12) \cdot \varphi(17) = \varphi(2^2 \cdot 3) \cdot 16 = 16 \cdot (2^2 - 2) \cdot 2 = 32 \cdot 2 = 64;$

10). $\varphi(24 \cdot 28 \cdot 45) = \varphi(2^3 \cdot 3 \cdot 2^2 \cdot 7 \cdot 3^2 \cdot 5) = \varphi(2^5 \cdot 3^3 \cdot 5 \cdot 7) = (2^5 - 2^4)(3^3 - 3^2) \cdot 4 \cdot 6 = 24 \cdot 16 \cdot 18 = 6912.$

133. $\frac{a}{m}$; $a \leq m$; $(a; m) = 1$, tarifiga ko’ra bunday kasrlar soni $\varphi(m)$ ta.

134. Berilgan oraliqda jami 120 ta natural son bor. Shulardan 120 bilan o’zaro tublari $\varphi(120) == \varphi(2^3 \cdot 3 \cdot 5) = (2^3 - 2^2) \cdot 2 \cdot 4 = 32$ ta. Shuning uchun ham izlanayotgan natural sonlarning soni $120 - 32 = 88$ ta.

135.a). $\varphi(2^\alpha) = 2^\alpha - 2^{\alpha-1} = 2^{\alpha-1}(2 - 1) = 2^{\alpha-1}$.

b). $\varphi(p^\alpha) = p^\alpha - p^{\alpha-1} = p^{\alpha-1}(p - 1) = p^{\alpha-1}\varphi(p)$.

c). $\varphi(m^\alpha) = m^{\alpha-1}\varphi(m)$ni isbotlash uchun m ning kanonik yoyilmasi $m = p_1^{\gamma_1} p_2^{\gamma_2} \dots p_k^{\gamma_k}$ ni qaraymiz. Bundan $m^\alpha = p_1^{\alpha \cdot \gamma_1} p_2^{\alpha \cdot \gamma_2} \dots p_k^{\alpha \cdot \gamma_k}$ va

$$\varphi(m^\alpha) = m^\alpha \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = m^{\alpha-1} \cdot m \cdot \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$= m^{\alpha-1} \cdot \varphi(m).$

136. Agar $(m, 2) = 1$ bo'lsa, Eyler funksiyasi multiplikativ bo'lgani uchun $\varphi(2m) = \varphi(2)\varphi(m) = \varphi(m)$.

Agar $(m, 2) > 1$ bo'lsa,$(m, 2) = 2$ bo'ladi. Bu holda $m = 2^{\alpha} \cdot m_1$, $(m_1; 2) = 1$deb yozib olamiz va $\varphi(2m) = \varphi(2^{\alpha+1} \cdot m_1) = \varphi(2^{\alpha+1})\varphi(m_1) = 2^{\alpha}\varphi(m_1) = 2\varphi(2^{\alpha}) \cdot \varphi(m_1) = 2\varphi(2^{\alpha} \cdot m_1) = 2\varphi(m)$.

137.a). $\varphi(4n + 2) = \varphi\big(2(2n + 1)\big) = \varphi(2)\varphi(2n + 1) = \varphi(2n + 1)$.

b). Agar $(n, 2) = 1$ bo'lsa, u holda $(n, 4) = 1$ bo'ladi. Shuning uchun ham $\varphi(4n) = \varphi(4)\varphi(n) = 2\varphi(n)$.

Agarda $n = 2^{\alpha} \cdot k$, $(k; 2) = 1$ bo'lsa, u holda $\varphi(4n) = \varphi(2^{\alpha+2} \cdot k) = \varphi(2^{\alpha+2}) \cdot \varphi(k) = 2^{\alpha+1} \cdot \varphi(k) = 2\varphi(2^{\alpha+1}) \cdot \varphi(k) = 2\varphi(2^{\alpha+1} \cdot k) = 2\varphi(2n)$.

138. a).$\varphi(5^x) = 100 \rightarrow 5^x - 5^{x-1} = 100 \rightarrow 5^{x-1} \cdot 4 = 100 \rightarrow 5^{x-1} = 5^2 \rightarrow x = 3$.

b).$\varphi(7^x) = 294 \Rightarrow 7^{x-1} \cdot 6 = 294 \Longrightarrow 7^{x-1} = 49 \Longrightarrow 7^{x-1} = 7^2 \Rightarrow x - 1 = 2 \Rightarrow x = 3$.

c). $\varphi(p^x) = p^{x-1} \Rightarrow p^{x-1}(p - 1) = p^{x-1}$. Bu tenglama $p > 2$ bo'lsa yechimga ega emas. $p = 2$ da ixtiyoriy natural son x tenglamaning yechimi bo'ladi.

d). $\varphi(3^x \cdot 5^y) = 600 \Rightarrow \varphi(3^x) \cdot \varphi(5^y) = 600 \Rightarrow 3^{x-1} \cdot 2 \cdot 5^{y-1} \cdot 4 = 600 \Rightarrow 3^{x-1} \cdot 5^{y-1} = 75 \Rightarrow 3^{x-1} \cdot 5^{y-1} = 3 \cdot 5^2 \Rightarrow x - 1 = 1;\ y - 1 = 2 \Rightarrow x = 2;\ y = 3$.

139. $m = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$ bo'lsin u holda

$$\varphi(m) = p_1^{\varphi_1 - 1}(p_1 - 1)p_2^{\varphi_2 - 1}(p_2 - 1) \dots p_k^{\varphi_k - 1}(p_k - 1)$$

bo'ladi. Bu yerda har bir toq p_k ko'paytuvchiga juft $p_i - 1$ ko'paytuvchi mos keladi va $\varphi(m)$ juft son bo'ladi. Agarda $m = 2^{\alpha} > 3$ ko'rinishida bo'lsa, $\varphi(m) = \varphi(2^{\alpha}) = 2^{\alpha-1}$juft son bo'ladi.

140. $x = m$ soni $\varphi(x) = a$ ning ildizi bo'lsa, u holda $\varphi(m) = a$ bajariladi. Bu holda $\varphi(2m) = \varphi(m) = a$; chunki shartga ko'ra $(2; m) = 1$. Bu yerdan $x = 2m$ soni ham berilgan tenglamaning ildizi ekanligi kelib chiqadi.

141. m ning ham n ning ham bo'luvchisi bo'lgan p tub soniga $\varphi(mn)$ da bitta $(1 - \frac{1}{p}) < 1$ ko'paytuvchi mos keladi. $\varphi(m)\,\varphi(n)$da esa ikkita shunday ko'paytuvchi $\left(1 - \frac{1}{p}\right)^2$ mos keladi. $\left(1 - \frac{1}{p}\right)^2 < 1 - \frac{1}{p}$ bo'lgani uchun $(m; n) > 1$ bo'lsa, $\varphi(m)\varphi(n) < \varphi(mn)$ bo'ladi. Xususiy holda $\varphi^2(m) \leq \varphi(m^2)$, bu yerda tenglik faqat $m = 1$ da bajariladi.

142. $q_1, q_2, \dots \dots \dots . q_t$ lar faqat m ning kanonik yoyilmasiga kiruvchi tub sonlar, $p_1, p_2, \dots, p_k$ lar$m$ va n larning ikkalasining ham kanonik yoyilmasiga kiruvchi tub sonlar,$l_1, l_2, \dots, l_s$ lar faqat n ning kanonik yoyilmasiga kiruvchi tub sonlar bo'lsinlar. U holda

$$\varphi(m\cdot n) = mn\prod_{i=1}^{t}\left(1-\frac{1}{q_i}\right)\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\prod_{i=1}^{s}\left(1-\frac{1}{l_i}\right) =$$

$$\left\{m\prod_{i=1}^{t}\left(1-\frac{1}{q_i}\right)\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\right\}\cdot\left\{n\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\prod_{i=1}^{s}\left(1-\frac{1}{l_i}\right)\right\}\frac{d}{d\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)}$$

$$= \varphi(m)\varphi(n)\cdot\frac{d}{\varphi(d)}.$$

Izoh: Agar $\frac{d}{\varphi(d)} \geq 1$ ekanligini inobatga olsak, isbotlangan tenglikdan 11-misoldagi munosabat to'gridan to'g'ri kelib chiqadi.

143. $[m;n] = \frac{mn}{(m;n)} \to \mu\cdot\delta = mn$ bo'lgani uchun $\varphi(mn) = \varphi(\mu\delta) =$
$\varphi(\mu)\varphi(\delta)\cdot\frac{d}{\varphi(d)} = \varphi(\mu)\cdot\varphi(\delta)$. $\quad d = (\mu;\delta) = \left(\frac{mn}{(m;n)}; mn\delta\right) = 1$

144. Yig'indini bevosita $\varphi(p^{\propto}) = p^{\propto} - p^{\propto-1}$ formuladan foydalanib hisoblaymiz: $\varphi(1) + \varphi(p) + \varphi(p^2) + \cdots \dots . + \varphi(p^{\propto}) =$

$$= 1 + p - 1 + p^2 - p + \cdots + p^{\propto} - p^{\propto-1} = p^{\propto}.$$

145. Agar a natural sonning kanonik yoyilmasi $a = p_1^{\propto_1}p_2^{\propto_2}\dots p_k^{\propto_k}$ va θ multiplikativ funksiya bo'lsa, u holda

$$\sum_{d\backslash a}\theta(d) = \left(1 + \theta(p_1) + \theta(p_1^2) + \cdots + \theta\left(p_1^{\propto_1}\right)\right)\times$$

$$\times\left(1 + \theta(p_2) + \theta(p_2^2) + \cdots + \theta\left(p_2^{\propto_2}\right)\right)\times\dots$$

$$\times\left(1 + \theta(p_k) + \theta(p_k^2) + \cdots + \theta\left(p_k^{\propto_k}\right)\right) \qquad (1)$$

ayniyat o'rinli. Haqiqatan ham (1) ning o'ng tomonidagi qavs ichidagi ifodalarni ko'paytirib, qavslarni ochsak va $\theta(a)$ ning multiplikativligidan foydalanib quyidagiga ega bo'lamiz:

$$\prod_{i=1}^{k}\left(1 + \theta(p_i) + \theta\left(p_i^2\right) + \cdots + \theta\left(p_i^{\propto_1}\right)\right)$$

$$= 1 + \theta(p_1) + \theta(p_2) + \cdots + \theta(p_k) + \cdots + \theta\left(p_1^{\propto_1}\right)\theta\left(p_2^{\propto_2}\right)\dots\theta\left(p_k^{\propto_k}\right)$$

$$= \sum_{\beta_1}^{\propto_1}\sum_{\beta_2}^{\propto_2}\dots\sum_{\beta_k}^{\propto_k}\theta(p_1^{\beta_1}p_2^{\beta_2\cdot}\dots p_k^{\beta_k}) = \sum_{d/a}\theta(d).$$

Bu yerda biz $a = p_1^{\propto_1} p_2^{\propto_2} \dots p_k^{\propto_k}$ sonining ixtiyoriy bo'luvchisi d ni $d = p_1^{\propto_1} p_2^{\propto_2} \dots p_k^{\propto_k}$, $0 \le \beta_i \le \propto_i$, $i = 1,2,\dots,k$ ko'rinishidagi ifodalash mumkinligidan foydalandik. Endi (1) da $\theta(d) = \varphi(d)$ deb olamiz. U holda (1) dan

$$\sum_{d/a} \varphi(d) = \prod_{i=1}^{k} \left(1 + \varphi(p_i) + \varphi\left(p_i^2\right) + \dots + \varphi\left(p_i^{\propto_i}\right)\right)$$

$$= \prod_{i=1}^{k} \left(1 + p_i - 1 + p_i^2 - p_i + \dots + p_i^{\propto_i} - p_i^{\propto_i - 1}\right) = \prod_{i=1}^{k} p^{\propto_i} = a.$$

146. Avvalo, agar $(a; m) = 1$ bo'lsa, u holda $(a; m - a) = 1$ ekanligini ko'rsatamiz. $(a; m - a) = d > 1$ bo'lsin deb faraz etaylik. U holda $a = da_1, m - a = d \cdot t$ deb yoza olamiz. Bu yerdan $m = a + dt = d(a_1 + t)$ ga, ya`ni $(a; m) = d > 1$ga ega bo'lamiz. Bu esa $(a; m) = 1$ ga qarama-qarshidir.

Endi m dan kichik va m bilan o'zaro tub sonlarni o'sib borish tartibida yozib chiqamiz:

$$1, \quad a_1, \quad a_2, \quad \dots, m - a_2, \quad m - a_1, \quad m - 1. \qquad (2)$$

Bularning soni $\varphi(m)$ ta. Bu yerda har bir a_i ga birta $m - a_i$ soni mos keladi. Ularning yig'indisi $a_i + (m - a_i) = m$ ga teng. Bunday juftliklar soni $\frac{1}{2}\varphi(m)$ ta. Shunday qilib (2) dagi sonlar yig'indisini S deb belgilasak, $S = \frac{1}{2} m\varphi(m)$ ga ega bo'lamiz.

147. 16 – masalada isbotlangan formuladan foydalanib,

$$S_1 = \frac{1}{2} p\varphi(p) = \frac{1}{2} p(p - 1); \quad S_2 = \varphi(p^2) = p^2 - p = p(p - 1)$$

$$\frac{S_2}{S_1} = \frac{p(p-1)}{\frac{1}{2} p(p-1)} = 2$$

topamiz.

148. 1). $\varphi(x) = p - 1$, $x = p^{\propto} \cdot y$, $(p; y) = 1$ deb olamiz. $\varphi(x) = \varphi(p^{\propto} \cdot y) = p^{\propto - 1}(p - 1)\varphi(y) = p - 1$ yoki$p^{\propto - 1} \cdot \varphi(y) = 1$ hosil bo'ladi. Bundan $\alpha = 1$, $\varphi(y) = 1$ yoki $y = 1$ va $y = 2$. $\alpha = 1$ va $y = 1$ da $x = p = 2$ tenglama bitta yechimi, $p > 2$ bo'lsa, tenglama 2 ta p va $2p$ yechimga ega bo'ladi.

2). $\varphi(x) = 14 \Rightarrow \varphi(x) = 2 \cdot 7dan\varphi(x) \vdots 7$ ya`ni x ning yoyilmasida 7 qatnashishi kerak, u holda $\varphi(x) \vdots 6$, lekin $\varphi(x)$ ifoda 6 bo'linmaydi. Demak tenglama yechimga ega emas.

3). $\varphi(x) = 8 = 2^3 \Rightarrow \varphi(x) \vdots 2$, $\varphi(x) \vdots 4$, $\varphi(x) \vdots 8$.

a) $x = 2^{\propto} \cdot 3^{\beta} \cdot 5^{\gamma}$ bo'lsin u holda $\varphi(x) = (2^{\propto} - 2^{\propto - 1})(3^{\beta} - 3^{\beta - 1})(5^{\gamma} - 5^{\gamma - 1}) = 8 \Rightarrow 2^{\propto - 1} \cdot 3^{\beta - 1} \cdot 5^{\gamma - 1} \cdot 2 \cdot 4 = 8 \Rightarrow 2^{\propto - 1} \cdot 3^{\beta - 1} \cdot 5^{\gamma - 1} = 1 \Rightarrow \alpha = 1$; $\beta = 1$; $\gamma = 1$ va $x = 30$, $\varphi(30) = 4 \cdot 2 = 8$;

$b)\ x=2^{\propto}\cdot 3^{\beta}\ \Rightarrow\quad \varphi(x)=2^{\propto-1}\cdot 3^{\beta-1}\cdot 2=8\ \Rightarrow 2^{\propto-1}\cdot 3^{\beta-1}=4\Rightarrow \alpha=3,\ \beta=1\Rightarrow\ x=8\cdot 3=24.$

c). $x=2^{\propto}\cdot 5^{\gamma}\Rightarrow \varphi(x)=2^{\propto-1}\cdot 5^{\gamma-1}\cdot 4=8\Rightarrow 2^{\propto-1}\cdot 5^{\gamma-1}=2\ \Rightarrow\ \gamma=1;\ \propto=2\Rightarrow x=4\cdot 5=20.$

$d).\ x=3^{\beta}\cdot 5^{\propto}\Rightarrow\ \varphi(x)=3^{\beta-1}\cdot 5^{\gamma-1}\cdot 8=8\Rightarrow 3^{\beta-1}\cdot 5^{\gamma-1}=1\Rightarrow\ \beta=1,$

$\gamma=1\Rightarrow x=15.$

$e).\ x=2^{\propto};\ \ \varphi(x)=2^{\propto-1}=8=2^3\Rightarrow\ \propto=4\ \Rightarrow x=16.$

Demak, **javob** $x=15;16;20;24;30.$

4). $\varphi(x)=12=2^2\cdot 3$. Mumkin bo'lgan hollarni qarab chiqamiz.

a) $x=2^2\cdot 3^{\beta}\cdot 7^{\gamma}\Rightarrow \varphi(x)=2^{\propto-1}\cdot 2\cdot 3^{\beta-1}\cdot 6\cdot 7^{\gamma-1}=12\ \Rightarrow\ 2^{\propto-1}\cdot 3^{\beta-1}\cdot 7^{\gamma-1}=1\Rightarrow\propto=1,\beta=1,\gamma=1\ \Rightarrow\ x=42.$

b) $x=2^{\propto}\cdot 3^{\beta}\Rightarrow \varphi(x)=2^{\propto-1}\cdot 2\cdot 3^{\beta-1}=12\ \Rightarrow\ 2^{\propto}\cdot 3^{\beta-1}=2^2\cdot 3\Rightarrow\propto=2;\ \beta=2\ \Rightarrow\ x=36.$

c)$x=2^{\propto}\cdot 7^{\gamma}\Rightarrow\ \ \varphi(x)=2^{\propto-1}\cdot 6\cdot 7^{\gamma-1}=12\Rightarrow 2^{\propto-1}\cdot 7^{\gamma-1}=2;\ \gamma=1;\ \propto=2\Rightarrow x=28.$

d) $x=3^{\beta}\cdot 7^{\gamma}\ \Rightarrow \varphi(x)=3^{\beta-1}\cdot 2\cdot 6\cdot 7^{\propto-1}=12\Rightarrow 3^{\beta-1}\cdot 7^{\gamma-1}=1\Rightarrow\ \ \beta=1,\gamma=1\ \Rightarrow x=21.$

e) $x=2^{\propto}\cdot 13^{\delta}\Rightarrow \varphi(x)=2^{\propto-1}\cdot 13^{\delta-1}\cdot 12=12\Rightarrow 2^{\propto-1}\cdot 13^{\delta-1}=1\Rightarrow \delta=1,\ \propto=1\ \Rightarrow x=26.$

g)$x=13^{\delta}\ \Rightarrow\ \varphi(x)=13^{\delta-1}\cdot 12=12\Rightarrow 13^{\delta-1}=1\Rightarrow \delta=1\Rightarrow x=13.$

Javob: $x=\ 5;13;21;26;28;36;42.$

149. 1). $\varphi(x)=2^{\propto};\ \ x=2^k\cdot 3^l\cdot 5^m$

a) $x=2^k\Rightarrow \varphi(x)=2^{k-1}=2^{\propto}\ \Rightarrow k-1=\propto\Rightarrow\ \ k=\propto+1\Rightarrow x=2^{\propto+1}.$

b) $x=2^k\cdot 5^m\Rightarrow \varphi(x)=2^{k-1}\cdot 5^{m-1}\cdot 4=2^{\propto}\Rightarrow\ \ m-1=0; m=1,\ \ k+1=\propto,\ k=\propto-1\Rightarrow x=2^{\alpha-1}\cdot 5.$

c) $x=2^k\cdot 3^l\Rightarrow\ \varphi(x)=2^{k-1}\cdot 3^{l-1}\cdot 2=2^{\propto}\Rightarrow\ k=\propto,\ l=1\Rightarrow\ x=2^{\propto}\cdot 3.$

d)$x=3^l\cdot 5^m\ \Rightarrow \varphi(x)=3^{l-1}\cdot 2\cdot 5^{m-1}\cdot 4=2^{\propto}\Rightarrow\propto=3\,;l=1\,;\ m=1\Rightarrow$

$x=15\,.$

e) $x=2^k\cdot 3^l\cdot 5^m\Rightarrow \varphi(x)=2^{k-1}\cdot 3^{l-1}\cdot 2\cdot 5^{m-1}\cdot 4=2^{\propto}\Rightarrow$

$2^{k+2}\cdot 3^{l-1}\cdot 5^{m-1}=2^{\propto}\Rightarrow k=\propto-2\,;\ \ l=1\,;\ \ m=1\Rightarrow\ \ x=2^{\propto-2}\cdot 15.$

Javob: $x=2^{\propto+1};\ 2^{\alpha-1}\cdot 5;\ \ 2^{\propto}\cdot 3;\ 15;\ \ 2^{\propto-2}\cdot 15.$

2). $\varphi(p^x)=6\cdot p^{x-2}\ \Rightarrow\ p^{x-1}(p-1)=6p^{x-2}\ \Rightarrow\ \ p(p-1)=6\Rightarrow\ p=3$ ixtiyoriy x qanoatlantiradi $p\neq 3$ da yechimi yoq.

150. $\varphi(m)=3600,$ bunda $m=3^{\propto}\cdot 5^{\beta}\cdot 7^{\gamma}.\ 3600=2^4\cdot 3^2\cdot 5^2\Rightarrow\ \ \varphi(m)=3^{\propto-1}\cdot 2\cdot 5^{\beta-1}\cdot 4\cdot 7^{\gamma-1}\cdot 6=2^4\cdot 3^2\cdot 5^2\ \Rightarrow 3^{\propto-1}\cdot 5^{\beta-1}\cdot 7^{\gamma-1}=3\cdot 5^2\Rightarrow\propto-1=1\,;\ \propto=2\,;\ \beta-1=2\,;\ \ \beta=3\,;\ \ \gamma=1\ \Rightarrow\ m=3^2\cdot 5^3\cdot 7=7875.$

151. $\varphi(x) = 120,\ \ x = p_1 \cdot p_2$ va $p_1 - p_2 = 2 \Rightarrow \varphi(x) = (p_1 - 1)(p_2 - 1) = 120\ ;\ p_1 = p_2 + 2\ \Rightarrow (p_2 + 1)(p_2 - 1) = 120\ \ \Rightarrow\ \ p_2 = 11\ ;\ p_1 = 13\ ;\ \ x = 143.$

152. Masalaning shartiga ko'ra: $\varphi(m) = 11424\ ;\ \ m = p_1^2 \cdot p_2^2$. Bulardan va $11421 = 2^5 \cdot 3 \cdot 7 \cdot 17$ ekanligidan $\varphi(p_1^2 \cdot p_2^2) = (p_1^2 - p_1)(p_2^2 - p_2) = p_1(p_1 - 1)p_2(p_2 - 1) = 2^5 \cdot 3 \cdot 7 \cdot 17 = 16 \cdot 17 \cdot 7 \cdot 6$ hosil bo'ladi. Bundan esa $p_1 = 17\ ;\ p_2 = 7;\ \ m = (p_1 \cdot p_2)^2 = 119^2 = 14161$ ni hosil qilamiz.

153. *a).*$\varphi(x) = \varphi(px)$da agar $p = 2$ bo$'$lsa $\varphi(x) = \varphi(2)\varphi(x) = \varphi(x),\ \ x$ ning barcha toq qiymatlari qanoatlantiradi, chunki bu holda
$(2; x) = 1\ ; p \geq 3$ bo$'$lsa $x = p_1^{\propto_1} \cdot p_2^{\propto_2} \dots . p_k^{\propto_k}$desak, $\varphi(x) = \left(p_1^{\propto_1} - p_1^{\propto_1 - 1}\right)\left(p_2^{\propto_2} - p_2^{\propto_2 - 1}\right) \dots \left(p_k^{\propto_k} - p_k^{\propto_k - 1}\right).$

Agar $(p; x) = 1$ bo'lsa, $\varphi(px) = \varphi(x)(p - 1) \neq \varphi(x)$. Agarda $(p; x) = p;\ (p = p_i)$ bolsa, $px = p_1^{\propto_1} \cdot p_2^{\propto_2} \dots p_i^{\propto_i + 1} \cdots p_k^{\propto_k}$ va $\varphi(px) = \left(p_1^{\propto_1} - p_1^{\propto_1 - 1}\right)\left(p_2^{\propto_2} - p_2^{\propto_2 - 1}\right) \dots \left(p_i^{\propto_i + 1} - p_i^{\propto_i}\right) \cdots \left(p_k^{\propto_k} - p_k^{\propto_k - 1}\right) = p_i \cdot \varphi(x) \neq \varphi(x)$
Demak, $p = 2$ da berilgan tenglamani x ning barcha toq qiymatlari qanoatlantiradi; $p \geq 3$ bo'lsa tenglama yechimga ega emas.

b). $\varphi(px) = p\varphi(x)$. 1). Agar $(x; p) = 1$ bo'lsa, $\varphi(px) = \varphi(p)\varphi(x) = (p - 1)\varphi(x) \Rightarrow \varphi(x)(p - 1) = p\varphi(x)\ \ \Rightarrow\ \ \varphi(x) = 0.$ Demak yechimi yo'q.

2). $x = p_1^{\propto_1} \cdot p_2^{\propto_2} \dots . p_k^{\propto_k}$ bo'lsa, $(p; x) = p\ ;\ \ (p = p_i)\ ;\ \ \varphi(px) = p_i \cdot \varphi(x) = p\varphi(x)$. Demak bu holda berilgan tenglamani x ning p ga karra natural qiymatlari qanoatlantiradi.

c).$\varphi(p_1 \cdot x) = \varphi(p_2 \cdot x);\ p_1 \neq p_2\ ;\ \ x = q_1^{\propto_1} \cdot q_2^{\propto_2} \dots . q_k^{\propto_k}\ ;$

1) $p_1 \neq q_i$; ya`ni$(x; p) = 1$ bo$'$lsa $\varphi(p_1\, x) = (p_1 - 1)\, \varphi(x)$, agarda $(x; p_1) = p_1$bo$'$lsa, $\varphi(p_1 x) = p_1\varphi(x)$.

2) $p_2 \neq q_i$ya`ni $(x; p_2) = 1\ \Rightarrow\ \varphi(p_2 x) = (p_2 - 1)\varphi(x)$; agarda $(x;\ p_2) = p_2\ ;\ \varphi(p_2 x) = p_2\varphi(x)$.

Bulardan quyidagi tenglamalarni hosil qilamiz:

1) $(p_1 - 1)\varphi(x) = (p_2 - 1)\varphi(x)$; 3) $(p_2 - 1)\varphi(x) = p_1\varphi(x)$;

2) $(p_1 - 1)\varphi(x) = p_2\varphi(x)$; 4) $p_2\varphi(x) = p_1\varphi(x)$.

1) dan $(p_1 - 1 - p_2 + 1)\varphi(x) = 0 \Rightarrow (p_1 - p_2)\varphi(x) = 0\ \Rightarrow\ p_1 - p_2 \neq 0.$

Demak, $\varphi(x) = 0$ bo'lishi kerak bu holda tenglama yechimga ega emas.

2) dan $(p_1 - p_2 - 1)\varphi(x) = 0\ ;\ \ p_1 = p_2 + 1\ ;\ p_1 = 3\ ;\ p_2 = 2$ da tenglamani x ning berilgan shartilarini qanoatlantiruvchi, ya`ni $(x; 3) = 1$ va $(x; 2) = 2$ (x ning 2 ga bo'linib, 3 ga bo'linmaydigan qiymatlari) tenglamani qanoatlantiradi.

3) dan $((p_1 - p_2 - 1)\varphi(x) = 0.$ Bundan yuqoridagi singari $p_1 = 2\ ;\ p_2 = 3$ da bajariladi. Ya`ni x ning 3 gabo'linib 2 bilano'zaro tub qiymatlarining berilgan tenglamani qanoatlantirishi kelib chiqadi.

4) dan $(p_2 - p_1)\varphi(x) = 0$; $p_1 \neq p_2$ bo'lgani uchun bu holda tenglama yechimga ega emas.

154. $a).\varphi(x) = \frac{x}{2} \Rightarrow \frac{x}{2}$ – butun son bo'lishi kerak. Shuning uchun ham $x = 2^{\propto} \cdot q$, $(q; 2) = 1$ deb yozish mumkin. Bu holda $\varphi(x) = 2^{\propto-1} \cdot \varphi(\mathrm{q}) = 2^{\propto-1} \cdot \mathrm{q} \Rightarrow \varphi(q) = q \Rightarrow q = 1$. Bundan $x = 2^{\propto}$ tenglamaning yechimi ($\propto \geq 1$) bo'ladi.

b).$\varphi(x) = \frac{x}{3} \Rightarrow x = 3^{\beta} \cdot q \Rightarrow \varphi(x) = 3^{\beta-1} \cdot 2 \cdot \varphi(q) = \frac{3^{\beta} \cdot q}{3} \Rightarrow \varphi(q) = \frac{q}{2} \Rightarrow q = 2^{\propto} \Rightarrow x = 2^{\propto} \cdot 3^{\beta}$.

c).$\varphi(x) = \frac{x}{4} \Rightarrow x = 2^{\propto} \cdot q$; $\propto \geq 2$; $(2^{\propto}; q) = 1 \Rightarrow \varphi(x) = 2^{\propto-1} \cdot \varphi(q) = \frac{2^{\propto} \cdot q}{4} = 2^{\propto-2} \cdot q$. Bundan $\varphi(q) = \frac{q}{2}$ ni hosil qilamiz. Bundan esa a) ga asosan $q = 2^{k}$ kelib chiqadi, lekin bizda $(2^{\propto}; \mathrm{q}) = 1$ bo'lishi kerak edi, bu qarama qarshilikdan berilgan tenglama ni yechimga ega emas degan xulosa kelib chiqadi.

155. $\varphi(p^{x}) = a \rightarrow p^{x-1}(p-1) = a \Rightarrow$

$$(x-1)\, ln\, p = ln\frac{a}{p-1} \Rightarrow x = 1 + \frac{ln\frac{a}{p-1}}{ln\, p} ,$$

bundan *a* birga teng yoki juft son .

156. $p_i (i = 1,2 \dots, \mathrm{k})$ barcha tub sonlaar bo'lsin. U holda $a = p_1 p_2 \dots p_k$ soni uchun

$$\varphi(a) = (p_1 - 1)(p_2 - 1) \dots (p_k - 1). \qquad (*)$$

Ikkinchi tomondan esa har bir $\leq a$ natural son $p_1, p_2, \dots, p_k$ tub sonlarning birortasiga bo'linadi va *a* bilan o'zaro tub emas.Shuning uchun ham $\varphi(a) = 1$. Shunday qilib (*) ga asosan $(p_1 - 1)(p_2 - 1) \dots (p_k - 1) = 1$ hosil bo'ladi. Bunday bo'lishi mumkin emas. Bu qarama-qarshilik tub sonlar soni chekli *k* ta bo'lsin deganimizdan kelib chiqdi. Demak, tup sonlar soni cheksiz ko'p.

157. $\frac{a}{b}$; $(a; b) = 1$; $0 < a < b$ musbat, to'g'ri, qisqarmas kasr berilgan bo'lsin. Maxraji b ga teng musbat, to'g'ri, qisqarmas kasrlar soni $\varphi(b)$ ta. Shuning uchun ham izlanayotgan son $\varphi(2) + \varphi(3) + \dots + \varphi(n)$ ga teng bo'ladi.

158. $x \leq 300$ va $(x; 300) = 20$ bajarilishi kerak. Bundan $\left(\frac{x}{20}; 15\right) = 1$.

$y = \frac{x}{20}$ deb olsak, $(y; 15) = 1$ va $y \leq 15$ bo'lishi kerak, bunday y lar soni $\varphi(15) = 8$ ta. Bular $y = 1, 2, 4, 7, 8, 11, 13, 14$ va bunga mos x lar $x = 20, 40, 80, 140, 160, 220, 260, 280$ lardan iborat.