VI-BOB. UZLUKSIZ KASRLAR VA ULARNING TADBIQLARI

1 -§. Chekli uzluksiz kasrlar.

Agar $\frac{a}{b}$ −qisqarmas (to'g'ri yoki noto'g'ri) oddiy kasr berilgan bo'lsa, uni Evklid algoritmi yordamida

$$\frac{a}{b} = q_0 + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \ddots + \cfrac{1}{q_n}}} \qquad (1)$$

ko'rinishida ifodalash mumkin (I.2- paragrafga qarang). (1) ga $\frac{à}{b}$- ratsional sonining chekli Uzluksiz (zanjirli) kasrga yoyilmasi deyiladi. Bunda q_0 − butun son, $q_1, q_2, \ldots, q_n$ lar natural sonlar, q_i larga chala bo'linmalar ham deyiladi. (1) yozuv o'rniga

$$\frac{a}{b} = (q_0, q_1, q_2, \ldots, q_n) \qquad (2)$$

qisqa yozuv ham ishlatiladi. Agarda biz $q_n>1$, bo'lishini talab qilsak (2) yagonadir. Aks holda yagona bo'lmaydi, chunki $q_n = (q_n - 1) + \frac{1}{1}$.

To'g'ri musbat kasrni uzluksiz kasrga yoysak, $q_0 = 0$ bo'ladi. Agarda manfiy kasrni uzluksiz kasrga yoysak, birinchi elementi $q_0 < 0$ bo'ladi, chunki manfiy sonning butun qismi manfiy, kasr qismi esa hamma vaqt musbat sondir.

Shuningdek, har qanday butun sonni $m=(m)$ bir elementli uzluksiz kasr deb, har qanday $\frac{1}{m}$ ko'rinishdagi tog'ri kasrni esa $\frac{1}{m} = (0, m)$ deb qarash mumkin.

Uzluksiz kasrlarning tatbiqlarida munosib kasrlar deb ataluvchi ushbu

$$\delta_0 = q_0, \quad \delta_1 = q_0 + \frac{1}{q_1}, \quad \delta_2 = q_0 + \cfrac{1}{q_1 + \cfrac{1}{q_2}}, \ldots, \quad \delta_n = q_0 + \cfrac{1}{q_1 + \ddots + \cfrac{1}{q_n}}$$

yoki

$$\delta_0 = q_0, \quad \delta_1 = (q_0, q_1), \quad \delta_2 = (q_0, q_1, \quad q_2),$$
$$\ldots, \delta_n = (q_0, q_1, q_2, \quad \ldots, q_n)$$

kasrlar muhim hamiyatga ega. Tushunarliki.

$$\delta_n = (q_0, q_1, q_2, \dots, q_n) = \frac{a}{b}.$$

δ_k ga odatda k-tartibli munosib kasr deyiladi. Endi $\delta_k = \frac{P_k}{Q_k}$ deb olsak, uning surat va maxrajini hisoblsh uchun quyidagi rekurent formula

$$\begin{cases} P_k = P_{k-1}q_k + P_{k-2} \\ Q_k = Q_{k-1}q_k + Q_{k-2} \end{cases}, \quad k = 0,1,2,\dots$$

o'rinli. Bunda $P_{-2} = 0$, $P_{-1} = 1$ va $Q_{-2} = 1$, $Q_{-1} = 0$ deb olinadi. Munosib kasrlarni hisoblashda quyidagi javdal ancha qulay

q_i			q_0	q_1	…	q_{k-2}	q_{k-1}	q_k	…	q_n
P_i	$P_{-2} = 0$	$P_{-1} = 1$	$P_0=q_0$	P_1	…	P_{k-2}	P_{k-1}	P_k	…	P_n
Q_i	$Q_{-2} = 1$	$Q_{-1} = 0$	$Q_0=1$	Q_1	…	Q_{k-2}	Q_{k-1}	Q_k	…	Q_n

Munosib kasrlar va berilgan $\frac{a}{b}$ kasr orasida quyidagi munosabatlar o'rinli:

$$\frac{P_0}{Q_0} < \frac{P_2}{Q_2} < \frac{P_4}{Q_4} < \dots < \frac{a}{b} < \dots < \frac{P_5}{Q_5} < \frac{P_3}{Q_3} < \frac{P_1}{Q_1}.$$

Bu yerdan ko'rinadiki, $\frac{a}{b}$ – kasr doimo ikkita qo'shni munosib kasr orasida joylashgan bo'ladi. Bunda munosib kasrlarning tartibi o'sishi bilan ular orasidagi interval kichrayib boradi. $\frac{a}{b}$ – kasrni $\frac{P_k}{Q_k}$ – munosib kasr bilan almashtirishdan hosil bo'ladigan xatolikni baholash uchun

$$\left|\frac{a}{b} - \frac{P_k}{Q_k}\right| \le \frac{1}{Q_k Q_{k+1}}$$

munosabatdan foydalanamiz.

348. Berilgan kasrlarni uzluksiz kasrga yoying:

1) $\frac{127}{52}$, 2) $\frac{24}{35}$, 3) 1,23 , 4) $\frac{29}{37}$.

349. Berilgan chekli uzluksiz kasrlarga mos qisqarmas oddiy kasrni toping:

1) (1,1,2,1,2,1,2), 2) (0,1,2,3,4,5), 3) (5,4,3,2,1), 4) (a, a, a, a, a),
5) (a, b, a, b, a), 6) (2,1,1,3,1,2), 7) (1,1,2,3,4), 8) (2,5,3,2,1,4,2,3).

350. Quyidagi kasrlarni uzluksiz kasrlarga yoyishdan foydalanib qisqartiring:

1) $\frac{3587}{2743}$, 2) $\frac{1043}{3427}$, 3) $\frac{3653}{3107}$, 4) $\frac{11281}{6583}$, 5) $\frac{1491}{2247}$.

351. Tenglamalarni yeching: 1) $(x,2,3,4)=\frac{73}{30}$, 2)$7(xyz+x+z)=10(yz+1)$.

352. Berilgan kasrlarni uzluksiz kasrga yoying va uni $\frac{P_5}{Q_5}-$ munosib kasr bilan almashtirib xatoligini aniqlang hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozing:

$$1)\ \frac{29}{37},\quad 2)\ \frac{163}{159},\ 3)\ \frac{648}{385},\ 4)\ \frac{1882}{1651}.$$

353. Berilgan kasrlarni uzluksiz kasrga yoying va uni $\frac{P_5}{Q_5}-$ munosib kasr bilan almashtirib xatoligini aniqlang hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozing:

$$1)\ \frac{571}{359},\quad 2)\ \frac{2341}{1721}.$$

354. Tishlari sonining nisbati $\frac{571}{359}$ ga teng bo'lgan ikkita shesterna yordamida tishli uzatma qurish talab etiladi. Tishlari sonining berilgan nisbatini surat maxraji eng kichik bo'lgan va xatoligi 0,001 dan oshmaydigan uzatmani qurish texnik jihatidan mumkinmi?

355. $(2,2,2,\dots,2)$ uzluksiz kasrni 2 ga bo'lishdan hosil bo'lgan bo'linmani toping.

356. $(a,a,a,\dots,a)$ uzluksiz kasrni 2 ga bo'lishdan hosil bo'lgan bo'linmani toping.

357. Tenglikni isbotlang:

$$\left(\frac{P_{n+2}}{P_n}-1\right)\cdot\left(1-\frac{P_{n-1}}{P_{n+1}}\right)=\left(\frac{Q_{n+2}}{Q_n}-1\right)\left(1-\frac{Q_{n-1}}{Q_{n+1}}\right).$$

358. Agar P_i va Q_i lar $(q_1,q_2,\ \dots,q_n)-$ uzluksiz kasrning munosib kasrlarining elementlari bo'lib, $n\geq 1$ bo'lsa,

$$\frac{P_n}{P_{n-1}}=(q_n,q_{n-1},\dots q_1)\text{va}\ \frac{Q_n}{Q_{n-1}}=(q_n,q_{n-1},\dots q_2)$$

ekanligini ko'rsating.

359. $\frac{P_n}{P_{n-1}}$ va $\frac{Q_n}{Q_{n-1}}$ larning qisqarmas kasr ekanligini isbotlang.

360. Isbotlang:

$$\left(\underbrace{2,2,2,\dots,2}_{nta}\right)=\frac{\left(1+\sqrt{2}\right)^{n+1}-\left(1-\sqrt{2}\right)^{n+1}}{\left(1+\sqrt{2}\right)^{n}-\left(1-\sqrt{2}\right)^{n}}$$

361. $P_nQ_{n-1}-Q_nP_{n-1}=(-1)^{n-1}$ munosabatdan foydalanib, ikki noma'lumli birinchi darajali aniqmas tenglamalarni yechish usulini bayon qiling.

362. 361- misolda bayon qilingan usuldan foydalanib quyidagi tenglamalarni yeching: 1) $38x+117y=209$, 2) $122x+129y=2$, 3) $119x-68y=34$,4) $258x-175y=113$, 5) $41x+114y=5$, 6) $70x+33y=1$.

363. Agar a natural son bo'lsa, $\frac{a^4+3a^2+1}{a^3+2a}$ ning qisqarmas kasr ekanligini isbotlang.

364. Simmetrik uzluksiz kasr ($q_n = q_1,\ q_{n-1} = q_2, \ldots$) lar uchun $P_{n-1} = Q_n$ munosabatning o'rinli ekanligini isbotlang.

365. Agar $n \geq 2$ bo'lsa, $Q_n \geq 2^{\frac{n-1}{2}}$ ekanligini isbotlang.

366. $P_nQ_{n-1} - Q_nP_{n-1} = (-1)^n$ munosabatdan foydalanib, $ax \equiv b(modm)$ taqqoslamaning $(a, m) = 1$ bo'lgandagi yechimini topish uchun formula keltirib chiqaring.

367. 366- misolda bayon qilingan usuldan foydalanib quyidagi taqqoslamalarni yeching: 1) $95x \equiv 59(mod308)$, 2) $91x \equiv 1(mod132)$.