IV.1-§.

248. a). Bu holda 3 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2 dan iborat. Bu sonlarni berilgan taqqoslama qo'yib sinab ko'ramiz va $x_1 = 1, x_2 = 2$ larning uni qanoatlantirishiga ishonch hosil qilamiz. Demak, berilgan taqqoslamaning yechimlari $x \equiv 1(mod 3)$ va $x \equiv 2(mod 3)$ yoki buni $x = 1 + 3t\ , t\epsilon Z$ va $x = 2 + 3t, t\epsilon Z$ ko'rinishda yozishimiz mumkin.

b). 5 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4. Bu sonlarni berilgan taqqoslamaga qo'ysak, ulardan $x_1 = 1$ va$x_2 = 2$ lar uni qanoatlantirishini ko'ramiz. Shuning uchun ham yechimlar $x \equiv 1(mod 5)$ va $x \equiv 2(mod 5)$ lardan iborat. **Javob** $x = 1 + 5t\ , t\epsilon Z$ va $x = 2 + 5t, t\epsilon Z$.

c). 3 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2 lardan iborat. Bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Demak, taqqoslama yechimga ega emas.

d). 5 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4 lardan iborat. Bularni berilgan taqqoslamaga qo'yib sinab ko'rsak, $x_1 = 3$ uni qanoatlantiradi. Demak, javob $x \equiv 3(mod 5)$, ya'ni $x = 3 + 5t, t\epsilon Z$.

e). 7 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4,5,6 lardan iborat. Bularni taqqoslamaga bevosita olib borib qo'ysak, $x_1 = 1$ va$x_2 = 2$ lar uni qanoatlantiradi. **Javob:** $x = 1 + 7t, x = 2 + 7t, t\epsilon Z$.

f).15 moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasi $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$ lardan iborat. Bularni berilgan taqqoslamaga qo'yib sinab ko'rib, $x_1 = 11$ ning uni qanoatlantirishini topamiz. Demak, $x \equiv 11(mod 15)$, ya'ni $x = 11 + 15t, t\epsilon Z$ berilgan taqqoslamaning yechimi.

Izoh. Bu holda $x = 1,2, \dots 14$ larning barchasi emas, balki $2x > 15$ shartni qanoatlantiruvchilari $x = 8,9,10,11,12,13,14$ni ham tekshirish kifoya bo'ladi.

249. 7 moduli boyicha chegirmalarning to'la sistemasini, tekshirish qulay bo'lishi uchun uni absolyut qiymati jihatidan eng kichik chegirmalar sistemasi ko'rinishida $0, \pm 1, \pm 2, \pm 3$ yozib olamiz. Berilgan taqqoslamaga bu sonlarni qo'yib tekshirsak, faqat 1 uni qanoatlantiradi, demaк, $x \equiv 1(mod 7)$ berilgan taqqoslamaning yagona yechimi.

250. Bu yerda 3 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi $0, \pm 1$ dan iborat, lekin bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi, ya'ni berilgan taqqoslama yechimga ega emas.

251.a). Avvalo koeffitsiyentlarini berilgan 15 moduli bo'yicha bo'yicha absolyut qiymati jihatidan eng kichik chegirmalar bilan almashtiramiz. Bunda$90 = 15 \cdot 6 + 0,\ \ 46 = 15 \cdot 3 + 1,\ \ 52 = 15 \cdot 3 + 7$ bo'lgani uchun berilgan taqqoslama $x^2 - 7x + 1 \equiv 0(mod15)$ taqqoslamaga teng kuchli. Endi 15 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7$ larni qo'yib, tekshirib ko'ramiz. U holda $x_1 = -4$ ning berilgan taqqoslamani qanoatlantiradi. Demak, berilgan taqqoslamaning yechimi $x = -4 + 15t, t\epsilon Z$.

b). Bunda $25 = 12 \cdot 2 + 1, 36 = 12 \cdot 3 + 0, 18 = 12 \cdot 1 + 6, 13 = 12 \cdot 1 + 1$bo'lgani uchun berilgan taqqoslama $3x^3 - 6x + 1 \equiv 0(mod12)$ taqqoslamaga teng kuchli.Endi 12 moduli bo'yicha chegirmalarning to'la sistemasi $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6$ larni qo'yib tekshirib ko'ramiz. Bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Taqqoslamaning yechimi yo'q.

Izoh. Buni quyidagicha izohlash ham mumkin. $3x^3 - 6x + 1 \equiv 0(mod12)$ taqqoslama

$$\begin{cases} 3x^3 - 6x + 1 \equiv 0(mod3) \\ 3x^3 - 6x + 1 \equiv 0(mod4) \end{cases}$$

ga teng kuchli. Bu yerda birinchi taqqoslama $1 \equiv 0(mod3)$ ziddiyatli taqqoslama bo'lgani uchun sistema va demak, berilgan taqqoslama ham yechimga ega emas.

c). $21x + 4 \equiv 7(mod6) \to 3x - 2 \equiv 1(mod6) \to 3x \equiv 3(mod6) \to x \equiv 1(mod2), x = 1 + 2t,$ tϵZ, $x \equiv 1,3,5(mod6)$. Yechimlar $x \equiv 1(mod6), x \equiv -2(mod6), x \equiv -1(mod6)$,ya'ni $x = 1 + 6t, x = -2 + 6t, x = -1 + 6t, t\epsilon Z$

d). $x^5 - 2x^3 + 13x - 1 \equiv 0(mod4) \to x^5 - 2x^3 + x - 1 \equiv 0(mod4)$. $x = \pm 1, \pm 2$ larni qo'yib tekshiramiz. U holda bularning birortasi ham bu taqqoslamani qanoatlantirmaydi va berilgan taqqoslama yechimga ega emas.

252. Bunda $12 = 3 \cdot 4 + 0$, $24 = 3 \cdot 8 + 0$ va $7 = 3 \cdot 2 + 1$ bo'lganligi uchun koeffitsiyentlarini absolyut qiymati jihatidan eng kichik chegirmalar bilan almashtirib, $x^3 - x \equiv 0(mod3)$ ni hosil qilamiz. Ferma teoremasiga ko'ra p tub son bo'lganda $x^p - x \equiv 0(modp)$ bajariladi. Bizda p=3, ya'ni oxirgi taqqoslama va demak, berilgan taqqoslama ham ayni taqqoslama. Shuning uchun ham noma'lum x ning barcha butun qiymatlari berilgan taqqoslamani qanoatlantiradi.

253. a).$x^3 - x + 6 \equiv 0(mod3)$. Bunda Ferma teoremasiga ko'ra $x^3 - x \equiv 0(mod3)$ va $6 \vdots 3$. Shuning uchun berilgan taqqoslama x ning ixtiyoriy butun qiymatida o'rinli.

b). $x(x^2 - 1) \equiv 0(mod6)$. Bu taqqoslamani $(x - 1)x(x + 1) \equiv 0(mod6)$ ko'rinishda yozib olish mumkun . Bu yerda chap tomondagi ifoda uchta ketma-ket sonlarning ko'paytmasi sifatida 6 ga bo'linadi, ya'ni berilgan taqqoslama x ning ixtiyoriy butun qiymatida o'rinli.

c). $20x^6 + x^5 - 10x^3 - x + 15 \equiv 0(mod5)$. Bunda koeffitsiyentlarni absolyut qiymat jihatdan eng kichik chegirmalar bilan almashtirib soddalashtiramiz.U holda $x^5 - x \equiv 0(mod5)$ ayniy taqqoslamaga ega bo'lamiz.

d). $x^{13} - 26x^{12} - x \equiv 0(mod13) \to x^{13} - x \equiv 0(mod13)$. Bu taqqoslama x ning ixtiyoriy butun qiymatlarida bajariladigan ayniy taqqoslama.

254. a). Berilgan $5x \equiv 4(mod5)$taqqoslama $0 \equiv 4(mod5)$ ziddiyatli taqqoslamaga teng kuchli. Shuning uchun taqqoslama yechimga ega emas.

b). $x^2 - 2x + 3 \equiv 0(mod4)$. Bu yerda 4 moduli bo'yicha chegirmalarning to'la sistemasi $0, 1, 2, 3$ larni qo'yib, tekshirib ko'ramiz. U holda ularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Demak, taqqoslama yechimga ega emas.

c). $20x^5 + 5x^4 - 10x^3 - 6 \equiv 0(mod5)$ taqqoslama $-1 \equiv 0(mod5)$ ziddiyatli taqqoslamaga teng kuchli. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

d). $x^{13} - 26x^{12} - x + 5 \equiv 0(mod13)$ taqqoslama $x^{13} - x + 5 \equiv 0(mod13)$ taqqoslamaga teng kuchli. Bu yerda $x^{13} - x \equiv 0(mod13)$ ayniy taqqoslama bo'lganligi uchun berilgan taqqoslama $5 \equiv 0(mod13)$ ziddiyatli taqqoslamaga teng kuchli bo'ladi. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

255. *a).*Bu yerda $y = x + a$ almashtirish olib berilgan taqqoslamaga qo'yamiz, u holda $(y + a)^n + a_1(y + a)^{n-1} + a_2(y + a)^{n-2} + \dots + a_n \equiv 0(modm)$ ni hosil qilamiz. Oxirgi taqqoslamada y ning bir xil darajalari oldidagi koeffitsiyentlarni yig'sak $y^n + (a_1 + na)y^{n-1} + \dots + (a^n + a_1 \cdot a^{n-1} + \dots + a_n) \equiv 0(modm)$ hosil bo'ladi. a ixtiyoriy bo'lgani uchun uni $a_1 + na \equiv 0(modm)$ shart bajariladigan qilib tanlaymiz. U holda $y^n + b_2y^{n-2} + \dots + b_n \equiv 0(modm)$ taqqoslamaga ega bo'lamiz.

b). $x^3 + 5x^2 + 6x - 8 \equiv 0(mod13)$. $a_1 = 5, m = 13\ , n = 3$a) qismdagiga asosan $a_1 + na \equiv 0(modm)$ dan $5 + 3a \equiv 0(mod13)$ ga ega bo'lamiz . Bundan $3a \equiv -5(mod13) \to -10a \equiv -5(mod13) \to 2a \equiv 1(mod13) \to 2a \equiv 14(mod13) \to a \equiv 7(mod13)$. Demak, $a = 7$ va biz $x = y + 7$ almashtirish bajaramiz u holda $(y + 7)^3 + 5(y + 7)^2 + 6(y + 7) - 8 = (y + 7)(y^2 + 14y + 49 + 5y + 35 + 6) - 8 = (y + 7)(y^2 + 19y + 90) - 8 = y^3 + 19y^2 + 90y + 7y^2 + 133y + 630 - 8 = y^3 + 26y^2 + 223y + 622 \equiv y^3 + 2y - 2 \equiv 0(mod13)$. Demak, izlanayotgan taqqoslama $y^3 + 2y - 2 \equiv 0(mod13)$ dan iborat.

256. Eyler teoremasiga ko'ra berilgan taqqoslamani $(x, 60) = 1$ shartni qanoatlantiruvchi barcha x lar qanoatlantiradi, ya'ni $\varphi(60) = \varphi(2^2 \cdot 3 \cdot 5) = \varphi(2^2) \cdot \varphi(3) \cdot \varphi(5) = (2^2 - 2) \cdot (3 - 1) \cdot (5 - 1) = 2 \cdot 2 \cdot 4 = 16$ ta yechimga ega. Bu yechimlar x ning $x \leq 60, (x, 60) = 1$ shartlarni qanoatlatiruvchi qiymatlari $x \equiv 1,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43, 47,\ 49,\ 53,\ 59\ (mod 60)$ dan iborat.

IV.2-§.

257. a). Bunda $(5,6) = 1$. Shuning uchun ham taqqoslama yagona yechimga ega. Bu yechimni tanlash usuli bilan topish uchun 6 moduli bo'yicha chegirmalarning to'la sistemasini biror ko'rinishda (masalan: $0, 1, 2, 3, 4, 5$) yozib olib bu sonlarni berilgan taqqoslamaga qo'yib tekshiramiz. x_1=3 berilgan taqqoslamani qanoatlantiradi. Shuning uchun berilgan taqqoslamaning yechimi $x \equiv 3(mod\ 6)$, ya`ni $x = 3 + 6t, t \in Z$.

b).$8x \equiv 3(mod\ 8)$ taqqoslamada $(8,10) = 2$, lekin 3 soni 2ga bo'linmaydi. Shuning uchun ham taqqoslama yechimga ega emas.

c). $2x \equiv 6(mod\ 8)$ taqqoslamada $(2,8) = 2$ va 6 soni 2 ga bo'linadi. Shuning uchun ham berilgan taqqoslama 2 ta yechimga ega. Bu holda berilgan taqqoslama $x \equiv 3(mod\ 4)$ga teng kuchli. Demak, berilgan taqqoslamaning yechimlari $x \equiv 3{,}7(mod\ 8)$, ya'ni $x = 3 + 8t$ va $x = 7 + 8t, t \in Z$ lardan iborat bo'ladi.

d). $3x \equiv -6(mod\ 7)$ning o'ng tomoniga 7 ni (modulni) qo'shsak, $3x \equiv 1(mod\ 7)$ taqqoslama hosil bo'ladi. Bunda $(3,7) = 1$ bo'lgani uchun u yagona yechimga ega. 7 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4,5,6 larni taqqoslamaga qo'yib, tekshirib ko'rib $x \equiv 5(mod\ 7)$,ya'ni $x = 5 + 7t, t \in Z$ ning berilgan taqqoslamaning yechimi ekanligini topamiz.

e). $4x \equiv 3(mod 12)$da $(4,12) = 4$,lekin 3 soni 4 ga bo'linmagani uchun ham taqqoslama yechimga ega emas.

f). $6x \equiv 5(mod\ 9)$ da $(6,9) = 3$ va 5 soni 3 ga bo'linmaydi, shuning uchun ham berilgan taqqoslama yechimga ega emas.

g). Bu yerda $(5,8) = 1$ va 8 –moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2, \pm3,\ 4$ dan iborat. Bularni qo'yib tekshirib, $x \equiv 3(mod 8)$ berilgan taqqoslamani yechimi ekanligini aniqlaymiz.

258. a). $5x \equiv 3(mod\ 7)$ taqqoslamada $(5,7) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bu yechimni taqqoslamalarning xossalaridan foydalanib topish uchun taqqoslamaning o'ng tomoniga modulni qo'shamiz. U holda $5x \equiv 10(mod\ 7)$ ni hosil qilamiz. Bu taqqoslamaning ikkala tomonini 5 ga qisqartiramiz.($(5; 7) = 1$ bo'lgani uchun bu ishni amalga oshirish mumkin). U holda $x \equiv 2(mod\ 7)$ ya'ni $x = 2 + 7t, t \in Z$ berilgan taqqoslamaning yechimiga ega bo'lamiz.

b). $8x \equiv 3(mod 11)$ taqqoslamada $(8; 11) = 1$. Demak, taqqoslama yagona yechimga ega. Bu taqqoslamaning o'ng tomonidan 11 ni ayirsak $8x \equiv -8(mod\ 11)$ taqqoslama hosil bo'ladi. Oxirgi taqqoslamaning ikkala tomonini 8 ga (chunki

(8;11)=1) qisqartirsak $x \equiv -1(mod11)$ taqqoslamaga ega bo'lamiz. Demak, berilgan taqqoslamaning yechimi $x = -1 + 11t, t \in Z$.

c). $4x \equiv 6(mod8)$ taqqoslamada $(4{,}8) = 4$ va 6 soni 4 ga bo'linmaydi, shuning uchun ham berilgan taqqoslama yechimga ega emas.

d). $4x \equiv 25(mod13)$ da $(4;13) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bu taqqoslamaning o'ng tomonidan 13 ni ayirib, hosil bo'lgan taqqoslamani ikkala tomonini 4 ga bo'lsak $x \equiv 3(mod13)$ taqqoslama hosil bo'ladi. Demak, berilgan taqqoslamaning yechimi $x = 3 + 13t, t \in Z$.

e). $11x \equiv 3(mod12)$ taqqoslamada $(11{,}12) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Berilgan taqqoslama yagona yechimga ega. Berilgan taqqoslamaning chap tomonidan uning modulini ayirsak, $-x \equiv 3(mod\ 12)$ yoki $x \equiv -3(mod\ 12)$ taqqoslama hosil bo'ladi. Bundan $x = -3 + 12t,\ t \in Z$ berilgan taqqoslamaning yechimi ekanligi kelib chiqadi.

f).$7x \equiv 5(mod\ 9)$ taqqoslamada $(9{,}7) = 1$. Demak, berilgan taqqoslama yagona yechimga ega. Bu taqqoslamaning ikkala tomonidan modul 9 niayiramiz. U holda $-2x \equiv -4(mod\ 9)$ hosil bo'ladi. Bundan $x \equiv 2(mod\ 9)$, ya'ni $x = 2 + 9t, t \in Z$ ekanligi kelib chiqadi.

g). Bunda $(5{,}8) = 1$ bo'lganli uchun taqqoslama yagona yechimga ega. Taqqoslamaning istalgan tomoniga modulga karrali sonni qo'shish yoki ayirish mumkin. Shuning uchun ham $5x \equiv 7 + 8(\mathrm{mod}\,8) \to 5x \equiv 15(\mathrm{mod}\,8)$.

Taqqoslamaning ikkala tomonini modul bilan o'zaro tub songa qisqartirish mumkin bo'lgani uchun $(5;15) = 5$ va $(5,8) = 1$ ekanligini e'tiborga olib oxirgi taqqoslamaning ikkala tomonini 5 ga qisqartiramiz. U holda $x \equiv 3(mod8)$ yechimni hosil qilamiz.

h). $(7,\ 15) = 1$ bo'lganli uchun bu taqqoslama yagona yechimga ega.

$$7x \equiv 6 + 15(\mathrm{mod}\ 15),\quad 7x \equiv 21(\mathrm{mod}\ 15),\quad (7,\ 21) = 7 \quad \text{va} \quad (7{,}15)=1$$

ekanligini e'tiborga olib, oxirgi taqqoslamaning ikkala tomonini 7ga qisqartiramiz. U holda $x \equiv 3(mod15)$ yechimni hosil qilamiz.

259. a). $13x \equiv 3(mod19)$ taqqoslamada $(13{,}19) = 1$ bo'lgani uchun yagona yechimga ega. Ma'lumki, agar $(a, m) = 1$ bo'lsa, $ax \equiv b(mod\ m)$ taqqoslamaning yechimini $x \equiv a^{\varphi(m)-1} \cdot b(modm)$ taqqoslamadan foydalanib topish mumkin. Bizning misolimizda $a = 13, b = 3, m = 19$ bo'lgani uchun

$x \equiv 13^{\varphi(19)-1} \cdot 3(mod\ 19)$ bo'ladi. Bunda $\varphi(19)$=18 va 13^{17}=$(13^{2})^{8}$·13== 169^{8}·13 =$(19 \cdot 9 - 2)^{8}$·13 bo'lgani uchun $x \equiv (-2)^{8}$·3·13(mod 19) ≡

$$256 \cdot 133(mod\ 19) \equiv (19 \cdot 13 + 9) \cdot 3 \cdot 13(mod19) \equiv\equiv 3 \cdot 9 \cdot 13(mod\ 19)$$

$$\equiv 3 \cdot 117(mod\ 19) \equiv (19 \cdot 6 + 3) \cdot 3(mod\ 19) \equiv 9(mod\ 19).$$

Demak, berilgan taqqoslamaning yechimi $x = 9 + 19t, t \in Z$.

b).$27x \equiv 7(mod\ 58)$ taqqoslamada $(27,58) = 1$ va $\varphi(58) = \varphi(2 \cdot 29) = \varphi(2) \cdot \varphi(29) = 28$ bo’lgani uchun $x \equiv 27^{27} \cdot 7(mod\ 58) \equiv 3^{81} \cdot 7(mod\ 58) \equiv (3^4)^{20} \cdot 21(mod\ 58) \equiv 23^{20} \cdot 21(mod\ 58) \equiv (58 \cdot 7 + 7)^{20} \cdot 21(mod\ 58) \equiv 7^{10} \cdot 21(mod\ 58) \equiv (7^3)^3 \cdot 147(mod\ 58 \equiv 343^3 \cdot (2 \cdot 58 + 31)(mod\ 58) \equiv (5 \cdot 58 + 53)^3 \cdot 31(mod\ 58) \equiv 53^3 \cdot 31(mod\ 58) \equiv 53^2 \cdot 53 \cdot 31(mod\ 58) \equiv (-5)^3 \cdot 31(mod\ 58) \equiv -9 \cdot 31(mod\ 58) \equiv -279(mod\ 58) \equiv 11(mod\ 58)$.

Demak, berilgan taqqoslamaning yechimi $x \equiv 11(mod\ 58)$, ya’ni $x = 11 + 58t, t \in Z$.

c). $5x \equiv 7(mod\ 10)$ taqqoslamada $(5,10) = 5$, lekin 7 soni 5 ga bo’linmaydi. Shuning uchun ham taqqoslama yechimga ega emas.

d). $3x \equiv 8(mod\ 13)$, bunda$(3,13) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bizda $a = 3, b = 8, m = 13$ va $\varphi(13)$=12 bo’lgani uchun $x \equiv 3^{11} \cdot 8(mod\ 13) \equiv (3^3)^3 \cdot 3^2 \cdot 8(mod\ 13) \equiv 72(mod\ 13) \equiv 7(mod\ 13)$. Demak berilgan toqqoslamaning yechimi $x = 7 + 13t, t \in Z$.

e). $25x \equiv 15(mod\ 17)$ da $(25,17) = 1$ bo’lgani uchun u yagona yechimga ega va $(5,17) = 1$ bo’lganidan taqqoslamaning ikkala tomonini 5 ga qisqartirish mumkin. U holda $5x \equiv 3(mod\ 17)$ taqqoslama hosil bo’ladi. Shuning uchun ham $x \equiv 5^{\varphi(17)-1} \cdot 3(mod\ 17) \equiv 5^{15} \cdot 3(mod\ 17) \equiv (5^3)^5 \cdot 3(mod\ 17) \equiv (17 \cdot 7 + 6)^5 \cdot 3(mod\ 17) \equiv 6^5 \cdot 3(mod\ 17) \equiv$

$$\equiv (6^2)^2 \cdot 18(mod\ 17) \equiv 2^2 mod(17) \equiv 4(mod\ 17).$$

Demak, izlanayotgan yechim $x = 4 + 17t, t \in Z$.

f). $29x \equiv 35(mod\ 12)$, bunda $(29,12) = 1$ bo’lgani uchun berilgan taqqoslama yagona yechimga ega. Bu taqqoslamaning koeffisiyentlari moduldan katta bo’lgani uchun ularni 12 moduli bo’yicha eng kichik manfiy bo’lmagan chegirmalar bilan almashtiramiz. Bunda $29 \equiv 12 \cdot 2 + 5, 35 \equiv 12 \cdot 2 + 11$ bo’lgani uchun, berilgan taqqoslamani $5x \equiv 11(mod\ 12)$ ko’rinishida yozib olish mumkin. $\varphi(12)=\varphi(2^2 \cdot 3)= =(2^2 - 2) \cdot 2=4$ va $x \equiv 5^{\varphi(12)-1} \cdot 11\ (mod\ 12) \equiv 5^3 \cdot 11(mod\ 12) \equiv 125 \cdot (-1)(mod12) \equiv -5(mod12) \equiv 7(mod\ 12)$. Demak, izlanayotgan yechim $x = 7 + 12t, t \in Z$.

g). Bu yerda $x \equiv a^{\varphi(m)-1} \cdot b(\mathrm{mod}\, m)$ formuladan foydalanamiz. Bizda *a=3*, *b=7*, *m=11* bo’lgani uchun $x \equiv 3^{\varphi(11)-1} \cdot 7(mod11)$ ni hosil qilamiz. Bunda $\varphi(11) = 11 - 1 = 10$ bo’lganidan $x \equiv 3^9 \cdot 7(mod11)$. Endi oxirgi taqqoslamaning o’ng tomonidagi ifodani eng kichik musbat chegirma ko’rinishiga keltiramiz. $3^9 \cdot 7(mod11) = (3^3)^3 \cdot 7(mod11) \equiv 27^3 \cdot 7(mod11) \equiv 5^3 \cdot 7(mod11) \equiv 4 \cdot 7 \equiv 6(mod11)$. Shunday qilib berilgan taqqoslamaning yechimi: $x \equiv 6(\mathrm{mod}11)$.

260. a). Berilgan $13x \equiv 1(mod\ 27)$ taqqoslamada $(13; 27) = 1$ bo’lgani uchun u yagona yechimga ega. Bu yechimni munosib kasrlardan foydalanib, ya’ni $x \equiv$

$(-1)^{n-1} \cdot bP_{n-1}(modm)$ $(*)$ formuladan foydalanib topish uchun,avvalo P_{n-1} ni (oxirgidan oldingi munosib kasrning suratini) aniqlashimiz kerak. Buning uchun esa $\frac{m}{a} = \frac{27}{13}$ kasrni uzluksiz kasrga yoyamiz. Bunda $27 = 13 \cdot 2 + 1$, $13 = 1 \cdot 13 + 0$ lardan $\frac{27}{13} = 2 + \frac{1}{13} = (2; 13)$. Endi P_{n-1} ni aniqlaymiz:

q_i		2	13
P_i	$P_0 = 1$	$P_1 = 2$	27

Jadvaldan $P_{n-1} = 2$ va $n = 2$. Bularni (*) ga olib borib qo'ysak, $x \equiv (-1)^{2-1} \cdot 2 \cdot 1(mod\ 27) \equiv -2(mod\ 27)$, ya`ni $x = -2 + 27t,\ t \in Z$.

Tekshirish: $13 \cdot (-2) \equiv 1(mod\ 27) \rightarrow 1 \equiv 1(mod\ 27)$ doimo bajariladi.

b). Berilgan $37x \equiv 25(mod\ 117)$ taqqoslamada $(37; 117) = 1$ bo'lgani uchun u yagona yechimga ega. P_{n-1} ni aniqlaymiz. Buning uchun $\frac{117}{37}$ kasrnii uzluksiz kasrlarga yoyamiz. $117 = 37 \cdot 3 + 6, 37 = 6 \cdot 6 + 1$, $6 = 1 \cdot 6 + 0$ lardan q_1=3, $q_2 = 6$, $q_3 = 6$ ekanligigini topamiz. U holda $\frac{117}{37} = (3{,}6{,}6)$ va

q_i		3	6	6
P_i	$P_0 = 1$	3	19	117

bo'lganidan $n = 3, P_{n-1} = P_2 = 19$ hamda $x \equiv (-1)^2 \cdot 19 \cdot 25(mod\ 117) \equiv 475(mod\ 117) \equiv 7(mod\ 117)$. Demak, izlanayotgan yechim $x = 7 + 117t, t \in Z$. **Tekshirish:** $37 \cdot 7 \equiv 259 \equiv (117 \cdot 2 + 25)\ (mod\ 117) \equiv 25(mod\ 117)$, ya'ni yechim to'g'ri topilgan.

c) $113x \equiv 89(mod\ 311)$ taqqoslamada $(113, 311) = 1$ bo'lgani uchun u yagona yechimga ega. Endi $\frac{311}{113}$ kasrni uzluksiz kasrlarga yoyamiz: $311 = 113 \cdot 2 + 85, 113 = 85 \cdot 1 + 28$, $85 = 28 \cdot 3 + 1$, $28 = 1 \cdot 28 + 0$. Demak,

$q_1 = 2$, $q_2 = 1$, $q_3 = 3$, $q_4 = 28$ va $\frac{311}{113} = (2; 1; 3; 28)$ hamda

q_i		2	1	3	28
P_i	$P_0 = 1$	2	3	11	311

bo'lganligi uchun P_{n-1}=11, $n = 4$ va $x \equiv (-1)^3 \cdot 11 \cdot 89(mod\ 311) \equiv -979(mod\ 311) \equiv -46(mod\ 311)$, ya'ni $x = -46 + 311t, t \in Z$.

Tekshirish: $113 \cdot (-46)\ (mod\ 311) \equiv -5198\ (mod\ 311) \equiv (311 \cdot 16 + 222)\ (mod\ 311) \equiv -222(mod\ 311) \equiv 89\ (mod\ 311)$. Demak, taqqoslamaning yechimi to'g'ri topilgan.

d) $221x \equiv 111(mod\ 360)$. Bunda $221 = 13 \cdot 17$ va $360 = 36 \cdot 10 = 2^2 \cdot 3^2 \cdot 2 \cdot 5 = 2^3 \cdot 3^2 \cdot 5$, ya'ni $(221; 360) = 1$. Shuning uchun ham berilgan taqqoslama yagona yechimga ega. Shu yechimni topish uchun $\frac{360}{221}$ kasrni uzluksiz kasrga yoyamiz. $360 = 221 \cdot 1 + 139, 221 = 139 \cdot 1 + 82, 139 = 82 \cdot 1 + 57, 82 = 57 \cdot 1 + 25, 57 = 25 \cdot 2 + 7, 25 = 7 \cdot 3 + 4, 7 = 4 \cdot 1 + 3, 4 = 1 \cdot 3 + 1, 3 = 1 \cdot 3 + 0.$ Bulardan $q_1 = 1, q_2 = 1, q_3 = 1, q_4 = 1,\ q_5 = 2,\ q_6 = 3,\ q_7 = 1,\ q_8 = 1,\ q_9 = 3$ va $\frac{360}{221} = (1, 1, 1, 1, 2, 3, 1, 1, 3)$ larni topamiz. Endi P_{n-1} ni aniqlaymiz.

q_i		1	1	1	1	2	3	1	1	3
P_i	P_0	1	2	3	5	13	44	57	101	360

Bundan P_{n-1}=101, $n = 9$, va $x \equiv (-1)^8 \cdot 101 \cdot 111 (mod\ 360) \equiv 11100 + +111 (mod\ 360) \equiv (30 \cdot 360 + 300 + 111)(mod\ 360) \equiv 411 (mod\ 360) \equiv 51 (mod\ 360)$. Demak, berilgan taqqoslamaning yechimi $x = 51 + 360t, t \in Z$.

Tekshirish: $221 \cdot 51 \equiv 11271 \equiv (360 \cdot 31 + 111) \equiv 111 (mod\ 360)$.

Bu yerdan ko'rinadiki, berilgan misolning yechimi to'g`ri topilgan.

e) $39x \equiv 84 (mod\ 93)$ da $39 = 3 \cdot 13,\ 93 = 3 \cdot 31$, ya'ni $(39; 93) = 3$ bo'lgani va 84 soni 3 ga bo'lingani uchun berilgan taqqoslama 3 ta yechimga ega bo'ladi. Berilgan taqqoslamani 3 ga qisqartirib, yagona yechimga ega bo'lgan $13x \equiv 28 (mod\ 31)$ taqqoslamani hosil qilamiz. Endi $\frac{31}{13}$ kasrni uzluksiz kasrlarga yoyamiz. Bunda $31 = 13 \cdot 2 + 5,\ 13 = 5 \cdot 2 + 3, 5 = 1 \cdot 3 + 2, 3 = 1 \cdot 2 + 1,\ 2 = 1 \cdot 2$ bo'lgani uchun q_1=2, q_2=2, q_3=1, q_4=1, q_5=2 va $\frac{31}{13} = (2, 2, 1, 1, 2)$. Endi P_{n-1}ni aniqlaymiz.

q_i		2	2	1	1	2
P_i	1	2	5	7	2	31

Bundan $n = 5$, P_{n-1}=12 va $x \equiv (-1)^4 \cdot 12 \cdot 28 (\text{mod } 31) \equiv 12 \cdot (-3)\ (\text{mod } 31) \equiv -5 (mod\ 31)$. Demak, berilgan taqqoslamaning yechimlari $x \equiv -5,\ 26, 57 (mod\ 93)$ ya'ni $x = -5 + 93t, x = 26 + 93t,\ x = 57 + 93t,\ t \in Z$.

Tekshirish: $x_1 = -5$ bo'lsa, $39 \cdot (-5) = -195\ (mod\ 93)\ 84 (mod 93)$; $x_2 = 26$ bo'lsa, $39 \cdot 26 = 1014 \equiv 93 \cdot 23 + 84 \equiv 84 (mod 93)$;

$x_3 = 57$ bo'lsa, $39 \cdot 57 = 2223 = 93 \cdot 23 + 84 \equiv 84 (mod 93)$. Bulardan ko'rinadiki, uchala yechim ham to'g'ri topilgan.

f). $143x \equiv 41 (mod 221)$ taqqoslamada $143 = 11 \cdot 13$, $221 = 13 \cdot 17$ bo'lgani uchun $(143, 221) = 13$, lekin 41 soni 13ga bo'linmaydi. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

g). Bu yerda $x \equiv (-1)^{n-1} b P_{n-1} (\bmod m)$(1) formuladan foydalanamiz. Avvalo P_{n-1} ni aniqlab olamiz. Buning uchun $\frac{m}{a} = \frac{43}{20}$ kasrni uzluksiz kasrga yoyamiz va munosib kasrlarini topamiz, u holda $43 = 20 \cdot 2 + 3$, $20 = 3 \cdot 6 + 2$, $3 = 2 \cdot 1 + 1$, $2 = 1 \cdot 2$. Demak, $q_1 = 2,\ q_2 = 6,\ q_3 = 1,\ q_4 = 2$. $\frac{m}{a} = \frac{43}{20} = (2,\ 6,\ 1, 2)$. Endi munosib kasrlarning suratlarini hisoblab P_{n-1}ni topamiz.

q_i		$q_1 = 2$	$q_2 = 6$	$q_3 = 1$	$q_4 = 2$
P_i	$P_0 = 1$	$P_1 = 2$	$P_2 = 13$	$P_3 = 15$	$P_4 = 43$

Demak, $n = 4, P_3 = 15$. Topilgan qiymatlarni (1) ga olib borib qo'ysak $x \equiv (-1)^3 \cdot 15 \cdot 13(mod 43) \equiv -195(mod 43) \equiv -195 + 43 \cdot 5(mod 43) \equiv 20(mod 43)$ hosil bo'ladi. **Javob**: $x = 20 + 43t,\ t \in$ Z .

261. a). $12x \equiv 9(mod\ 15)$ taqqoslamada $(12,15) = 3$ va 9 soni 3 ga karrali bo'lgani uchun u 3 ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 3ga qisqartirsak $4x \equiv 3(mod\ 5)$ taqqoslama hosil bo'ladi. Bunda $(4,5) = 1$ bo'lgan uchun u yagona yechimga ega. Uning yechimini aniqlaymiz. $4x \equiv (3 + 5)(mod\ 5)$ yoki $4x \equiv 8(mod\ 5)$. Keyingi taqqoslamaning ikkala tomonini 4 ga bo'lsak, $x \equiv 2(mod\ 5)$ hosil bo'ladi. Demak, berilgan taqqoslamaning yechimlari $x \equiv 2, 7, 12(mod\ 15)$, ya'ni
$x = 2 + 15t\ , x = 7 + 15t\ , x = 12 + 15t, t \in \mathbb{Z}$.

b). $12x \equiv 9(mod\ 18)$taqqoslamada $(12,18) = 6$, lekin 9 soni 6 ga bo'linmaydi. Shuning uchun berilgan taqqoslama yechimga ega emas.

c). $20x \equiv 10(mod\ 25)$ taqqoslamada $(10,35) = 5$ va 25 soni 5 gabo'linadi. Demak, taqqoslama 5 ta yechimga ega berilgan taqqoslamaning ikkala tomonini va modulini 5ga bo'lib, $4x \equiv 2(mod\ 5)$ taqqoslamani hosil qilamiz. Bundan $4x \equiv (2 + 5 \cdot 2)(mod\ 5) \to 12(mod\ 5) \to x \equiv 3(mod\ 5)$. Demak, taqqoslamaning yechimlari $x \equiv 3, 3, 13, 18, 23(mod\ 15)$, ya'ni $x = 3 + 25t\ , x = 8 + 25t\ ,$ $x = 13 + 25t, x = 18 + 25t, x = 23 + 25tt \in \mathbb{Z}$ lardan iborat bo'ladi.

d). $10x \equiv 25(mod\ 35)$ taqqoslamada $(10\ , 35) = 5$ va 25 soni 5 ga bo'linadi. Shuning uchun ham berilgan taqqoslama 5 ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 5 ga qisqartib yagona yechimga ega bo'lgan $2x \equiv 5(mod\ 7)$ taqqoslamaga ega bo'lamiz. Bundan $2x \equiv (5 - 7)(mod\ 7) \to 2x = -2(mod\ 7) \to x = -1(mod\ 7)$. Demak, berilgan taqqoslamaning yechimlari $x \equiv -1, 6, 13, 20, 27(mod\ 7)$, ya'ni $x = -1 + 7t\ , x = 6 + 7t,\ x = 13 + 7t,\ x = 20 + 7t,\ x = 27 + 7t, t \in \mathbb{Z}$dan iborat.

e). $39x \equiv 84(mod\,93)$ va $(39{,}93) = 3$ hamda 84 soni 3ga bo'linadi. Shuning uchun ham berilgan taqqoslama 3 ta yechimga ega. Berilgan taqqoslamani 3 ga bo'lib, yagona yechimga ega bo'lgan $13x \equiv 28(mod\,31)$ taqqoslamaga ega bo'lamiz. Bundan $13x \equiv (28 - 31)(mod\,31) \to 13x \equiv -3(mod\,31), 13x \equiv (-3 - 2 \cdot 31(mod\,31),\ 13x = -65(mod\,31) \to x \equiv -5(mod\,31)$. Demak, topilgan yechimlar $x \equiv -5, 26, 57(mod\,93)$, ya'ni

$x = -5 + 93t\,, x = 26 + 93t\,, x = 57 + 93t, t \in \mathbb{Z}$.

f). $90x + 18 \equiv 0(mod\,138)$ dan $90x \equiv -18(mod\,138)$ bo'lgani uchun (90,138)=6 va -18 soni 6 ga bo'linadi. Demak, berilgan taqqoslama 6 ta yechimga ega. Berilgan taqqoslamani 6ga bo'lib yagona yechimga ega bo'lgan $15x \equiv -3(mod\,23)$ taqqoslamaga kelamiz. Bundan $15x \equiv (-3 + +23)(mod\,23) \to 15x \equiv 20(mod\,23)$. Bu yerda $(15{,}20) = 5$va $(23{,}5) = 1$ bo'lgani uchun $3x \equiv 4(mod23) \to 3x \equiv (4 + 23)(mod\,23) \to x \equiv 9(mod\,23)$ni hosil qilamiz. Demak, berilgan taqqoslamaning yechimlari

$x \equiv 9,\ 32,\ 55,\ 78, 101, 124(mod\,138)$,ya'ni$x = 9 + 138t,\ x = 32 + +138t,$ $x = 55 + 138t, x = 78 + 138t,\ x = 101 + 138t, x = 124 + 138t, t \in \mathbb{Z}$ dan iborat.

g). Bu yerda $(15, 35) = 5$ va 55 soni 5 ga bo'linadi. Demak, berilgan taqqoslama 5ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 5 ga qisqartirib, $3x \equiv 7(mod11)$ ni hosil qilamiz. Buni taqqoslamalarning xossalaridan foydalanib, koeffitsientlarini almashtirish usuli bilan yechamiz. U holda $3x \equiv 7 + 11(\mathrm{mod}\ 11), \quad x \equiv 6(\mathrm{mod}\ 11).$ Bundan berilgan taqqoslamaning yechimlari $x \equiv 6, 17, 28, 39, 50\ (mod55)$ ekanligi kelib chiqadi.

262. a). Ma'lumki, $ax \equiv b(modm)$ taqqoslama $ax = b + my,\ y \in Z,$ ga teng kuchli. Bundan $ax - my = b\,,\ x, y \in Z$ tenglamani hosil qilamiz. Shunday qilib $ax + m(-y) = b\,,\ y \in Z,$ tenglama $ax \equiv b(modm)$ taqqoslamaga teng kuchli ekan. Shunga asosan $5x + 4y = 3 \leftrightarrow 5x \equiv 3(mod\,4) \leftrightarrow (5 - 4)x \equiv 3(mod\,4) \to x \equiv 3(mod\,4)$, ya'ni $x = 3 + 4t$, bu holda $4y = 3 - 5x$ bo'lgani uchun $4y = 3 - 5(3 + 4t) = 3 - 15 - 20t = -12 - 20t$, bundan $y = -3 - 5t, t \in \mathbb{Z}$ berilgan tenglama yechimi.

Tekshirish: $5(3 + 4t) + 4(-3 - 5t) = 15 + 20t - 12 - 20t = 3,$ ya'ni topilgan yechimlar tenglamani qanoatlantiradi.

b). $17x + 13y = 1$ dan $17x \equiv 1(mod\,13) \to 4x \equiv 1(mod\,13) \to 4x \equiv -12(mod\,13) \to x \equiv -3(mod\,13) \to x \equiv -3(mod\,13) \to x =$
$= -3 + 13t, t \in \mathbb{Z}$.

Endi y ni aniqlaymiz. Berilgan tenglamadan$13y = 1 - 17x = 1 - 17(-3 + 13t) = 52 - 221t$. Bundan $y = 4 - 17t\,, t \in \mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x = -3 + 13t, y = 4 - 17t\,, t \in \mathbb{Z}$.

Tekshirish: $17(-3+13t)+13(4-17t)=-51+52=1$. Demak, topilgan yechimlar berilgan tenglamani qanoatlantiradi.

c). $91x-28y=35$tenglamada $91=7\cdot 13;\ 28=2^2\cdot 7$, ya'ni $(91,28)=7$va 35 soni 7ga bo'linadi. Demak, taqqoslamaning ikkala tomonini 7ga bo'lsak, $13x-4y=5$ tenglama hosil bo'ladi. Bundan $13x\equiv 5(mod 4)$. Shunday qilib yechimlar $x=1+4t, y=2+13t,\ t\in\mathbb{Z}$.

Tekshirish: $13(1+4t)-4(2+13t)=13-8=5$. Demak, topilgan yechim berilgan tenglamani qanoatlantiradi.

d). $2x+3y=4\rightarrow 2x\equiv 4(mod\ 3)\rightarrow (2,3)=1$ va $x\equiv 2(mod\ 3)$, ya'ni$x=2+3t, t\in\mathbb{Z}$. Endi y ni aniqlaymiz. $2(2+3t)+3y=4,\ 3y=-6t$,bundan$y=-2t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=2+3t, y=-2t, t\in\mathbb{Z}$.

Tekshirish: $2(2+3t)+3(-2t)=4$. Demak topilgan yechim berilgan tenglamani qanoatlantiradi.

e). $4x-3y=2\rightarrow 4x\equiv 2(mod\ 3)\rightarrow (4-3)x\equiv 2(mod\ 3)\rightarrow x\equiv 2(mod\ 3)$, ya'ni $x=2+3t, t\in\mathbb{Z}$.Endi x ning qiymatini tenglamaga qo'yib y ni aniqlaymiz. $y=2+4t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=2+3t,\ y=2+4t, t\in\mathbb{Z}$.

Tekshirish: $4(2+3t)-3(2+4t)=2$, ya'ni topilgan yechim berilgan tenglamani qanoatlantiradi.

f). $3x-7y=1\rightarrow 3x\equiv 1(mod\ 7)\rightarrow (3-7)x\equiv (1+7)(mod\ 3)\rightarrow$

$\rightarrow -4x\equiv 8(mod\ 3)\rightarrow x=-2(mod 7)$, ya'ni $x=-2+7t, t\in\mathbb{Z}$. U holda $7y=3x-1=3(-2+7t)-1=-7+21t$, yoki bundan $y=-1+3t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=-2+7t,\ y=-1+3t,\ t\in\mathbb{Z}$.

Tekshirish: $3(-2+7t)-7(-1+3t), t\in\mathbb{Z}$, ya'ni topilgan yechim berilgan tenglamani qanoatlantiradi.

g). $7x\equiv 11(mod 6)\rightarrow (7-6)x\equiv (11-6)(mod 6)\rightarrow x\equiv 5(mod 6)$. Bundan $x=5+6t,\ t\in Z$. Buni berilgan tenglamaga qo'ysak, $7(5+6t)+6y=11\ \rightarrow 6y=11-35-42t\ \rightarrow y=-4-7t,\ t\in Z$. Demak berilgan tenglamaning yechimi $x=5+6t,\ y=-4-7t,$
$t\in Z$.

263. a) Avvalo 6-misoldagi singari berilgan $8x-13y=-6$ aniqmas tenglamaning butun sonlardagi umumiy yechimini aniqlaymiz. $8x\equiv -6(mod\ 3)\rightarrow 4x\equiv\equiv -3(mod\ 13)\rightarrow 4x\equiv 10(mod\ 13)\rightarrow 2x\equiv 5(mod\ 13)\rightarrow 2x\equiv 18(mod\ 13)\rightarrow x\equiv 9(mod\ 13)$ya'ni$x=9+13t, t\in\mathbb{Z}. x$ ning topilgan qiymatini berilgan tenglamaga qo'yib y ni topamiz. $13y=8(9+13t)+6,\ y=6+8t, t\in\mathbb{Z}$. Shunday qilib berilgan to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar

$x = 9 + 13t,\ y = 6 + 8t, t \in \mathbb{Z}$ ekan. Endi bular orasidan $-100 \le x \le 150$ shartni qanoatlantiruvchilarni ajratib olamiz. $-100 \le 9 + 13t \le 150 \to -109 \le 13t \le 141 \to -\frac{109}{13} \le t \le \frac{141}{13} \to -8{,}38 \le t \le 10{,}85, t \in \mathbb{Z}$. Bu oraliqdagi butun sonlar soni $10 + 8 + 1 = 19$ ta.

b). $5x - 7y = 8$ tenglamaning umumiy yechimini topamiz $5x \equiv 8(mod\ 7) \to (5 - 7)x \equiv 8(mod\ 7) \to -2x \equiv 8(mod\ 13) \to x \equiv -4(mod\ 7)$, ya'ni $x = -4 + +7t, t \in \mathbb{Z}$. Endi y ni aniqlaymiz. $7y = 5(-4 + 7t) - 8$ bundan$y = -4 + 5t, t \in \mathbb{Z}$. Demak, berilgan to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar $x = -4 + +7t, y = -4 + 5t, t \in \mathbb{Z}$. Endi bular orasidan $1 \le x \le 200$ shartni qanoatlantiruvchilarini ajratib olamiz. $1 \le -4 + 7t \le 200 \to 5 \le 7t \le 204 \to \frac{5}{7} \le t \le \frac{204}{7} \to 0{,}7 \le t \le 29{,}1\ , t \in \mathbb{Z}$. Bu oraliqdasi t ning butun qiymatlari 29 ta. Shunday qilib, $x = 1$ va $x = 100$ to'g'ri chiziqlar orasida joylashgan $5x - 7y = 8$ to'g'ri chiziqdagi butun koordinatali nuqtalar soni 29 ta ekan.

264. a) $f(x) = \frac{9x-1}{7}$ funksiya`ning butun bo'lishi uchun $9x - 1$ ifoda 7 ga bo'linishi kerak, ya'ni $9x - 1 \equiv 0(mod\ 7)$ bajarilishi kerak. Bundan $9x \equiv 1(mod\ 7)$. Demak, $x = 4 + 7\mathrm{t}$ qiymatlarida $f(x)$ funksiya butun qiymat qabul qiladi. Haqiqatdan ham $f(4 + 7t) = \frac{9(4+7t)-1}{7} = \frac{35+63t}{7} = 5 + 9t\ , t \in \mathbb{Z}$.

b). $f(x) = \frac{9x-1}{15}$dan$7x \equiv 1(mod\ 15) \to 7x \equiv$(1-15)$(mod\ 15) \to 7x \equiv -14(mod\ 15) \to x \equiv -2(mod\ 15)$, ya'ni $x = -2 + 15t\ , t \in \mathbb{Z}$. Haqiqatan ham $f(-2 + 15t) = \frac{7(-2+15t)-1}{15} = \frac{-15+105t}{15} = -1 + 7t\ ,$

$t \in \mathbb{Z}$.

c). $2x \equiv 1(mod\ 11) \to 2x \equiv$12$(mod\ 11) \to x \equiv 6(mod\ 11)$, ya'ni

$x = 6 + 11t\ , t \in \mathbb{Z}$. Bu qiymatda $f(6 + 11t) = \frac{2(6+11t)-1}{11} = \frac{-11+22t}{11} = -1 + 2t\ , t \in \mathbb{Z}$.

265. a). 60 kg lik qoplar sonini x, 80 kg lik qoplar sonini esa y bilan belgilab masalani $60x + 80y = 440$ tenglamaning natural sonlardagi yechimlarini topishga keltiramiz. Hosil bo'lgan tenglamani 20 ga qisqartib, $3x + 4y = 22$ tenglamaga keltiramiz va bu tenglamani 6-misoldagi usul bilan yechamiz. $3x \equiv 22(mod\ 4) \to 3x \equiv 2(mod\ 4) \to 3x \equiv 6(mod\ 4) \to x \equiv 2(mod\ 4)$,ya'ni $x = 2 + 4t\ , t \in \mathbb{Z}$ ni hosil qilamiz. Endi y ni aniqlaymiz. $4y = 22 - 3x = 22 - 3(2 + 4t) = 16 - 12t$yoki bundan $y = 4 - 3t\ ,\ t \in \mathbb{Z}$. Endi $x = 2 + 4t\ , y = 4 - 3t\ , t \in \mathbb{Z}$ umumiy yechimdan masalaning shartini qanoatlantiruvchi natural yechimlarni ajratib olamiz.$t = 0$ da $x = 2, y = 4$, t=1 da $x = 6, y = 1$ lardan boshqa x va y larning natural qiymatlari yo'q. Demak, 2 ta 60 kg lik va 4 ta 80 kg lik qopyoki 6 ta 60 kg lik va 1 ta 80 kg lik qop kerak bo'lar ekan.

b) agar 30 so'mlik markalar sonini x bilan, 50 so'mlik markalar sonini y bilan belgilasak. Bu masalani yechishni $30x + 50y = 1490$ tenglamani natural sonlarda yechishga keltiriladi. Bundan$3x + 5y = 149 \rightarrow 3x \equiv 149(mod\ 5) \rightarrow 3x \equiv 4(mod\ 5) \rightarrow 3x \equiv 9(mod\ 4) \rightarrow x \equiv 3(mod\ 5)$,ya'ni$x = 3 + 5t\ , t \in \mathbb{Z}$ni hosil qilamiz. Topilgan qiymatni tenglamaga qo'yib, y ni topamiz. $5y = 149 - 3x = 149 - 3(3 + 5t) = 140 + 15tdany = 28 - 3t\ , t \in \mathbb{Z}$.

Endi topilgan $x = 3 + 5t\ y = 28 - 3t\ , t \in \mathbb{Z}$ umumiy yechimlardan masalaning shartini qanoatlantiruvchi natural yechimlarini ajratib olamiz.

$t = 0$ da $x = 3, y = 28$, $t = 1$ da $x = 8, y = 25$, $t = 2$ da $x = 13, y = 22, t = 3$ da $x = 18$, $y = 19$, $t = 4$ da $x = 23, y = 16$, $t = 5$ da $x = 28, y = 13$,

$t = 6$ da $x = 33, y = 10$, $t = 7$ da $x = 38, y = 7, t = 8$ da $x = 43, y = 4$, $t = 9$ da $x = 48, y = 1$.

Demak, markalarni 9 xilda turlicha qilib xarid qilish mumkin ekan.

c). 200 so'mlik daftarlar sonini x bilan, 250 so'mlik daftarlar sonini y bilan belgilasak, $200x + 250y = 6000$ aniqmas tenglama hosil bo'ladi. Bundan $20x + 25y = 600 \ \rightarrow\ 4x + 5y = 120\ \rightarrow 4x \equiv 120(mod5) \rightarrow 4x \equiv 0(mod5)\ \rightarrow x \equiv 0(mod5) \rightarrow x = 5t$; $4 \cdot 5t + 5y = 120\ \rightarrow \mathrm{y} = 24 - 4\mathrm{t},\ \mathrm{t} \in \mathrm{Z}$. Masalaning javobini jadval ko'rinishida yozamiz.

t	0	1	2	3	4	5	6
x	0	5	10	15	20	25	30
y	24	20	16	12	8	4	0

266. a) 523 sonining o'ng tomoniga yozilgan 3 ta raqamdan hosil bo'lgan sonni x bilan belgilasak, u holda$523 \cdot 10^3 + x \equiv 0(mod\ 7 \cdot 8 \cdot 9)$ bajarilishi kerak. Bundan $x \equiv -523000\ (mod\ 504) \equiv -(1038 \cdot 504 - 152)(mod\ 504) \equiv 152(mod\ 504)$, yoki $x = 152 + 504t, t \in \mathbb{Z}. x$ uch xonali son bo'lgani uchun $t = 0$ da $x = 152$, t=1 da $x = 656$ bo'lishi mumkin.

Tekshirish: 523152 soni 7, 8, 9, larga bo'linadi, shuningdek, 523656 soni ham $7, 8, 9$ larga bo'linadi.

b). 32 sonining o'ng tomoniga yozilgan 2 ta raqamli sonni x bilan belgilasak, u holda $32 \cdot 10^2 + x \equiv 0(mod\ 7 \cdot 3) \rightarrow x \equiv -3200(mod\ 21) \equiv -3200 + 21 \cdot$ $\cdot\ 153(\mathrm{mod}\ 21) \equiv -32(\mathrm{mod}\ 21)\ \equiv 13(mod\ 21)$ yoki $x = 13 + 21t, t \in \mathbb{Z}$.

Bu yerda x ikkita raqamdan tuzilgan son bo'lgani uchun $t = 0$ da $x = 13$, $t = 1$ da x=34, t=2 da $x = 55$, t=3 da $x = 76$,t=4 da $x = 97$. Demak, izlanayotgan sonlar 3213, 3234, 3255, 3276, 3297 lardan iborat. Bularning hammasi 3 va7 ga bo'linadi.

IV.3-§.

267. 1). Birinchi taqqoslamani $x = 6 + 15t_1,\ t_1\epsilon Z$ tenglik ko`rinishida yozib olib, 2-taqqoslamadagi $x$ ning joyiga olib borib qo`yamiz va t_1 ga nisbatan yechamiz:$6 + 15t_1 \equiv 18(mod\ 21) \to 15t_1 \equiv 12(mod\ 21)$. Bunda $(15, 21) = 3$ va $12 \vdots 3$ bo`lgani uchun taqqoslamani 3 ga qisqartirib, $5t_1 \equiv 4(mod\ 7)$ ni hosil qilamiz. Bundan $5t_1 \equiv 4 + 3 \cdot 7)(mod\ 7) \to 5t_1 \equiv 25(mod\ 7) \to t_1 5(mod\ 7)$, ya`ni $t_1 = 5 + 7t_2,\ \ t_2\epsilon Z.$ t_1 ning bu ifodasini $x = 6 + 15t_1$ ga qo`ysak, $x = 6 + 15(5 + 7t_2) = 81 + 105t_2, t_2\epsilon Z$ hosil bo'ladi. Endi $x$ ning ifodasini 3-taqqoslamaga qo`yib t_2 ni aniqlaymiz: $81 + 105t_2 \equiv 3(mod\ 12) \to 105t_2 \equiv -78(mod\ 12) \to (105, 12) = 3$ va $78 \vdots 3$ bo`lgani uchun taqqoslamani 3 ga qisqartirib $35t_2 \equiv -26(mod\ 4)$ ni,bundan esa $3t_2 \equiv 2(mod\ 4) \to 3t_2 \equiv 6(mod\ 4) \to t_2 \equiv 2(mod\ 4)$, ya`ni $t_2 = 2 + 4t_3, t_3\epsilon Z$ ni hosil qilamiz. Shunday qilib $x = 81 + 105(2 + 4t_3) = 291 + 420t_3$, ya`ni $x = 291 + 420t_3,\ t_3\epsilon Z$ berilgan sistemaning yechimiga ega bo`lamiz.

2). $x \equiv 13(mod\ 14) \to x \equiv -1(mod\ 14) \to x = -1 + 4t_1, t_1\epsilon Z$. Buni ikkinchi taqqoslamaga qo`yib, $t_1$ni aniqlaymiz:$-1 + 4t_1 \equiv 6(mod\ 35) \to 4t_1 \equiv 7(mod\ 35) \to 4t_1 \equiv 7 - 35(mod\ 35) \to 4t_1 \equiv -28(mod\ 35) \to t_1 \equiv -7(mod\ 35)$ , ya`ni $t_1 = -7 + 35t_2, t_2\epsilon Z.$ $t_1$ning bu qiymatini $x = -1 + 4t_1$ ga qo`yamiz, u holda $x = -1 + 4(-7 + 35t_2) = -29 + 140t_2$. Endi $x$ ning bu qiymatini 3-taqqoslamaga qo`yib, t_2 ni topamiz: $-29 + 140t_2 \equiv 26(mod\ 45) \to 140t_2 \equiv 55(mod\ 45)$. Bunda $(140, 45) = 5$ va $55 \vdots 5$. Shuning uchun ham bu taqqoslamani 5 ga qisqartirib, $28t_2 \equiv 11(mod\ 9)$ni yoki bundan $t_2 \equiv 2(mod\ 9)$ ni hosil qilamiz. Demak, $t_2 = 2 + 9t_3,\ t_3\epsilon Z$. Shunday qilib $x = -29 + 140(2 + 9t_3) = 251 + 1260t_3,\ t_3\epsilon Z$.

3). $\begin{cases} x \equiv 19(mod\ 56) \\ x \equiv 3(mod\ 24) \\ x \equiv 7(mod\ 20) \end{cases} \to \quad x = 19 + 56t_1, t_1\epsilon Z,\ 19 + 56t_1 \equiv 3(mod\ 24) \to$

$56t_1 \equiv -16(mod\ 24)$. Bunda $(56, 24) = 8$ va $16 \vdots 8$, shuning uchun ham oxirgi taqqoslamaning ikkita tomoni va modulini 8 ga qisqartirilib, $7t_1 \equiv -2(mod\ 3) \to t_1 \equiv 1(mod\ 3)$, ya`ni $t_1 = 1 + 3t_2,\ t_2\ \epsilon Z$ ni hosil qilamiz . Buni $x = 19 + 56t_1$, ga qo`ysak $x = 19 + 56(1 + 3t_2) = 75 + 168t_2$ kelib chiqadi. x ning bu qiymatini 3-taqqoslamaga qo`yib, $t_2$ ni aniqlaymiz. $75 + 168t_2 \equiv 7(mod\ 20) \to 168t_2 \equiv -68(mod\ 20) \to 8t_2 \equiv -8(mod\ 20) \to (8, 20) = 4$ va $8 \vdots 4$ bo`lgani uchun $2t_2 \equiv -2(mod\ 5) \to t_2 \equiv -1(mod\ 5)$, ya`ni $t_2 = -1 + 5t_3,\ t_3\epsilon Z$. Shunday qilib $x = -93 + 840t_3, t_3\epsilon Z$ ni hosil qilamiz .

4). $\begin{cases} x \equiv 4(mod\ 5) \\ x \equiv 1(mod\ 12) \\ x \equiv 7(mod\ 14) \end{cases} \rightarrow \quad x = 4 + 5t_1\ , t_1 \epsilon Z,\ 4 + 5t_1 \equiv 1(mod\ 12) \rightarrow 5t_1 \equiv$
$-3(mod\ 12) \rightarrow 5t_1 \equiv (-3 + 12 \cdot 4)(mod\ 12) \rightarrow 5t_1 \equiv 45(mod\ 12) \rightarrow t_1 \equiv$ $9(mod\ 12)$ ya`ni $t_1 \equiv 9 + 12t_2, t_2\ \epsilon Z$. Bundan $x = 4 + 5(9 + 12t_2) = 49 +$ $60t_2$. $x$ ning bu qiymatini 3-taqqoslamaga qo`yib t_2 ni aniqlaymiz:$49 + 60t_2 \equiv$ $7(mod\ 14) \rightarrow 60t_2 \equiv 7 - 49(mod\ 14) \rightarrow 60t_2 \equiv -42(mod\ 14)$. Bunda $(60; 14) = 2$ va $42 \vdots 2$ bo`lgani uchun $30t_2 \equiv -21(mod\ 7)$, yoki bundan $2t_2 \equiv 0(mod\ 7) \rightarrow t_2 \equiv 0(mod\ 7)$, ya`ni $t_2 = 7t_3\ , t_3 \epsilon Z$. Buni $x = 49 + 60t_2$ ga qo`yib, $x = 49 + 60 \cdot 7t_3 = 49 + 420t_3$ni, ya`ni $x = 49 + 420t_3,\ t_3 \epsilon Z$ ni hosil qilamiz .

5). $x \equiv 13(mod\ 16) \rightarrow x = 3 + 16t_1\ , t_1 \epsilon Z.$ x ning bu qiymatini $2-$ taqqoslamaga qo′yib $t_1$ni aniqlaymiz. $-3 + 16t_1 \equiv 3(mod\ 10) \rightarrow 16t_1 \equiv$ $6(mod\ 10\ )$. Bunda $(16\ , 10) = 2$ va $6 \vdots 2$ bo`lgani uchun $8t_1 \equiv 3(mod\ 5) \rightarrow$ $8t_1 \equiv 8(mod\ 5) \rightarrow t_1 \equiv 1(mod\ 5)$, ya`ni $t_1 = 1 + 5t_2,\ t_2 \epsilon Z.$ x ning bu qiymatini 3-taqqoslamaga qo`yib $t_2$ ni aniqlaymiz: $x = -3 + 16(1 + 5t_2) = 13 + 80t_2$ . Buni 3-taqqoslamaga olib borib qo`yib, t_2 ni topamiz. $13 + 80t_2 \equiv 9(mod\ 14) \rightarrow 80t_2 \equiv$ $-4(mod\ 14)$. Bunda $(80,\ 14) = 24 \vdots 2$ bo`lgani uchun bo`lgani uchun taqqoslamaning ikkala tomoni va modulini 2 ga qisqartirib, $40t_2 \equiv -2(mod\ 7)$ ni, yoki bundan $-2t_2 \equiv -2(mod\ 7) \rightarrow t_2 \equiv 1(mod\ 7)$, ya`ni $t_2 = 1 + 7t_3\ , t_3 \in Z$ ga ega bo`lamiz. Demak, $x = 13 + 80t_2 = 13 + 80(1 + 7t_3) = 93 + 560t_3$, ya`ni $x = 93 + 560t_3\ , t_3 \in Z$ ni hosil qilamiz.

6). $x = 9 + 10t_1\ , 9 + 10t_1 \equiv 10(mod\ 15) \rightarrow 10t_1 \equiv 1(mod\ 15)$. Bundan $(10, 15) = 5$, lekin 1 soni 5 ga bo`linmaydi. Demak taqqoslamalar sistemasi yechimga ega emas .

7). $x = 7 + 9t_1, t_1 \in Z.$ x ning bu qiymatini $2-$ taqqoslamaga
qo′yib $t_1$ni aniqlaymiz. $7 + 9t_1 \equiv 2(mod\ 7) \rightarrow 2t_1 \equiv 2(mod\ 2) \rightarrow t_1 \equiv$ $1(mod\ 7) \rightarrow t_1 = 1 + 7t_2\ , t_2 \in Z$.Demak,$x = 7 + 9(1 + 7t_2) = 16 +$ $63t_2, t_2Z.$ Buni 3 taqqoslamaga olib borib qo`yib, t_2 ni topamiz.

$16 + 63t_2 \equiv 3(mod\ 12) \rightarrow 63t_2 \equiv -13(mod\ 12) \rightarrow 3t_2 \equiv -1(mod\ 12)$. Bunda $(3, 12) = 3$ va lekin 1 soni 3 ga bo`linmaydi. taqqoslamalar sistemasi yechimga ega emas.

8). $x = 5 + 12t_1,\ 5 + 12t_1 \equiv 2(mod\ 8) \rightarrow 12t_1 \equiv -3(mod\ 8) \rightarrow 4t_1 \equiv$ $5(mod\ 8)$ bunda $(4,\ 8) = 4$, lekin 5 soni 8 ga bo`linmaydi shuning uchun ham taqqoslamalar sistemasi yechimga ega emas.

9). $x = 7 + 10t_1, t_1 \in Z.$ x ning bu qiymatini $2-$ taqqoslamaga
qo′yib, $t_1$ni aniqlaymiz. $7 + 10t_1 \equiv 2(mod\ 5), 10t_1 \equiv -5(mod\ 5)$. Bunda $(10, 5) = 5$ va $5 \vdots 5$ bo`lgani uchun oxirgi taqqoslamani 5 ga qisqartirib $2t_1 \equiv$

$-1(mod\ 1)$doimo bajariladigan taqqoslamaga ega bo`lamiz. Demak, $t_1 = t_2$ deb olish mumkin, u holda $x = 7 + 10t_2$ ni hosil qilamiz, buni 3-taqqoslamaga qo`ysak, $7 + 10t_2 \equiv 8(mod\ 9) \to 10t_2 \equiv 1(mod\ 9) \to t_2 \equiv 1(mod\ 9)$, ya`ni $t_2 = 1 + 9t_3, t_3 \in Z$. Bundan $x = 7 + 10t_2 = 7 + 10(1 + 9t_3) = 17 + 90t_3, t_3 \in Z$ ni hosil qilamiz.

10). $x \equiv 8(mod\ 7) \to x \equiv 1(mod\ 7) \to x = 1 + 7t_1, t_1 \in Z$. x ning bu qiymatini $2-$taqqoslamaga qo'yib, $t_1$ni aniqlaymiz. $1 + 7t_1 \equiv 3(mod\ 11) \to 7t_1 \equiv 2(\mathrm{mod}\ 11) \to 7t_1 \equiv (2 + 11 \cdot 3)(mod\ 11) \to t_1 \equiv 5(mod\ 11)$, ya`ni $t_1 = 5 + 11t_2, t_2 \epsilon Z$. Buni $x = 1 + 7t_1$ ga olib borib qo`ysak, $x = 1 + 7(5 + 11t_2) = 36 + 77t_2$. x ning bu qiymtini 3-taqqoslamaga qo`ysak, $36 + 77t_2 \equiv 9(mod\ 13) \to 77t_2 \equiv -27(mod\ 13) \to (77 - 6 \cdot 13)t_2 \equiv (-27 + 2 \cdot 13)(mod\ 13) \to t_2 \equiv -1(mod\ 13) \to t_2 \equiv 1(mod\ 13) \to t_2 = 1 + 13t_3$, $t_3 \epsilon Z$. Shunday qilib $x = 36 + 77(1 + 13t_3) = 113 + 1001t_3, t_3 \epsilon Z$. Ya`ni $x = 113 + 1001t_3, t_3 \epsilon Z$ berilgan sistemaning yechimi .

11). Bu sistemadagi har bir taqqoslama alohida-alohida x ga nisbatan yechilgan holda berilgan. Shuning uchun ham 1-taqqoslamaning yechimlari $x = 2 + 5t_1$, $t_1 \in Z$ larning orasidan 2- taqqoslamani qanoatlantiruvchilarini ajratib olamiz. Buning uchun $\mathrm{x} = 2 + 5t_1$ ni 2-taqqoslamaga qo'yib, t_1 ni aniqlaymiz: $2 + 5t_1 \equiv 8(\mathrm{mod}11) \Rightarrow 5t_1 \equiv 6(\mathrm{mod}11) \Rightarrow 5t_1 \equiv -5(\mathrm{mod}11)$,

$t_1 \equiv -1(\mathrm{mod}11), t_1 = -1 + 11t_2, t_2 \in Z$. $t_1$ning topilgan ifodasini x ga olib borib qo'yamiz. U holda x=2+5(-1+11t_2)=-3+55t_2, ya'ni $x = -3 + 55t_2, t_2 \in Z$ ga ega bo'lamiz. $x$ning bu ifodasini 3-taqqosslamaga olib borib qo'yib, t_2 ni aniqlaymiz.

$$-3 + 55t_2 \equiv 12(\mathrm{mod}15) \Rightarrow 55t_2 \equiv 15(\mathrm{mod}15) \Rightarrow 10t_2 \equiv 0(\mathrm{mod}15)$$

bunda $(10,15) = 5$ bo'lganidan $2t_2 \equiv 0(\mathrm{mod}\,3)\,yoki\quad t_2 \equiv 0(\mathrm{mod}\,3)$, bundan $t_2 = 15t_3, t_2 = 3 + 15t_3, t_2 = 6 + 15t_3, t_2 = 9 + 15t_3, t_2 = 12 + 15t_3, t_3 \in Z$ larni hosil qilamiz . U holda berilgan sistemaning yechimlari: $x_1 = -3 + 825t_3$,

$x_2 = 162 + 825t_3, x_3 = 327 + 825t_3, x_4 = 492 + 825t_3, x_5 = 657 + 825t_3, t_3 \in Z$ ga ega bo'lamiz.

268.1). Bizning misolimizda $m_1 = 6$,$m_2 = 7, m_3 = 11$, $M_1 = 77, M_2 = 66$, $M_3 = 42$ $\left(M_i = \frac{M}{m_i}\right)$, $b_1 = 1$,$b_2 = 2$,$b_3 = 3$, M_i' larni $M_iM_i' \equiv 1(mod\ m_i)$ $(i = 1, 2, 3, \dots)$ taqqoslamadan aniqlaymiz. $77M_1' \equiv 1(mod6) \to 5M_1' \equiv 1(mo\square\ 6) \to 5M_1' \equiv (1 + 4 \cdot 6)(mod\ 6\ ) \to M_1' \equiv 5(mod\ 6)$. Demak, $M_1' = 5$; $66M_2' \equiv 1(mod\ 7) \to 3M_2' \equiv 1(mod\ 7) \to 3M_2' \equiv -6(mod\ 7) \to M_2' \equiv -2(mod\ 7)$; $M_2' = -2$;

$42M_3' \equiv 1(mod\ 11) \to -2M_3' \equiv 12(mod\ 11) \to M_3' \equiv 6(mod\ 11)$, $\square_3' = -6$ deb olishimiz mumkin. Endi $x_0 = M_1M_1'b_1 + M_2M_2'b_2 + M_3M_3'b_3(1)$ formuladan x_0 ni aniqlaymiz. $x_0 = 77 \cdot 5 \cdot 1 + 66 \cdot (-2) \cdot 2 + 42 \cdot (-6) \cdot 3 = 385 - 264 + 756 == -635$. Demak, sistemaning yechimi $x \equiv -635(mod\ 462) \equiv 289(mod\ 462)$.

2). Avvalo berilgan $2x \equiv 1(mod\ 5), x \equiv 2(mod\ 7), 3x \equiv 4(mod\ 11)$ taqqoslamalarni x ga nisbatan yechib olamiz. U holda

$\begin{cases} x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 7) \\ x \equiv 5(mod\ 11) \end{cases}$ sistemaga ega bo`lamiz va bu sistemani

1-misoldagi singari mulohaza yuritib yechamiz. Bunda $M = 385,\ M_1 = 77,\ M_2 = 55,\ M_3 = 35, b_1 = 3, b_2 = 2, b_3 = 5.$ M_i' larni aniqlaymiz. $77M_1' \equiv 1(mod\ 5) \rightarrow 2M_1' \equiv 1(mod\ 5) \rightarrow M_1' = 3$; $55M_1' \equiv 1(mod\ 7) \rightarrow -M_2' \equiv 1(mod\ 7) \rightarrow M_2' = -1$; $35M_3' \equiv 1(mod\ 11) \rightarrow 2M_3' \equiv 1(mod\ 11) \rightarrow M_3' = 6$; Endi x_0 ni aniqlaymiz.
$x_0 = 77 \cdot 3 \cdot 3 + 55 \cdot (-1) \cdot 2 + 35 \cdot 6 \cdot 5 = 693 - 110 + +1050 = 1633$ va
$x \equiv 1633(mod\ 385) \equiv 93(mod\ 385)$. Demak, berilgan sistemaning yechimi
$x \equiv 93(mod\ 385)$.

4) 2-misoldagi singari mulohaza yuritib, berilgan sistemaning $x \equiv 6(mod\ 17), x \equiv 2(mod\ 5), x \equiv -2(mod\ 9)$ ko`rinishga keltirib olamiz. Bunda $m_1 = 17, m_2 = 5, m_3 = 9$ va $M = 765, M_1 = 45, M_2 = 153, M_3 = 85,\ b_1 = 6,\ b_2 = 12,\ b_3 = -2.$ $M_1',\ M_2', M_3'$ larni aniqlaymiz. $45M_1' \equiv 1(mod\ 17) \rightarrow 11M_1' \equiv 1(mod\ 17) \rightarrow 11M_1' = (1 - 17 \cdot 2)(mod\ 17) \rightarrow M_1' \equiv -3(mod\ 17) \rightarrow M_1' = -3$;
$153M_2' \equiv 1(mod\ 5) \rightarrow 3M_2' \equiv 6(mod\ 5) \rightarrow M_2' = 2.$

$85M_3' \equiv 1(mod\ 9) \rightarrow 4M_3' \equiv 10(mod\ 9) \rightarrow 2M_3' = -2.$

Bularga asosan

$x_0 = 45 \cdot (-3) \cdot 6 + 153 \cdot 2 \cdot 2 + 85 \cdot (-2) \cdot (-2) = -810 + 612 + 340 = 142.$
Demak, berilgan sistemaning yechimi $x \equiv 142(mod\ 765)$.

5) Yuqoridagi misollar singari mulohaza yuritib, berilgan sistemani $x \equiv 4(mod\ 9)$, $x \equiv 4(mod\ 13), x \equiv 6(mod\ 11)$ ko`rinishiga keltirib olamiz. Bunda $m_1 = 9, m_2 = 13, m_3 = 11$ va $M = 1287,\ M_1 = 143,\ M_2 = 99,\ \ M_3 = 117,\ b_1 = 4,\ b_2 = 4,\ b_3 = 6.$ $M_1',\ M_2', M_3'$ larni aniqlaymiz.$143M_1' \equiv 1(mod\ 9) \rightarrow -M_1' \equiv 1(mod\ 9) \rightarrow M_1' = -1(mod\ 9) \rightarrow M_1' = -3$;$99M_2' \equiv 1(mod\ 13) \rightarrow 8M_2' \equiv 14(mod\ 13) \rightarrow 4M_2' \equiv 7(mod\ 13) \rightarrow M_2' \equiv 5(mod\ 13) \rightarrow M_2' = 5.117M_3' \equiv 1(mod\ 11) \rightarrow -5M_3' \equiv 1(mod\ 11) \rightarrow 6M_3' \equiv 12(mod\ 11) \rightarrow M_3' = 2$. Bularga asosan

$x_0 = 143 \cdot (-3) \cdot 4 + 99 \cdot 4 \cdot 5 + 117 \cdot 2 \cdot 6 = -1716 + 1980 + 1404 = 1668.$
Demak, berilgan sistemaning yechimi $x \equiv 1668(mod\ 1287) \equiv 381(mod 1287)$.

5). $\begin{cases} 6x \equiv 1(mod\ 35) \\ 3x \equiv 4(mod\ 17) \\ 10x \equiv 7(mod\ 13) \end{cases} \leftrightarrow \begin{cases} x \equiv 6(mod\ 35) \\ x \equiv 7(mod\ 17). \\ x \equiv 2(mod\ 13) \end{cases}$

Bundan $m_1 = 35, m_2 = 17, m_3 = 13$ va $M = 7735, M_1 = 221, M_2 = 455, M_3 = 595,\ b_1 = 6,\ b_2 = 7,\ b_3 = 2.\ M_1', M_2', M_3'$ larni aniqlaymiz. $221M_1' \equiv 1(mod\,35) \to (221 - 6 \cdot 35)M_1' \equiv 1(mod\,35) \to 11M_1' \equiv 1(mod\,35) \to 24M_1' \equiv 3(mod\,35) \to -2M_1' \equiv 3(mod\,35) \to M_1' = 16; 455M_2' \equiv 1(mod\,17)$
$\to -4M_2' \equiv -16(mod\,17) \to M_2' \equiv 4; 595M_3' \equiv 1(mod\,13) \to (595 - 13 \cdot 46)M_3' \equiv 1(mod\,13) \to -3M_3' \equiv -12(mod\,13) \to M_3' \equiv 4$. Endi x_0 ni aiqlaymiz.

$x_0 = 221 \cdot 16 \cdot 6 + 455 \cdot 4 \cdot 7 + 595 \cdot (-1) \cdot 2 = 21216 + 12740 + 4760 = 38716$ va demak $x \equiv 38716(mod\,7735) \equiv (38716 - 5 \cdot 7735)(mod\,7735) \equiv (mod\,7735)$. Shunday qilib, $x \equiv 41(mod\,7735)$ berilgan sistemaing yechimi.

6). $\begin{cases} 8x \equiv 7(mod\,17) \\ 5x \equiv 11(mod\,6) \\ x \equiv -1(mod\,19) \end{cases} \leftrightarrow \begin{cases} x \equiv 3(mod\,17) \\ x \equiv 1(mod\,6) \\ x \equiv -1(mod\,19) \end{cases}.$

Bundan $m_1 = 17, m_2 = 6, m_3 = 19$ va $M = 1938, M_1 = 114, M_2 = 323, M_3 = 102, b_1 = 3, b_2 = 1, b_3 = -1.\ \ M_1', M_2', M_3'$ larni aniqlaymiz.$114M_1' \equiv 1(mod\,17) \to (114 - 7 \cdot 17)M_1' \equiv 1(mod\,) \to -5M_1' \equiv 35(mod\,17) \to M_1' = -7; 323M_2' \equiv 1(mod\,6) \to (323 - 54 \cdot 6)M_2' \equiv 1(mod\,17) \to M_2' \equiv -1; 102M_3' \equiv 1(mod\,19) \to (102 - 19 \cdot 5)M_3' \equiv 1(mod\,19) \to 7M_3' \equiv 1(mod\,19) \to 7M_3' \equiv (1 - 3 \cdot 19)(mod\,19) \to M_3' \equiv -8(mod\,19) \to M_3' \equiv -8$. Endi x_0 ni aiqlaymiz.

$x_0 = 114 \cdot (-7) \cdot 3 + 323 \cdot (-1) \cdot 11 + 102 \cdot (-8) \cdot (-1) = -2394 - 323 + 816 = -1901.$

Demak, $x \equiv -1901(mod\,1938) \equiv 37(mod\,1938)$, ya\`ni $x \equiv 37(mod\,1938)$ berilgan taqqoslamalar sistemasi yechimi.

7). $\begin{cases} 11x \equiv -4(mod\,18) \\ 7x \equiv 1(mod\,11) \\ 3x \equiv 5(mod\,7) \end{cases} => \begin{cases} -7x \equiv 14(mod\,18) \\ -4x \equiv 12(mod\,11) \\ 3x \equiv 12(mod\,7) \end{cases} => \begin{cases} x \equiv -2(mod\,18) \\ x \equiv -3(mod\,11). \\ x \equiv 4(mod\,7) \end{cases}$

Bundan $m_1 = 18, m_2 = 11, m_3 = 7$ va $M = 1386,\ M_1 = 77,\ M_2 = 126,\ M_3 = 198, b_1 = -2, b_2 = -3, b_3 = 4\ \ M_1', M_2', M_3'$ larni aniqlaymiz.$77M_1' \equiv 1(mod\,18) \to 5M_1' \equiv 1(mod\,18) \to 5M_1' \equiv (1 - 2 \cdot 18)(mod\,18) \to M_1' = -7; 126M_2' \equiv 1(mod\,11) \to (126 - 11 \cdot 11)M_2' \equiv 1(mod\,11) \to 5M_2' \equiv 1(mod\,11) \to -6M_2' \equiv 12(mod\,11) \to M_2' \equiv -2(mod\,11) \to M_2' = -2; 198M_3' \equiv 1(mod\,7) \to (198 - 28 \cdot 7)M_3' \equiv 1(mod\,7) \to 2M_3' \equiv 1(mod\,7) \to M_3' = 4.$ Endi x_0 ni aniqlaymiz.$x_0 = 77 \cdot (-7) \cdot (-2) + 126 \cdot (-2) \cdot (-3) + 198 \cdot 4 \cdot 4 = 1078 + 756 + +3168 = 5002.$ Demak, $x \equiv 5002(mod\,1386) \equiv 844(mod\,1386)$ berilgan taqqoslamalar sistemasining yechimi.

8). $\begin{cases} 21x \equiv -2(mod\ 23) \\ 12x \equiv 3(mod\ 9) \\ x \equiv 6(mod\ 11) \end{cases} \leftrightarrow \begin{cases} x \equiv 1(mod\ 23) \\ x \equiv 1(mod\ 9) \\ x \equiv 6(mod\ 11) \end{cases}$.

Bundan $m_1 = 23, m_2 = 9, m_3 = 11$ va $M = 2277, M_1 = 99, M_2 = 253, M_3 = 207, b_1 = 1, b_2 = 1, b_3 = 6. M_1', M_2', M_3'$ larni aniqlaymiz.$99M_1' \equiv 1(mod\ 23) \rightarrow (99 - 4 \cdot 23)M_1' \equiv 1(mod\ 23) \rightarrow 7M_1' \equiv 1(mod\ 23) \rightarrow 7M_1' \equiv (1 + 3 \cdot 23)(mod\ 23) \rightarrow 7M_1' \equiv 70(mod\ 23) \rightarrow M_1' = 10;$ $253M_2' \equiv 1(mo\square\ 9) \rightarrow (-28 \cdot 9 + 253)M_2' \equiv 1(mod\ 9) \rightarrow M_2' \equiv 1;$ $207M_3' \equiv 1(mod\ 11) \rightarrow (207 - 19 \cdot 11)M_3' \equiv 1(mod\ 11) \rightarrow M_3' \equiv -6$. Bulardan foydalanib, x_0 ni topamiz.

$x_0 = 99 \cdot 10 \cdot 1 + 253 \cdot 1 \cdot 1 + 207 \cdot (-6) \cdot 6 = 990 + 255 - 7452 = -6209.$
Demak $x \equiv -6209(mod\ 2277)$ berilgan sistemaning yechimi.

9). $\begin{cases} x \equiv 3(mod\ 29) \\ x \equiv -5(mod\ 12) \\ 2x \equiv 7(mod\ 11) \end{cases} \leftrightarrow \begin{cases} x \equiv 3(mod\ 29) \\ x \equiv -5(mod\ 12) \\ x \equiv 9(mod\ 11) \end{cases}$. Bundan $m_1 = 29, m_2 = 12, m_3 = 11$ va $M = 3828, M_1 = 132, M_2 = 319, M_3 = 318, b_1 = 3, b_2 = -5,$ $b_3 = 9. M_1', M_2', M_3'$ larni aniqlaymiz.

$132M_1' \equiv 1(mod\ 29) \rightarrow -13M_1' \equiv 30(mod\ 29) \rightarrow 16M_1' \equiv 30(mod\ 29) \rightarrow 8M_1' \equiv 15(mod\ 29) \rightarrow 8M_1' \equiv 44(mod\ 29) \rightarrow 2M_1' \equiv 11(mod\ 29) \rightarrow M_1' \equiv 2(mod\ 29) \rightarrow M_1' \equiv -9(mod\ 29) \rightarrow M_1' = 9;319M_2' \equiv 1(mod\ 12) \rightarrow (319 - 27 \cdot 12)M_2' \equiv 1(mod\ 12) \rightarrow -5M_2' \equiv (1 + 2 \cdot 12)(mod\ 12) \rightarrow M_2' = -5;348M_3' \equiv 1(mod\ 11) \rightarrow (348 - 11 \cdot 32)M_3' \equiv 1(mod\ 11) \rightarrow -4M_3' \equiv 12(mod\ 11) \rightarrow M_3' \equiv -3.$ Endi x_0 ni aiqlaymiz.$x_0 = 132 \cdot (-9) \cdot 3 + 319 \cdot (-5) \cdot (-5) + 348 \cdot (-3) \cdot 9 = -3564 + 7975 - 9396 = -4985$.

Demak, $x \equiv -4985(mod\ 3828) \equiv 2671(mod\ 3828)$ berilgan taqqoslamalar sistemasining yechimi bo`ladi.

10). $\begin{cases} 6x \equiv 5(mod\ 35) \\ x \equiv -2(mod\ 17) \\ 5x \equiv 3(mod\ 13) \end{cases} \leftrightarrow \begin{cases} x \equiv 6(mod\ 35) \\ x \equiv -2(mod\ 17) \\ x \equiv 6(mod\ 13) \end{cases}$.

Bundan $m_1 = 31, m_2 = 29, m_3 = 27$ va $M = 24273, M_1 = 783, M_2 = 837, M_3 = 899, b_1 = 6, b_2 = -2, b_3 = 6. M_1', M_2', M_3'$ larni aniqlaymiz. $783M_1' \equiv 1(mod\ 31) \rightarrow (783 - 31 \cdot 25)M_1' \equiv 1(mod\ 31) \rightarrow 8M_1' \equiv 32(mod\ 31) \rightarrow M_1' \equiv 4(mod\ 31) \rightarrow M_1' = 4;$

$837M_2' \equiv 1(mod\ 29) \rightarrow (837 - 29 \cdot 29)M_2' \equiv 1(mod\ 29) \rightarrow -4M_2' \equiv 1(mod\ 29) \rightarrow -2M_2' \equiv 15(mod\ 29) \rightarrow M_2' \equiv 22(mod\ 29) \rightarrow M_2 = 7;$

$899M_3' \equiv 1(mod\ 27) \rightarrow (899 - 27 \cdot 33)M_3' \equiv 1(mod\ 27) \rightarrow 8M_3' \equiv 1(mod\ 27) \rightarrow 8M_3' \equiv 28(mod\ 27) \rightarrow 2M_3' \equiv 7(mod\ 27) \rightarrow M_3' \equiv 17(mod\ 27) \rightarrow M_3' \equiv 10.$ Endi x_0 ni aniqlaymiz. $x_0 = 783 \cdot 46 + 837 \cdot 7 \cdot (-2) + 899 \cdot (-10) \cdot$

$6 = 18792 - 11718 - 53940 = 46866$. Bundan $x \equiv -46866(mod\ 24273) \equiv 1680(mod\ 24273)$ berilgan sistemaning yechimi ekanligi kelib chiqadi.

11). Bu yerda $m_1 = 7, m_2 = 9, m_3 = 11$ va $M = 693,\ M_1 = 99,\ M_2 = 77,\ M_3 = 63,\ b_1 = 1, b_2 = 3, b_3 = 5$.Endi M_1', M_2', M_3' larni aniqlaymiz.

$99M_1' \equiv 1(mod 7) \rightarrow (99 - 7 \cdot 14)M_1' \equiv 1(mod\ 7) \rightarrow M_1' \equiv 1(mod\ 7) \rightarrow M_1' = 1; 77M_2' \equiv 1(mod\ 9) \rightarrow (77 - 9 \cdot 8)M_2' \equiv 1(mod\ 9) \rightarrow 5M_2' \equiv 1(mod\ 9) \rightarrow 5M_2' \equiv 10(mod\ 9) \rightarrow M_2' \equiv 2(mod\ 9) \rightarrow M_2' = 2; 63M_3' \equiv 1(mod\ 11) \rightarrow (63 - 11 \cdot 6)M_3' \equiv 1(mod\ 11) \rightarrow -3M_3' \equiv 1(mod\ 11) \rightarrow -3M_3' \equiv 12(mod\ 11) \rightarrow M_3' \equiv -4(mod\ 11) \rightarrow M_3' \equiv 7(mod\ 11) \rightarrow M_3' \equiv 7$. Endi x_0 ni aniqlaymiz.$x_0 = 99 \cdot 1 \cdot 1 + 77 \cdot 2 \cdot 3 + 63 \cdot 7 \cdot 5 = 99 + 462 + 2205 = 2766$. Bundan $x \equiv 2766(mod\ 693) \equiv -6(mod\ 693)$ berilgan sistemaning yechimi ekanligi kelib chiqadi.

269. 1). Bu masala taqqoslamaning ta'rifiga ko'ra shunday x ni topishimiz kerakki, u $\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 2(mod\ 8) \\ x \equiv 3(mod\ 9) \end{cases}$ taqqoslamalar sistemasini qanoatlantiruvchi eng kichik natural son bo'lishi kerak. Berilgan sistemani yechamiz. Buning uchun bizga berilgan sistemada modullar o'zaro tub bo'lganligi sababli 2–misoldagi (1) formuladan foydalansak bo'ladi. Bizda $m_1 = 7, m_2 = 8, m_3 = 9$ va $M = 7 \cdot 8 \cdot 9 = 504,\ M_1 = 72, M_2 = 63,\ M_3 = 56, b_1 = 1, b_2 = 2, b_3 = 3. M_1', M_2', M_3'$ larni aniqlaymiz.$72M_1' \equiv 1(mod\ 7) \rightarrow 2M_1' \equiv 8(\square od\ 7) \rightarrow M_1' \equiv 4; 63M_2' \equiv 1(mod\ 8) \rightarrow -M_2' \equiv 1(mod\ 8) \rightarrow M_2' \equiv 1(mod\ 8) \rightarrow M_2' = -1; 56M_3' \equiv 1(mod\ 8) \rightarrow 2M_3' \equiv 10(mod\ 9) \rightarrow M_3' \equiv 5(mod\ 9) \rightarrow M_3' \equiv 5.$

Bulardan foydalanib, x_0 ning qiymatini aniqlaymiz:

$x_0 = 72 \cdot 41 + 63 \cdot (-1) \cdot 2 + 56 \cdot 5 \cdot 3 = 288 - 126 + 840 = 1002.$

Demak, $x \equiv 1002(mod\ 504) \equiv -6(mod\ 504)$, ya`ni $x = -6 + 504t$ , $t\epsilon Z$ berilgan berilgan sistemaning umumiy yechimi. Endi shular orasidan $x$ ning eng kichik natural son bo`ladigan qiymatini aniqlab olamiz. Agar $t \leq 0$ bo`lsa, $\square < 0$ bo`ladi; $t = 1$ da $x = 498$ izlanyotgan qiymatga ega bo`lamiz.

2). $\begin{cases} x \equiv 1(mod\ 3) \\ x \equiv 2(mod\ 4) \\ x \equiv 3(mod\ 5) \end{cases} \rightarrow$ bundan

$b_1 = 1,\ b_2 = 2,\ b_3 = 3,\ m_1 = 3,\ m_2 = 4,\ m_3 = 5,\ M = 60,\ M_1 = 20,\ M_2 = 15,\ M_3 = 12 . M_1', M_2', M_3'$larni aniqlaymiz.

$20M_1' \equiv 1(mod\ 3) \rightarrow 2M_1' \equiv 1(mod\ 3) \rightarrow M_1' \equiv 2(mod\ 3) \rightarrow M_1' = 2;$

$15M_2' \equiv 1(mod\ 4) \rightarrow -M_2' \equiv 1(mod\ 4) \rightarrow M_2' \equiv -1(mod\ 4) \rightarrow M_2' = -1;$

$12M_3' \equiv 1(mod\ 5) \rightarrow 2M_3' \equiv 6(mod\ 5) \rightarrow M_3' \equiv 3(mod\ 5) \rightarrow M_3' \equiv 5$. Endi x_0 ni topamiz. $x_0 = 20 \cdot 2 \cdot 1 + 15 \cdot (-1) \cdot 2 + 12 \cdot 3 \cdot 3 = 40 - 30 + 108 = 118.$

Demak, sistemaning umumiy yechimi $x \equiv 118(mod\, 60)$, yoki buni $x \equiv -2(mod\, 60)$, ya`ni $x = -2 + 60t$, $t \epsilon Z$ ko`rinishida yozish mumkin. Bundan izlanayotgan eng kichik natural qiymat 58 ga teng ekanligi kelib chiqadi.

3). $\begin{cases} x \equiv 3(mod\, 9) \\ x \equiv 5(mod\, 10) \\ x \equiv 6(mod\, 13) \end{cases}$ → dan $b_1 = 3$, $b_2 = 5$, $b_3 = 6$, $m_1 = 9$, $m_2 = 10$, $m_3 = 13$, $M = 1170$, $M_1 = 130$, $M_2 = 117$, $M_3 = 90$. M_1', M_2', M_3' larni aniqlaymiz.

$130M_1' \equiv 1(mod\, 9) \to (130 - 14 \cdot 9)M_1' \equiv 1(mod\, 9) \to 4M_1' \equiv 10(mod\, 9) \to 2M_1' \equiv 5(mod\, 9) \to M_1' \equiv 7(mod\, 9) \to M_1' = 7; 117M_2' \equiv 1(mod\, 10) \to (117 - 10 \cdot 12)M_2' \equiv -9(mod\, 10) \to M_2' \equiv 3(mod\, 10) \to M_2' = 3; 90M_3' \equiv 1(mod\, 13) \to (90 - 7 \cdot 13)M_3' \equiv 1(mod\, 13) \to M_3' \equiv -1(mod\, 5) \to M_3' = -1$.
Endi x_0 ni topamiz. $x_0 = 130 \cdot 7 \cdot 3 + 117 \cdot 3 \cdot 5 + 90 \cdot (-1) \cdot 6 = 2730 + 1755 - 540 = 3945$. Bu holda umumiy yechim $x \equiv 3945(mod\, 1170)$
$\equiv 435(mod\, 1170)$, ya`ni $x = 435 + 1170t$, $t \epsilon Z$. Bundan eng kichik natural yechim $x = 435$.

4). $\begin{cases} x \equiv 2(mod\, 9) \\ x \equiv 3(mod\, 10) \\ x \equiv 4(mod\, 13) \end{cases}$ → bu sistema 3-misoldagi sistemadan b_1, b_2, b_3 ning qiymatlari bilan farq qiladi. Shuning uchun ham $x_0 = 910 \cdot b_1 + 351 \cdot b_2 - 90b_3$, ya`ni $x_0 = 910 \cdot 2 + 351 \cdot 3 + 90 \cdot 4 = 1820 + 1053 - 360 = 2513$ va $x \equiv 2513(mod\, 1170) \equiv 173(mod\, 1170)$, ya`ni sistemaning umumiy yechimi $x = 173 + 1170t$, $t \epsilon Z$. Eng kichik natural yechim $x = 173$.

5). $\begin{cases} x \equiv 2(mod\, 3) \\ x \equiv 4(mod\, 7) \\ x \equiv 5(mod\, 8) \end{cases}$ → dan $b_1 = 4$, $b_2 = 4$, $b_3 = 5$, $m_1 = 3$, $m_2 = 7$, $m_3 = 8$, $M = 168$, $M_1 = 56$, $M_2 = 24$, $M_3 = 21$. M_1', M_2', M_3'larni aniqlaymiz. $56M_1' \equiv 1(mod\, 3) \to 2M_1' \equiv 1(mod\, 3) \to M_1' \equiv 2(mod\, 3) \to M_1' = 2; 24M_2' \equiv 1(mod\, 7) \to 3M_2' \equiv 1(mod\, 7) \to 3M_2' \equiv 15(mod\, 7) \to M_2' \equiv 5(mod\, 7) \to M_2' = 5; 21M_3' \equiv 1(mod\, 8) \to -3M_3' \equiv 9(mod\, 8) \to M_3' \equiv -3(mod\, 8) \to M_3' \equiv -3$.
Shuning uchun ham $x_0 = 56 \cdot 2 \cdot 2 + 24 \cdot 5 \cdot 4 + 21 \cdot (-3) \cdot 5 = 224 + 480 - 315 = 389$ va $x \equiv 389(mod\, 168) \equiv 53(mod\, 168)$ sistemaning umumiy yechimi. Endi $x = 53 + 166t$, $\square \epsilon Z$ dan eng kichik natural sonni aniqlaymiz. $t = 0$ da $x = 53$ izlanayotgan son.

6). $\begin{cases} x \equiv 4(mod\, 7) \\ x \equiv 9(mod\, 13) \\ x \equiv 1(mod\, 17) \end{cases}$ → dan $b_1 = 4, b_2 = 9, b_3 = 1, m_1 = 7, m_2 = 13$, $m_3 = 17$, $M = 7 \cdot 13 \cdot 17 = 1547$, $M_1 = 221$, $M_2 = 119$, $M_3 = 91$. M_1', M_2', M_3' larni aniqlaymiz. $221M_1' \equiv 1(mod\, 7) \to (221 - 31 \cdot 7)M_1' \equiv 1(mod\, 7) \to 4M_1' \equiv$

$8(mod\ 7) \to M_1' \equiv 2(mod\ 7) \to M_1' = 2; 119M_2' \equiv 1(mod\ 13) \to -(119 - 9 \cdot 13)M_2' \equiv 1(mod\ 13) \to 2M_2' \equiv 14(mod\ 13) \to M_2 \equiv 7(mod\ 13) \to M_2' = 7; 91M_3' \equiv 1(mod\ 17) \to (91 - 5 \cdot 17)M_3' \equiv 1(mod\ 17) \to 6M_3' \equiv -18(mod\ 17) \to M_3' \equiv 3(mod\ 17) \to M_3' \equiv 3.$ Bulardan foydalanib, x_0 ni topamiz. $x_0 = 221 \cdot 2 \cdot 4 + 119 \cdot 7 \cdot 9 + 91 \cdot 3 \cdot 1 = 1786 + 7497 + 273 = 9538.$ Demak,
$x \equiv 9538(mod\ 1547) \equiv (9538 - 6 \cdot 1547)(mod\ 1547) \equiv 256(mod\ 1547)$, ya`ni $x = 256 + 1547t\ , t\epsilon Z$ taqqoslamalar sistemasining umumiy yechimi. Bu holda eng kichik natural yechim 256 dan iborat.

7). $\begin{cases} x \equiv 9\ (mod\ 13) \\ x \equiv 1\ (mod\ 21) \\ x \equiv 13\ (mod 23) \end{cases}$ bo’lgani uchun $b_1 = 9, b_2 = 1, b_3 = -10,\ m_1 = 13,\ m_2 = 21,\ m_3 = 23$ u holda $M = 6279, M_1 = 483,\ M_2 = 299,\ M_3 = 273.$ Endi bulardan foydalanib, M_1', M_2', M_3'larni aniqlaymiz.

$483M_1' \equiv 1(mod\ 13) \to (483 - 37 \cdot 13)$ $M_1' \equiv 1(mod\ 13) \to 2M_1' \equiv 14(mod\ 13) \to M_1' \equiv 7(mod\ 13) \to M_1' = 7;$ $299M_2' \equiv 1(mod\ 21) \equiv (299 - 14 \cdot 21)M_2' \equiv 1(mod\ 21) \to 5M_2' \equiv -20(mod\ 21) \to M_2' \equiv -4(mod\ 21) \to M_2' = -4;\ 273M_3' \equiv 1(mod\ 23) \to (273 - 23 \cdot 12)M_3' \equiv 1(mod\ 23) \to -3M_3' \equiv 1(mod\ 23) \to M_3' \equiv -8(mod\ 23) \to M_3' = -8.$

Bulardan foydalanib, x_0 ni topamiz. $x_0 = 483 \cdot 7 \cdot 9 + 299 \cdot (-4) \cdot 1 + 273 \cdot (-8) \cdot (-10) = 30429 - 1196 + 21840 = 51073 = 8 \cdot 6279 + 841.$

Demak, $x_0 \equiv 841(mod 6279)$, ya`ni $x = 841 + 6279t, t\epsilon Z$ taqqoslamalar sistemasining umumiy yechimi. Eng kichik natural yechim 841 ga teng.

8). $\begin{cases} x \equiv 2(mod 3) \\ x \equiv 4(mod 5) \\ x \equiv 1(mod 8) \end{cases}$ dan $b_1 = -1,\ b_2 = 1,\ b_3 = 1,\ m_1 = 3, m_2 = 5,\ m_3 = 8$ deb olishimiz mumkin. Bu holda $M = 120, M_1 = 40,\ M_2 = 24,\ M_3 = 15.$ Endi M_1', M_2', M_3' larni aniqlaymiz: $40M_1' \equiv 1(mod\ 3) \to M_1' \equiv 1(mod\ 3) \to M_1' = 1; 24M_2' \equiv 1(mod\ 5) \to -M_2' \equiv 1(mod\ 5) \to M_2' \equiv -1(mod\ 5) \to M_2' = -1; 15M_3' \equiv 1(mod 8) \to -M_3' \equiv 1(mod 8) \to M_3' \equiv -1(mod 8) \to M_3' = -1.$

topilganlardan foydalanib x_0 ni hisoblaymiz.

$x_0 = 40 \cdot 1 \cdot (-1) + 24 \cdot (-1) \cdot (-1) + 15 \cdot (-1) \cdot 1 = -40 + 24 - 15 = -31.$

Demak, $x \equiv -31(mod 120) \equiv 89(mod 120)$, ya`ni $x = 89 + 120t, t \in$ Z taqqoslamalar sistemasining umumiy yechimi. Bundan masala shartini qanoatlantiradigan eng kichik natural son 89 ekanligi kelib chiqadi.

9). $\begin{cases} x \equiv 1(mod3) \\ x \equiv 4(mod5) \\ x \equiv 7(mod8) \end{cases}$, bundan ko'rinadiki, bu sistema 8-misoldagi sistemadan faqat $b_1, b_2,\ b_3$ larning qiymatlari bilan farq qiladi. Shuning uchun ham 8-misolda qarab chiqilganiga asosan $x_0 = 40b_1 - 24b_2 - 15b_3 = 40 \cdot 1 - 24 \cdot 4 - 15 \cdot 7 = 40 - 96 - 105 = -167$ va $x \equiv -161(mod120) \equiv -41(mod120) \equiv 79(mod120)$ qaralayotgan taqqoslamalar sistemasining yechimi $x = 79 + 120t, t \in Z$ bo'lganligi uchun masala shartini qanoatlantiruvchi eng kichik natural son 79 bo'ladi.

10). $\begin{cases} x \equiv 4(mod\ 5) \\ x \equiv 6(mod\ 7) \\ x \equiv 1(mod\ 9) \end{cases}$, bo'lgani uchun $b_1 = -1, b_2 = -1, b_3 = 1$ deb olishimiz mumkin. Bizda $m_1 = 5, m_2 = 7, m_3 = 9, M = 315,\ M_1 = 63, M_2 = 45,\ M_3$=35. Endi M_1', M_2', M_3' larni aniqlaymiz.

$63M_1' \equiv 1(mod5) \to 3M_1' \equiv 6(mod5) \to M_1' \equiv 2(mod5) \to M_1' = 2;$
$45M_2' \equiv 1(mod7) \to 3M_2' \equiv 15(mod7) \to M_2' \equiv 5(mod7) \to M_2' = -2;$
$35M_3' \equiv 1(mod9) \to -M_3' \equiv 1(mod9) \to M_3' \equiv 1(mod9) \to M_3' = -1.$

Bularga asosan $x_0 = 63 \cdot 2 \cdot (-1) + 45 \cdot (-2) \cdot (-1) + 35 \cdot (-1) \cdot 1 = -126 + 90 - 35 = -71$ va $x \equiv -71(mod15) \equiv 244(mod315)$.

Shunday qilib izlanayotgan natural son 244 dan iborat.

11). Bu masala taqqoslamaning ta'rifiga ko'ra shunday x ni topishi-miz kerakki, u $\begin{cases} x \equiv 6(mod\ 7) \\ x \equiv 12(mod\ 13) \\ x \equiv 16(mod17) \end{cases}$ taqqoslamalar sistemasini qanoatlantiruvchi eng kichik natural son bo'lishi kerak. Berilgan sistemani yechamiz. Buning uchun bizga berilgan sistemada modullar o'zaro tub bo'lganligi sababli 2 – misolda (1) formuladan foydalansak bo'ladi. Bizda $m_1 = 7, m_2 = 13, m_3 = 17, M = 1517,\ M_1 = 63, M_2 = 45,\ M_3$=35. 6-misolga asosan

$x_0 = 442 \cdot b_1 + 833 \cdot b_2 + 273 \cdot b_3 = 442 \cdot (-1) + 833 \cdot (-1) + 273 \cdot (-1) = -1548.$

Demak, $x \equiv -1548(mod\ 1547) \equiv -1(mod\ 1547)$, ya`ni $x = 1546 + 1547t\ , t\epsilon Z$ taqqoslamalar sistemasining umumiy yechimi. Bu holda eng kichik natural yechim 1546 dan iborat.

270.1). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin.

$$\begin{cases} x \equiv 5(\mathrm{mod}\, 18) \\ x \equiv 8(\mathrm{mod}\, 21) \\ x \equiv a(\mathrm{mod}\, 35), \end{cases} \rightarrow x = 5 + 18□_1, t_1 \in Z, 5 + 18t_1 \equiv 8(mod21) \rightarrow 18t_1 \equiv$$

$3(mod21) \rightarrow 6t_1 \equiv 1(mod7) \rightarrow -t_1 \equiv 1(mod7) \rightarrow t_1 \equiv 1,8,15\ (mod21)$. Bundan $x = 5 + 18(-1 + 21t_2) = -13 + 378t_2, t \in Z$ bo′ladi. Buni 3-tenglamaga qo`ysak, $-13 + 378t_2 \equiv a(mod35) \rightarrow 378t_2 \equiv a + 13(mod35) \rightarrow (378 - 10 \cdot 35)t_2 \equiv a + 13(mod35) \rightarrow 28t_2 \equiv a + 13(mod35)$

Bunda $(28,35) = 7$ va demak, taqqoslama yechimga ega bo'lishi uchun $a + 13 \equiv 0(mod7)$ bajarilishi kerak. Bundan $a \equiv -13(mod7) \rightarrow a \equiv 1(mod\ 7)$, ya`ni $a = 7k + 1, k \in Z$ ko'rinishda bo'lishi kerak ekanligi kelib chiqadi.

Izoh. Masalaning shartida a ning qanday qiymatida berilgan taqqoslamalar sistemasi yechimga ega, deb so'ralgan,(ya'ni sistemaning barcha yechimlarini topish so'ralmagan) shuning uchun ham a ning so'ralgan qiymatini topdik.

2) a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin.

$$\begin{cases} x \equiv a(mod\ 7) \\ x \equiv 2(mod\ 9) \\ x \equiv 7(mod\ 11) \end{cases} \rightarrow x = 7 + 11t_1,\ t_1 \in Z.$$

Bundan $7 + 11t_1 \equiv 2(mod\ 9) \rightarrow 2t_1 \equiv -5(mod\ 9) \rightarrow 2t_1 \equiv 4(\mathrm{mod}9) \rightarrow \mathrm{t}_1 \equiv 2(mod9)$ yoki $t_1 = 2 + 9\mathrm{t}_2,\ t_2 \in Z.$ U holda $x = 7 + 11(2 + 9\mathrm{t}_2) = 29 + 99t_2,\ t_2 \epsilon Z.\ 29 + 99t_2 \equiv a(mod7) \rightarrow 99t_2 \equiv a - 29(mod\ 7) \rightarrow t_2 \equiv a - 1(mod\ 7)$,
bundan $t_2 = (a - 1) + 7t_3$. Buni x ning ifodasiga olib borib qo′ysak, $x = 29 + 99\big((a - 1) + 7t_3\big) = 29 + +99(a - 1) + 693t_3,\ t_3 \epsilon Z.$ Demak, berilgan sistema $a -$ ning ixtiyoriy $a\ \epsilon\ Z$ qiymatlarida yechimga ega.

3). $x \equiv 5(mod\ 12) \rightarrow x = 5 + 12t_1 \rightarrow 5 + 12t_1 \equiv 3(mod\ 15) \rightarrow 12t_1 \equiv -2(mod\ 15) \rightarrow 12t_1 \equiv 13(mod\ 15)$. Bunda$(12,15) = 3$,
lekin 13 soni 3 ga bo′linmaydi. Shuning uchun ham berilgan sistema $a$ning birorta ham qiymatida yechimga ega emas.

4).$x = 11 + 20t_1, t \in Z$ dan $11 + 20t_1 \equiv 1(mod\ 15) \rightarrow 20t_1 \equiv -10(mod\ 15) \rightarrow 20t_1 \equiv 5(mod\ 15) \rightarrow 4t_1 \equiv 1(mod3)$, ya′ni $t_1 = 1 + 3t_2, t_2 \in Z$ va $x = 11 + 20(1 + 3t_2) = 31 + 60t_2, t_2 \in Z$.

3-taqqoslamadan $31 + 60t_2 \equiv a(mod18) \rightarrow 60t_2 \equiv a - 31(mod18) \rightarrow 6t_2 \equiv a - 31(mod18)$, bunda$(6, 18) = 6$ bo′lgani uchun berilgan taqqoslamalar sistemasi yechimga ega bo′lishi uchun $(a - 31) \vdots 6$ bo′lishi, ya'ni $a - 31 \equiv 0\ (mod\ 6)$, yoki bundan $a \equiv 1\ (mod\ 6)$ ning bajarilishi kerak ekanligi kelib chiqadi. Shunday qilib $a \equiv 6k+1, k \in Z$ ko'rinishda bo'lsa, berilgan sistema yechimga ega bo'lar ekan.

5). $x \equiv 19(mod\ 24) \to x = 19 + 24t_1, t_1 \in Z$. x ning bu qiymatini ikkinchi taqqoslamaga qo'yib, t_1 ni aniqlaymiz: $19 + 24t_1 \equiv 10(mod\ 21)$. $3t_1 \equiv -9(mod\ 21\ ) \to t_1 \equiv -3(mod\ 7)$, ya'ni $t_1 = -3 + 7t_2, t_2 \in$ □. Bu holda $x = 19 + 24(-3 + 7t_2 = -53 + 168t_2, t_2 \in Z$. Buni 3-taqqoslamaga qo'yib, t_2 ni aniqlashga harakat qilamiz. $-53 + 168t_2 \equiv a(mod\ 9) \to 168t_2 \equiv a + 53(mod\ 9) \to (168 - 18 \cdot 9)t_2 \equiv a + 53\ \ (mod\ 9) \to 6t_2 \equiv a - 1(mod\ 9)$. Bunda $(6.9) = 3$, demak, u yechimga ega bo'lishi uchun$(a-1) \vdots 3$, ya`ni $a \equiv 1(mod\ 3)$ bajarilishi kerak ekan. Demak, agar $a = 3k+1, k \in Z$ ko'rinishda bo'lsa, berilgan taqqoslamalar sistemasi yechimga ega bo'ladi.

6). $x = 6 + 15t_1, t_1 \in Z$. Buni ikkinchi taqqoslamaga qo'yib, $t_1$ni aniqlaymiz: $6 + 15t_1 \equiv 18(mod\ 21) \to 15t_1 \equiv 12(mod\ 21) \to 5t_1 \equiv 4(mod\ 7) \to 5t_1 \equiv 25(mod\ 7) \to t_1 \equiv 5(mod\ 7)$, ya`ni$t_1 = 5 + 7t_2, t_2 \in Z$. Bu holda $x = 6 + 15(5 + 7t_2) = 81 + 105t_2, t_2 \in Z$. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz. $81 + 105t_2 \equiv a(mod 11) \to 6t_2 \equiv a - 4(mod\ 11)$. Bunda $(6,11) = 1$ bo'lgani uchun a ning ixtiyoriy butun qiymatida berilgan taqqoslama yagona yechimga ega va demak, berilgan taqqoslamalar sistemasi ham a ixtiyoriy butun qiymatida yechimga ega.

7). $x = 19 + 56t_1, 19 + 56t_1 \equiv 3(mod 24) \to 8t_1 \equiv -16(mod\ 24)t_1 \equiv -2(mod\ 3) \to t_1 \equiv 1(mod\ 3), t_1 \in Z$.Ya`ni$t_1 = 1 + 3t_2,\ t_2 \in Z$. Buni inobatga olsak
$x = 19 + 56(1 + 3t_2) =$
$75 + 168t_2$. x ning bu qiymatini 3 taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $75 + 168t_2 \equiv a\ (mod\ 20) \to 8t_2 \equiv a + 5(mod\ 20)$. Bunda $(8; 20) = 4$ va taqqoslama, yechimga ega bo'lishi uchun$(a+5) \vdots 4,\ k \in Z$,ya`ni $a \equiv -5\ (mod\ 4)$. Yoki bundan $a \equiv 3(mod\ 4)$ shartni qanoatlantirishi kerak. Demak, agar $a = 4k+3, k \in Z$ ko'rinishidagi butun son bo'lsa, berilgan taqqoslamaa yechimga ega bo'ladi.

8). $x = 3 + 5t_1, t_1 \in Z$. Buni ikkinchi taqqoslamaga qo'yibt_1 ni aniqlaymiz: $3 + 5t_1 \equiv 2(mod\ 7) \to 5t_1 \equiv -1(mod\ 7) \to 5t_1 \equiv -15(mod\ 7) \to t_1 \equiv -3(mod\ 7)$, ya`ni$t_1 = -3 + 7t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo'ysak , $x = 3 + 5(-3 + 7t_2) = -12 + 35t_2, t_2 \in Z$. $12 + 35t_2 \equiv a(mod\ 9) - t_2 \equiv a + 3(mod\ 9) \to t_2 \equiv -(a+3)(mod\ 9)$. x ning bu qiymatini 3-taqqoslamaga qo'yib, t_2 ni aniqlaymiz: $-12 + 35t_2 \equiv a(mod\ 9) \to$
$-t_2 \equiv a + 3(mod\ 9) \to t_2 \equiv -(a+3)(mod\ 9)$. Demak, $t_2 = -(a+3) + 9t_3$, $t_3 \in$ Z. Bundan $x = -12 + 35[-(a+3) + 9t_3] = -12 - 35(a+3) + 315t_3, t_3$ Z .

Shunday qilib, berilgan taqqoslamalar sistemasi □ ning ixtiyoriy butun qiymatida yechimga ega.

9). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1-misolda tanlangan usuldan foydalanishimiz mumkin. 1-taqqoslamadan $x = 1 + 3t_1, t_1 \in$ Z.x ning bu qiymatini 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz:

$1 + 3t_1 \equiv 5(mod\ 7) \to 3t_1 \equiv 4(mod\ 7) \to t_1 - 1(mod\ 7)$, ya'ni$t_1 = -1 + 7t_2, t_2 \in Z$.

Buni x ning ifodasiga olib borib qo`yib, $x = 1 + 3(-1 + 7t_2) = -2 + 21t_2, t_2 \in Z$ ga ega bo'lamiz. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz:

$-2 + 21t_2 \equiv a(mod\ 11) \to 10t_2 \equiv (a + 2)(mod\ 11) \to -t_2 \equiv (a + 2)(mod\ 11) \to t_2 \equiv -(a + 2)(mod\ 11)$. Demak, a ning ixtiyoriy butun qiymatida berilgan taqqoslamalar sistemasi yechimga ega.

10). 1-taqqoslamadan $x = 14 + 19t_1, t_1 \in Z. x$ ning bu qiymatini 2-taqqoslamaga olib borib qo'yib, t_1 ni aniqlaymiz:$14 + 19t_1 \equiv 5(mod\ 25) \to 19t_1 \equiv -9(mod\ 25) \to -6t_1 \equiv 16(mod 25) \to -3t_1 \equiv 8(mod\ 25) \to -3t_1 \equiv 33(mod\ 25) \to t_1 \equiv -11(mod\ 25) \to t_1 = -11 + 25t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo`yib, $x = 14 + 19(-11 + 25t_2) = -195 + 475t_2, t_2 \in Z$ ga ega bo'lamiz. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz:

$-195 + 475t_2 \equiv a(mod 10) \to 5t_2 \equiv \square + 5(mod 10)$. Bunda $(5; 10) = 5$. Demak, taqqoslama yechimga ega bo'lishi uchun $(a + 5) \vdots 5$, ya`ni $a \equiv -5(mod 5)$ bo'lishi kerak. Bundan $a \equiv 0(mod\ 5)$, ya'ni $a = 5k, k \in Z$. Demak, agar $a = 5k, k \in Z$ ko'rinishda bo'lsa, berilgan sistema yechimga ega bo'ladi .

11). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin.1-taqqoslamadan $x = 5 + 11t_1, t_1 \in Z. x$ ning bu qiymatini 2-taqqoslamaga olib borib qo'yib, t_1 ni aniqlaymiz:$5 + 11t_1 \equiv 4(mod\ 7) \to 4t_1 \equiv -1(mod\ 7) \to -3t_1 \equiv 6(mod 7) \to t_1 \equiv -2(mod 7) \to t_1 = -2 + 7t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo`yib $x = 5 + 11(-2 + 7t_2) = -17 + 77t_2, t_2 \in Z$ ga ega bo'lamiz. $x$ning bu qiymatini 3-taqqoslamaga qo'yib, $t_2$ni aniqlaymiz:$-17 + 77t_2 \equiv a(mod 9) \to 5t_2 \equiv a + 17(mod 9) \to 5t_2 \equiv a - 1(mod 9)$. Bunda $(5; 9) = 1$. Demak a ning ixtiyoriy butun qiymatida berilgan taqqoslamalar sistemasi yechimga ega. $2 \cdot 5t_2 \equiv 2 \cdot (a - 1)(mod 9) \to t_2 \equiv 2 \cdot (a - 1)(mod 9)$.

271.1). Buning uchun berilgan to'g'ri chiziqning kesishish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko'rinishda yozib olib, uning yechimini topamiz.

$\begin{cases} x \equiv 2(mod\ 5) \\ x \equiv 1(mod\ 8) \\ x \equiv 3(mod\ 11) \end{cases} \to x = 2 + 5t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib, t_1 ni aniqlaymiz: $2 + 5t_1 \equiv 1(mod\ 8) \to 5t_1 \equiv -1(mod\ 8) \to 5t_1 \equiv (-1 -$

$3\cdot 8)(mod\ 8) \to 5t_1 \equiv -25\ (mod\ 8) \to t_1 \equiv -5(mod\ 8),$ ya$'$ni $t_1 = 3 + 8t_2,\ t_2 \in Z.\ t_1$ning bu ifodasini x ning ifodasiga olib borib qo'ysak $x = 2 + 5(3 + 8t_2)$=$17 + 40t_2,\ t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib, t_2 ni aniqlaymiz:

$17 + 40t_2 \equiv 3(mod\ 11) \to -4t_2 \equiv 8(mod\ 11) \to t_2 \equiv -2(mod\ 11) \to t_2 = -2 + 11t_3, t_3 \in Z.$ Buni x ning ifodasidagi t_2 ning o'rniga olib borib qo'ysak, $x = 17 + 40(-2 + 11t_3) = -63 + 440t_3,\ t_3 \in Z$ hosil bo'ladi. Demak, absitssasi $x = -63 + 440t_3, t_3 \in Z$ nuqtadan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi. Bu nuqtalarni ordinatalarini to'g'ri chiziq tenglamasidan topamiz. Birinchit tenglamadan

$-63 + 440t_3 = 2 + 5y \to 5y = -65 + 440t_2 \to y = -13 + 88t_3.$ Ikkinchi tenglamadan

$-63 + 440t_3 = 1 + 8y \to -64 + 440t_3 = 8y \to y = -8 + 55t_3.$ Uchinchi tenglamadan

$-63 + 440□_3 = 3 + 11y \to -66 + 440t_3 = 11y \to y = -6 + 40t_3.$ Shunday qilib bu nuqtalarning koordinatalari $(-63 + 440t_3; -13 + 88t_3), (-63 + 440t_3; -8 + 55t_3),\ (-63 + 440t_3 - 6 + 40t_3),\ t_3 \in Z.$

2).Buning uchun berilgan to'g'ri chiziqning kesisish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko'rinishda yozib olib uning yechimini topamiz.

$\begin{cases} 4x \equiv 9(mod\ 7) \\ 2x \equiv 15(mod\ 9) \\ 5x \equiv 12(mod\ 13) \end{cases} \to \begin{cases} x \equiv 4(mod\ 7) \\ x \equiv 3(mod\ 9) \\ x \equiv 5(mod\ 13) \end{cases}.$ Endi bu sistemani yechamiz.

Sistemaning 1-taqqoslamasidan $x = 4 + 7t_1, t_1 \in Z.$ Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $4 + 7t_1 \equiv 3(mod\ 9) \to 7t_1 \equiv -1(mod\ 9) \to 16t_1 \equiv 8(mod\ 9) \to 2t_1 \equiv 1(mod\ 9) \to t_1 \equiv 5(mod\ 9) \to t_1 = 5 + 9t_2, t_2 \in Z.\ t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qo`ysak, $x = 4 + 7(5 + 9t_2)$=$39 + 63t_2$,$t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib, $t_2$ni aniqlaymiz: $39 + 63t_2 \equiv 5(mod 13) \to -2t_2 \equiv 5(mod\ 13) \to t_2 \equiv -9(mod\ 13) \to\ t_2 \equiv 4(mod\ 13),$ ya`ni$t_2 = 4 + 13t_3, t_3 \in Z.$ Buni x ning ifodasidagi t_2 ning o'rniga olib borib qo'ysak, $x = 39 + 63(4 + 13t_3) = 39 + 252 + 819t_3 = 291 + 819t_3,\ t_3 \in Z$ ni hosil qilamiz. Bundan $x = 291 + 819t_3.$ Demak, absitssasi $x = 291 + 819t_3,\ t_3 \in Z$ nuqtadan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

3) Buning uchun berilgan to'g'ri chiziqning kesishish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko'rinishda yozib olib, uning yechimini topamiz.

$\begin{cases} 3x \equiv 1(mod\ 5) \\ 2x \equiv 3(mod\ 3) \\ 5x \equiv 7(mod\ 7) \end{cases} \rightarrow \begin{cases} x \equiv 2(mod\ 5) \\ x \equiv 0(mod\ 3) \\ x \equiv 0(mod\ 7) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 5t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 5t_1 \equiv 0(mod\ 3) \rightarrow 5t_1 \equiv -2(mod\ 3) \rightarrow 2t_1 \equiv 1(mod\ 3) \rightarrow 2t_1 \equiv 4(mod\ 3) \rightarrow t_1 \equiv 2(mod\ 3) \rightarrow t_1 = 2 + 3t_2, t_2 \in Z. t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qo`ysak, $x = 2 + 5(2 + 3t_2) = 12 + 15t_2, t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz:

$12 + 15t_2 \equiv 0(mod7) \rightarrow 15t_2 \rightarrow -12(mod\ 7) \rightarrow t_2 \equiv 2(mod\ 7) \rightarrow t_2 = 2 + 7t_3, t_3 \in Z$. Buni x ning ifodasidagi t_2 ning o'rniga olib borib qo'ysak $x = 12 + 15(2 + 7t_3) = 42 + 105t_3$ ni hosil qilamiz. Bundan

$x = 42 + 105t_3,\ t_3 \in Z$. Demak abtsitsalari o'qining $x = 42 + 105t_3, t_3 \in Z$ nuqtadan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

4). $\begin{cases} x \equiv 2(mod\ 7) \\ x \equiv 3(mod\ 5) \\ 2x \equiv 6(mod\ 7) \end{cases} \rightarrow \begin{cases} x \equiv 2(mod\ 7) \\ x \equiv 3(mod\ 5) \\ x \equiv 3(mod\ 7) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 7t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 7t_1 \equiv 3(mod\ 5) \rightarrow 2t_1 \equiv 1(\square od\ 5) \rightarrow t_1 \equiv 3(mod\ 5)$, ya'ni

$t_1 = 3 + 5t_2, t_2 \in Z. t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qo`ysak , $x = 2 + 7(3 + 5t_2) = 23 + 35t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib, $t_2$ni aniqlaymiz: $23 + 35t_2 \equiv 3(mod\ 7) \rightarrow\ 0 \cdot t_2 \equiv 1(mod\ 7)$. Bu taqqoslamani qanoatlantiruvchi t_2 qiymatlari mavjud emas va demak, masalaning shartini qanoatlantiruvchi nuqtalar ham mavjud emas.

Izoh: Bunday nuqtalarning mavjud emasligini $x \equiv 2(mod\ 7)$va $x \equiv 3(mod\ 7)$ taqqoslamaning bir vaqtda bajarilmasligi bilan ham asoslash mumkin.

5). $\begin{cases} 2x \equiv 1(mod\ 3) \\ x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 11) \end{cases} \rightarrow \begin{cases} x \equiv 2(\ mod\ 3) \\ x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 11) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 3t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 3t_1 \equiv 3(mod\ 5) \rightarrow 3t_1 \equiv 1(mod\ 5) \rightarrow t_1 \equiv 2(mod\ 5) \rightarrow t_1 = 2 + 5t_2, t_2 \in Z. t_1$ ning topilgan qiymatini$x$ ning ifodasiga olib borib qoysak $x = 2 + 3(2 + 5t_2) = 8 + 15t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib, $t_2$ni aniqlaymiz: $8 + 15t_2 \equiv 2(mod\ 11) \rightarrow 4t_2 \equiv -6(mod\ 11) \rightarrow 2t_2 \equiv -3(mod\ 11) \rightarrow t_2 \equiv 4(mod\ 11) \rightarrow t_2 = 4 + 11t_3, t_3 \in Z$ ni hosil qilamiz. Bundan $x = 8 + 15(4 + 11t_3) = 68 + 165t_3, t_3 \in Z$. Demak, abtsitsalari o’qining $x=68+165t_3,\ t_3\in Z$ nuqtasidan OX o’qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

6). $\begin{cases} 11x\equiv 6(mod\ 5) \\ 10x\equiv 9(mod\ 11) \\ 12x\equiv -1(mod\ 13) \end{cases} \to \begin{cases} x\equiv 1(mod\ 5) \\ x\equiv 2(mod\ 11) \\ x\equiv 1(mod\ 13) \end{cases}$. Endi bu sistemani yechamiz.

Sistemaning 1-taqqoslamasidan$x=1+5t_1, t_1\in Z$. Buni 2-taqqoslamaga olib borib qo’yib t_1 ni aniqlaymiz:

$1+5t_1\equiv 2(mod\ 11)\to 5t_1\equiv 1(mod\ 11)\to -6t_1\equiv 12(mod\ 11)\to t_1\equiv -2(mod\ 11)\to t_1=-2+11t_2, t_2\in Z. t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qo`ysak,

$x=1+5(-2+11t_2)=-9+55t_2, t_2\in Z$ hosil bo’ladi. $x$ning bu qiymatini 3-taqqoslamaga qo’yib $t_2$ni aniqlaymiz:$-9+55t_2\equiv 1(mod\ 13)\to 3t_2\equiv -3(mod\ 13\ )\to t_2\equiv -1(mod\ 13)\to t_2=-1+13t_3, t_3\in Z$. Buni x ning ifodasiga qo’yib, $x=-9+55(-1+13t_3)=-64+715t_3, t_3\in Z$ ga ega bo’lamiz. Demak, abtsitsalari o’qining $x=-64+715t_3, t_3\in Z$ nuqtasidan OX o’qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

7). $\begin{cases} 3x\equiv 5(mod\ 7) \\ 5x\equiv 4(mod\ 8) \\ 11x\equiv -2(mod\ 13) \end{cases} \to \begin{cases} x\equiv 4(mod\ 7) \\ x\equiv 4(mod\ 8) \\ x\equiv 1(mod\ 13) \end{cases}$.

Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x=4+7t_1, t_1\in Z$. Buni 2-taqqoslamaga olib borib qo’yib t_1 ni aniqlaymiz:

$4+7t_1\equiv 4(mod\ 8)\to 7t_1\equiv 0(mod\ 8)\to t_1\equiv 0(mod\ 8)\to t_1=8t_2, t_2\in Z. t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qo`ysak, $x=4+56t_2, t_2\in Z$ hosil bo’ladi. x ning bu qiymatini 3-taqqoslamaga qo’yib $t_2$ni aniqlaymiz: $4+56t_2\equiv 1(mod\ 13)\to 4t_2\equiv -3(mod\ 13)\to t_2=9+13_3\ , t_3\in Z$. Buni x ning ifodasiga qo’yib, $x=4+56(9+13t_3)=508+728t_3, t_3\in Z$ ga ega bo’lamiz. Demak, abtsitsalari o’qining $x=508+728t_3, t_3\in Z$ nuqtasidan OX o’qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

8). $\begin{cases} 10x\equiv 1(mod\ 9) \\ x\equiv 3(mod\ 7) \\ x\equiv 2(mod\ 5) \end{cases} \to \begin{cases} x\equiv 1(mod\ 9) \\ x\equiv 3(mod\ 7) \\ x\equiv 2(mod\ 5) \end{cases}$. Endi bu sistemani

yechamiz. Sistemaning 1-taqqoslamasidan $x=1+9t_1, t_1\in Z$. Buni 2-taqqoslamaga olib borib qo’yib, t_1 ni aniqlaymiz: $1+9t_1\equiv 3(mod\ 7)\to 9t_1\equiv 2(mod\ 7)\to t_1\equiv 1(mod\ 7)\to t_1=1+7t_2,\ t_2\in Z. t_1$ ning topilgan qiymatini$x$ ning ifodasiga olib borib qo`ysak,

$x = 1 + 9(1 + 7t_2) = 10 + 63t_2,\ t_2 \in Z$hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib, $t_2$ni aniqlaymiz:$10 + 63t_2 \equiv 2(mod\ 5) \to 3t_2 \equiv -3(mod\ 5) \to t_2 \equiv\ -1(mod\ 5) \to t_2 =\ -1 + 5t_3, t_3 \in Z$. Buni x ning ifodasiga qo'yib $x = 10 + 63(-1 + 5t_3) = -53 + 315t_3, t_3 \in Z$ga ega bo'lamiz. Demak, abtsitsalari o'qining $x = -53 + 315t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

9). $\begin{cases} 11x \equiv 5(mod\ 17) \\ 19 \equiv 1(mod\ 37\ ) \\ 11x \equiv 4(mod\ 7) \end{cases} \to \begin{cases} x \equiv 2(mod\ 17) \\ x \equiv 2(mod\ 37) \\ x \equiv 1(mod\ 7) \end{cases}$.Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 17t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib, t_1 ni aniqlaymiz:$2 + 17t_1 \equiv 2(mod37)$ $\to 17t_1 \equiv 0(mod\ 37) \to t_1 \equiv 0(mod\ 37) \to t_1 = 37t_2,\ t_2 \in Z. t_1$ ning topilgan qiymatini$x$ ning ifodasiga olib borib qo\`ysak, $x = 2 + 629t_2,\ t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $2 + 629t_2 \equiv 1(mod\ 7) \to (629 - 7 \cdot 90)t_2 \equiv -1(mod\ 7) \to -t_2 \equiv -1(mod\ 7) \to t_2 \equiv 1(mod\ 7) \to t_2 = 1 + 7t_3, t_3 \in Z.$ Buni x ning ifodasiga qo'yib, $x = 2 + 629 \cdot (1 + 7t_3) = 631 + 4403t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abtsitsalari o'qining $x = 631 + 4403t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

10). $\begin{cases} x \equiv 2(mod\ 10) \\ 5x \equiv 2(mod\ 13\ ) \\ 10x \equiv -3(mod\ 13) \end{cases} \to \begin{cases} x \equiv 2(mod\ 19) \\ x \equiv 3(mod\ 13) \\ x \equiv 1(mod\ 13) \end{cases}$. Bu sistema ziddiyatli sistema.
Shuning uchun ham bu holda masala shartini qanoatlantiruvchi nuqtalar yo'q (4-masaladan keying izohni qarang).

11). Buning uchun $\begin{cases} x - 7y = 5 \\ 3x + 8y = 7 \\ x = 11 + 3y \end{cases}$ sistemaning yechimini topish kifoya.

Bu sistema ushbu $\begin{cases} x \equiv 5(\mathrm{mod}\,7) \\ 3x \equiv 7(\mathrm{mod}\,8) \\ x \equiv 11(\mathrm{mod}\,3) \end{cases} \to \begin{cases} x \equiv 5(\mathrm{mod}\,7) \\ x \equiv 5(\mathrm{mod}\,8) \\ x \equiv 11(\mathrm{mod}\,3) \end{cases}$ taqqoslamalar sistemasiga teng kuchli. Endi shu taqqoslamalar sistemasini yechamiz. Sistemaning 1-taqqoslamasidan $x = 5 + 7t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib, t_1 ni aniqlaymiz: $5 + 7t_1 \equiv 5(mod8) \to 7t_1 \equiv 0(mod\ 8) \to t_1 \equiv 0(mod\ 8) \to t_1 = 8t_2,\ t_2 \in Z. t_1$ ning topilgan qiymatini$x$ ning ifodasiga olib borib qo\`ysak, $x = 5 + 56t_2,\ t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $5 + 56t_2 \equiv 11(mod3) \to 2t_2 \equiv 0(mod\ 3) \to t_2 = 3t_3, t_3 \in Z.$ Buni x ning ifodasiga qo'yib, $x = 5 + 168t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abtsitsalari o'qining $x = 5 + 168t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

272. a).$56 = 8 \cdot 7$ va $(8,7) = 1$ bo'lgani uchun masala shartiga ko'ra

$4x87y6 \equiv 0(mod\, 8), 4x87y6 \equiv 0(mod\, 7)$ taqqoslamalar o'rinli bo'lishi kerak. 8 ga bo'linish belgisiga asosan birinchi taqqoslamadan $7y6 \equiv (mod\, 8) \to 7 \cdot 10^2 + 10y + 6 \equiv 0(mod\, 8) \to (-1) \cdot 2^2 + 2y - 2 \equiv 0(mod\, 8) \to 2y \equiv 6(mod\, 8) \to$ $y \equiv\ 3(mod\, 4) \to y = 3 + 4k,\ k \in Z$.Bu yerda y raqam bo'lganligi uchun $y = 3$ va $y = 7$. y ning bu topilgan qiymatlarni yuqoridagi 2-taqqoslamaga qo'yib $4x8736 \equiv 0(mod\, 7)$ va $4x8776 \equiv 0(mod\, 7)$ larni hosil qilamiz. Bularning birinchisidan: $4 \cdot 10^5 + x \cdot 10^4 + 8 \cdot 10^3 + 7 \cdot 10^2 + 3 \cdot 10 + 6 \equiv 0(mod\, 7) \to 4 \cdot 3^5 + x \cdot 3^4 + 8 \cdot 3^3 + 7 \cdot 3^2 + 3 \cdot 3 - 1 \equiv 0(mod\, 7) \to 4 \cdot (-1) \cdot 3^2 + x \cdot 3(-1) + 8(-1) + 1 \equiv 0(mod\,) \to -1 - 3x - 1 + 1 \equiv 0(mod\, 7) \to 3x \equiv -1\ (mod\, 7) \to$ $x \equiv 2(mod\, 7)$. Bundan $x_1 = 2, x_2 = 9$. Endi ikkinchi taqqoslamani yechamiz:

$4 \cdot 10^5 + x \cdot 10^4 + 8 \cdot 10^3 + 7 \cdot 10^2 + 7 \cdot 10 + 6 \equiv 0(mod\, 7) \to 4 \cdot 3^5 + x \cdot 3^4 + 1 \cdot 3^3 - 1 \equiv 0(mod\, 7) \to 4 \cdot (-1) \cdot 3^2 - 3x - 1 - 1 \equiv 0(\square od\, 7) \to 3x \equiv -3(mod\, 7\ ) \to x \equiv 6(mod\, 7)$. Bundan $x_3 = 6$. Endi x ning topilgan qiymatlarini olib borib oqiga qo'ysak, $428736,\ 498776,\ 468776$ sonlarini hosil bo'ladi.

c) Shartga ko'ra

$$\begin{cases} xyz138 \equiv 0(mod\, 7) \\ 138xyz \equiv 6(mod\, 13) \\ x1y3z8 \equiv (mod 11) \end{cases}$$ bajariladi.

1-taqqoslamadan $xyz \cdot 10^3 + 138 \equiv 0(mod\, 7) \to xyz \cdot 3^3 + 5 \equiv 0(mod\, 7) \to$ $xyz \equiv 5(mod\, 7)$. (1) .

2-taqqoslamadan

$$138 \cdot 10^3 + xyz \equiv 6(mod\, 13) \to 8 \cdot (-3)^3 + xyz \equiv 6(mod\, 13) \to xyz \equiv 1(mod\, 13). \quad (2).$$

(1) va (2) taqqoslamalarni birgalikda yechib xyz ni aniqlaymiz. (1) dan $xyz = 5 + 7t_1, \quad t_1 \in Z$. Buni (2)ga olib borib qoyamiz. U holda

$5 + 7t_1 \equiv 1(mod\, 13) \to 7t_1 \equiv -4(mod\, 13) \to -6t_1 \equiv -4(mod\, 13) \to 3t_1 \equiv 2(mod\, 13) \to t_1 \equiv 5(mod\, 13) \to t_1 = 5 + 13t_2, t_2 \in Z$. Demak, $xyz = 5 + 35 + 91t_2 = 40 + 91t_2, t_2 \in Z$. Bundan $t = 1,2,3, \dots 10$ larda uch xonali sonlar

$$x = 131, 222, 313,\ \dots, 950 \quad (3)$$

sonlarini hosil qilamiz. Endi 3-taqqoslamaga qaraymiz .

$x \cdot 10^5 + 10^4 + y \cdot 10^3 + 3 \cdot 10^2 + z \cdot 10 + 8 \equiv 5(mod\ 11)$
$\rightarrow -x + 1 - y + 3 - z + 8 \equiv 5(mod\ 11) \rightarrow x + y + z \equiv 7(mod\ 11)$
$\rightarrow x + y + z = 7 + 11\mathrm{t}_1, t_1 \in Z, \qquad (4)$

bu yerda x, y, z lar raqamlar bo'lganligi uchun $0 < x + y + z < 27$ bo'lganligi uchun (4) dan $t = 0$ va $t = 1$ da $x + y + z = 7$ va $x + y + z = 18$ larni hosil qilamiz. Endi (3) sonlar ketma-ketligidan shu shartlarni qanoatlantiruvchilarini ajratib olamiz. Ular 313,495. Demak, izlanayotgan sonlar 313138, 495138 .

c). $792 = 8 \cdot 9 \cdot 11$ va shart bo'yicha $13xy45z \equiv 0(mod\ 792)$. Bu oxirgi taqqoslama

$\begin{cases} 13xy45z \equiv 0(mod\ 8) \\ 13xy45zz \equiv 0(mod\ 9) \\ 13xy45z \equiv 0(mod\ 11) \end{cases}$ taqqoslamalar sistemasiga teng kuchli.

1-taqqoslamadan 8 ga bo'linish belgisiga asosan $45z \equiv 0(mod\ 8) \rightarrow 450 + z \equiv 0(mod\ 8) \rightarrow z \equiv 6(mod\ 8)$. Demak $z = 6$ va uni 2 va 3- taqqoslamalarga qo'ysak: $\begin{cases} 13xy456 \equiv 0(mod\ 9) \\ 13xy456 \equiv 0(mod\ 11) \end{cases}$ hosil bo'ladi. 9 ga bo'linish belgisiga asosan bu yerdagi 1-taqqoslamadan $19 + x + y \equiv 0(mod\ 9) \rightarrow x + y + 1 \equiv 0(mod\ 9)$.
2- taqqoslamadan $13 \cdot 10^5 + x \cdot 10^4 + y \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 6 \equiv 0(mod\ 11) \rightarrow 2 \cdot (-1) + x - y + 4 - 5 + 6 \equiv 0(mod\ 11) \rightarrow 0(mod\ 11) \rightarrow x - y + 3 \equiv 0(mod\ 11)$. Bulardan x va y lar raqam bo'lganligi uchun $\begin{cases} x + y = 8 \\ x - y = 8 \end{cases} \rightarrow$ $x = 8, y = 0$. Shunday qilib izlanayotgan son 1380456.

273. a). 2-taqqoslamadan $x = 3 + 7t_1$, u holada buni 1-taqqoslamaga qo'ysak $3 + 3y \equiv 5(mod\ 7) \rightarrow 3y \equiv 9(mod\ 7) \rightarrow y \equiv 3(mod\ 7)$.
Javob: $x = 3 + 7t_1, y = 3 + 7t_1, t_1 \in Z$.

b). $\begin{cases} 9y \equiv 15(mod\ 12\,) \\ 7x - 3y \equiv 1(mod\ 12) \end{cases}$.

1-taqqoslamadan $3y \equiv 5(mod\ 4) \rightarrow y \equiv 3(mod\ 4) \rightarrow y \equiv 3,7,11\ (mod\ 12)$. Bundan va berilgan sistemadan quyidagi 3 ta sistemani hosil qilamiz:
$\begin{cases} y \equiv 3(mod\ 12) \\ 7x \equiv 10(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 7(mod\ 12) \\ 7x \equiv 10(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 11(mod\ 12) \\ 7x \equiv -2(mod\ 12) \end{cases}$. Bular mos ravishda quyidagi sistemalarga teng kuchli:
$\begin{cases} y \equiv 3(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 11(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}$. Shunday qilib yechimlar
: $\begin{cases} x \equiv 10 \\ y \equiv 3 \end{cases} (mod\ 12)$; $\begin{cases} x \equiv 10 \\ y \equiv 7 \end{cases} (mod\ 12)$; $\begin{cases} x \equiv 10 \\ y \equiv 11 \end{cases} (mod\ 12)$.

c). $\begin{cases} x \equiv 2(mod\ 4) \\ -2y \equiv -1(mod\ 4) \end{cases} \rightarrow \begin{cases} x \equiv 2(mod\ 4) \\ 2y \equiv 3(mod\ 4) \end{cases}$;

bu yerdagi ikkinchi taqqoslamada $(2:4) = 2$, lekim 3 soni 2 ga bo'linmaydi, taqqoslama yechimga ega emas. Shuning uchun sistema ham yechimga ega emas.

d). $\begin{cases} 9y \equiv 15(mod\ 12) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to \begin{cases} 3y \equiv 5(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to$

$$\begin{cases} 3y \equiv 1(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to \begin{cases} 3y \equiv 9(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases}$$

$$\to \begin{cases} y \equiv 3(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to \begin{cases} y \equiv 3,7,11(mod\ 12) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases}.$$

Bundan $\begin{cases} y \equiv 3(mod\ 12) \\ 3x - 21 \equiv 1(mod\ 12) \end{cases} \to \begin{cases} \square \equiv 3(mod\ 12) \\ 3x \equiv 10(mod\ 12) \end{cases}$

Bu yerda 2-taqqoslama yechimga ega emas.

$\begin{cases} y \equiv 7 \\ 3x \equiv 50\ (mod\ 12) \end{cases}$. Bu yerda ham 2-taqqoslama yechimga ega emas.

$$\begin{cases} y \equiv 11(mod\ 12) \\ 3x \equiv 78(mod\ 12) \end{cases} \to \begin{cases} y \equiv 11(mod\ 12) \\ 3x \equiv 6(mod\ 12) \end{cases} \to \begin{cases} y \equiv 11(mod\ 12) \\ x \equiv 2(mod\ 4) \end{cases} \to$$

$$\begin{cases} y \equiv 11(mod\ 12) \\ x \equiv 2,6,10(mod\ 12) \end{cases}.$$

Demak, yechimlar $\begin{cases} x \equiv 2(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$; $\begin{cases} x \equiv 6(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$;

$\begin{cases} x \equiv 10(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$.

e). $\begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ 9y \equiv 15(mod\ 12) \end{cases} \to \begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ 3y \equiv 5(mod\ 4) \end{cases} \to$

$\begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ y \equiv 3(mod\ 4) \end{cases} \to \begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ y \equiv 3,7,11(mod\ 4) \end{cases}$. Bundan

$\begin{cases} 3x \equiv 16(mod\ 12) \\ y \equiv 3(mod\ 4) \end{cases} \to$ Bu yerda $(3,12) = 3$, lekin 16 soni 3 ga bo'linmaydi, ya'ni sistema yechimga ega emas.

$$\begin{cases} y \equiv 7(mod\ 12) \\ 3x \equiv 36(mod\ 12) \end{cases} \to \begin{cases} y \equiv 7(mod\ 12) \\ 3x \equiv 0(mod\ 12) \end{cases} \to \begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0\ (mod\ 4) \end{cases} \to$$

$\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0,4,8\ (mod\ 12) \end{cases}$. Bundan yechimlar

$\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 4(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 8\ (mod\ 12) \end{cases}$ bo'lar ekan.

274. a).$\begin{cases} x + 2y \equiv 3(mod\ 5) \\ 4x + y \equiv 2(mod\ 5) \end{cases}$ dagi ikkinchi taqqoslamaning ikkala tomonini 2 ga (2,5)=1 ko'paytiramiz, ularni hadlab ayiramiz. U holda $-7x \equiv -1(mod\ 5) \to 3x \equiv 4(mod\ 5) \to x \equiv 3(mod\ 5)$ni hosil qilamiz. Buni berilgan sistemaga qo'ysak, $y \equiv 2 - 4x(mod\ 5) \to y \equiv -10(mod\ 5) \to y \equiv 0(mod\ 5)$ kelib chiqadi. Demak yechim
$\begin{cases} x \equiv 3(mod\ 5) \\ y \equiv 0(mod\ 5) \end{cases}$.

b).$\begin{cases} x + 2y \equiv 0(mod\ 5) \\ 3x + 2y \equiv 2(mod\ 5) \end{cases}$ sistemadagi taqqoslamalarni hadlab ayiramiz. U holda$-2x \equiv -2(mod\ 5) \to x \equiv 1(mod\ 5)$. Buni berilgan sistemaga qo'ysak, $2y \equiv -x(mod\ 5) \to\ 2y \equiv -1(mod\ 5) \to\ 2y \equiv 4(mod\ 5) \to\ y \equiv 2(mod\ 5)$ kelib chiqadi. Demak, yechim $\begin{cases} x \equiv 1(mod\ 5) \\ y \equiv 2(mod\ 5) \end{cases}$.

c).$\begin{cases} 3x + 4y \equiv 29(mod\ 143) \\ 2x - 9y \equiv -84(mod\ 143) \end{cases} \to \begin{cases} 6x + 8y \equiv 58(mod\ 143) \\ 6x - 27y \equiv -252(mod\ 143) \end{cases}$

$\to 35y \equiv 310(mod\ 143) \to 35y \equiv 24(mod\ 143)$. Bu yerda $(35; 143) = 1$ bo`lgani uchun taqqoslama yagona yechimga ega.Bu yechimni topish uchun $\frac{143}{5}$ ni uzluksiz kasrga yoysak, $\frac{143}{35} = (4,11,1,2)$ hosil bo'ladi. Bundan munosib kasrlarning suratini aniqlasak,

q_i		4	11	1	2
P_i	$P_0 = 1$	4	45	49	143

Bundan $P_{n-1} = 49,\ n = 4$ bo`ladi va $y \equiv (-1)^3 \cdot 49 \cdot 24(mod\ 143) \equiv -1176(mod\ 143) \equiv\ (-1176 + 1144)(mod\ 143) \equiv -32(mod\ 143) \equiv 111(mod\ 143)$.

Demak, $3x + 4 \cdot 111 \equiv 29(mod\ 143) \to 3x \equiv -415(mod\ 143) \to\ 3x \equiv 14(mod\ 143) \to 3x \equiv (14 - 143)(mod\ 143) \to$

$3x \equiv -129(mod\ 143) \to x \equiv -43(mod\ 143) \to x \equiv 100(mod\ 143)$.

Javob: $\begin{cases} x \equiv 100 \\ y \equiv 111 \end{cases}(mod\ 143)$.

d). $\begin{cases} x+2y=4(mod\ 5) \\ 3x+y\equiv 2(mod\ 5) \end{cases}$.

Bu yerdagi ikkinchi taqqoslamaning ikkala tomonini 2 ga $(2,5)=1$ ko’paytiramiz ularni hadlab ayiramiz. U holda$-5x\equiv 0(mod\ 5)\rightarrow x\equiv 0(mod\ 5)$. Buni berilgan sistemaga qo’ysak $y\equiv 2(mod\ 5)$ hosil bo’ladi. Demak, sistemaning yechimi
$\begin{cases} x\equiv 0(mod\ 5) \\ y\equiv 2(mod\ 5) \end{cases}$.

e). $\begin{cases} x+5y\equiv 5(mod\ 6) \\ 5x+3y\equiv 1(mod\ 6) \end{cases}$. Bu yerdagi birinchi taqqoslamaning ikkala tomonini 5 ga $(6,5)=1$ ko’paytiramiz va ularni hadlab ayiramiz. U holda
$22y\equiv 24(mod\ 6)\rightarrow 4y\equiv 0(mod\ 6)\rightarrow 2y\equiv 0(mod\ 3)\rightarrow\equiv 0(mod\ 3)\rightarrow$
$y\equiv 0,3(mod\ 6)$. y ning bu qiymatlarini berilgan sistemaga qo’yib x ni aniqlaymiz: $x\equiv 2,5(mod\ 6)$. Demak, yechim $\begin{cases} x\equiv 5(mod\ 6) \\ y\equiv 0(mod\ 6) \end{cases}$; $\begin{cases} x\equiv 2(mod\ 6) \\ y\equiv 3(mod\ 6) \end{cases}$.

f). $\begin{cases} 5x-y\equiv 3(mod\ 6) \\ 2x+2y\equiv -1(mod\ 6) \end{cases}$. Sistemaning birinchi taqqoslamasidan $5x-y\equiv 3(mod\ 6)\rightarrow y\equiv 5x-3(mod\ 6)$. Buni ikkinchi taqqoslamaga qo’ysak, $2x+2y\equiv -1(mod\ 6)\rightarrow 2x+2(5x-3)\equiv -1(mod\ 6)\rightarrow 12x\equiv 5(mod 6)$ hosil bo’ladi. Bu yerda $(12,6)=6$, lekinda 5 soni 6ga bo’linmaydi. Shuning uchun ham bu taqqoslama va demak, berilgan sistema ham yechimga ega emas.

g). $\begin{cases} x-y\equiv 2(mod\ 6) \\ 4x+2y\equiv 2(mod\ 6) \end{cases}$. Bu yerdagi birinchi taqqoslamadan $x-y\equiv 2(mod\ 6)\rightarrow x\equiv y+2(mod\ 6)$. Buni ikkinchi taqqoslamaga qo’ysak,

$4x+2y\equiv 2(mod\ 6)\rightarrow 4(y+2)+2y\equiv 2(mod\ 6)\rightarrow 6y\equiv -6(mod 6)$ hosil bo’ladi. Bu taqqoslama ayniy taqqoslama bo’lgani uchun uni y ning ixtiyoriy qiymati qanoatlantiradi. Shuning uchun ham $x\equiv y+2(mod\ 6)$, ya’ni sistemaning yechimlari to’plami $x-y\equiv 2(mod\ 6)$ taqqoslamaning yechimlari bilan bir xil.

h). $\begin{cases} 4x-y\equiv 2(mod\ 6) \\ 2x+2y\equiv 0(\square od\ 6) \end{cases}$.

Bu yerdagi birinchi taqqoslamadan

$4x-y\equiv 2(mod\ 6)\rightarrow y\equiv 4x-2(mod\ 6)$.

Buni ikkinchi taqqoslamaga qo’ysak,

$2x+2y\equiv 0(mod\ 6)\rightarrow x+y\equiv 0(mod\ 3)\rightarrow x+4x-2\equiv 0(mod\ 3)\rightarrow$
$5x\equiv 2(mod\ 3)\rightarrow x\equiv 1(mod\ 3)\rightarrow x\equiv 1,4(mod\ 6)$. Demak, sistemaning yechimlari: $\begin{cases} x\equiv 1(mod\ 6) \\ y\equiv 2(mod\ 6) \end{cases}$, $\begin{cases} x\equiv 4(mod\ 6) \\ y\equiv 2(mod\ 6) \end{cases}$.

275. *a)*. Ma’lumki, (1) dan

$$Dx\equiv D_1(mod\ m) \text{va } Dy\equiv D_2(mod\ m). \qquad (*)$$

Agar $(m,D)=1$ bu ikkala taqqoslama ham yagona yechimga ega.

$$x \equiv D^{\varphi(m)-1}.D_1(modm)\text{va}x \equiv D^{\varphi(m)-1}D_2(modm).$$

b). (*) dan$(D;m) = d > 1$ bo'lib D_1 va D_2 larning ikkalasi ham

$d$ga bo'linsa, ularning har biri d ta yechimga ega bo'ladi. Agar D_1va D_2larning birortasi d ga bo'linmasa sistema yechimga ega emas. Shunday qilib, berilgan sistemaning yechimga bo'lmasligi sharti (*) dagi D_1 yok$i$ D_2larning birortasining $(D;m) = d$ ga bo'linmasligidir.

c). $D \equiv D_1 \equiv D_2 \equiv 0(mod\ m)$ bajarilsa, (1) dagi 2-taqqoslama birinchisining natijasi bo'ladi. Haqiqatdan ham 1-taqqoslamaning ikkala tomonini a_2 ko'paytirsak,

$$a_1a_2x + b_1a_2y \equiv c_1a_2(mod\ m) \qquad (2)$$

hosil bo'ladi. $D \equiv 0$ va$D_2 \equiv 0(mod\ m)$ lardan $\left.\begin{matrix} a_2b_1 \equiv a_1b_2 \\ a_2b_1 \equiv a_1c_2 \end{matrix}\right\}(mod\ m).$

U holda (2) dan $a_1a_2x + a_1b_2y \equiv a_1c_2(mod\ m)$. Bu yerda $(a_1, m) = 1$ bo'lganligi uchun oxirgi taqqoslamaning ikkala tomonini a_1 ga qisqartirib, $a_2x + by \equiv c_2(mod\ m)$ni, ya'ni (1) dagi 2-taqqoslamani hosil qilamiz.

IV. 4-§.

276. *a).* Avvalo berilgan taqqoslamaning koeffitsiyentlaridan modulga karrali sonlarni chiqarib soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $x^{10} - 2x + 1 \equiv 0(mod\ 5)$. Bu taqqoslamada Ferma teoremasiga ko'ra, $x^5 \equiv x(mod\ 5)$ ekanligidan foydalanib darajasini pasaytiramiz: $(x^5)^2 - 2x + 1 \equiv 0(mod\ 5) \rightarrow x^2 - 2x + 1 \equiv 0(mod\ 5)$. Demak, berilgan taqqoslama oxirgi taqqoslamaga teng kuchli ekanligidan oxirgi taqqoslamani yechamiz: $(x-1)^2 \equiv 0(mod\ 5) \rightarrow x - 1 \equiv 0(mod\ 5) \rightarrow x \equiv 1(mod\ 5)$. Shunday qilib berilgan taqqoslamaning yechimi $x \equiv 1(mod\ 5)$ dan iborat.

Izoh:$x^2 - 2x + 1 \equiv 0(mod\ 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi chegirmalarni qo'yib, sinab ko'rish yo'li bilan ham yechish mumkin .

Tekshirish: $6^2 - 2 \cdot 6 + 1 \equiv 25(mod\ 5) \equiv 0(mod\ 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi .

Javob:$x \equiv 1(mod\ 5)$.

b). Bu taqqoslamada Ferma teoremasiga ko'ra, $x^5 \equiv x(mod\ 5)$ ekanligidan foydalanib, darajasini pasaytiramiz: U holda quyidagiga ega bo'lamiz: $x^5 - 2x^3 + x^2 - 2 \equiv 0(mod\ 3) \rightarrow x^3 \cdot x^2 + x^3 + x^2 + 1 \equiv 0(mod\ 3) \rightarrow x^3 + x + x^2 + 1 \equiv x + x + x^2 + 1 \equiv x^2 + 2x + 1 \equiv 0(mod\ 3) \rightarrow (x+1)^2 \equiv 0(mod\ 3) \rightarrow x + 1 \equiv 0(mod\ 3)$. Bundan $x \equiv -1(mod\ 3)$.

Tekshirish: $(-1)^5 - 2(-1)^3 + (-1)^2 - 2 \equiv 0(mod\ 3)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob:** $x \equiv -1(mod\ 3)$.

a) Avvalo berilgan taqqoslamani $x^3 \cdot x^2 - x^3 \cdot x - x + 1 \equiv x^3 - x^2 - x + 1 \equiv 0(mod\, 3)$ ko'rinishda yozib olamiz va bunda $x^2(x-1)-(x-1)=(x-1)(x^2-1)=(x-1)^2(x+1)$ bo'lgani uchun berilgan taqqoslama

$(x-1)^2(x+1) \equiv 0(mod\, 3)$ gat eng kuchli. Bundan $x_1 \equiv 1(mod\, 3)$ va $x_2 \equiv -1(mod\, 3)$ lar berilgan taqqoslamaning yechimlari ekanligi kelib chiqadi.

Tekshirish: 1) $1^5 - 1^4 - 1 + 1 \equiv 0(mod\, 3)$;

2)$(-1)^5 - (-1)^4 + 1 - 1 \equiv 0(mod\, 3)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob:**$x_1 \equiv 1(mod\, 3)$va$x_2 \equiv -1(mod\, 3)$.

d). Avvalo berilgan taqqoslamani $x^5 \cdot x^2 - x^5 \cdot x + 2 \equiv 0(mod\, 5) \rightarrow x^3 - x^2 + 2 \equiv 0(mod\, 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda$x_1 \equiv -1(mod\, 5)$va$x_2 \equiv -2(mod\, 5)$ lar berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:

1) $(-1)^7 - (-1)^6 + 5 \cdot (-1)^2 - 3 = -1 - 1 + 5 - 3 = 0 \equiv 0(mod\, 5)$;

2) $(-2)^7 - (-2)^6 + 5 \cdot (-2)^2 - 3 = 64(-3) + 20 - 3 = 65(-3) + 20 \equiv 0(mod\, 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x_1 \equiv -1(mod\, 5)$va$x_2 \equiv -2(mod\, 5)$.

e). Avvalo berilgan taqqoslamani $x^5 + x^4 + x^3 - x^2 - 2 \equiv 0(mod\, 5) \rightarrow x + x^4 + x^3 - x^2 - 2 \equiv 0(mod\, 5) \rightarrow x^4 + x^3 - x^2 + x - 2 \equiv 0(mod\, 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv 2(mod\, 5)$va$x_2 \equiv -2(mod\, 5)$ lar berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:

1) $2^5 + 2^4 + 2^3 - 2^2 - 2 = 32 + 16 + 8 - 4 - 2 = 50 \equiv 0(mod\, 5)$;

2) $(-2)^5 + (-2)^4 + (-2)^3 - (-2)^2 - 2 = -32 + 16 - 8 - 4 - 2 = -30 \equiv 0(mod\, 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x_1 \equiv 2(mod\, 5)$ va $x_2 \equiv -2(mod\, 5)$.

b) Berilgan taqqoslamani $x^7 - 6 \equiv 0(mod\, 5) \rightarrow x^5 \cdot x^2 - 6 \equiv x^3 - 6 \equiv 0(mod\, 5)$ ko'rinishda yozib olamiz.

Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 1(mod\, 5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:$1 - 6 = -5 \equiv 0(mod\, 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob**: $x \equiv 1(mod\, 5)$.

g).Berilgan taqqoslamani $x^8 + 2x^7 + x^5 - x + 3 \equiv 0(mod\, 5) \rightarrow x^5 \cdot x^3 + 2x^5 \cdot x^2 + x^5 - x + 3 \equiv 0(mod\, 5) \rightarrow x^4 + 2x^3 + x - x + 3 \equiv 0(mod\, 5) \rightarrow 2x^3 +$

$4 \equiv 0(mod\,5) \to x^3 + 2 \equiv 0(mod\,5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 2(mod\,5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:$2^8 + 2^8 + 2^5 - 2 + 3 = 512 + 32 + 1 = 545 \equiv 0(mod\,5)$.

Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x \equiv 2(mod\,5)$.

h). Berilgan taqqoslamani $6x^4 + 17x^2 - 16 \equiv 0(mod\,3) \to 2x^2 - 1 \equiv 0(mod\,3)$ ko'rinishda yozib olamiz. Bu taqqoslamani 3 moduli bo'yicha chegirmalarning to'la sistemasidagi 0, ±1 ni qo'yib, tanlash usuli bilan yechamiz. Bu sonlarning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Shuning uchun ham berilgan taqqoslama yechimga ega emas. **Javob:** taqqoslama yechimga ega emas.

i). Berilgan taqqoslamani $4x^7 - 2x^3 + 8 \equiv 0(mod\,5) \to -x^5 \cdot x^2 - 2x^3 - 2 \equiv (mod\,5) \to -x^3 - 2x^3 - 2 \equiv 0(mod\,5) \to 3x^3 + 2 \equiv 0(mod\,5) \to x^3 - 1 \equiv 0(mod\,5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 1(mod\,5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish : $4 \cdot 1^7 - 2 \cdot 1^3 + 8 = 10 \equiv 0(mod\,5) \equiv 0(mod\,5)$.

Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x \equiv 1(mod\,5)$.

j). Berilgan taqqoslamani $3x^7 - 2x^6 + 2x^2 + 13 \equiv 0(mod\,5) \to -2x^5 \cdot x^2 - 2x^5 \cdot x + 2x^2 - 2 \equiv (mod\,5) \to -2x^3 - 2x^2 + 2x^2 - 2 \equiv 0(mod\,5) \to x^3 + 1 \equiv 0(mod\,5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib, tanlash usuli bilan yechamiz. U holda $x \equiv -1(mod\,5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish: $3 \cdot (-1)^7 - 2 \cdot (-1)^6 + 2 \cdot (-1)^2 + 13 = -3 - 2 + 2 + 13 = 10 \equiv 0(mod\,5) \equiv 0(mod\,5)$. Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob:** $x \equiv -1(mod\,5)$.

277. *a).* Berilgan taqqoslamani $f(x) = x^3 + 4x^2 - 3 \equiv 0(mod\,5) \to x^3 - x^2 + 2 \equiv 0(mod\,5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv -1(mod\,5) va x_2 \equiv -2(mod\,5)$ lar berilgan taqqoslamaning yechimlari bo'lgani uchun $x^3 + 4x^2 - 3 \equiv (x+2)(x+1)h(x) \equiv 0(mod\,5)$ bo'lishi kerak. $h(x)$ ni aniqlash uchun $x^3 + 4x^2 - 3$ ni $(x+2)(x+$

$1) = x^2 + 3x + 2$ ga bo'lamiz va biz quyidagiga ega bo'lamiz:$f(x) = (x+2)(x+1)(x+1) + (-5x+5)$, ya'ni

$f(x) \equiv (x+2)(x+1)^2 (mod\, 5)$.

Javob: $f(x) \equiv (x+2)(x+1)^2 (mod\, 5)$.

b). Berilgan taqqoslamani soddalashtirib $f(x) = x^4 + x^3 - x^2 + x - 2 \equiv 0(mod\, 5) \rightarrow x^3 - x^2 + x - 1 \equiv 0(mod\, 5)$ ko'rinishda yozib olamiz. $f(x) = x^3 - x^2 + x - 1 \equiv 0(mod\, 5)$ ni quyidagicha yozish mumkin: $f(x) = x^2(x-1) + (x-1) = (x-1)(x^2+1) \equiv 0(mod\, 5)$. Endi $x^2 + 1 \equiv 0(mod\, 5)$ taqqoslamaning yechimini izlaymiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv 2(mod\, 5)$ va $x_2 \equiv -2(mod\, 5)$lar berilgan taqqoslamaning yechimlari. Shuning uchun ham $f(x) \equiv (x+2)(x-1)(x-2)(mod\, 5)$. **Javob:** $f(x) \equiv (x+2)(x-1)(x-2)(mod\, 5)$.

c). Berilgan $x^4 + x + 4 \equiv (mod\, 11)$ taqqoslamani 11 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar

$0, \pm1, \pm2, \pm3, \pm4, \pm5$ (1)

larni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 2(mod\, 11)$ berilgan taqqoslamaning bitta yechimi ekanligini topamiz. U holda $x^4 + x + 4 = (x-2)(x^3 + 2x^2 + 4x - 2) + 22(mod\, 11) \equiv (x-2)(x^3 + 2x^2 + 4x - 2)(mod\, 11)$ ni hosil qilamiz. Endi $x^3 + 2x^2 + 4x - 2 \equiv 0(mod\, 11)$ taqqoslamanig yechimini izlaymiz. (1) dagi chegirmalarni tekshirib ko'ramiz. U holda $x_2 \equiv 2(mod\, 11)$, $x_3 \equiv 3(mod\, 11), x_4 \equiv 4(mod\, 11)$ lar uning yechimi ekanligini topamiz. Demak, $x^4 + x + 4 \equiv (x-2)^2(x-3)(x-4)(mod\, 11)$ bo'lar ekan.

Javob: $f(x) \equiv (x-2)^2(x-3)(x-4)(mod\, 11)$.

d). Berilgan $x^2 + 2x + 2 \equiv 0(mod 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\, 5)$, $x_2 \equiv 2(mod\, 5)$ lar taqqoslamaning yechimlari boladi. Shuning uchun ham $x^2 + 2x + 2 = (x-1)(x-2)(mod 5)$.

Javob: $f(x) \equiv (x-1)(x-2)(mod 5)$.

e). Berilgan $3x^3 - 1 \equiv 0(mod\, 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ ni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv -2(mod\, 5)$ taqqoslamani qanoatlantirishini topamiz. Shuning uchun ham $3x^3 - 1 \equiv (x+2)(3x^2 - 6x + 2)(mod\, 5)$. Endi $3x^2 - x + 2 \equiv 0(mod 5)$ taqqoslamaning yechiminiizlaymiz. Bu taqqoslama yechimga emas. Shuning uchun ham $3x^2 - 1 \equiv (x+2)(3x^2 - x + 2)(mod\, 5)$.

Javob:$f(x) \equiv (x+2)(3x^2 - x + 2)(mod 5)$.

f).$f(x) = 2x^4 + x^3 - 3x^2 + 2x - 2 \equiv 0(mod 11)$ ni qaraymiz. Bu taqqoslamani 11 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar

$0, \pm1, \pm2, \pm3, \pm4, \pm5$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 11)$ ning berilgan taqqoslamani qanoatlantirishini ko'ramiz. $f(x)$ ni $(x-1)$ gabo'lamiz. U holda $f(x) = (x-1)(2x^3+3x^2+2) \equiv 0(mod 11)$ ga ega bo'lamiz. Endi $2x^3+3x^2+2 \equiv 0(mod 11)$ taqqoslamaning yechimini izlaymiz. Bu taqqoslamani tanlash usuli bilan yechib uning yechimi yo'q ekanligiga ishonch hosil qilamiz. Shunday qilib, $f(x) = (x-1)(2x^3+3x^2+2) \equiv 0(mod 11)$. **Javob:**$f(x) = (x-1)(2x^3+3x^2+2) \equiv 0(mod 11)$.

g).$f(x) = x^4 - 7x^3 + 13x^2 + 21x + 23 \equiv 0(mod 7)$ ni qaraymiz. Buni soddalashtirib $f(x) = x^4 - x^2 + 2 \equiv 0(mod 7)$ ni hosil qilamiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv 2(mod\ 7)$, $x_2 \equiv -2(mod\ 7)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib $f(x)$ ni$(x+2)(x-2) = x^2 - 4$ ga bolib, $f(x) = x^4 - x^2 + 2 \equiv (x+2)(x-2)(x^2 + 3) + 14 \equiv (x+2)(x-2)(x^2+3)(mod 7)$ ni hosil qilamiz. Endi $x^2 + 3 \equiv 0(mod 7)$ ning yechimini izlaymiz. Bu yerda $x^2 \equiv -3(mod 7) \to x^2 \equiv 4(mod 7)$ bo'lgani uchun $x_3 \equiv 2(mod\ 7)$, $x_4 \equiv -2(mod\ 7)$ lar oxirgi taqqoslamaning yechimi bo'ladi. Demak, $f(x) \equiv (x+2)^2(x-2)^2(mod 7)$.

Javob: $f(x) \equiv (x+2)^2(x-2)^2(mod 7)$.

h). $f(x) = 2x^4 + x^3 - 3x^2 + 2x - 2 \equiv 0(mod 5)$ ni qaraymiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2$ larni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 5)$,

$x_2 \equiv 2(mod\ 5)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib $f(x)$ ni $(x-1)(x-2) = x^2 - 3x + 2$ ga bo'lib

$f(x) = (x-1)(x-2)(2x^2 + 7x + 14) + 30(x-1)(mod 5) \equiv (x-1)(x-2)(2x^2 + 2x - 1)(mod 5)$ ni hosil qilamiz. Endi $2x^2 + 2x - 1 \equiv 0(mod 5)$ ning yechimlarini izlaymiz. Bu taqqoslama yechimga ega emas. Shuning uchun ham $f(x) \equiv (x-1)(x-2)(2x^2+2x-1)(mod 5)$ deb yoza olamiz.

Javob: $f(x) \equiv (x-1)(x-2)(2x^2+2x-1)(mod 5)$.

i).$f(x) = 2x^3 + 5x^2 - 2x - 3 \equiv 0(mod 7)$ ni qaraymiz. Buni soddalashtirib, $f(x) = 2x^3 - 2x^2 - 2x - 3 \equiv 0(mod 7)$ ni hosil qilamiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni qo'yib, tanlash usuli bilan yechamiz. U holda bu sonlarning birortasi ham berilgan taqqoslamani qanoatlantirmasligini ko'ramiz, ya'ni taqqoslama yechimga ega emas. Shuning uchun ham $f(x)$ ko'paytuvchilarga ajralmaydi.

Javob: $f(x)$ ko'paytuvchilarga ajralmaydi.

j).$f(x) = x^4 - 2x^2 + x + 4 \equiv 0(mod 7)$ ni qaraymiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni qo'yib, tanlash usuli bilan yechamiz. U holda $x \equiv 2(mod\ 7)$ ning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib, $f(x)$ ni $x-2$ ga bo'lib $f(x)=(x-2)(x^3+2x^2+2x+5)+14\equiv(x-2)(x^3+2x^2+2x-2)(mod7)$ ni hosil qilamiz. Endi $x^3+2x^2+2x-2\equiv 0(mod7)$ ning yechimlarini izlaymiz. $x\equiv 3(mod\ 7)$ uning yechimi bo'lgani uchun oxirgu taqqoslama $x-3$ ga bo'linadi, ya'ni $x^3+2x^2+2x-2=(x-3)(x^2+5x+17)+49\ (mod7)\equiv(x-3)(x^2-2x+3)(mod7).x^2-2x+3\equiv 0(mod7)$ ni qaraymiz. Bu taqqoslama yechimga ega emas, ya'ni ko'paytuvchilarga ajralmaydi. Shunday qilib, $f(x)=(x-2)(x-3)(x^2-2x+3)(mod7)$.

Javob: $f(x)=(x-2)(x-3)(x^2-2x+3)(mod7)$.

277. *a).* Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x)\equiv 8x^{13}\cdot x^7-(13+2)x^{13}\cdot x^6+7x^{13}\cdot x^5+(13\cdot 2+2)\,x^{13}\cdot x^4-4x^{13}\cdot x^3+(2\cdot 13+4)x^{13}\cdot x^2+10x^6-4x^3+(13+10)x^2-(13+8)x-11\equiv 8x^8-2x^7+7x^6+2x^5-4x^4+4x^3-3x^6-4x^3-3x^2+5x+2(mod13)\equiv -5x^8-2x^7+4x^6+2x^5-4x^4-3x^2+5x+2(mod13)$.

Demak, biz$-5x^8-2x^7+4x^6+2x^5-4x^4-3x^2+5x+2\equiv 0(mod13)$ taqqoslamani yechishimiz kerak. $m=13$ moduli bo`yicha chegirmalarning to'la sistemasidagi sonlar $0,\pm1,\pm2,\pm3,\pm4,\pm5,\pm6$ larni sinab ko`rsak ularning birortasi ham berilgan taqqoslamani qonoatlantirmaydi. Demak, taqqoslama yechimga ega emas. **Javob:** taqqoslama yechimga ega emas.

b). Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x)\equiv x^7\cdot x^3+x^7\cdot x+x^7-x^4-x^2+4x-3\equiv x^4+x^2+x-x^4-x^2+4x-3\equiv 5x-3(mod7)$. Demak, biz $5x-3\equiv 0(mod7)$ taqqoslamaning yechimlarini izlashimiz kerak: $5x\equiv 3(mod7)\rightarrow 5x\equiv 10(mod7)$. Bu yerda $(5,7)=1$ bo`lgani uchun $x\equiv 2(mod7)$. **Javob:** $x\equiv 2(mod7)$.

c). Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x)\equiv x^{101}+3x^{15}+x^{11}-3x^5+9x^2+10x-5\equiv(x^{11})^9\cdot x^2+3x^{11}\cdot x^4+x^{11}-3x^5-2x^2-x-5\equiv x+3x^5+x-3x^5-2x^2-x-5\equiv -2x^2+x-5\equiv 0(mod11)$. Demak, berilgan taqqoslama $2x^2-5x+5\equiv 0(mod11)$ ga teng kuchli ekan. 11 modul bo`yicha chegirmalarning to'la sistemasidagi sonlar $0,\pm,\pm2,\pm3,\pm4,\pm5$ larni sinab ko`rish yo`li bilan yechsak, $x_1\equiv 2(mod11)$va$x_2\equiv 4(mod11)$ yechimlarga ega bo`lamiz.

Javob: $x_1\equiv 2(mod11)$va$x_2\equiv 4(mod11)$.

e) Berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz:

f) $f(x)\equiv 2x^{35}-17x^{15}+13x^8-3x^5+12x+5\equiv 0(mod11)\rightarrow 2(x^{10})^3\cdot x^5+5x^{10}\cdot x^5+2x^8-3x^5+x+5\equiv 2x^5+5x^5+2x^8-3x^5+x+5\equiv 2x^8+4x^5+x+5\equiv 0(mod11)$. Demak, berilgan taqqoslama $2x^8+4x^5+x+5\equiv 0(mod11)$ ga teng kuchli. 11 modul bo`yicha $0,\pm1,\pm2,\pm3,\pm4,\pm5$ larni sinab ko`rish yo`li bilan yechsak, $x_1\equiv 3(mod11)$ va $x_2\equiv 5(mod11)$ lar taqqoslamaning yechimlari ekanligiga ishonsh hosil qilamiz. **Javob:**$x_1 \equiv 3(mod 11)$ va $x_2 \equiv 5(mod 11)$.

e).Berilgan $x^{12} - 2x^7 + x^3 + 1 \equiv 0(mod 5)$ soddalashtirib,

$(x^5)^2 \cdot x^2 - 2x^5 \cdot x^2 + x^3 + 1 \equiv 0(mod 5) \to x^4 - 2x^3 + x^3 + 1 \equiv 0(mod 5)$ $\to x^4 - x^3 + 1 \equiv 0(mod 5)$. Demak, berilgan taqqoslama $x^4 - x^3 + 1 \equiv 0(mod 5)$ ga teng kuchli. 5 modul bo`yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2$ larni sinab ko`rish yo`li bilan yechsak, $x \equiv -2(mod 5)$ yechimga ega bo`lamiz. **Javob:** $x \equiv -2(mod 5)$.

279. Bizga ma`lumki, $p$-tub modulbo`yicha $x^p - x$ ni $f(x)$ ga bo`lishdan chiqqan qoldiq $R(x)$ ning barcha koeffitsiyentlari $p$ ga bo`linishi kerak. $R(x)$ ni aniqlaymiz: $x^7 - x = (x^3 + ax + b)(x^4 - ax^2 - bx + a^2) + 2abx^2 + (b^2 - a^3 - 1)x - a^2 b$. Demak,

$$\begin{cases} 2ab \equiv 0(mod 7) \\ b^2 - a^3 - 1 \equiv (mod 7) \\ a^2 b \equiv (mod 7) \end{cases}$$

bajaralishi kerak, shartga ko`ra $a \not\equiv 0(mod 7)$ va $b \not\equiv 0(mod 7)$ bo`lgani uchun $ab \not\equiv (mod 7)$ ya`ni birinchi shart bajarilmaydi. Shunday qilib berilgan taqqoslama 3 ta yechimga ega bo`la olmaydi.

280. $x^p - x$ ni $x^n - a$ ga bo`lib, $x^p - x = (x^n - a) \cdot (x^{p-n} - ax^{p-2n}) + ax^{p-n} - x$ ni hosil qilamiz. Birinchi qoldiq $ax^{p-n} - x$ ga teng. Bo'lish jarayonidagi ikkinchi qoldiq $a^2 x^{p-2n} - x$ va hokazo k-qo'ldiq $a^k x^{p-kn} - x$ larni hosil qilamiz. Faraz etaylik k- qoldiq oxirgisi bo'lsin. U holda $R(x) = a^k x^{p-kn} - x$ bo`ladi. 279-misolga asosan $x^n \equiv a(mod p)$ ning $n$ ta yechimga ega bo`lishi uchun $R(x)$ ning barcha koeffisentlari p ga bo`linishi kerak. Agar $p - nk > 1$ bo`lsa, a^k va 1 koeffitsiyentlar p ga bo`linmaydi va demak bu holda $x^n \equiv a(mod p)$ taqqoslama $n$ ta yechimga ega bo`lmaydi.

Agarda $p - nk = 1$ bo`lsa $R(x) = (a^k - 1)x$ bo`lib$x^n \equiv a(mod p)$ taqqoslamaning n ta yechimga ega bo`lishi uchun $a^k - 1 \equiv 0(mod p)$ yoki

$$a^k \equiv 1(mod p) \to a^{\frac{p-1}{n}} \equiv 1(mod p) \qquad (1)$$

bajarilishi kerak ekan. Shunday qilib, $x^n \equiv a(mod p)$, $n < p$ va $(a, p) = 1$ taqqoslamaning n ta yechimga ega bo`lishi uchun $\frac{p-1}{n}$ butun son bo`lib (1) shartning bajarilishi zarur va yetarli ekan.

281. *a).* $x^3 \equiv 1(mod 7)$ ni qaraymiz. 280-misoldagi $a^{\frac{p-1}{n}} \equiv 1(mod p)$ shartni tekshiramiz $1^{\frac{7-1}{3}} \equiv 1(mod 7)$ bajariladi. Demak, berilgan taqqoslama 3 ta yechimga ega. Endi shu yechimlarni topamiz. Buning uchun 7 modul bo`yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2, \pm 3$ larni sinab ko`rish yo`li bilan yechsak, u holda $x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3(mod7)$ larning berilgan taqqoslamani qanoatlantirishini ko`ramiz.

Javob: $x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3(mod7)$.

b). $x^2 \equiv 2(mod5)$ ni qaraymiz. (1) dan $2^{\frac{5-1}{2}} \equiv 2^2 \equiv 4(mod5)$. Demak, berilgan taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

c). $x^5 \equiv 10(mod11)$ ni qaraymiz. (1) dan $a^{\frac{11-1}{5}} \equiv 10^2 \equiv 1(mod11)$, Demak, berilgan taqqoslama 5 ta yechimga ega. Endi shu yechimlarni topamiz. Buning uchun 11 modul bo`yicha chegirmalarning to'la sistemasidagi
sonlar $0, \pm1, \pm2, \pm3, \pm4, \pm5$ larni sinab ko`rish yo`li bilan yechsak, u holda
$x_1 \equiv -1, x_2 \equiv +2, x_3 \equiv -3, x_4 \equiv -4, x_5 \equiv -5(mod11)$ larning berilgan taqqoslamani qanoatlantirishini ko`ramiz.

Javob: $x_1 \equiv -1, x_2 \equiv 2, x_3 \equiv -3, x_4 \equiv -4, x_5 \equiv -5(mod11)$.

d). $x^4 \equiv 1(mod11)$ ni qaraymiz. (1) dan $1^{\frac{11-1}{4}} \equiv 1^{\frac{5}{2}} \equiv 1$ bo'lishi kerak. Lekin bu yerda $\frac{p-1}{n}$ butun son emas shuning uchun ham berilgan taqqoslama 4 ta yechimga ega deya olamiz. Taqqoslamaning yechimlarini topamiz. Buning uchun

$0, \pm1, \pm2, \pm3, \pm4, \pm5$ larni qo`yib tekshiramiz. U holda berilgan taqqoslama 2 ta $x_1 \equiv -2(mod11),\ x_2 \equiv 2(mod11)$ yechimlarga ega ekanligiga ishonch hosil qilamiz.

Javob: $x_1 \equiv -2(mod11),\ x_2 \equiv 2(mod11)$.

c) $x^6 \equiv 3(mod7)$ ni qaraymiz. (1) dan$3^{\frac{7-1}{6}} \equiv 3(mod7)$. Demak berilgan taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

f). $x^4 \equiv 3(mod13)$ ni qaraymiz. (1) dan $3^{\frac{13-1}{4}} \equiv 3^3 \equiv 1(mod13)$. Demak, berilgan taqqoslama 4 ta yechimga ega. Taqqoslamaning yechimlarini topamiz. Buning uchun $0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni qo`yib tekshiramiz. U holda $x_1 \equiv -2$, $x_2 \equiv 2,\ x_3 \equiv -3,\ x_4 \equiv 3(mod13)$ larning berilgan taqqoslamaning yechimi ekanligiga ishonch hosil qilamiz.

282. Vilson teoremasiga asosan p tub soni uchun

$$(p-1)! + 1 \equiv 0(modp) \rightarrow (p-2)!\,(p-1) \equiv -1(modp) \rightarrow$$
$$(p-2)! \equiv 1(m\square dp)$$

bajariladi.

283. Faraz etaylik, $p$va $p+2$ lar tub sonlar bo`lsinlar. Vilson teoremasiga ko`ra $(p-1)! + 1 \equiv 0(modp)$. Buning ikkala tomonini 4 ga ko`paytirib hosil bo`lgan taqqoslamani $p \equiv 0(modp)$ ayniy taqqoslama qo`shamiz. U holda

$$4 \cdot [(p-1)! + 1] + p \equiv 0(modp) \qquad (2)$$

taqqoslamaga ega bo`lamiz. Endi $p+2\equiv 0(modp+2)$ taqqoslamaniga qaraymiz. Bundan $p\equiv -2(modp+2)$. Buning ikkala tomonini $(p+1)$ ga ko`paytirsak, $p(p+1)\equiv -2(p+1)=-2\big((p+2)-1\big)\equiv -2(p+2)+2\equiv 2(modp+2)$

hosil bo`ladi, ya'ni $p(p+1)\equiv 2(modp+2)$. Oxirgi taqqoslamaning ikkala tomonini $(p-1)!\cdot 2$ ga ko`paytirib, hosil bo`lgan taqqoslamaning ikkala tomoniga $4+p$ ni qo`shamiz. U holda

$$2\cdot(p+1)!+4+p\equiv(p-1)!\cdot 4+4+p(mod\ p+2)$$
$$\to 2[(p+1)!+1]+(p+2)$$
$$\equiv 4[(p-1)!+1]$$
$$+p(modp+2). \qquad (3)$$

Agar p tub son bo`lsa, Vilson teoremasiga ko`ra $(p+1)!+1\equiv 0(modp+2)$ bo'lishi kerak. Shuning uchun ham (3) dan $4[(p-1)!+1]+p\equiv 0(modp+2)$ (4)

kelib chiqadi. (2)va (4) dan

$$4[(p-1)!+1]+p\equiv 0(modp(p+2)) \qquad (5)$$

ni hosil qilamiz.

Aksincha, agar (5) shart bajarilsa, (4) ning bajarilishi kelib chiqadi. Bundan (3) ga asosan $(p+1)!+1\equiv 0(modp+2)$ hosil bo`ladi. Bu esa Vilson teoremasiga asosan $p+2$ soni tub p son degani.

284. $p=4n+1$ tub son bo`lsin. U holda Vilson teoremasiga asosan:

$(p-1)!\equiv -1(modp)\to(4n)!\equiv -1(modp)$. Bu yerda

$1\cdot 2\cdot 3\cdot\dots\cdot 2n\cdot(2n+1)(2n+2)\cdot\dots\cdot(4n-1)\cdot 4n=(2n)!\,(p-2n)(p-2n+1)\dots(p-2)(p-1)=(2n)!\,[(-1)(-2)\dots(-2n)+pt](modp)\equiv(-1)^{2n}((2n)!)^2\ (modp)\equiv((2n)!)^2\ (modp)$ bo`lgani uchun $\big((2n)!\big)^2\equiv -1(modp)$ ga bo`lamiz.

285. *a).* Vilson teoremasiga asosan

$$(p-1)!\equiv -1(modp). \qquad (6)$$

Bundan $a(p-1)!\equiv -a(modp)$. Ferma teoremasiga asosan

$$a^p\equiv a(modp). \qquad (7)$$

Bu ikki taqqoslamani hadlab qo`shsak, $a^p+a(p-1)!\equiv 0(modp)$hosil bo`ladi.

b). (6) va (7) ni hadlab ko'paytirsak,

$$a^p(p-1)!\equiv -a(modp)\to a^p(p-1)!+a\equiv 0(modp)$$

hosil bo`ladi.

286. $p>2$tub son bo`lsin. U holda (6) dan$(p-1)!+1=(p-2)!\,(p-1)+1=(p-2)!\,p-(p-2)!+1\equiv 0(modp)$ yoki $(p-2)!-1\equiv 0(modp)$.

287. $$x^p-x=f(x)Q(x)+R(x) \qquad (8)$$

ayniyatni qaraymiz. Bu yerda $Q(x)$ va $R(x)$ lar butun koeffitsiyentli ko`phadlar bo`lib, deg $Q(x) = p - n$, deg$R(x) \leq n - 1$. Agar $f(x) \equiv 0(modp)$ taqqoslama n ta yechimga ega bo'lsa, u yechimlar $R(x) \equiv 0(modp)$ni ham qanoatlantirishi kerak. deg.$R(x) \leq n - 1$bo`lgani uchun 2- teoremaning natijasiga asosan $R(x)$ ning barcha koeffitsiyientlari $p$ ga bo`linishi kerak.

Aksincha, $R(x)$ning barcha koeffitsiyientlari p ga bo`linsa, (8) dan $f(x)Q(x) \equiv 0(modp)$ hosil bo`ladi. Demak, $f(x) \equiv 0(modp)$ taqqoslama *n* ta yechimga ega.

288. *a).* x^5-x ning berilgan $x^2 + 2x + 2$ ko'phadga bo'lib qoldiqni topamiz:

$$x^5 - x = (x^2 + 2x + 2) \cdot (x^3 - 2x^2 + 2x) + (-5x).$$

Bundan $R(x) = -5x$ bo'lib, -5 soni 5 ga bo'linadi. Berilgan taqqoslama 2 ta yechimga ega. Haqiqatdan ham, 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 5)$ va $x_2 \equiv 2(mod\ 5)$ lar berilgan $x^2 + 2x + 2 \equiv 0(mod\ 5)$ taqqoslamani qanoatlantirishini topamiz. **Javob:**$x_1 \equiv 1(mod5)$, $x_2 \equiv 2\ (mod5)$.

b). Avvalo, berilgan $3x^3 - 4x^2 - 2x - 4 \equiv 0(mod7)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan taqqoslama bilan almashtiramiz.

$3a \equiv 1(mod7) \rightarrow a \equiv 5(mod7) \rightarrow a \equiv -2(mod7)$ bo'lgani uchun berilgan taqqoslamaning ikkala tomonini (-2) ga ko'paytiramiz:

$$-6x^3 + 8x^2 + 4x + 8 \equiv x^3 + x^2 - 3x + 1(mod7)$$

bo'lgani uchun berilgan taqqoslama $x^3 + x^2 - 3x + 1 \equiv 0(mod7)$ga teng kuchli. Endi $R(x)$ ni aniqlaymiz: $x^7 - x = (x^3 + x^2 - 3x + 1) \cdot (x^4 - x^3 + 4x^2 - 8x + 21) + (-49x^2 + 70x - 21)$ bo'lgani uchun $R(x) = -49x^2 + 70x - 21$. $R(x)$ ning barcha koeffitsiyentlari 7 ga karrali, shuning uchun ham berilgan $x^3 + x^2 - 3x + 1 \equiv 0(mod7)$ taqqoslama 3 ta yechimga ega. Bu yerda 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2, \pm 3$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3\ (mod7)$ lar uni qanotlantiradi. **Javob:**$x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3\ (mod7)$.

IV.5-§.

289. 1). Berilgan taqqoslama ushbu

$$\begin{cases} 3x^3 + 4x^2 - 7x - 6 \equiv 0(mod3) \\ 3x^3 + 4x^2 - 7x - 6 \equiv 0(mod5) \end{cases}$$

sistemaga teng kuchli. Bu sistemani soddalashtiramiz:

$$\begin{cases} x^2 - x \equiv 0(mod3) \\ 2x^3 + x^2 + 2x + 1 \equiv 0(mod5). \end{cases}$$

Bu sistemadagi 1-taqqoslamaning yechimlari $x \equiv 0, 1\ (mod3)$; ikkinchisiniki esa $x \equiv -2, 2(mod5)$ lardan iborat, natijada quydagi 4ta sistemaga ega bo'lamiz:

a) $\begin{cases} x \equiv 0(mod3) \\ x \equiv -2(mod5) \end{cases}$ b) $\begin{cases} x \equiv 0(mod3) \\ x \equiv 2(mod5) \end{cases}$

c) $\begin{cases} x \equiv 1(mod3) \\ x \equiv -2(mod5) \end{cases}$ d) $\begin{cases} x \equiv 1(mo\square 3) \\ x \equiv 2(mod5) \end{cases}$.

a)dan $\begin{cases} x = 3t_1 \\ 3t_1 \equiv -2(mod5) \end{cases} \rightarrow \begin{cases} t_1 \equiv 1(mod\ 5) \\ t_1 = 1 + 5t_2,\ t_2 \in Z. \end{cases} \rightarrow x = 3(1 + 5t_2) = 3 + 15t_2,\ t_2 \in Z;$

b)dan $\begin{cases} x = 3t_1 \\ 3t_1 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} x \equiv 3t_1 \\ t_1 \equiv -1(mod5) \end{cases} \rightarrow x = 3(-1 + 5t_2) = -3 + 15t_2, t_2 \in Z$

c) dan $\begin{cases} x = 1 + 3t_1 \\ 1 + 3t_1 \equiv -2(mod5) \end{cases} \rightarrow \begin{cases} x \equiv 1 + 3t_1 \\ 3t_1 \equiv -3(mod5) \end{cases} \rightarrow \begin{cases} x = 1 + 3t_1 \\ t_1 = -1 + 5t_2 \end{cases}$

$$\rightarrow x = 1 + 3(-1 + 5t_2) = -2 + 15t_2, t_2 \in Z.$$

d) dan $\begin{cases} x = 1 + 3t_1 \\ 1 + 3t_1 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} x = 1 + 3t_1 \\ 3t_1 \equiv 1(mod5) \end{cases} \rightarrow \begin{cases} x = 1 + 3t_1 \\ t_1 = 2 + 5t_2 \end{cases}$

$$\rightarrow x = 1 + 3(2 + 5t_2) = 7 + 15t_2, t_2 \in Z.$$

Shunday qilib berilgan taqqoslama 4 ta yechimga;

$x \equiv 3, -3, -2, 7\ (mod15)$ ega ekan.

Javob: $x \equiv 3, -3, -2, 7\ (mod15)$.

2). Berilgan taqqoslama $6x^3 - 3x^2 - 13x - 10 \equiv 0(mod30)$ ushbu taqqoslamalar

$$\begin{cases} 6x^3 - 3x^2 - 13x - 10 \equiv 0(mod2) \\ 6x^3 - 3x^2 - 13x - 10 \equiv 0(mod3) \\ 6x^3 - 3x^2 - 13x - 10 \equiv 0(mod5) \end{cases}$$

sistemasiga teng kuchli. Bu sistema

$$\begin{cases} x^2 - x \equiv 0(mod2) \\ 2x - 1 \equiv 0(mod3) \\ x^3 + 2x^2 + 2x \equiv 0(mod5) \end{cases}$$

ga teng kuchli. Bu yerdagi birinchi taqqoslamaning yechimi $x \equiv 0{,}1(mod2)$, ikkinchisiniki $x \equiv 2(mod3)$, uchunchisiniki esa $x \equiv 0, 1, 2(mod5)$ dan iborat.

Bulardan

a) $\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases}$ b) $\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases}$

$$\text{c)}\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases} \quad \text{d)}\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 2(mod5) \end{cases}$$

$$\text{e)}\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases} \quad \text{f)}\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 2(mod5) \end{cases}$$

chiziqli taqqoslamalar sistemalariga ega bo'lamiz.

a) dan $\begin{cases} x = 2t_1 \\ 2t_1 \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} x = 2t_1 \\ t_1 = 1 + 3t_2 \\ x \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} x = 2 + 6t_2 \\ x \equiv 0(mod5) \end{cases} \rightarrow$

$2 + 6t_2 \equiv 0(mod5) \rightarrow \;\; 3t_2 \equiv -1(mod5) \rightarrow 3t_2 \equiv 9(mod5)t_2 = 3 + 5t_3, \;\; t_3 \in Z$

$\rightarrow x = 2 + 6(3 + 5t_3) \;\;\; = 20 + 30t_3$

b) dan $\begin{cases} x = 1 + 2t_1 \\ 1 + 2t_1 \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} x = 1 + 2t_1 \\ t_1 = 2 + 3t_2 \\ x \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} x = 1 + 4 + 6t_2 \\ 5 + 6t_2 = 0(mod5) \\ t_2 \equiv 5t_3 \end{cases}$

$$x = 5 + 6(5t_3) = 5 + 30t_3.$$

c) dan $\begin{cases} x = 2t_1 \\ 2t_1 \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases} \rightarrow \begin{cases} x = 2t_1 \\ t_1 = 1 + 3t_2 \\ x \equiv 1(mod5) \end{cases} \rightarrow \begin{cases} x = 2 + 6t_2 \\ 2 + 6t_2 \equiv 1(mod5) \\ t_2 \equiv -1(mod5) \end{cases}$

$$x = 2 + 6(-1 + 5t_3) = -4 + 30t_3.$$

d) dan $\begin{cases} x = 5 + 6t_2 \\ x \equiv 2(mod5) \end{cases} \rightarrow 2 + 6t_2 \equiv 2(mod5) \rightarrow 6t_2 \equiv 0(mod5)$

$$\rightarrow t_2 \equiv 0(mod5) \rightarrow t_2 = 5t_3 \rightarrow x = 2 + 30t_3.$$

e) dan $\begin{cases} x = 5 + 6t_2 \\ 5 + 6t_2 \equiv 1(mod5) \end{cases} \rightarrow \begin{cases} t_2 \equiv -4(mod5) \\ t_2 \equiv 1 + 5t_3 \end{cases} \rightarrow$

$$x = 5 + 6(1 + 5t_3) \equiv 11 + 30t_3\,, t_3 \;\in Z.$$

f) dan $\begin{cases} x = 5 + 6t_2 \\ 5 + 6t_2 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} x = 5 + 6t_2 \\ t_2 \equiv 2 + 5t_3 \end{cases} \rightarrow x = 17 + 30t_3, \, t_3 \;\in Z.$

Shunday qilib, berilgan taqqoslamaning yechimlari
$x \equiv -13, -10, -4, 2, 5, 11 \; (mod30)$ lardan iborat ekan.
Javob:$x \equiv -13, -10, -4, 2, 5, 11 \; (mod30)$.
3).Berilgan taqqoslama$x^4 - 33x^2 + 8x - 26 \equiv 0(mod35)$ ushbu taqqoslamalar

$$\begin{cases} x^4-33x^2+8x-26\equiv 0(mod5) \\ x^4-33x^2+8x-26\equiv 0(mod7)\end{cases} \to \begin{cases} x^4+2x^2+3x-1\equiv 0(mod5) \\ x^4+2x^2+x+2\equiv 0(mod7)\end{cases}$$

sistemasiga teng kuchli. Bu yerda birinchi taqqoslamaning yechimi $x\equiv 1(mod5)$; ikkinchisining yechimi esa$x\equiv 2(mod7)$ dan iborat. Bulardan $\begin{cases} x\equiv 1(mod5) \\ x\equiv 2(mod7)\end{cases}$ sistemaga kelamiz.Buni yechib

$$\begin{cases} x=1+5t_1 \\ 1+5t_1\equiv 2(mod7)\end{cases} \to \begin{cases} x=1+5t_1 \\ 5t_1\equiv 1(mod7)\end{cases} \to \begin{cases} x=1+5t_1 \\ 5t_1\equiv 15(mod7)\end{cases}$$

$$\to \begin{cases} x=1+5t_1 \\ t_1\equiv 3(mod7)\end{cases} \to x=1+5(3+7t_2) \to x=16+35t_2,\ t_2\in Z.$$

Demak,berilgan sistemaning yechimi $x\equiv 16(mod35)$ dan iborat.

Javob: $x\equiv 16(mod35)$.

4) Berilgan taqqoslama $x^5-3x^4+5x^3+9x^2+4x-12\equiv 0(mod42)$ ushbu taqqoslamalar

$$\begin{cases} x^5-3x^4+5x^3+9x^2+4x-12\equiv 0(mod2) \\ x^5-3x^4+5x^3+9x^2+4x-12\equiv 0(mod3) \\ x^5-3x^4+5x^3+9x^2+4x-12\equiv 0(mod7)\end{cases}$$

sistemasiga teng kuchli. Bu sistemadagi taqqoslamalarni soddalashtirib quyidagi sistemaga ega bo’lamiz:

$$\begin{cases} (x^2)^2\cdot x-(x^2)^2+x^2\cdot x+x^2\equiv 0(mod2) \\ (x^3)\cdot x^2-x^3+x\equiv 0(mod3) \\ x^5-3x^4-2x^3+2x^2-3x+2\equiv 0(mod7)\end{cases} \to$$

$$\begin{cases} x^3+x^2\equiv 0(mod2) \\ x\equiv 0(mod3) \\ x^5-3x^4-2x^3+2x^2-3x+2\equiv 0(mod7)\end{cases}.$$

Birinchi taqqoslamani ixtiyoriy $x\in Z$ soni qanoatlantiradi. Ikkinchisining yechmi $x\equiv 0(mod3)$ dan iborat. Uchinchi taqqoslamani yechamiz. 0, $\pm 1, \pm 2, \pm 3$ lardan faqat 2 uni qanoatlantiradi. Demak, bu taqqoslama yagona $x\equiv 2(mod7)$ yechimga ega. Shunday qilib, ushbu

$$\begin{cases} x\equiv 0(mod3) \\ x\equiv 2(mod7)\end{cases}$$

sistemani hosil qildik. Buni yechib

$$\begin{cases} x=3t \\ 3t\equiv 2(mod7)\end{cases} \to \begin{cases} x=3t \\ t=3+7t\end{cases} \to x=3+21t$$

$x\equiv 3, 24\ (mod42)$ ga ega bolamiz. **Javob:**$x\equiv 3, 24\ (mod42)$.

5).Berilgan taqqoslama $x^5+x^4-3x^3+2x-2\equiv 0(mod77)$ ushbu taqqoslamalar

$\begin{cases} x^5+x^4-3x^3+2x-2\equiv 0(mod7) \\ x^5+x^4-3x^3+2x-2\equiv 0(mod11) \end{cases}$ sistemasiga teng kuchli. Bu sistemadagi taqqoslamalarni yechib quyidagilarga ega bo'lamiz: $0,\ \pm1,\ \pm2, \pm3$ lardan birortasi ham qanotlantirmaydi. Berilgan taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

6). Berilgan taqqoslama $3x^3+6x^2+x+10\equiv 0(mod15)$ ushbu taqqoslamalar $\begin{cases} 3x^3+6x^2+x+10\equiv 0(mod3) \\ 3x^3+6x^2+x+10\equiv 0(mod5) \end{cases}$ sistemasiga teng kuchli. Bu sistemadagi birinchi taqqoslamani tanlash usuli bilan yechamiz. Buning uchun 0, ±1 larni unga qo'yib tekshirib ko'rish kifoya. Holda $x\equiv -1(mod3)$ ning birinchi taqqoslamani qanoatlantirishini ko'ramiz. Sistemadagi ikkinchi taqqoslamaning yechimlari esa $x\equiv 0,1,2(mod5)$ lardan iborat. Natijada quydagi 3 ta sistemaga ega bo'lamiz:

a)$\begin{cases} x\equiv -1(mod3) \\ x\equiv 0(mod5) \end{cases}$ b) $\begin{cases} x\equiv -1(mod3) \\ x\equiv 1(mod5) \end{cases}$ c)$\begin{cases} x\equiv -1(mod3) \\ x\equiv 2(mod5) \end{cases}$.

Shuning uchun ham berilgan taqqoslamaning yechimini $x\equiv x_0=M_1M_1'b_1+M_2M_2'b_2$(mod15) ko'rinishda izlash mumkin. Bizda $M_1=5,\ M_2=3$ bo'lgani uchun $5M_1'\equiv 1(mod3)\to M_1'=2$ va $3M_2'\equiv 1(mod5)\to M_2'=2$ ga ega bo'lamiz. Bulardan foydalanib $x\equiv x_0=5\cdot 2b_1+3\cdot 2b_2(mod15)\equiv -5b_1+6b_2(mod15)$. Endi bunda $b_1=-1,\ b_2=0,1,2$ deb olib berilgan taqqoslamaning yechimlari $x_1\equiv 5(mod15),\ x_2\equiv 11(mod15), x_3\equiv 2(mod15)$ ni hosil qilamiz.

Javob: $x\equiv 2{,}5{,}11(mod15)$.

7). Berilgan taqqoslama $37x\equiv 17(mod180)$ da $180=36\cdot 5$ bo'lgani uchun ushbu taqqoslamalar $\begin{cases} 37x\equiv 17(mod5) \\ 37x\equiv 17(mod36) \end{cases}\to \begin{cases} x\equiv 1(mod5) \\ x\equiv 17(mod36) \end{cases}$ sistemasiga teng kuchli. Shuning uchun ham berilgan taqqoslamaning yechimini $x\equiv x_0=M_1M_1'b_1+M_2M_2'b_2$(mod15) ko'rinishda izlash mumkin. Bizda $M_1=36,\ M_2=5$ bo'lgani uchun $36M_1'\equiv 1(mod5)\to M_1'=1$ va $5M_2'\equiv 1(mod36)\to M_2'=-7$ ga ega bo'lamiz.

Bulardan foydalanib $x\equiv x_0=36\cdot b_1+5\cdot(-7)b_2(mod15)\equiv 36b_1-35b_2(mod180)$. Endi bunda $b_1=1,\ b_2=17$ deb olib berilgan taqqoslamaning yechimlari $x\equiv -19(mod180)$, ni hosil qilamiz. **Javob:** $x\equiv -19(mod180)$.

290. 1). $4x^3-8x-13\equiv 0(mod\ 27)$ taqqoslamani yechishimiz kerak. Bu yerda $27=3^3$ bo'lgani uchun avvalo, $4x^3-8x-13\equiv 0(mod3)$ taqqoslamani qaraymiz. Bundan x^3+x-1≡0(mod 3). Bu taqqoslamaning yechimlarini topish uchun 3 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0,\pm1$ ni qo'yib tekshirib ko'ramiz. U holda $x\equiv -1(mod\ 3)$ uning yechimi ekanligini ko'ramiz.

Endi bu $x \equiv -1+3t_1$ yechimni nazariy qismdagi

$$\frac{f(x_1)}{p}+t_1 f'(x_1) \equiv 0(mod p) \qquad (7)$$

ga olib borib qo'yamiz: U holda, bizda $p=3,\ x_1=-1, f(-1)=-9, f'(x)=12x^2-8$ va$f'(-1)\ =4$ bo'lgani uchun $\frac{-9}{3}$+$4t_1$≡$0(mod\ 3) \to t_1$≡$3(mod\ 3) \to t_1=3+3t_2 \to x=-1+9+9t_2=8+9t_2$. Endi yana (7) dan foydalanib, (nazariy qismdagi (9) ga qarang)

$$\frac{f(x_2)}{p^2}+t_2 f'(x_2) \equiv 0(mod\ p) \qquad (9)$$

ni tuzib olamiz. Bizda $p=3, x_2=8. f(8)\ =1971,\ f'(8)=760$ bo'lgani uchun $\frac{1971}{9}+760t_2 \equiv 0(mod\ 3) \to t_2 \equiv -219(mod\ 3) \to t_2 \equiv 0(mod\ 3)$, ya'ni $t_2=3t_3$. Bu qiymatni x ning ifodasiga olib borib qo'ysak, $x=8+9(3t_3)$= $8+27t_3$,$t_3 \epsilon Z$, ya'ni $x \equiv 8(mod\ 27)$ berilgan taqqoslamaning yechimiga ega bo'lamiz.

Javob: $x \equiv 8(mod\ 27)$.

2). $f(x)=x^4-3x^3+2x^2-5x-10 \equiv 0(mod\ 343)$ taqqoslamani yechishimiz kerak. Bu yerda $343=7^3$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod7)$ taqqoslamani qaraymiz. Bu $x^4-3x^3=2x^2+2x-3 \equiv 0(mod\ 7)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 7 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, \pm 1, \pm 2, \pm 3$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv 3(mod\ 7)$ uning yechimi ekanligini ko'ramiz. Demak, $x \equiv 3(mod\ 7)$ berilgan taqqoslamaning yechimi. Endi bu $x \equiv 3+7t_1$ yechimni nazariy qismdagi

$$\frac{f(x_1)}{p}+t_1 f'(x_1) \equiv 0(modp) \qquad (7)$$

ga olib borib qo'yamiz: U holda, bizda $f(3)=3^4$-3$\cdot 27+18-15-10=-7; f'(x)\ =4x^3-9x^2+4x-5$ va $f'(3)$= $108-81+12-5=34$ bo'lgani uchun (7) dan $\frac{(-7)}{7}+34t_1 \equiv 0(mod\ 7)$→$34t_1 \equiv 1(mod\ 7)$→$t_1 \equiv -1(mod\ 7)$, ya'ni $t_1 \equiv -1+7t_2$. Buni x ning ifodasiga qo'ysak, $x=3+7(-1+7t_2)=-4+49t_2$ hosil bo'ladi. Endi bundan foydalanib, (9) ni tuzamiz. Bunda $f(-4)=(-4)^4$ $-3\cdot(-4)^3+2 \cdot. 16+20-10=256+192+32+20-10=490. f'(-4)\ =-256-9\cdot 16-16-5=-421$ bo'lgani uchun $\frac{490}{49}+t_2(-421) \equiv 0(mod\ 7) \to\ -421t_2 \equiv -10(mod\ 7)$→ - $t_2 \equiv -3(mod\ 7)$→$t_2\ 3(mod\ 7)$→$t_2=3+7t_3,\ \ t_3 \in Z$. Hosil bo'lgan qiymatni x ning ifodasiga olib borib qo'ysak, $x=-4+49(3+7t_3)\ =-4+147+343t_3 \equiv 143+343t_3,\ t_3 \epsilon Z$ ga ega bo'lamiz, ya'ni $x \equiv 143(mod\ 343)$ berilgan taqqoslamaning yechimiga ega bo'lamiz.

Javob: $x \equiv 143(mod\ 343)$.

3). $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 25)$ taqqoslamani yechishimiz kerak. Bu yerda $25 = 5^2$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod 5)$ taqqoslamani, ya'ni $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 5)$ ni qaraymiz. Bu $x^4 + x^3 + 2x^2 + x + 1 \equiv 0(mod\ 5)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 5 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, \pm1, \pm2$ ni qo'yib, tekshirib ko'ramiz. U holda $x \equiv 2(mod\ 5)$ va $x \equiv -2(mod\ 5)$ lar uning yechimlari ekanligini ko'ramiz.

Endi avvalo, $x = 2 + 5t_1$ yechimni nazariy qismdagi (7)-formulaga olib borib qo'yamiz. U holda, bizda $p = 5, x_1 = 2, f(2) = 0, f'(x) = 4x^3 + 12x^2 + 4x + 1$ va $f'(2) = -7$ bo'lgani uchun (7) dan $\frac{0}{5} + 4t_1 \equiv 0(mod\ 3) \to t_1 \equiv 0(mod\ 5) \to t_1 = 5t_2 \to x = 2 + 5 \cdot 5t_2 = 2 + 25t_2, t_2 \epsilon Z$ yoki $x_1 \equiv 2(mod\ 25)$. Ikkinchi yechimi $x \equiv -2(mod\ 5)$ uchun $\frac{f(-2)}{5} + t_1 f'(-2) \equiv 0(mod\ 5)$. Bunda $f(-2) = 16 + 32 + 8 - 2 + 6 = 60$, $f'(-2) = -32 - 48 - 8 + 1 = -87$ bo'lgani uchun $\frac{60}{5} + t_1(-87) \equiv 0(mod\ 5) \to 12 + 3t_1 \equiv 0(mod\ 5) \to 3t_1 \equiv 3(mod\ 5) \to t_1 \equiv 1(mod\ 5)$, ya'ni $t_1 = 1 + 5t_2 \to x = -2 + 5(1 + 5t_2) = 3 + 25t_2$. Demak, berilgan taqqoslamaning ikkinchi yechimi $x_2 \equiv 3(mod\ 25)$. **Javob:** $x_1 \equiv 2(mod\ 25)$, $x_2 \equiv 3(mod\ 25)$.

4). $9x^2 + 29x + 62 \equiv 0(mod\ 64)$ taqqoslamani yechishimiz kerak. Bu yerda $64 = 2^6$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod 2)$ taqqoslamani, ya'ni $f(x) = 9x^2 + 29x + 62 \equiv 0(mod\ 2)$ ni qaraymiz. Bu $x^2 + x \equiv 0(mod\ 2)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 2 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, 1$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv 0(mod\ 2)$ va $x \equiv 1(mod\ 2)$ lar uning yechimlari ekanligini ko'ramiz.

a). $x \equiv 0(mod\ 2) \to x \equiv 2t_1$ yechim uchun $\frac{f(0)}{2} + f'(0)t_1 \equiv 0(mod\ 2)$ dan $f(0) = 62; f'(x) = 18x + 29$, $f'(0) = 29$ bo'lgani uchun $31 + 29t_1 \equiv 0(mod\ 2) \to t_1 \equiv -1(mod\ 2) \to t_1 = -1 + 2t_2,\ t_2 \in Z. x = 2(-1 + 2t_2) = -2 + 4t_2,\ t_2 \in Z$. $x = -2 + 4t_2$ dan foydalanib (9) ga asoslanib quyidagilarga ega bo'lamiz: $\frac{f(-2)}{4} + t_2 f'(-2) \equiv 0(mod\ 2)$, bunda $f(-2) = 36 - 58 + 62 = 40$, $f'(-2) = -7$ bo'lgani uchun $10 - 7t_2 \equiv 0(mod\ 2) \to t_2 \equiv 0(mod 2) \to t_2 = 2t_3 \to x = -2 + 4(2t_3) = -2 + 8t_3,\ t_3 \in Z$.

Endi $x = -2 + 8t_3,\ t_3 \in Z$dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(-2)}{8} + t_3 f'(-2) \equiv 0(mod\ 2)$ni hosil qilamiz. Bunda $f(-2) = 40$, $f'(-2) = -7$ bo'lgani uchun $5 - 7t_3 \equiv 0(mod\ 2) \to 7t_3 \equiv 5(mod\ 2) \to t_3 \equiv 1(mod\ 2)$, $t_3 = 1 + 2t_4 \to x = -2 + 8(1 + 2t_4) = 6 + 16t_4, t_4 \epsilon Z$.Endi $x = 6 + 16t_4,\ t_4 \in Z$dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(6)}{16} +$

$t_4 f'(6) \equiv 0(mod\ 2)$ ga ega bo'lamiz. Bu yerda $f(6) = 9 \cdot 36 + 29 \cdot 6 + 62 = 324 + 174 + 62 = 560, f'(6) = 18 \cdot 6 + 29 = 108 + 29 = 137$ bo'lgani uchun $35 + t_4 \cdot 137 \equiv 0(mod\ 2) \to t_4 \equiv 1(mod\ 2) \to t_4 = 1 + 2t_5 \to x = 6 + 16(1 + 2t_5) = 22 + 32t_5\ , t_5 \epsilon Z$.

Endi $x = 22 + 32t_5,\ t_5 \in Z$dan foydalanib oxirgi qadamni amalga oshiramiz. U holda $\frac{f(22)}{32} + t_5 f'(22) \equiv 0(mod\ 2)$ ga ega bo'lamiz. Bu yerda $f(22) = 9 \cdot 22^2 + 29 \cdot 22 + 62 = 4356 + 638 + 62 = 5056,\ f'(22) = 18 \cdot 22 + 29 = 425$ bo'lgani uchun $\frac{5056}{32} + 425t_5 \equiv 0(mod\ 2) \to 158 + t_5 \equiv 0(mod\ 2) \to t_5 \equiv 0(mod\ 2) \to t_5 = 2t_6 \to x = 22 + 32(2t_6) = 22 + 64t_6\ , t_6 \epsilon Z$. Demak, berilgan taqqoslamaning bitta yechimi $x \equiv 22(mod\ 64)$ ekan.

b). Endi $x \equiv 1(mod\ 2)$ ga mos yechimini izlaymiz: $x = 1 + 2t_1$yechim uchun $\frac{f(1)}{2} + f'(1)t_1 \equiv 0(mod\ 2)$ni tuzib olamiz. Bu yerda $f(1) = 9 + 29 + 62 = 100,\ f'(1) = 18 + 29 = 47$ bo'lgani uchun $50 + 47t_1 \equiv 0(mod\ 2)$

$t_1 \equiv 0(mod\ 2)$, $t_1 = 2t_2 \to x = 1 + 2t_1 = 1 + 4t_2$.$x$ning bu qiymatidan foydalanib quyidagilarga ega bo'lamiz: $\frac{f(1)}{4} + f'(1)t_2 \equiv 0(mod\ 2) \to 25 + 47t_2 \equiv 0(mod\ 2) \to t_2 \equiv -1(mod\ 2 \to t_2 = -1 + 2t_3 \to x = 1 + 4(-1 + 2t_3) = -3 + 8t_3$. $x$ning bu topilgan qiymatidan foydalanib quyidagiga ega bo'lamiz: $\frac{f(-3)}{8} + f'(-3)t_3 \equiv 0(mod\ 2)$. Bunda$f(-3) = 9 \cdot 9 + 29(-3) + 62 = 81 - 87 + 62 = 56$ va$f'(-3) = 18(-3) + 29 = -54 + 29 = -25$ ekanligini e'tiborga olib $7 - 25t_3 \equiv 0(mod\ 2) \to 1 + t_3 \equiv 0(mod\ 2) \to t_3 = 1 + 2t_4 \to x = -3 + 8(1 + 2t_4) = 5 + 16t_4$. $x = 5 + 16t_4$ dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(5)}{16} + f'(5)t_4 \equiv 0(mod\ 2)$. Bunda $f(5) = 9 \cdot 25 + 29 \cdot 5 + 62 = 225 + 145 + 62 = 432$ va $f'(5) = 18 \cdot 5 + 29 = 119$ bo'lgani uchun $27 + 119t_4 \equiv 0(mod\ 2) \to t_4 \equiv -1(mod\ 2) \to t_4 = -1 + 2t_5 \to x = 5 + 16(-1 + 2t_5) = -11 + 32t_5$. x ning bu qiymatidan foydalanib t_5 ni topish uchun quyidagi taqqoslamani hosil qilamiz:

$\frac{f(-11)}{32} + f'(-11)t_5 \equiv 0(mod\ 2)$. Bu yerda $f(-11) = 9 \cdot 121 - 29 \cdot 11 + 62 = 1089 - 319 + 62 = 832$ va $f'(-11) = 18 \cdot (-11) + 29 = -169$ ekanligini e'tiborga olib $26 - 169t_5 \equiv 0(mod\ 2) \to t_5 \equiv 0(mod\ 2) \to t_5 = 2t_6 \to x = -11 + 32(2t_6) = -11 + 64t_6,\ t_6 \epsilon Z$.

Demak, berilgan taqqoslamaning ikkinchi yechimi $x \equiv -11(mod\ 64)$ dan iborat ekan.

Javob: $x \equiv 22(mod\ 64),\ x \equiv 53(mod\ 64)$.

5). $6x^3 - 7x - 11 \equiv 0(mod\ 125)$ taqqoslamani qaraymiz. Bu yerda $125 = 5^3$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$6x^3-7x-11\equiv 0(mod\ 5)$→$x^3-2x-1\equiv 0(mod\ 5)$.

Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0,\pm 1,\pm 2$ dagi sonlarni qo'yib, sinab ko'rish usuli bilan yechamiz. U holda $x_1\equiv -1(mod\ 5)$,$x_2\equiv -2(mod\ 5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

a). Avvalo, $x\equiv -1(mod\ 5)$ ya'ni $x=-1+5t_1$ yechimni qaraymiz. (7) ga asosan $\frac{f(-1)}{5}+f'(-1)t_1\equiv 0(mod\ 5)$ taqqoslamani hosil qilamiz. Bu yerda $f(-1)=-6+7-11=-10$, $f'(-1)=18\cdot(-1)^2-7=11$ bo'lgani uchun$-2+11t_1\equiv 0(mod\ 5)\to t_1\equiv 2(mod\ 5)\to t_1=2+5t_2\to x=-1+5(2+5t_2)=9+25t_2$ ni hosil qilamiz. Endi 25 moduli bo'yicha taqqoslamadan 125 moduli bo'yicha taqqoslamaga o'tish uchun $\frac{f(9)}{25}+f'(9)t_2\equiv 0(mod\ 5)$ ni tuzib olamiz. Bunda $f(9)=6\cdot 9^3-7\cdot 9-11=6\cdot 729-63-11=4300$ va$f'(9)=18\cdot 9^2-7=18\cdot 81-7=1451$ ekanligini e'tiborga olsak, $172+1451t_2\equiv 0(mod\ 5)$ ga ega bo'lamiz. Bundan $t_2\equiv -2(mod\ 5)$→$t_2=-2+5t_3$→$x=9+25(-2+5t_3)=-41+125t_3$. Demak, berilgan taqqoslamaning bitta yechimi $x\equiv -41(mod\ 125)$.

b). Endi $x\equiv -2(mod\ 5)$ yechimni qaraymiz. Bundan $x=-2+5t_1$ va (7) ga asosan $\frac{f(-2)}{5}+f'(-2)t_1\equiv 0(mod 5)$. Bunda $f(-2)=6\cdot(-8)-7\cdot(-2)-11=-48+14-11=-45$ va $f'(-2)=18\cdot 4-7=65$ ekanligini inobatga olsak, $-9+65t_1\equiv 0(mod\ 5)\to 65t_1\equiv 9(mod\ 5)$. Bu yerda $(65,5)=5$, lekin 9 soni 5 ga bo'linmaydi, shuning uchun bu taqqoslama yechimga ega emas.

Javob: $x\equiv -4(mod\ 125)$.

6).$x^3+3x^2-5x+16\equiv 0(mod\ 125)$ taqqoslamani qaraymiz. Bu yerda $125=5^3$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$x^3+3x^2-5x+16\equiv 0(mod\ 5)$→$x^3+3x^2+1\equiv 0(mod\ 5)$. Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0,\pm 1,\pm 2$ dagi sonlarni qo'yib sinab ko'rish usuli bilan yechamiz. U holda $x_1\equiv 1(mod\ 5)$,$x_2\equiv -2(mod\ 5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

$a)$. $x\equiv 1(mod\ 5)$ ni,ya'ni $x=-1+5t_1$ yechimni qaraymiz. (7) ga asosan ya'ni $\frac{f(1)}{5}+f'(1)t_1\equiv 0(mod\ 5)$. Bu yerda $f(1)=15$, $f'(1)=3\cdot 1^2+6\cdot 1-5=4$ bo'lgani uchun $3+4t_1\equiv 0(mod\ 5)$→$4t_1\equiv 2(mod\ 5)$→$2t_1\equiv 1(mod\ 5)$→$t_1\equiv 3(mod\ 5)\to t_1=3+5t_2$→$x=1+5(3+5t_2)=16+25t_2$. x ning bu qiymatidan foydalanib, t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(16)}{25}+f'(16)t_2\equiv 0(mod\ 5)$. Bu yerda $f(16)=16^3+3\cdot 16^2-5\cdot 16+16=16(256+48-4)=4800$ va $f'(16)=3\cdot 256+96-5=859$ ekanligini e'tiborga olsak, $192+859t_2\equiv 0(mod\ 5)$hosil bo'ladi. Bundan $t_2\equiv 2(mod\ 5)$→$t_2=2+$

$5t_3 \to x = 16 + 25(2 + 5t_3) = 66 + 125t_3 \to x \equiv 66(mod\ 125)$. Demak, berilgan taqqoslamaning bitta yechimi $x \equiv 66(mod\ 125)$ dan iborat.

b). Endi $x \equiv -2(mod\ 5)$ yechimni qaraymiz: $x = -2 + 5t_1$ dan (7) ga asosan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod\ 5)$. Bunda $f(-2) = -8 + 12 + 10 + 16 = 30$ va $f'(-2) = 3 \cdot 4 + 6 \cdot (-2) - 5 = -5$ ekanligini e'tiborga olib $6 - 5t_1 \equiv 0(mod\ 5)$ ni hosil qilamiz. Bundan $5t_1 \equiv 1(mod\ 5)$, bunda (5,5) = 5 bo'lib, 1 soni 5 ga bo'linmagani uchun taqqoslama yechimga ega emas.

Javob: $x \equiv 66(mod\ 125)$.

7). $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod\ 625)$ taqqoslamani qaraymiz. Bu yerda $625 = 5^4$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod 5) \to x^4 - x^3 + 2x^2 + x + 2 \equiv 0(mod\ 5)$. Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0, \pm1, \pm2$ dagi sonlarni qo'yib sinab ko'rish usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 5)$, $x_2 \equiv -1(mod\ 5)$ va $x_3 \equiv 2(mod\ 5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

a). $x \equiv 1(mod\ 5)$yechimni qaraymiz. $x = 1 + 5t_1$ dan (7) ga asosan, ya'ni $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(1) = 20$, $f'(1) = 21$ bo'lgani uchun $4 + 21t_1 \equiv 0(mod\ 5) \to t_1 \equiv 1(mod\ 5) \to t_1 = 1 + 5t_2 \to x = 1 + 5(1 + 5t_2) = 6 + 25t_2$. $x$ning bu qiymatidan foydalanib, t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(6)}{25} + f'(6)t_2 \equiv 0(mod\ 5)$. Bunda $f(6) = 90$ va

$f'(6) = 4 \cdot 6^3 + 12 \cdot 6^2 + 4 \cdot 6 + 1 = 824 + 432 + 25 = 1281$ bo'lgani uchun $90 + 1281t_2 \equiv 0(mod\ 5) \to t_2 \equiv 0(mod\ 5) \to t_2 = 5t_3 \to x = 6 + 125t_3$. Endi $x$ning bu qiymatidan foydalanib, t_3 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(6)}{125} + f'(6)t_3 \equiv 0(mod\ 5)$. Bundan $18 + 1281t_3 \equiv 0(mod\ 5) \to t_3 \equiv 2(mod\ 5) \to t_3 = 2 + 5t_4 \to x = 6 + 125(2 + 5t_4) = 256 + 625t_4$. Demak, berilgan taqqoslamaning bitta yechimi $x = 256 + 625t_4$.

b) Endi $x \equiv -1(mod\ 5)$, ya'ni $x = -1 + 5t_1$ yechimni qaraymiz. Bu holda (7) ga asosan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod\ 5)$ni hosil qilamiz. Bu yerda $f(-1) = 1 - 4 + 2 - 1 + 12 = 10$ va $f'(-1) = -4 + 12 - 4 + 1 = 5$ bo'lgani uchun

$2 + 5t_1 \equiv 0(mod\ 5) \to 5t_1 \equiv 3(mod\ 5)$ ga ega bo'lamiz. Bu yerda (5,5) = 5 va 3 soni 5 ga bo'linmaydi, shuning uchun taqqoslama yechimga ega emas.

c) $x \equiv 2(mod 5)$, ya'ni $x = 2 + 5t_1$ yechimni qaraymiz. Bu holda (7) ga asosan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod\ 5)$ni hosil qilamiz. Bu yerda $f(2) = 16 + 32 + 8 + 2 + 12 = 70$ va $f'(x) = 4x^3 + 12x^2 + 4x + 1 \to f'(2) = 32 + 48 + 9 = 89$ bo'lgani uchun $14 + 89t_1 \equiv 0(mod 5) \to 4t_1 \equiv 1(mod\ 5) \to t_1 \equiv -1(mod\ 5) \to t_1 = -1 + 5t_2 \to x = 2 + 5(1 + 5t_2) = -3 + 25t_2$ ni hosil qilamiz. $x$ning bu qiymatidan foydalanib, t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(-3)}{25}+f'(-3)t_2 \equiv 0(mod\ 5)$. Bu yerda$f(-3)=81-108+18-3+12=0$ va $f'(-3)=4\cdot(-27)+12\cdot 9-12+1=-108+108-11=-11$ bo'lgani uchun $-11\mathrm{t}_2 \equiv 0(\mathrm{mod}\ 5)$→$\mathrm{t}_2 \equiv 0(\mathrm{mod}\ 5)$ → $\mathrm{t}_2=5\mathrm{t}_3$→$x=-3+25\cdot 5\mathrm{t}_3=-3+125\mathrm{t}_3$. Bundan foydalanib, t_3 ni topish uchun $\frac{f(-3)}{125}+f'(-3)t_3 \equiv 0(mod\ 5)$ taqqoslamani tuzib olamiz. Bu yerdan $-11\mathrm{t}_3 \equiv 0(\mathrm{mod}\ 5)$ →$\mathrm{t}_3=5\mathrm{t}_4$ → $x=-3+625\mathrm{t}_4$ kelib chiqadi.

Javob:$x=256+625t_4$ $x=-3+625\mathrm{t}_4,\ \mathrm{t}_4 \in \mathrm{Z}$.

8). $f(x)=2x^4+5x-1,\ f(x)\equiv 0\ (\mathrm{mod}\ 27)$ taqqoslamani yeching.

$27=3^3$. $f(x)\equiv 0(\mathrm{mod}\,3)$, $(0,\pm 1)$ taqqoslama birta $x\equiv 1(\mathrm{mod}\,3)$ yechimga ega. Bu yerda $f'(x)=8x^3+5$ va $f'(x_1)=13$ va 13 soni 3 ga bo'linmaydi. Demak A holga to'g'ri keladi.

$x=1+3t_1,\ f(1)+3t_1\cdot f'(1)\equiv 0(\mathrm{mod}\,9),\ 6+3t_1\cdot 13\equiv 0\,(\mathrm{mod}\,9), 13t_1\equiv -2(\mathrm{mod}\,3),\ t_1\equiv -2\,(\mathrm{mod}\,3),$

$t_1=-2+3t_2.$ Demak, $x=1+3(-2+3t_2)=-5+9t_2,\ \ f(-5)+9t_2\,f'(-5)\equiv 0\,(\mathrm{mod}\,27)$

$1224+9t_2(-995)\equiv 0\,(\mathrm{mod}\,27),\ \ -995t_2\equiv -136(\mathrm{mod}\,3),\ t_2\equiv -1(\mathrm{mod}\,3),\ t_2=-1+3t_3,\ t_3\in Z$

$x=-5+9(-1+3t_3)=-14+27t_3.$ Demak, $x\equiv 13(\mathrm{mod}\,27)$

berilgan taqqoslamaning yechimi.

291. 1). $x^4+4x^3+2x^2+x+12 \equiv 0(mod\ 45)$ ni qaraymiz.

Bu taqqoslama

$$\begin{cases} x^4+4x^3+2x^2+x+12 \equiv 0(mod\ 5) \\ x^4+4x^3+2x^2+x+12 \equiv 0(mod\ 9) \end{cases}$$

ga teng kuchli.Buning birinchisining yechimlari: $x \equiv \pm 1(mod\ 5)$ va $x \equiv 2(mod\ 5)$. (290.1) misolning ishlanishiga qarang). Endi sistemadagi ikkinchi taqqoslamani yechamiz.Buning uchun avvalo, $x^4+4x^3+2x^2+x+12 \equiv 0(mod\ 3)$ taqqoslamani yechamiz. Bu taqqoslama $x^4+x^3-x^2+x \equiv 0(mod\ 3)$ ga teng kuchli. Buni 3 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 0(mod\ 3), \text{ya'ni}$ $x=3t_1$uning yechimi ekanligini topamiz. Bu holda (7) ga asosan $\frac{f(0)}{3}+f'(0)t_1 \equiv 0(mod\ 3)$ ni hosil qilamiz. Bu yerda $f(0)=12$va $f'(0)=1$ bo'lgani uchun $4+\mathrm{t}_1 \equiv 0(\mathrm{mod}\ 3)$ → $\mathrm{t}_1 \equiv -1(\mathrm{mod}\ 3)$→$\mathrm{t}_1=-1+3\mathrm{t}_2$ → $x \equiv -3+9\mathrm{t}_2$ ni hosil qilamiz. Natijada biz berilgan taqqoslama quyidagi taqqoslamalar sistemasiga ekvivalent deya olamiz:

$$a)\begin{cases} x \equiv 1(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases}; \qquad b)\begin{cases} x \equiv -1(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases}; \qquad c)\begin{cases} x \equiv 2(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases}.$$

Bu sistemalarni yechamiz.U holda :

$a)$ $\begin{cases} x = 1 + 5t_1 \\ 1 + 5t_1 \equiv -3(mod\ 9) \end{cases} \rightarrow \begin{cases} x = 1 + 5t_1 \\ 5t_1 \equiv -4(mod\ 9) \end{cases} \rightarrow \begin{cases} x = 1 + 5t_1 \\ t_1 \equiv 1(mod\ 9) \end{cases} \rightarrow$

$x = 1 + 5(1 + 9\mathrm{t}_2) = 6 + 45\mathrm{t}_2$, ya'ni $x \equiv -21$(mod 45).

b)$\begin{cases} x = -1 + 5t_1 \\ -1 + 5t_1 \equiv -3(mod\ 9) \end{cases} \rightarrow \begin{cases} x = -1 + 5t_1 \\ 5t_1 \equiv -4(mod\ 9) \end{cases} \rightarrow \begin{cases} x = -1 + 5t_1 \\ t_1 \equiv -4 + 9t_2 \end{cases} \rightarrow$

$x = -1 + 5(-4 + 9\mathrm{t}_2) = 6 + 45\mathrm{t}_2 \ \rightarrow \ x \equiv -21$(mod 45).

c)$\begin{cases} x = 2 + 5t_2 \\ 2 + 5t_1 \equiv -3(mod\ 9) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ t_1 = -1 + 9t_2 \end{cases} \rightarrow x = 2 + 5(-1 + 9t_2) = -3 + 43t_2 \ \rightarrow x \equiv -3(mod\ 45)$.

Javob: $x \equiv 6, 24, 42(mod\ 45)$.

2). $f(x) = x^4 - 3x^3 - 4x^2 - 2x - 2 \equiv 0(mod\ 50)$ taqqoslamani qaraymiz. Bu yerda $50 = 2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 2) \\ f(x) \equiv 0(mod\ 25) \end{cases}$$

ga teng kuchli. Bunda birinchisining yechimlari: $x \equiv 0(mod\ 2)$va $x \equiv 1(mod\ 5)$. Endi $f(x) = 0(mod\ 25)$ taqqoslamani yechamiz. Buning uchun esa $f(x) \equiv x^4 + 2x^3 + 2x^2 + 3x - 2 \equiv 0(mod\ 5)$ ni qaraymiz. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak, $x \equiv -1, 1, 2(mod\ 5)$ lar uning yechimi ekanligini topamiz.

a). Avvalo, $x \equiv \ 1(mod\ 5)$, ya'ni $x = 1 + 5t_1$ yechimni qaraylik. Bu holda (7) ga asosan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(m\square d\ 5)$ ni hosil qilamiz. Bu yerda $f(1) = 10$ va $f'(1) = 4 \cdot 1^3 - 9 \cdot 1^2 - 8 \cdot 1 - 2 = -15$ bo'lgani uchun $2 - 15t_1 \equiv 0(mod\ 5)$, $15t_1 \equiv 2(mod\ 5) \rightarrow \ (15,5) = 5$, lekin 2 soni 5ga bo'linmaydi, shuning uchun taqqoslama yechimga ega emas.

b). $x \equiv -1(mod\ 5)$ ni, ya'ni $x = -1 + 5t_2$ ni qaraymiz. (7) ga asosan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod\ 5)$.

Bunda $f'(-1) = 1 + 3 - 4 + 2 - 2 = 0$ va

$f'(-1) = 4 \cdot (-1)^3 - 9 \cdot (-1)^2 - 8 \cdot (-1) - 2 = -7$ bo'lgani uchun

$$-7t_1 \equiv 0(mod\ 5) \rightarrow t_1 \equiv 0(mod\ 5) \rightarrow t_1 = 5t_2 \rightarrow x = -1 + 5(5t_2)$$
$$= -1 + 25t_2 \ \rightarrow x \equiv -1(mod\ 25).$$

c) Endi $x \equiv 2(\text{mod } 5)$, ya'ni $x = 2 + 5t$ yechimni qaraylik. Bu holda (7) ga asosan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod\ 5)$ ni hosil qilamiz. Bu yerda $f(2) = -30$ va $f'(2) = -22$ bo'lgani uchun $-6 - 22t_1 \equiv 0(mod\ 5) \rightarrow$ $3t_1 \equiv 1(mod\ 5) \rightarrow t_1 \equiv 2(mod\ 5) \rightarrow\ t_1 = 2 + 5t_2 \rightarrow x = 2 + 5(2 + 5t_2) =$ $12 + 25t_2$.

Demak, berilgan taqqoslamani yechishni

$$\begin{cases} x \equiv 0(mod\ 2) \\ x \equiv -1(mod\ 25) \end{cases};$$

$$\begin{cases} x \equiv 0(mod\ 2) \\ x \equiv 12(mod\ 25) \end{cases}; \begin{cases} x \equiv 1(mod\ 2) \\ x \equiv -1(mod\ 25) \end{cases}; \begin{cases} x \equiv 2(mod\ 2) \\ x \equiv 12(mod\ 25) \end{cases}$$

taqqoslamalar sistemalarini yechishga keltirdik. Buning birinchisidan

$$\begin{cases} x = 2t_1 \\ 2t_1 \equiv -1(mod\ 25) \end{cases} \rightarrow \begin{cases} x = 2t_1 \\ t_1 \equiv 12(mod\ 25) \end{cases} \rightarrow x = 24 + 50t_2.$$

Ikkinchisidan

$$\begin{cases} x = 2t_1 \\ 2t_1 \equiv 12(mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv 6(mod\ 25) \\ x = 12 + 50t_2 \end{cases}.$$

Uchinchisidan

$$\begin{cases} x = 1 + 2t_1 \\ 1 + 2t_1 \equiv -1(mod\ 25) \end{cases} \rightarrow \begin{cases} x = 2t_1 \\ t_1 \equiv -1(mod\ 25) \end{cases} \rightarrow x = -1 + 50t_2.$$

To'rtinchisidan

$$\begin{cases} x = 1 + 2t_1 \\ 1 + 2t_1 \equiv 12(mod\ 25) \end{cases} \rightarrow \begin{cases} x = 1 + 2t_1 \\ t_1 = 18 + 25t_2 \end{cases} \rightarrow x = 1 + 2(18 + 25t_2) = 37 +$$
$50t_3$.

Javob: $x \equiv 12, 24, 37, 49$ (mod 50).

3).$f(x) = x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 \equiv 0(mod\ 147)$ taqqoslamani qaraymiz. Bu yerda $147 = 7^2 \cdot 3$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 3) \\ f(x) \equiv 0(mod\ 7^2) \end{cases}$$

ga teng kuchli. Buning birinchisidan $f(x)=x^5-5x^4-5x^3+25x^2+4x-20\equiv 0(mod\ 3)\to x^5+x^4+x^3+x^2+x+1\equiv 0(mod\ 3)\to x+x^2+x+x^2+x+1=2x^2+3x+1\equiv 2x^2+1\equiv 0(mod\ 3)$. Uning yechimlari: $x\equiv -1(mod\ 3)$ va $x\equiv 1(mod\ 3)$.

Endi $f(x)=0(mod\ 49)$ taqqoslamani yechamiz. Buning uchun esa $f(x)\equiv 0(mod\ 7)$ ning yechimini topishimiz kerak. Bundan $x^5+2x^4+2x^3+4x^2+4x+1\equiv 0(mod\ 7)$. Buni 7moduli bo'yicha chegirmalarning to'la sistemasi $0,\pm 1,\pm 2,\pm 3$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x\equiv -1,1,-2,2(mod\ 7)$ lar uning yechimi ekanligini topamiz. Bu 7 moduli bo'yicha topilgan yechimlardan 49 moduli bo'yicha yechimga o'tish uchun ularning har birini alohida-alohida qarab chiqamiz.

a). $x\equiv 1(\text{mod } 7)$ yechim uchun (7) dan $\frac{f(1)}{7}+f'(1)t_1\equiv 0(mod\ 7)$ ni hosil qilamiz. Bu yerda $f(1)\ =0$ va $f'(1)=5\cdot 1^4-20\cdot 1^3-15\cdot 1^2+50\cdot 1+4=24$ bo'lgani uchun $24t_1\equiv 0(mod\ 7)\to\ t_1\equiv 0(mod\ 7)\to \mathrm{t}_1=7\mathrm{t}_2\to x=1+49\mathrm{t}_2\to x\equiv 1(\text{mod } 49)$.

b). Endi ikkinchi yechimni $x\equiv -1(\text{mod } 7)\to x=-1+7\mathrm{t}_1$ qaraymiz.

(7) dan $\frac{f(-1)}{7}+f'(-1)t_1\equiv 0(mod\ 7)$ ni hosil qilamiz. Bu yerda $f(-1)\ =0$ va $f'(-1)=-36$ bo'lgani uchun $36\mathrm{t}_1\equiv 0(\text{mod } 7)\to \mathrm{t}_1=7\mathrm{t}_2\to x=-1+49\mathrm{t}_2\ \to$ $x\equiv -1(\text{mod } 49)$.

c). Uchinchi yechim $x\equiv 2(mod\ 7)\to x=2+7t_1$uchun (7) dan $\frac{f(2)}{7}+f'(2)t_1\equiv 0(mod\ 7)$. Bunda$f(2)=0$ va$f'(2)=-36$ bo'lgani uchun$-36t_1\equiv 0(mod\ 7)\to t_1=7t_2\to x=2+49t_2\ \to x\equiv 2(mod\ 49)$.

e). To'rtinchi yechim $x\equiv -2(\text{mod } 7)\to x=-2+7\mathrm{t}_1$uchun $\frac{f(-2)}{7}+f'(-2)t_1\equiv 0(mod\ 7)$, bunda $f(-2)=0$va $f'(-2)=84$ bo'lgani uchun $84t_1\equiv 0(mod\ 7)\,,t_1\epsilon Z\ \to x=-2+49t_2$ ga ega bo'lamiz.

Bulardan

$$\begin{cases}x\equiv -1(mod\ 3)\\ x\equiv 1(mod\ 49)\end{cases},\ \begin{cases}x\equiv -1(mod\ 3)\\ x\equiv -1(mod\ 49)\end{cases},\ \begin{cases}x\equiv -1(mod\ 3)\\ x\equiv 2(mod\ 49)\end{cases},$$

$$\begin{cases}x\equiv -1(mod\ 3)\\ x\equiv -2(mod\ 49)\end{cases},\begin{cases}x\equiv 1(mod\ 3)\\ x\equiv 1(mod\ 49)\end{cases},\begin{cases}x\equiv 1(mod\ 3)\\ x\equiv -1(mod\ 49)\end{cases},$$

$$\begin{cases}x\equiv 1(mod\ 3)\\ x\equiv 2(mod\ 49)\end{cases},\ \begin{cases}x\equiv 1(mod\ 3)\\ x\equiv -2(mod\ 49)\end{cases}$$

chiziqli taqqoslamalar sistemalariga ega bo'lamiz. Bularni yechib:

$a_1)\ \begin{cases} x=-1+3t_1 \\ -1+3t_1\equiv 1(mod\ 49)\end{cases}\rightarrow\begin{cases} x=-1+3t_1 \\ 3t_1\equiv 2(mod\ 49)\end{cases}\rightarrow\begin{cases} t_1=17+49t_2 \\ x=50+147t_2\end{cases}.$

$a_2)\begin{cases} x=-1+3t_1 \\ -1+3t_1\equiv -1(mod\ 49)\end{cases}\rightarrow\begin{cases} t_1=49t_2 \\ x=-1+147t_2\end{cases}.$

$a_3)\ \begin{cases} x=-1+3t_1 \\ -1+3t_1\equiv 2(mod\ 49)\end{cases}\rightarrow\begin{cases} t_1\equiv 1(mod\ 49) \\ x=2+147t_2\end{cases}.$

$a_4)\ \begin{cases} x=-1+3t_1 \\ -1+3t_1\equiv -2(mod\ 49)\end{cases}\rightarrow\begin{cases} 3t_1\equiv -1(mod\ 49) \\ x=-1+3t_1\end{cases}\rightarrow$

$\begin{cases} t_1\equiv 16+49t_2 \\ x=-1+48+147t_2\end{cases}\rightarrow x=47+147t_2.$

$a_5)\ \begin{cases} x=1+3t_1 \\ 1+3t_1\equiv 1(mod\ 49)\end{cases}\rightarrow\begin{cases} t_1\equiv 0(mod\ 49) \\ x=1+147t_2\end{cases}.$

$a_6)\ \begin{cases} x=1+3t_1 \\ 1+3t_1\equiv -1(mod\ 49)\end{cases}$
$\rightarrow\begin{cases} 3t_1\equiv -2(mod\ 49) \\ x=1+3t_1\end{cases}\rightarrow\begin{cases} t_1\equiv -17+49t_2 \\ x=-50+147t_2\end{cases}.$

$a_7)\quad \begin{cases} x=1+3t_1 \\ 1+3t_1\equiv 2(mod\ 49)\end{cases}$
$\rightarrow\begin{cases} 3t_1\equiv 1(mod\ 49) \\ x=1+3t_1\end{cases}$
$\rightarrow\begin{cases} t_1\equiv -16+49t_2 \\ x=-47+147t_2\end{cases}.\ a_8)\begin{cases} x=1+3t_1 \\ 1+3t_1\equiv -2(mod\ 49)\end{cases}$
$\rightarrow\begin{cases} 3t_1\equiv -3(mod\ 49) \\ x=1+3t_1\end{cases}\rightarrow\begin{cases} t_1=-1+49t_2 \\ x=-2+147t_2\end{cases}.$

Javob: $x\equiv -50,-47,-2,-1,1,2,47,50\ (mod\ 147)$.

4). $f(x)=x^5+3x^4-7x^3+4x^2+4x-10\equiv 0(mod\ 175)$ taqqoslamani qaraymiz. Bu yerda $175=7\cdot 5^2$ bo’lgani uchun bu taqqoslama

$$\begin{cases} f(x)\equiv 0(mod\ 7) \\ f(x)\equiv 0(mod\ 25)\end{cases} \qquad (1)$$

ga teng kuchli. Buning birinchisidan $x^5+3x^4-3x^2-3x-3\equiv 0(mod\ 7)$. Buni 7 moduli bo’yicha chegirmalarning to’la sistemasi $0,\pm1,\pm2,\pm3$ dagi chegirmalarni sinab ko’rish usuli bilan yechsak, $x\equiv 1,-2,-3(mod\ 7)$ laruning yechimi ekanligini topamiz.

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\text{mod } 25)$ ni, ya'ni $x^5 + 3x^4 - 7x^3 + 4x^2 + 4x - 10 \equiv 0(mod\ 5)$ ni qaraymiz. Bu taqqoslama $3x^4 - 2x^3 - x^2 \equiv 0(mo\square\ 5) \to x^2\ (3x^2 - 2x - 1) \equiv 0(mod\ 5)$ ga teng kuchli. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak, $x \equiv 0,1,-2(mod\ 5)$ lar uning yechimi ekanligini topamiz.

Shuning uchun ham birinchi yechim $x \equiv 0(mod\ 5)$ uchun (7) ga asosan $\frac{f(0)}{5} + f'(0)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(0) = -10$ va $f'(0) = 4$ bo'lganidan

$$-2 + 4t_1 \equiv 0(mod\ 5) \to\ 4t_1 \equiv 2(mod\ 5) \to\ 2t_1 \equiv 1(mod\ 5) \to t_1 \equiv 3(mod\ 5)$$
$$\to t_1 = 3 + 5t_2 \ \ \to\ \ x = 15 + 25t_2.$$

Ikkinchi yechim $x \equiv 1(mod\ 5)$ uchun $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(1) = -5$ va $f'(1) = 8$ bo'lganidan $-1 + 8t_1 \equiv 0(mod\ 5) \to\ 8t_1 \equiv 1(mod\ 5) \to\ -2t_1 \equiv 6(mod\ 5) \to t_1 \equiv -3(mod\ 5) \to t_1 = -3 + 5t_2 \ \to\ x = 1 + 5t_1 = 1 - 15 + 25t_2 = -14 + 25t_2 \ \to x \equiv 11(mod 25)$.

Uchinchi yechim $x \equiv -2(mod\ 5)$ uchun $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(-2) = 70$ va $f'(-2) = -112$ bo'lganidan $14 - 112t_1 \equiv 0(mod\ 5) \to -1 +\ 3t_1 \equiv 0(mod 5) \to\ 3t_1 \equiv 1(mod 5) \to 3t_1 \equiv 6(mod 5) \to t_1 \equiv 2(mod 5) \to x = -2 + 5t_1 = -2 + 5(2 + 5t_2) = 7 + 25t_2.$

Shunday qilib, berilgan taqqoslamani yechishni quyidagi 1-darajali bir noma'lumli taqqoslamalar sistemalarni yechishga keltirdik:

$$a_1)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 15(mod\ 25) \end{cases}, \ a_2)\begin{cases} x \equiv -2(m\square d\ 7) \\ x \equiv 15(mod\ 25) \end{cases}, \ a_3)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 15(mod\ 25) \end{cases}$$

$$a_4)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}, \ a_5)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}, \ a_6)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}$$

$$a_7)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}, \ a_8)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}, \ a_9)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}.$$

Bularni yechib: $a_1)\begin{cases} x = 1 + 7\text{t}_1 \\ 1 + 7\text{t}_1 \equiv 15(\text{mod } 25) \end{cases} \to \begin{cases} 7\text{t}_1 \equiv 14(\text{mod } 25) \\ x = 1 + 7\text{t}_1 \end{cases} \to$
$\begin{cases} \text{t}_1 \equiv 2(\text{mod } 25) \\ x = 1 + 7\text{t}_1 \end{cases} \to \begin{cases} \text{t}_1 \equiv 2 + 25\text{t}_2 \\ x = 1 + 7\text{t}_1 \end{cases} \to x = 1 + 7(2 + 25\text{t}_2) = 15 + 175\text{t}_2.$

ya'ni $x \equiv 15(\text{mod } 175)$.

$$a_2)\begin{cases} x = -2 + 7t_1 \\ -2 + 7t_1 \equiv 15(mod\ 25) \end{cases} \to \begin{cases} 7t_1 \equiv 17(mod\ 25) \\ x = -2 + 7t_1 \end{cases} \to$$

$$\begin{cases} \mathrm{t}_1 \equiv 6(\mathrm{mod}\ 25) \\ x = -2 + 7(6 + 25\mathrm{t}_2) = 40 + 175\mathrm{t}_2 \end{cases}, \text{ ya'ni } x \equiv 40(\mathrm{mod}\ 175).$$

$$a_3)\begin{cases} x = -3 + 7t_1 \\ -3 + 7t_1 \equiv 15(mod\ 25) \end{cases} \to \begin{cases} 7t_1 \equiv 18(mod\ 25) \\ x = -3 + 7t_1 \end{cases} \to$$

$$\begin{cases} t_1 \equiv -1(mod\ 25) \\ x = -3 + 7(-1 + 25t_2) = -10 + 175t_2 \end{cases} \to x \equiv -10(\mathrm{mod}\ 175).$$

$$a_4)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 1 + 7t_1 \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 7t_1 \equiv 10(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv 1 + 7t_1 \\ t_1 \equiv 5(mod\ 25) \end{cases} \to x \equiv 1 + 7(5 + 25t_2) = 36 + 175t_2.$$

$$a_5)\ \begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv -2 + 7t_1 \\ -2 + 7t_1 \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv -2 + 7t_1 \\ 7t_1 \equiv 13(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -2 + 7t_1 \\ t_1 \equiv 9(mod\ 25) \end{cases} \to x \equiv -2 + 7(9 + 25t_2) = 61 + 175t_2.$$

$$a_6)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -3 + 7t_1 \\ -3 + 7t_1 \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv -3 + 7t_1 \\ 7t_1 \equiv 14(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -3 + 7t_1 \\ t_1 \equiv 2(mod\ 25) \end{cases} \to x \equiv -3 + 7(2 + 25t_2) = 11 + 175t_2.$$

$$a_7)\begin{cases} x \equiv 1(mod\ 7) \\ \square \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 1 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 7t_1 \equiv 6(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv 1 + 7t_1 \\ t_1 \equiv 8(mod\ 25) \end{cases} \to x \equiv 1 + 7(8 + 25t_2) = 57 + 175t_2.$$

$$a_8)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -2 + 7t_1 \\ -2 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv -2 + 7t_1 \\ 7t_1 \equiv 9(mod\ 25) \end{cases} \to$$

$\begin{cases} x \equiv -2 + 7t_1 \\ t_1 \equiv 12(mod\ 25) \end{cases} \to x \equiv -2 + 7(12 + 25t_2) = 82 + 175t_2.$

$a_9) \begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}$
$\to \begin{cases} x \equiv -3 + 7t_1 \\ -3 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv -3 + 7t_1 \\ 7t_1 \equiv 10(mod\ 25) \end{cases} \to$

$\begin{cases} x \equiv -3 + 7t_1 \\ t_1 \equiv 5(mod\ 25) \end{cases} \to x \equiv -3 + 7(5 + 25t_2) = 32 + 175t_2.$

Javob: $x \equiv -10, 11,15, 32, 36, 40, 57, 61\ 82(\text{mod } 175).$

5). $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod135)$ taqqoslamani qaraymiz. Bu yerda $135 = 3^3 \cdot 5$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 5) \\ f(x) \equiv 0(mod\ 27) \end{cases} \qquad (1)$$

ga teng kuchli. Buning birinchisidan $x^4 - 4x^3 + 2x^2 + x + 6 \equiv x^4 + x^3 + 2x^2 + x + 1 \equiv 0(mod\ 5)$. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak, $x \equiv 2, -2(mod\ 5)$ laruning yechimi ekanligini topamiz.

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\text{mod } 3)$ ni, ya'ni $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 3)$ ni qaraymiz. Bu taqqoslama $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 3) \to x^4 - x^3 - x^2 + x \equiv 0(mod3) \to x(x^3 - x^2 - x + 1) \equiv 0(mod3) \to x(-x^2 + 1) \equiv 0(mod3)$. Bundan $x \equiv 0, \pm 1(mod3)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$a_1). x \equiv 0(mod3)$ – ni qaraymiz. $x = 3t_1$ bo'lgani uchun 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(0)}{3} + f'(0)t_1 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda $f(0) = 6$ va $f'(x) = 4x^3 - 12x^2 + 4x + 1$, $f'(0) = 1$ bo'lgani uchun$2 + t_1 \equiv 0(mod3) \to t_1 \equiv 1(mod3) \to t_1 \equiv 1 + 3t_2 \to x = 3 + 9t_2$. x ning bu qiymatidan foydalanib, navbatdagi qadamni amalga oshiramiz. U holda t_2 ga nisbatan $\frac{f(3)}{9} + f'(3)t_2 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda $f\ (3) = 0, f'(3) = 13$ bo`lgani uchun $13t_2 \equiv 0(mod3)$ ga ega bo'lamiz. Bundan $\text{t}_2 = 3\text{t}_3 \to x = 3 + 27\text{t}_3 \in \text{Z}$.

$a_2)$. Endi ikkinchi yechim$x \equiv 1(mod3)$ ni qaraymiz. Bundan $x = 1 + 3t_1$ bo'lgani uchun 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda $f(1) = 6$ va $f'(x) = f'(1) = -3$ bo'lgani uchun$2 - 3t_1 \equiv 0(mod3) \to\ 3t_1 \equiv 2(mod3)$ ni hosil qilamiz. Bu yerda $(3,3) = 3$, lekin 2 soni 3ga bo’linmaydi.Shuning uchun ham oxirgi taqqoslama yechimga ega emas.

a_3). Uchinchi yechim $x \equiv -1(mod3)$ ni qaraymiz. Bu holda $f(-1) = 12$, $f'(-1) = -19$ bo’lgani uchun $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ dan $4 - 19t_1 \equiv 0(mod3) \rightarrow 19t_1 \equiv 4(mod3) \rightarrow t_1 \equiv 1(mod3) \rightarrow t_1 = 1 + 3t_2$ ni hosil qilamiz. Buni x ning ifodasiga olib borib qo’ysak, $x = -1 + 3t_1 = 2 + 9t_2$ ifoda hosil bo’ladi. x ning bu ifodasidan foydalanib, keyingi qadamni amalga oshiramiz. U holda t_2 ga nisbatan $\frac{f(2)}{9} + f'(2)t_2 \equiv 0(mod3)$ taqqoslama kelib chiqadi. Bunda $f(2) = 0$, $f'(2) = -7$ bo’lgani uchun $-7t_2 \equiv 0(mod3) \rightarrow t_2 \equiv 0(mod3) \rightarrow t_2 = 3t_3 \rightarrow x = 2 + 27t_3$ ga ega bo’lamiz.

Shunday qilib berilgan taqqoslamani yechishni quyidagi birinchi darajali taqqoslamalar sistemasini yechishga keltirdik:

$$b_1)\begin{cases} x \equiv -2(mod5) \\ x \equiv 3(mod27) \end{cases} \qquad b_2)\begin{cases} x \equiv -2(mod5) \\ x \equiv 2(mod27) \end{cases}$$

$$b_3)\begin{cases} x \equiv 2(mod5) \\ x \equiv 3(mod27) \end{cases} \qquad b_4)\begin{cases} x \equiv 2(mod5) \\ x \equiv 2(mod27) \end{cases}.$$

Endi bularning har birining yechimini izlaymiz.

b_1) dan $\begin{cases} x = -2 + 5t_1 \\ -2 + 5t_1 \equiv 3(mod27) \end{cases} \rightarrow \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 5(mod27) \end{cases} \rightarrow \begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 1(mod27) \end{cases} \rightarrow$

$\begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 1 + 27t_2 \end{cases} \rightarrow x = 3 + 135t_2,\ t_2 \in Z.$

b_2) dan

$$\begin{cases} x = -2 + 5t_1 \\ -2 + 5t_1 \equiv 2(mod27) \end{cases} \rightarrow \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 4(mod27) \end{cases} \rightarrow \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 85(mod27) \end{cases} \rightarrow$$

$\begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 17 + 27t_2 \end{cases} \rightarrow x = 83 + 135t_2,\ t_2 \in Z.$

b_3) dan

$$\begin{cases} x = 2 + 5t_1 \\ 2 + 5t_1 \equiv 3(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 1(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 55(mod27) \end{cases} \rightarrow$$

$$\begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 11 + 27t_2 \end{cases} \rightarrow x = 57 + 135t_2,\ t_2 \in Z.$$

b_4) dan

$$\begin{cases} x = 2 + 5t_1 \\ 2 + 5t_1 \equiv 2(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 0(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 0(mod27) \end{cases} \rightarrow$$

$$\begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 27t_2 \end{cases} \rightarrow x = 2 + 135t_2, t_2 \in Z.$$

Javob: $x \equiv 2{,}3{,}57{,}83(mod135)$.

6). $f(x) = 4x^3 + 7x^2 - 7x - 10 \equiv 0(mod225)$ taqqoslamani qaraymiz. Bu yerda $225 = 3^2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 3^2) \\ f(x) \equiv 0(mod\ 5^2) \end{cases} \qquad (1)$$

ga teng kuchli.

a). $f(x) = 4x^3 + 7x^2 - 7x - 10 \equiv 0(mod3^2)$ taqqoslamani yechamiz. Buning uchun $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod3) \rightarrow x^3 + x^2 - x - 1 \equiv 0(mod3) \rightarrow x^2 - 1 \equiv 0(mod3) \rightarrow (x + 1)(x - 1) \equiv 0(mod3)$ taqqoslamani yechishimiz kerak. Buning yechimlari $x \equiv -1, 1(mod3)$ dan iborat.

a_1) $x = 1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(1) = -6$, $f'(x) = 12x^2 + 14x - 7$, $f'(1) = 19$ bo'lgani uchun $-2 + 19t_1 \equiv 0(mod3) \rightarrow t_1 \equiv -1(mod3) \rightarrow t_1 = -1 + 3t_2 \rightarrow x = -2 + 9t_2$.

a_2) $x = -1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(-1) = 0$, $f'(-1) = -9$ bo'lgani uchun $-9t_1 \equiv 0(mod3) \rightarrow 0 \cdot t_1 \equiv 0(mod3)$.Bu taqqoslama ayniy taqqoslama bo'lib, t_1 ning ixtiyoriy qiymati qanoatlantiradi.

Endi (1) dagi ikkinchi taqqoslamani $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod5) \rightarrow -x^3 + 2x^2 - 2x \equiv 0(mod5)$ ni qaraymiz. Bundan $x(-x^2 + 2x - 2) \equiv 0(mod5) \rightarrow x \equiv 0, -1, -2(mod5)$.

b_1) $x \equiv 0(mod5) \rightarrow x = 5t_1$ ni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(0)}{5} + f'(0)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(0) = -10$, $f'(0) = -7$ bo'lgani uchun $-2 - 7t_1 \equiv 0(mod5) \rightarrow 3t_1 \equiv -3(mod5) \rightarrow t_1 = -1 + 5t_2 \rightarrow x = -5 + 25t_2$.

$b_2)$ $x \equiv -1(mod 5) \to x = -1 + 5t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod 5)$ taqqoslamani tuzib olamiz. Bunda $f(-1) = 0,\ f'(-1) = -9$ bo'lgani uchun $-9t_1 \equiv 0(mod 5) \to t_1 \equiv 0(mod 5) \to t_1 = 5t_2 \to x = -1 + 25t_2$.

$b_3)$ $x \equiv -2(mod 5) \to x = -2 + 5t_1$ yechimni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod 5)$ taqqoslamani tuzib olamiz. Bunda $f(-2) = 0,\ f'(-2) = 13$ bo'lgani uchun $13t_1 \equiv 0(mod 5) \to t_1 \equiv 0(mod 5) \to t_1 = 5t_2 \to x = -2 + 25t_2$.

Bulardan quyidagi chiziqli tenglamalar sistemasiga kelamiz.

$c_1)\begin{cases} x \equiv 7(mod 9) \\ x \equiv -5(mod 25) \end{cases}; c_2)\begin{cases} x \equiv 7(mod 9) \\ x \equiv -1(mod 25) \end{cases}; c_3)\begin{cases} x \equiv 7(mod 9) \\ x \equiv -2(mod 25) \end{cases}.$

Bularni yechamiz:

$c_1)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -5(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -12(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ 3t_1 \equiv -4(mod 25) \end{cases} \to$$

$$\begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv 7(mod 25) \end{cases} \to x = 7 + 9(7 + 25t_2) = 70 + 225t_2.$$

$c_2)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -1(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -8(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv 117(mod 25) \end{cases} \to$$

$$\begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv 13(mod 25) \end{cases} \to x = 7 + 9(13 + 25t_2) = 124 + 225t_2.$$

$c_3)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -2(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -9(mod 25) \end{cases} \to \begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv -1(mod 25) \end{cases} \to$$

$$\to x = 7 + 9(-1 + 25t_2) = -2 + 225t_2.$$

Javob: $x \equiv 70; 124; 223(mod 225)$.

7). $31x^4 + 57x^3 + 96x + 191 \equiv 0(mod 225)$ taqqoslamani qaraymiz. Bu yerda $225 = 3^2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 3^2) \\ f(x) \equiv 0(mod\ 5^2) \end{cases} \qquad (1)$$

ga, ya'ni

$$\begin{cases}31x^4+57x^3+96x+191\equiv 0(mod3^2)\\31x^4+57x^3+96x+191\equiv 0(mo\square 5^2)\end{cases}$$

ga teng kuchli. Birinchi taqqoslamani qaraymiz:

$31x^4+57x^3+96x+191\equiv 0(mod3)\rightarrow x^4-1\equiv 0(mod3)\rightarrow(x^2-1)(x^2+1)\equiv 0(mod3)\rightarrow(x-1)(x+1)(x^2+1)\equiv 0(mod3)$ bo'lgani uchun bu taqqoslamaning yechimlari $x\equiv -1,1(mod3)$ dan iborat.

$a_1)$ $x=-1+3t_1$ yechimni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(-1)}{3}+f'(-1)t_1\equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(-1)=69$, $f'(x)=124x^2+171x+96$, $f'(-1)=143$ bo'lgani uchun$23+143t_1\equiv 0(mod3)\rightarrow 2t_1\equiv 1(mod3)\rightarrow t_1\equiv 2(mod3)\rightarrow t_1=-1+3t_2\rightarrow x=-4+9t_2$.

$a_2)$ $x=1+3t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(1)}{3}+f'(1)t_1\equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(1)=375$, $f'(x)=124x^2+171x+96$, $f'(1)=391$ bo'lgani uchun $125+391t_1\equiv 0(mod3)\rightarrow t_1\equiv -2(mod3)\rightarrow t_1\equiv 1(mod3)\rightarrow t_1=1+3t_2\rightarrow x=4+9t_2$. Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x)\equiv 0(\mathrm{mod}\ 5)$ ni, ya'ni $31x^4+57x^3+96x+191\equiv 0(mod\ 5)$ ni qaraymiz. Bu taqqoslama $x^4+2x^3+x+1\equiv 0(mod\ 5)$ ga teng kuchli. Bundan $x\equiv 1,2(mod5)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$b_1)$ $x\equiv 1(mod5)\rightarrow x=1+5t_1$ ni qaraymiz. Bundan foydalanib, t_1 ga nisbatan $\frac{f(1)}{5}+f'(1)t_1\equiv 0(m\square d5)$ taqqoslamani tuzib olamiz. Bunda $f(1)=375$, $f'(1)=391$ bo'lgani uchun $75+391t_1\equiv 0(mod5)\rightarrow t_1\equiv 0(mod5)\rightarrow t_1=5t_2\rightarrow x=1+25t_2$.

$b_2)x\equiv 2(mod5)\rightarrow x=2+5t_1$ ni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(2)}{5}+f'(2)\square_1\equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(2)=1335$, $f'(1)=1772$ bo'lgani uchun $267+1772t_1\equiv 0(mod5)\rightarrow 2t_1\equiv -2(mod5)\rightarrow t_1=-1+5t_2\rightarrow x=-3+25t_2$.

Bulardan quyidagi chiziqli tenglamalar sistemasiga kelamiz:

$$c_1)\begin{cases}x\equiv 5(m\square d9)\\x\equiv 1(mod25)\end{cases};c_2)\begin{cases}x\equiv 5(mod9)\\x\equiv -3(mod25)\end{cases};$$

$$c_3)\begin{cases}x\equiv 4(mod9)\\x\equiv 1(mod25)\end{cases};c_4)\begin{cases}x\equiv 4(mod9)\\x\equiv -3(mod25)\end{cases}.$$

Bularni yechamiz:

$c_1)$ dan

$$\begin{cases}x\equiv 5+9t_1\\5+9t_1\equiv 1(mod25)\end{cases}\rightarrow\begin{cases}x\equiv 5+9t_1\\9t_1\equiv -4(m\square d25)\end{cases}\rightarrow\begin{cases}x\equiv 5+9t_1\\9t_1\equiv 21(mod25)\end{cases}\rightarrow$$

$$\begin{cases} x = 5 + 9t_1 \\ 3t_1 \equiv 7(mod 25) \end{cases} \to \begin{cases} x = 5 + 9t_1 \\ 3t_1 \equiv -18(mod 25) \end{cases} \to \begin{cases} x = 5 + 9t_1 \\ t_1 \equiv -6(mod 25) \end{cases} \to$$

$$x = -49 + 225t_2, t_2 \in Z.$$

c_2) dan

$$\begin{cases} 5 + 9t_1 \equiv -3(mod 25) \\ x = 5 + 9t_1 \end{cases} \to \begin{cases} 9t_1 \equiv -8(mod 25) \\ x = 5 + 9t_1 \end{cases} \to \begin{cases} 9t_1 \equiv -33(mod 25) \\ x = 5 + 9t_1 \end{cases} \to$$

$$\begin{cases} 3t_1 \equiv -11(mod 25) \\ x = 5 + 9t_1 \end{cases} \to \begin{cases} 3t_1 \equiv -36(mod 25) \\ x = 5 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv -12(mod 25) \\ x = 5 + 9t_1 \end{cases} \to$$

$$x = -103 + 225t_2, t_2 \in Z.$$

c_3) dan

$$\begin{cases} 4 + 9t_1 \equiv 1(mod 25) \\ x = 4 + 9t_1 \end{cases} \to \begin{cases} 9t_1 \equiv -3(mod 25) \\ x = 4 + 9t_1 \end{cases} \to \begin{cases} 3t_1 \equiv -1(mod 25) \\ x = 4 + 9t_1 \end{cases}$$

$$\begin{cases} t_1 \equiv 8(mod 25) \\ x = 4 + 9(5 + 25t_2) = 76 + 225t_2 \end{cases} \to x = 76 + 225t_2, t_2 \in Z.$$

c_4) dan

$$\begin{cases} 4 + 9t_1 \equiv -3(mod 25) \\ x = 4 + 9t_1 \end{cases} \to \begin{cases} 9t_1 \equiv -7(mod 25) \\ x = 4 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv 2(mod 25) \\ x = 4 + 9t_1 \end{cases} \to$$

$x = 22 + 225t_2, t_2 \in Z$. **Javob**: $x \equiv -103, -49, 22, 76(mod 225)$.

8). $f(x) = 2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\, 441)$ taqqoslamani qaraymiz. Bu yerda $441 = 3^2 \cdot 7^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\, 9) \\ f(x) \equiv 0(mod\, 49) \end{cases} \quad (1)$$

ga teng kuchli. Buning birinchisidan $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\, 3^2)$ ni yechish uchun $-x^6 - x^2 - 1 \equiv 0(mod\, 3) \to -x^2 + 1 \equiv 0(mod\, 3)$. Buni 3 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1$, dagi chegirmalarni sinab ko'rish usuli bilan yechsak, $x \equiv 1, -1(mod\, 3)$ lar uning yechimi ekanligini topamiz.

a_1) $x \equiv 1(mod\, 3) \to x = 1 + 3t_1$ yechimni qaraymiz. 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod 3)$ taqqoslamaga ega bo'lamiz. Bunda $f(1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x$, $f'(1) = -26$ bo'lgani uchun$-5 - 26t_1 \equiv 0(mod 3) \to t_1 \equiv 2(mod 3) \to t_1 \equiv -1 + 3t_2 \to x = -2 + 9t_2, t_2 \in Z$.

$a_2)\ x \equiv -1(mod\ 3) \rightarrow x = -1 + 3t_1$ yechimni qaraymiz. 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda $f(-1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x,\ f'(-1) = 26$ bo'lgani uchun$-5 + 26t_1 \equiv 0(mod3) \rightarrow 2t_1 \equiv 2(mod3) \rightarrow t_1 \equiv 1 + 3t_2 \rightarrow x = 2 + 9t_2, t_2 \in Z.$

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\mathrm{mod}\ 7)$ ni, ya'ni $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 7)$ ni qaraymiz. Bu taqqoslama $2x^6 + x^4 + 3 \equiv 0(mod\ 7)$ ga teng kuchli. Buni 7 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2, \pm3$ larni qo'yib tanlash usuli bilan yechsak $x \equiv \pm2(mod7)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$\square_1)\ x \equiv 2(mod\ 7) \rightarrow x = 2 + 7t_1$ yechimni qaraymiz. 290-misoldagi (7– formulaga asosan t_1 ga nisbatan $\frac{f(2)}{3} + f'(2)t_1 \equiv 0(mod7)$ taqqoslamaga ega bo'lamiz. Bunda $f(2) = 0$ va $f'(12) = 164$ bo'lgani uchun$164t_1 \equiv 0(mod7) \rightarrow t_1 \equiv 0(mod7) \rightarrow t_1 \equiv 7t_2 \rightarrow x = 2 + 49t_2, t_2 \in Z.$

$b_2)\ x \equiv -2(mod\ 7) \rightarrow x = -2 + 7t_1$ yechimni qaraymiz. 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(-2)}{3} + f'(-2)t_1 \equiv 0(mod7)$ taqqoslamaga ega bo'lamiz. Bunda $f(-1) = 0$ va $f'(-1) = -164$ bo'lgani uchun $-164t_1 \equiv 0(mod7) \rightarrow t_1 \equiv 0(mod7) \rightarrow t_1 \equiv 7t_2 \rightarrow x = -2 + 49t_2, t_2 \in Z.$

Shunday qilib, quyidagi chiziqli taqqoslamalar sistemasini yechishga keldik:

$$c_1)\begin{cases} x \equiv -2\ \ (mod\ 9) \\ x \equiv 2\ \ (mod\ \ 49) \end{cases} c_2)\begin{cases} x \equiv -2\ \ (mod\ 9) \\ x \equiv -2\ \ (mod\ \ 49) \end{cases}$$

$$c_3)\begin{cases} x \equiv 2\ \ (mod\ 9) \\ x \equiv 2\ \ (mod\ \ 49) \end{cases} c_4)\begin{cases} x \equiv 2\ \ (mod\ 9) \\ x \equiv -2\ \ (mod\ \ 49) \end{cases}$$

$c_1)$ dan

$$\begin{cases} x = -2 + 9t_1 \\ -2 + 9t_1 \equiv 2\ (mod\ 49) \end{cases} \rightarrow \begin{cases} 9t_1 \equiv 4\ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \rightarrow$$

$$\begin{cases} 9t_1 \equiv -45\ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv -5\ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv -5 + 49t_2 \\ x = -47 + 441t_2 \end{cases} \rightarrow$$

$x = -47 + 441t_2, t_2 \in Z.$

$c_2)$ dan

$$\begin{cases} x = -2 + 9t_1 \\ -2 + 9t_1 \equiv -2 \ (mod \ \ 49) \end{cases} \rightarrow \begin{cases} 9t_1 \equiv 0 \ (mod\ 49) \\ t_1 \equiv 0 \ (mod\ 49) \end{cases} \rightarrow \begin{cases} t_1 \equiv \ 49t_2 \\ x = -2 + 441t_2 \end{cases} \rightarrow x =$$
$-2 + 441t_2, t_2 \in Z$.

c_3) dan

$$\begin{cases} x = 2 + 9t_1 \\ 2 + 9t_1 \equiv 2 \ (mod\ 49) \end{cases} \rightarrow \begin{cases} 9t_1 \equiv 0 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv 0 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases}$$

$$\rightarrow \begin{cases} t_1 = 49t_2 \\ x = 2 + 9t_1 \end{cases} x = 2 + 441t_2, t_2 \in Z.$$

c_4) dan

$$\begin{cases} x = 2 + 9t_1 \\ 2 + 9t_1 \equiv -2 \ (mod\ 49) \end{cases} \rightarrow \begin{cases} 9t_1 \equiv -4 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \rightarrow$$
$$\begin{cases} 9t_1 \equiv \ 45 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv 5 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv 5 + 49t_2 \\ x = 2 + 9t_1 \end{cases} \rightarrow x =$$
$47 + 441t_2, t_2 \in Z$.

Javob: $x \ \equiv \ -47, \ -2, \ 2, \ 47 \ \ (mod \ \ 441)$.

9). $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0 \ (mod \ \ 1225)$taqqoslamani qaraymiz. Bu yerda $1225 = 5^2 \cdot 7^2$ bo’lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 25) \\ f(x) \equiv 0(mod\ 49) \end{cases} \qquad (1)$$

ga teng kuchli. (1) dagi 2-taqqoslamani 291.8) misolda yechgan edik. Uning yechimlari $x \equiv -2 \ (mod \ \ 49)$va $x \equiv 2 \ (mod\ 49)$ iborat edi. Shuning uchun ham (1) dagi 1-taqqoslama $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5^2)$ ni yechamiz. Buning uchun $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5) \rightarrow 2x^6 - x^4 - 2x^2 + 1 \equiv 0(mod\ 5) \rightarrow 2x^2 - 1 - 2x^2 + 1 \equiv 0(mod\ 5)$ni qaraymiz. Bu taqqoslama $2x^6 - x^4 - 2x^2 + 1 \equiv 0(mod\ 5) \rightarrow 2x^2 - 1 - 2x^2 + 1 \equiv 0(mod\ 5)$ ayniy taqqoslamaga teng kuchli bo’lgani uchun uning yechimlari $x \equiv -1, 1, 2, -2 \ (mod\ 5)$ dan iborat. Endi bu yechimlardan foydalanib, $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5^2)$ ning yechimini topishga harakat qilamiz.

a_1) $x \equiv -1 \ (mod\ 5\ )$, $x = -1 + 5t_1$ni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod 5)$ taqqoslamaga ega bo’lamiz. Bunda $f(-1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x$, $f'(-1) = 26$ bo’lgani uchun $-3 + 26t_1 \equiv 0(mod 5) \rightarrow t_1 \equiv 3(mod 5) \rightarrow t_1 \equiv 3 + 5t_2 \rightarrow x = 14 + 25t_2, t_2 \in Z$.

$a_2)$ $x \equiv 1(mod\ 5) \to x = 1 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod 5)$ taqqoslamaga ega bo'lamiz. Bunda $f(1) = -15$ va $f'(-1) = -26$ bo'lgani uchun $-3 - 26t_1 \equiv 0(mod5) \to t_1 \equiv -3(mod5) \to t_1 \equiv -3 + 5t_2 \to x = -14 + 25t_2, t_2 \in Z$.

$a_3)$ $x \equiv 2(mod\ 5) \to x = 2 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) – formulaga asosan t_1 ga nisbatan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo'lamiz. Bunda $f(2) = 0$ va $f'(2) = 164$ bo'lgani uchun $164t_1 \equiv 0(mod5) \to t_1 \equiv 0(mod5) \to t_1 \equiv 5t_2 \to x = 2 + 25t_2, t_2 \in Z$.

$a_4)$ $x \equiv -2(mod\ 5) \to x = -2 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo'lamiz. Bunda $f(-2) = 0$ va $f'(-2) = -164$ bo'lgani uchun $164t_1 \equiv 0(mod5) \to t_1 \equiv 0(mod5) \to t_1 \equiv 5t_2 \to x = -2 + 25t_2, t_2 \in Z$. Shunday qilib, quyidagi chiziqli taqqoslamalar sistemasini yechishga keldik:

$c_1)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv 14\ (mod\ 25) \end{cases}$; $c_2)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv -14\ (mod\ 25) \end{cases}$;

$c_3)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv 2\ (mod\ 25) \end{cases}$; $c_4)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv -2\ (mod\ 25) \end{cases}$;

$c_5)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv 14\ (mod\ 25) \end{cases}$; $c_6)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv -14\ (mod\ 25) \end{cases}$;

$c_7)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv 2\ (mod\ 25) \end{cases}$; $c_8)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv -2\ (mod\ 25) \end{cases}$.

$c_1)$ dan $\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv 14\ (mod\ 25) \end{cases} \to \begin{cases} 49t_1 \equiv 16\ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \to$

$\to \begin{cases} t_1 \equiv 9\ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \to x = -2 + 49(9 + 25t_2) = 439 + 1225t_2, t_2 \in Z.$

$c_2)$ dan $\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv -14\ \ (mod\ \ 25) \end{cases} \to \begin{cases} x = -2 + 49t_1 \\ 49t_1 \equiv -12\ \ (mod\ \ 25) \end{cases} \to$

$\to \begin{cases} t_1 \equiv 12\ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \to x = -2 + 49 \cdot (12 + 25t_2) = 586 + 1225t_2, t_2 \in \square.$

c_3) dan $\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv 2\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv -4\ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \to x = -2 +$

$49(-4 + 25\ t_2) = -198 + 1225\ t_2, \qquad t_2 \in Z.$

c_4) dan $\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv -2\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv 0\ (mod\ 25) \\ t_1 = 25t_2 \end{cases} \to x = -2 +$ $1225t_2\ , t_2 \in Z.$

c_5) dan $\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv 14\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv -12\ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \to x = -586 +$ $1225t_2\ , t_2 \in Z.$

c_6) dan $\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv -14\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv 16\ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \to x = 786 +$ $1225t_2\ , t_2 \in Z.$

c_7) dan $\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv 2\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv 0\ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \to t_1 = 25t_2 \to$

$$x = 2 + 1225t_2\ , t_2 \in Z.$$

c_8) dan $\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv -2\ (mod\ 25) \end{cases} \to \begin{cases} t_1 \equiv 4\ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \to \begin{cases} t_1 \equiv 4 + 25t_2 \\ x = 2 + 49t_1 \end{cases} \to$

$$x = 198 + 1225t_2, t_2 \in Z.$$

Shunday qilib, berilgan taqqoslamaning yechimlari: $x \equiv -586,\ -198,\ -2,\ 2,$ $198,\ 439,\ 586,\ 786(mod\ \ 1225).$

IV.6-§.

292.1). Avvalo, berilgan $2x^2 + 4x - 1 \equiv 0(mod5)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $2a \equiv 1(mod5) \to a \equiv 3(mod5)$ bo`lgani uchun berilgan taqqoslamaning ikkala tomonini 3 ga ko`paytiramiz. U holda $6x^2 + 12x - 3 \equiv 0(mod5)$ tenglama hosil bo`ladi. Bundan $x^2 + 2x + 2 \equiv 0(mod5) \to (x+1)^2 + 1 \equiv 0(mod5) \to (x+1)^2 \equiv -1(mod5) \to (x+1)^2 \equiv 4(mod5) \to x + 1 \equiv 2(mod5)vax + 1 \equiv -2(mod5) \to x \equiv 1(mod5)vax \equiv -3(mod5)$ hosil bo'ladi.

Javob: $x \equiv -3, 1\ (mod5).$

2). Avvalo, berilgan $3x^2+2x\equiv 1(mod7)$ taqqoslamani bosh hadining koeffisiyenti 1 ga teng bo’lgan holga keltiramiz. $3a\equiv 1(mod7)\to a\equiv 5(mod7)$ bo`lgani uchun $3x^2+2x\equiv 1(mod7)$ taqqoslamaning ikkala tomonini 5 ga ko`paytiramiz. U holda $15x^2+10x-5\equiv 0(mod7)\to x^2+10x+9\equiv 0(mod7)\to (x+5)^2-16\equiv 0(mod7)\to (x+5)^2\equiv 16(mod7)\to x+5\equiv 4(mod7)$ va $x+5\equiv -4(mod7)\to x\equiv -1(mod7)$va $x\equiv -2(mod7)$ hosil bo’ladi. **Javob:** $x\equiv -1,-2(mod7)$.

3). Avvalo, berilgan $2x^2-2x-1\equiv 0(mod7)$ taqqoslamani bosh hadining koeffisenti 1 ga teng bo’lgan holga keltiramiz. $2a\equiv 1(mod7)\to a\equiv 4(mod7)$ bo`lgani uchun $2x^2-2x-1\equiv 0(mod7)$ taqqoslamaning ikkala tomonini 4 ga ko`paytiramiz. U holda $8x^2-8x-4\equiv 0(mod7)$, $x^2-x+3\equiv 0(mod7)\to x^2+6x+10\equiv 0(mod7)\to (x+3)^2+1\equiv 0(mod7)\to (x+3)^2\equiv -1(mod7)\to (x+3)^2\equiv 6(mod7)$. Bu taqqoslamaga 7 moduli bo`yicha chegirmalarning to’la sistemasidagi chegirmalarning to’la sistemasi $0,\pm 1,\pm 2,\pm 3$ qo’yib tekshirsak, taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

4). Berilgan $3x^2-x\equiv 0(mod5)$ taqqoslamani bosh hadining koeffisiyenti 1 ga teng bo’lgan holga keltiramiz. $3a\equiv 1(mod5)\to a\equiv 2(mod5)$ bo`lgani uchun $3x^2-x\equiv 0(mod5)$ taqqoslamaning ikkala tomonini 2 ga ko`paytiramiz. U holda $6x^2-2x\equiv 0(mod5)\to x^2-2x\equiv 0(mod5)\to (x-1)^2-1\equiv 0(mod5)\to (x-1)^2\equiv 1(mod5)\to x-1\equiv 1(mod5)$ va $x-1\equiv -1(mod5)\to x\equiv 2(mod5)$va $x\equiv 0(mod5)$ hosil bo’ladi. **Javob:** $x\equiv 0,2(mod5)$.

5) Berilgan $3x^2+7x+8\equiv 0(mod17)$ taqqoslamani bosh hadining koeffisiyenti 1 ga teng bo’lgan holga keltiramiz. $3a\equiv 1(mod17)\to a\equiv 6(mod17)$ bo`lgani uchun $3x^2+7x+8\equiv 0(mod17)$ taqqoslamaning ikkala tomonini 6 ga ko`paytiramiz. U holda $18x^2+42x+48\equiv 0(mod17)\to x^2+8x+48\equiv 0(mod17)\to (x+4)^2+32\equiv 0(mod17)\to (x+4)^2\equiv 2(mod17)\to (x+4)^2\equiv 36(mod17)\to x+4\equiv 6(mod17)$va $x+4\equiv -6(mod17)\to x\equiv 2(mod17)$va $x\equiv 7(mod17)$ hosil bo’ladi. **Javob**: $x\equiv 2,7(mod17)$.

6). Berilgan $3x^2+4x+7\equiv 0(mod31)$ taqqoslamani $9x^2+12x+21\equiv 0(mod31)\to (3x+2)^2+17\equiv 0(mod31)\to (3x+2)^2\equiv 14+31\cdot 5(mod31)\to (3x+2)^2\equiv 169(mod31)\to 3x+2\equiv 113(mod31)$va $3x+2\equiv -13(mod31)\to 3x\equiv 11(mod31)$va $3x\equiv -15(mod31)\to x\equiv 14(mod31)$ va $x\equiv -5(mod31)$. **Javob:** $x-5,14(mod31)$.

7). $4x^2-11x-3\equiv 0(\mathrm{mod}13)$ taqqoslamani ikkihadli taqqoslama ko’rinishiga keltirib, keyin yeching.

$$4x^2-24x-16\equiv 0(\mathrm{mod}13)\to x^2-6x-4\equiv 0(\mathrm{mod}13)\to (x-3)^2\equiv 0(\mathrm{mod}13)\to x\equiv 3(\mathrm{mod}\ 13).$$

8). $3x^2+7x+8\equiv 0(\mathrm{mod}17)$ taqqoslamani ikkihadli taqqoslama ko'rinishiga keltirib, keyin yeching.

$$3x^2+24x-9\equiv 0(\mathrm{mod}17)\to x^2+8x-3\equiv 0(\mathrm{mod}17)\to (x+4)^2\equiv 19(\mathrm{mod}17)\to$$

$$(x+4)^2\equiv 36(\mathrm{mod}17)\to x+4\equiv 6(\mathrm{mod}17)\ va\ \ x+4\equiv -6(\mathrm{mod}17)\to\ x\equiv 6-4(\mathrm{mod}17)$$

$$va\ \ x\equiv -10(\mathrm{mod}17)\to x\equiv 2(\mathrm{mod}17)\ va\ x\equiv 7(\mathrm{mod}17).$$

293.1). Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo`limishi kerak, ya`ni $x^2+2x+7\equiv 0(mod55)$ bajarilishi kerak. Bundan $(x+1)^2\equiv -6(mod55)\to\begin{cases}(x+1)^2\equiv -6(mod5)\\(x+1)^2\equiv -6(mod11)\end{cases}\to$

$\begin{cases}(x+1)^2\equiv 4(mod5)\\(x+1)^2\equiv 16(mod11)\end{cases}\to\begin{cases}x+1\equiv \pm 2(mod5)\\x+1\equiv \pm 4(mod11)\end{cases}$ larga ega bo'lamiz. Keyingi sistemadagi birinchi taqqoslamaning yechimlari $x\equiv 1,2(mod5)$, ikkinchi taqqoslamaning yechimlari $x\equiv -5,3(mod11)$ dan iborat ekanligi kelib chiqadi. Bulardan quyidagi taqqoslamalar sistemalarini hosil qilamiz:

$$a_1\begin{cases}x\equiv 1(mod5)\\x\equiv 6(mod11)\end{cases};\ a_2)\ \begin{cases}x\equiv 1(mod5)\\x\equiv 3(mod11)\end{cases};\qquad a_3)\begin{cases}x\equiv 2(mod5)\\x\equiv 6(mod11)\end{cases};$$

$$a_4)\begin{cases}x\equiv 2(mod5)\\x\equiv 3(mod11)\end{cases}.$$

Bu sistemalarni yechmiz. U holda

$a_1)$ dan$\begin{cases}x\equiv 1+5t_1\\1+5t_1\equiv 6(mod11)\end{cases}\to\begin{cases}5t_1\equiv 5(mod11)\\x\equiv 1+5t_1\end{cases}\to\begin{cases}t_1\equiv 1(mod11)\\x\equiv 1+5t_1\end{cases}\to$

$$x\equiv 6+5t_2,\qquad t_2\in Z.$$

$a_2)$ dan $\begin{cases}x\equiv 1+5t_1\\1+5t_1\equiv 3(mod11)\end{cases}\begin{cases}5t_1\equiv 2(mod11)\\x\equiv 1+5t_1\end{cases}\to\begin{cases}t_1\equiv 7(mod11)\\x\equiv 1+5t_1\end{cases}\to$

$x=36+55t_2,\ t_2\in Z.$

$a_3)$ dan $\begin{cases}x\equiv 2+5t_1\\2+5t_1\equiv 6(mod11)\end{cases}\to\begin{cases}5t_1\equiv 4(mod11)\\x\equiv 2+5t_1\end{cases}\to\begin{cases}t_1\equiv 3(mod11)\\x\equiv 2+5t_1\end{cases}\to$

$x=17+55t_2,\ t_2\in Z.$

$a_4)$ dan$\begin{cases}x\equiv 2+5t_1\\2+5t_1\equiv 3(mod11)\end{cases}\begin{cases}5t_1\equiv 1(mod11)\\x\equiv 2+5t_1\end{cases}\to\begin{cases}t_1\equiv -2(mod11)\\x\equiv 2+5t_1\end{cases}\to$

$x=-8+55t_2,\ t_3\in Z.$

Javob: $x=6+55t,\ x=17+55t,\ x=36+55t,\ x=47+55t, t\in Z.$

4) Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo`limishi kerak, ya`ni$x^2+3x+1\equiv 0(mod25)$ bajarilishi kerak. $x^2+3x+1\equiv 0(mod5)$ ni qaraymiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasi $0,\pm 1,\pm 2$ dagi sonlarni sinab ko'rish usuli bilan yechsak, $x\equiv 1(mod5)$

uning yechimi ekanligini topamiz. Endi 5 moduldan 25 modulga o'tamiz. Buning uchun 290-misoldagi singari ish tutamiz. U holda (7)-formulaga asosan $\frac{f(1)}{5}+f'(1)t_1 \equiv 0(mod25)$ ga ega bo'lamiz. Bu yerda

$f(1)=5, f'(1)=2x+3$ va $f'(1)=5$ bo'lgani uchun $1+5t_1 \equiv 0(mod5) \rightarrow 5t_1 \equiv 4(mod5)$ ga ega bo'lamiz. Bunda$(5,25)=5$, lekin 4 soni 5 ga bo`linmagani uchun bu taqqoslama yechimga ega emas, ya`ni berilgan ifoda butun qiymat qabul qiladigan x ning natural qiymatlari mavjud emas.

Javob: berilgan ifoda butun qiymat qabul qiladigan x ning natural qiymatlari mavjud emas.

5) Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo`limishi kerak, ya`ni $x^2+3x++5 \equiv 0(mod15)$ bajarilishi kerak. Bu taqqoslama ushbu

$$\begin{cases} x^2+3x+5 \equiv 0(mod3) \\ x^2+3x+5 \equiv 0(mod5) \end{cases}$$

taqqoslamalar sistemasiga teng kuchli. Bu sistemaning 1-taqqoslamasini yechamiz. U holda $x^2+3x+5 \equiv 0(mod3) \rightarrow x^2-1 \equiv 0(mod3) \rightarrow x \equiv -1$ va$x \equiv 1(mod3)$ larni hosil qilamiz.

Endi 2- taqqoslamasini yechamiz:

$$x^2+3x+5 \equiv 0(mod5 \rightarrow x^2-2x \equiv 0(mod5) \rightarrow x \equiv 0{,}2(mod5).$$

Bularga asoslanib quyidagi sistemalarni tuzib olamiz:

$a_1)\ \begin{cases} x \equiv -1(mod3) \\ x \equiv 0(mod5) \end{cases}; a_2)\ \begin{cases} x \equiv -1(mod3) \\ x \equiv 2(mod5) \end{cases}; a_3)\ \begin{cases} x \equiv 1(mod3) \\ x \equiv 0(mod5) \end{cases}$

$a_4)\ \begin{cases} x \equiv 1(mod3) \\ x \equiv 2(mod5) \end{cases}.$

Bu sistemalarni yechib, yechimlarini topamiz:

$a_1)$ dan $\begin{cases} x \equiv -1+3t_1 \\ -1+3t_1 \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv 1(mod5 \\ x = -1+3t \end{cases} \rightarrow \begin{cases} t_1 \equiv 2(mod5) \\ x = -1+3t_1 \end{cases} \rightarrow$
$x = 5+15t_2, t_2 \in Z.$

$a_2)$ dan $\begin{cases} x \equiv -1+3t_1 \\ -1+3t_1 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv 3(mod5 \\ x = -1+3t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv 1(mod5) \\ x = -1+3t_1 \end{cases}$
$x = 2+15t_2, t_2 \in Z.$

$a_3)$ dan $\begin{cases} x \equiv 1+3t_1 \\ 1+3t_1 \equiv 0(mod5) \end{cases} \begin{cases} 3t_1 \equiv -1(mod5) \\ x = 1+3t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv -2(mod5) \\ x = 1+3t_1 \end{cases}$
$x = -5+15t_2,\ t_2 \in Z.$

$a_4)$ dan $\begin{cases} x \equiv 1+3t_1 \\ 1+3t_1 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv 1(mod5) \\ x = 1+3t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv 2(mod5) \\ x = 1+3t_1 \end{cases}$

$$x = 7 + 15t_2, \qquad t_2 \in Z.$$

Javob: $x = 2 + 15t_2,\ x = 5 + 15t_2,\ x = 7 + 15t_2,\ x = 10 + 15t_2, t_2 Z.$

294. $x^2 \equiv a(modp)$ taqqoslama yechimga ega bo`lishi uchun Eyler kriteriyasiga asosan $a^{\frac{p-1}{2}} \equiv 1(modp)$ bajarilishi kerak. Budan $p = 7$ da $a^3 \equiv 1(mod7)$ ga ega bo`lamiz 7 moduli bo`yicha 1,2,3,4,5,6 dan iborat . Bularni Eyler kriteriyasiga qo`yib, tekshirib ko`ramiz; $1^3 \equiv 1, 2^3 \equiv 1, 3^3 \equiv -1, 4^3 \equiv 1, \quad 5^3 \equiv -1, 6^3 \equiv -1.$ Demak, 1,2,4 sonlari 7 modul bo`yicha kvadratik chegirma, qolganlari, ya'ni 3,5,6 lar esa kvadratik chegirma emas.

295. 1). $p = 11$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kriteriyasi $a^{\frac{p-1}{2}} \equiv 1(modp) \to a^5 \equiv 1(mod11)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^5 \equiv 1(mod11),$

$$2^5 \equiv -1(mod11), 3^5 \equiv 81 \cdot 3 \equiv 1(mod11), 4^5 \equiv 16 \cdot 16 \cdot 4 \equiv 5 \cdot 5 \cdot 4 \equiv 3 \cdot 4 \equiv 1(mod11), 5^5 \equiv 25 \cdot 25 \cdot 5 \equiv 3 \cdot 3 \cdot 5 \equiv 1(mod11),$$

$$6^5 \equiv 36 \cdot 36 \cdot 6 \equiv 3 \cdot 3 \cdot 6 \equiv -1(mod11),\ 7^5 \equiv 49 \cdot 49 \cdot 7 \equiv 5 \cdot 5 \cdot 7 \equiv -1(mod11),\ 8^5 \equiv 64 \cdot 64 \cdot 8 \equiv (-2) \cdot (-2) \cdot 8 \equiv -1(mod11),$$

$9^5 \equiv 81 \cdot 81 \cdot 9 \equiv 4 \cdot 4 \cdot (-2) \equiv 1(mod11).$ Bizga ma'lumki, $p > 2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida $1,3,4,5,9$larning $p = 11$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1 + 11k, 3 + 11k, 4 + 11k, 5 + 11k, 9 + 11k$ lar $p = 11$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1 + 11k, 3 + 11k, 4 + 11k, 5 + 11k, 9 + 11k, k \in Z.$

2). $p = 13$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kiriteryasi $a^{\frac{p-1}{2}} \equiv 1(modp) \to a^6 \equiv 1(mod13)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10,11,12$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^6 \equiv 1(mod13),$

$$2^6 \equiv -1(mod13), 3^6 \equiv 27 \cdot 27 \equiv 1(mod13), 4^6 \equiv 16 \cdot 16 \cdot 16 \equiv 27 \equiv 1(mod13), 5^6 \equiv 25 \cdot 25 \cdot 25 \equiv -1(mod13),$$

$6^6 \equiv 36 \cdot 36 \cdot 36 \equiv -3 \cdot (-3) \cdot (-3) \equiv -1(mod13),\ 7^6 \equiv 49 \cdot 49 \cdot 49 \equiv -3 \cdot (-3) \cdot (-3) \equiv -1(mod13),\ 8^6 \equiv 64 \cdot 64 \cdot 64 \equiv (-1) \cdot (-1) \cdot (-1) \equiv -1(mod13), 9^6 \equiv 81 \cdot 81 \cdot 81 \equiv 3 \cdot 3 \cdot 3 \equiv 1(mod13), \quad 10^6 \equiv 100 \cdot 100 \cdot 100 \equiv -3 \cdot (-3) \cdot (-3) \equiv -1(mod13),\ 11^6 \equiv 121 \cdot 121 \cdot 121 \equiv 4 \cdot 4 \cdot 4 \equiv$

$-1(mod13)$, $12^6 \equiv 144 \cdot 144 \cdot 144 \equiv 1(mod13)$. Bizga ma'lumki, $p > 2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida 1,3,4,9,10,12 larning $p = 13$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1 + 13k, 3 + 13k, 4 + 13k, 9 + 13k, 10 + 13k, 12 + 13k$ lar $p = 13$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1 + 13k, 3 + 13k, 4 + 13k, 9 + 13k, 10 + 13k, 12 + 13k, k \in Z$.

3). $p = 17$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kiriteryasi $a^{\frac{p-1}{2}} \equiv 1(modp) \rightarrow a^8 \equiv 1(mod17)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^8 \equiv 1(mod17)$, $2^8 \equiv 2^4 \cdot 2^4 \equiv 1(mod17), 3^8 \equiv 81 \cdot 81 \equiv -4(-4) \equiv 1(mod17), 4^8 \equiv 16 \cdot 16 \cdot 16 \cdot 16 \equiv 1(mod17), 5^8 \equiv 25 \cdot 25 \cdot 25 \cdot 25 \equiv 8^2 \cdot 8^2 \equiv 1(mod17), 6^8 \equiv 36^4 \equiv 2^4 \equiv -1(mod17)$, $7^8 \equiv 49^4 \equiv (-2)^4 \equiv -1(mod17)$, $8^8 \equiv 64^4 \equiv (-4)^4 \equiv 1(mod17), 9^8 \equiv 81^4 \equiv (-4)^4 \equiv 1(mod17)$, $10^8 \equiv 100^4 \equiv (-2)^2 \equiv -1(mod17)$, $11^8 \equiv 121^4 \equiv 2^4 \equiv -1(mod17)$, $12^8 \equiv 144^4 \equiv 8^4 \equiv 64 \cdot 64 \equiv -1(mod17)$, $13^8 \equiv 169^4 \equiv (-1)^4 \equiv 1(mod17)$, $14^8 \equiv (-3)^8 \equiv 81^2 \equiv (-4)^2 \equiv -1(mod17)$, $15^8 \equiv (-2)^8 \equiv 2^4 \cdot 2^4 \equiv 1(mod17)$, $16^8 \equiv (-1)^8 \equiv 1(mod17)$. Bizga ma'lumki, $p > 2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida 1,2,4,8,9,13,15,16 larning $p = 17$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1 + 17k, 2 + 17k, 4 + 17k, 9 + 17k, 9 + 17k, 13 + 17k, 15 + 17k, 16 + 17k$ lar $p = 17$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1 + 17k, 2 + 17k, 4 + 17k, 9 + 17k, 9 + 17k, 13 + 17k, 15 + 17k$, $16 + 17k$ $k \in Z$.

296. 1).7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 2(mod7)$ ga qo'yib tekshirsak, $x \equiv \pm3(mod7)$ ning uni qanoatlantirishini ko'ramiz. **Javob:** $x \equiv \pm3(mod7)$.

2). 7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak,$\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 4(mod7)$ga qo'yib tekshirsak, $x \equiv \pm2(mod7)$ ning uni qanoatlantirishini ko'ramiz. **Javob:** $x \equiv \pm2(mod7)$.

3). 7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 3(mod7)$ga qo'yib tekshirsak, ularning birortasi ham uni qanoatlantirmasligini ko'ramiz. **Javob:** taqqoslama yechimga ega emas.

4). 13 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 3(mod 13)$ga qo'yib tekshirsak, $x \equiv \pm4(mod 7)$ ning uni qanoatlantirishini ko'ramiz. **Javob:** $x \equiv \pm4(mod 13)$.

5). 11 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3, \pm4, \pm5$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 4(mod 11)$ga qo'yib tekshirsak, $x \equiv \pm2(mod 11)$ ning uni qanoatlantirishini ko'ramiz. **Javob:** $x \equiv \pm2(mod 11)$.

297. Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz.

1).$\left(\frac{63}{131}\right) = \left(\frac{3^2 \cdot 7}{131}\right)$ dan 4^0 –xossaga asosan $\left(\frac{3^2 \cdot 7}{131}\right) = \left(\frac{3^2}{131}\right) \cdot \left(\frac{7}{131}\right)$ ni hosil qilamiz. Ta'rifga ko'ra $\left(\frac{3^2}{131}\right) = \left(\frac{3}{131}\right)^2 = 1$, shuning uchun ham $\left(\frac{3^2 \cdot 7}{131}\right) = \left(\frac{7}{131}\right)$. Oxirgi tenglikning o'ng tomoniga kvadratik chegirmalarning o'zgalik qonuni 6^0 –xossani qo'llaymiz. U holda $\left(\frac{7}{131}\right) = (-1)^{\frac{131-1}{2} \cdot \frac{7-1}{2}} \left(\frac{131}{7}\right) = -\left(\frac{18 \cdot 7+5}{7}\right)$ hosil bo'ladi. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{7}{131}\right) = -\left(\frac{5}{7}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonida yana bir marta 6^0 –xossadan foydalanamiz: $\left(\frac{7}{131}\right) = -\left(\frac{5}{7}\right) = -(-1)^{\frac{5-1}{2} \cdot \frac{7-1}{2}} \left(\frac{7}{5}\right) = -\left(\frac{5 \cdot 1+2}{5}\right) = -\left(\frac{2}{5}\right)$.

Bunga $5^0 -$ xossani qo'llaymiz. U holda $-\left(\frac{2}{5}\right) = -(-1)^{\frac{5^2-1}{8}}$=1. Demak, $\left(\frac{63}{131}\right) = 1$. **Javob:**1.

2).$\left(\frac{35}{97}\right) = \left(\frac{5 \cdot 7}{97}\right)$ dan 4^0 –xossaga asosan $\left(\frac{5 \cdot 7}{97}\right) = \left(\frac{5}{97}\right) \cdot \left(\frac{7}{97}\right)$ ni hosil qilamiz. Oxirgi tenglikning o'ng tomonida har bir ko'paytuvchi uchun kvadratik chegirmalarning o'zgalik qonuni 6^0 –xossani qo'llaymiz. U holda $\left(\frac{5}{97}\right) \cdot \left(\frac{7}{97}\right) = (-1)^{\frac{97-1}{2} \cdot \frac{5-1}{2}} \cdot \left(\frac{97}{5}\right) \cdot (-1)^{\frac{97-1}{2} \cdot \frac{7-1}{2}} \cdot \left(\frac{97}{7}\right) = \left(\frac{97}{5}\right) \cdot \left(\frac{97}{7}\right) = \left(\frac{5 \cdot 19+2}{5}\right) \cdot \left(\frac{7 \cdot 13+6}{7}\right)$ hosil bo'ladi. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{2}{5}\right) \cdot \left(\frac{6}{7}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonidagi birinchi ko'paytuvchiga $5^0 -$ xossani qo'llaymiz, ikkinchisini esa $\left(\frac{6}{7}\right) = \left(\frac{2}{7}\right)\left(\frac{3}{7}\right)$ deb yozish mumkin. Shuning uchun ham $\left(\frac{2}{5}\right) \cdot \left(\frac{6}{7}\right) = (-1)^{\frac{5^2-1}{8}} \cdot (-1)^{\frac{7^2-1}{8}} \cdot \left(\frac{3}{7}\right) = -1 \cdot 1 \cdot \left(\frac{3}{7}\right) = -\left(\frac{3}{7}\right)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonida yana bir marta 6^0 –xossadan foydalanamiz: $-\left(\frac{3}{7}\right) = -(-1)^{\frac{3-1}{2} \cdot \frac{7-1}{2}} \left(\frac{7}{3}\right) = \left(\frac{3 \cdot 2+1}{3}\right) = \left(\frac{1}{5}\right) = 1$. Demak, $\left(\frac{35}{97}\right) = 1$. **Javob:** 1.

3).$\left(\frac{47}{73}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{47-1}{2}\cdot\frac{73-1}{2}}\cdot\left(\frac{73}{47}\right)=\left(\frac{47\cdot1+26}{47}\right)$ ni hosil qilamiz. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{47\cdot1+26}{47}\right)=\left(\frac{26}{47}\right)=\left(\frac{2}{47}\right)\left(\frac{13}{47}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonidagi birinchi ko'paytuvchiga 5^0-xossani, ikkinchisiga esa 6^0 –xossani qo'llaymiz. $\left(\frac{2}{47}\right)\left(\frac{13}{47}\right)=(-1)^{\frac{47^2-1}{8}}\cdot(-1)^{\frac{13-1}{2}\cdot\frac{47-1}{2}}\left(\frac{47}{13}\right)=\left(\frac{47}{13}\right)=\left(\frac{13\cdot3+8}{13}\right)$ deb yozish mumkin. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{8}{13}\right)=\left(\frac{2^2\cdot2}{13}\right)=\left(\frac{2^2}{13}\right)\cdot\left(\frac{2}{13}\right)=\left(\frac{2}{13}\right)$. Shuning uchun ham $\left(\frac{2}{13}\right)=(-1)^{\frac{13^2-1}{8}}=-1$ ga ega bo'lamiz. Demak, $\left(\frac{47}{73}\right)=-1$. **Javob: -**1.

4).$\left(\frac{29}{383}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{383-1}{2}\cdot\frac{29-1}{2}}\cdot\left(\frac{383}{29}\right)=\left(\frac{13\cdot29+6}{29}\right)$ ni hosil qilamiz. bo'ladi. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{13\cdot29+6}{29}\right)=\left(\frac{6}{29}\right)=\left(\frac{2}{29}\right)\cdot\left(\frac{3}{29}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonidagi birinchi ko'paytuvchiga 5^0-xossani, ikkinchisiga esa 6^0 –xossani qo'llaymiz. $\left(\frac{2}{29}\right)\cdot\left(\frac{3}{29}\right)=(-1)^{\frac{29^2-1}{8}}\cdot(-1)^{\frac{3-1}{2}\cdot\frac{29-1}{2}}\left(\frac{29}{3}\right)=\left(\frac{29}{3}\right)=\left(\frac{9\cdot3+2}{3}\right)$ deb yozish mumkin. Bu yerda 1^0 –xossadan foydalansak, $\left(\frac{9\cdot3+2}{3}\right)=\left(\frac{2}{3}\right)=(-1)^{\frac{3^2-1}{8}}=-1$. Demak, $\left(\frac{29}{383}\right)=-1$. **Javob: -**1.

5).$\left(\frac{241}{593}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{593-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{593}{241}\right)=\left(\frac{593}{241}\right)=\left(\frac{241\cdot2+111}{241}\right)=\left(\frac{111}{241}\right)=\left(\frac{37\cdot3}{241}\right)=\left(\frac{37}{241}\right)\cdot\left(\frac{3}{241}\right)$ ni hosil qilamiz. bo'ladi. Bu tenglikning o'ng tomonidagi ikkala ko'paytuvchiga ham 6^0 –xossani qo'llaymiz. U holda $\left(\frac{37}{241}\right)\cdot\left(\frac{3}{241}\right)=(-1)^{\frac{37-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{241}{37}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{241}{3}\right)=\left(\frac{241}{37}\right)\cdot\left(\frac{241}{3}\right)=\left(\frac{37\cdot6+19}{37}\right)\cdot\left(\frac{3\cdot80+1}{3}\right)$. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{37\cdot6+19}{37}\right)\cdot\left(\frac{3\cdot80+1}{3}\right)=\left(\frac{19}{37}\right)\cdot\left(\frac{1}{3}\right)=\left(\frac{19}{37}\right)$. Endi bunga yana 6^0 –xossani qo'llaymiz. U holda $\left(\frac{19}{37}\right)=(-1)^{\frac{37-1}{2}\cdot\frac{19-1}{2}}\cdot\left(\frac{37}{19}\right)=\left(\frac{37}{19}\right)=\left(\frac{19\cdot1+18}{19}\right)$. Bunga 1^0 –xossani tadbiq etsak, $\left(\frac{19\cdot1+18}{19}\right)=\left(\frac{18}{19}\right)=\left(\frac{2\cdot3^2}{19}\right)=\left(\frac{2}{19}\right)\cdot\left(\frac{3^2}{19}\right)=\left(\frac{2}{19}\right)$. Endi oxirgi tenglikning o'ng tomoniga 5^0-xossani qo'llab $\left(\frac{2}{19}\right)=(-1)^{\frac{19^2-1}{8}}=-1$. Demak, $\left(\frac{241}{593}\right)=-1$. **Javob: -**1.

6). Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$\left(\frac{257}{571}\right) \overset{6^0}{\triangleq} (-1)^{\frac{571-1}{2}\cdot\frac{257-1}{2}} \cdot \left(\frac{571}{257}\right) = \left(\frac{571}{257}\right) = \left(\frac{257\cdot2+57}{257}\right) \overset{1^0}{\triangleq} \left(\frac{57}{257}\right) = \left(\frac{3\cdot19}{257}\right) \overset{4^0}{\triangleq} \left(\frac{3}{257}\right) \cdot$
$\left(\frac{19}{257}\right) \overset{6^0}{\triangleq} (-1)^{\frac{3-1}{2}\cdot\frac{257-1}{2}} \cdot \left(\frac{257}{3}\right) \cdot (-1)^{\frac{19-1}{2}\cdot\frac{257-1}{2}} \cdot \left(\frac{257}{19}\right) = \left(\frac{257}{3}\right) \cdot \left(\frac{257}{19}\right) = \left(\frac{3\cdot85+2}{3}\right) \cdot$
$\left(\frac{19\cdot13+10}{19}\right) \overset{1^0}{\triangleq} \left(\frac{2}{3}\right) \cdot \left(\frac{10}{19}\right) \overset{5^0}{\triangleq} (-1)^{\frac{3^2-1}{8}} \cdot \left(\frac{2\cdot5}{19}\right) = -\left(\frac{2}{19}\right) \cdot \left(\frac{5}{19}\right) \overset{5^0,6^0}{\triangleq} -(-1)^{\frac{19^2-1}{8}} \cdot$
$(-1)^{\frac{19-1}{2}\cdot\frac{5-1}{2}} \left(\frac{19}{5}\right) = \left(\frac{19}{5}\right) = \left(\frac{5\cdot3+4}{5}\right) \overset{1^0}{\triangleq} \left(\frac{4}{5}\right) = \left(\frac{2^2}{5}\right) = 1.$ **Javob:**1.

7). Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$$\left(\frac{251}{577}\right) \overset{6^0}{\triangleq} (-1)^{\frac{577-1}{2}\cdot\frac{251-1}{2}} \cdot \left(\frac{577}{251}\right) = \left(\frac{577}{251}\right) = \left(\frac{251\cdot2+75}{251}\right) \overset{1^0}{\triangleq} \left(\frac{75}{251}\right) = \left(\frac{5^2\cdot3}{251}\right)$$
$$\overset{4^0}{\triangleq} \left(\frac{5^2}{251}\right) \cdot \left(\frac{3}{251}\right) \overset{6^0}{\triangleq} (-1)^{\frac{3-1}{2}\cdot\frac{251-1}{2}} \cdot \left(\frac{251}{3}\right) = -\left(\frac{251}{3}\right) =$$
$$-\left(\frac{3\cdot83+2}{3}\right) \overset{1^0}{\triangleq} -\left(\frac{2}{3}\right) \overset{5^0}{\triangleq} -(-1)^{\frac{3^2-1}{8}} = 1.$$ **Javob:**1.

8).Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$\left(\frac{342}{677}\right) = \left(\frac{2\cdot3^2\cdot19}{677}\right) \overset{4^0}{\triangleq} \left(\frac{2}{677}\right) \cdot \left(\frac{3^2}{677}\right) \cdot \left(\frac{19}{677}\right) = \left(\frac{2}{677}\right) \cdot \left(\frac{19}{677}\right) \overset{5^0,6^0}{\triangleq} (-1)^{\frac{677^2-1}{8}} \cdot$
$(-1)^{\frac{677-1}{2}\cdot\frac{19-1}{2}} \cdot \left(\frac{677}{19}\right) = -\left(\frac{677}{19}\right) = -\left(\frac{19\cdot35+12}{19}\right) \overset{1^0}{\triangleq} -\left(\frac{12}{19}\right) = -\left(\frac{2^2\cdot3}{19}\right) \overset{4^0}{\triangleq} -\left(\frac{2^2}{19}\right) \cdot$
$\left(\frac{3}{19}\right) = -\left(\frac{3}{19}\right) \overset{6^0}{\triangleq} -(-1)^{\frac{3-1}{2}\cdot\frac{19-1}{2}} \cdot \left(\frac{19}{3}\right) = \left(\frac{19}{3}\right) = \left(\frac{3\cdot6+1}{3}\right) \overset{1^0}{\triangleq} \left(\frac{1}{3}\right) = 1.$ **Javob:**1.

298. 1).Lejandr simvolidan foydalanib, berilgan $x^2 \equiv 6(mod7)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 6(mod7)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{6}{7}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{6}{7}\right) = \left(\frac{2}{7}\right) \cdot \left(\frac{3}{7}\right) = (-1)^{\frac{7^2-1}{8}} \cdot \left(\frac{3}{7}\right) = \left(\frac{3}{7}\right) = (-1)^{\frac{3-1}{2}\cdot\frac{7-1}{2}} \cdot \left(\frac{7}{3}\right) = -\left(\frac{3\cdot2+1}{3}\right) =$
$-\left(\frac{1}{3}\right) = -1.$ Demak, berilgan taqqoslama yechimga ega emas.

Javob: berilgan taqqoslama yechimga ega emas.

2). Lejandr simvolidan foydalanib, berilgan $x^2 \equiv 3(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa, uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 3(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{6}{7}\right)$ ning qiymatini aniqlaymiz.

$$\left(\frac{3}{11}\right) = (-1)^{\frac{3-1}{2}\cdot\frac{11-1}{2}} \cdot \left(\frac{11}{3}\right) = -\left(\frac{3\cdot3+2}{3}\right) = -\left(\frac{2}{3}\right) = -(-1)^{\frac{3^2-1}{8}} = 1.$$

Demak, berilgan taqqoslama 2 ta yechimga ega.

Berilgan taqqoslamaning yechimlarini topish uchun 11 moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar $\pm1, \pm2, \pm3, \pm4, \pm5$ larni taqqoslamaga qo`yib, sinab ko'rishimiz yoki taqqoslamalarning xossalaridan foydalanishimiz mumkin. Biz bu yerda birinchi yo'ldan boramiz va berilgan taqqoslamaning yechimlari $x \equiv \pm5(mod 11)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm5(mod 11)$ dan iborat.

3).Lejandr simvolidan foydalanib, berilgan $x^2 \equiv 12(mod 13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa, uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 12(mod 13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{12}{13}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{12}{13}\right) = \left(\frac{2^2 \cdot 3}{13}\right) = \left(\frac{2^2}{13}\right) \cdot \left(\frac{3}{13}\right) = \left(\frac{3}{13}\right) = (-1)^{\frac{3-1}{2} \cdot \frac{13-1}{2}} \cdot \left(\frac{13}{3}\right) = \left(\frac{3 \cdot 4+1}{3}\right) = \left(\frac{1}{3}\right) = 1.$

Demak, berilgan taqqoslama 2 ta yechimga ega.Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 12(mod 13) \rightarrow x^2 \equiv 25(mod 13) \rightarrow x \equiv \pm5(mod 13)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm5(mod 13)$ dan iborat.

6) Lejandr simvolidan foydalanib berilgan $x^2 \equiv 3(mod 13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 3(mod 13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{12}{13}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{3}{13}\right) = (-1)^{\frac{3-1}{2} \cdot \frac{13-1}{2}} \cdot \left(\frac{13}{3}\right) = \left(\frac{3 \cdot 4+1}{3}\right) = \left(\frac{1}{3}\right) = 1.$ Demak, berilgan taqqoslama 2 ta yechimga ega.Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 3(mod 13) \rightarrow x^2 \equiv 16(mod 13) \rightarrow x \equiv \pm4(mod 13)$ ekanligini topamiz. **Javob:** berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm4(mod 13)$ dan iborat.

7) Lejandr simvolidan foydalanib, berilgan $x^2 \equiv 5(mod 11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa, uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 5(mod 11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{5}{11}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{5}{11}\right) = (-1)^{\frac{5-1}{2} \cdot \frac{11-1}{2}} \cdot \left(\frac{11}{5}\right) = \left(\frac{5 \cdot 2+1}{5}\right) = \left(\frac{1}{5}\right) = 1.$ Demak, berilgan taqqoslama 2 ta yechimga ega. Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 5(mod 11) \rightarrow x^2 \equiv$

$16(mod11) \to x \equiv \pm 4(mod11)$ ekanligini topamiz. **Javob:** berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 4(mod11)$ dan iborat.

6). Lejandr simvolidan foydalanib, berilgan $x^2 \equiv 13(mod17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa, uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 13(mod17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{13}{17}\right)$ ning qiymatini aniqlaymiz. $\left(\frac{13}{17}\right) = (-1)^{\frac{13-1}{2}\cdot\frac{17-1}{2}} \cdot \left(\frac{17}{13}\right) = \left(\frac{13\cdot 1+4}{13}\right) = \left(\frac{4}{13}\right) = \left(\frac{2^2}{13}\right) = 1.$ Demak, berilgan taqqoslama 2 ta yechimga ega. Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 13(mod17) \to x^2 \equiv 13 + 17 \cdot 3(mod17) \to x^2 \equiv 64(mod17) \to x \equiv \pm 8(mod17)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 8(mod17)$ dan iborat.

8) Lejandr simvolidan foydalanib berilgan $x^2 \equiv 5(mod17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 5(mod17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{5}{17}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{5}{17}\right) = (-1)^{\frac{5-1}{2}\cdot\frac{17-1}{2}} \cdot \left(\frac{17}{5}\right) = \left(\frac{5\cdot 3+2}{5}\right) = \left(\frac{2}{5}\right) = (-1)^{\frac{5^2-1}{8}} = -1$ Demak, berilgan taqqoslama yechimga ega emas. **Javob:** berilgan taqqoslama yechimga ega emas.

299. 1). Berilgan taqqoslama $x^2 \equiv a(mod5)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(mo\square p)$ taqqoslama yechimga bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a^2 \equiv 1\ (mod5) \to a^2 - 1 \equiv\ (mod5) \to (a-1)(a+1) \equiv 0(mod5) \to a - 1 \equiv 0(mod5)$ yoki $a + 1 \equiv 0(mod5) \to a \equiv \pm 1(mod5)$. **Javob:** $a = \pm 1 + 5t, t \in Z$.

2). Berilgan taqqoslama $x^2 \equiv a(mod7)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechimga bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a^3 \equiv 1\ (mod7)$. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning keltirgan sistemasi $\pm 1, \pm 2, \pm 3$ larni taqqoslamaga qo`yib sinab ko'ramiz. U holda

$a \equiv -3, 1, 2(mod7)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. **Javob:** $a = -3 + 5t, a = 1 + 5t, a = 2 + 5t, t \in Z$.

3). Berilgan taqqoslama $x^2 \equiv a(mod11)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechimga bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a^5 \equiv 1\ (mod11)$. Bu taqqoslamani 11 moduli bo'yicha chegirmalarning keltirgan sistemasi $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ larni taqqoslamaga qo`yib sinab ko'ramiz. U holda

$a \equiv\ 1,3,4,5,9(mod11)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. **Javob:** $x = 1 + 11t, a = 3 + 11t, a = 4 + 11t, a = 5 + 11t, a = 9 + 11t, t \in Z.$

4). Berilgan taqqoslama$x^2 \equiv a(mod13)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechimga, bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (mo\square p)$ shartni qanoatlantirishi kerak. Bundan $a^6 \equiv 1\ (mod13)$. Bu taqqoslamani 13 moduli bo'yicha chegirmalarning keltilgan sistemasi $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni taqqoslamaga qo`yib sinab ko'ramiz. U holda $a \equiv\ \pm1, \pm3, \pm4\ (mod13)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. **Javob:** $a = 1 + 13t, a = 3 + 13t, a = 4 + 13t, a = 9 + 13t, a = 10 + 13t, a = 10 + 13t, t \in Z.$

5). Berilgan taqqoslama$x^2 \equiv a(mod3)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechimga ega bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a \equiv 1\ (mod3)$. **Javob:** $a = 1 + 3t, t \in Z.$

300. $x^2 + 1 \equiv 0(modp)$ taqqoslama yechimga ega bo'lishi uchun $(-1)^{\frac{p-1}{2}} \equiv 1\ (modp)$ shart bajarilishi kerak. Agar $p = 4n + 1$ ko'rinishidagi tub son bo'lsa, u holda $(-1)^{2n} \equiv 1\ (modp)$ bajariladi va taqqoslama ikkita yechimga ega.

Endi agar berilgan taqqoslama $x^2 + 1 \equiv 0(modp)$ yechimga ega bo'lsa, $p = 4n + 1$ ko'rinishidagi tub son bo'lishini ko'rsatamiz. Butun sonlarni 4ga bo'lgandagi qoldiqlar bo'yicha yozsak: $4n, 4n + 1, 4n + 2, 4n + 3$ ko'rinishlarda bo'ladi. $p -$ tub son bo'lsa, $p = 4n + 1$ yoki $p = 4n + 3$ ko'rinishda bo'lishi mumkin. Agar $p = 4n + 3$ ko'rinishda bo'lsa, Eyler kriteriyasiga ko'ra $(-1)^{\frac{p-1}{2}} \equiv 1\ (modp) \to (-1)^{2n+1} \equiv 1\ (modp)$ bo'lishi kerak, lekin bu taqqoslama o'rinli emas. Shuning uchun ham $p = 4n + 1.$

301. $a^2 + b^2 \equiv 0(modp)$ bo'lsa, $(a, b) = 1$ bo'lgani uchun $a \not\equiv 0(modp)$ va $b \not\equiv 0(modp)$ bo'lishi kerak. Faraz etaylik, x soni $bx \equiv 1(modp)$ taqqoslamaning yechimi bo'lsin. U holda $(bx)^2 \equiv 1(modp)$ va $(ax)^2 + (bx)^2 \equiv (ax)^2 + 1 \equiv 0(modp)$ bo'lishi kerak. 300-misolga asosan $(ax)^2 + 1 \equiv 0(modp)$ taqqoslama faqat va faqat $p = 4n + 1$ ko'rinishidagi tub son bo'lsagina o'rinli.

302. $x(x + 1) \equiv 1(mod13)$ desak, $x^2 + x - 1 \equiv 0(mod13) \to \left(x + \frac{1}{2}\right)^2 - \frac{5}{4} \equiv 0(mod13) \to (2x + 1)^2 \equiv 5(mod13)$. Bu taqqoslama yechimga ega emas. Chunki, Lejandr simvolining qiymati $\left(\frac{5}{13}\right) = (-1)^{\frac{13-1}{2} \cdot \frac{5-1}{2}} \cdot \left(\frac{13}{5}\right) = \left(\frac{5 \cdot 2+3}{5}\right) = \left(\frac{3}{5}\right) = (-1)^{\frac{3-1}{2} \cdot \frac{5-1}{2}} \cdot \left(\frac{5}{3}\right) = \left(\frac{3 \cdot 1+2}{3}\right) = \left(\frac{2}{3}\right) = (-1)^{\frac{9-1}{8}} = -1$ ga teng.

303. 302-misolga asosan berilgan $x(x + 1) \equiv a(mod13)$ taqqoslamani $(2x + 1)^2 \equiv 4a + 1(mod13)$ ko'rinishida yozish mumkin. Ma'lumki, $p > 2 -$ moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2, \ldots, \pm \frac{p-1}{2}$ dagi chegirmalarning yarmi kvadratik chegirma, qolgan yarmi esa kvadratik chegirma emas bo'ladi. Kvadratk chegirmalar sifatida $0, 1, 2, \ldots, \frac{p-1}{2}$ larning kvadratlarini olish mumkin. Shuning uchun ham $4a + 1 \equiv 0, 1, 4, 9, 3, 12, 10 (mod 13) \to 4a \equiv -1, 0, 3, 8, 2, 11, 9 (mod 13) \to a \equiv 3, 0, 4, 2, 7, 6, 12 (mod 13)$. Shunday qilib, $a = 13t$, $a = 2 + 13t, 3 + 13t, 4 + 13t, 6 + 13t, 7 + 13t, 12 + 13t, t \in Z$.

Javob: $a = 13t, a = 2 + 13t, 3 + 13t, 4 + 13t, 6 + 13t, 7 + 13t, 12 + 13t, t \in Z$.

304. Faraz qilaylik, bunday tub sonlarning soni chekli bo'lib, ular $p_1, p_2, \ldots, p_k$ lardan iborat bo'lsin. $N = (p_1 p_2 \cdots p_k)^2 + 1$ sonini qaraymiz. Bu son 300-misolga asosan faqat $4n + 1$ ko'rinishidagi tub sonlarga bo'linadi. Lekin N soni $p_1, p_2, \ldots, p_k$ ning birortasiga ham bo'linmaydi. Shuning uchun ham N ning o'zi tub son yoki u biror p_{k+1} tub bo'luvchiga ega. Demak, $4n + 1$ ko'rinishidagi tub sonlar soni cheksiz ko'p.

305. 1).$4x^2 - 5y = 6 \to 4x^2 = 6 + 5y \to 4x^2 \equiv 6 (mod 5) \to 2x^2 \equiv 3 (mod 5) \to 2x^2 \equiv 8 (mod 5) \to x^2 \equiv 4 (mod 5) \to x \equiv \pm 2 (mod 5) \to$ $x = \pm 2 + 5t, t \in Z. x$ ning topilgan qiymatini berilgan tenglamaga qo'yib, y ning qiymatini aniqlaymiz: $4(\pm 2 + 5t)^2 - 5y = 6 \to 4(4 \pm 20t + 25t^2) - 5y = 6 \to 5y = 10 \pm 80t + 100t^2 \to y = 2 \pm 16t + 20t^2$. Shunday qilib, izlanayotgan yechim $(\pm 2 + 5t, 2 \pm 16t + 20t^2), t \in Z$.

Javob:$(\pm 2 + 5t, 2 \pm 16t + 20t^2), t \in Z$.

2). $5x^2 = 11y + 7 \to 5x^2 \equiv 7 (mod 11) \to x^2 \equiv 8 (mod 11)$. Oxirgi taqqoslamada $\left(\frac{8}{11}\right) = \left(\frac{4 \cdot 2}{11}\right) = \left(\frac{2}{11}\right) = (-1)^{\frac{121-1}{8}} = -1$ bo'lgani uchun u yechimga ega emas. Demak, berilgan egri chiziq butun koordinatali nuqtadan o'tmaydi. **Javob:** $\emptyset$.

3). $x^2 - 10x + 5 = 11y \to (x - 5)^2 - 20 = 11y \to (x - 5)^2 \equiv 20 (mod 11) \to$ $(x - 5)^2 \equiv 9 (mod 11) \to x - 5 \equiv \pm 3 (mod 11) \to x \equiv 5 \pm 3 (mod 11) \to x \equiv 2 (mod 11)$ va $x \equiv 8 (mod 11) \to x = 2 + 11t$ va $x = 8 + 11t, t \in Z. x$ ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x = 2 + 11t$ ga mos y ning qiymatini aniqlaymiz Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz:

$$(2 + 11t)^2 - 10(2 + 11t) + 5 = 11y \to 11y = 121t^2 - 66t - 11 \to$$
$$y = 11t^2 - 6t - 1.$$

Endi $x = 8 + 11t$ ga mos y ning qiymatini aniqlaymiz:

$$(8 + 11t)^2 - 10(8 + 11t) + 5 = 11y \to 11y = 121t^2 + 176t + 64 - 80 -$$

$-110t+5 \to 11y=121t^2+66t-11 \to y=11t^2+6t-1.$ Demak, yechimlar $(2+11t, 11t^2-6t-1)$ va $(8+11t, 11t^2+6t-1), t \in Z$. **Javob:** $(2+11t, 11t^2-6t-1)$ va $(8+11t, 11t^2+6t-1), t \in Z$.

4).$x^2-21x+110=13y \to x^2-21x+110 \equiv 0(mod13) \to x^2-8x+6 \equiv 0(mod13) \to (x-4)^2 \equiv 10(mod13) \to (x-4)^2 \equiv 36(mod13) \to x-4 \equiv \pm 6(mod13) \to x \equiv 4 \pm 6(mod13) \to x \equiv -2(mod13)$va $x \equiv 10(mod13) \ \to x=-2+13t$ va $x=10+13t, t \in Z.$ x ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x=-2+13t$ ga mos y ning qiymatini aniqlaymiz Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz: $(-2+13t)^2-21(-2+13t)+110=13y \to 4-52t+169t^2+42-273t+110=13y \to 169t^2-325t+156=13y \to y=13t^2-25t+12, t \in Z.$ Endi $x=10+13t$ ga mos y ning qiymatini aniqlaymiz: $(10+13t)^2-21(10+13t)+110=13y \to 13y=169t^2+260t+100-210-273t+110 \to 13y=169t^2-13t \to y=13t^2-t, t \in Z.$ Demak, yechimlar $(-2+13t, 13t^2-25t+12)$ va $(10+13t, 13t^2-t), t \in Z$.

Javob: $((-2+13t, 13t^2-25t+12)$ va $(10+13t, 13t^2-t), t \in Z$.

5).$15x^2-7y^2=9 \to 15x^2 \equiv 9(mod7) \to x^2 \equiv 9(mod7) \to x=\pm 3+7t.$ x ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x=-3+7t$ ga mos y ning qiymatini aniqlaymiz. Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz: $15(-3+7t)^2-7y^2=9 \to 15(9-42t+49t^2)-7y^2=9 \to 135-630t+735t^2-7y^2=9 \to 126-630t+735t^2=7y^2 \ \to y^2=105t^2-90t+18.$ Bunda oxirgi ifodaning o'ng tomonidagi uchhadning diskriminanti 540 ga teng va shuning uchun ham u to'liq kvadratni bermaydi, ya'ni y ning butun qiymatlari mavjud emas. Endi $x=3+7t$ ga mos y ning qiymatini aniqlaymiz: : $15(3+7t)^2-7y^2=9 \to 15(9+42t+49t^2)-7y^2=9 \to 135+630t+735t^2-7y^2=9 \to 126+630t+735t^2=7y^2 \ \to y^2=105t^2+90t+18.$ Bunda ham oxirgi ifodaning o'ng tomonidagi uchhadning diskriminanti 540 ga teng va shuning uchun ham u to'liq kvadratni bermaydi, ya'ni y ning butun qiymatlari mavjud. Demak, berilgan tenglama butun sonlarda yechimga ega emas. **Javob:** berilgan tenglama yechimga ega emas.

306.1). Lejandr simvolining ta'rifiga asosan$\left(\frac{5}{p}\right)=(-1)^{\frac{p-1}{2} \cdot \frac{5-1}{2}} \cdot \left(\frac{p}{5}\right)=\left(\frac{p}{5}\right)$ bo'lganidan, $a=5$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan $p^{\frac{5-1}{2}} \equiv 1(mod5) \to p^2 \equiv 1(mod5)$ ning bajarilishi zarur va yetarlidir. Bundan $p^2 \equiv 16(mod5) \to p \equiv \pm 4(mod5) \to p \equiv \pm 1(mod5)$ ni hosil qilamiz. Buni $p=\pm 1+5k$ ko'rinishida yozish mumkin.

Umuman, butun sonlarni 5 moduli bo'yicha 5 ta: $5k, 5k+1, 5k+2, 5k+3, 5k+4$ sinfga ajratish mumkin bo'lgani uchun, agar $p=5k+1$ yoki $p=5k+4$

ko'rinishdagi tub son bo'lsa, $a=5$ soni p-tub moduli bo'yicha kvadratik chegirma, agar $p=5k+2$ yoki $p=5k+3$ ko'rinishdagi tub son bo'lsa, $a=5$ soni p-tub moduli bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a=5$ soni $p=5k+1$ va $p=5k+4$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p=5k+2$ va $p=5k+3$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi.

2) Lejandr simvolining ta'rifiga asosan $\left(\frac{-3}{p}\right)=\left(\frac{-1\cdot 3}{p}\right)=\left(\frac{-1}{p}\right)\left(\frac{3}{p}\right)=(-1)^{\frac{p-1}{2}}\cdot(-1)^{\frac{p-1}{2}\cdot\frac{3-1}{2}}\cdot\left(\frac{p}{3}\right)=\left(\frac{p}{3}\right)$ bo'lganidan, $a=-3$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan $p^{\frac{3-1}{2}}\equiv 1(mod3)\rightarrow p\equiv 1(mod3)$ ning bajarilishi zarur va yetarlidir. Buni $p=1+3k$ ko'rinishida yozish mumkin.

Umuman, butun sonlarni 3 moduli bo'yicha 3 ta: $3k, 3k+1, 3k+2$ sinfga ajratish mumkin bo'lgani uchun, agar $p=3k+1$ ko'rinishdagi tub son bo'lsa, $a=-3$ soni p-tub moduli bo'yicha kvadratik chegirma, agar $p=3k+2$ yoki ko'rinishdagi tub son bo'lsa, $a=-3$ soni p-tub moduli bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a=-3$ soni $p=3k+1$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma, $p=3k+2$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma emas bo'ladi.

3).Lejandr simvolining ta'rifiga asosan $\left(\frac{3}{p}\right)=(-1)^{\frac{p-1}{2}\cdot\frac{3-1}{2}}\cdot\left(\frac{p}{3}\right)=(-1)^{\frac{p-1}{2}}\cdot\left(\frac{p}{3}\right)$ bo'lganidan,agar$p=3k+1$ ko'rinishida bo'lsa,

$$\left(\frac{3}{p}\right)=(-1)^{\frac{3k}{2}}\cdot\left(\frac{3k+1}{3}\right)=(-1)^{\frac{3k}{2}}\cdot\left(\frac{1}{3}\right)=(-1)^{\frac{3k}{2}} \qquad (*)$$

bo'ladi. Agar bunda $k=4q$ bo'lsa, (*) dan $\left(\frac{3}{p}\right)=1$ hosil bo'ladi, ya'ni 3 soni $p=12q+1$ ko'rinishdagi tub moduli bo'yicha kvadratik chegirma bo'ladi. 12 moduli bo'yicha barcha butun sonlarni 12 ta sinfga ajratish mumkin. Bulardan $12q+1, 12q+5, 12\square+7, 12q+11$ sinflardagina tub sonlar bo'ladi. Agar $p=12q+5$ bo'lsa, u holda

$\left(\frac{3}{p}\right)=(-1)^{\frac{12q+4}{2}}\cdot\left(\frac{12q+5}{3}\right)=\left(\frac{3\cdot(4q+1)+2}{3}\right)=\left(\frac{2}{3}\right)=(-1)^{\frac{9-1}{8}}=-1;$ agarda $p=12q+7$ bo'lsa,

$$\left(\frac{3}{p}\right)=(-1)^{\frac{12q+6}{2}}\cdot\left(\frac{12q+7}{3}\right)=-\left(\frac{3\cdot(4q+2)+1}{3}\right)=-\left(\frac{1}{3}\right)=-1;$$

agarda $p=12q+11$ bo'lsa,

$$\left(\frac{3}{p}\right)=(-1)^{\frac{12q+10}{2}}\cdot\left(\frac{12q+11}{3}\right)=-\left(\frac{3\cdot(4q+3)+2}{3}\right)=-\left(\frac{2}{3}\right)=1$$

larni hosil qilamiz. Shunday qilib, 3 soni $p=12q+1, p=12q+11$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p=12q+5, p=12q+7$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: 3 soni $p=12q+1, p=12q+11$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p=12q+5, p=12q+7$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi.

4). $a=2$ ning p moduli bo'yicha kvadratik chegirma bo'lishi uchun Lejandr simvolining ta'rifiga asosan $\left(\frac{2}{p}\right)=(-1)^{\frac{p^2-1}{8}}=1$ bajarilishi kerak. Buning uchun esa $\frac{p^2-1}{8}=2q \to p^2=1+16q \to p^2 \equiv 1(mod16)$ ko'rinishida bo'lishi kerak. Oxirgi taqqoslamani $p=16k+1, p=16k+3, p=16k+5, p=16k+7,\ p=16k+9, p=16k+11, p=16k+13, p=16k+15$ lardan foydalanib tekshirsak, $p=16k+1, p=16k+7,\ p=16k+9,\ p=16k+15$ uni qanoatlantiradi. Qolganlari qanoatlantirmaydi. Shuning uchun ham 2 soni $p=16k+1, p=16k+7,\ p=16k+9,\ p=16k+15$ modullar bo'yicha kvadratik chegirma, $p=16k+3, p=16k+5,\ p=16k+11, p=16k+13$ modullar bo'yicha kvadratik chegirma emas bo'ladi. Bularni 8 moduli birlashtirib, yozib olishimiz mumkin. U holda 2 soni $p=8k+1, p=8k+7$modullar bo'yicha kvadratik chegirma, $p=8k+3, p=8k+5$,modullar bo'yicha kvadratik chegirma emas bo'ladi.

Javob: 2 soni $p=8k+1, p=8k+7$ modullar bo'yicha kvadratik chegirma, $p=8k+3, p=8k+5,$ modullar bo'yicha kvadratik chegirma emas bo'ladi.

5). Lejandr simvolining ta'rifiga asosan$\left(\frac{-7}{p}\right)=\left(\frac{-1\cdot 7}{p}\right)=\left(\frac{-1}{p}\right)\left(\frac{7}{p}\right)=(-1)^{\frac{p-1}{2}}\cdot(-1)^{\frac{p-1}{2}\cdot\frac{7-1}{2}}\cdot\left(\frac{p}{7}\right)=\left(\frac{p}{7}\right)$ bo'lganidan, $a=-7$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan $p^{\frac{7-1}{2}} \equiv 1(mod7) \to p^3 \equiv 1(mod7)$ ning bajarilishi zarur va yetarlidir. Buni $p=1+7k,\ p=2+7k, p=3+7k, p=4+7k, p=5+7k, p=6+7k$ larni qo'yib tekshirsak, $p=1+7k,\ p=2+7k, p=4+7k$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi. Demak, $a=-7$ soni $p=1+7k,\ p=2+7k, p=4+7k$ modullar bo'yicha kvadratik chegirma, $p=3+7k,\ p=5+7k, p=6+7k$ modullari bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a=-7$ soni $p=1+7k,\ p=2+7k, p=4+7k$ modullar bo'yicha kvadratik chegirma, $p=3+7k,\ p=5+7k, p=6+7k$ modullari bo'yicha kvadratik chegirma emas bo'ladi.

307.1). Berilgan taqqoslamadan $x(x+1) \equiv 1(modp) \to x^2+x-1 \equiv 0(modp) \to \left(x+\frac{1}{2}\right)^2-\frac{1}{4}-1 \equiv 0(modp) \to (2x+1)^2 \equiv 5(modp).$ Bu taqqoslama yechimga ega bo'lishi uchun $\left(\frac{5}{p}\right)=(-1)^{\frac{p-1}{2}\cdot\frac{5-1}{2}}\cdot\left(\frac{p}{5}\right)=\left(\frac{p}{5}\right)=1$ bo'lishi kerak. $p=1+5k,\ p=2+5k, p=3+5k, p=4+5k$ larni qo'yib tekshirsak, $p=1+5k,\ \ p=4+5k$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi.

Javob: $p=1+5k,\ \ p=4+5k$ modullar bo'yicha berilgan taqqoslama yechimga ega, $p=2+5k, p=3+5k$ modullar bo'yicha taqqoslama yechimga ega.

2). Berilgan taqqoslamadan $x(x-1)\equiv 2(modp)\rightarrow x^2-x-2\equiv 0(modp)\rightarrow \left(x-\frac{1}{2}\right)^2-\frac{1}{4}-2\equiv 0(modp)\rightarrow(2x-1)^2\equiv 9(modp)$. Bu yerda $\left(\frac{9}{p}\right)=1$ bo'lgani uchun. Ixtiyoriy $p>2$ tub modul uchun berilgan taqqoslama yechimga ega bo'ladi.

Javob: Ixtiyoriy $p>2$ modul bo'yicha berilgan taqqoslama yechimga ega.

3).Berilgan taqqoslamadan$x(x-1)\equiv 3(modp)\rightarrow x^2-x-3\equiv 0(modp)\rightarrow\left(x-\frac{1}{2}\right)^2-\frac{1}{4}-3\equiv 0(modp)\rightarrow(2x-1)^2\equiv 13(modp)$. Bu taqqoslama yechimga ega bo'lishi uchun $\left(\frac{13}{p}\right)=(-1)^{\frac{p-1}{2}\cdot\frac{13-1}{2}}\cdot\left(\frac{p}{13}\right)=\left(\frac{p}{13}\right)=1$ bo'lishi kerak. $p=1+13k,\ p=3+13k,\ p=4+13k,\ p=9+13k, p=10+13k, p=12+13k$ va $p=13$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi.

Javob:$p=1+13k,\ p=3+13k,\ p=4+13k,\ p=9+13k, p=10+13k,$ $p=12+13k$ va $p=13$ modullar bo'yicha taqqoslama yechimga ega. $p=2+13k,\ p=5+13k,\ p=6+13k,\ p=7+13k, p=8+13k,\ p=11+13k$ modullar bo'yicha berilgan taqqoslama yechimga ega emas.

308.1). Agar $x^2\equiv 13(modp)$ yoki $x^2\equiv 17(modp)$ lardan birortasi o'rinli bo'lsa, berilgan taqqoslama $(x^2-13)(x^2-17)(x^2-221)\equiv 0(modp)$ yechimga ega bo'ladi. Agar ularning ikkalasi ham yechimga ega bo'lmasa, $\left(\frac{13}{p}\right)=\left(\frac{17}{p}\right)=-1$ bajarilishi kerak. Bundan $\left(\frac{13}{p}\right)\cdot\left(\frac{17}{p}\right)=\left(\frac{221}{p}\right)=1$ kelib chiqadi. Bu esa $x^2\equiv 221(modp)$ bajariladi degani. Demak, berilgan taqqoslama ixtiyoriy □ >2 tub modul bo'yicha o'rinli.

2). Agar $x^2\equiv 3(modp)$, yoki $x^2\equiv 5(modp)$,yoki$x^2\equiv 7(modp)$,yoki $x^2\equiv 11(modp)$ lardan birortasi o'rinli bo'lsa, berilgan taqqoslama $(x^2-3)(x^2-5)(x^2-7)(x^2-11)(x^2-1155)\equiv 0(modp)$ yechimga ega bo'ladi. Agar ularning to'rtalasi ham yechimga ega bo'lmasa, $\left(\frac{3}{p}\right)=\left(\frac{5}{p}\right)=\left(\frac{7}{p}\right)=\left(\frac{11}{p}\right)=-1$ bajarilishi kerak. Bundan $\left(\frac{3}{p}\right)\cdot\left(\frac{5}{p}\right)\cdot\left(\frac{7}{p}\right)\cdot\left(\frac{11}{p}\right)=1$ kelib chiqadi. Bu esa $x^2\equiv$

$1155(modp)$ bajariladi degani. Demak, berilgan taqqoslama ixtiyoriy $p > 2$ tub modul bo'yicha o'rinli.