2 -§. Cheksiz uzliksiz kasrlarning yaqinlashuvchanligi.

Munosib kasrlar quyidagi xossalarga ega:

1^0. $\frac{P_k}{Q_k} - \frac{P_{k-1}}{Q_{k-1}} = \frac{(-1)^{k-1}}{Q_k Q_{k-1}}$ yoki $\Delta_k = P_k Q_{k-1} - P_{k-1} Q_k = (-1)^{k-1}$.

Bu yerdan $\frac{P_k}{Q_k}$ qisqarmas kasr degan xulosaga kelamiz, chunki $(P_k, Q_k)=1$.

2^0. Munosib kasrlarning tartibining o'sib borishi bilan ularning juft tartiblilari o'sadi, toq tartiblilari esa kamayadi. Bunda har bir juft tartibli munosib kasr ixtiyoriy toq tartibli munosib kasrdan kichik bo'ladi.

3^0. $\alpha = (q_0, q_1, \dots, q_k, \alpha_{k+1}) = \frac{P_k \alpha_{k+1} + P_{k-1}}{Q_k \alpha_{k+1} + Q_{k-1}}$, $k = 1,2,\dots$ va $\alpha_{k+1} = (q_{k+1}, q_{k+2}, \dots)$.

4^0. $\alpha = (q_0, q_1, \dots, q_k, \dots) -$ irratsional soni uchun

$$\frac{P_0}{Q_0} < \frac{P_2}{Q_2} < \frac{P_4}{Q_4} < \dots < \alpha < \dots < \frac{P_5}{Q_5} < \frac{P_3}{Q_3} < \frac{P_1}{Q_1}$$

va

$$\alpha = \lim_{k\to\infty} \frac{P_k}{Q_k}$$

munosabatlar o'rinli. $\frac{P_k}{Q_k} -$ munosib kasr $\alpha -$ haqiqiy soni uchun eng yaxshi ratsional yaqinlashish bo'ladi, ya'ni maxraji $y \le Q_k$ shartni qanoatlantiruvchi birorta ham $\frac{x}{y}$ ratsional kasr $\alpha -$ haqiqiy soniga $\frac{P_k}{Q_k} -$ munosib kasrga qaragan yaqin bo'la olmaydi. $\frac{P_k}{Q_k} -$kasr $\alpha -$ haqiqiy soniga $\frac{1}{Q_k Q_{k+1}}$ aniqlik bilan yaqinlashadi. $\alpha -$ haqiqiy soniga berilgan ε aniqlik bilan yaqinlashadigan munosib kasrni aniqlash uchun $Q_k > \sqrt{\frac{1}{\varepsilon}}$ bajariladigan qilib olish kerak bo’ladi. Shuni ham ta’kidlash kerakki, bunday aniqlikni kichikiroq tartibli munosib kasrlar ham ta’minlashi mumkin.

368. Quyidagi sonlarni 4-tartibli munosib kasrlar bilan almashtiring va buning natijasida hosil bo’ladigan xatolikni baholang:

$$1).\ \frac{587}{103},\quad 2).\ 3{,}14159,\quad 3).\frac{-1+\sqrt{5}}{2},\quad 4).\ \frac{2-\sqrt{3}}{5},$$
$$5).\frac{1+\sqrt{5}}{2},\quad 6)\ \frac{-1+\sqrt{2}}{2}.$$

369. $\frac{1261}{881}-$ sonini imkoni boricha kichik maxrajli munosib kasr bilan almashtiringki, bunda xatolik 0,0001 dan katta bo’lmasin.

370. Berilgan sonlarga 0,001 gacha aniqlikdagi eng yaxshi yaqinlashishni toping: $1).\sqrt{2},\quad 2).\sqrt{3},\quad 3).\sqrt{7},\quad 4).\sqrt{11}.$

371. Berilgan tenglamalarning ildizlariga 0,0001 gacha aniqlikdagi eng yaxshi yaqinlashishni toping:

$$1).\ x^2-5x+2=0,\qquad 2).4x^2+20x+23=0,$$
$$3).\ x^2+9x+6=0,\qquad 4).\ 2x^2-3x-6=0.$$

372. Avvalo $\frac{P_n}{Q_n}$ va $\frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}$ larning ikkalasi ham α ning bir tomonida yotishiga ishonch hosil qiling va $\left|\alpha-\frac{P_n}{Q_n}\right| > \frac{1}{Q_n(Q_n+Q_{n+1})}$ tengsizlikning o’rinli ekanligini isbotlang.

373. Agar $q_n -$ chala bo’linma bir necha birlikga ortsa n-tartibli munosib kasr ortadimi yoki kamayadimi?

374. $n \geq 1$ bo’lsa, quyidagi tengsizliklardan hech bo’lmasa birtasining o’rinli ekanligini isbotlang: $\left|\alpha-\frac{P_n}{Q_n}\right| < \frac{1}{2Q_n^2}$ yoki $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right| < \frac{1}{2Q_{n-1}^2}$.