5-§. Murakkab modul bo'yicha yuqori darajali taqqoslamalar.

Murakkab modulli taqqoslamani yechishni tub modul bo'yicha taqqoslamani yechishga keltirish mumkin. Bunda ushbu teoremadan foydalaniladi:

Teorema. Agar $f(x)\equiv 0(\mathrm{mod}\, M)$ (1)

taqqoslamaning moduli M juft-jufti bilan o'zaro tub bo'lgan ko'paytuvchilarga ajratilgan $M=m_1 m_2\ldots m_k,\quad (m_i,m_j)=1$ bo'lsa, u holda:

1). (1) taqqoslama ushbu taqqoslamalar sistemasi

$$f(x)\equiv 0(\mathrm{mod}\, m_1),\ f(x)\equiv 0(\mathrm{mod}\, m_2)\ ,\cdots,\ f(x)\equiv 0(\mathrm{mod}\, m_k) \qquad (2)$$

ga teng kuchlidir.

2). Agarda (1) taqqoslama N ta yechimga ega bo'lib, (2) ning birinchisi n_1, ikkinchisi n_2 va h.k. Oxirgisi $n_к$ ta yechimga ega bo'lsa, u holda

$N=n_1\cdot n_2\cdot\cdots\cdot n_k$ bo'ladi.

Yuqoridagi teoremaga asosan murakkab modul boyicha taqqoslamani hamma vaqt

$$f(x)\equiv 0(\mathrm{mod}\, p^{\alpha}), \qquad p-\text{туб сон},\ \alpha\geq 1 \qquad (1')$$

ko'rinishdagi taqqoslamani yechishga keltirish mumkin. Bu taqqoslamani tanlash usuli bilan yechish p^{α} katta son bo'lganda ancha noqulay (1) ni yechishni

$$f(x)\equiv 0(\mathrm{mod}\, p) \qquad (3)$$

ni yechishga keltirish mumkin. Ma'lumki (1') ni qanoatlantiruvchi har bir x_1 soni (3) ni ham qanoatlantiradi. Shuning uchun ham (1') ning yechimlarini (3) ning yechimlari orasidan qidirish kerak. Buni ketma-ket (3) dan p bo'yicha, keyin p^2 va h.k. taqqoslamalarga o'tib bajarish mumkin.

Faraz etaylik, (3) ning birorta yechimi topilgan bo'lsin:

$$x\equiv x_1(\mathrm{mod}\, p)\quad ya'ni \qquad x=x_1+pt_1, \qquad t\in Z \qquad (4)$$

(4) dan $f(x)\equiv 0(\mathrm{mod}\, p^2)$ (5)

taqqoslamani qanoatlantiruvchilarini ajratib olamiz.

$$f(x_1+pt_1)\equiv 0(\mathrm{mod}\, p^2).$$

Bu taqqoslamaning chap tomonini hisoblash uchun $f(x_1+pt_1)$ ning Teylor qator yoyilmasidan foydalanish qulay:

$$f(x_1)+pt_1\cdot f'(x_1)+\frac{(pt_1)^2}{2!}f''(x_1)+\ldots+\frac{(pt_1)^k}{k!}f^{(k)}(x_1),$$

bu yerdagi har bir qo'shiluvchi butun son. Bundan foydalanib, oxirgi taqqoslamani quyidagicha yozish mumkin:

$$f(x_1)+pt_1\cdot f'(x_1)\equiv 0(\mathrm{mod}\, p^2) \qquad (6)$$

Bu yerda $p \setminus f(x_1)$ bo'lgan uchun

$$\frac{f(x_1)}{p} + t_1 f'(x_1) \equiv 0(\text{mod}\, p)$$

Yoki
$$t_1 f'(x_1) \equiv -\frac{f(x_1)}{p}(\text{mod}\, p). \qquad (7)$$

Bu yerda quyidagi uchta hol bo'lishi mumkin:

A.$p \nmid f'(x_1)$ bo'lsa, (7) dan $t_1 \equiv t'\left(\text{mod}\, p\right)$, ya'ni $t_1 = t' + pt_2, \quad t_2 \in Z.$

Buni (4) ga qo'ysak, $x = x_1 + p(t' + pt_2) = x_1 + pt' + p^2 t_2$ hosil bo'ladi. $t_2 = 0$ äà $x = x_1 + pt'$. Bundan (5) ning bitta $x_2 = x_1 + pt'$ yechimi hosil bo'ladi. Demak $x = x_2 + p^2 t_2$. Buni

$$f(x) \equiv 0(\text{mod}\, p^3) \qquad (8)$$

taqqoslamaga olib borib qo'yib, yuqoridagi singari mulohaza yuritib, t_2 ni topamiz.

$f\left(x_2 + p^2 t_2\right) \equiv 0(\text{mod}\, p^3)$ yoki $f\left(x_2\right) + p^2 t_2 f'\left(x_2\right) \equiv 0(\text{mod}\, p^3)$, bu yerda $p^2 \mid f(x_2)$

bo'lgani uchun

$$\frac{f\left(x_2\right)}{p^2} + t_2 f'\left(x_2\right) \equiv 0(\text{mod}\, p). \qquad (9)$$

$x_2 \equiv x_1\ (modp)$ bo'lgani uchun $f'(x_1) \equiv f'(x_2)(modp)$. Bunda shart bo'yicha $f'(x_1) \not\equiv 0(modp)$ va demak, $f'(x_2) \not\equiv 0(modp)$.

Demak, (9) yagona yechimga ega. $t_2 \equiv t_2'\left(\text{mod}\, p\right)$, ya'ni $t_2 = t_2' + pt_3, \quad t_3 \in Z$. U holda $x = x_2 + p^2(t_2' + pt_3) = x_2 + p^2 t_2' + p^3 t_3$ yoki

$x = x_3 + p^3 t_3$, ya'ni $x \equiv x_3(\text{mod}\, p^3)$.

Shu jarayonni takrorlab $x \equiv x_\alpha(modp^\alpha)$ ni hosil qilamiz.

Shunday qilib, $p \nmid f'(x_1)$ holda (3) ning har bir yechimi (1') ning birta yechimiga olib keladi.

Б. Agarda $p \mid f'(x_1)$ bo'lib, (7) ning o'ng tomoni esa p ga bo'linmasa (7) va demak (5) va (1') ham yechimga ega emas.

B. Agarda $p \mid f'(x_1)$ bo'lib, (7) ning o'ng tomoni ham p ga bo'linsa, (7) ayniy taqqoslamaga aylanadi, uni (4) dagi ixtiyoriy butun son t_1 qanoatlantiradi. Lekin bu yechimlar p^2 moduli bo'yicha p ta sinfga tegishli bo'ladi, ya'ni (5) taqqoslama p ta yechimga ega bo'ladi. Keyin bu yechimlardan umumiy usul bilan p^3 moduli bo'yicha taqqoslamani qanoatlantiruvchilarini ajratib olamiz va h..k.

289. Quyidagi taqqoslamalarni yeching:

1) $3x^3 + 4x^2 - 7x - 6 \equiv 0(mod15)$;

2) $6x^3 - 3x^2 - 13x - 10 \equiv 0(mod30)$;
3) $x^4 - 33x^3 + 8x - 26 \equiv 0(mod35)$;
4) $x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod42)$;
5) $x^5 + x^4 - 3x^3 + x^2 + 2x - 2 \equiv 0(mod77)$;
6) $3x^3 + 6x^2 + x + 10 \equiv 0 \ (\mathrm{mod}15)$
7) $37x \equiv 17\left(\mathrm{mod}180\right)$

290. Taqqoslamalarni yeching:
1) $4x^3 - 8x - 13 \equiv 0(mod27)$;
2) $x^4 - 3x^3 + 2x^2 - 5x - 10 \equiv 0(mod343)$;
3) $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod25)$;
4) $9x^2 + 29x + 62 \equiv 0(mod64)$;
5) $6x^3 - 7x - 11 \equiv 0(mod125)$;
6) $x^3 + 3x^2 - 5x + 16 \equiv 0(mod125)$;
7) $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod625)$;
8). $2x^4 + 5x - 1 \equiv 0(\mathrm{mod}\,27)$.

291. Taqqoslamalarni yeching:
1)$x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod45)$;
2) $x^4 - 3x^3 - 4x^2 - 2x - 2 \equiv 0(mod50)$;
3) $x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 \equiv 0(mod147)$;
4) $x^5 + 3x^4 - 7x^3 + 4x^2 + 4x - 10 \equiv 0(mod175)$;
5) $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod135)$;
6) $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod225)$;
7) $31x^4 + 57x^3 + 96x + 191 \equiv 0(mod225)$;
8) $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod441)$;
9) $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod1225)$.