3-§. Berilgan sonning bo'luvchilari soni va bo'luvchilari yig'indisini ifodalovchi funksiyalar.

$\tau(n)$ va $\sigma(n)$ funksiyalari n ning barcha natural qiymatlarida aniqlangan bo`lib, mos ravishda $n$ ning barcha natural bo`luvchilari sonini va barcha natural bo`luvchilari yig`indisini ifodalaydi.Ta`rifdan $\tau(1) = \sigma(1) = 1$ ekanligi kelib chiqadi. Agar $n$ ning kanonik yoyilmasi $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$ bo`lsa, $\tau(n)$ va $\sigma(n)$ lar mos ravishda quydagi formulalar yordamida topiladi:

$$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1), \quad (1)$$

$$\sigma(n) = \frac{p_1^{\alpha_1+1}-1}{p_1-1} \cdot \frac{p_2^{\alpha_2+1}-1}{p_2-1} \cdot \dots \cdot \frac{p_k^{\alpha_k+1}-1}{p_k-1}. \quad (2)$$

Ikkala funksiya ham multiplikativ funksiya ya`ni $(m, n) = 1$ lar uchun

$$\tau(m \cdot n) = \tau(m) \cdot \tau(n), \sigma(m \cdot n) = \sigma(m) \cdot \sigma(n)$$

tengliklar o`rinli.

108. Quyidagi sonlarning barcha natural bo`luvchilari soni va bo`luvchilari yig`indisini toping: 1) 375; 2) 720; 3) 957; 4) 988;
5) 990; 6) 1200; 7) 1440; 8) 1500; 9) 1890; 10) 4320.

109. Berilgan sonlarning barcha natural bo`luvchilarini toping:
1) 360; 2) 720; 3) 954; 4) 988; 5) 600.

110. Noma'lum natural son x faqat ikkita tub bo`luvchiga ega ekanligi va uning bo`luvchilari soni 6 ga, bo`luvchilarining yig`indisi 28 ga teng bo`lsa, shu sonni toping.

111. $N = p^{\alpha} \cdot p^{\beta}$(p, q lar turli tub sonlar) bo`lsin. Agar $N^2$ soni 15 ta har xil bo`luvchilarga ega bo`lsa, $N^3$nechta natural bo`luvchilarga ega bo`ladi.

112. $\tau(x)$ va $\sigma(x)$ larning grafigini sxematik tasvirlang.

113. Har bir egizak tub sonlar juftligi $p_1 < p_2$ uchun $\sigma(p_1) = \varphi(p_2)$ ekanligini isbotlang. Bunda $\varphi(a)$ −Eyler funksiyasi.

114. $\sigma(m) = 2m - 1$ tenglamaning m natural sonlarda cheksiz ko`p yechimga ega ekanligini isbotlang.

115.1). Agar $(m, n) = d > 1$ bo`lsa, $\tau(m\,n)$ va $\tau(m)\tau(n)$ larda qaysi katta?
2). Agar $(m, n) = d > 1$ bo`lsa, $\sigma(m\,n)$ va $\sigma(m)\sigma(n)$ lardan qaysi katta?

116. m natural sonining barcha natural bo`luvchilarining ko'paytmasi $\delta(m)$uchun formula chiqaring va $\delta(10)$ ni toping.

117. O`zining natural bo`luvchilarining ko`paytmasiga teng bo`lgan barcha natural sonlar to`plami barcha tub sonlar to`plami bilan ustma-ust tushishini isbotlang.

118. $n = p_1^{\alpha_1}\, p_2^{\alpha_2} \ldots\, p_k^{k}$ sonining bo`luvchilarining $k$- darajalarining yig`indisi $\sigma_k(n)$ uchun formula chiqaring.

119. $\sigma_k(n)$uchun (118-misoldagi) formuladan foydalanib hisoblang:
1) $\sigma_2(12)$; 2) $\sigma_2(18)$, 3) $\sigma_3(36)$, 4) $\sigma_2(16)$, 5) $\sigma_3(8)$.

120. $\sigma(n) = 2n$ tenglikni qanoatlantiruvchi n natural sonlarga mukammal sonlar deyiladi. $28,\ 496,\ 8128$ sonlarining mukammal sonlar ekanligini tekshiring.

121. $\sigma(n) < 2n$ shartni qanoatlantiruvchi n soniga yetarli sondagi bo`luvchilarga ega emas, $\sigma(n) > 2n$ shartni qanoatlantiruvchilarga esa ortiqcha bo`luvchilarga ega bo`lgan son deyiladi. $N = p^n$ sonining yetarli bo`luvchilarga ega emasligini isbotlang. Bunda p tup son, n −natural son.

122. $N = p^{\alpha} \cdot q^{\beta}$ ko`rinishdagi toq natural sonning yetarli bo`luvchilariga ega emasligini isbotlang. Bunda p, q lar turli tub sonlar, α, β lar natural sonlar.

123. 1). Barcha bo`luvchilarining ko`paytmasi 5832 ga teng bo`lgan n natural sonini toping.

2). Barcha bo`luvchilarining ko`paytmasi $3^{30} \cdot 5^{40}$ ga teng bo`lgan n natural sonini toping.

124. $N = p_1^{\alpha_1}\, p_2^{\alpha_2} \ldots\, p_k^{\alpha_k}$ $-$ko`rinishdagi kanonik yoyilmaga ega bo`lgan sonni necha xilda 2 ta har xil ko`paytuvchiga ajratish mumkun.

125. Agar $5N$ soni N soniga qaraganda 8 ta ko`p, $7N$ soni $N$ soniga qaraganda 12 ta ko`p, $8N$ soni ega N soniga qaraganda 18 ta ko`p bo`luvchiga ega bo`lsa, $N = 2^{\alpha} \cdot 5^{\beta} \cdot 7^{\gamma}$ sonini toping.

126. N soni $N = 2^x \cdot 3^y \cdot 5^z$ ko`rinishiga ega. Agar $N$ ni 2 ga bo`lsak, hosil bo`lgan sonning bo`luvchilari soni N ning bo`luvchilari sonidan $30$ taga kam. Agar $N$ ni 3 ga bo`lsak, hosil bo`lgan sonning bo`lnuvchilari soni N ning bo`luvchilari sonidan $35$ taga kam. Agarda $N$ sonini $5$ ga bo`lsak, hosil bo`lgan sonning bo`linuvchilari soni N ning bo`linuvchilari sonidan $42$ ta kam bo`ladi. Shu N sonini toping.

127. Agar $2^{\alpha+1} - 1$ soni tub son bo`lsa, u holda $2^{\alpha}(2^{\alpha+1} - 1)$ sonining mukammal son ekanligini isbotlang (Evklid teoremasi).

128. Agar $2^{\alpha+1} - 1$ tub son bo`lsa, $2^{\alpha}(2^{\alpha+1} - 1)$ ning yagona juft mukammal son ekanligini isbotlang (Eyler teoremasi).

129. Bo`luvchilar yig`indisi o`zidan 3 marta katta bo`lgan $2^{\alpha} \cdot p_1 \cdot p_2$, $(p_1, p_2$ lar toq tub sonlar) ko`rinishidagi eng kichik sonni toping. (Ferma masalasi).

130. Berilgan natural sonning aniq kvadrat bo`lishi uchun, uning har xil natural bo`luvchilari sonining toq bo`lishi zarur va yetarli ekanligini isbotlang.