Chapter 2:

1. Conditional Prob:

The conditional probability of B, given A, denoted by $P(B|A)$ is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad \text{provided} \quad P(A) > 0.$$

Conditions:

1. __A___ if,____B___ P(A|B)

2. ___A__ given that B P(A|B)

3. If _A__, what is prob __B__ P(B|A)

4. Given that ___A___, what is prob _B_ P(B|A)

2. Bayes Law:

If mentioned that find the whole prob of event then use this:

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i).$$

If its mentioned that find prob. from specific location then use this:

(Bayes' Rule) If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for } r = 1, 2, \ldots, k.$$

Chapter 3:

1. Discrete Prob Distribution:

The set of ordered pairs $(x, f(x))$ is a **probability function, probability mass function, or probability distribution** of the discrete random variable X if, for each possible outcome x,

1. $f(x) \geq 0$,
2. $\sum_x f(x) = 1$,
3. $P(X = x) = f(x)$.

Steps:

1. Draw the table of x,F(x)

2. If you have to find value of any variable then use Condition-2 according to prob function.

2. Cummulative Distribution Function for Discrete:

The **cumulative distribution function** $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -00 < x < 00.$$

e.g, F(x)= f(0) ____Condition___

f(1) ______Condition_

f(2)=f(0)+f(1) _____Condition__

f(3)= f(0)+f(1)+f(2) ___Condition____

3. Continuous Prob Density Function:

2. $\int_{-\infty}^{\infty} f(x)\, dx = 1.$

3. $P(a < X < b) = \int_a^b f(x)\, dx.$

4. Cummulative Distribution Function for Continuous:

The **cumulative distribution function** $F(x)$ **of a** continuous random variable **A'** with density function $f(x)$ is

$$F(x) = P(X \leq a) = \int_{-\infty}^{x} f(t)\, dt \quad \text{for } -00 < x < \infty.$$

5. **Joint Prob Distribution of Discrete:**

The function $f(x,y)$ is a **joint probability distribution** or **probability mass function** of the discrete random variables X and Y if

1. $f(x,y) \geq 0$ for all (x,y),
2. $\sum_x \sum_y (x,y) = 1,$
3. $P(X = x, Y = y) = f(x,y).$

For any region A in the xy plane, $P[(X, Y) \in A] = \sum\sum_A f(x,y).$

Steps:
1. Draw Table of x,y and put prob of their corresponding values.

2. Write g(x) and h(y) according to prob.

These g(x) and h(y) are called marginal distribution of X,Y.

6. Joint Density Function of Continuous:

The function $f(x,y)$ is a **joint density function** of the continuous random variables X and Y if

1. $f(x, y) \geq 0$, for all (x, y),
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$,
3. $P[(X, Y) \in A] = \int\int_A f(x, y)\, dx\, dy$,

for any region A in the xy plane.

7. Marginal Distributions:

The **marginal distributions** of X alone and of Y alone are

$$g(x) = \sum_{y} f(x, y) \quad \text{and} \quad h(y) = \sum_{x} f(x, y),$$

for the discrete case, and

$$g(x) = \int_{-\infty}^{\infty} f(x, y)\, dy \quad \text{and} \quad h(y) = \int_{-\infty}^{\infty} f(x, y)\, dx,$$

for the continuous case.

8. **Conditional Prob distribution:**

Let X and Y be two random variables, discrete or continuous. The **conditional distribution** of the random variable Y given that $X = x$ is

$$f(y|x) = \frac{f(x, y)}{g(x)}, \quad g(x) > 0.$$

Similarly the conditional distribution of the random variable X given that $Y = y$ is

$$f(x|y) = \frac{f(x, y)}{h(y)}, \quad h(y) > 0.$$

If want to find prob. Then

If we wish to find the probability that the discrete random variable X falls between a and b when it is known that the discrete variable $Y = y$, we evaluate

$$P(a < X < b | Y = y) = \sum_{x} f(x|y),$$

where the summation extends over all values of X between a and b. **When** X and Y are continuous, we evaluate

$$P(a < X < b | Y = y) = \int_{a}^{b} f(x|y)\, dx.$$

9. **Independence:**

The random variables X and Y are said to be statistically independent if and only if

$$f(x, y) = g(x)h(y)$$

for all (x, y) within their range.

Otherwise not independent.

Chapter 4:

1. Mean of Random Variable:

Let X be a random variable with probability distribution $f(x)$. The **mean** or **expected value** of X is

$$\mu = E(X) = \sum_x x f(x)$$

if X is discrete, and

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

if X is continuous.

Property:

1.

Let X be a random variable with probability distribution $f(x)$. The expected value of the random variable $g(X)$ is

$$\mu_{g(X)} = E[g(X)] = \sum_x g(x) f(x)$$

if X is discrete, and

$$\mu_{g(X)} = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

if X is continuous.

2.

Let X and Y be random variables with joint probability distribution $f(x, y)$. The mean or expected value of the random variable $g(X, Y)$ is

$$\mu_{g(X,Y)} = E[g(X,Y)] = \sum_x \sum_y g(x, y) f(x, y)$$

if X and Y are discrete, and

$$\mu_{g(X,Y)} = E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y)\, dx\, dy$$

if X and Y are continuous.

2. Variance:

Condition #1:

Let X be a random variable with probability distribution $f(x)$ and mean μ. The variance of X is

$$\sigma^2 = E[(X - \mu)^2] = \sum_x (x - \mu)^2 f(x), \quad \text{if } X \text{ is discrete, and}$$

$$\sigma^2 = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx, \quad \text{if X is continuous.}$$

The positive square root of the variance, σ, is called the **standard deviation** of X.

For Discrete: Var(X)= E(X^2) - $(E(X))^2$

For Continuous: E(X^2) = Integral(x^2 f(x)) from infinity to – infinity.

E(x)= Integral(x f(x)) from infinity to – infinity.

And S.D is Sq. root of Variance.

Condition #2:

Let X be a random variable with probability distribution $f(x)$. The variance of the random variable $g(X)$ is

$$\sigma^2_{g(X)} = E\{[g(X) - \mu_{g(X)}]^2\} = \sum_x [g(x) - \mu_{g(X)}]^2 f(x)$$

if X is discrete, and

$$\sigma^2_{g(X)} = E\{[g(X) - \mu_{g(X)}]^2\} = \int_{-\infty}^{\infty} [g(x) - \mu_{g(X)}]^2 f(x) dx$$

if X is continuous.

3. Correlation Co-efficient:

Let X and Y be random variables with covariance σ_{XY} and standard deviations σ_X and σ_Y, respectively. The correlation coefficient of X and Y is

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}.$$

4. Co-variance:

For Continous and Discrete Both.

$$\sigma_{XY} = COV(X,Y) = E(XY) - E(X)E(Y)$$

Properties:

Mean	Variance
1. E(a)=a where a is const	1. Var(a)=0
2. E(x±a)=E(x)±a	2. Var(x±a)= Var(x)

3. E(ax)=aE(x)	3. Var(ax)=a^2Var(x)
4. E(ax+b)=aE(x)+b	4. Var(ax+b)= a^2Var(x)

If X,Y are Random Variables:

Mean	Variance
1. E(x+y)=E(X)+E(Y)	1. Var(x+y)= Var(x)-Var(y)
2. E(x-y)= E(X)- E(Y)	2. Var(X-Y)= Var(X)+ Var(Y)
3. E(XY)= E(X)E(Y)	3. Var(XY)= Var(X) Var(Y)

Chapter 5:

1. Discrete Uniform Distribution:

If the random variable X assumes the values $x_1, x_2, \ldots, x_k$, with equal probabilities, then the discrete uniform distribution is given by

$$f(x;k) = \frac{1}{k}, \quad x = x_1, x_2, \ldots, x_k.$$

Mean and Var:

The mean and variance of the discrete uniform distribution $f(x;k)$ are

$$\mu = \frac{1}{k}\sum_{i=1}^{k} x_i, \quad \text{and} \quad \sigma^2 = \frac{1}{k}\sum_{i=1}^{k}(x_i - \mu)^2.$$

2. Bernoulli Distribution:

$$f(x) = n_{C_x} q^{n-x}\, p^x$$

Where x ranges from 0 to n

n is number of trials, p is prob of successful event, q is prob of failure of event.

It is used when selection is done with replacement and independent trials.

Mean and Variance:

Mean=np and Var=npq

2.1. Multinomial Distributions:

Multinomial Distribution: If a given trial can result in the k outcomes $E_1, E_2, \ldots, E_k$ with probabilities $p_1, p_2, \ldots, p_k$, then the probability distribution of the random variables $X_1, X_2, \ldots, X_k$, representing the number of occurrences for $E_1, E_2, \ldots, E_k$ in n independent trials is

$$f(x_1, x_2, \ldots, x_k; p_1, p_2, \ldots, p_k, n) = \binom{n}{x_1, x_2, \ldots, x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k},$$

3. Hyper Geometric Distributions:

$$f(x) = \frac{k_{C_x}\, N - k_{C_{n-x}}}{N_{C_n}}$$

Where n is total selected items, N is total items, k is total favourable items.

N-k= total unfavourable items, k is total favourable items.

x is random variable.

It is used when selection is done without replacement. And it does not require independence.

Mean and Variance:

Mean=nk/N. and Var= (N-n)/(N-1)*n*k/N(1-k/N)

3.1. Multivariate Hyper Geometric Distributions:

Multivariate Hypergeometric Distribution: If N items can be partitioned into the A: cells $A_1, A_2, \ldots A_k$ with $a_1, a_2, \ldots a_k$ elements, respectively, then the probability distribution of the random variables $X_1, X_2, \ldots, X_k$, representing the number of elements selected from $A_1, A_2, \ldots, A_k$ in a random sample of size n, is

$$f(x_1, x_2, \ldots x_k; a_1, a_2, \ldots, a_k, N, n) = \frac{\binom{a_1}{x_1}\binom{a_2}{x_2}\cdots\binom{a_k}{x_k}}{\binom{N}{n}},$$

with $\sum_{i=1}^{k} x_i = n$ and $\sum_{i=1}^{k} a_i = A'$.

4. Poisson Distributions:

When n is very large and p is very small. i.e, n>30 and p<0.05

$$f(x) = \frac{-e^{\mu}\mu^{x}}{x!}$$

Where µ=np. And var is np.

5. Negative Binomial Distributions:

When repeated independent trials and number of trial on which kth success occurs

$$* \, b(x,k,p) = x - 1_{C_{k-1}}\, p^k q^{k-1}$$

5.1. Geometric Distributions:

When repeated independent trials and number of trials on which 1st success occurs

$$g(x,p) = pq^{x-1}$$

Mean and Variance:

The mean and variance of a random variable following the geometric distribution are

$$\mu = \frac{1}{P}; \quad \sigma^2 = \frac{1-p}{P^2}.$$