Örnek 5.1. Doğrusal olmayan sistem aşağıdaki formüllerle verilmiştir.

$$\dot{x}_1 = x_1^2 \mu_1 + x_2$$
$$\dot{x}_2 = \sin(x_1 x_2)\mu_2 + u \qquad (5.24)$$

burada $\mu = [\mu_1, \mu_2]^T$ constant parameter vector
Amaç, $Y = [x_1, x_2]^T$ olan GASP problemini çözen denetleyici bulmak. Algoritma (5.1)-i takip edelim

μ ile çarpılan f fonksiyonlarını ayıralım

$$f_1^T(x_1)\mu = x_1^2 \mu_1 \qquad f_2^T(x_1\, x_2)\mu = \sin(x_1 x_2)\mu_2$$

Adım 1. s_1 ve τ_1 seçelim.

$$s_1(x_1, \hat{\mu}) = -x_1^2 \hat{\mu}_1 - 2x_1$$
$$\tau_1(x_1\, \hat{\mu}) = \begin{bmatrix} x_1^2 \\ 0 \end{bmatrix}$$

Adım 2. $\mathcal{X}_2 = x_2 - s_1(x_1\, \hat{\mu}) = x_2 + x_1^2 \hat{\mu}_1 + 2x_1$

q_2-yi hesapla formül (5.20)'den

$$q_2(\vec{x}_2, \hat{\mu}) = f_2(\vec{x}_2) - \underbrace{\frac{\partial s_1}{\partial x_1} f_1(x_1)}$$

$$\frac{\partial s_1}{\partial x_1} f_1(x_1) = \frac{\partial}{\partial x_1}(-x_1^2 \hat{\mu}_1 - 2x_1) f_1(x_1) = -(2x_1\hat{\mu}_1 + 2)\begin{bmatrix} x_1^2 \\ 0 \end{bmatrix} = \begin{bmatrix} -(2x_1\hat{\mu}_1+2)x_1^2 \\ 0 \end{bmatrix}$$

$$q_2 = f_2 - \begin{bmatrix} -(2x_1\hat{\mu}_1+2)x_1^2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sin(x_1 x_2) \end{bmatrix} - \begin{bmatrix} -(2x_1\hat{\mu}_1+2)x_1^2 \\ 0 \end{bmatrix} = \begin{bmatrix} (2x_1\hat{\mu}_1+2)x_1^2 \\ \sin(x_1 x_2) \end{bmatrix}$$

τ_2 hesabı

(5.18) formülünü kullanalım

$$\tau_2 = \tau_1 + \Lambda(\overset{\mathfrak{X}_2}{x_2 - s_1}) q_2 \qquad \Lambda = I$$

$$\tau_2 = \tau_1 + \underbrace{(x_2 - s_1)} q_2$$

$$x_2 - s_1 = x_2 + x_1^2 \hat{\mu}_1 + 2x_1$$

Sonra τ_1 yerine koy

$$\tau_2 = \begin{bmatrix} x_1^2 \\ 0 \end{bmatrix} + (x_2 + x_1^2 \hat{\mu}_1 + 2x_1) \begin{bmatrix} (2x_1\hat{\mu}_1 + 2) x_1^2 \\ \sin(x_1 x_2) \end{bmatrix} =$$

$$= \begin{bmatrix} x_1^2 + (2x_1\hat{\mu}_1 + 2) x_1^2 (x_2 + x_1^2\hat{\mu}_1 + 2x_1) \\ \sin(x_1 x_2)(x_2 + x_1^2 \hat{\mu}_1 + 2x_1) \end{bmatrix}$$

v_2 hesabı

$$v_2 = \underbrace{\frac{\partial s_1}{\partial x_1}} x_2 + \underbrace{\frac{\partial s_1}{\partial \hat{\mu}}} \tau_2$$

$$\frac{\partial s_1}{\partial x_1} = -(2x_1\hat{\mu}_1 + 2) \qquad \frac{\partial s_1}{\partial \hat{\mu}} = \frac{\partial}{\partial \hat{\mu}}(-x_1^2 \hat{\mu}_1 - 2x_1) = [-x_1^2 \quad 0]$$

$$v_2 = -(2x_1\hat{\mu}_1 + 2) x_2 + [-x_1^2 \quad 0] \begin{bmatrix} \tau_{2,1} \\ \tau_{2,2} \end{bmatrix} =$$

$$= -(2x_1\hat{\mu}_1 + 2) x_2 - \underbrace{x_1^2 \tau_{2,1}}_{\text{açalım}}$$

$$\rightarrow \ v_2 = -(2x_1\hat{\mu}_1 + 2) x_2 - x_1^4 - (2x_1\hat{\mu}_1 + 2) x_1^4 (x_2 + x_1^2 \hat{\mu}_1 + 2x_1)$$

S_2 hesabı

$$S_2(x_2, \hat{\mu}) = \underbrace{-q_2^T(\vec{x_2}, \hat{\mu})\hat{\mu}} - \beta_2 \underbrace{(x_2 - s_1)} + \underline{\underline{v_2(\vec{x_2}, \hat{\mu})}}$$

$\beta_2 = 9/4$ seçelim

$$q_2^T \hat{\mu} = \left[(2x_1\mu + 2)x_1^2 \quad \sin(x_1 x_2)\right]\begin{bmatrix}\hat{\mu}_1 \\ \hat{\mu}_2\end{bmatrix} = (2x_1\hat{\mu}_1 + 2)x_1^2\hat{\mu}_1 + \sin(x_1 x_2)\hat{\mu}_2$$

$$x_2 - s_1 = x_2 + x_1^2\hat{\mu}_1 + 2x_1$$

$$S_2 = -\left((2x_1\hat{\mu}_1 + 2)x_1^2\hat{\mu}_1 + \sin(x_1 x_2)\hat{\mu}_2\right) - \frac{9}{4}(x_2 + x_1^2\hat{\mu}_1 + 2x_1) + v_2$$

Artık

S_2, τ_2 bulunduğuna göre Theorem 5.2-e göre kontrolcümüzü aşağıdaki gibi tasarlayabiliriz

$$u = S_2(x_1, x_2, \hat{\mu})$$

$$\dot{\hat{\mu}} = \tau_2(x_1, x_2, \hat{\mu})$$