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Chapter 11

Electrostatics

11.1 Electric Charge

"Electric charge is a basic property of elementary particles that causes them to exert forces on one another."

Alternatively, we can define electric charge as, ***"A physical quantity whose presence produces an electric field."***

Explanation

Charge is a fundamental property of some elementary particles like electron, proton etc. There are two kind of charges; positive and negative. Protons are present inside the nucleus of an atom, possess positive charge and electrons are clouding around the nucleus of an atom, carry negative charge. All protons are alike and have the same charge $+e = 1.60219 \times 10^{-19}$ coulomb. Similarly all electrons are alike and have the same charge $-e$. e is the fundamental charge.

11.1.1 Properties of Electric Charge

Electrification

"Electrification is the process in which a neutral body is charged by the removal or addition of electrons."

In nature, majority objects are in neutral state because the number of electrons is equal to the number of protons in them. If electrons are removed, a positive charge appears on the body. Similarly, if electrons are added, a negative charge appears on the body. It is important to note that during the process, charge can never be created nor destroyed.

Conservation of Electric Charge

Conservation of charge means that, ***"Charge can never be created nor destroyed in a process."*** We can also state it as, ***"the total charge of an isolated system remains conserved."***

Quantization of Electric Charge

Another very basic property of electric charge is that charge is quantized. Quantization of charge means that it exists in discrete packets rather than in continuous amounts. It means that charge on a certain body can be developed due to addition or removal of one or two or three or 'n' electrons. Any charge q, no matter what is its origin, is an integral multiple of the minimum elementary charge e. Mathematically, 'q' charge on a body can be expressed as:

$$q = ne \quad \text{and n} = 1, 2, 3, \ldots$$

> **☑ Checkpoint 11.1**
>
> We say that charge always exists in an integral multiple of 'e' but we also believe in the existence of quarks (having charge 2/3 e, 1/3 e etc.). How is this possible? (Answers of checkpoints are given at the end of the chapter)

Action between Two Charges

It has been experimentally proved that like charges repel each other and unlike charges do attract. This means that a positive charge will repel a positive and so does a negative. And positive attracts a negative one and a negative does attract a positive one.

Sciences of Electric Charges

On the basis of state of rest or motion of electric charges, science of charges is divided into two branches i.e. electrostatics and electrodynamics.

Electrostatics/Static Electricity

"It is the study of electric charges at rest". In this chapter, we will deal with this branch. We will study some basic laws like Coulomb's law, Gauss's law etc, and some fundamental concepts electric field, electric potential and further capacitors and their related concepts.

Electrodynamics/Current Electricity

"It is the study of electric charges in motion." We will encounter this in next chapter. We will study some basic concepts of current flow, resistance and laws like Ohm's law and Kirchoff's laws.

11.2 Coulomb's law

Background

As we know that electric charges attract or repel each other. To quantify this attaraction or repulsion, Charles-Augustin de Coulomb[1], in 1785, first measured the force of interac-

[1] ***Charles Augustin de Coulomb (1736 – 1806):*** Coulomb, a French physicist, began his career as a military engineer in the West Indies. In 1776, here turned to Paris and retired to a small estate to

tion (attraction or repulsion) between electric charges and deduced the law that governs them, called as Coulomb's law. He used for this purpose an apparatus called Torsion's balance.

Statement

"Two stationary point[2] charges attract or repel each other with a force which is directly proportional to the product of the magnitude of the charges and is inversely propotional to the square of the distance between them, and this force acts along the line connecting the charges."

Mathematical Form

Let us consider two electric charges 'q_1' and 'q_2' seperated by distance 'r', these charges will exert forces on one another and this force according to Coulomb's law will be:

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2} \tag{11.1}$$

Where k is the constant of propotionality and is called 'Coulomb's constant' (we will discuss it in detail later). If r is the unit vector along the line connecting the charges, then Coloumb's law expression will be written as:

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \tag{11.2}$$

This is the vectorial form of Coulomb's law, which shows that force is directed along the unit verctor $\hat{r}$. We have two charges q_1 and q_2. If q_1 is considered as source charge(the charge exerting the force) and q_2 is considered as field charge (the charge experiencing the force), then the unit vector $\hat{r}$ is directed from q_1 to q_2 and vice versa. It is important to note that the direction of force is according to source charge.

Attractive or Repulsive Force

For like charges, the product $q_1 q_2$ is positive; the force is repulsive and is directed away from the source charge. For unlike charges, the force is attractive as $q_1 q_2$ is negative and the force is directed towards the source charge.

do his scientific research. He invented a torsion balance to measure the quantity of a force and used it for determination of forces of electric attraction or repulsion between small charged spheres. He thus arrived in 1785 at the inverse square law relation, now known as Coulomb's law. The law had been anticipated by Priestley and also by Cavendish earlier, though Cavendish never published his results. Coulomb also found the inverse square law of force between unlike and like magnetic poles.

[2]As word point suggests an entity of no dimension. In practice,two charges are said to be point charges if their dimesions (means their radii) are very very smaller as compared to the distance between them. Coloumb's law has its validity over the point charges.

Constant of propotionality 'k'

We used the constant of propotionality 'k' in coulomb's law expression. The constant 'k' depends upon:

(a) System of units used (in which q, r and F are measured)

(b) Properties of medium surrounding the charges

In SI units,force, charge and distance are measured in newtons, coloumbs and metres. As far as the choice of the medium is concerned, we start with vacuum or free space. In free space and SI units, the constant 'k' is written as:

$$k = \frac{1}{4\pi\epsilon_o} \tag{11.3}$$

Where 'ϵ_o' is an electrical constant (read as 'epsilon not') and called as permittivity of free space (permittivity of a medium is the property of the medium which determines how much that medium affects the force between the charges). The value of 'ϵ_o' is measured experimentally and is found to be 8.85×10^{-12} C^2/Nm^2 (rounded to two decimal places). Using the value of 'ϵ_o' in equation 11.3, we get value of 'k' equal to 8.99×10^9 Nm^2/C^2. So, in SI units and for charges placed in vacuum (or free space), Coulomb's law in equation 11.4 can be written as:

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}\hat{r} \tag{11.4}$$

And in magnitude form, (using 'vac' in subscript for representation of force in vacuum):

$$F_{vac} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2} \tag{11.5}$$

11.2.1 Coloumb's law in Material Media

As the constant of propotionality 'k' in coulomb's law expression depends on the medium around the charges. Therefore, if the charges are placed in a medium of permittivity ϵ, then coloumbic force will be given by:

$$F_{med} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \tag{11.6}$$

From equation 11.6, it implies that a material medium with high permittivity is a medium which reduces appreciably the force between the charges compared with the vacuum. For air, ϵ_{air} is only slightly greater than ϵ_o (1.006) and for many practical purposes, is taken equal to ϵ_o. In order to make force more simplified, we introduce a term relative permittivity ϵ_r, which is defined as, "the permittivity of a medium compared with the permittivity of vacuum." So, it is given by the ratio:

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} \tag{11.7}$$

So, 'ϵ_r' is a dimensionless constant. It is also called the dielectric constant of the medium. For vacuum or free space, $\epsilon_r = 1$, and hence $\epsilon = \epsilon_o$. From equation 11.17:

$$\epsilon = \epsilon_o \epsilon_r \tag{11.8}$$

Putting this value in equation 11.6, we get:

$$F_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1 q_2}{r^2} \tag{11.9}$$

Here, ''ϵ_o' is a constant and has a fixed value. It is important to note that the value of dielectric constant 'ϵ_r' for any material medium is always greater than one. So force in a mterial medium is always less than that of vacuum. This becomes more obvious, if we take ratio of equations (11.9) and (11.5):

$$\frac{F_{med}}{F_{vac}} = \frac{1}{\epsilon_r} \tag{11.10}$$

which gives:

$$F_{med} = \frac{F_{vac}}{\epsilon_r} \tag{11.11}$$

Since $\epsilon_r > 1$ for any material medium, therefore force in a medium is less than force in vacuum for two given charges by 'ϵ_r' times. This equation gives us a sense that how much times force in a medium is reduced as compared with the vacuum for two charges. The above equation can also be written as:

$$\epsilon_r = \frac{F_{vac}}{F_{med}} \tag{11.12}$$

This equation provides us with another definition of dielectric constant i.e. ***"the ratio of force between two point charges placed in vacuum to the force between the same charges when the medium surrounding them is material medium, is called dielectric constant/relative permittivuty of that medium."***

☑ Checkpoint 11.2

Distilled water has a dielectric constant of nearly 80. If force between the two charges placed in vacuum is 80 N. How much the force will be when the same charges be placed in water?

11.2.2 Units of Electric Charge

SI Unit

The S.I unit of electric charge is coulomb (C), which can be defined in a number of ways stated under. One way of defining one coloumb charge is using Coulomb's law expression i.e. ***"The charge on a body is said to be one coulomb when it exerts a force of*** 9×10^9 ***N on a similar charge at a distance of one metre from it in vacuum."***

Alternatively, we can also define it in terms of charge of an electron. As we know that:

$$charge\ of\ one\ electron = e = 1.602 \times 10^{-19}C$$

So one coloumb charge will contain 6.25×10^{18} electrons. Hence, one coulomb can be defined as, ***"The charge of*** 6.25×10^{18} ***electrons is said to be one coloumb."***

Another way of defining one coulomb charge is in terms of current (flowing charge) i.e. ***"The amount of charge that flows through a given cross section of a wire in one second if there is a steady current of one ampere in the wire."***

Multiples

Since coloumb is a very large unit (imagine it as the charge of 6.25×10^{18} electrons), so its submultiples are commonly used.

$$1\ \mu C = 10^{-6}\ C,\ 1\ nC = 10^{-9}\ C,\ 1\ pC = 10^{-12}\ C$$

11.2.3 Coloumb's Law and Newton's Third Law

As we know that charges exert forces on each other, hence coloumic force is a mutual force. To show that the force that one charge exerts on the other is equal in magnitude but opposite in direction to the force that it experiences by the other charge, let us consider two point charges (assume them to be positive for convenience). The distance between them is r, as shown in figure 11.1.

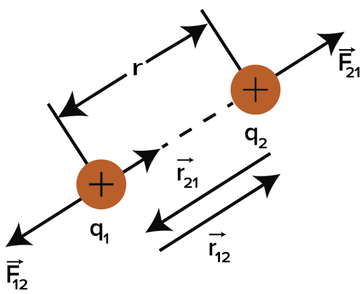


Figure 11.1: Force exerted by each charge, $\vec{F_{12}}$ is the force exerted on charge 'q_1' by 'q_2', $\vec{F_{21}}$ is the force exerted on charge 'q_2' by 'q_1'

Let $\vec{F_{12}}$ is the force exerted by charge 'q_2' on charge 'q_1' (subscript as, '12' means by 2 on 1) and $\vec{r_{12}}$ is the position vector directed from 'q_2' to 'q_1'. Similarly, $\vec{F_{21}}$ is the force exerted by charge 'q_1' on 'q_2' and $\vec{r_{12}}$ is the unit vector directed from 'q_1' to 'q_2'. Since,

$$\vec{r_{12}} = r_{12}\hat{r_{12}}$$

And,

$$\vec{r_{21}} = r_{21}\hat{r_{21}}$$

So, $\vec{F_{12}}$ according to Coloumb's law will be given by:

$$\vec{F_{12}} = k\frac{q_1 q_2}{r_{21}^2}\hat{r_{21}}$$

And,

$$\vec{F_{21}} = k\frac{q_1 q_2}{r_{12}^2}\hat{r_{12}}$$

Since,

$$\vec{r_{12}} = -\vec{r_{21}}$$

Therefore,

$$\vec{F_{12}} = -\vec{F_{21}}$$

So we concluded that the force between two charges is mutual and the forces pair is a Newton's third law pair.

11.2.4 Principle of Superposition

Coulombic force obeys the principle of superposition i.e. ***"The net force acting on a charge by an assembly of charges is the vectorial sum of all the forces exerted by the number of charges."***

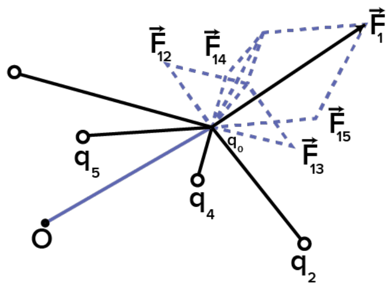


Figure 11.2: Net force exerted on charge 'q_1' is the vectorial sum of all the forces due to each charge on 'q_1'.

From the figure 11.2, for 'n' charges exerting a force on the charge 'q_1', we can say that:

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + ... + \vec{F_{1n}}$$

☑ Checkpoint 11.3

(a) We have two charges, 'q_1' and 'q_2'. Let the force of interaction between them is 4 N. Then we placed another charge 'q_3' in the vicinity of the charges. What would be the force of attraction between 'q_1' and 'q_2' now?

(b) Show that for two opposite charges, 'q_1' and 'q_2' using usual notations,

$$\vec{F_{12}} = -\vec{F_{21}}$$

11.3 Electric Field and Electric Field Intensity

Introduction

The concept of electric field was introduced by ***Michael Faraday***. He stated that the charge 'q' produces an electric field surrounding it and when a charge 'q_o' is brought into its field, the field of charge 'q' interacts with that of 'q_o' and exerts force on it.

Definition of Electric Field

"The region around a charge in which a test charge experiences an electric force, is called electric field."

Explanation

As we know that charges exert forces on each other. Electric field is actually the region around a source charge upto which it can exert a force on a test charge. In order not to distort the field of a source charge, the test charge should be small. So the strength and direction of electric field can be determined by placing a unit positive test charge in that field.

The strength and direction of field at a point in space is determined by the force that a unit positive charge will experience at that point. The direction of field is that direction in which test charge moves or tends to move at a given point.

Electric Field Intensity

"A single vector quantity containing information about the field strength and directiom at a given point is called electric field intensity."

Mathematical Form

If '$\vec{F}$' is the force exerted on a unit positive test charge (it is a convention to take test charge positive) 'q_o' by a source charge 'q', then the electric field intensity '$\vec{E}$' is given by:

$$\vec{E} = \frac{\vec{F}}{q_o} \tag{11.13}$$

Thus, it is the force per unit charge. Defininig in this way, electric intensity is independent of test charge 'q_o'. Since 'q_o' is always positive, therefore is always along the direction of force on that test charge.

Units and Dimensions

The S.I unit of electric field intensity is NC^{-1}. Another unit is volt per metre (V/m), we will dicuss this in electric potential. Dimensions are $[MLT^{-3}A^{-1}]$.

11.3.1 Field Intensity due to a Point Charge

In order to find an expression for field intensity due to a point charge, consider point charges 'q' and 'q_o' and at a distance 'r' from each other. As an example, we find the intensity of field which exists in air around the isolated charge 'q'.

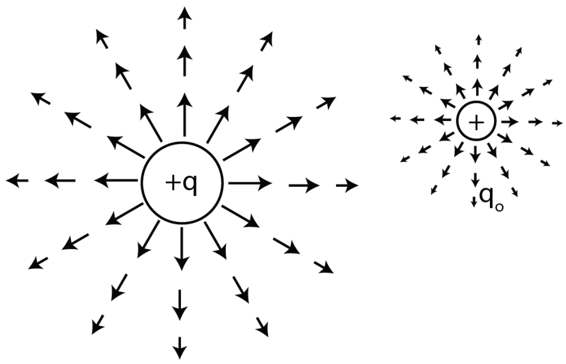


Figure 11.3: Field of a source charge 'q' influencing test charge 'q_o'

Here 'q_o' is a small test charge. Coulomb's force due to 'q' on 'q_o' is:

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{r^2}\hat{r} \tag{11.14}$$

From the definition of electric field intensity i.e. the force per unit charge, we can write:

$$\vec{E} = \frac{\vec{F}}{q_o}$$

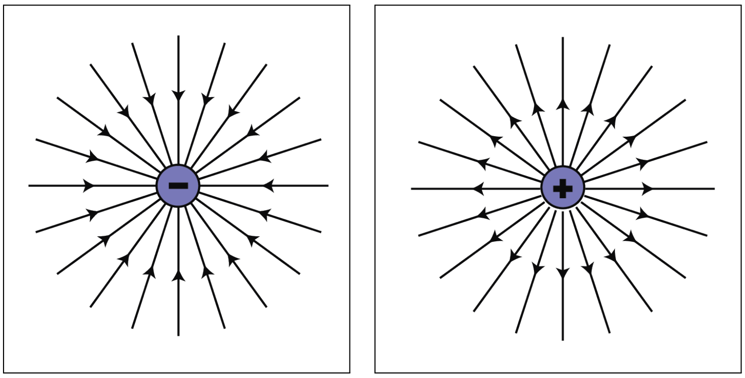
Hence,

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}} \tag{11.15}$$

From above equation, electric field varies directly with the magnitude of source charge and is inversely propotional to the square of the distance from the source charge. Moreover, electric field is directed along the radius outward if 'q' is positive and radially inward if 'q' is negative as shown in figure 11.4.

If medium surrounding the charge 'q' is other than vacuum or air, having dielectric constant 'ϵ_r', then electric intensity will be given by:

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q}{r^2}\hat{r}} \tag{11.16}$$



(a) Positive Charge (b) Negative Charge

Figure 11.4: Electric field pattern for an isolated positive & negative charge

11.3.2 Electric Lines of Force/Electric Field Lines

Definition

"An electric line of force is a curve so drawn that a tangent to the curve at any point shows the direction of the field at that point."

Explanation

Electric lines of force are imaginary lines to show the strength and direction of a real field. As the strength of the field means electric field intensity, so electric lines of force represents the magnitude and direction of electric field. The mapping of electric field by field lines helps in visualizing the electric field.

The magnitude of the electric field at a certain point is determined by the density of lines (number of lines per unit area). The greater the density of lines, the greater will be the magnitude of electric field. The direction of the field at a given is determined by drawing a tangent to the field line at that point. Actually, the field line tells us about the movement of a test charge at each and every point inside an electric field.

Properties of Electric Field Lines

(i) The density of lines show the direction of electric field.

(ii) The arrowhead shows the direction of field.

(iii) The field lines begin from positive charge and terminate on negative charge, they are continuous in region containing no charge.

(iv) The lines of force do never cross.If they did cross, then electric field would have two different directions at the point of intersection which is not possibe. Put another way, at the point of intersection, there would be two tangents possible. As at a given point, the tangent to the curve represents the direction of field or the direction in which the test charge will move along at that point, so the test charge will have to move along two different directions at the same point and time, which is not possible.

(v) The lines of force strike the surface of conductor perpendicularly.

(vi) The lines of force can not pass through the conductor. As the presence of lines implies that there would be an electric field inside the conductor. As conductor have free charges in them, so the existence of electric field corresponds the force on those charges, leading to the flow of charges (electric current), but no such perpetual currents exist. Hence no electric field exists inside the conductor body hence no field lines.

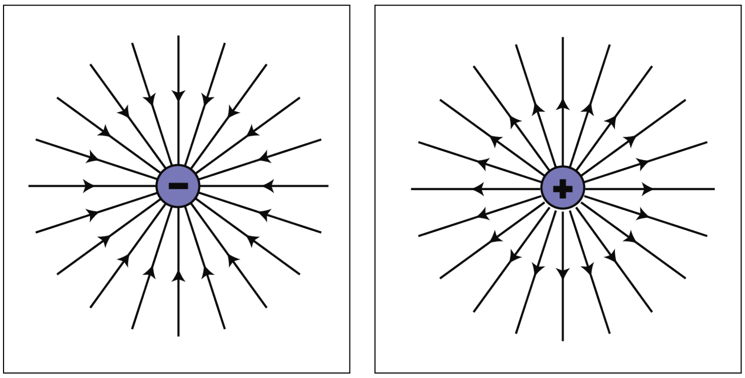
Neutral point/Null point

"The point where the net electric field intensity is zero is called neutral point/null point." It is denoted by 'N' (we will discuss its example in next section).

11.3.3 Field Lines of Some Electrostatic Charge Distributions

For an Isolated Charge

We have discussed already that field lines for an isolated positive charge are radially outward and radially inward for a negative charge extending in space as shown:



(a) Positive Charge (b) Negative Charge

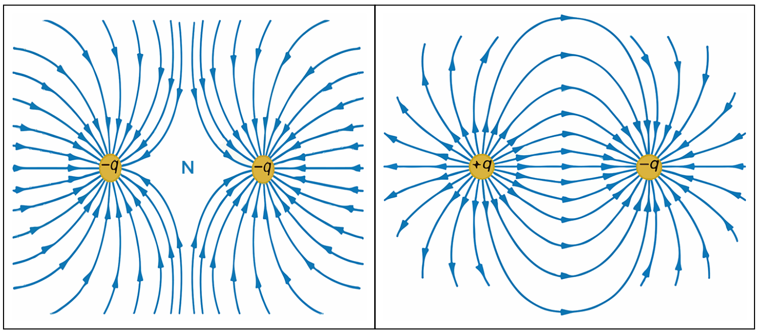
Figure 11.5: Electric field pattern for an isolated positive & negative charge

For Two Charges of Same Magnitude

For two charges of same sign and same magnitude (for convenience, we take them negative), configuration is as shown in 11.6a. The point 'N' is neutral point. The number of lines per unit area shows the strength of field. So, no lines means no net electric field. Here, in this case, we assumed both charges of same sign and same magnitude, hence the point of zero electric field will lie in mid of these chrarges. For two charges of same magnitudes but opposite signs, field lines are shown in figure 11.6b.

Electric Lines Due to a Positive Charge Near a Metal Plate

Consider an electric charge '$+q$' placed near a metal plate. The positive charge will attract the negative charges (electrons) in the metal plate resulting in the motion of the charges



(a) For two negative charges (b) For a positive & negative charge

Figure 11.6: Electric field pattern for two charges of same magnitude

until some of them reach that surface of the metal which is near the '$+q$' charge where they will be at rest. Thus the field lines starting from '$+q$' charge will terminate on the negative charges of the metal plate. Furthermore,these lines starting from '$+q$' charge are always perpendicular to the conductor. The electric lines of force can not pass through the metal. Electric field is zero inside a conductor under electrostatic conditions. If it was not so, electric field would exert forces on electrons causing them to flow establishing current. Since, no such currents do exist, hence field inside the metal will be zero. The field configuration of above discussed case is as shown:

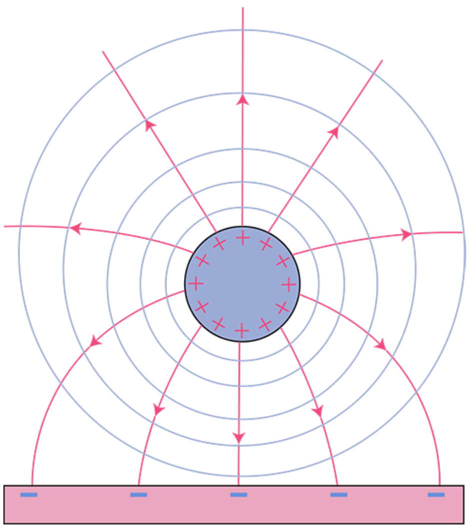


Figure 11.7: Field pattern for a positive charge near a metal plate

Uniform Electric Field

Definition: ***"A field is said to be uniform in a certain region if it has the same magnitude and direction in that region."***

Explanation: From the definition of uniform electric field, it is clear that the magnitude of field must be constant and direction as well. As we know that the magnitude of electric field depends upon the number of lines per unit area,so for uniform electric field, the field lines are uniformly spaced. The direction the field is is represented by the arrows, hence for same direction, they (lines) must point in the same direction.

Production of Uniform Electric Field: A uniform electric field can be produced by connecting the terminals of a baatery connected to two large parallel metal plates. If the plates are of finite length,then the lines of force are bulging at the ends of the plates. This non-uniform field at the edges of the plates is called 'fringing field'. In order to avoid fringing field, the plates must be of infinite length. Practically, plates are said to be of infinite length when the distance between them is much smaller than their dimensions. The field pattern is shown in figure 11.8.

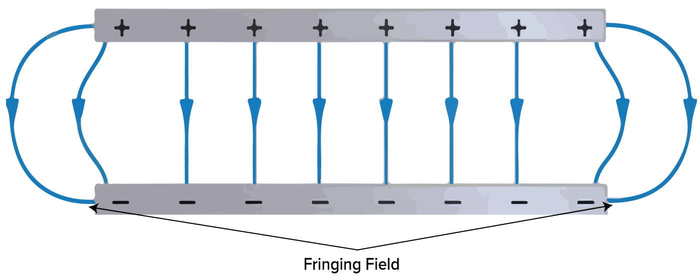


Figure 11.8: Uniform Electric Field with a 'fringing field'

11.4 Electric Flux

"The number of electric lines passing through the area placed in the electric field,is called electric flux."

Symbol

It is denoted by Φ_E (Greek alphabet phi) where the subscript 'E' is for electric.

Explanation

In the above definition,we have discussed two vector quantities, electric field lines mean electric field intensity. The magnitude of electric field intensity is, ***"the number of lines per unit area placed perpendicular to $\vec{E}$."*** The second vector involved in the definition of electric flux is Area. The magnitude of the area vector is equal to the area of the plane occupied and direction is always normal to the surface/plane shown by '$\hat{A}$' or '$\hat{n}$'. Here we considered a flat circular surface, area vector is shown, the direction is shown by normal unit vector '$\hat{n}$'.

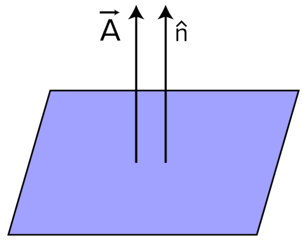


Figure 11.9: Vector area shown for a flat rectangular surface. The direction is normal to the surace.

Mathematical Form

If $\vec{E}$ is the electric field intensity which is uniform over a certain area, then the electric flux will be given by:

$$\Phi_E = \vec{E}.\vec{A} \tag{11.17}$$

Hence, it is the dot product of previously dicussed two vector quantities $\vec{E}$ and $\vec{A}$. Using usual notations we can write it as:

$$\Phi_E = EAcos\phi \tag{11.18}$$

where $Acos\phi$ is the component of area held perpendicular to $\vec{E}$ (But its direction will be along $\vec{E}$). This equation is the mathematical form of flux at any angle. In the figure, the area component which is perpendicular to field lines is '$A_\perp$'. Note that only this component contributes to electric flux, the lines of forces just skim through '$A_\parallel$'. Resolving area vector into its $\hat{i}$ and $\hat{j}$ components, the component which is along the direction of electric field is $Acos\phi\,\hat{i}$ which is actually the vector associated with the same area component that is held perpendicular to electric field lines and contributing to flux (As its direction is perpendicular to '$A_\parallel$'). So we defined electric flux as the product of electric intensity and the component of area vector along the direction of $\vec{E}$.

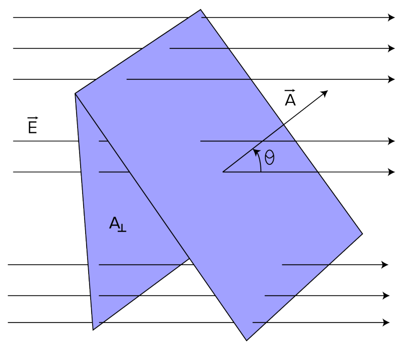


Figure 11.10: Surface held at an angle to field. Its vector area $\vec{A}$ is perpendicular to it. Lines of force pass through $A_\perp$ and just skim through $A_\parallel$.

Maximum Flux

If the surface is placed perpendicular to electric field such that surface area vector is parallel to the direction of electric field $\vec{E}$, then maximum number of lines of force will pass through the surface. Consequently, maximum electric flux will pass through the surface. (Note that ϕ is the smaller angle between $\vec{E}$ and area vector $\vec{A}$ which is perpendicular to the plane of area). This is shown in figure 11.11:

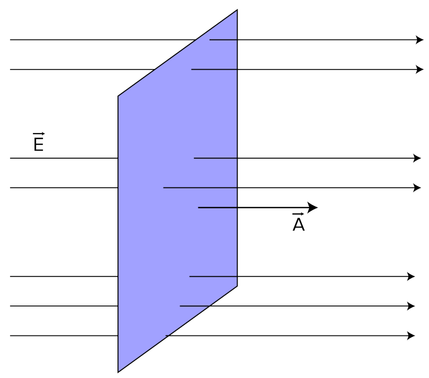


Figure 11.11: Maximum flux lines pass through the surface when it is held perpendicular to the field lines

Using flux definition:

$$\Phi_E = EAcos\phi$$

Here $\phi = 0°$, So,

$$\Phi_E = EAcos(0°)$$

$$\Phi_E = EA$$

This can be physically understood by another way. Since '$A_\perp$' is the only area component contributing to flux and there is no parallel component of area '$A_\parallel$'. So, only the vector area related with '$A_\perp$' will be involved in defining flux i.e. $Acos\,\phi$, which is equal to the whole area vector, and writing in magnitude form we have flux equal to EA.

Zero Flux

If the surface is placed parallel to the electric field lines such that area vector $\vec{A}$ is normal to the electric field $\vec{E}$, then the maximum number of lines will pass through the surface. Consequently, no electric flux will pass through the surface, as shown in figure 11.12. Mathematically,

$$\Phi_E = EAcos\phi$$

Here $\phi = 90°$, So,

$$\Phi_E = EAcos(90°)$$

$$\Phi_E = 0$$

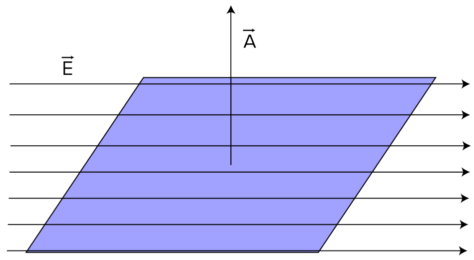


Figure 11.12: Zero flux lines pass through the surface when it is held parallel to the field lines

Note: We defined '$A_\perp$ and '$A_\parallel$' be the components of the surface on the basis of their orientation relative to field lines.

Nature

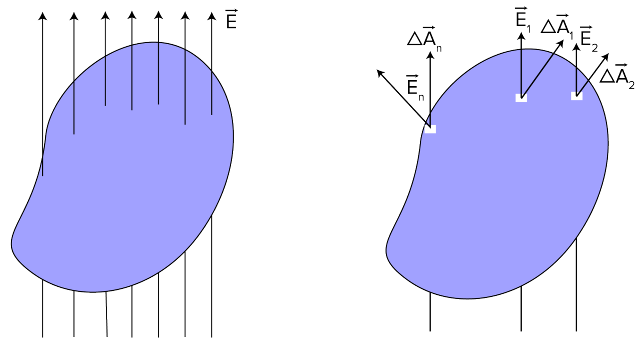
Since,electric flux is the dot product of $\vec{A}$ and $\vec{A}$, so it is a scalar quantity.

Units

The S.I unit of electric flux is Nm^2C^{-1}.

11.4.1 Flux through an Arbitary Shaped Body

We defined flux to be equal to $\vec{E}.\vec{A}$ which is applicable only if '$\vec{E}$' is uniform over the whole area A, i.e. for flat surfaces. If the surface is not flat, then equation $\vec{E}.\vec{A}$ can not be applied directly, because the angle between $\vec{E}$ and $\vec{A}$ changes from place to place. In such cases, the product $\vec{E}.\vec{A}$ defines flux only for small area piece which is considered flat. Therefore, to find flux for an uneven surface (open or closed), we divide the surface into small flat pieces ΔA_1, ΔA_2, $\Delta A_3, ...,$ ΔA_n. as shown:



(a) Field lines passing through the surface

(b) Surface is divided into 'n' small patches. Each patch has an associated electric field and area vector.

Figure 11.13: Flux for an arbitrary shaped surface.

The area vectors of these area components are $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, ..., \Delta\vec{A}_n$. The electric field intensity at these area pieces are $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3, ...,$ $\vec{E}_n$ repectively. The small amount of flux '$\Delta\Phi_1$' through 'ΔA_1' is defined as:

$$\Delta\Phi_1 = \vec{E}_1.\Delta\vec{A}_1$$

Similarly,

$$\Delta\Phi_2 = \vec{E}_2.\Delta\vec{A}_2$$

Hence for n$^{\text{th}}$ patch,

$$\Delta\Phi_n = \vec{E}_n.\Delta\vec{A}_n$$

So, the total flux through the surface will be:

$$\Phi_T = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi_T = \vec{E}_1.\Delta\vec{A}_1 + \vec{E}_2.\Delta\vec{A}_2 + \vec{E}_3.\Delta\vec{A}_3 + ... + \vec{E}_n.\Delta\vec{A}_n$$

which in compressed notation:

$$\Phi_T = \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i \tag{11.19}$$

11.4.2 Flux through a Closed Arbitrary Surface

Let us consider a closed sufrace of arbitrary with the positive normal taken outward from the volume beig closed. In case of closed surface, the electric flux may be positive, negative or zero depending upon the number of lines entering or leaving the surface. We discuss it as:

(i) The electric flux is positive, if net number of lines are leaving the surface. Since positive charge is a source of field lines, so it means that there is a source of lines inside the surface, i.e. positive charge. Mathematically, for a closed surace of abitrary shape:

$$\Phi_T = \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i$$

And this implies that there is a source of field lines inside the surface. The number of lines leaving the surface are more than number of lines entering, flux would be negative for lines entering and it would be positive for lines leaving and hence net flux would be positive.

(ii) The electric flux through a closed surface will be negative, if net number of lines are entering the surface or more field lines are entering than leaving the surface; there is a sink of field lines in the closed surface i.e. a negative charge as field lines terminate on negative charge.

(iii) The electric flux will be zero if number of lines entering is equal to the number of lines leaving the surface or no field lines intercpepting the surface. This is possible when there is no net charge because net charge is a source or sink of lines.

11.5 Gauss's Law

Background

The electric field of a given charge distribution can be calculated using Coloumb's law. But sometimes field calculation using Coulomb's law becomes very difficult. An alternative method to calculate the electric field of a given charge distribution relies on theorem called 'Gauss's law' given by Karl Friedrich Gauss[3]. It provides a realtionship between the net electric flux through the closed surface and the net charge enclosed by that surface.

Statement

"The net electric flux through any closed surface is equal to $\frac{1}{\epsilon_o}$ times the charge 'q' enclosed by that surface."

Explanation

Gauss's law provides a simple relationship between the electric flux through any closed surface and the total charge inside that surface. The charges inside the surface can be either a single point charge or a number of charges.

In order to derive an expression for Gauss's law, let us consider a closed surface (for convenince we take a sphere) of radius 'r' having a point charge 'q' at its centre as shown in figure 11.14.

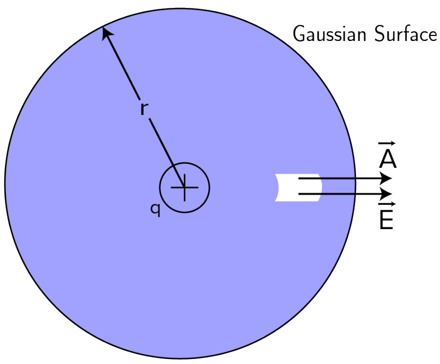


Figure 11.14: Flux for a closed surface; sphere

As the direction of electric field intensity E varies from place to place, so in order to calculate electric flux, we divide the whole surface into 'n' number of small pieces

[3]German mathematician and astronomer (1777-1855) Gauss received a doctoral degree in Mathematics from the University of Helmsted in 1799. In addition to his work in electromagnetism, he made contributions in mathematics, number thoery, statistics and mecahnics.

having area ΔA_1, ΔA_2, $\Delta A_3, ...,$ ΔA_n. Here, we considered a sphere, so two things are important:

(i) Electric field intensity is same at every point as they are equidistant from the charge.

(ii) As field is radial, it means that at every point $\vec{E}$ and $\vec{A}$ are in same direction i.e. the angle between E and A at every point will be zero degree.

Now, we know that the total flux through area ΔA_1 will be:

$$\Delta\Phi_1 = \vec{E_1}.\Delta\vec{A_1}$$

Similarly,

$$\Delta\Phi_2 = \vec{E_2}.\Delta\vec{A_2}$$

Hence for n$^{\text{th}}$ patch,

$$\Delta\Phi_n = \vec{E_n}.\Delta\vec{A_n}$$

Since,

$$E_1 = E_2 = E_3 = E_n = E$$

And the total flux will be:

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi = \vec{E}.\Delta\vec{A_1} + \vec{E}.\Delta\vec{A_2} + \vec{E}.\Delta\vec{A_3} + ... + \vec{E}.\Delta\vec{A_n}$$

Since, field is radial, hence:

$$\Phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_{3+...+E\Delta A_n}$$

$$\Phi = E(\Delta A_1 + \Delta A_2 + \Delta A_{3+...+\Delta A_n})$$

$$\Phi = E\sum_{surface}\Delta A \tag{11.20}$$

Since, electric field at a distance 'r' due to charge 'q' will be:

$$E = \frac{q}{4\pi\epsilon_o r^2}$$

Also we know that in case of sphere:

$$\sum_{surface}\Delta A = 4\pi r^2$$

Putting these values in equation 11.20

$$\Phi = \frac{q}{4\pi\epsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\epsilon_o} \tag{11.21}$$

Equation 11.21 defines Gauss's law for point charge. It is clear that electric flux is independent of:

(i) The shape of closed surface.

(ii) The radius(size) of the closed surface.It means that if radius is made very small or large, still $\frac{q}{\epsilon_o}$ lines will come from 'q'.

And equation also tells that the flux depends upon:

(i) The amount of charge enclosed.

(ii) The medium surrounding the charge.

Conclusion

We conclude that each positive charge must have $\frac{q}{\epsilon_o}$ lines coming from it. A negative charge will have the same number of lines going through it. Another important thing is that whatever the shape of the closed surface is flux will be same for a given charge placed inside the surface in a medium. We assumed the closed surface as sphere which helped in making our calculation easy. At last we found that it does not matter that what is the shape of surface, it should be just closed and must enclose a charge to give out flux.

11.5.1 Electric Flux due to Many Charges

To formulate Gauss's law for many charges, let us consider point charges q_1, q_2, q_3,..., q_n inside a closed surface S, of some arbitrary shape, as shown:

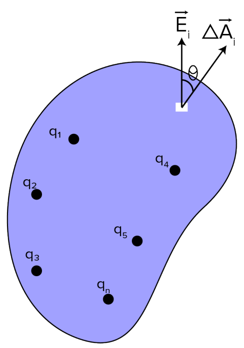


Figure 11.15: Flux due to many charges inside a closed surface

We know that all the flux lines coming from 'q_1' passes through the surface 'S', therefore from Gauss's law, flux due to 'q_1' will be:

$$\Phi_1 = \frac{q_1}{\epsilon_o}$$

Similarly,

$$\Phi_2 = \frac{q_2}{\epsilon_o}$$

And,

$$\Phi_3 = \frac{q_3}{\epsilon_o}$$

Hence due to nth charge:

$$\Phi_n = \frac{q_n}{\epsilon_o}$$

As total flux is given by:

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi = \frac{1}{\epsilon} \times (q_1 + q_2 + q_3 + ... + q_n)$$

Since,

$$q_1 + q_2 + q_3 + ... + q_n) = Q$$

So, we get:

$$\Phi = \frac{Q}{\epsilon_o} \tag{11.22}$$

Note that 'Q' is the net charge which is the algebraic sum of all the charges. It is also important to note that the electric field due to a charge outside the surface contributes zero electric flux through the surface because as many field lines due to that charge enter as leave it.In section before Gauss's law, we discussed positive, negative and zero electric flux through an arbitrary closed surface by using definition of electric flux. Now, we can relate the same using Gauss's law. We can further specify equation 11.22 by writing it as:

$$\Phi = \frac{Q_{enclosed}}{\epsilon_o} \tag{11.23}$$

Now we discuss three cases:

(i) The algebraic sum of '$+q$' and '$-q$' charges, if equal to zero i.e. $Q = 0$, then net flux using above equation will be zero. In other case, if there is no charge enclosed, then flux will again be zero.

(ii) If there is a positive charge or sum of charges taking into account their signs comes out to be positive, then electric flux would be positive. More number of lines will be leaving thean the number of lines entering.

(iii) If the closed surface has a negative charge or a net negative charge, the flux will be negative. The flux will be inward flux, i.e. more lines will be enteing than the leaving lines provided the normal is taken outward.

11.5.2 Applications of Gauss's Law

Gauss's law provides a convenient method to calculate electric field in case of sufficiently symmetric charge distribution. Here we discuss some applications of Gauss's law:

Absence of Electric Field and Charge Inside a Condctor and Location of Excess Charge on a Conductor

Under steady state conditions, electric field is zero inside a metal or conductor. This statement is proved by following considerations; Conductors have charges in them, which are free to move. If a resultant field E exists inside the conductor, these charges will experience a force due to the field. They will move and internal currents will set-up. Eventually, within fractions of second, electrostatic equilibrium is achieved. The internal currents will stop. When no electric current flows, resultant force on the charges (electrons) in the conductor must be zero. Since, electric field is responsible for exerting force on the charges, hence, when electrostatic equilibrium is achieved, the electric field in the interior of conductor must be zero. In fact, Gauss's law can be used to show that an excess charge placed on an insulated conductor, resides on its outer surface. For this purpose, consider an isolated conductor carrying an excess charge 'q'. Take Gaussian surface 'S' shown by dotted line. 'S' lies inside just below the actual suface of the conductor as shown:

As under steady state conditions, the electric field inside a conductor is zero. Applying Gauss's law to the Gaussian's surface 'S', we have:

$$\sum_{surface} \vec{E}.\Delta\vec{A} = \frac{Q}{\epsilon_o} \tag{11.24}$$

As 'E' is zero, so $\sum_{surface} \vec{E}.\Delta\vec{A} = 0$. Now, $\epsilon_o \neq \infty$, as it has a finite value equal to $8.85 \times 10^{-12}\, C^2/Nm^2$, so above equation implies:

$$Q = 0$$

This means that there is no charge inside the Gaussian's surface 'S'. As this surface lies just below the actual surface of the conductor, having no charge inside it. This means that the charge is on the actual surface of the conductor.

Electric Field Intensity due to a Charged Conducting Spherical Shell

Shell: ***"Any hollow enclosure with a covering is called a shell."***

Calculation of Electric Field: We are interested in the calculation of electric field due to a charged conducting sphericall shell. For this purpose, consider a metallic shell of radius 'R' having a positive charge 'Q'. We know that the charge will distribute itself uniformly on the conducting surface. As shell has an interior and exterior, therefore, we will find electric intensity seperately for interior and exterior of charged conducting spherical shell.

Field at the Interior: Suppose we want to find the intensity of the field at an interior point 'B' at a distance $r_B < R$ from the centre of the shell. Through the point 'B' construct a 'Gaussian's surface' (Sphere) of radius 'r' with centre at the centre of the shell, as shown:

Since there is no charge inside the gaussian surface, hence Gauss's law gives:

$$\sum_{surface} \vec{E_B}.\Delta\vec{A} = \frac{Q}{\epsilon_o} = 0$$

The charge distribution is spherically symmetric. This implies that $\vec{E_B}$ is radial and has a constant magnitude E_B at all points on the gaussian's surface at the radial distance 'r_B' from the centre of the shell. The vector $\Delta\vec{A}$ at every point is also radial, So we can write above equation as:

$$E_B \sum_{surface} \Delta A = 0$$

As $\sum_{surface} \Delta A \neq 0$ and equal to $4\pi r^2$, So

$$E_B(4\pi r^2) = 0$$

Hence,

$$E_B = 0$$

As 'B' was chosen as an arbitrary point, therefore, we conclude that the field inside a charged conducting spherical shell is zero.

Field at the Exterior: Let us how find the field at an arbitrary point P, which is outside the shell and at a ditance 'r_p' from the centre of the shell. Through P, construct a gaussian surface of radius 'r_p' concentric with the shell, as shown: Applying Gauss's law:

$$\sum_{surface} \vec{E_P}.\Delta\vec{A} = \frac{Q}{\epsilon_o}$$

Because of spherical symmetry, $\vec{E_p}$ must be radial. $\Delta\vec{A}$ is also radial. So, above equation implies:

$$E_P \sum_{surface} \Delta A = \frac{Q}{\epsilon_o}$$

As $\sum_{surface} \Delta A = 4\pi r^2$, So

$$E_P(4\pi r^2) = \frac{Q}{\epsilon_o}$$

$$E_P = \frac{Q}{4\pi\epsilon_o r^2} \tag{11.25}$$

Conclusion: The equation is identical with Coulomb's law for a point charge. This means that the field outside a charged sphere is the same as that of the field due to an equal-magnitude point charge placed at the centre of the sphere.

Distribution of electric Charge on a Hollow Conductor having a Charge in its Cavity

Here we discuss two cases:

Case 1: The Hollow Conductor is Uncharged: Consider an uncharged hollow conductor with '$+q$' charge placed inside it. We insulate it so that no charges can jump from one surface to another. We consider a gaussian's surface 'S' of same geometry as shown:

Now, using Gauss's law, we can conclude that the flux through the gaussian's surface is zero, which means charge enclosed must be zero. So, in order to maintain the neutral status, '$-q$' charge will appear on the surface of the conductor. As charge is always conserved, so '$+q$' must lie on the outside surface of the conductor (see figure **??**).

Case 2: When the Hollow Conductor is Already Charged If in the previous case, we take a charged conductor having a charge 'Q' on it. As charge resides on outer surface,so its outer surface will have 'Q' charge. Now, if 'q' is inserted inside it, then again by taking a gaussian's surface, the flux and hence the charge inside that must be zero. So, in order to maintain neutral status, a charge having opposite sign as that of charge inside the cavity will appear on the inside surface, so the net charge on the outer surface of the conductor will be algebraic sum of 'Q' and 'q'. Example is shown in figure **??**

Electric Field Intensity due to an Infinite Sheet of Charge

Definition of Infinite sheet: ***"The sheet of charge is said to be infinite with respect to a point 'P' if the dimensions of the sheet are very very greater than than the distance of 'P' from the sheet."***

Calculation of Electric Field: To calculate the electric field intensity due to an infinite sheet of charge, let us consider a sheet of positive charge having constant surface charge density 'σ' (charge per unit area). We have to calculate the field at point 'P' close to the sheet. Imagine a gaussian's surface in the form of cylinder passing through the sheet as shown:

The cylinder has cross sectional area 'A'. The surface charge density of the sheet (assumed constant) is given by:

$$\sigma = \frac{Q}{A}$$

Now, it is clear that electric field is parallel to the area vectors of the right and left faces. For curved surface, consider the surface to be composed of small area components. The electric field is always perpendicular to each '$\Delta\vec{A}$' vector on each point on the curved surface. Now let the charge enclosed by the cylindrical gaussian surface is 'q', then 'q' will be equal to:

$$q = \sigma A \tag{11.26}$$

Now, total flux will be equal to the flux through right and left end faces, because the curved surface contributes no flux (lines can not pass through the curved surface). For each face, $\vec{E}$ and $\vec{A}$ are in same direction, so flux due to each left and right face will be EA. Hence, total flux will be:

$$\Phi = EA + EA + 0$$

$$\Phi = 2EA \tag{11.27}$$

From Gauss's law:

$$\Phi = \frac{q}{\epsilon_o} \tag{11.28}$$

Comparing equations 11.27 & 11.28:

$$2EA = \frac{q}{\epsilon_o}$$

Putting equation 11.26 in above equation, we get:

$$E = \frac{\sigma}{2\epsilon_o} \tag{11.29}$$

If '$\hat{n}$' is the unit vector directed normally outwards from the sheet, then we can write:

$$\vec{E} = \frac{\sigma}{2\epsilon_o}\hat{n} \tag{11.30}$$

Note that the 'E' is independent of 'r'. This result is correct approximately for real sheets (not infinite) if 'r' is very close to the sheet.

Electric Field Intensity between Two Oppositely Charged Parallel Metal Plates

To calculate the intensity of electric field between two oppositely charged metal plates, let us consider two oppositely charged parallel metal plates. The charge densities are '$+\sigma$' and '$-\sigma$' (assumed constant) with respect to the charge on the plate. The electric field is uniform in the region between the plates, and is normal to them. We have to calculate the field at an arbitrary point 'P'. For this purpose, we consider gaussian surface in the form of a box as shown:

Let 'A' be the cross sectional area of the box. As the surface must enclose a net charge (let 'Q' be the charge enlosed), so the top face of the box is inside the upper plate. There are six faces of the box. The area vectors of the left, right, front and back faces are perpendicular to $\vec{E}$, hence contributes no electric flux. The top face is inside the plate so electric field is zero, hence no flux will be out of there. So the only contributing surface is bottom face, in which $\vec{E}$ and area vector are in same direction. So flux will be:

$$\Phi = EA \tag{11.31}$$

As,

$$q = \sigma A \tag{11.32}$$

And from Gauss's law:

$$\Phi = \frac{Q}{\epsilon_o} \tag{11.33}$$

So from equations 11.31, 11.32 & 11.33, we can write:

$$EA = \frac{\sigma A}{\epsilon_o}$$

which implies:

$$E = \frac{\sigma}{\epsilon_o} \tag{11.34}$$

This is the expression for electric field of two oppositely charged parallel metal plates. It is independent of position at which you are to find the field strength.

Note: As we know that electric field is bulging out at the ends of plates. So, to avoid this bulging (fringing field), the plates should be of infinite length. So, for practical purposes, the plates are said to be infinite if the distance between them is much smaller than their dimensions.

Alternate Method: Another interesting method for calculation of electric field between the oppositely charged metal plates, let us consider them to be infinite sheets of charge seperately. For positively charged sheet, $\vec{E}$ is directed upward and directed downward below it, having magnitude equal to $\frac{\sigma}{2\epsilon_o}$ below and above. Similarly, for negatively charged sheet, the field above it is directed downward, and below it, is directed upward having magnitude equal to $\frac{\sigma}{2\epsilon_o}$, below and above. We can understand it by figure below:

$$E = \frac{\sigma}{2\epsilon_o} + \frac{\sigma}{2\epsilon_o}$$
$$E = \frac{\sigma}{\epsilon_o}$$

This method is more efficient for such cases. We can also show that the electric field due to these plates will be zero above and below the plates. Similarly, we can also calculate for two same charge plates.

11.6 Electric Potential

"The amount of work done in moving a unit positive charge from infinity to a point inside the electric field against the electric field is called electric potential."

Explanation

Let us consider a positive charge '$+q_o$' placed in between oppositely charged plates.A charge experiences a force q_oE in an electric field. If the charge is allowed to move freely inside the electric field, it will move from 'A' to 'B' and gain kinetic energy. If we have to move the charge against the electric field, we have to apply external force. Now, in order to move the charge from 'B' to 'A' without giving acceleration, an external force must be applied which will be equal and opposite to q_oE as shown: Let W_{BA} be the amount of work done by external force in carrying 'q_o' from 'B' to 'A', without disturbing the equilibrium state of the charge. Change in potential energy of 'q_o' is defined to be the work done by the force applied in carrying it from one point to other against the electric field i.e.

$$\Delta U = W_{BA}$$

And,

$$U_A - U_B = W_{BA} \tag{11.35}$$

Where ΔU is the change in potential energy and U_A and U_B are the electric potential energies at ponts 'A' and 'B' respectively. Electric potential energy at a point is equal to the amount of work done in moving a charge from infinity to that point inside the electric field against the field, i.e. at point 'A' and 'B':

$$U_A = W_{\infty \to A} \, and \, U_B = W_{\infty \to B} \tag{11.36}$$

Now, dividing equation 11.35 by 'q_o':

$$\frac{U_A}{q_o} - \frac{U_B}{q_o} = \frac{W_{BA}}{q_o} \tag{11.37}$$

But the potential energy at a point per unit charge is potential, denoted by 'V'. Hence,

$$\frac{U_A}{q_o} = V_A \;\; and \;\; \frac{U_B}{q_o} = V_B \tag{11.38}$$

From equation 11.36, we can write:

$$V_A = \frac{W_{\infty \to A}}{q_o} \tag{11.39}$$

And,

$$V_B = \frac{W_{\infty \to B}}{q_o} \tag{11.40}$$

So, for any point, we drop subscripts and get:

$$V = \frac{W}{q_o} \tag{11.41}$$

which is the actual definition of electric potential at a given point.

Now, Putting values from equation 11.38 into equation 11.39, we can write:

$$V_A - V_B = \frac{W_{BA}}{q_o}$$

Writing $V_A - V_B = \Delta V$ (change in potential, called as potential difference), we get:

$$\Delta V = \frac{W_{BA}}{q_o} \tag{11.42}$$

So, potential difference between two points in electric field is defined as, ***"The work done in bringing a unit positive charge from one point to another inside the electric field keeping the charge in electrostatic equilibrium."***

Unit of Potential Difference

The S.I unit of potential difference is joule per coulomb known as volt (V), after great scientist Volta.

One Volt

"One volt is the amount of potential difference between two points in an electric field if one joule of work is done in moving one coulomb of charge from one point to the other."

Submultiples are:

$$1 \text{ millivolt} = 10^{-3} \text{ V}, 1 \text{ microvolt} = 10^{-6} \text{ V}$$
$$1 \text{ Gigavolt} = 10^{9} \text{ V}, 1 \text{ kilovolt} = 10^{3} \text{ V}$$

11.6.1 Electric Potential at a Point due to a Point charge

Definition

"The potential at a point at some distance 'r' from a point charge 'q' is the amount of work done per unit charge required to bring from infinity distance to that point."

Mathematical Derivation

Let us consider a charge 'Q' fixed in space. If a test charge 'q' is placed at infinity, the force on the test charge due to charge 'Q' will be zero. As test charge is chosen positive, so when it is moved from infinity towards 'Q', the force of repulsion acts on it. So, work is required to be done to bring it to point 'B'. Hence, when the charge is moved towards point 'B', an amount of electric potential energy will be stored in it.

As we know that work done on a body is given by $\vec{F}.\vec{d}$ i.e. work done between two points is given as:

$$\Delta W = \vec{F}.\vec{d} \tag{11.43}$$

The force is required to move charge against the field, so

$$\vec{F} = -q\vec{E} \tag{11.44}$$

Putting 11.44 into 11.43:

$$\Delta W = -q\,\vec{E}.\vec{d} \tag{11.45}$$

Since E and d are oppositely directed, so equation 11.45 reduces to:

$$\Delta W = qEd \tag{11.46}$$

Now, electric field due to a point charge at a distance 'r' is given by:

$$E = \frac{Q}{4\pi\epsilon_o r^2} \tag{11.47}$$

It varies inversely as square of the distance. So, it does not remain constant over the distance. In order to use above equation, E must be constant, so we divide the distance over which the test charge is moved into infinitesimally small displacements 'Δr'. We consider two points 'A' and 'B' for convenience and charge is moved between these points over small displacements 'Δr' as shown:

So equation 11.46 gives:

$$\Delta W = qE\Delta r \tag{11.48}$$

Putting value of E from equation 11.47 into equation 11.48, we get:

$$\Delta W = \frac{Qq}{4\pi\epsilon_o r^2}\Delta r \tag{11.49}$$

Now, let the test charge is moved through small intervals from r_A to r_1, ($\Delta r = r_A - r_1$). We assume that ponts are very closer, still we have to take the average of E. As E at a distance 'r_A' is:

$$E_A = \frac{Q}{4\pi r_A^2}$$

And 'E' at a distance 'r_1' will be:

$$E_1 = \frac{Q}{4\pi r_1^2}$$

Hence at the beginning of the interval, E varies as $\frac{1}{r_A^2}$ and at the end of the interval, it is varying with $\frac{1}{r_1^2}$. In order to find the average value of E, we have to find an average value of r in equation 11.49. i.e.

$$r = \frac{r_A + r_1}{2}$$

Squaring on both sides, we get:

$$r^2 = (\frac{r_A + r_1}{2})^2 \tag{11.50}$$

As $\Delta r = r_A - r_1$, which implies

$$r_1 = r_A + \Delta r$$

Putting this value of r_1 in equation 11.50, we get:

$$r^2 = (\frac{r_A + (r_A + \Delta r)}{2})^2$$

$$r^2 = \frac{4r_A^2 + \Delta r^2 - 4r_A\Delta r}{4}$$

For smaller Δr, we have $\Delta r^2 = 0$ so the above equation reduces to:

$$r^2 = \frac{4r_A^2 - 4r_A\Delta r}{4}$$

$$\implies r^2 = r_A^2 - r_A\Delta r \tag{11.51}$$

Put back value of Δr in 11.51:

$$\implies r^2 = r_A^2 - r_A^2 + r_A r_1$$

$$\implies r^2 = r_A r_1$$

Now putting value of Δr and r^2 in 11.49, we get:

$$\Delta W = \frac{Qq}{4\pi\epsilon_o}\frac{r_A - r_1}{r_A r_1}$$

which implies:

$$\Delta W_{r_A \to r_1} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_1} - \frac{1}{r_A}) \tag{11.52}$$

Now continuing in this manner, from r_1 to r_2, equation 11.52 implies:

$$\Delta W_{r_1 \to r_2} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_2} - \frac{1}{r_1}) \tag{11.53}$$

Similarly from r_{n-1} to r_n,

$$\Delta W_{r_{n-1} \to r_n} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_n} - \frac{1}{r_{n-1}}) \tag{11.54}$$

And finally from r_n to r_B,

$$\Delta W_{r_n \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{r_n}) \tag{11.55}$$

Now, total work will fron point 'A' to 'B' will be:

$$W = \Delta W_{r_A \to r_1} + \Delta W_{r_1 \to r_2} + ... + \Delta W_{r_{n-1} \to r_n} + \Delta W_{r_n \to r_B}$$

Putting respective values:

$$W = \frac{Qq}{4\pi\epsilon_o}(-\frac{1}{r_A} + \frac{1}{r_1} - \frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_2} + ... - \frac{1}{r_n} + \frac{1}{r_n} + \frac{1}{r_B})$$

which will result:

$$W_{r_A \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{r_A}) \tag{11.56}$$

This gives the on a test charge q when it is brought from one point to another point inside the field against its direction. Now if point 'A' is chosen at infinity, then equation 11.56 implies:

$$W_{\infty \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{\infty})$$

So,

$$W_{\infty \to r_B} = \frac{Qq}{4\pi\epsilon_o}\frac{1}{r_B}$$

Since 'B' is an arbitrary point, so we drop subscripts and above equation will be written as:

$$W = \frac{Qq}{4\pi\epsilon_o}\frac{1}{r} \tag{11.57}$$

This work is stored as 'Electric Potential Energy'. Since, electric potential is given by:

$$V = \frac{W}{q}$$

So, putting equation 11.57 in the potential equation:

$$V = \frac{Qq}{4\pi\epsilon_o q}\frac{1}{r}$$

which implies:

$$V = \frac{Q}{4\pi\epsilon_o r} \tag{11.58}$$

This the mathematical form of potential at a distance 'r' from a point charge. It varies as 1/r and is inependent of test charge. This is relative to infinity (which is a point of zero electric potential). Moreover, electric potential is a scalar quantity. It can be positive, negative depending upon the sign of charge. Potential of a point charge 'Q' at distance 'r' is considered positive and considered negative for a negative charge.

As potential is a scalar quantity, therefore electric potential due to several point charges q_1, q_2, q_3,..., q_n would be the algreabraic sum of the individual potentials due to each charge i.e.

$$V = V_1 + V_2 + V_3 + ... + V_n \tag{11.59}$$

Note:

(i) The potential very near the positive charge is large and decreases towards zero the further away from the charge. If the charge producing the field is negative, the potential is also negative and increases towards zero at larger distances.

(ii) Variation of potential is an inverse propotionality with distance and variation of electric field is inverse square propotionality with distance.

(iii) In case of uniform field, e.g. field between two parallel plates, the potential decreases uniformly as we move from positive hand plate to negative hand plate.

(iv) Electric field is a conservative field i.e. work done is independent of the path followed, it is not important that which way you move the charge rather it depends on the initial and final positions of the charge. Also the work done on the closed path will be zero.

(v) We have an idea of positive work done that when the energy of the system increases work done is said to be positive and if the energy decreases the work done will be negative. So work done against the field is said to be positive as it increases the potential energy of the system and vice versa.

11.7 Potential Gradient and Electric Field Intensity

"The maximum rate of change of electric potential with respect to position is called potential gradient."

The purpose of this topic is to establish a relationship between electric field intensity and the electric potential at a certain point.

Mathematical Derivation

Let us consider oppositely charged parallel plates. The electric field between the plates is uniform, hence field intensity 'E' is constant. Let a positive charge 'q_o' is moved from point 'A' to 'B' as shown: The potential difference between points 'A' and 'B' according to the definition will be:

$$\Delta V = \frac{W_{AB}}{q_o} \tag{11.60}$$

And,

$$W_{AB} = \vec{F}.\vec{d} \tag{11.61}$$

In this case,

$$\vec{F} = q_o\vec{E} \tag{11.62}$$

Putting in equation 11.62:

$$W_{AB} = q_o\vec{E}.\vec{d} \tag{11.63}$$

Here $\vec{E}$ and $\vec{d}$ are acting in opposite direction, hence equation 11.63 implies:

$$W_{AB} = -q_oEd \tag{11.64}$$

Putting in equation 11.60, we get:

$$\Delta V = \frac{-q_oEd}{q_o} \tag{11.65}$$

which gives:

$$E = -\frac{\Delta V}{d} \tag{11.66}$$

For a small separation of two points denoted by 'Δr', the equation 11.66 results:

$$E = -\frac{\Delta V}{\Delta r} \tag{11.67}$$

As Δr is the perpendicular distance between the plates, hence the shortest distance. So 11.67 will give maximum rate of change of potential. This maximum rate of change of potential with respect to the distance Δr is called potential gradient. Hence, we can define electric field as, ***"The negative rate of change of potential with respect to distance,"*** or, ***"Electric field is the negative gradient of potential."***

Another unit of Electric Field

From equation 11.67, the unit of electric field intensity is volt per meter (V/m) which is equal to N/C.

Significance of Negative Sign

The negative sign indicates that the direction of field is opposite to the direction in which the potential is increasing. To understand this, we can think of a positive charge as a source charge. When we bring a positive test charge towards it, the potential will increase i.e. towards the charge and the field is radially outward away from the charge. Similarly, potential increases as we move the test charge away from a negative charge and the electric field is radially inward towards the negative charge.

Note:

It is important to note that gradient of a quantity is the change in that quantity with distance. Potential is a scalar quantity but gradient of potential is a vector.

☑ Checkpoint 11.4

Show that $\frac{V}{m}$ is equal to $\frac{N}{C}$.

11.8 Equipotential Surface/Equipotential Line

"The surface/line passing through such points which have the same value of electric potential is called equipotential surface/line."

Electric field is very difficult to represent in diagram. Both strength and direction of field must be properly indicated at every point in the field. As an alternative to field line diagrams, 'contour maps' of the electric field can be drawn using equipotential lines. These lines connect all the points in space where the potential of an electric field is the same as shown:

Show that Two Points on an Equipotential Line have Same Potential

Consider two points 'A' and 'B' on an equipotential line at a distance 'r' from the point charge 'q' as shown:

The electric potential due to a point charge at a distance 'r' is given by:

$$V = \frac{Q}{4\pi\epsilon_o r}$$

Now, work done on test charge 'q_o' between points 'A' and 'B' on the equipotential surface will be:

$$W_{AB} = q_o \Delta V$$

$$W_{AB} = \frac{Qq_o}{4\pi\epsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

As $r_A = r_B$, hence,

$$W_{AB} = \frac{Qq_o}{4\pi\epsilon_o}(\frac{1}{r} - \frac{1}{r})$$

which implies:

$$W_{AB} = 0$$

It means that if a charge is moved over an equipotential line, work done on the charge will be zero. Also we can say that potential difference between two points on an equipotential line is zero. Zero potential difference means points are at same potential.

11.8.1 Characteristics of Equipotential Surface/Line

Some properties related with equipotential line/surface are discussed as:

(i) Two equipotential lines can not intersect. Since over the whole equipotential line, a charge if mooved, bears the same potential.So at the intersection,the charge will have two values of potential which is not possible.

(ii) No work is done in moving a charge from one point to another on an equipotential surface.

(iii) Work is done in moving a charge from one surface to other equipotential surface.

(iv) If there are two or more charges, then electric potential at any point is the sum of the potential due to the individual charges. e.g. for a positive and negative charge placed a distance apart, equipotential lines are as shown:

(v) Electric field lines and equipotential lines are always perpendicular to each other e.g. field pattern for an isolated charge and between oppositely charges plates along with equipotential lines is as shown:

(vi) Closer the equipotential lines, the stronger will be the field at a given point. So the potential energy must be changing by large amounts in small distances and there must be a large force acting.

Example

The boundary of a conductor is an example of an equipotential surface in electrostatics.

11.9 The Electron-Volt

"It is the amount of energy gained or lost by an electron when it is displaced across two points between which the potential difference is one volt."

Explanation

We know that when a charge 'q' is moved up across a potential difference 'ΔV', then the electric potential energy of the charge increases by an amount given as:

$$\Delta U = q\Delta V$$

Also, when the same charge 'q' is allowed to move inside the electric field through the same potential difference ΔV, it loses its potential energy and gains the kinetic energy i.e.

$$\Delta K.E = q\Delta V$$

As by definition, when $q = 1e$ and $\Delta V = 1V$, then:

$$\Delta K.E = 1\,eV$$

Relationship with Joule

$$1\,eV = 1.602 \times 10^{-19}C \times 1V$$

which gives:

$$1\,eV = 1.602 \times 10^{-19}\,J$$

From above relation, it is clear that the electron volt (eV) is another unit of energy like joule. Its small size is enough to be convenient for the energies of elementary particles. Its submultiples are:

$$1 \text{ MeV} = 10^6 \text{ eV (million electron volts)}$$
$$1 \text{ GeV} = 10^9 \text{ eV (billion electron volts)}$$

11.10 Capacitor

"A device used for storing electric charges is called capacitor." or, ***"A device in which electrical energy is stored."*** For example, the batteries in a camera store energy in the photoflash unit by charging a capcaitor. The batteries can supply energy at only a modest rate, too slowly for the photoflash unit to emit a flash of light. However, once the capacitor is charged, it can supply energy at a much greater rate when the photoflash unit is triggered___enough energy to allow the unit to emit a burst of bright light. In this topic we will understand some basics related with capacitors including their charging discharging and their applications.

Construction

Capacitors have many different forms. One form is parallel plate capacitor, which consists of two parallel metal plates, seperated by small distance. The medium between the plates is air or a sheet os some insulator. This medium is known as 'dielectric'. Other forms include cylindrical and spherical capacitors. In electrical circuits, capacitors are denoted by '⊣⊢'. This symbol is for parallel plate actually but in practice we denote the capacitor of any geometry by this symbol. The vertical lines represent the two conductors and parallel lines represent connection in circuit.

Figure 11.16: An assortment of capacitors.

Charging of a Capacitor

The capacitor is commonly charged by connecting its plates for while to the opposite terminals of the battery. In this way, some of the electrons are transferred through the battery from the postive plate to the negative plate. Charge 'Q' and '-Q' appears on the plates. Mutual attraction between the charges keep them bound on the inner surfaces of the both plates and thus the charge remains stored in the capacitor even after the removal of the battery.

Mechanism of Charging

A battery is a device that maintains a certain potential difference between its terminals (points at which charge can enter or leave the battery) by means of internal electrochemical reactions in which electric forces can move internal charge (we will study this in detail in next chapter). To understand the process of charging, let us consider a capacitor C, connected in an electrical circuit through a battery B and a switch 'S' as shown:

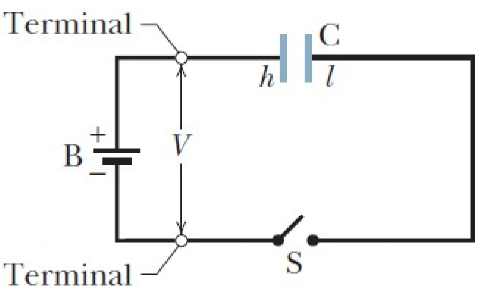


Figure 11.17: A battery B connected to capacitor with switch 'S' in a circuit.

The battery has two terminals positive terminal (+) at high potential and negative terminal (-) at low potential.The battery maintains a constant potential difference 'V' between its terminals. The circuit shown in figure above is said to be incomplete when the switch 'S' is open. When the switch is closed, electrically connecting those wires, the circuit is complete and the charge can then flow through the switch and the wires.When the circuit is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires. The field drives electrons from left plate to the positive terminal of the battery, thus left plate, losing electrons, becomes positively charged. The field drives just as many electrons from the negative terminal of the battery to right plate, gaining electrons, becomes negatively charged.

Initially, when the plates are uncharged, the potential difference between them is zero. As the plates become oppositely charged, that potential difference increases until it equals the potential difference of the battery.Then both the plates are at same potential, and there is no longer an electric field in the wire between them. Capacitor is then said to be fully charged.

11.10.1 Capacitance of the Capacitor

"The capability of a capacitor to store charges is called capacitance of that capacitor."

Explanation

As we know that if we increase the potential difference, the charge 'Q' on the plates increases which increases the potential difference 'V' between the plates i.e.

$$Q \propto V$$

And,

$$Q = CV \tag{11.68}$$

Where 'C' is the cinstant of propotionality and is constant for a particular capacitor and is called its capacitance, and is defined according to the equation below:

$$C = \frac{Q}{V} \tag{11.69}$$

Hence, ***"The ratio of the magnitude of the charge on the either plate of the capacitor to the potential difference between the plates."***

Note:

We observed that charges '$+q$' and '$-q$' are present on the plates of the capacitor. It does not mean that net charge is zero. Whenever we talk about charge on a capacitor we should refer to charge on either plate. The picture becomes more obvious if we say net charge is absolute value of charge on either plate.

☑ Checkpoint 11.5:

Does the capacitance of a capacitor C increase or decrease or remain constant if :

(a) The charge 'q' on it is doubled

(b) Potential difference across it is tripled

Dependence of Capacitance

The capacitance of a capacitor depends upon:

(i) Size of the capacitor

(ii) Shape of the capacitor

(iii) Seperation between the plates

(iv) Dielectric medium between them

Unit of Capacitance

The unit of capacitance is coulomb/volt known as farad (F) in the honour of great physicist Michael Faraday.

One Farad

"A capacitor has a capacitance of one farad if a charge of one coulomb will give its plates a potential difference of one volt." Convenient multiples are:

$$1\ \mu\text{F} = 10^{-6}\ \text{F},\ 1\ \text{pF} = 10^{-12}\ \text{F}$$

11.10.2 Capacitance of a Parallel Plate Capacitor

Parallel plate Capacitor

"A capacitor having two parallel metal plates, whose size is greater than the distance between the plates, is called parallel plate capacitor."

Mathematical Expression for Capacitance

Let us consider a parallel plate capacitor of capacitance 'C', plates area 'A' and distance between the plates 'd'. Let us charge the capacitor to 'Q'. As a result, a potential difference 'V' is created between the plates. As we know from previous sections:

$$V = Ed \tag{11.70}$$

And surface charge density 'σ' is given by:

$$\sigma = \frac{Q}{A}$$

As we considered capacitors plate of infinite length, so electric field will be uniform between the plates and is given by:

$$E = \frac{\sigma}{\epsilon_o}$$

Putting value of σ, we get:

$$E = \frac{Q}{\epsilon_o A}$$

Putting value of E in equation 11.70 we get:

$$V = \frac{Qd}{\epsilon_o A} \tag{11.71}$$

As we know from equation 11.69:

$$C = \frac{Q}{V}$$

Putting value of 'V' from equation 11.71,

$$C = \frac{Q\epsilon_o A}{Qd}$$

which gives us the value of capacitance, hence we can write:

$$\boxed{C = \frac{A\epsilon_o}{d}} \tag{11.72}$$

From here, it is found that capacitance of a parallel plate capacitor is:

(a) Directly propotional to the area A of the plates. Larger the area for a given seperation between the plates of a capacitor placed in vacuum, larger will be the ability to store charges, hence larger will be the capacitance.

(b) Inversly related with seperation of plates. Smaller will be the seperation for a capacitor of fixed plate area, the more will be the ability of that capacitor to hold on charges over the plates, thereby increasing capacitance.

Above equation is for capacitance of a parallel plate capacitor when the medium is free space or air or vacuum, so we can write:

$$\boxed{C_{vac} = \frac{A\epsilon_o}{d}} \tag{11.73}$$

If instead of vacuum, if there is placed a dielectric medium of relative permittivity or dielectric constant 'ϵ_r' between the plates of a parallel plate capacitor, then capacitance will increase by a factor of 'ϵ_r', and will be given by:

$$\boxed{C_{med} = \frac{A\epsilon_o\epsilon_r}{d}} \tag{11.74}$$

From above equation, it is also clear that a capacitor with a medium of high dielectric constant will have a large capacitance than the case when the same capacitor is placed in a medium of relatively low dielectric constant. Now taking ratio of equations 11.74 & 11.73, we get:

$$\epsilon_r = \frac{C_{med}}{C_{vac}} \tag{11.75}$$

This equation tells us that when a dielectric medium is introduced between the plates of capacitor, then the capacitance of that capacitor increases by a factor of 'ϵ_r', and the above equation also provides us with another definition of relative permittivity or dielectric constant or specific inductive capacity: ***"The ratio of the capacitance of a capacitor with a given material filling the space between the conductors to the capacitance of the same capacitor when the space is evacuated is the relative permittivity of the material."***

11.10.3 Combination of Capacitors

In elctrical circuits,sometimes we need to connect two or more capacitors either in series or in parallel to get the desired value of capacitance. Here we discuss:

(i) Series combination of capacitors

(ii) Parallel combination of capacitors

Series Combination of Capacitors

"When the capacitors are connected plate to plate i.e. the right plate of one capacitor is connected to the left plate of the next capacitor and so on, they are said to be in connected in series."

Diagram: Let us consider capacitors C_1, C_2, C_3 connected in series as shown in figure 11.18.

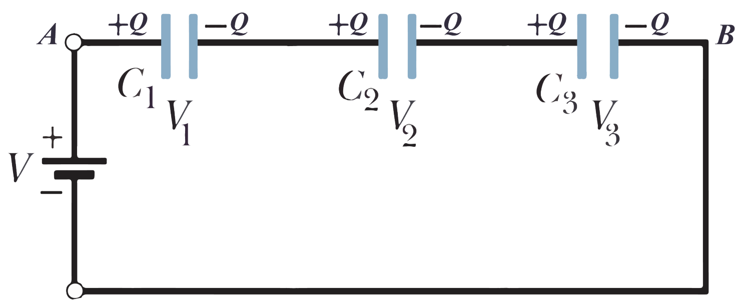


Figure 11.18: Capacitors in series.

Characteristics of Series Combination of Capacitors: Following are the characteristics of series combination:

(i) A battery of voltage 'V' is connected between points 'A' and 'B' so that the voltage divides between the capacitors.

(ii) The battery supplies charges 'Q' on the left plates of 'C_1', it induces charge '-Q' on its right plates,it induces 'Q' charge on the left plate of capacitor 'C_2' and this process goes on, hence each capacitor gets an equal charge 'Q' on each of its plates.

(iii) The potential difference 'V' in series combination is equal to the sum of potential differences across each capacitor i.e.

$$V = V_1 + V_2 + V_3$$

(iv) The equivalent capacitance of the series combination is less than the individual capacitances of the combination.

(v) The reciprocal of the equivalent capacitance of the combination is equal to the sum of the reciprocals of the individual capacitances.

Equivalent Capacitance of Series Combination: ***"A single capacitance which when connected in place of the capacitors of the combination so that there is no change in potential difference across the circuit and charge is called equivalent of a combination."***

Derivation: Let C_1, C_2, C_3 be the three capacitors shown in figure. The charge 'Q' is deposited on the each plate of each capacitor. The voltage applied is equal to the sum of potential differences between the plates of capacitors i.e.

$$V = V_1 + V_2 + V_3 \tag{11.76}$$

Let 'C_{eq}' be the equivalent capacitance of the circuit as shown:

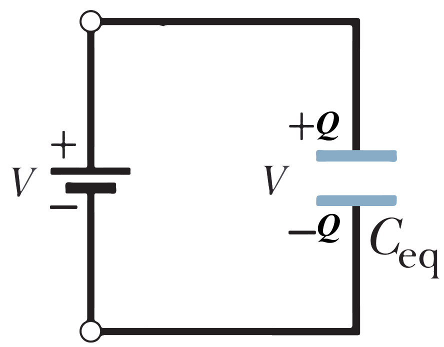


Figure 11.19: Equivalent capacitance of series combination.

The voltage across it is 'V' and charge is 'Q'. From equation 11.68, we can write:

$$Q = C_{eq}V$$

Now as the charge deposited on the plates of each capacitor is same. Hence their voltages can be written in terms of their capacitances. Hence,

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

and,

$$V = \frac{Q}{C_{eq}}$$

Putting respective values in equation 11.76, we get:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

which readily gives:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{11.77}$$

For two capacitors C_1 and C_2,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

which implies:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \tag{11.78}$$

For 'n' capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{11.79}$$

Conclusion: From above results, it is obvious that:

(i) In series combination, equivalent capacitance is less than the individual capacitances of the combination, or we can say that the equivalent capacitance is smaller than the smallest capacitance of the combination.

(ii) The reciprocal of the equivalent capacitance is equal to the sum of reciprocals of the individual capacitances of the combination.

(iii) Whenever it is desired to decrease the capacitance or a capacitance of low farads is required than the present capacitances, then we connect them in series combination to get our required capacitance.

Note:

(i) If we have two capacitors both of capacitance C in series, then their equivalent capacitance will be $\frac{C}{2}$.

(ii) If we have n capacitors each of capacitance C in series, then their equivalent capacitance will be $\frac{C}{n}$.

Parallel Combination of Capacitors

"When the capacitors are connected between the same two points in a circuit, they are said to be connected in parallel."

Diagram Let us consider capacitors C_1, C_2, C_3 connected in parallel between two points 'A' and 'B' as shown in figure 11.20.

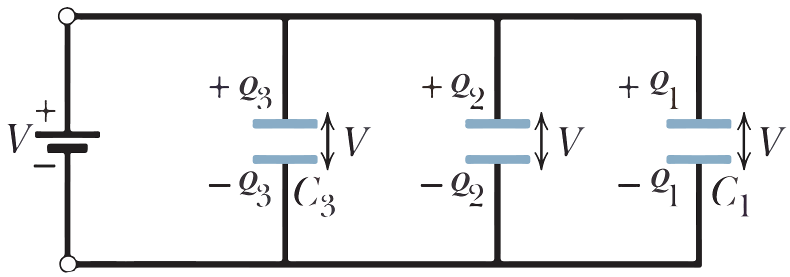


Figure 11.20: Capacitors in parallel.

Characteristics of Parallel Combination of Capacitors Following are the characteristics of parallel combination of capacitors:

(i) The voltage across the circuit is equal to potential difference between the plates of each capacitor as they are connected to two same points.

(ii) The charge 'Q' divides in parallel combination and is equal to the sum of charge deposited on the plates of each capacitor i.e.

$$Q = Q_1 + Q_2 + Q_3$$

(iii) The equivalent capacitance of the parallel combination of the capacitors is equal to the sum of individual capacitances of the combination.

(iv) The equivalent capacitance of the parallel combination is greater than any one of the capacitance of the combination.

Derivation of Equivalent Capacitance of Parallel Combination To derive an expression of equivalent capacitance 'C_{eq}' of the combination, consider the figure 11.21:

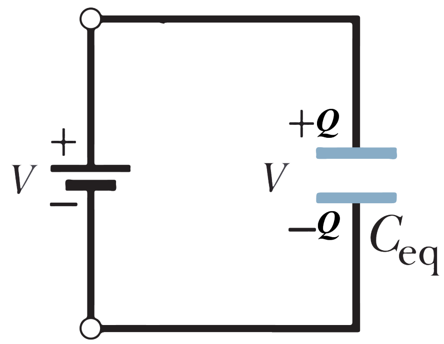


Figure 11.21: Equivalent capacitance of parallel combination.

The voltage across each capacitor is same. Charge flowing from the voltage source deposits according to the capacitance of each capacitance and is equal to the sum of the charge deposited on the plates of each capacitor i.e.

$$Q = Q_1 + Q_2 + Q_3 \tag{11.80}$$

From equation 11.68, we have,

$$Q = CV$$

For 'C'_{eq}, charge will be 'Q' and voltage will be 'V', hence we can write:

$$Q = C_{eq}V,\ Q_1 = C_1V,\ Q_2 = C_2V, Q_3 = C_3V$$

Putting all these values in 11.80, we get:

$$C_{eq}V = C_1V + C_2V + C_3V$$

which implies:

$$\boxed{C_{eq} = C_1 + C_2 + C_3} \tag{11.81}$$

For two capacitors C_1 and C_2,

$$\boxed{C_{eq} = C_1 + C_2} \tag{11.82}$$

For 'n' capacitors in parallel:

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + ... + C_n} \tag{11.83}$$

Conclusion: From above results, we can conclude that:

(i) The equivalent capacitance of the parallel combination is equal to the sum of individual capacitances of the combination.

(ii) The equivalent capcitance of the parallel combination is greater than the individual capacitance of the combination or we can say that the equivalent capacitance is larger than the largest capacitance of the combination.

(iii) Whenever we want an increased capacitance as compared with the individual capacitances, then we connect them in parallel.

Note:

(i) If we have two capacitors both of capacitance C in parallel, then their equivalent capacitance will be $2C$.

(ii) If we have n capacitors each of capacitance C in parallel, then their equivalent capacitance will be nC.

☑ Checkpoint 11.6

How many capacitors of capacitances 100 μF be connected in parallel with 100 V across each capacitor to store 2 C charge?

11.11 Energy Stored in a Capacitor

A capacitor is a device to store charge. As we defined capacitor as a device for storing electrical energy. The charge on each plate possesses electrical potential energy, which arises because work is required to be done to deposit charge on the plates. Initially, when the capacitor is uncharged, the potential difference between the conductors (capacitors plates are in general called as conductors) is zero volts. When the charge deposits, the potential difference rises gradually and when the potential difference becomes V, the capacitor is said to be fully charged which is the maximum value of potential difference. Hence the average potential difference between the plates will be:

$$V_{avg} = \frac{0 + V}{2} = \frac{1}{2}V$$

As the electrical potential energy 'U' is given by:

$$U = charge \times potential\ difference$$

Hence, energy will be given by:

$$U = \frac{1}{2}QV$$

Where Q is the charge on capacitor and V is the applied voltage. Also we know that for a capacitor:

$$Q = CV$$

Hence, we can write 'U' as:

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} \tag{11.84}$$

It is also possible to regard the energy being stored as the energy stored in the electric field between the plates rather than the potential energy of the charges on the plates. This helps us when the electric field E between the plates is considered rather than the charges on the plates causing the field to be considered. We know that:

$$V = Ed$$

And,

$$C = \frac{A\epsilon o\epsilon_r}{d}$$

Substituting these values in equation 11.84, we get:

$$U = \frac{1}{2}\frac{A\epsilon_o\epsilon_r}{d} \times (Ed)^2$$

which implies:

$$U = \frac{1}{2}\epsilon_o\epsilon_r E^2(Ad)$$

The product Ad represents volume between the plates. Let u denotes the energy density (energy per unit volume), then:

$$u = \frac{U}{Ad}$$

which implies:

$$\boxed{u = \frac{1}{2}\epsilon_o\epsilon_r E^2 \qquad (11.85)}$$

11.12 Electric Polarization

"The process of seperation of positive and negative charges from each other within a dielectric when placed inside an electric field is called electric polarization."

Explanation

An insulating medium commonly used between two plates of a capacitor is known as dielectric. The dielectric constant is represented in terms of relative permittivity, ϵ_r which is known as dielectric constant.

When a dielectric is inserted between the plates of an initially charged capacitor, due to positive and negative plates, a field exists directed from positive plate to negative, the electrons in the atoms of the dielectric medium are slightly displaced towards the positive plate. Similarly the nuclei are slighly displaced towards the negative field. These charges are fixed and can not move, hence called bound charges. Hence the electrical centres of positive and negative charges is not coinciding anymore and the material is said to be polarized as shown:

Effect on Electric Field

In case of polarization, the electric field due to induced charges is directed in a direction opposite to that of electric field due to the plates of capacitor(as shown in figure). Let $\vec{E_{ind}}$ be the electric field due to the induced charges and $\vec{E}$ denotes the electric field due to the plates of capacitor. Now when there was no dielectric present, only $\vec{E}$ was acting the role, now the two fields are oppositely directed giving rise to a net field $\vec{E_{net}}$ which is obviously less than $\vec{E}$. The strength of the field is reduced according to the dielectric material i.e the strength is reduced by a factor of ϵ_r compared to the case when there was vacuum. So, we can write:

$$\vec{E_{net}} = \vec{E} - \vec{E_{ind}}$$

And,

$$\vec{E_{net}} = \frac{\vec{E}}{\epsilon_r}$$

Hence,

$$\vec{E_{ind}} = \vec{E} - \frac{\vec{E}}{\epsilon_r}$$

$$\vec{E_{ind}} = \vec{E}(1 - \frac{1}{\epsilon_r}) \tag{11.86}$$

Result

"Whenever a dielectric medium is inserted between the plates of a charged capacitor the electric field of that capacitor is reduced by a factor of ϵ_r."

Effect on Voltage

As we know that:

$$V = Ed$$

Now, for a given capacitor, if dielectric is places between its plates, Electric field goes down by a factor of ϵ_r, hence voltage also goes down by a factor of ϵ_r.

Effect on Capacitance

As charges on the dielectric is bound and charges on the plates are also not moving, hence according to:

$$C = \frac{Q}{V} \tag{11.87}$$

As V goes down by ϵ_r, hence capacitance will go up (increase) by a factor of ϵ_r.

11.13 Charging and Discharging of a Capacitor

11.13.1 Process of Charging

"The process of storage of electric charges on the plates of a capacitor is called charging."

Explanation

To understand the process of charging of a capacitor, let us consider an arrangement in which a capacitor and a resistor are connected as shown in figure 11.22.

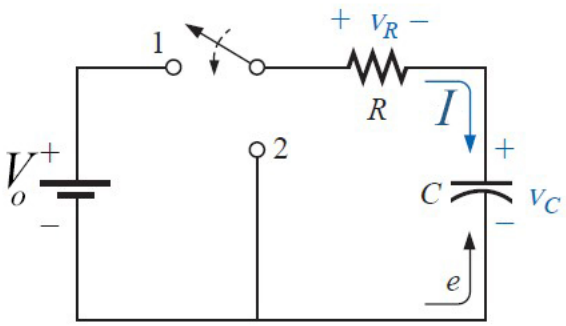


Figure 11.22: An arrangement for charging the capacitor.

This circuit is called Resistor-Capacitor, RC circuit. A battery of voltage V is connected as a power supply. When the switch is closed at position 1, charging circuit is closed, the battery starts charging the capacitor through the resistor R. The resistor is introduced to prevent instantaneous charging of capacitor. It takes sometime to charge the capacitor depending upon the value of resistor for a given capacitor. Finally the capcitor is charged to maximum charge of q_o and a voltage of V_o. Note that actually the capcitor never charges completely, which will be understood after equations of charging.

Graphical Representation

Suppose at $t = 0$, charge on the plate of capacitor is zero i.e. $q = 0$. With the passage of time, charge builds up gradually on the plates. It is found that charge at any instant of time 't' is given by the exponential relation:

$$q = q_o(1 - e^{\frac{-t}{RC}}) \tag{11.88}$$

where 'e' is Euler's constant having value 2.71828.... Looking over the equation 11.88: when $t = 0$,

$$q = q_o(1 - e^0) = 0$$

and when $t = \infty$, we have:

$$q = q_o(1 - 0) = q_o$$

This gives us a useful information that a capacitor requires infinite time to charge completely. Since there is no real value of time to charge a capacitor completely, hence a capacitor never charges completely. The curve shown for charging of a capacitor below also shows that the curve never crosses or touches the maximum value qo rather continues to move on alongside the parallel line through q_o (assymptotic behaviour[4]). The graph is shown in figure 11.23.

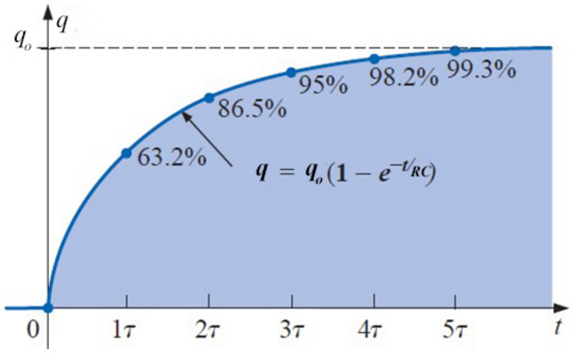


Figure 11.23: q versus t during the charging phase.

Voltage Curve for Capacitor during Charging

As we know that:

$$q = CV_C$$

Hence, equation 11.88 implies:

$$CV_C = CV_o(1 - e^{\frac{-t}{RC}})$$

which gives:

$$V_C = V_o(1 - e^{\frac{-t}{RC}}) \tag{11.89}$$

Here V_o is the maximum voltage upto which capacitor can charge. Hence the curve will have the same shape as for charge, only will be scaled.

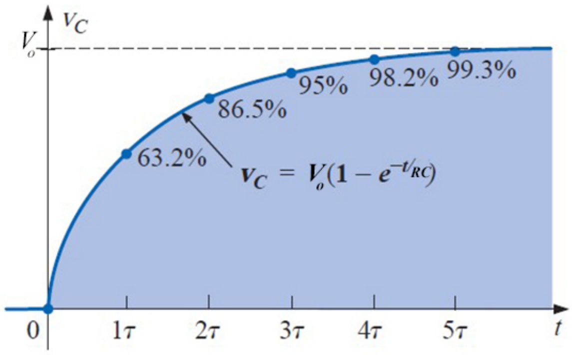


Figure 11.24: V_C versus t during the charging phase.

[4]If a curve follows a line upto infinity being parallel to it but never touches it, that line will be considerd as assymptote of that curve. Here the line $q = q_o$ will be assymptote to the charging curve.

Voltage Curve for Resistor during Charging

As we know that voltage from the supply V_o divides between capacitor and resistor and all the times their sum must be equal to V_o i.e.

$$V_o = V_R + V_C$$

Putting value of ''V_C' from equation 11.89, we get:

$$V_o = V_R + V_o(1 - e^{\frac{-t}{RC}})$$

which gives:

$$V_R = V_o e^{\frac{-t}{RC}} \tag{11.90}$$

This curve will start from V_o at $t = 0$ and goes to zero as t approches ∞ as shown in figure 11.25.

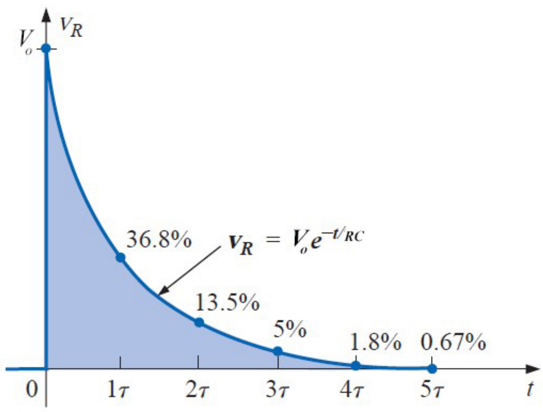


Figure 11.25: V_R versus t during the charging phase.

Current Curve during Charging

As both the components 'R' and 'C' are in series, so same current passes through both capacitor and resistor. From resistor concepts, we know that:

$$V_R = IR$$

Hence, we can write for I:

$$I = \frac{V_R}{R}$$

Putting value of 'V_R' from equation 11.90, we get:

$$I = \frac{V_o}{R} e^{\frac{-t}{RC}} = I_0 e^{\frac{-t}{RC}} \tag{11.91}$$

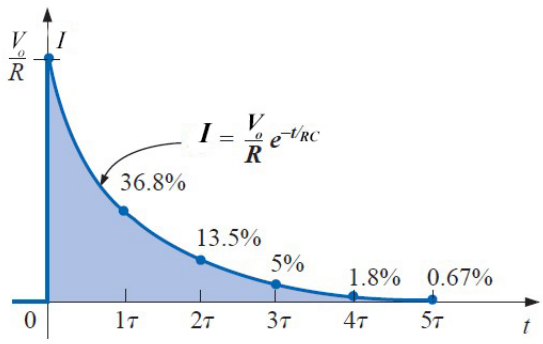


Figure 11.26: I versus t during the charging phase.

It is also worth noting that current curve for the capacitor will be same.

Time constant (RC)

"The factor RC is called time constant of the RC circuit." It is denoted by τ.
Time constant can be defined in case of charging as, ***"It is the duration of time in which 63.2% of its maximum value, charge is deposited on the plates of capacitor."*** From equation 11.88,

$$q = q_o(1 - e^{\frac{-t}{RC}})$$

If $t = RC$, then:

$$q = q_o(1 - e^{-1})$$
$$q = q_o(0.632)$$
$$q = 0.632q_o$$
$$\frac{q}{q_o} = 63.2\%$$

In short, we can say that the instant when the plates having charge 63.2% of the maximum charge, the time from the start up to that instant is time constant.

Approximate Time for Charging

As we said that a capacitor never charges 100%, so a capacitor is said to be fully charged after $t = 5RC$, to a value of 99.3% of the maximum value i.e.

$$q = q_o(1 - e^{-5})$$
$$q = q_o(0.993)$$
$$q = 0.993q_o$$
$$\frac{q}{q_o} = 99.3\%$$

11.13.2 Process of Discharging

"The removal of charge from the plates of the capacitor is called discharging."

Explanation

To understand the process of discharging, we consider again the circuit of figure 11.22. When the switch is placed in position 1, the capacitor will charge towards the supply voltage, as described in the charging section. At any point in the charging process, if the switch is moved to position 2, the capacitor will begin to discharge at a rate sensitive to the same time constant $\tau = RC$. The arrangement for discharging is shown in figure 11.27.

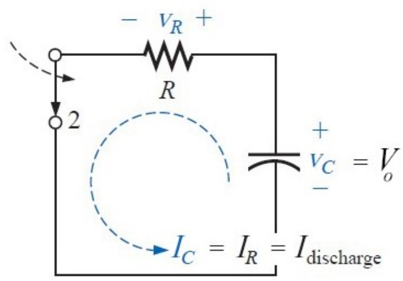


Figure 11.27: Discharging circuit.

The established voltage across the capacitor will create a flow of charge in the closed path that will eventually discharge the capacitor completely. In essence, the capacitor functions like a battery with a decreasing terminal voltage. Note in particular that the current I has reversed direction, changing the polarity of the voltage across R.

Graphical Representation

If the capacitor had charged to the full battery voltage as indicated in figure 11.27, the equation for the decaying charge across the capacitor would be the following:

$$q = q_o e^{\frac{-t}{RC}} \tag{11.92}$$

When $t = 0$, we have

$$q = q_o(e^0) = q_o$$

When $t = \infty$, we have

$$q = q_o(0) = 0$$

Note that initially, capacitor has charge 'q_o' on its plates. During discharging process it reduces below 'q_o'. Also it takes infinite time to reduce charge on the plates of the capacitor to zero. The curve for charge resembles voltage with V_o and V_c replaced with q_o and q.

Voltage Curve for Capacitor during Discharging

We can write the voltage equation for capacitor during charging as:

$$V_C = V_o e^{\frac{-t}{RC}} \tag{11.93}$$

Voltage Curve for Resistor during Discharging

As from the circuit:

$$V_R + V_C = 0$$

Hence,

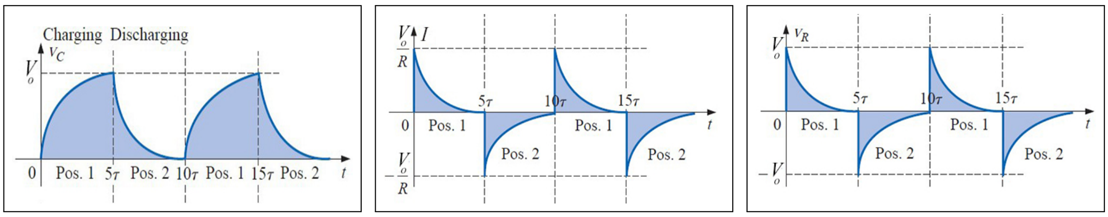
$$V_R = -V_o e^{\frac{-t}{RC}} \tag{11.94}$$

Current Curve during Discharging

From previous knowledge, we can write:

$$I = -\frac{V_o}{R} e^{\frac{-t}{RC}} = -I_o e^{\frac{-t}{RC}} \tag{11.95}$$

The current curve starts from '$-I_o$' as the direction of current is reversed, and it finally becomes zero when the capacitor has fully discharged. The same curve will be for current in capacitor as it is a series RC circuit.



(a) Voltage across capacitor during discharging (b) Current flowing through the circuit during discharging (c) Voltage across resistor during discharging

Figure 11.28: Graphs of various quantities during discharging phase compared with charging

Time Constant (RC)

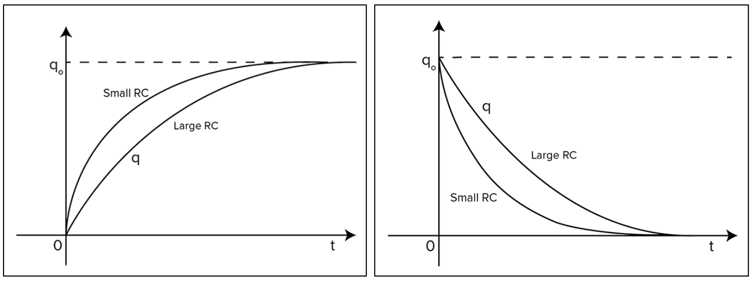
Time constant can also be defined from discharging. ***"It is the duration of time in which 63.2% of its maximum value, charge is decayed from the plates of capacitor."***

Hence, charge remaining on the plate can be found by putting $t = RC$ in equation 11.92, i.e.

$$
\begin{aligned}
q &= q_o(e^{-1}) \\
q &= q_o(0.368) \\
q &= 0.368 q_o \\
\frac{q}{q_o} &= 36.8\%
\end{aligned}
$$

Effect of Larger & Smaller $\tau = RC$

Larger the time constant $\tau = RC$, longer the curve will take to rise to its maximum value. Similarly, with larger values of RC, the curve will take longer time to decay to its minimum value. Reverse will happen when RC will be small. This is shown in figure 11.29:



(a) q curve for charging (b) q curve for discharging

Figure 11.29: Comparison of small and large RC.

Note:

Capacitor behaves as short circuit at start when charged and behaves as open circuit when fully charged. That's why we say that a capacitor blocks D.C. This behaviour is not observed when an alternating current passes through the capacitor.

Applications of Charging & Discharging

Charging and Discharging of capacitors find many applications. For example, in most of automobiles, the windshield wipers work automatically after small pauses during light drizzle. The timing of the ON-OFF cycle of these wipers is determined by the time constant of the RC circuit.

Some Important Concepts of Mcqs Regarding Capacitor

A. For a parallel plate capacitor with no dielectric placed in it and the power supply was disconnected after charging:

As we know that:

$$V = Ed \tag{i}$$

$$C = \frac{Q}{V} \tag{ii}$$

$$E = \frac{\sigma}{\epsilon} = \frac{QA}{\epsilon} \tag{iii}$$

When ‘d’ is changed (let say increased):

- ☐ Charge remains same (As charge is trapped)
- ☐ Electric field will not change as Q doesn’t change (σ is fixed) and ϵ is constant.
- ☐ As E remains same and d is increased, so V must increase according to equation i.
- ☐ As Q is same and V is increased so capacitance would decrease according to equation ii.

Result: When a capacitor with no dielectric in it is charged and power supply is removed; then by increasing distance between the plates will increase the P.D and hence a decrease in capacitance.

B. Now if a dielectric is inserted after the power supply was disconnected and distance between the plates is not changed:

- ☐ Electric field will reduce by ϵ_r according to equation iii.
- ☐ As distance is fixed and field is changed so according to equation i, V must reduce by a factor of ϵ_r.
- ☐ As charge is fixed (same) so ‘C’ will increase by a factor of ϵ_r. because V goes down by ϵ_r and according to equation ii, ‘C’ goes up by ϵ_r.

Result: When a charged capacitor with power supply disconnected, and a dielectric of permittivity ϵ_r is introduced then E and V decrease by ϵ_r and capacitance increases by ϵ_r.

C. Consider a capacitor of capacitance C with no dielectric in it (same as case A) but now the power supply is not disconnected (remains connected after charging) and d is increased:

- ☐ As battery is connected then V must not change (remains same).
- ☐ From equation i, if d is increased then E will be reduced.
- ☐ From equation iii, Q will be decreased hence capacitance will be decreased.

Result: If a capacitor charged and power supply remains connected then V will not change, and if d is increased then E will be reduced, C will decrease and Q will decrease and charge will flow from capacitor plates to the source.

D. Now a capacitor with power supply connected and distance between the plates fixed and a dielectric is inserted between the conductors:

- ☐ V will not change
- ☐ Important is that electric field will not change as from equation i, d and V both are fixed.
- ☐ As E remains same so charge must increase in order to make E constant according to equation iii.
- ☐ Q increases and V is constant so C must increase according to equation ii.

Result: When a charged capacitor with power supply connected and a dielectric is inserted keeping plates separation fixed then E and V remains same and capacitance and charge will increase by a factor of ϵ_r and charge will flow towards the plates.

Chapter 12

Electrodynamics/Current Electricity

12.1 Electric Current

"The flow of charge per unit time is called electric current".

OR

"Whenever electric charge flows, current is said to exist."

Symbol

It is denoted by 'I'.

Mathematical Form

If 'Q' is the amount of charge flowing through a wire of cross section 'A' in time 't', then current 'I' will be given by:

$$I = \frac{Q}{t} \tag{12.1}$$

Explanation

We know that substances which conduct electricity are known as conductors and which do not, called as insulators. Before the discovery of electron, proton etc., scientist supposed that the current is due to the flow of positive charges. The concept of flow of positive charges was developed because in physics, the positive terminal is at high potential with respect to a negative terminal. With the discovery of electron and nucleus, it is clear that:

(i) In metal conductors, the current is due to the flow of electrons only.

(ii) In liquids, current is due to flow of negative and positive charges, e.g in case of electrolytes.

(iii) In discharge through gaseous, the current is due to the flow of both charges.

When we talk about electric current, we often study the behaviour of metallic conductors. In metallic conductors, actually the electrons flow from negative terminal to positive terminal. However,prior to electron theory, it was assumed that positive charge flows only. But it was observed that a negative charge flowing in one direction has an equivalent effects if the same positive charge flows in opposite direction. So, the concept of flow of positive charge was retained as a convention and so called conventional current. The current due to electrons was called electronic current. The direction of electronic current is from negative terminal of the battery while that of conventional current is taken from positive to negative terminal of the battery. From now, we will take the direction of current as conventional direction i.e. from positive end to negative end.

12.2 Drift Velocity of Electrons in a Metallic Conductor

Every metal has a huge number of free electrons which wander randomly within the body of the conductor. The average speed of free electrons is sufficiently high, approximately of the order of 10^5 m/s. During random motion, the free electrons collide with the atoms of conductor again and again and after each collision, their direction of motion changes. As many electrons move in one direction, as much electrons move in opposite direction. So due to random motion, there is no net flow of charges in a particular direction, so no current exists in the case when no power supply is connected.

Motion of Charges When the Potential Difference is Applied

When a potential difference is applied across the ends of a conductor, it sets up an electric field at every point along the wire. As inside the conductor, there are free charges (free electrons in case of metallic conductors), so electrons experience a force in a direction opposite to the electric field given by:

$$\vec{F} = -e\vec{E} \tag{12.2}$$

Due to this force, electrons gain velocity and hence accelerate, then Newton's second law gives:

$$\vec{F} = m\vec{a} \tag{12.3}$$

This net force is actually the force exerted on electrons by the electric field, hence comparing equations 12.2 & 12.3, we get:

$$m\vec{a} = -e\vec{E}$$

$$\vec{a} = -\frac{e\vec{E}}{m} \tag{12.4}$$

So,direction of acceleration is in a direction opposite to electric field.As electrons move along the wire, they continuously collide with atoms, and due to the force by electric field, they acquire a net velocity along a direction, this velocity is called drift velocity and is defined as, ***"the average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called drift velocity."*** Symbolically, the drift velocity is denoted by $\vec{V_d}$.

Order of Drift Velocity

The drift velocity of electrons in a metallic conductor is of the order of 10^{-5} m/s.

Effect Transfer

As drift velocity of electrons is of the order of 10^{-5} m/s but when we switch on the bulb,it immediately glows. Although the drift velocity is negligible but the effect of electrons movement travel around the circuit with a speed approximately equal to the speed of light that is of the order of 10^8 m/s. That's why, when we switch on the bulb, the bulb glows in fractions of a second.

12.3 Sources of Current

"A device that supply a constant current by maintaining a constant potential difference between its two terminals is called a source of current."

Explanation

It is a firmly established convention that a positively charged body is at higher potential than a negatively charged body. When a body at a higher potential is connected to a body at a lower potential through a metallic wire, electric current will flow from higher potential to lower potential as shown in the figure below:

The current will stop when both the bodies come at the same potential. To maintain a steady current through the wire, the ends of the wire must be maintained at a constant potential difference. So, a device or agent must be present that will maintain the required potential difference. This device will convert some other forms of energy into electrical energy. Such a device is called a 'power supply' (or source of emf, we will talk about emf in this chapter). Examples are:

(i) In electrical cells, chemical energy is converted into electrical energy. Similar is the principle of battery.

(ii) Electric generator convert mechanical energy into electrical energy (we will talk about it in chapter 14).

(iii) Thermocouple convert heat energy into light energy (we will discuss thermocouple in this chapter).

(iv) Solar cells convert light energy int electrical energy (we will encounter solar cell in chapter **??**).

12.4 Elecroencephalogram(EEG)

12.5 Ohm's Law

Background

In previous section, we studied that when a potential difference is maintained across the ends of a wire, current flows. The relationship between potential difference (V) and electric current (I) in a D.C circuit was first discovered by German scientist ***George Simon Ohm*** in 1826 in the form of a law known as 'Ohm's law'. This law is known to be the fundamental law of electricity.

Statement

"The magnitude of electric current 'I' in a metallic wire is proportional to the applied voltage 'V' provided the physical state of conductor is constant."

Mathematical Form

Consider an electric circuit as shown:

When potential difference 'V' is maintained in the circuit, current 'I' flows and by Ohm's law:

$$I \propto V$$

$$I = (constant).V$$

$$I = \frac{1}{R}V$$

which implies:

$$V = IR \tag{12.5}$$

where, 'R' is a constant and called as resistance of wire and it depends upon:

(i) Nature of material of the wire

(ii) Physical state of the material

(iii) Dimensions of the wire

The above equation is the mathematical form of Ohm's law.

Validity

Ohm's law is valid only for metallic conductors. It does not hold for electron tubes, discharge through gaseous, filament of bulb etc. Those materials which obey Ohm's law are called ohmic materials while those which do not are termed as non-ohmic.

12.5.1 I-V Characteristics for Ohmic and Non-ohmic Materials

"The curves which are drawn between current and voltage for the given materials are called I-V characteristics." We will discuss here separately for ohmic and non-ohmic conductors.

Ohmic Conductors

From Ohm's law, we know that:

$$V = IR$$
$$\frac{1}{V} = \frac{1}{R}$$

So a graph between 'I' and 'V' will have a slope '1/R'. From Ohm's law, 'R' is constant if the physical conditions are kept fixed, then it is obvious that '1/R' will also be a constant i.e. I-V graph will have a constant slope. So the graph will be a straight line passing through the origin as shown:

Non-ohmic Conductors

The I-V graphs for non-ohmic conductors are discussed as:

Filament of an Electric Bulb: The I-V graph for a filament lamp is shown in the figure below:

The graph bends over as 'V' and 'I' increases. This shows that a given change of 'V' causes a smaller change in 'I' at larger values of 'V'. This means that the slope decreases with the increase of voltage. As '1/R' indicates the slope, hence resistance will increase as the current raises the temperature of the filament.

Thermistor: Thermistor is a 'thermoresistor'. The graph for a thermistor is shown as:

It is clear that graph bends upward. Slope increases, hence resistance decreases sharply as the temperature rises. Thermistors are made up of semiconductor materials.

Semiconductor Diode: The I-V graph for a semiconductor diode is shown as:

The graph is a non-linear curve, hence, non-ohmic. If the voltage is reversed, the current is nearly zero. It conducts in one direction only.

Other Examples of Non-ohmic Materials: Electron-vacuum tubes,electric arcs, neon gas and liquid electrolytes are some other examples of non-ohmic materials; disobeying Ohm's law.

12.6 Electrical Resistance

"The opposition offered by a substance to the flow of electric current is called electrical resistance."

Explanation

As an example, let us consider a metallic conductor. We know that in metals, current is due to the flow of electrons. It is also known that inside the metallic bulk, positively charged centres are at fixed positions. They vibrate about their mean positions with a certain amplitude of vibration. When electrons move, they collide with the positive charged metal ions, due to which there is a reduction in their flow. So when the flow is reduced, we say that an intrinsic electrical resistance is present in every substance.

Symbol

Symbolically it is denoted by 'R'. In electrical circuits, it is represented as '–\/\/\/–'.

Mathematical Form

From Ohm's law, keeping the physical conditions same, the ratio of voltage applied to the current flown is constant i.e.

$$\frac{V}{I} = constant$$

This constant is called resistance.

$$R = \frac{V}{I} \tag{12.6}$$

So we can also define resistance as ***"the ratio of voltage applied across the ends of a conductor to the current flowing through the conductor."***

Dependence

Resistance of a material depends upon:

(i) Nature of material of the conductor

(ii) Physical state of the conductor

(iii) Dimensions of the conductor

Unit of Resistance

The S.I unit of resistance is ohm symbolized as Ω (capital Greek alphabet omega), in regard of great scientist Simon Ohm.

One Ohm

"The resistance of a wire is said to be one ohm,when one volt potential difference across the two ends of wire causes a current of one ampere to flow through it."

Multiples

$$1\ \mathrm{k}\Omega = 10^3\Omega\ 1\ \mathrm{M}\Omega = 10^6\Omega$$

12.6.1 Specific Resistance/Resistivity

As we know that resistance is the opposition offered by a material to the flow of current.During the collisions, electrons in a metal lose energy. It has been experimentally observed that electrons lose more energy while moving along a longer path while keeping the cross-sectional area constant, and less will be the resistance if we increase the cross-sectional area of a wire of given length, hence,

$$R \propto L$$

$$R \propto \frac{1}{A}$$

Introducing constant of proportionality:

$$R = \rho \frac{L}{A} \tag{12.7}$$

where, Greek alphabet 'ρ' (Rho) is called specific resistance or resistivity and it depends upon:

(i) Nature of the material of the wire

(ii) Temperature of the material

An important point to be remembered is that, ***"Resistance is a property of a particular wire but resistivity is a property of a particular material."*** Now if we put $A = 1\,m^2$, $L = 1\,m$ in equation 12.7, we get:

$$R = \rho$$

Hence, we can define resistivity quantitatively as: ***"The resistance of a wire of unit cross-section and unit length is called resistivity/specific resistance of that wire."*** Resistivity of a material is identity of material. Substances having high resistivity are poor conductors of electricity and those having low resistivities are good conductors

Unit of Resistivity

The S.I unit of resistivity is ohm-metre (Ω m).

12.7 Conductance

"The reciprocal of resistance of a conductor is called conductance of that conductor"".

Symbol

Its symbol is 'G'.

Mathematical Form

If a conductor has resistance 'R', then its conductance 'G' is given by:

$$G = \frac{1}{R} \tag{12.8}$$

Unit

As it is the reciprocal of resistance, so its unit is ohm^{-1} or mho (ohm spelt backward). A practical unit is siemen denoted by S.

12.8 Conductivity

"The reciprocal of resistivity is called conductivity of a conductor."

Symbol

It is denoted by 'σ'.

Mathematical Form

If 'ρ' is the resistivity of a conductor, then its conductivity 'σ' is given by:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} \tag{12.9}$$

Unit

The S.I unit of conductivity is $ohm^{-1}\,m^{-1}$ or $mho\,m^{-1}/S\,m^{-1}$.

12.9 Effect of Temperature on Resistance

Consider a material having certain resistance. As we know that resistance is a result of collision of electrons with atoms vibrating about their mean positions. Also from kinetic molecular studies, we know that temperature is the measure of average kinetic energy of vibrational and other motions. When temperature is increased, the atoms vibrate with higher amplitude, hence collisions with electrons increase and hence resistance increases. Note that here, we considered the case of metals. In case of pure metals, resistance increases with temperature fairly regular for normal range of temperatures.

12.9.1 Temperature Coefficient of Resistance

Let us consider a conductor having resistance 'R_o' at $0°C$ and 'R_T' at an elevated temperature '$T°C$'. It has been found that in the normal range of temperatures, the change in resistance '$R_T - R_o$' is:

- Directly proportional to the original temperature i.e.

$$R_T - R_o \propto R_o \tag{12.10}$$

- Directly proportional to the rise in temperature 'ΔT', so,

$$R_T - R_o \propto \Delta T \tag{12.11}$$

Combining relations 12.10 & 12.11:

$$R_T - R_o \propto R_o \Delta T$$

$$R_T - R_o = \alpha R_o \Delta T \tag{12.12}$$

where 'α' is the constant of proportionality and is called ***"temperature coefficient of resistance".*** Its value depends upon:

(i) Nature of the material

(ii) Temperature

Now, rearranging equation 12.12, we get:

$$R_T = Ro\,(1 + \alpha \Delta T)$$

This equation is used to find the resistance of a material at an elevated temperature. For definition of temperature coefficient of resistance 'α', rearrange (12.12) as:

$$\alpha = \frac{R_T - R_o}{R_o \Delta T} \tag{12.13}$$

Hence, we can define temperature coefficient of resistance as, ***"Change in resistance per unit original resistance per degree/kelvin rise in temperature."*** or ***"Fractional change in resistance per degree/kelvin rise in temperature."***

Unit

The units are $°C_{-1}$ or K_{-1}.

Positive and Negative Temperature Coefficient of Resistance

The materials whose resistance increases with the rise in temperature i.e $R_T - R_o$ is a positive quantity have positive coefficient of resistance, e.g. metals have positive value of α. While others with negative α, their resistance decreases with the rise in temperature e.g. in case of semiconductors. It is due to the fact that when temperature is increased for a semiconductor material, the number of electrons increase, and vacancy of electron (hole) is created, hence material conducts more (The details are discussed in chapter 17). Hence with respect to R_o, the quantity $R_T - R_o$ is a negative quantity, hence value of α is negative. Same is observed in case of non-metals, since at higher temperatures, electrons are shaken loose and leave their places for conduction.

12.9.2 Variation of Resistivity with Temperature

The resistivity of most materials increase with increase in temperature linearly. Looking over equation 12.12, putting respective values of resistance:

$$\frac{\rho_T L}{A} - \frac{\rho_o L}{A} = \frac{\rho_o L}{A}\alpha\Delta T$$

which implies

$$\rho_T - \rho_o = \rho_o \alpha \Delta T$$

so,

$$\alpha = \frac{\rho_T - \rho_o}{\rho_o \Delta T} \tag{12.14}$$

Where 'α' is called temperature coefficient of resistivity and defined as ***"fractional change in resistivity per degree/kelvin rise in temperature".*** The concept of positive and negative temperature coefficient of resistivity is same as for temperature coefficient of resistance.

Uses

The temperature coefficient of resistivity of materials can be used to distinguish between metals, e.g Iron and Platinum can have the same resistivity but different coefficients of resistivity

12.10 Types of Resistors

Resistors are required for many purposes in electrical circuits. Although resistors dissipate energy but still useful in circuits. There may exist several types of resistors but three main types are

(i) Carbon Resistors

(ii) Wire-Wound Resistors

(iii) Thermistors

Here we discuss only wire-wound resistors and thermistors.

12.10.1 Wire-Wound Resistors

In a wire wound resistor, a long wire is wound to occupy a minimum space. Wires of alloys such as manganin, constantan, eureka etc. are used for such resistors because of their high resistances. Resistance boxes in laboratory consists of coils, which can be connected in series by plugs or switches to give the required value. High accuracy and stability resistors are always wire-wound. They can be fixed or variable.

Variable Wire-Wound Resistors

In this type, a wire of high resistivity is wrapped around an insulating core to get a variable resistance out of it.

Construction

To design a variable wire-wound resistor, generally Nickel or Chromium is used because of its very small coefficient of resistance. Wire-wound resistors can safely operate at higher temperatures than carbon-type resistors.

Uses

Wire-wound resistors can be used in two different ways:

(i) Rheostats

(ii) Potential Divider

i. Rheostats: ***"A device used for controlling current in the circuit".***

Working Rule: By adjusting the length of the wire-wound resistor, current us controlled.

Working: The working is very simple. To use it as a current-control device, one of its fixed terminal, here A, and other sliding terminal, here C, are inserted as shown: In this way, the resistance between the sliding terminal C and fixed terminal A is used. If the sliding contact C is shifted away from terminal A towards B, then the length of the effective path increases, hence resistance to be used increases. If the sliding contact is moved towards A, the length hence resistance decreases.Adjusting the resistance in the circuit thus controls the current in the circuit.

ii. Potential Divider: ***"A device which helps us to provide a variable potential difference from a fixed potential difference".***

Basic Principle: The underlying principle is conversion of large battery to provide us a small battery by varying the resistance of the circuit.

Working: The arrangement for a potential divider is shown in the figure below With the help of a battery, a potential difference 'V' IS applied across the ends 'A' and 'B' of the resistor. Let 'R' be the resistance of the wire AB. The current passing through AB will be according to Ohm's law, given by

$$I = \frac{V}{R}$$

If RBC is the resistance of the portion BC of the wire which is adjustable and the current passing through this portion is I. The potential difference between the points 'B' and 'C' is given by

$$V_{BC} = IR_{BC}$$

$$V_{BC} = \frac{V}{R}R_{BC}$$

$$V_{BC} = \frac{R_{BC}}{R} V \tag{12.15}$$

Depending upon the position of the sliding contact C, the value of the fraction $\frac{R_{BC}}{R}$ can be varied from 0 to 1 (when $R_{BC} = R$, $V_{BC} = V$ and when $R_{BC} = 0$, $V_{BC} = 0$. When the sliding contact is moved towards 'B' the length of the portion BC decreases, hence the resistance of small portion decreases which reduces the obtained voltage V_{BC}. On the other hand, if the sliding contact 'C' is moved towards 'A' the length and hence the resistance of the portion BC increases and hence the voltage V_{BC} increases.

12.10.2 Thermistor

"A resistor made of semiconductors having resistance that varies rapidly and noticeably with temperature is known as thermistor".

Explanation

A thermistor is actually a 'thermal resistor'. It is a heat sensitive device usually made of a semiconductor material whose resistance varies rapidly with the variation of temperature.

Properties

A thermistor has mainly the following properties:

i. Resistance: The resistance of a thermistor changes very rapidly with change of temperature.

ii. High Temperature Coefficient: The temperature coefficient of thermistor is very high

iii. Negative and Positive Temperature Coefficients: The temperature coefficient of thermistor can be both positive and negative. Positives have the property of rise of their resistance with rise in temperature and negative temperature coefficient thermitors are those whose resistance decreases with rise in temperature.

Construction

Thermistors are made by heating under high pressure semiconductor ceramic made from mixture of metallic oxides of manganese, iron, nickel,cobalt etc. They are generally in the form of discs or rods. Pair of Platinum leads are attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed.

Applications of Thermistor

Following are some of the applications of thermistors:

i. Safeguard against Surges in a Circuit: A thermistor with negative temperature coefficient of resistance may be used to safeguard against surges in a circuit where this could be harmful, e.g in a circuit where the heater of radio valves are in series as shown. A thermistor is included in the circuit. When the supply voltage is switched on the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warms up, the thermistor resistance drops appreciably and an increased current then flow through the heaters.

ii. Use in Modern Appliances: Modern appliances, communication tools and accessories like mobile phone, computers, LCD displays, CPUs, rechargeable batteries, and medical and patient monitoring equipment are all equipped with thermistors so they can be used continuously without fear of overheating and appliance damage.

iii. Use for Alarms in Windings: A thermistor with a negative temperature coefficient (NTC) can be used to issue an alarm for excessive temperature of winding of motors,transformers and generators. When the temperature of the windings is low, the thermistor is cool and its resistance is high. Hence, only a small amount of current flows through the thermistor. When the temperature of the windings is high, the thermistor is hot and its resistance is low, hence a large current flows in the coil to close the contact.

iv. Temperature Control in Various Devices: Some appliances such as washing machines, clothes dryers, refrigerators and freezers as well as appliances like hair dryers, curling irons, ovens, toasters, thermostats, air conditioners and fire alarms also have NTCs for their temperature control.

12.11 Electromotive Force

"The amount of work done in moving a unit positive charge from negative terminal to the positive terminal of the battery."

Symbol

It is denoted by 'ε'.

Mathematical form

If 'W' is the amount of work done in moving a charge 'q' from negative terminal to positive terminal of the battery , then emf 'ε' will be given by:

$$E = \frac{W}{q}$$

Unit

The S.I unit of emf is volt(V).

Source of Emf

In our daily life, we need a constant current in an electric circuit. To maintain a constant current, a constant potential difference needed across the conductor. This potential difference can be maintained if some device change some non-electrical energy into electrical energy. This device is called source of emf. We can define the source of emf as ***"A device which converts non-electrical energy into electrical energy in order to maintain a constant potential difference in the external circuit is called a source of emf."*** The figure below shows a source with emf ε connected to a resistor R. Inside the battery the current is from negative to positive terminal, while in the outer circuit, it is from positive terminal to negative terminal of the battery. The battery maintains its upper terminal at a high potential and its lower terminal at zero potential acting as a source of emf.

Sources of emf

The work done on the charges in the source of emf to push them along it must be derived from a source of energy. There are many sources of emf, some are:

(i) Batteries or cells convert chemical energy into electrical energy.

(ii) Electrical generators convert mechanical energy into electrical energy.

(iii) Thermocouples convert heat energy into electrical energy.

(iv) Radiant(a solar cell) converts sunlight directly into electrical energy.

12.12 Potential Difference

"It is the amount of energy per unit charge converted from electrical energy into a non-electrical energy across the ends of a conductor."

Explanation

As we discussed that battery supplies energy to a charge ,this charge when passes through a conductor (or resistor), it loses energy and comes to lower potential, this dissipation of energy per unit charge is termed as potential difference. It is denoted by V. It has also units same as emf i.e. volt.

12.12.1 Internal Resistance of a Power Supply

All power supplies have some internal resistance, a negligible resistance. When the circuit is open i.e. the power supply is delivering no current, then potential difference across the terminals of the battery is equal to the emf of the supply. When a load of resistance 'R' is connected across the terminal of the battery, then current 'I' starts flowing through the circuit as shown: Due to the flow of current, there is a voltage drop across internal resistance 'r' of the supply so that terminal voltage 'V' will be less than ε by Ir. So , we can develop a relationship between 'V' and 'ε' can be easily established. As we know that:

Chapter 13

Electromagnetism

13.1 History

As early as 600 B.C. the Greeks knew that a certain form of iron ore, now known as magnetite or lodestone, had the property of attracting small pieces of iron. Later, during the Middle Ages, crude navigational compasses were made by attaching pieces of lodestone to wooden splints. These splints always come to rest pointing in a N—S direction, and were the forerunners of the modern aircraft and ship compasses.

The word 'lodestone' is derived from an old English word meaning way, and refers to the directional property of the stone mentioned above. Chemically, it consists of iron oxide having the formula Fe_3O_4. The word magnetism is derived from Magnesia, the place where magnetic iron ore was first discovered.

In 1820, a Danish Physicist Hans Christian Oersted made one of the most important discovery of all times. He determined that when a current carrying wire is held near a compass needle, the needle is deflected. This discovery leads to the entire field of electromagnetism.

13.2 Definition of Electromagnetism

"The branch of Physics which deals with the study of magnetic effects of electric current is called electromagnetism."

Explanation

The electric and magnetic fields are different aspects of electromagnetism but intrinsically related. Thus, a changing electric field generates a magnetic field and conversely a changing magnetic field generates an electric field. The latter effect is called electromagnetic induction and is the basic operation for electric generators, induction motors and transformers and is studied in electromagnetism. (we will cover it in next chapter).

13.3 Magnetic Field

"The space around a magnet or current carrying conductor, where a test magnet can feel a force of attraction or repulsion is called magnetic field."

13.3.1 Forces of Magnets

Magnets exert forces on each other. These forces are either attraction or repulsion. The effects may be summarised in the law of magnets:

"Like poles repel and unlike poles attract."

13.3.2 Magnetic Field Lines

"Magnetic field lines are the curves drawn so that the tangent to a given curve at a point gives the direction of magnetic field at that point."

Properties of Field Lines

Magnetic field are not visible but they can be represented by lines of magnetic force extending in three dimensions. The properties of magnetic lines of force are given as:

(i) The magnetic field lines start at a north pole and end at a south pole.

(ii) These lines are smooth curves,they never cross or touch.(Can you state why?)

(iii) The strength of the field is indicated by the distance between the lines_closer lines mean a stronger field and vice versa.

(iv) Magnetic field lines always form closed curves.

13.3.3 Magnetic Field of an Electric current

As it is known that all electric currents produce magnetic fields. The size and shape of magnetic field depends on the size of the current and the shape(configuration) of the conductor through which the current is travelling.

Magnetic field of a Straight Current Carrying Wire

The magnetic field due to a straight wire may be plotted using the apparatus shown in the figure below. Iron fillings are sprinkled on a horizontal board and current is passed through the wire as a result of which a magnetic field will be produced.Iron fillings will be in the indicated pattern showing the magnetic field around a straight current carrying wire will be in the form of concentric circles. The separation of lines increases with the distance from the wire, indicating the field is decreasing in strength as we move away from the wire.The field also increases as the current is increased in the wire. The direction of field can be found by placing magnetic compasses or using right hand rule which states that:

"Imagine hold the conductor in the right hand with the thumb pointing in the direction of the current, the curled fingers will point in the direction of field."

13.4 Force on a current carrying conductor

The interaction of magnetic fields produced by two magnets causes force of attraction or repulsion between the two. If a conductor is placed between the poles and a current is passed through the conductor, the magnetic fields of the current-carrying conductor and the magnet may interact, causing forces between them. In order to explain, we will demonstrate it by the following experiment:

Place a straight wire between the poles of a magnet. When a current flows in the wire, a force is exerted on the wire. In first demonstration, the current flows inward direction (into the page), the wire experiences a downward push. This force is neither parallel to magnetic field nor parallel to the wire. Instead this force is directed at right angle to the magnetic field and wire. Now, if the current is reversed (out of the page), the direction of push will also be reversed i.e. upward. It is found that the direction of force is always perpendicular to the wire and also perpendicular to the direction of field.

These demonstrations lead us to define a rule for the direction of force, i.e.

"Outstretch the fingers of your right hand in the direction of current, then bend the fingers in the direction of magnetic field, the extend thumb will indicate the direction of the force in the current carrying wire"

Magnitude of Force

It is found experimentally that the magnitude of the force is directly proportional to :

(i) Current in the wire

(ii) Length of the wire inside the magnetic field

(iii) Strength of the field

i.e.

$$F \propto ILB \tag{13.1}$$

Secondly, it was also found that:

- When the wire was perpendicular to the field, the force was maximum
- When the wire as parallel to to the field, there was no force at all
- At any angle, it varies with the sine of the angle between $\vec{L}$ and $\vec{B}$.

So, we can write:

$$F \propto sin\theta \tag{13.2}$$

Combining relations 13.1 and 13.2:

$$F \propto ILBsin\theta$$

Here constant of proportionality is 1, so

$$F = ILBsin\theta \tag{13.3}$$

Generalized Form

The magnitude as well as the direction of the magnetic force on a current carrying wire can be described in vector notations by the following cross product:

$$\vec{F} = I\vec{L} \times \vec{B} \tag{13.4}$$

$$\vec{F} = ILBsin\theta\,\hat{n} \tag{13.5}$$

Where $\vec{L}$ is a vector whose magnitude is the length of the wire and whose direction is along the wire (assumed straight) in the direction of current. The unit vector $\hat{n}$ is along the direction of $\vec{F}$ and is perpendicular to $\vec{L}$, $\vec{B}$ and plane determined by $\vec{L}$ and $\vec{B}$.

Fleming's Left-Hand Rule

The rule we discussed earlier is known to us already if we have a knowledge of cross product (extending fingers in the direction of first vector in the cross product and curl them towards other vector, thumb will indicate the direction of the resultant of this cross product i.e. the direction of $\hat{n}$. An alternate rule for the direction of force is the Fleming's left hand rule which states that:
"If the forefinger, central finger and thumb of left hand are held mutually perpendicular with the Forefinger pointing in the direction of Field, central finger in the direction of Current, the thumb would indicate the Motion of conductor(the direction of force on conductor)."

Definition of $\vec{B}$

$\vec{B}$ being a vector has magnitude as well as direction.

Direction of $\vec{B}$

The direction of $\vec{B}$ at any point of the magnetic field is the direction in which the force acting on a straight current carrying wire, placed at that point, is zero, i.e. $\vec{L} \parallel \vec{B}$.

As we know that when the wire is in the direction of field, it experiences no force, so we move wire so that the point comes when it experiences no force, we say that the direction of the wire at that point will be the direction of field.

Magnitude of $\vec{B}$

The magnitude of $\vec{B}$ is defined when angle between $\vec{B}$ and $\vec{L}$ is 90° and force is maximum, hence,

$$B = \frac{F_{max}}{IL} \tag{13.6}$$

So, ***"It is the maximum force acting on a conductor of unit length when one ampere current passes through it"***

Unit of Magnetic Field

The SI unit of magnetic field is tesla (T) and :

$$1\,T = 1\,NA^{-1}m^{-1} \tag{13.7}$$

One Tesla

"Magnetic field at any point is said to be one tesla if it exerts a force of 1 N on one metre length of the conductor placed at right angles to the field when a current of 1 A passes through it."

Other Units

An older name of tesla is weber per metre squared i.e. $\frac{Wb}{m^2}$. Another commonly used unit is gauss (G). And,

$$1\,G = 10^{-4}\,T$$

Magnetic field of earth is half a gauss:

$$\tfrac{1}{2}G = 0.5 \times 10^{-4}\,T$$

13.5 Magnetic Flux

"The number of magnetic field lines passing through a surface is known as magnetic flux"

OR

"The dot product of magnetic induction $\vec{B}$ and vector area element $\vec{A}$ is known as magnetic flux."

13.5.1 Symbol

Its symbol is Φ_B.

Mathematical Form

If $\vec{B}$ is the magnetic induction and $\vec{A}$ is the area vector (a vector having magnitude equal to the area of surface and direction normal to the area element), then the flux would be:

$$\Phi_B = BAcos\theta \tag{13.8}$$

where θ is the smaller angle between $\vec{B}$ and $\vec{A}$.

Explanation

As magnetic flux for magnetic induction $\vec{B}$ and element of area $\Delta\vec{A}$ is given by:

$$\Phi_B = B\Delta A cos\theta$$

We know that area vector is normal to the plane of area. If the area is not a flat surface i.e. the angle between area vector and magnetic induction is different at different points, thus we divide area into smaller 'n' elements. So, the total magnetic flux through the whole area placed in a field of magnetic induction $\vec{B}$ is the sum of the contributions from the individual area elements is given by:

$$\Phi_T = \sum_{i=1}^{n} \Delta\Phi_i$$

$$\Phi_T = \sum_{i=1}^{n} B_i \Delta A_i cos\theta_i \tag{13.9}$$

In a uniform field,

$$\Phi_T = B\sum_{i=1}^{n} \Delta A_i cos\theta_i \tag{13.10}$$

Maximum Flux

If the surface area is held normal to the field lines such that area vector $\vec{A}$ is parallel to the field, then the maximum lines of force will pass and flux will be maximum i.e.

$$\Phi_B = BAcos0^\circ$$

$$\Phi_B = BA$$

Minimum Flux

If the surface area is placed such that it is parallel to the lines of force, so that area vector $\vec{A}$ is normal to the field, no lines will pass, θ will be 90 degrees, flux will be zero.

$$\Phi_B = BAcos90^\circ$$
$$\Phi_B = 0$$

Unit

Unit of magnetic flux is Tm^2 called as weber (Wb).

13.5.2 Magnetic Flux Density

Using equation

$$B = \frac{\Phi}{A} \tag{13.11}$$

So magnetic induction B can also be defined as: ***"Magnetic flux per unit area."*** Hence it is also called magnetic flux density. It has unit $\frac{Wb}{m^2}$ (T).

13.6 Ampere's Circuital Law

Background

We know that a current carrying wire has a magnetic field around it. The direction of the field can be determined by right hand rule. The magnitude of the field can be determined by a relation called Ampere's circuital law.

Statement

"The sum of the dot products 'B' and 'L' around a closed path in the magnetic field of a current is equal to μ_o times the current enclosed by the path."

Mathematical Form

$$\sum \vec{B} \cdot \Delta \vec{L} = \mu_0 I \tag{13.12}$$

Explanation

Let us first consider a special case of the magnetic field of a long straight-current carrying wire as shown:

From experiments as well as from the cylindrical symmetry of the wire, it is obvious that the magnitude of magnetic induction is constant on a circle of radius 'r' centred on wire. It is further observed that 'B' around a long straight current-carrying wire is directly proportional to the current 'I' and inversely proportional to the distance 'r' from the wire i.e.

$$B \propto I \tag{13.13}$$

$$B \propto \frac{1}{r} \tag{13.14}$$

Combining relations 13.13 and 13.14:

$$B \propto \frac{I}{r}$$

Introducing constant of proportionality,

$$B = \frac{\mu_0 I}{2\pi r} \tag{13.15}$$

Where, $\frac{\mu_0}{2\pi}$ is constant of proportionality and its value is $4\pi \times 10^{-7}\, WbA^{-1}m^{-1}$ and is called its permeability of free space. Equation shows radial dependency of B. This radial dependence is used to derive expression for Ampere's law.

Derivation

Let us consider a circle of radius 'r' around current carrying wire as shown below:

To find $\vec{B} \cdot \vec{L}$, where L is the circumference of circle, we divide this path into small segments $\Delta\vec{L}_1$, $\Delta\vec{L}_2$, $\Delta\vec{L}_3$, ..., $\Delta\vec{L}_n$. Then:

$$\vec{B} \cdot \vec{L} = \sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{L}_i$$

As it is clear from the figure that $\vec{B}$ is parallel to the $\Delta\vec{L}$ at each point, therefore:

$$\sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{L}_i = \sum_{i=1}^{n} B_i \Delta L_i$$

As B is constant, so we pull it out of summation:

$$\sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{L}_i = B \sum_{i=1}^{n} \Delta L_i$$

As,

$$\sum_{i=1}^{n} \Delta L_i = 2\pi r$$

and $B = r$ (for circle) and $B = \frac{\mu_0 I}{2\pi r}$

$$\sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{L}_i = \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

$$\sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{L}_i = \mu_0 I \tag{13.16}$$

where, 'I' is the current closed.From equation, it is clear that $\sum \vec{B} \cdot \Delta\vec{L}$ is independent of the shape or size of the closed path. It can be applied to closed path of any shape.

Note:

(i) Ampere's law can be applied to any assembly of currents. The closed path in the magnetic field is called amperian loop.

(ii) If there is no current enclosed within the amperian path, the amperian summation of $\sum \vec{B} \cdot \Delta \vec{L}$ is zero.

13.7 Magnetic Field due to a Current Carrying Solenoid

Solenoid

"If a straight wire is wrapped in the form of several closely spaced loops, the resulting device is called solenoid."

13.8 Magnetic Force on a Moving Charge

We know that force acting on a conductor of length 'L' having current 'I' placed at right angles to the magnetic field 'B' is given by:

$$F_{max} = ILB \tag{13.17}$$

It is also our well-established knowledge that the conductor contains charges and current in a conductor is due to drift of the charges. Therefore, we conclude that the force on the conductor is due to the force on the charges in motion in the conductor. If the charge on a particle is '$+q$' and there are 'n' charges per unit length of the wire moving with velocity 'V', then:
Distance moved by charges in one second will be:

$$S = V(1) = V\ m$$

The charges contained in 'V' metres will pass through the section PP' in one second which makes the current 'I'. The charge contained in V m will be nqV, so current will be:

$$I = nqV \tag{13.18}$$

Put value of 'I' in equation 13.17, we get:

$$F_{max} = nqLVB$$

The number of charges in unit length will be 'n' and in length L, they will be 'nL'. So, force on 'nL' charges is:

$$F_{max} = nqLVB$$

The force on each charge will be:

$$F_{max} = \frac{nqLVB}{nL}$$

$$F_{max} = qVB \tag{13.19}$$

This was the special case when the conductor was placed perpendicular to the field,in our case, '$\vec{B}$' is perpendicular to '$\vec{V}$', but if there is an angle 'θ' between '$\vec{B}$' and '$\vec{V}$', then:

$$F = qVBsin\theta \tag{13.20}$$

The equation gives the magnitude of the magnetic force on a particle of charge 'q' moving with velocity 'V' in a magnetic field of strength 'B' and 'θ' is the angle between '$\vec{B}$' and '$\vec{V}$'.

Maximum Force

When $\theta = 90°$, $sin90° = 1$, then:

$$F_{max} = qVB$$

This means that the force is maximum when the charge particle moves perpendicular to the magnetic field.

Minimum Force

When $\theta = 0°$, $sin0° = 0$, then:

$$F = qVBsin0°$$

$$F = 0$$

This means that when the particle moves parallel to the field direction, the force will be minimum i.e zero.

Direction

We know that force is a vector quantity, so it must have a specific direction. Equation 13.20 can be written in vector form as:

$$\vec{F} = q(\vec{V} \times \vec{B}) \tag{13.21}$$

$$\vec{F} = qVBsin\theta\hat{n} \tag{13.22}$$

The direction of the force is perpendicular to velocity and magnetic field and the plane formed by them and can be determined by right hand rule of cross product. Interestingly, you can extend Fleming's left hand rule to determine the direction of this force, ***"Hold your forefinger, middle finger and thumb of your left hand mutually perpendicular such that forefinger points in the direction of field, middle finger in the direction of velocity of a charged particle, then the thumb would indicate the direction of force on that moving charge."***

Note:

(i) If the force on '$+q$' is upward, then force on '$-q$' will be downward in the same field.

$$\vec{F} = -qVBsin\theta\hat{n}$$

$$\vec{F} = qVBsin\theta(-\hat{n})$$

This means that the force on negative charge is opposite to that of positive charge.

(ii) As the force is perpendicular to the direction of motion,therefore it can only change the direction of motion of the charged particle. It can neither speeds up or slowed down the particle.

Applications

The deflection of charged particles by magnetic field is used in T.V tubes, electron microscopes, spectrographs and charged particles accelerators like Cyclotron and Betatron.

13.8.1 Circular Trajectory of a Charged Particle in a Magnetic Field

Let us consider a particle of charge '$+q$' thrown perpendicular to a uniform magnetic field of magnetic flux density 'B' with velocity 'v' as shown:

The magnetic force in the above case:

$$F_m = qvB \tag{13.23}$$

As this force is all the time perpendicular to the direction of velocity and magnetic field, therefore it compels the charge to move in circular path. So, magnetic force is the necessary centripetal force i.e.

$$F_m = F_c$$

Hence, from]refeq:13.23:

$$F_m = \frac{mv^2}{r}$$

Putting values, we get:

$$\frac{mv^2}{r} = qVB$$

$$r = \frac{mv}{qB} \tag{13.24}$$

This equation gives the radius of the circular path in which a charge 'q' of mass 'm' and velocity 'V' will move in a magnetic field when projected perpendicularly. Also,

$$v = r\omega$$

Put this value of 'r' from equation 13.24, we get:

$$v = \frac{mr\omega}{qB}$$

which gives:

$$\omega = \frac{qB}{m} \tag{13.25}$$

This is the angular frequency of the circulating body. As,

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{qB}{2\pi m} \tag{13.26}$$

And,

$$f = \frac{1}{T}$$

So,

$$T = \frac{2\pi m}{qB} \tag{13.27}$$

From the equation, it is clear that the time period of the particle is independent of the radius of the path followed by the charge. Smaller the radius, less will be the velocity and larger radius will increase the velocity as a result time period for a given charge will remain same. The frequency found in equation 13.26 is called 'cyclotron frequency' (cyclotron is a device used for accelerating charged particles) of the circulating particle.

13.8.2 Helical Trajectory of a Charged Particle

If the direction of velocity is not perpendicular to the magnetic field, then instead of circular trajectory, charged particle adopts a helical path as shown:

Actually, velocity has two components, the vertical component and horizontal component is affected by magnetic force describing circular path and horizontal moves it straight. As a result, it follows a spiral path.

13.8.3 Determination of e/m for an Electron

Principle

Circular trajectory of charged particle in magnetic field.

Mathematical Derivation

A narrow beam of electrons moving with constant velocity 'v' is projected at right angles to a uniform magnetic field 'B' as shown:

The magnetic force provides the necessary centripetal force, So,

$$\frac{mv^2}{r} = eVB$$

$$\frac{e}{m} = \frac{v}{Br} \tag{13.28}$$

Knowing v, B and r, value of $\frac{e}{m}$ can be calculated.

Determination of radius

The radius is measured by making the electrons trajectory visible. This is done by filling a glass tube with a gas such as hydrogen at low pressure. The tube is placed in a region occupied by a uniform magnetic field of known value. As the electrons are shot into this tube, they begin to move along a circle under the action of magnetic force. As the electrons move, they collide with the atoms due to which they emit light and their path becomes visible as a circular ring. The diameter of this ring can be easily determined.

Determination of Velocity

We know that the kinetic energy gained by electrons is due to the electric potential.

$$Gain\ in\ Kinetic\ Energy = \frac{1}{2}mv^2 = eV$$

So,

$$v = \sqrt{\frac{2eV}{m}}$$

Now, to find $\frac{e}{m}$, squaring above equation:

$$v^2 = \frac{2eV}{m}$$

Squaring equation 13.28:

$$\frac{e^2}{m^2} = \frac{v^2}{B^2r^2} \tag{13.29}$$

Putting value of v^2 in equation 13.29:

$$\frac{e^2}{m^2} = \frac{2eV}{mB^2r^2}$$

$$\boxed{\frac{e}{m} = \frac{2V}{B^2r^2}} \tag{13.30}$$

Using potential which is known and substituting all values '$\frac{e}{m}$' can be calculated. The accurately known value of '$\frac{e}{m}$' for electron is $1.77588 \times 10^{11}\ \frac{C}{kg}$.

13.8.4 Charge in Combined Electric and Magnetic Field

Electric Force

We know that force on a charge 'q' placed in an electric field of intensity 'E' is given by:

$$F = qE \tag{13.31}$$

Also, from Newton's 2nd law:

$$F = ma \tag{13.32}$$

If charges are free to move, then acceleration will be:

$$ma = qE$$

$$a = \frac{qE}{m} \tag{13.33}$$

If the electric field is uniform,the force is constant. The acceleration produced will be uniform. Therefore, the position and velocity of the particle at any instant of time can be determined by using the equations for uniformly accelerated motion. The figure shows how a beam of electrons is deflected by the uniform electric field.

☑ Checkpoint 13.1

Prove that electrons would follow a parabolic path when deflected by electric field.

Magnetic Force

The force on a charge 'q' moving with velocity 'v' in a region of magnetic field 'B' is:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

As this force is always perpendicular to the motion of charged particle. So it will move the particle in circular path as shown:

Combined Electric and Magnetic Field

If a charge particle 'q' is projected in a region with velocity 'v' where there is magnetic field 'B' and electric field 'E', then force on the particle will be:

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \tag{13.34}$$

This force is called Lorentz force.

Note:

The electric field can:

(a) Deflect the charge particle

(b) Speed up or slow down the particle or impart energy to it

whereas magnetic field can:

(a) Deflect the charge particle

(b) Not speed up or slow down the particle means can not change kinetic energy of the moving charge. This is because magnetic force is perpendicular to v.

13.8.5 Velocity Selector

"When the electric field is perpendicular to the magnetic field such that a charge particle with a particular velocity passes undeflected, the arrangement is called velocity selector."

Construction

A velocity selector consists of a tube in which electric field 'E' is oriented perpendicular to the magnetic field 'B' as shown:

The field strengths 'E' and 'B' are so oriented that the electric force and the magnetic force act in opposite direction. A charged particle that enters the tube in a direction perpendicular to both 'E' and 'B' with speed $v = \frac{E}{B}$ will pass undeflected.

Derivation

As discussed above, the electric and magnetic forces are equal and opposite for velocity selector, so,

$$F_E = F_m$$

$$qE = qvB$$

$$v = \frac{E}{B} \tag{13.35}$$

So charge particles with this velocity will pass undeflected and those with velocities other than this will be deflected either upwards or downwards.

Chapter 14

AC Circuits

14.1 Direct Current

"That type of current that flows in the circuit elements in one direction."
It is important to note that the direction of D.C remains constant but the magnitude is not necessarily constant. On this basis, D.C can be divided into two types:

1. Pure D.C
2. Pulsating D.C

14.1.1 Pure D.C

"That type of D.C in which neither the direction nor the magnitude changes is called pure D.C."
e.g. Battery is a source of pure D.C.

Graphical Representation

14.1.2 Pulsating D.C

"That type of D.C whose direction remains same but magnitude changes is called pulsating D.C."

Graphical Representation

Graphically, pulsating D.C is shown by a wavy line with the passage of time. It is important to note that voltage or current is positive all the time i.e. the current direction remains same.

14.2 Alternating Current

"That type of current whose magnitude changes and direction reverses many times in a second is called alternating current."

OR

"A voltage which changes its polarity at regular intervals of time is called alternating voltage and current produced by that voltage is called alternating current."

Explanation

When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction, the direction of current at any instant depends upon the polarity of the voltages. Consider the figure below in which an alternating voltage source is connected to a resistor 'R' as shown:

When the upper terminal of the A.C source is positive and the lower terminal is negative, the current flows in one direction shown in figure **??**. After half the time period, the polarity of the voltage source is reversed, the current flows in opposite direction. Such a current is called alternating current because the current flows in the alternate directions in the circuit.

14.3 Sinusoidal Alternating Voltage and Current

"The current which reverses its direction many times in a second, is known as alternating current."

Sinusoidal means a curve which obeys graph of sine function. Sinusoidal voltage can be produced by rotating a coil with a uniform angular velocity. A.C voltage switches polarity over time. When graphed over time, the "wave" traced by this voltage of alternating polarity from an alternator takes on a distinct shape, known as sine wave.

Graphical Demonstration

The sinusoidal waveform is shown in figure 14.1.

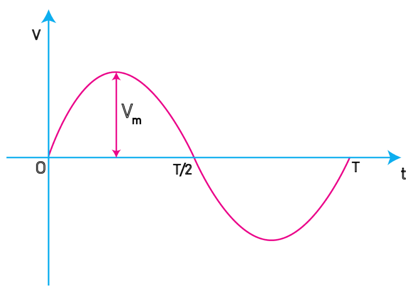


Figure 14.1: Sinusoidal waveform.

Mathematical Form

The output at any instant of time 't' is represented by:

$$V = V_m sin\omega t \tag{14.1}$$

where,
V = Instantaneous voltage
V_m= maximum or peak value or the amplitude of sinusoidal voltage
ω = angular frequency of A.C source (rad/s)
t = time
Also,

$$\omega = 2\pi f$$

So, equation 14.1 implies:

$$V = V_m sin 2\pi f t \tag{14.2}$$

Similarly, sinusoidal current can be represented as:

$$I = I_m sin\omega t \tag{14.3}$$

From the figure 14.1, it is obvious that voltage or current not only changes direction at regular intervals but the magnitude is also changing continuously.

14.3.1 Average Value of Alternating Voltage and Current

"The average of the alternating voltage or current over one cycle is called mean value/average value."

OR

"The sum of entire positive and negative values over one cycle divided by the time period."

Explanation

As the current and voltage is varying sinusoidally, so we should take average value to measure it, the graph is shown in figure 14.2.

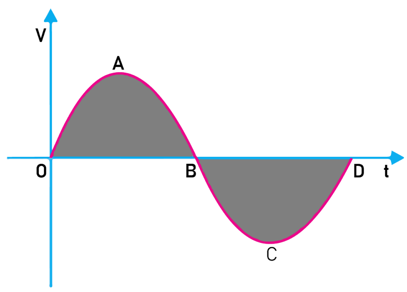


Figure 14.2: Sinusoidal voltage.

To find the average value, we have to add values in the positive as well as the negative cycle and divided by time for one cycle. From the graph, it is clear that:

(i) The sum of all the positive values in one cycle is the area of the upper lobe (shaded region OAB).

(ii) The sum of all the negative values is the area of all the lower lobe (shaded area BCD).

So,

$$MeanValue = \frac{AreaunderOAB - AreaunderBCD}{TimePeriod} \tag{14.4}$$

As,

$$\text{Area OAB} = \text{Area BCD}$$

Hence, mean value of 'V' or 'I' over one cycle is zero.

Using Calculus (average value of a function over an interval [a,b] by mean value theorem):

$$\begin{aligned}
\overline{V} &= \frac{1}{b-a}\int_a^b sinx\, dx \\
&= \frac{1}{2\pi - 0}\int_0^{2\pi} sinx\, dx \\
&= \frac{1}{2\pi}(cos2\pi - cos0) \\
&= \frac{1}{2\pi}(1-1) \\
\overline{V} &= 0
\end{aligned}$$

Same is for I.

Note:

This result is valid only for sinusoidal voltage or current.

14.3.2 Root mean square value

"It is the value of that direct current which would produce heat at the same rate as the alternating current in a given resistor."

Explanation

As we know that the mean value of alternating current ($< I >$) or voltage ($< V >$) is zero over a cycle but practically, a heater operated with alternating current produces heat. If the effective value of current is zero, how would a heater could produce heat. As,

$$P = I^2R = \frac{V^2}{R} \tag{14.5}$$

Equation 12.5 uses squares of I and V rather than I and V, so it is obvious that we should find the mean value of I^2 and V^2 rather than I and V, which are called mean square values (average values of the squared quantities).

Consider the figure showing a graph of sinusoidal current and its squared value I^2 varying with time t.

From the figure, squared values of positive numbers as well as negative numbers are positive, therefore the values of I^2 are positive for the positive as well as negative half cycle. As it is known (You can find it by using integral from 0 to π) that value of $sin^2\omega t$ is $\frac{1}{2}$ in one cycle. So,

$$< I^2 >=< I_0^2 sin^2\omega t >$$

$$< I^2 >= \frac{1}{2} I_0^2$$

Taking square root on both sides of the equation:

$$\sqrt{< I^2 >} = \frac{I_0}{\sqrt{2}}$$

And this value $\sqrt{< I^2 >}$ (The root of an average of a square) is called root mean square value of I and is denoted by I_{rms}. Hence, we can write:

$$I_{rms} = \frac{I_0}{\sqrt{2}} \tag{14.6}$$

$$I_{rms} = 0.707 I_0 \tag{14.7}$$

Relation between I_{dc} and I_{rms}

I_{dc} is the effective value of DC which would deliver the same power to a resistor as the AC to the same resistor i.e

$$P_{dc} = P_{ac}$$

And,

$$P_{dc} = I^2 R$$

$$P_{ac} =< I^2 > R$$

So,

$$I_{dc}^2 R =< I^2 > R$$

$$I_{dc}^2 =< I^2 >$$

$$I_{dc} = I_{rms} \tag{14.8}$$

Similarly,

$$V_{dc} = V_{rms} \tag{14.9}$$

Also, power in A.C circuit will be:

$$P = I_{rms}^2 R \tag{14.10}$$

From equations 14.8, 14.9, 14.10, it is clear that rms values and DC values of current and voltage produce the same effect i.e. heating or performing the same work.

14.4 Single Element A.C Circuit

Circuit

"A combination of electrical components that form a close conducting path is called circuit."

Circuit Elements

"Resistors, inductors, capacitors, transistors or other devices used in making the circuit are called circuit elements." In this section, we will examine the behaviour of the individual circuit elements like resistor, capacitor and inductor when an A.C flows through them.

14.4.1 Resistor in A.C Circuit

To study the behaviour of resistor in A.C circuit, let us consider a resistance 'R' connected with an alternating voltage source as shown in figure 14.3.

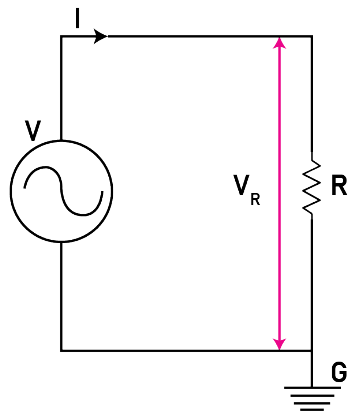


Figure 14.3: Resistor in A.C circuit.

We know that the instantaneous voltage 'V_R' across R is given by:

$$V_R = V_m sin\omega t \tag{14.11}$$

From Ohm's law:

$$V_R = IR$$

$$I = \frac{V_R}{R}$$

Hence,

$$I = \frac{V_m sin\omega t}{R}$$

Since,

$$\frac{V_m}{R} = I_m$$

Hence,

$$I = I_m sin\omega t \tag{14.12}$$

Waveform

From equation 14.11 and 14.12, it is obvious that the waveform of voltage and current is the same, as shown:

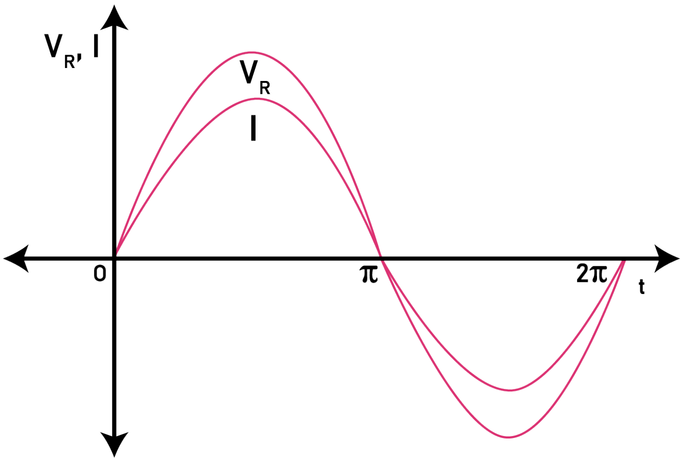


Figure 14.4: Voltage and current waveform for a pure resistive circuit.

Phase

The factor ωt (argument of sine) tells us about the starting point of the waveform and is called phase of current or voltage. From equations 14.11 and 14.12, it is clear that current and voltage are in phase, means reach the maximum and minimum value at the same time. So we can say that in a purely resistive circuit, I and V are in phase.

Figure 14.5: Phasor for a pure resistive circuit.

Power Loss

The power curve for a pure resistive circuit is obtained from the product of the corresponding instantaneous values of voltage and current.

Consider the figure 14.6 showing that power is always positive where I and V are positive and zero for I and V zero. It means that the voltage source is constantly delivering power to the circuit, which is consumed by the circuit.

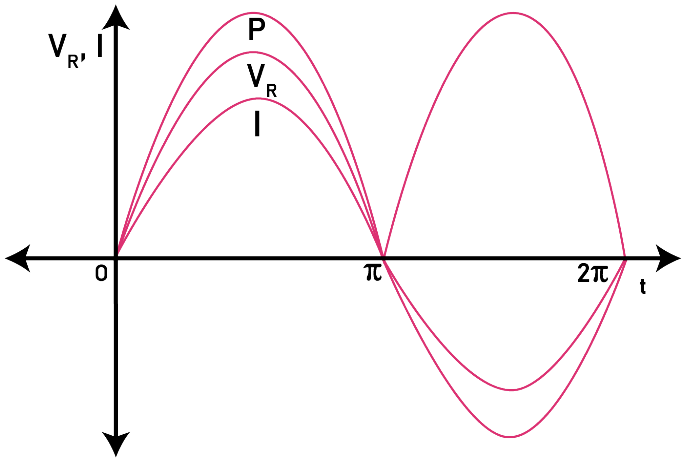


Figure 14.6: Power loss in a pure resistive circuit.

Mathematical Form

The average power dissipated in a resistor over the complete cycle of A.C is:

$$P_{avg} = < VI >$$

$$P_{avg} = < V_m sin\omega t I_m sin\omega t >$$

$$P_{avg} = V_m I_m < sin^2\omega t >$$

$$P_{avg} = \frac{V_m I_m}{2}$$

$$P_{avg} = V_{rms} I_{rms} \tag{14.13}$$

So, the average value of power dissipated in a pure resistive circuit is the product of rms values of voltage and current.

14.4.2 Capacitor in an A.C Circuit:

Capacitor

"A device for storing electric charges and hence energy is called capacitor."

Capacitor in D.C Circuit

If a D.C source is connected to a capacitor, the plates of the capacitor quickly acquire equal and opposite charges and the current in the circuit dies quickly. Afterwards, there is no current in the circuit.

Capacitor with A.C

Let us consider a capacitor of capacitance C connected in an A.C circuit as shown in figure 14.7.

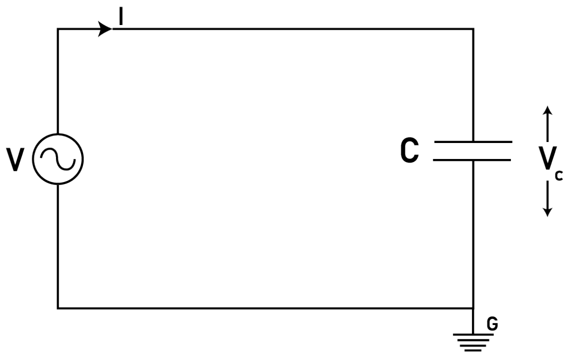


Figure 14.7: Capacitor in A.C circuit.

We are interested in discussing the voltage created across the plates of capacitor and the current in the circuit. So when an A.C voltage is first turned on, charge begins to flow and one plate acquires a positive charge and the other a negative charge. Now, when the voltage reverses itself, the current flows in opposite direction. Thus we can say that an alternating voltage creates and AC in the circuit.

Derivation

As we know that the instantaneous voltage across the capacitor 'C' is given by:

$$V_C = V_m sin\omega t$$

Also,

$$Q = CV_C$$

Putting value of 'V_C', we get:

$$Q = CV_m sin\omega t$$

Take rate of change (derivative):

$$\frac{dQ}{dt} = \frac{CV_m dsin\omega t}{dt} \tag{14.14}$$

As $\frac{dQ}{dt} = I$ and $\frac{d}{dt}(sin\omega t) = \omega cos\omega t$, hence equation 14.14 implies:

$$I = CV_m \omega cos\omega t$$

If $\omega t = 0$, current will be maximum i.e.

$$I_m = CV_m \omega$$

So,

$$\boxed{I = I_m cos\omega t \qquad (14.15)}$$

Equivalently,

$$I = I_m sin(\omega t + \frac{\pi}{2}) \tag{14.16}$$

Shape of the Graph

The voltage varies with the sine of 'ωt' and current as cosine of 'ωt' as shown:

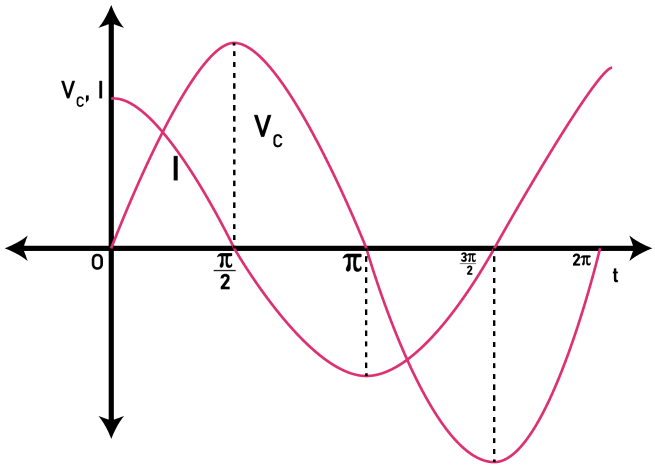


Figure 14.8: Waveform of V and I for a pure capacitive circuit.

Phase

From equations 14.15 and 14.16, it is obvious that the current is starting (90°) quarter cycle before the voltage and it is also clear from the graph.
Let us explain:

- ☐ When $\omega t = 0°$, $I = I_m$, $V_C = 0$
- ☐ When $\omega t = 90°$, $V_C = V_m$, $I = 0$
- ☐ When $\omega t = 180°$, $V_C = 0$, $I = -I_m$
- ☐ When $\omega t = 270°$, $V_C = -V_m$, $I = 0$
- ☐ When $\omega t = 360°$, $V_C = 0$, $I = I_m$

From the above data, it is concluded that current is flowing through the circuit and reaches its maximum value yet no voltage is developed across the plates of capacitor. When the voltage across the plates reaches its maximum value, the current in the circuit ceases to zero. Now, when the applied voltage direction is reversed, the current in the circuit reverses its direction too. The current in the circuit reaches its maximum value when the voltage across the plates becomes zero. So all the time, ***the current through the circuit is leading the voltage by*** $90°$ ***or*** $\frac{\pi}{2}\, rad$**.**

Phasor Diagram

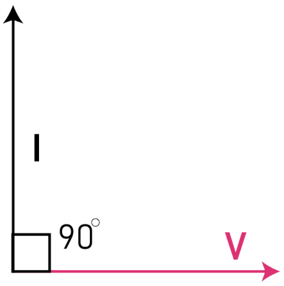


Figure 14.9: Phasor Diagram for pure capacitive circuit.

Reactance of a Capacitor

In analogy with Ohm's law, the ratio $\frac{V}{I}$ is the opposition to the flow of current in A.C circuit. In case of a capacitor introduced in the circuit, this ratio us called reactance so as to distinguish from resistance and us defined as, ***"The capacitive reactance is the measure of opposition offered by the capacitor to the flow of AC."*** It is denoted by X_c.

Mathematical Form

$$X_c = \frac{V_m}{I_m}$$

Putting $I_m = CV_m\omega$, we get:

$$X_c = \frac{V_m}{CV_m\omega}$$

$$X_c = \frac{1}{\omega C} \tag{14.17}$$

$$X_c = \frac{1}{2\pi f C} \tag{14.18}$$

Hence for D.C, $f = 0$, hence $X_c = \infty$. We can say ***"Capacitance offers infinite opposition to D.C."*** For A.C, X_c decreases as f and C increase. Certain capacitors will have a large reactance at low frequency. So the magnitude of the opposition offered by it will be large and the current in the circuit will be small. On the other hand, at high frequency, the reactance will be low and high frequency current through the same capacitor will be large.

Power Loss in a Capacitor

The instantaneous power is given by:

$$P = IV$$

and Average power is given by:

$$P_{avg} = V_m I_m < sin\omega t cos\omega t >$$

As $< sin\omega t cos\omega t >= 0$, Hence:

$$\boxed{P_{avg} = 0}$$

It means that average power dissipated in a pure capacitive circuit is zero. Energy from the source is fed into the capacitor where it is stored in the electric field between the plates. As the field decreases, the energy returns to the source. ***Thus no power is dissipated in a pure capacitive circuit.***

Power Curve

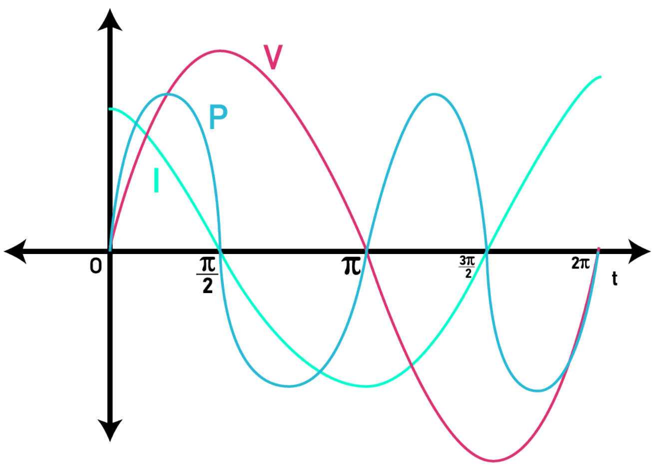


Figure 14.10: Power curve for pure capacitive circuit.

From the figure 14.10, it is clear that per cycle, as much the curve is positive (the power is delivered to capacitor), by the same amount, it is negative (power is delivered back to source). Positive power is equal to the negative power, so power is absorbed by capacitor in one cycle is zero.

14.4.3 Inductor in an A.C Circuit

Inductor

"A coil wound from a thick wire so that it has a large value of self-inductance 'L' and has a negligible resistance is known as inductor."

Inductor with A.C

Let us consider an inductor 'L' connected with an A.C source as shown:

When an A.C voltage is applied across an inductor, it must oppose the flow of current which is continuously changing, therefore the current developed in the inductor lags behind the applied voltage.

Mathematical Derivation

When a sinusoidal current 'I' flows in time 't', then a back emf is induced due to the inductance of the coil. The back emf at each instant opposes the change in current through the coil. As there is no drop in potential, so the applied voltage has to overcome the back emf.

$$Applied\ voltage = back\ emf$$

As the current through the circuit is given by:

$$I = I_m sin\omega t$$

This sinusoidally changing current sets up a back emf in the coil, and is given by:

$$\mathcal{E} = L\frac{dI}{dt}$$

To maintain a constant current, a constant voltage equal to back emf must be applied i.e.

$$V = L\frac{dI}{dt}$$

Putting value of I:

$$V = L\frac{d(I_m sin\omega t)}{dt}$$

So,

$$V = LI_m\omega cos\omega t$$

Writing $V_m = LI_m\omega$, we get:

$$V = V_m cos\omega t \tag{14.19}$$

Equivalently,

$$V = V_m sin(\omega t + \frac{\pi}{2}) \tag{14.20}$$

Shape of the Graph

Both current and voltage are varying sinusoidally. Current is varying with sine of 'ωt' and voltage with cosine of 'ωt' as shown:

Phase

From the equations of voltage and current, it is clear that voltage leads the current by $\frac{\pi}{2}$ rad or 90° or current lags behind the voltage by 90°. Let us explain:

- ☐ When $\omega t = 0°$, $I = I_m(0) = 0$, $V_L = V_m(1) = V_m$

- ☐ When $\omega t = 90°$, $V_L = 0$, $I = I_m$
- ☐ When $\omega t = 180°$, $V_L = V_m$, $I = 0$
- ☐ When $\omega t = 270°$, $V_L = 0$, $I = I_m$
- ☐ When $\omega t = 360°$, $V_L = V_m$, $I = 0$

So from the above results, we can say that when voltage is applied to the inductor, no current flows at once because of back emf. The current reaches its maximum value when the voltage is zero. ***All the time, the voltage through circuit is leading the current by 90° or $\frac{\pi}{2}$ rad.***

Phasor Diagram

Inductive Reactance

As from the pure inductive circuit voltage V_m is given by:

$$V_m = \omega L I_m \tag{14.21}$$

From Ohm's law:

$$V_m = I_m R \tag{14.22}$$

Comparing equations 14.21 and 14.22:

$$\omega L I = I R$$

Hence ,

$$R = \omega L \tag{14.23}$$

The factor here in replacement of 'R' in an inductive circuit is called inductive reactance and is the measure of opposition offered by the inductor to the flow of A.C.
So, we can write:

$$\boxed{X_L = \omega L} \tag{14.24}$$

Unit: The unit for X_L is same as for resistance (ohms).

Frequency Response: From equation 14.24:

$$X_L = \omega L$$

Putting $\omega = 2\pi f$:

$$X_L = 2\pi f L$$

For D.C, $f = 0$, hence,

$$X_L = 0$$

For A.C, X_L ***decreases as f decreases and increases for increasing f.***

Power Dissipation

The average power dissipated in an inductor over one cycle is :

$$P = IV$$

$$P_{avg} = V_m I_m < sin\omega t cos\omega t >$$

$$\boxed{P_{avg} = 0}$$

So the average power in a pure inductive circuit over one whole cycle is zero.

Power Curve

From the graph, we see that for first 90○ of the cycle, both voltage and current are positive and the power dissipated is positive. Therefore, power flows from the source to the coil. Similarly, for the next 90○ of the cycle, power flows from the coil to the source. For next 90○, it is again positive and in last 90○, it is again negative. So over one cycle, the positive power is equal to the negative power. So the resultant power over each cycle is zero. Energy from the source is fed into the inductor where it is stored in the magnetic field. As the field decreases, the energy returns to the source. Thus no power is dissipated in the pure inductive circuit.

14.5 Choke Coil

"It is an inductor having high reactance and low resistance used for reducing high frequency component of alternating signal."

Explanation

It consists of a thick copper wire wounded closely in a large number of turns over a soft iron laminated core. This makes the inductance 'L' of the coil very large and resistance very small. Thus it consumes a little power.

In general, a choke is used to prevent electrical signals along undesired paths. The choke is used as a filter in power supply to prevent ripple. It also prevents unwanted signals to enter other parts of the circuits e.g radio frequency choke (RFC) prevents radio frequency signals from entering audio frequency circuits, thus undesired signals and noise can be attenuated.

14.6 Impedance

"It is the combined opposition offered by the circuit elements to the alternating current."

Symbol

It is symbolized as 'Z'.

Explanation

We know that the resistance 'R' offers opposition to the flow of D.C as well as A.C. On the other hand, inductor does not oppose D.C and capacitor has an infinite opposition to D.C.

In case of A.C, an inductance 'L' or a capacitance 'C' offers finite opposition which is measured by the reactances 'X_L' and 'X_C' respectively. If an A.C circuit consists of a resistance 'R', an inductance 'L' and a capacitance 'C', the combined opposition of these elements will be called as impedance of the circuit.

Mathematical Form

Impedance is the ratio of rms values of applied voltage and alternating current.

$$Z = \frac{V_{rms}}{I_{rms}} \tag{14.25}$$

Unit

Similar to resistance, impedance is the resistance so it is also expressed in ohms.

14.7 RL-Series Circuit

"The circuit in which resistor 'R' and inductor 'L' are connected in series."

Circuit Diagram

Explanation

For RL circuit, we are interested in finding the impedance, current and phase angle between the current and voltage. As there is same current through each component, so we consider the current as the reference phasor. Now to understand the RL circuit, we know in resistor, voltage and current are in phase. So, V_R vector will be parallel to I. Also, we know that the potential difference across inductor 'V_L' leads I by 90 degrees. Therefore phasor V_L is in upward direction and V_R will be parallel to I. The resultant V will be the vector sum of V_R and V_L.

Mathematical Derivation

To calculate the current, impedance and phase angle, let us consider the above statements in a diagram. The vector V_R is in phase with I as represented by magnitude and direction by phasor $\vec{OP}$. The voltage V_L leads the current by 90 degrees and is represented in magnitude and direction by phasor $\vec{PM}$ as shown:

ϕ is the angle between V_L and V_R. Now we calculate:

Current(I)

Applying Pythagoras Theorem:

$$V^2 = V_L^2 + V_R^2$$

Put $V_R = IR$ and $V_L = IX_L$, we get:

$$V^2 = I^2X_L^2 + I^2R^2$$

$$V^2 = I^2(X_L^2 + R^2)$$

$$V = I\sqrt{(X_L^2 + R^2)}$$

Hence,

$$I = \frac{V}{\sqrt{(X_L^2 + R^2)}} \tag{14.26}$$

This is the equation for the current.

Impedance

From the definition of Impedance:

$$Z = \frac{V}{I}$$

Put value of I from equation 14.26:

$$Z = \sqrt{(X_L^2 + R^2)} \tag{14.27}$$

The quantity $\sqrt{X_L^2 + R^2}$ is the opposition offered by the RL circuit components to the current flow and is called impedance of the circuit.

Phase Angle

From the diagram, the current I (which is in the horizontal x-direction; the reference direction) lags behind the voltage V of the circuit by ϕ, which is less than 90°.

From figure:

$$tan\phi = \frac{V_L}{V_R}$$

Put $V_R = IR$ and $V_L = IX_L$:

$$tan\phi = \frac{IX_L}{IR}$$

$$\phi = tan^{-1}\frac{X_L}{R} \tag{14.28}$$

From equation 14.28, it is also clear that:

(i) When $\phi \to 90°$, $\omega \to \infty$.

(ii) When $\phi \to 0°$, $\omega \to 0$.

It means that RL-series A.C circuit will behave like a pure inductive circuit for very high frequency and pure resistive circuit for very low frequency.

Power in RL Circuit

To calculate the power in RL circuit, let us write equation for the circuit current I and voltage V. As,

$$I = I_m sin(\omega t - \phi)$$

And

$$V = V_m sin\omega t$$

It means that voltage leads the current by ϕ.Now,

$$P_{avg} =< IV >$$

$$P_{avg} =< I_m sin(\omega t - \phi) V_m sin\omega t >$$

$$P_{avg} = I_m V_m < sin\omega t sin(\omega t - \phi) >$$

Use formula from trigonometry:

$$sin(a - b) = sinacosb - cosasinb$$

So, we can write P_{avg} as:

$$P_{avg} = I_m V_m < sin\omega t(sin\omega t cos\phi - cos\omega t sin\phi) >$$

$$P_{avg} = I_m V_m < sin^2\omega t cos\phi - cos\omega t sin\omega t sin\phi) >$$

Pulling constants out of average:

$$P_{avg} = I_m V_m (cos\phi < sin^2\omega t > - sin\phi < cos\omega t sin\omega t >)$$

Since $< sin^2\omega t >= \frac{1}{2}$ and $< cos\omega t sin\omega t >= 0$, Hence,

$$P_{avg} = I_m V_m (\frac{cos\phi}{2} - 0)$$

$$P_{avg} = \frac{I_m V_m cos\phi}{2}$$

$$\boxed{P_{avg} = IV cos\phi \qquad (14.29)}$$

Note

- ☐ We are writing I and V, it is understood that the values we will take into account are rms values of voltage and current.
- ☐ It is also important that in analogy to D.C where $P = I^2R$, here average power is equal to I_{rms}^2R, normally written as I^2R in A.C circuits.

Power Curve

The power curve for a phase angle of 30 degrees is shown as:

Note: As $tan\phi = \frac{X_L}{R}$, when ϕ is large i.e greater the phase angle, larger will be the reactance compared to the resistance and less will be the power consumption.

14.8 RC Series A.C Circuit

"The circuit in which resistor R, capacitor C are connected in series with an alternating voltage."

Circuit Diagram

Let us consider a capacitor C and resistor R connected in series with an alternating voltage V as shown:

Explanation

As the same current flows through each component, so will consider current to be our reference phasor. V_R is the voltage across the resistor and V_C is the voltage across the capacitor. As in resistor, voltage and current are in phase, so V_R must be parallel to I. Also we know that the potential difference across C is V_C and it lags behind I by 90 degrees. Hence the vector V_C must be in downward direction. The vector sum of V_C and V_R equals the potential difference applied V.

Mathematical Derivation

To calculate current, impedance and phase angle, we will draw a diagram based on above explanation.

Current

Applying Pythagoras Theorem:

$$V^2 = V_C^2 + V_R^2$$

Put $V_R = IR$ and $V_C = IX_C$:

$$V^2 = I^2X_C^2 + I^2R^2$$

$$V^2 = I^2(X_C^2 + R^2)$$

$$V = I\sqrt{(X_C^2 + R^2)}$$

Hence,

$$I = \frac{V}{\sqrt{(X_C^2 + R^2)}} \tag{14.30}$$

This is the equation for the current.

Impedance

From the definition of impedance :

$$Z = \frac{V}{I}$$

Put value of I from equation 14.30:

$$Z = \sqrt{(X_C^2 + R^2)} \tag{14.31}$$

The quantity $\sqrt{X_C^2 + R^2}$ is the opposition offered by the RC circuit components to the current flow and is called impedance of the circuit.

Phase Angle

From the diagram, the current I(which is in the horizontal x-direction; the reference direction) leads the voltage V of the circuit by ϕ, which is less than 90°. From figure:

$$tan\phi = \frac{V_C}{V_R}$$

Put $V_R = IR$ and $V_C = IX_C$:

$$tan\phi = \frac{IX_C}{IR}$$

$$\phi = tan^{-1}\frac{X_C}{R} \tag{14.32}$$

From equation 14.31, it is also clear that:

(i) When $\phi \to 0°$, $\omega \to \infty$.

(ii) When $\phi \to 90°$, $\omega \to 0$.

It means that RL-series A.C circuit will behave like a pure inductive circuit for very high frequency and pure resistive circuit for very low frequency.

Power in RC Circuit

To calculate the power in RC circuit, let us write equation for the circuit current I and voltage V. As,

$$I = I_m sin(\omega t - \phi)$$

And

$$V = V_m sin\omega t$$

It means that voltage leads the current by ϕ. Now,

$$P_{avg} =< IV >$$

$$P_{avg} =< I_m sin(\omega t - \phi) V_m sin\omega t >$$

$$P_{avg} = I_m V_m < sin\omega t sin(\omega t - \phi) >$$

Use formula from trigonometry:

$$sin(a - b) = sinacosb - cosasinb$$

So, P_{avg} can be written as:

$$P_{avg} = I_m V_m < sin\omega t(sin\omega t cos\phi - cos\omega t sin\phi) >$$

$$P_{avg} = I_m V_m < sin^2\omega t cos\phi - cos\omega t sin\omega t sin\phi) >$$

Pulling constants out of average:

$$P_{avg} = I_m V_m (cos\phi < sin^2\omega t > -sin\phi < cos\omega t sin\omega t >)$$

Since $< sin^2\omega t = \frac{1}{2} >$ and $< cos\omega t sin\omega t >= 0$, Hence:

$$P_{avg} = I_m V_m (\frac{cos\phi}{2} - 0)$$

$$P_{avg} = \frac{I_m V_m cos\phi}{2}$$

$$P_{avg} = IV cos\phi \tag{14.33}$$

Note

In case of R.C circuit, the phase angle ϕ is in clockwise direction because V_C lies along negative y-axis. Some authors do derivations by writing V_C as $-V_C$ and hence ϕ negative, which is also an approach.

14.9 RLC Series A.C Circuit

"The circuit containing resistor, inductor and capacitor in series with an A.C source is called RLC series A.C circuit."

14.9.1 Explanation

Let us consider an inductor L, resistor R and capacitor C connected with an A.C source as shown: The purpose is to find the current, impedance and the phase angle of the voltage with current. find the required quantities, we must draw the phasor diagram for the circuit. We know that in series the current in all the circuit elements is same, therefore, it is taken as the reference phasor and is drawn horizontally directed to the right as shown:

Also we know that V_R is in phase with I. The voltage V_L is leads I by 90 degrees, whereas V_C lags behind the I by 90 degrees i.e V_L and V_C are out of phase by 180°. Now, there are two possibilities:

(i) If V_L is greater than V_C, the resultant $V_L - V_C$ is in the direction of V_L.

(ii) If V_C is greater than V_C, the resultant $V_L - V_C$ is in the direction of V_C.

It is shown as: Now we discuss:

Current

From either of Diagram, applying Pythagoras Theorem:

$$V^2 = (V_C - V_L)^2 + V_R^2$$

Since $(a-b)^2 = (b-a)^2$, Hence,

$$V^2 = (V_L - V_C)^2 + V_R^2$$

Putting $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$, we get:

$$V^2 = I^2(X_L^2 - X_C^2) + I^2R^2$$

$$V^2 = I^2((X_L^2 - X_C^2) + R^2)$$

$$V = I\sqrt{((X_L^2 - X_C^2) + R^2)}$$

Hence,

$$I = \frac{V}{\sqrt{(X_L^2 - X_C^2) + R^2)}} \tag{14.34}$$

This is the equation for the current.

Impedance

From the definition of Impedance:

$$Z = \frac{V}{I}$$

Put value of I from equation 14.33:

$$Z = \sqrt{((X_L^2 - X_C^2) + R^2)} \tag{14.35}$$

The quantity $\sqrt{(X_L^2 - X_C^2) + R^2)}$ is the opposition offered by the RC circuit components to the current flow and is called impedance of the circuit.

Phase Angle

From figure:

$$tan\phi = \frac{V_L - VC}{V_R}$$

Put $V_R = IR, V_L = IX_L$ and $V_C = IX_C$:

$$\phi = tan^{-1}\frac{X_L - X_C}{R} \tag{14.36}$$

Impedance Triangle

In analogy with phasor diagram, the impedance diagram is shown as:

Power Factor

As power factor is given by:

$$cos\phi = \frac{V_R}{V} = \frac{R}{Z} \tag{14.37}$$

Power Dissipation

Using formula:

$$P_{avg} = IV cos\phi \tag{14.38}$$

Now, we discuss some cases:

Case 1: The quantity $X_L - X_C$ is called the reactance of the circuit. When $X_L - X_C$ is positive, i.e. $X_L > X_C$, phase angle ϕ is positive and the circuit will be inductive. In other words, the circuit current I will lag behind the voltage V by ϕ.

Case 2: When X_L-X_C is negative, i.e. $X_L > X_C$, phase angle ϕ is negative and the circuit is capacitive. That is to say that circuit current I leads the applied voltage by ϕ.

Case 3

When $X_L - X_C = 0$, i.e. $X_L = X_C$, the circuit is purely resistive. In other words, circuit current I and applied voltage V will be in phase i.e. $\phi = 0°$, the power factor will be unity.

14.10 Series Resonance A.C Circuits

Definition

"The situation in which the current in the circuit is maximized is called electrical resonance."

OR

"The situation in which the inductive reactance is equal to the capacitive reactance."

Explanation

In the impedance equation, along with the equations for the inductive and capacitive reactances, we see that impedance has a rather complicated dependence on frequency. As,

$$Z = \sqrt{((X_L^2 - X_C^2) + R^2)} \tag{14.39}$$

And

$$I = \frac{V}{Z} \tag{14.40}$$

Now, the frequency of circuit can be set so that the capacitive reactance equals the inductive reactance i.e $X_L = X_C$ then according to equation 14.39, Z will be equal to R and by equation 14.40, I will be maximum. This condition is called resonance and is electrical analogue to resonance in harmonic oscillators such as swinging pendulum of a mass at the end of spring.

Resonance Frequency

As we know that at resonance,

$$X_L = X_C$$

Put $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$:

$$\frac{1}{\omega C} = \omega L$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \tag{14.41}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{14.42}$$

This is known as resonance frequency for a given L and C.

14.11 Electromagnetic Radiations

Electromagnetic radiations such as infrared are different from each other due to their properties. But they have some features in common such as electric field and magnetic field. Therefore it can be described in terms of electric and magnetic fields, and they all travel through vacuum with the same speed (the speed of light).

Fundamentally these radiations are different only in wavelength or frequency. The names given to them in figure below shows various regions of the spectrum along with given names. There are no gaps in the spectrum, nor their sharp boundaries between the various categories (certain regions of spectrum are assigned by law for commercial or other uses such as T.V, A.M, F.M broadcasting).

Let us consider some of these types of electromagnetic radiation in more details:

14.11.1 Light

The visible region of spectrum, most familiar to us, is the electromagnetic radiation emitted by sun. The wavelength of light ranges from about 400 nm (violet) to about 700 nm (red). Light is often emitted when the outer (or valence) electrons in atoms change their state of motion: for this reason, such transitions in the state of the electron are called optical transitions. The color of the light tells us something about the atoms or the object from which it was emitted. The study of the light emitted from the Sun and from distant stars gives information about their composition.

14.11.2 Infrared

Infrared radiation, which has wavelength stronger than the visible (from 0.7 m to about 1 mm), is commonly emitted by atoms or molecules when they change their rotational or vibrational motion. Infrared radiation is an important means of heat transfer and is sometimes called heat radiation. The warmth you feel when you place your hand near a glowing light bulb is primarily a result of the infrared radiation emitted from the bulb.

All Objects emit electromagnetic radiation (thermal radiation) because of their temperature. Objects of temperatures ranges from 3K to 3000K emit their most intense thermal radiation in the infrared region of the spectrum.

A remote control is a component of an electronics device, most commonly a television set, DVD player and home theater systems originally used for operating the device wirelessly from a short line-of-sight distance. The main technology used in home remote controls is infrared light. The signal between a remote control handset and the device it controls consists of pulses of infrared light, which is invisible to the human eye. Infrared radiation is also used for cooking the surface of food (the interior is then heated by convection and conduction).

14.11.3 Microwaves

Microwaves can be regarded as short radio waves, with typical wavelengths in the range 1mm to 1m. They are commonly produced by electromagnetic oscillators in electric circuits, as in the case of microwave ovens. Microwaves are often used to transmit telephone conversations. Figure shows a microwaves station that serves to relay telephone calls. Microwaves also reach us from extraterrestrial sources.

Neutral hydrogen atoms, which populace the regions between the stars in our galaxy, are common extraterrestrial source of Microwaves emitting radiation with a wavelength of 21cm.

14.11.4 Radio Waves

Radio waves have wavelengths longer than 1 m. They are produced from terrestrial sources through electrons oscillating in wires of electric circuits. By carefully choosing the geometry of these circuits, as in an-antenna, we can control the distribution in space of the emitted radiation (if the antenna acts as a transmitter) or the sensitivity of the detector (if the antenna acts as a receiver). Traveling outward at the speed of light, the expanding of TV signals transmitted on Earth.

Radio waves reach us from extraterrestrial sources, the sun being a major source that often interferes with radio or TV reception on Earth. Mapping the radio emissions from extraterrestrial sources, known as radio astronomy, has provided information about the universe that is often not obtainable using optical telescopes.

14.11.5 Ultraviolet

The radiations of wavelengths shorter than the visible begin with the ultraviolet (lnm to which can be produced in atomic transitions of the outer- electrons as well as in radiation from thermal sources such as the Sun. Because our atmosphere absorbs strongly at ultraviolet wavelengths, little of this radiation from the Sun reaches the ground. However, the principal agent of this absorption is atmospheric ozone, which has been depleted in recent years as a result of chemical reactions with fluorocarbons released from aerosol sprays, causes common sun burn but long-term exposure can lead to more serious effects, including skin cancer.

Ultraviolet Lamp(UV light)

A lamp producing ultraviolet(UV) radiation emitted through clear, pre-filtered , particle free water. This UV light is extremely effective in killing and eliminating bacteria, yeasts, viruses, molds and other harmful organisms known to man. It is used in industry and hospitals to treat water. It is also used as a post disinfecting method for residential water treatment.

14.11.6 X-rays

X-rays (typical wavelengths 0.01 nm to 10 nm) can be produced with discrete wavelengths In individual transitions among the inner (most tightly bound) electrons Of an atom, and they can also be produced when charged particles (such as electrons) are decelerated. X rays can easily penetrate soft tissue but are stopped by bone and other solid matter; for this reason they have found wide use in medical diagnosis.

14.11.7 Gamma rays

Gamma rays are electromagnetic radiations with the shortest wavelengths less than 10 pm. They are the most penetrating of electromagnetic radiations, and exposure to intense gamma radiation can have a harmful effect on the human body. These radiations can be emitted in transitions of an atomic nucleus from one state to another and can also occur in the decays of certain elementary particles. For example, a neutral ion can decay into two gamma rays according to

$$\pi^0 = \gamma + \gamma$$

Chapter 15

Nuclear Physics