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Chapter 11

Electrostatics

11.1 Electric Charge

"Electric charge is a basic property of elementary particles that causes them to exert forces on one another."

Alternatively, we can define electric charge as, *"A physical quantity whose presence produces an electric field."*

Explanation

Charge is a fundamental property of some elementary particles like electron, proton etc. There are two kind of charges; positive and negative. Protons are present inside the nucleus of an atom, possess positive charge and electrons are clouding around the nucleus of an atom, carry negative charge. All protons are alike and have the same charge +e = 1.60219×10^{-19} coulomb. Similarly all electrons are alike and have the same charge -e. e is the fundamental charge.

11.1.1 Properties of Electric charge

Electrification

"Electrification is the process in which a neutral body is charged by the removal or addition of electrons."

In nature, majority objects are in neutral state because the number of electrons is equal to the number of protons in them. If electrons are removed, a positive charge appears on the body. Similarly, if electrons are added, a negative charge appears on the body. It is important to note that during the process, charge can never be created nor destroyed.

Conservation of Electric charge

Conservation of charge means that, "Charge can never be created nor destroyed in a process." We can also state it as, "the total charge of an isolated system remains conserved."

Quantization of Electric charge

Another very basic property of electric charge is that charge is quantized. Quantization of charge means that it exists in discrete packets rather than in continuous amounts. It means that charge on a certain body can be developed due to addition or removal of one or two or three or ‘n’ electrons. Any charge q, no matter what is its origin, is an integral multiple of the minimum elementary charge e. Mathematically,’q’ charge on a body can be expressed as:

$$q = ne \quad \text{and n} = 1, 2, 3, \ldots$$

Checkpoint 11.1

We say that charge always exists in an integral multiple of ‘e’ but we also believe in the existence of quarks (having charge 2/3 e, 1/3 e etc.). How is this possible? (Answers of checkpoints are given at the end of the part).

Action between Two Charges

It has been experimentally proved that like charges repel each other and unlike charges do attract. This means that a positive charge will repel a positive and so does a negative. And positive attracts a negative one and a negative does attract a positive one.

Sciences of Electric Charges

On the basis of state of rest or motion of electric charges, science of charges is divided into two branches i.e. electrostatics and electrodynamics.

- **Electrostatics/Static Electricity:**
  “It is the study of electric charges at rest”. In this chapter, we will deal with this branch. We will study some basic laws like Coulomb’s law, Gauss’s law etc, and some fundamental concepts electric field, electric potential and further capacitors and their related concepts.

- **Electrodynamics/Current Electricity:**
  “It is the study of electric charges in motion.” We will encounter this in next chapter. We will study some basic concepts of current flow, resistance and laws like Ohm’s law and Kirchoff’s laws.

11.2 Coulomb’s law

Background

As we know that electric charges attract or repel each other. To quantify this attaraction or repulsion, Charles-Augustin de Coulomb[1], in 1785, first measured the force of interaction (attraction or repulsion) between electric charges and deduced the law that governs

[1] ***Charles Augustin de Coulomb (1736 – 1806):*** Coulomb, a French physicist, began his career as a military engineer in the West Indies. In 1776, here turned to Paris and retired to a small estate to do his scientific research. He invented a torsion balance to measure the quantity of a force and used it for

them, called as Coulomb's law. He used for this purpose an apparatus called Torsion's balance.

Statement:

"Two stationary point[2] charges attract or repel each other with a force which is directly proportional to the product of the magnitude of the charges and is inversely propotional to the square of the distance between them, and this force acts along the line connecting the charges."

Mathematical Form:

Let us consider two electric charges 'q_1' and 'q_2' seperated by distance 'r', these charges will exert forces on one another and this force according to Coulomb's law will be:

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2} \tag{11.1}$$

Where k is the constant of propotionality and is called 'Coulomb's constant' (we will discuss it in detail later). If r is the unit vector along the line connecting the charges, then Coloumb's law expression will be written as:

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \tag{11.2}$$

This is the vectorial form of Coulomb's law,which shows that force is directed along the unit verctor $\hat{r}$. We have two charges q_1 and q_2. If q_1 is considered as source charge(the charge exerting the force) and q_2 is considered as field charge (the charge experiencing the force),then the unit vector $\hat{r}$ is directed from q_1 to q_2 and vice versa. It is important to note that the direction of force is according to source charge.

Attractive or Repulsive Force

For like charges, the product q_1q_2 is positive; the force is repulsive and is directed away from the source charge. For unlike charges, the force is attractive as q_1q_2 is negative and the force is directed towards the source charge.

determination of forces of electric attraction or repulsion between small charged spheres. He thus arrived in 1785 at the inverse square law relation, now known as Coulomb's law. The law had been anticipated by Priestley and also by Cavendish earlier, though Cavendish never published his results. Coulomb also found the inverse square law of force between unlike and like magnetic poles.

[2]As word point suggests an entity of no dimension.In practice,two charges are said to be point charges if their dimesions (means their radii) are very very smaller as compared to the distance between them. Coloumb's law has its validity over the point charges.

Constant of propotionality 'k'

We used the constant of propotionality 'k' in coulomb's law expression. The constant 'k' depends upon:

(a) System of units used (in which q, r and F are measured)

(b) Properties of medium surrounding the charges

In SI units,force, charge and distance are measured in newtons, coloumbs and metres. As far as the choice of the medium is concerned, we start with vacuum or free space. In free space and SI units, the constant 'k' is written as:

$$k = \frac{1}{4\pi\epsilon_o} \tag{11.3}$$

Where 'ϵ_o' is an electrical constant (read as 'epsilon not') and called as permittivity of free space (permittivity of a medium is the property of the medium which determines how much that medium affects the force between the charges). The value of 'ϵ_o' is measured experimentally and is found to be 8.85×10^{-12} C^2/Nm^2 (rounded to two decimal places). Using the value of 'ϵ_o' in equation 11.3, we get value of 'k' equal to 8.99×10^9 Nm^2/C^2. So, in SI units and for charges placed in vacuum (or free space), Coulomb's law in equation 11.4 can be written as:

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}\hat{r} \tag{11.4}$$

And in magnitude form, (using 'vac' in subscript for representation of force in vacuum):

$$F_{vac} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2} \tag{11.5}$$

11.2.1 Coloumb's law in Material Media

As the constant of propotionality 'k' in coulomb's law expression depends on the medium around the charges. Therefore, if the charges are placed in a medium of permittivity ϵ, then coloumbic force will be given by:

$$F_{med} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \tag{11.6}$$

From equation 11.6, it implies that a material medium with high permittivity is a medium which reduces appreciably the force between the charges compared with the vacuum. For air, ϵ_{air} is only slightly greater than ϵ_o (1.006) and for many practical purposes, is taken equal to ϵ_o. In order to make force more simplified, we introduce a term relative permittivity ϵ_r, which is defined as, "the permittivity of a medium compared with the permittivity of vacuum." So, it is given by the ratio:

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} \tag{11.7}$$

So, 'ϵ_r' is a dimensionless constant. It is also called the dielectric constant of the medium. For vacuum or free space, $\epsilon_r = 1$,and hence $\epsilon = \epsilon_o$. From above equation:

$$\epsilon = \epsilon_o \epsilon_r \tag{11.8}$$

Putting this value in equation 11.6, we get:

$$F_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1 q_2}{r^2} \tag{11.9}$$

Here, 'ϵ_o' is a constant and has a fixed value. It is important to note that the value of dielectric constant εr for any material medium is always greater than one. So force in a mterial medium is always less than that of vacuum. This becomes more obvious, if we take ratio of equations (11.9) and (11.5):

$$\frac{F_{med}}{F_{vac}} = \frac{1}{\epsilon_r} \tag{11.10}$$

which gives:

$$F_{med} = \frac{F_{vac}}{\epsilon_r} \tag{11.11}$$

Since $\epsilon_r > 1$ for any material medium, therefore force in a medium is less than force in vacuum for two given charges by 'ϵ_r times'. This equation gives us a sense that how much times force in a medium is reduced as compared with the vacuum for two charges. The above equation can also be written as:

$$\epsilon_r = \frac{F_{vac}}{F_{med}} \tag{11.12}$$

This equation provides us with another definition of dielectric constant i.e. ***"the ratio of force between two point charges placed in vacuum to the force between the same charges when the medium surrounding them is material medium, is called dielectric constant/relative permittivuty of that medium."***

Checkpoint 11.2

Distilled water has a dielectric constant of nearly 80. If force between the two charges placed in vacuum is 80 N. How much the force will be when the same charges be placed in water?

11.2.2 Units of Electric Charge

SI Unit

The S.I unit of electric charge is coulomb (C), which can be defined in a number of ways stated under. One way of defining one coloumb charge is using Coulomb's law expression i.e. ***"The charge on a body is said to be one coulomb when it exerts a force of 9×10⁹ N on a similar charge at a distance of one metre from it in vacuum."***

Alternatively, we can also define it in terms of charge of an electron.As we know that:

$$charge\,of\,one\,electron = e = 1.602 \times 10^{-19}C$$

So one coloumb charge will contain 6.25×10^{18} electrons. Hence, one coulomb can be defined as, ***"The charge of 6.25×10^{18} electrons is said to be one coloumb."***

Another way of defining one coulomb charge is in terms of current (flowing charge) i.e. ***"The amount of charge that flows through a given cross section of a wire in one second if there is a steady current of one ampere in the wire."***

Multiples

Since coloumb is a very large unit (imagine it as the charge of 6.25×10^{18} electrons), so its submultiples are commonly used.

$$1\ \mu C = 10^{-6}\ C,\ 1\ nC = 10^{-9}\ C,\ 1\ pC = 10^{-12}\ C$$

11.2.3 Coloumb's Law and Newton's Third Law

As we know that charges exert forces on each other, hence coloumic force is a mutual force. To show that the force that one charge exerts on the other is equal in magnitude but opposite in direction to the force that it experiences by the other charge, let us consider two point charges (assume them to be positive for convenience). The distance between them is r, as shown in figure 11.1.

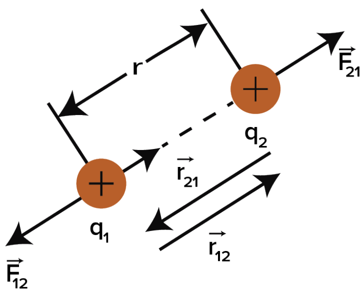


Figure 11.1: Force exerted by each charge, $\vec{F_{12}}$ is the force exerted on charge 'q_1' by 'q_2', $\vec{F_{21}}$ is the force exerted on charge 'q_2' by 'q_1'

Let $\vec{F_{12}}$ is the force exerted by charge 'q_2' on charge 'q_1' (subscript as, '12' means by 2 on 1) and $\vec{r_{12}}$ is the position vector directed from 'q_2' to 'q_1'. Similarly, $\vec{F_{21}}$ is the force exerted by charge 'q_1' on 'q_2' and $\vec{r_{12}}$ is the unit vector directed from 'q_1' to 'q_2'. Since,

$$\vec{r_{12}} = r_{12}\hat{r_{12}}$$

And,

$$\vec{r_{21}} = r_{21}\hat{r_{21}}$$

So, $\vec{F_{12}}$ according to Coloumb's law will be given by:

$$\vec{F_{12}} = k\frac{q_1 q_2}{r_{21}^2}\hat{r_{21}}$$

And,

$$\vec{F_{21}} = k\frac{q_1 q_2}{r_{12}^2}\hat{r_{12}}$$

Since,

$$\vec{r_{12}} = -\vec{r_{21}}$$

Therefore,

$$\vec{F_{12}} = -\vec{F_{21}}$$

So we concluded that the force between two charges is mutual and the forces pair is a Newton's third law pair.

11.2.4 Principle of Superposition

Coulombic force obeys the principle of superposition i.e. ***"The net force acting on a charge by an assembly of charges is the vectorial sum of all the forces exerted by the number of charges."***

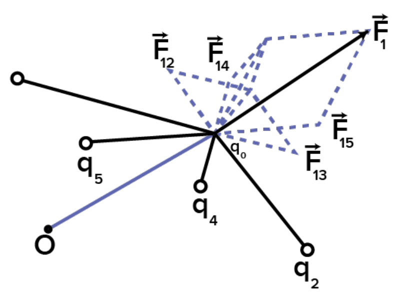


Figure 11.2: Net force exerted on charge 'q_1' is the vectorial sum of all the forces due to each charge on 'q_1'.

From the figure 11.2, for 'n' charges exerting a force on the charge 'q_1', we can say that:

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + ... + \vec{F_{1n}}$$

Checkpoint 11.3

(a) We have two charges, 'q_1' and 'q_2'. Let the force of interaction between them is 4 N. Then we placed another charge 'q_3' in the vicinity of the charges. What would be the force of attraction between 'q_1' and 'q_2' now?

(b) Show that for two opposite charges, 'q_1' and 'q_2' using usual notations,

$$\vec{F_{12}} = -\vec{F_{21}}$$

11.3 Electric Field and Electric Field Intensity

Introduction

The concept of electric field was introduced by ***Michael Faraday***. He stated that the charge ‘q’ produces an electric field surrounding it and when a charge ‘q_o’ is brought into its field, the field of charge ‘q’ interacts with that of ‘q_o’ and exerts force on it.

Definition of Electric Field

“The region around a charge in which a test charge experiences an electric force, is called electric field.”

Explanation

As we know that charges exert forces on each other. Electric field is actually the region around a source charge upto which it can exert a force on a test charge. In order not to distort the field of a source charge, the test charge should be small. So the strength and direction of electric field can be determined by placing a unit positive test charge in that field.

The strength and direction of field at a point in space is determined by the force that a unit positive charge will experience at that point. The direction of field is that direction in which test charge moves or tends to move at a given point.

Electric Field Intensity

“A single vector quantity containing information about the field strength and directiom at a given point is called electric field intensity.”

Mathematical Form

If ‘$\vec{F}$’ is the force exerted on a unit positive test charge (it is a convention to take test charge positive) ‘q_o’ by a source charge ‘q’, then the electric field intensity ‘$\vec{E}$’ is given by:

$$\vec{E} = \frac{\vec{F}}{q_o} \tag{11.13}$$

Thus, it is the force per unit charge. Defininig in this way, electric intensity is independent of test charge ‘q_o’. Since ‘q_o’ is always positive, therefore is always along the direction of force on that test charge.

Units and Dimensions

The S.I unit of electric field intensity is NC^{-1}. Another unit is volt per metre (V/m), we will dicuss this in electric potential. Dimensions are $[MLT^{-3}A^{-1}]$.

11.3.1 Field Intensity due to a Point Charge

In order to find an expression for field intensity due to a point charge, consider point charges ‘q’ and ‘q_o’ and at a distance ‘r’ from each other. As an example, we find the intensity of field which exists in air around the isolated charge ‘q’.

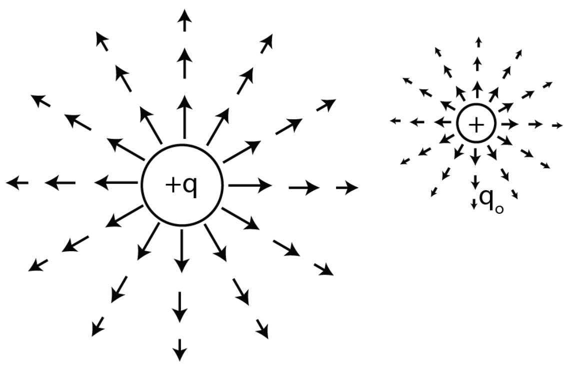


Figure 11.3: Field of a source charge ‘q’ influencing test charge ‘q_o’

Here ‘q_o’ is a small test charge. Coulomb’s force due to ‘q’ on ‘q_o’ is:

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{r^2}\hat{r} \tag{11.14}$$

From the definition of electric field intensity i.e. the force per unit charge, we can write:

$$\vec{E} = \frac{\vec{F}}{q_o}$$

Hence,

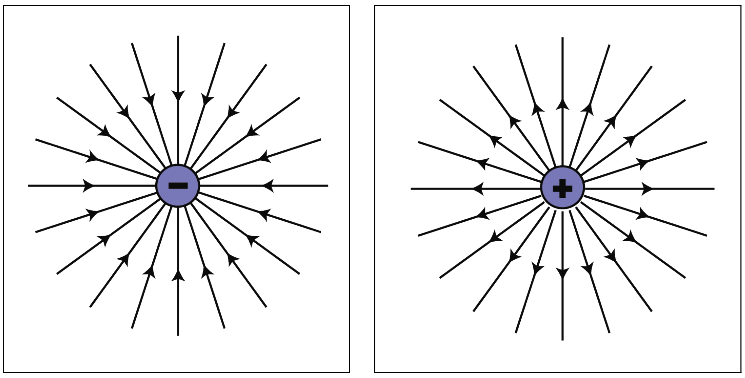
$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r} \tag{11.15}$$

From above equation, electric field varies directly with the magnitude of source charge and is inversely propotional to the square of the distance from the source charge. Moreover, electric field is directed along the radius outward if ‘q’ is positive and radially inward if ‘q’ is negative as shown in figure 11.4.

If medium surrounding the charge ‘q’ is other than vacuum or air, having dielectric constrant ‘ϵ_r’, then electric intensity will be given by:

$$\vec{E} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q}{r^2}\hat{r} \tag{11.16}$$

11.3.2 Electric Lines of Force/Electric Field Lines



(a) Positive Charge (b) Negative Charge

Figure 11.4: Electric field pattern for an isolated positive & negative charge

Definition

"An electric line of force is a curve so drawn that a tangent to the curve at any point shows the direction of the field at that point."

Explanation

Electric lines of force are imaginary lines to show the strength and direction of a real field. As the strength of the field means electric field intensity, so electric lines of force represents the magnitude and direction of electric field. The mapping of electric field by field lines helps in visualizing the electric field.

The magnitude of the electric field at a certain point is determined by the density of lines (number of lines per unit area). The greater the density of lines, the greater will be the magnitude of electric field. The direction of the field at a given is determined by drawing a tangent to the field line at that point. Actually, the field line tells us about the movement of a test charge at each and every point inside an electric field.

Properties of Electric Field Lines

(i) The density of lines show the direction of electric field.

(ii) The arrowhead shows the direction of field.

(iii) The field lines begin from positive charge and terminate on negative charge, they are continuous in region containing no charge.

(iv) The lines of force do never cross.If they did cross, then electric field would have two different directions at the point of intersection which is not possibe. Put another way, at the point of intersection, there would be two tangents possible. As at a given point, the tangent to the curve represents the direction of field or the direction in which the test charge will move along at that point, so the test charge will have to move along two different directions at the same point and time, which is not possible.

(v) The lines of force strike the surface of conductor perpendicularly.

(vi) The lines of force can not pass through the conductor. As the presence of lines implies that there would be an electric field inside the conductor. As conductor have free charges in them, so the existence of electric field corresponds the force on those charges, leading to the flow of charges (electric current), but no such perpetual currents exist. Hence no electric field exists inside the conductor body hence no field lines.

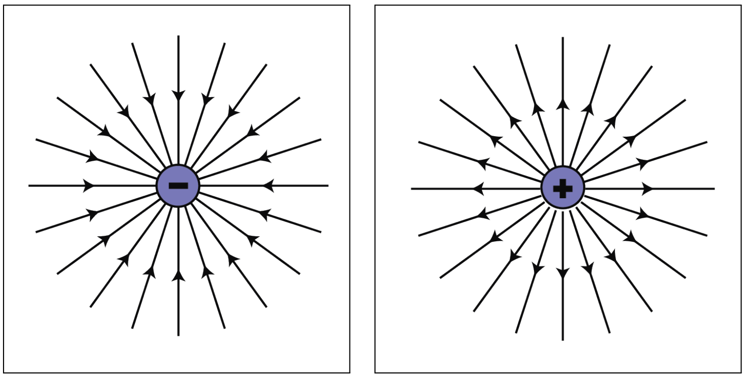
Neutral point/Null point

"The point where the net electric field intensity is zero is called neutral point/null point." It is denoted by 'N' (we will discuss its example in next section).

11.3.3 Field Lines of Some Electrostatic Charge Distributions

For an Isolated Charge

We have discussed already that field lines for an isolated positive charge are radially outward and radially inward for a negative charge extending in space as shown:



(a) Positive Charge (b) Negative Charge

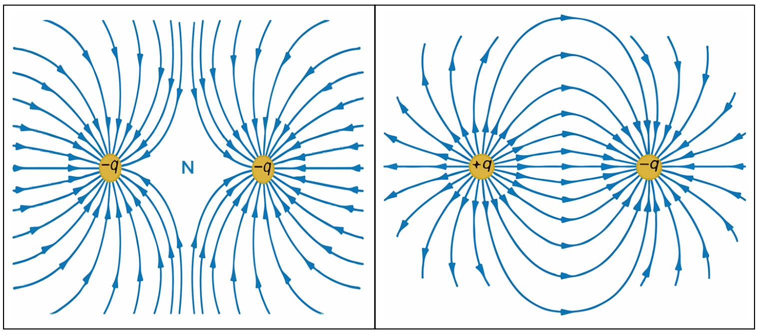
Figure 11.5: Electric field pattern for an isolated positive & negative charge

For Two Charges of Same Magnitude

For two charges of same sign and same magnitude (for convenience, we take them negative), configuration is as shown in 11.6a. The point 'N' is neutral point. The number of lines per unit area shows the strength of field. So, no lines means no net electric field. Here, in this case, we assumed both charges of same sign and same magnitude, hence the point of zero electric field will lie in mid of these chrarges. For two charges of same magnitudes but opposite signs, field lines are shown in figure 11.6b.

Electric Lines Due to a Positive Charge Near a Metal Plate

Consider an electric charge '+q' placed near a metal plate. The positive charge will attract the negative charges (electrons) in the metal plate resulting in the motion of the charges until some of them reach that surface of the metal which is near the '+q' charge where they will be at rest. Thus the field lines starting from '+q' charge will terminate on



(a) For two negative charges (b) For a positive & negative charge

Figure 11.6: Electric field pattern for two charges of same magnitude

the negative charges of the metal plate. Furthermore,these lines starting from '+q' charge are always perpendicular to the conductor. The electric lines of force can not pass through the metal. Electric field is zero inside a conductor under electrostatic conditions. If it was not so, electric field would exert forces on electrons causing them to flow establishing current. Since, no such currents do exist, hence field inside the metal will be zero. The field configuration of above discussed case is as shown:

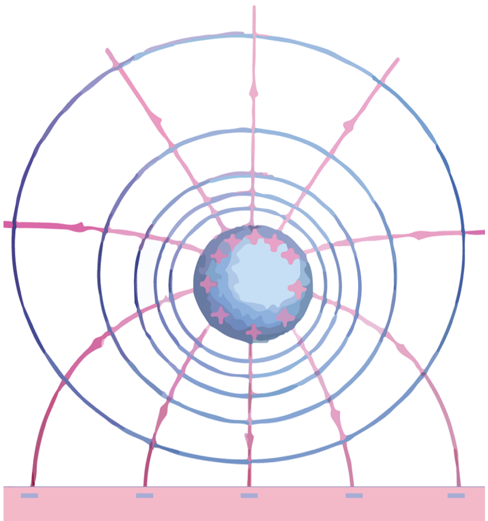


Figure 11.7: Field pattern for a positive charge near a metal plate

Uniform Electric Field

Definition: ***"A field is said to be uniform in a certain region if it has the same magnitude and direction in that region."***

Explanation: From the definition of uniform electric field, it is clear that the magnitude of field must be constant and direction as well. As we know that the magnitude of electric field depends upon the number of lines per unit area,so for uniform electric field, the field lines are uniformly spaced. The direction the field is is represented by the arrows, hence for same direction, they (lines) must point in the same direction.

Production of Uniform Electric Field: A uniform electric field can be produced by connecting the terminals of a baatery connected to two large parallel metal plates. If the plates are of finite length,then the lines of force are bulging at the ends of the plates. This non-uniform field at the edges of the plates is called 'fringing field'. In order to avoid fringing field, the paltes must be of infinite length. Practically, plates are said to be of infinite length when the distance between them is much smaller than their dimensions. The field pattern is shown in figure 11.8.

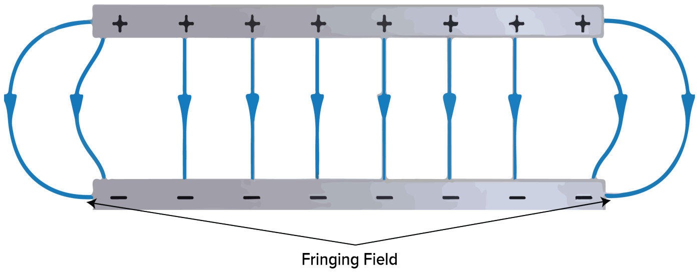


Figure 11.8: Uniform Electric Field with a 'fringing field'

11.4 Electric Flux

"The number of electric lines passing through the area placed in the electric field,is called electric flux."

Symbol

It is denoted by Φ_E (Greek alphabet phi) where the subscript 'E' is for electric.

Explanation

In the above definition,we have discussed two vector quantities, electric field lines mean electric field intensity. The magnitude of electric field intensity is, ***"the number of lines per unit area placed perpendicular to $\vec{E}$."*** The second vector involved in the definition of electric flux is Area. The magnitude of the area vector is equal to the area of the plane occupied and direction is always normal to the surface/plane shown by '$\hat{A}$' or '$\hat{n}$'. Here we considered a flat circular surface, area vector is shown, the direction is shown by normal unit vector '$\hat{n}$'.

Mathematical Form

If $\vec{E}$ is the electric field intensity which is uniform over a certain area, then the electric flux will be given by:

$$\Phi_E = \vec{E}.\vec{A} \tag{11.17}$$

Hence, it is the dot product of previously dicussed two vector quantities $\vec{E}$ and $\vec{A}$. Using usual notations we can write it as:

$$\Phi_E = EAcos\phi \tag{11.18}$$

where $Acos\phi$ is the component of area held perpendicular to E (But its direction will be along E). This equation is the mathematical form of flux at any angle. In the figure, the area component which is perpendicular to field lines is '$A_\perp$'. Note that only this component contributes to electric flux, the lines of forces just skim through '$A_\parallel$'. Resolving area vector into its $\hat{i}$ and $\hat{j}$ components, the component which is along the direction of electric field is $Acos\phi\,\hat{i}$ which is actually the vector associated with the same area component that is held perpendicular to electric field lines and contributing to flux (As its direction is perpendicular to '$A_\parallel$'). So we defined electric flux as the product of electric intensity and the component of area vector along the direction of $\vec{E}$.

Maximum Flux

If the surface is placed perpendicular to electric field such that surface area vector is parallel to the direction of electric field E, then maximum number of lines of force will pass through the surface. Consequently, maximum electric flux will pass through the surface. (Note that ϕ is the smaller angle between $\vec{E}$ and area vector $\vec{A}$ which is perpendicular to the plane of area). This is shown in figure **??**: Using flux definition:

$$\Phi_E = EAcos\phi$$

Here $\phi = 0°$, So,

$$\Phi_E = EAcos(0°)$$

$$\Phi_E = EA$$

This can be physically understood by another way. Since '$A_\perp$' is the only area component contributing to flux and there is no parallel component of area '$A_\perp$. So,only the vector area related with '...' will be involved in defining flux i.e. $Acos\ \phi$, which is equal to the whole area vector, and writing in magnitude form we have flux equal to EA.

Zero Flux

If the surface is placed parallel to the electric field lines such that area vector $\vec{A}$ is normal to the electric field $\vec{E}$, then the maximum number of lines will pass through the surface. Consequently, no electric flux will pass through the surface, as shown: Mathematically,

$$\Phi_E = EAcos\phi$$

Here $\phi = 90°$, So,

$$\Phi_E = EAcos(90°)$$

$$\Phi_E = 0$$

Note: We defined '$A_\perp$ and '$A_\parallel$' be the components of the surface on the basis of their orientation relative to field lines.

Nature

Since,electric flux is the dot product of $\vec{A}$ and $\vec{A}$, so it is a scalar quantity.

Units

The S.I unit of electric flux is Nm^2C^{-1}.

11.4.1 Flux Through an Arbitary Shaped Body

We defined flux to be equal to $\vec{E}.\vec{A}$ which is applicable only if 'E' is uniform over the whole area A, i.e. for flat surfaces. If the surface is not flat, then equation $\vec{E}.\vec{A}$ can not be applied directly, because the angle between E and A changes from place to place.In such cases, the product $\vec{E}.\vec{A}$ defines flux only for small area piece which is considered flat. Therefore, to find flux for an uneven surface (open or closed), we divide the surface into small flat pieces ΔA_1, ΔA_2, $\Delta A_3, ...,$ ΔA_n. as shown:

The area vectors of these area components are $\Delta\vec{A}_1$, $\Delta\vec{A}_2$, $\Delta\vec{A}_3, ...,$ $\Delta\vec{A}_n$. The electric field intensity at these area pieces are $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3, ...,$ $\vec{E}_n$ repectively. The small amount of flux '$\Delta\Phi_1$' through 'ΔA_1' is defined as:

$$\Delta\Phi_1 = \vec{E}_1.\Delta\vec{A}_1$$

Similarly,

$$\Delta\Phi_2 = \vec{E}_2.\Delta\vec{A}_2$$

Hence for n$^{\text{th}}$ patch,

$$\Delta\Phi_n = \vec{E}_n.\Delta\vec{A}_n$$

So, the total flux through the surface will be:

$$\Phi_T = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi_T = \vec{E}_1.\Delta\vec{A}_1 + \vec{E}_2.\Delta\vec{A}_2 + \vec{E}_3.\Delta\vec{A}_3 + ... + \vec{E}_n.\Delta\vec{A}_n$$

Which in compressed notation:

$$\Phi_T = \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i \tag{11.19}$$

11.4.2 Flux through a closed Arbitrary Surface

Let us consider a closed sufrace of arbitrary with the positive normal taken outward from the volume beig closed.In case of closed surface, the electric flux may be positive, negative or zero depending upon the number of lines entering or leaving the surface.We discuss it as:

(i) The electric flux is positive,if net number of lines are leaving the surface.Since positive charge is a source of field lines,so it means that there is a source of libes inside the surface,i.e.positive charge. Mathematically,for closed surace of abitrary shape:

$$\Phi_T = \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i$$

And this implies that there is a source of field lines inside the surface. The number of lines leaving the surface are more than number of lines entering,flux would be negative for lines entering and it would be positive for lines leaving and hence net flux would be positive.

(ii) The electric flux through a closed surface will be negative, if net number of lines are entering the surface or more field lines are entering than leaving the surface;there is a sink of field lines in the closed surface i.e. a negative charge as field lines terminate on negative charge.

(iii) The electric flux will be zero if number of lines entering is equal to the number of lines leaving the surface or no field lines intercpepting the surface. This is possible when there is no net charge because net charge is a source or sink of lines.

11.5 Gauss's Law

Background

The electric field of a given charge distribution can be calculated using Coloumb's law. But sometimes field calculation using Coulomb's law becomes very difficult. An alternative method to calculate the electric field of a given charge distribution relies on theorem called 'Gauss's law' given by Karl Friedrich Gauss[3]. It provides a realtionship between the net electric flux through the closed surface and the net charge enclosed by that surface.

Statement

"The net electric flux through any closed surface is equal to $\frac{1}{\epsilon_o}$ times the charge 'q' enclosed by that surface."

Explanation

Gauss's law provides a simple relationship between the electric flux through any closed surface and the total charge inside that surface. The charges inside the surface can be either a single point charge or a number of charges.

[3]German mathematician and astronomer (1777-1855) Gauss received a doctoral degree in Mathematics from the University of Helmsted in 1799. In addition to his work in electromagnetism, he made contributions in mathematics, number thoery, statistics and mecahnics.

In order to derive an expression for Gauss's law, let us consider a closed surface (for convenince we take a sphere) of radius 'r' having a point charge 'q' at its centre as shown in figure **??**. As the direction of electric field intensity E varies from place to place, so in order to calculate electric flux, we divide the whole surface into 'n' number of small pieces having area ΔA_1, ΔA_2, $\Delta A_3, ...,$ ΔA_n. Here,we considered a sphere,so two things are important:

(i) Electric field intensity is same at every point as they are equidistant from the charge.

(ii) As field is radial, it means that at every point E and A are in same direction i.e. the angle between E and A at every point will be zero degree.

Now, we know that the total flux through area ΔA_1 will be:

$$\Delta\Phi_1 = \vec{E_1}.\Delta\vec{A_1}$$

Similarly,

$$\Delta\Phi_2 = \vec{E_2}.\Delta\vec{A_2}$$

Hence for n$^{\text{th}}$ patch,

$$\Delta\Phi_n = \vec{E_n}.\Delta\vec{A_n}$$

Since,

$$E_1 = E_2 = E_3 = E_n = E$$

And the total flux will be:

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi = \vec{E}.\Delta\vec{A_1} + \vec{E}.\Delta\vec{A_2} + \vec{E}.\Delta\vec{A_3} + ... + \vec{E}.\Delta\vec{A_n}$$

Since, field is radial, hence:

$$\Phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_{3+...+E\Delta A_n}$$

$$\Phi = E(\Delta A_1 + \Delta A_2 + \Delta A_{3+...+\Delta A_n})$$

$$\Phi = E\sum_{surface}\Delta A \tag{11.20}$$

Since, electric field at a distance 'r' due to charge 'q' will be:

$$E = \frac{q}{4\pi\epsilon_o r^2}$$

Also we know that in case of sphere:

$$\sum_{surface}\Delta A = 4\pi r^2$$

Putting these values in equation 11.20

$$\Phi = \frac{q}{4\pi\epsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\epsilon_o} \tag{11.21}$$

Equation 11.21 defines Gauss's law for point charge. It is clear that electric flux is independent of:

(i) The shape of closed surface.

(ii) The radius(size) of the closed surface.It means that if radius is made very small or large, still $\frac{q}{\epsilon_o}$ lines will come from 'q'.

And equation also tells that the flux depends upon:

(i) The amount of charge enclosed.

(ii) The medium surrounding the charge.

Conclusion

We conclude that each positive charge must have $\frac{q}{\epsilon_o}$ lines coming from it. A negative charge will have the same number of lines going through it. Another important thing is that whatever the shape of the closed surface is flux will be same for a given charge placed inside the surface in a medium. We assumed the closed surface as sphere which helped in making our calculation easy. At last we found that it does not matter that what is the shape of surface, it should be just closed and must enclose a charge to give out flux.

11.5.1 Electric Flux due to Many Charges

To formulate Gauss's law for many charges, let us consider point charges q_1, q_2, q_3,..., q_n inside a closed surface S, of some arbitrary shape, as shown: We know that all the flux lines coming from q_1' passes through the surface 'S', therefore from Gauss's law, flux due to 'q_1' will be:

$$\Phi_1 = \frac{q_1}{\epsilon_o}$$

Similarly,

$$\Phi_2 = \frac{q_2}{\epsilon_o}$$

And,

$$\Phi_3 = \frac{q_3}{\epsilon_o}$$

Hence due to n^{th} charge:

$$\Phi_n = \frac{q_n}{\epsilon_o}$$

As total flux is given by:

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n$$

Putting respective values:

$$\Phi = \frac{1}{\epsilon} \times (q_1 + q_2 + q_3 + ... + q_n)$$

Since,

$$q_1 + q_2 + q_3 + ... + q_n) = Q$$

So, we get:

$$\Phi = \frac{Q}{\epsilon_o} \tag{11.22}$$

Note that 'Q' is the net charge which is the algebraic sum of all the charges. It is also important to note that the electric field due to a charge outside the surface contributes zero electric flux through the surface because as many field lines due to that charge enter as leave it.In section before Gauss's law, we discussed positive, negative and zero electric flux through an arbitrary closed surface by using definition of electric flux. Now, we can relate the same using Gauss's law. We can further specify equation 11.22 by writing it as:

$$\Phi = \frac{Q_{enclosed}}{\epsilon_o} \tag{11.23}$$

Now we discuss three cases:

(i) The algebraic sum of '+q' and '-q' charges, if equal to zero i.e. $Q = 0$,then net flux using above equation will be zero. In other case, if there is no charge enclosed, then flux will again be zero.

(ii) If there is a positive charge or sum of charges taking into account their signs comes out to be positive, then electric flux would be positive. More number of lines will be leaving thean the number of lines entering.

(iii) If the closed surface has a negative charge or a net negative charge, the flux will be negative. The flux will be inward flux, i.e. more lines will be enteing than the leaving lines provided the normal is taken outward.

11.5.2 Applications of Gauss's Law

Gauss's law provides a convenient method to calculate electric field in case of sufficiently symmetric charge distribution. Here we discuss some applications of Gauss's law:

Absence of Electric Field and Charge Inside a Condctor and Location of Excess Charge on a Conductor

Under steady state conditions, electric field is zero inside a metal or conductor. This statement is proved by following considerations; Conductors have charges in them, which are free to move. If a resultant field E exists inside the conductor, these charges will experience a force due to the field. They will move and internal currents will set-up. Eventually, within fractions of second, electrostatic equilibrium is achieved. The internal currents will stop. When no electric current flows, resultant force on the charges (electrons) in the conductor must be zero. Since, electric field is responsible for exerting force on the charges, hence, when electrostatic equilibrium is achieved, the electric field in the interior of conductor must be zero. In fact, Gauss's law can be used to show that an excess charge placed on an insulated conductor, resides on its outer surface. For this purpose, consider an isolated conductor carrying an excess charge 'q'. Take Gaussian surface 'S' shown by dotted line. 'S' lies inside just below the actual suface of the conductor as shown:

As under steady state conditions, the electric field inside a conductor is zero. Applying Gauss's law to the Gaussian's surface 'S', we have:

$$\sum_{surface} \vec{E}.\Delta\vec{A} = \frac{Q}{\epsilon_o} \tag{11.24}$$

As 'E' is zero.so $\sum_{surface} \vec{E}.\Delta\vec{A} = 0$. Now, $\epsilon_o \neq \infty$, as it has a finite value equal to $8.85 \times 10^{-12}\, C^2/Nm^2$, so above equation implies:

$$Q = 0$$

This means that there is no charge inside the Gaussian's surface 'S'. As this surface lies just below the actual surface of the conductor, having no charge inside it. This means that the charge is on the actual surface of the conductor.

Electric Field Intensity due to a Charged Conducting Spherical Shell

Shell: ***"Any hollow enclosure with a covering is called a shell."***

Calculation of Electric Field: We are interested in the calculation of electric field due to a charged conducting sphericall shell. For this purpose, consider a metallic shell of radius 'R' having a positive charge 'Q'. We know that the charge will distribute itself uniformly on the conducting surface. As shell has an interior and exterior, therefore, we will find electric intensity seperately for interior and exterior of charged conducting spherical shell.

Field at the Interior: Suppose we want to find the intensity of the field at an interior point 'B' at a distance $r_B < R$ from the centre of the shell. Through the point 'B' construct a 'Gaussian's surface' (Sphere) of radius 'r' with centre at the centre of the shell, as shown:

Since there is no charge inside the gaussian surface, hence Gauss's law gives:

$$\sum_{surface} \vec{E_B}.\Delta\vec{A} = \frac{Q}{\epsilon_o} = 0$$

The charge distribution is spherically symmetric. This implies that $\vec{E_B}$ is radial and has a constant magnitude E_B at all points on the gaussian's surface at the radial distance

'r_B' from the centre of the shell. The vector $\Delta\vec{A}$ at every point is also radial, So we can write above equation as:

$$E_B \sum_{surface} \Delta A = 0$$

As $\sum_{surface} \Delta A \neq 0$ and equal to $4\pi r^2$, So

$$E_B(4\pi r^2) = 0$$

Hence,

$$E_B = 0$$

As 'B' was chosen as an arbitrary point, therefore, we conclude that the field inside a charged conducting spherical shell is zero.

Field at the Exterior: Let us how find the field at an arbitrary point P, which is outside the shell and at a ditance 'r_p' from the centre of the shell. Through P, construct a gaussian surface of radius 'r_p' concentric with the shell, as shown: Applying Gauss's law:

$$\sum_{surface} \vec{E_P}.\Delta\vec{A} = \frac{Q}{\epsilon_o}$$

Because of spherical symmetry, E_p must be radial. $\Delta\vec{A}$ is also radial. So, above equation implies:

$$E_P \sum_{surface} \Delta A = \frac{Q}{\epsilon_o}$$

As $\sum_{surface} \Delta A = 4\pi r^2$, So

$$E_P(4\pi r^2) = \frac{Q}{\epsilon_o}$$

$$E_P = \frac{Q}{4\pi\epsilon_o r^2} \tag{11.25}$$

Conclusion: The equation is identical with Coulomb's law for a point charge. This means that the field outside a charged sphere is the same as that of the field due to an equal-magnitude point charge placed at the centre of the sphere.

Distribution of electric Charge on a Hollow Conductor having a Charge in its Cavity

Here we discuss two cases:

Case 1: The Hollow Conductor is Uncharged: Consider an uncharged hollow conductor with '+q' charge placed inside it. We insulate it so that no charges can jump from one surface to another. We consider a gaussian's surface 'S' of same geometry as shown:

Now, using Gauss's law, we can conclude that the flux through the gaussian's surface is zero, which means charge enclosed must be zero. So, in order to maintain the neutral status,'-q' charge will appear on the surface of the conductor. As charge is always conserved, so '+q' must lie on the outside surface of the conductor (see figure **??**).

Case 2: When the Hollow Conductor is Already Charged If in the previous case, we take a charged conductor having a charge 'Q' on it. As charge resides on outer surface,so its outer surface will have 'Q' charge. Now,if 'q' is inserted inside it, then again by taking a gaussian's surface, the flux and hence the charge inside that must be zero. So, in order to maintain neutral status, a charge having opposite sign as that of charge inside the cavity will appear on the inside surface, so the net charge on the outer surface of the conductor will be algebraic sum of 'Q' and 'q'. Example is shown in figure **??**

Electric Field Intensity due to an Infinite Sheet of Charge

Definition of Infinite sheet: ***"The sheet of charge is said to be infinite with respect to a point 'P' if the dimensions of the sheet are very very greater than than the distance of 'P' from the sheet."***

Calculation of Electric Field: To calculate the electric field intensity due to an infinite sheet of charge, let us consider a sheet of positive charge having constant surface charge density 'σ' (charge per unit area). We have to calculate the field at point 'P' close to the sheet. Imagine a gaussian's surface in the form of cylinder passing through the sheet as shown:

The cylinder has cross sectional area 'A'. The surface charge density of the sheet (assumed constant) is given by:

$$\sigma = \frac{Q}{A}$$

Now, it is clear that electric field is parallel to the area vectors of the right and left faces. For curved surface, consider the surface to be composed of small area components. The electric field is always perpendicular to each '$\vec{\Delta A}$' vector on each point on the curved surface. Now let the charge enclosed by the cylindrical gaussian surface is 'q', then 'q' will be equal to:

$$q = \sigma A \tag{11.26}$$

Now,total flux will be equal to the flux through right and left end faces, because the curved surface contributes no flux (lines can not pass through the curved surface). For each face, $\vec{E}$ and $\vec{A}$ are in same direction, so flux due to each left and right face will be EA. Hence, total flux will be:

$$\Phi = EA + EA + 0$$

$$\Phi = 2EA \tag{11.27}$$

From Gauss's law:

$$\Phi = \frac{q}{\epsilon_o} \tag{11.28}$$

Comparing equations 11.27 & 11.28:

$$2EA = \frac{q}{\epsilon_o}$$

Putting equation 11.26 in above equation, we get:

$$E = \frac{\sigma}{2\epsilon_o} \tag{11.29}$$

If '$\hat{n}$' is the unit vector directed normally outwards from the sheet, then we can write:

$$\vec{E} = \frac{\sigma}{2\epsilon_o}\hat{n} \tag{11.30}$$

red Note that the 'E' is independent of 'r'. This result is correct approximately for real sheets (not infinite) if 'r' is very close to the sheet.

Electric Field Intensity between Two Oppositely Charged Parallel Metal Plates

To calculate the intensity of electric field between two oppositely charged metal plates, let us consider two oppositely charged parallel metal plates. The charge densities are '$+\sigma$' and '$-\sigma$' (assumed constant) with respect to the charge on the plate. The electric field is uniform in the region between the plates, and is normal to them. We have to calculate the field at an arbitrary point 'P'. For this purpose, we consider gaussian surface in the form of a box as shown:

Let 'A' be the cross sectioal area of the box. As the surface must enclose a net charge (let 'Q' be the charge enlosed), so the top face of the box is inside the upper plate. There are six faces of the box. The area vectors of the left, right, front and back faces are perpendicular to $\vec{E}$, hence contributes no electric flux. The top face is inside the plate so electric field is zero, hence no flux will be out of there. So the only contributing surface is bottom face, in which E and area vector are in same direction. So flux will be:

$$\Phi = EA \tag{11.31}$$

As,

$$q = \sigma A \tag{11.32}$$

And from Gauss's law:

$$\Phi = \frac{Q}{\epsilon_o} \tag{11.33}$$

So from equations 11.31, 11.32 & 11.33, we can write:

$$EA = \frac{\sigma A}{\epsilon_o}$$

which implies:

$$E = \frac{\sigma}{\epsilon_o} \tag{11.34}$$

This is the expression for electric field of two oppositely charged parallel metal plates. It is independent of position at which you are to find the field strength.

Note: As we know that electric field is bulging out at the ends of plates. So, to avoid this bulging (fringing field), the plates should be of infinite length. So, for practical purposes, the plates are said to be infinite if the distance between them is much smaller than their dimensions.

Alternate Method: Another interesting method for calculation of electric field between the oppositely charged metal plates, let us consider them to be infinite sheets of charge seperately. For positively charged sheet, $\vec{E}$ is directed upward and directed downward below it, having magnitude equal to $\frac{\sigma}{2\epsilon_o}$ below and above. Similarly, for negatively charged sheet, the field above it is directed downward, and below it, is directed upward having magnitude equal to $\frac{\sigma}{2\epsilon_o}$, below and above. We can understand it by figure below:

$$E = \frac{\sigma}{2\epsilon_o} + \frac{\sigma}{2\epsilon_o}$$
$$E = \frac{\sigma}{\epsilon_o}$$

This method is more efficient for such cases.We can also show that the electric field due to these plates will be zero above and below the plates. Similarly, we can also calculate for two same charge plates.

11.6 Electric Potential

"The amount of work done in moving a unit positive charge from infinity to a point inside the electric field against the electric field is called electric potential."

Explanation

Let us consider a positive charge '+q_o' placed in between oppositely charged plates.A charge experiences a force q_oE in an electric field. If the charge is allowed to move freely inside the electric field, it will move from 'A' to 'B' and gain kinetic energy. If we have to move the charge against the electric field, we have to apply external force. Now, in order to move the charge from 'B' to 'A' without giving acceleration, an external force must be applied which will be equal and opposite to q_oE as shown: Let W_{BA} be the amount of work done by external force in carrying 'q_o' from 'B' to 'A', without disturbing the equilibrium state of the charge. Change in potential energy of 'q_o' is defined to be the work done by the force applied in carrying it from one point to other against the electric field i.e.

$$\Delta U = W_{BA}$$

And,

$$U_A - U_B = W_{BA} \tag{11.35}$$

Where ΔU is the change in potential energy and U_A and U_B are the electric potential energies at ponts 'A' and 'B' respectively. Electric potential energy at a point is equal to the amount of work done in moving a charge from infinity to that point inside the electric field against the field, i.e. at point 'A' and 'B':

$$U_A = W_{\infty \to A} \, and \, U_B = W_{\infty \to B} \tag{11.36}$$

Now, dividing equation 11.35 by 'q_o':

$$\frac{U_A}{q_o} - \frac{U_B}{q_o} = \frac{W_{BA}}{q_o} \tag{11.37}$$

But the potential energy at a point per unit charge is potental, denoted by 'V'. Hence,

$$\frac{U_A}{q_o} = V_A \;\; and \;\; \frac{U_B}{q_o} = V_B \tag{11.38}$$

From equation 11.36, we can write:

$$V_A = \frac{W_{\infty \to A}}{q_o} \tag{11.39}$$

And,

$$V_B = \frac{W_{\infty \to B}}{q_o} \tag{11.40}$$

So, for any point, we drop subscripts and get:

$$V = \frac{W}{q_o} \tag{11.41}$$

which is the actual definition of electric potential at a given point.
Now, Putting values from equation 11.38 into equation 11.39, we can write:

$$V_A - V_B = \frac{W_{BA}}{q_o}$$

Writing $V_A - V_B = \Delta V$ (change in potential, called as potential difference), we get:

$$\Delta V = \frac{W_{BA}}{q_o} \tag{11.42}$$

So, potential difference between two points in electric field is defined as, ***"The work done in bringing a unit positive charge from one point to another inside the electric field keeping the charge in electrostatic equilibrium."***

Unit of Potential Difference

The S.I unit of potential difference is joule per coulomb known as volt (V), after great scientist Volta.

One Volt

"One volt is the amount of potential difference between two points in an electric field if one joule of work is done in moving one coulomb of charge from one point to the other."

Submultiples are:

$$1 \text{ millivolt} = 10^{-3} \text{ V}, 1 \text{ microvolt} = 10^{-6} \text{ V}$$
$$1 \text{ Gigavolt} = 10^{9} \text{ V}, 1 \text{ kilovolt} = 10^{3} \text{ V}$$

11.6.1 Electric Potential at a Point due to a Point charge

Definition

"The potential at a point at some distance 'r' from a point charge 'q' is the amount of work done per unit charge required to bring from infinity distance to that point."

Mathematical Derivation

Let us consider a charge '+Q' fixed in space. If a test charge 'q' is placed at infinity, the force on the test charge due to charge '+Q' will be zero. As test charge is chosen positive, so when it is moved from infinity towards '+Q', the force of repulsion acts on it. So, work is required to be done to bring it to point 'B'. Hence, when the charge is moved towards point 'B', an amount of electric potential energy will be stored in it.

As we know that work done on a body is given by $\vec{F}.\vec{d}$ i.e. work done between two points is given as:

$$\Delta W = \vec{F}.\vec{d} \tag{11.43}$$

The force is required to move charge against the field, so

$$\vec{F} = -q\vec{E} \tag{11.44}$$

Putting 11.44 into 11.43:

$$\Delta W = -q\,\vec{E}.\vec{d} \tag{11.45}$$

Since E and d are oppositely directed, so equation 11.45 reduces to:

$$\Delta W = qEd \tag{11.46}$$

Now, electric field due to a point charge at a distance 'r' is given by:

$$E = \frac{Q}{4\pi\epsilon_o r^2} \tag{11.47}$$

It varies inversely as square of the distance. So, it does not remain constant over the distance. In order to use above equation, E must be constant, so we divide the distance over which the test charge is moved into infinitesimally small displacements 'Δr'. We consider two points 'A' and 'B' for convenience and charge is moved between these points over small displacements 'Δr' as shown:

So equation 11.46 gives:

$$\Delta W = qE\Delta r \tag{11.48}$$

Putting value of E from equation 11.47 into equation 11.48, we get:

$$\Delta W = \frac{Qq}{4\pi\epsilon_o r^2}\Delta r \tag{11.49}$$

Now, let the test charge is moved through small intervals from r_A to r_1, $(\Delta r = r_A - r_1)$. We assume that ponts are very closer, still we have to take the average of E. As E at a distance 'r_A' is:

$$E_A = \frac{Q}{4\pi r_A^2}$$

And 'E' at a distance 'r_1' will be:

$$E_1 = \frac{Q}{4\pi r_1^2}$$

Hence at the beginning of the interval, E varies as $\frac{1}{/r_A^2}$ and at the end of the interval, it is varying with $\frac{1}{r_1^2}$. In order to find the average value of E, we have to find an average value of r in equation 11.49. i.e.

$$r = \frac{r_A + r_1}{2}$$

Squaring on both sides, we get:

$$r^2 = (\frac{r_A + r_1}{2})^2 \tag{11.50}$$

As $\Delta r = r_A - r_1$, which implies

$$r_1 = r_A + \Delta r$$

Putting this value of r_1 in equation 11.50, we get:

$$r^2 = (\frac{r_A + (r_A + \Delta r)}{2})^2$$

$$r^2 = \frac{4r_A^2 + \Delta r^2 - 4r_A\Delta r}{4}$$

For smaller Δr, we have $\Delta r^2 = 0$ so the above equation reduces to:

$$r^2 = \frac{4r_A^2 - 4r_A\Delta r}{4}$$

$$\Longrightarrow r^2 = r_A^2 - r_A\Delta r \tag{11.51}$$

Put back value of Δr in 11.51:

$$\Longrightarrow r^2 = r_A^2 - r_A^2 + r_A r_1$$

$$\Longrightarrow r^2 = r_A r_1$$

Now putting value of Δr and r^2 in 11.49, we get:

$$\Delta W = \frac{Qq}{4\pi\epsilon_o}\frac{r_A - r_1}{r_A r_1}$$

which implies:

$$\Delta W_{r_A \to r_1} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_1} - \frac{1}{r_A}) \tag{11.52}$$

Now continuing in this manner, from r_1 to r_2, equation 11.52 implies:

$$\Delta W_{r_1 \to r_2} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_2} - \frac{1}{r_1}) \tag{11.53}$$

Similarly from r_{n-1} to r_n,

$$\Delta W_{r_{n-1} \to r_n} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_n} - \frac{1}{r_{n-1}}) \tag{11.54}$$

And finally from r_n to r_B,

$$\Delta W_{r_n \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{r_n}) \tag{11.55}$$

Now, total work will fron point ‘A’ to ‘B’ will be:

$$W = \Delta W_{r_A \to r_1} + \Delta W_{r_1 \to r_2} + ... + \Delta W_{r_{n-1} \to r_n} + \Delta W_{r_n \to r_B}$$

Putting respective values:

$$W = \frac{Qq}{4\pi\epsilon_o}(-\frac{1}{r_A} + \frac{1}{r_1} - \frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_2} + ... - \frac{1}{r_n} + \frac{1}{r_n} + \frac{1}{r_B})$$

which will result:

$$W_{r_A \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{r_A}) \tag{11.56}$$

This gives the on a test charge q when it is brought from one point to another point inside the field against its direction. Now if point ‘A’ is chosen at infinity, then equation 11.56 implies:

$$W_{\infty \to r_B} = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_B} - \frac{1}{\infty})$$

So,

$$W_{\infty \to r_B} = \frac{Qq}{4\pi\epsilon_o}\frac{1}{r_B}$$

Since ‘B’ is an arbitrary point, so we drop subscripts and above equation will be written as:

$$W = \frac{Qq}{4\pi\epsilon_o}\frac{1}{r} \tag{11.57}$$

This work is stored as ‘Electric Potential Energy’. Since, electric potential is given by:

$$V = \frac{W}{q}$$

So, putting equation 11.57 in the potential equation:

$$V = \frac{Qq}{4\pi\epsilon_o q}\frac{1}{r}$$

which implies:

$$V = \frac{Qq}{4\pi\epsilon_o r} \tag{11.58}$$

This the mathematical form of potential at a distance 'r' from a point charge. It varies as 1/r and is inependent of test charge. This is relative to infinity (which is a point of zero electric potential). Moreover, electric potential is a scalar quantity. It can be positive, negative depending upon the sign of charge. Potential of a point charge '+Q' at distance 'r' is considered positive and considered negative for a negative charge.

As potential is a scalar quantity, therefore electric potential due to several point charges q_1, q_2, q_3,..., q_n would be the algreabraic sum of the individual potentials due to each charge i.e.

$$V = V_1 + V_2 + V_3 + ... + V_n \tag{11.59}$$

Note:

(i) The potential very near the positive charge is large and decreases towards zero the further away from the charge. If the charge producing the field is negative, the potential is also negative and increases towards zero at larger distances.

(ii) Variation of potential is an inverse propotionality with distance and variation of electric field is inverse square propotionality with distance.

(iii) In case of uniform field ,e.g field between two parallel plates, the potential decreases uniformly as we move from positive hand plate to negative hand plate.

(iv) Electric field is a conservative field i.e. work done is independent of the path followed, it is not important that which way you move the charge rather it depends on the initial and final positions of the charge. Also the work done on the closed path will be zero.

(v) We have an idea of positive work done that when the energy of the system increases work done is said to be positive and if the energy decreases the work done will be negative. So work done against the field is said to be positive as it increases the potential energy of the system and vice versa.

11.7 Potential Gradient and Electric Field Intensity

"The maximum rate of change of electric potential with respect to position is called potential gradient."

The purpose of this topic is to establish a relationship between electric field intensity and the electric potential at a certain point.

Mathematical Derivation

Let us consider oppositely charged parallel plates. The electric field between the plates is uniform, hence field intensity 'E' is constant. Let a positive charge 'q_o' is moved from point 'A' to 'B' as shown: The potential difference between points 'A' and 'B' according to the definition will be:

$$\Delta V = \frac{W_{AB}}{q_o} \tag{11.60}$$

And,

$$W_{AB} = \vec{F}.\vec{d} \tag{11.61}$$

In this case,

$$\vec{F} = q_o\vec{E} \tag{11.62}$$

Putting in equation 11.62:

$$W_{AB} = q_o\vec{E}.\vec{d} \tag{11.63}$$

Here $\vec{E}$ and $\vec{d}$ are acting in opposite direction, hence equation 11.63 implies:

$$W_{AB} = -q_oEd \tag{11.64}$$

Putting in equation 11.60, we get:

$$\Delta V = \frac{-q_oEd}{q_o} \tag{11.65}$$

which gives:

$$E = -\frac{\Delta V}{d} \tag{11.66}$$

For a small separation of two points denoted by 'Δr', the equation 11.66 results:

$$E = -\frac{\Delta V}{\Delta r} \tag{11.67}$$

As Δr is the perpendicular distance between the plates, hence the shortest distance. So 11.67 will give maximum rate of change of potential. This maximum rate of change of potential with respect to the distance Δr is called potential gradient. Hence, we can define electric field as, ***"The negative rate of change of potential with respect to distance,"*** or, ***"Electric field is the negative gradient of potential."***

Another unit of Electric Field

From equation 11.67, the unit of electric field intensity is volt per meter (V/m) which is equal to N/C.

Significance of Negative Sign

The negative sign indicates that the direction of field is opposite to the direction in which the potential is increasing. To understand this, we can think of a positive charge as a source charge. When we bring a positive test charge towards it, the potential will increase i.e. towards the charge and the field is radially outward away from the charge. Similarly, potential increases as we move the test charge away from a negative charge and the electric field is radially inward towards the negative charge.

Note

It is important to note that gradient of a quantity is the change in that quantity with distance. Potential is a scalar quantity but gradient of potential is a vector.

Checkpoint 11.4

Show that $\frac{V}{m}$ is equal to $\frac{N}{C}$.

11.8 Equipotential Surface/Equipotential Line

"The surface/line passing through such points which have the same value of electric potential is called equipotential surface/line."

Electric field is very difficult to represent in diagram. Both strength and direction of field must be properly indicated at every point in the field. As an alternative to field line diagrams, 'contour maps' of the electric field can be drawn using equipotential lines. These lines connect all the points in space where the potential of an electric field is the same as shown:

Show that Two Points on an Equipotential Line have Same Potential

Consider two points 'A' and 'B' on an equipotential line at a distance 'r' from the point charge 'q' as shown:

The electric potential due to a point charge at a distance 'r' is given by:

Now, work done on test charge 'q_o' between points 'A' and 'B' on the equipotential surface will be: It means that if a charge is moved over an equipotential line, work done on th charge will be zero. Since, potential difference is the amount of work done on a unit positive charge between two points. So, no work means no potential difference and hence the points are at the same potential.

11.8.1 Characteristics of Equipotential Surface/Line

Some properties related with equipotential line/surface are discussed as:

(i) Two equipotential lines can not intersect. Since over the whole equipotential line, a charge if mooved, bears the same potential.So at the intersection,the charge will have two values of potential which is not possible.

(ii) No work is done in moving a charge from one point to another on an equipotential surface.

(iii) Work is done in moving a charge from one surface to other equipotential surface.

(iv) If there are two or more charges, then electric potential at any point is the sum of the potential due to the individual charges. e.g, for a positive and negative charge placed a distance apart, equipotential lines are as shown:

(v) Electric field lines and equipotential lines are always perpendicular to each other e.g. field pattern for an isolated charge and between oppositely charges plates along with equipotential lines is as shown:

(vi) Closer the equipotential lines, the stronger will be the field at a given point. So the potential energy must be changing by large amounts in small distances and there must be a large force acting.

Example

The surface of a conductor is an example of an equipotential surface in electrostatics.

11.9 The Electron-Volt

"It is the amount of energy gained or lost by an electron when it is displaced across two points between which the potential difference is one volt."

Explanation

We know that when a charge 'q' is moved up across a potential difference 'ΔV', then the electric potential energy of the charge increases by an amount given as:

$$\Delta U = q\Delta V \tag{11.68}$$

Also, when the same charge 'q' is allowed to move inside the electric field through the same potential difference ΔV, it loses its potential energy and gains the kinetic energy i.e.

$$\Delta K.E = q\Delta V \tag{11.69}$$

As by definition, when $q = 1e$ and $\Delta V = 1V$, then:

$$\Delta K.E = 1\,eV \tag{11.70}$$

Relationship with Joule

$$1\,eV = 1.602 \times 10^{-19} C \times 1V \tag{11.71}$$

which gives:

$$1\,eV = 1.602 \times 10^{-19}\,J \tag{11.72}$$

From above relation, it is clear that the electron volt (eV) is another unit of energy like joule. Its samll size is enough to be convenient for the energies of elementary particles. Its submultiples are:

$$1 \text{ MeV} = 10^6 \text{ eV (million electron volts)}$$
$$1 \text{ GeV} = 10^9 \text{ eV (billion electron volts)}$$

Chapter 12

Electrodynamics/Current Electricity

12.1 Electric Current

"The flow of charge per unit time is called electric current".

OR

"Whenever electric charge flows, current is said to exist."

Symbol

It is denoted by 'I'.

Mathematical Form

If 'Q' is the amount of charge flowing through a wire of cross section 'A' in time 't', then current 'I' will be given by:

$$I = \frac{Q}{t} \tag{12.1}$$

Explanation

We know that substances which conduct electricity are known as conductors and which do not, called as insulators. Before the discovery of electron, proton etc., scientist supposed that the current is due to the flow of positive charges. The concept of flow of positive charges was developed because in physics, the positive terminal is at high potential with respect to a negative terminal. With the discovery of electron and nucleus, it is clear that:

(i) In metal conductors, the current is due to the flow of electrons only.

(ii) In liquids, current is due to flow of negative and positive charges, e.g in case of electrolytes.

(iii) In discharge through gaseous, the current is due to the flow of both charges.

When we talk about electric current, we often study the behaviour of metallic conductors. In metallic conductors, actually the electrons flow from negative terminal to positive terminal. However,prior to electron theory, it was assumed that positive charge flows only. But it was observed that a negative charge flowing in one direction has an equivalent effects if the same positive charge flows in opposite direction. So, the concept of flow of positive charge was retained as a convention and so called conventional current. The current due to electrons was called electronic current. The direction of electronic current is from negative terminal of the battery while that of conventional current is taken from positive to negative terminal of the battery. From now, we will take the direction of current as conventional direction i.e. from positive end to negative end.

12.2 Drift Velocity of Electrons in a Metallic Conductor

Every metal has a huge number of free electrons which wander randomly within the body of the conductor. The average speed of free electrons is sufficiently high, approximately of the order of 10^5 m/s. During random motion, the free electrons collide with the atoms of conductor again and again and after each collision, their direction of motion changes. As many electrons move in one direction, as much electrons move in opposite direction. So due to random motion, there is no net flow of charges in a particular direction, so no current exists in the case when no power supply is connected.

Motion of Charges When the Potential Difference is Applied

When a potential difference is applied across the ends of a conductor, it sets up an electric field at every point along the wire. As inside the conductor, there are free charges (free electrons in case of metallic conductors), so electrons experience a force in a direction opposite to the electric field given by:

$$\vec{F} = -e\vec{E} \tag{12.2}$$

Due to this force, electrons gain velocity and hence accelerate, then Newton's second law gives:

$$\vec{F} = m\vec{a} \tag{12.3}$$

This net force is actually the force exerted on electrons by the electric field, hence comparing equations 12.2 & 12.3, we get:

$$m\vec{a} = -e\vec{E}$$

$$\vec{a} = -\frac{e\vec{E}}{m} \tag{12.4}$$

So,direction of acceleration is in a direction opposite to electric field.As electrons move along the wire, they continuously collide with atoms, and due to the force by electric field, they acquire a net velocity along a direction, this velocity is called drift velocity and is defined as, ***"the average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called drift velocity."*** Symbolically, the drift velocity is denoted by $\vec{V_d}$.

Order of Drift Velocity

The drift velocity of electrons in a metallic conductor is of the order of 10^{-5} m/s.

Effect Transfer

As drift velocity of electrons is of the order of 10^{-5} m/s but when we switch on the bulb,it immediately glows. Although the drift velocity is negligible but the effect of electrons movement travel around the circuit with a speed approximately equal to the speed of light that is of the order of 10^8 m/s. That's why, when we switch on the bulb, the bulb glows in fractions of a second.

12.3 Sources of Current

"A device that supply a constant current by maintaining a constant potential difference between its two terminals is called a source of current."

Explanation

It is a firmly established convention that a positively charged body is at higher potential than a negatively charged body. When a body at a higher potential is connected to a body at a lower potential through a metallic wire, electric current will flow from higher potential to lower potential as shown in the figure below:

The current will stop when both the bodies come at the same potential. To maintain a steady current through the wire, the ends of the wire must be maintained at a constant potential difference. So, a device or agent must be present that will maintain the required potential difference. This device will convert some other forms of energy into electrical energy. Such a device is called a 'power supply' (or source of emf, we will talk about emf in this chapter). Examples are:

(i) In electrical cells,chemical energy is converted into electrical energy. Similar is the principle of battery.

(ii) Electric generator convert mechanical energy into electrical energy (we will talk about it in chapter 14).

(iii) Thermocouple convert heat energy into light energy (we will discuss thermocouple in this chapter).

(iv) Solar cells convert light energy int electrical energy (we will encounter solar cell in chapter 17).

12.4 Elecroencephalogram(EEG)

12.5 Ohm's Law

Background

In previous section, we studied that when a potential difference is maintained across the ends of a wire, current flows. The relationship between potential difference (V) and electric current (I) in a D.C circuit was first discovered by German scientist ***George Simon Ohm*** in 1826 in the form of a law known as 'Ohm's law'. This law is known to be the fundamental law of electricity.

Statement

"The magnitude of electric current 'I' in a metallic wire is proportional to the applied voltage 'V' provided the physical state of conductor is constant."

Mathematical Form

Consider an electric circuit as shown:

When potential difference 'V' is maintained in the circuit, current 'I' flows and by Ohm's law:

$$I \propto V$$

$$I = (constant).V$$

$$I = \frac{1}{R}V$$

which implies:

$$\boxed{V = IR} \tag{12.5}$$

where, 'R' is a constant and called as resistance of wire and it depends upon:

(i) Nature of material of the wire

(ii) Physical state of the material

(iii) Dimensions of the wire

The above equation is the mathematical form of Ohm's law.

Validity

Ohm's law is valid only for metallic conductors. It does not hold for electron tubes, discharge through gaseous, filament of bulb etc. Those materials which obey Ohm's law are called ohmic materials while those which do not are termed as non-ohmic.

12.6 I-V characteristics for Ohmic and Non-ohmic materials

Chapter 13

Electromagnetism

13.1 History

As early as 600 B.C. the Greeks knew that a certain form of iron ore, now known as magnetite or lodestone, had the property of attracting small pieces of iron. Later, during the Middle Ages, crude navigational compasses were made by attaching pieces of lodestone to wooden splints. These splints always come to rest pointing in a N—S direction, and were the forerunners of the modern aircraft and ship compasses.

The word 'lodestone' is derived from an old English word meaning way, and refers to the directional property of the stone mentioned above.Chemically,it consists of iron oxide having the formula Fe_3O_4. The word magnetism is derived from Magnesia,the place where magnetic iron ore was first discovered.

In 1820, a Danish Physicist Hans Christian Oersted made one of the most important discovery of all times. He determined that when a current carrying wire is held near a compass needle, the needle is deflected. This discovery leads to the entire field of electromagnetism.

13.2 Definition of Electromagnetism

"The branch of Physics which deals with the study of magnetic effects of electric current is called electromagnetism."

Explanation

The electric and magnetic fields are different aspects of electromagnetism but intrinsically related. Thus, a changing electric field generates a magnetic field and conversely a changing magnetic field generates an electric field. The latter effect is called electromagnetic induction and is the basic operation for electric generators, induction motors and transformers and is studied in electromagnetism (we will cover it in next chapter).

13.3 Magnetic Field

"The space around a magnet or current carrying conductor,where a test magnet can feel a force of attraction or repulsion is called magnetic field."

13.3.1 Forces of Magnets

Magnets exert forces on each other.These forces are either attraction or repulsion.The effects may be summarised in the law of magnets:

"Like poles repel and unlike poles attract."

13.3.2 Magnetic Field Lines

"Magnetic field lines are the curves drawn so that the tangent to a given curve at a point gives the direction of magnetic field at that point."

Properties of Field Lines

Magnetic field are not visible but they can be represented by lines of magnetic force extending in three dimensions.The properties of magnetic lines of force are given as:

(i) The magnetic field lines start at a north pole and end at a south pole.

(ii) These lines are smooth curves,they never cross or touch.(Can you state why?)

(iii) The strength of the field is indicated by the distance between the lines_closer lines mean a stronger field and vice versa.

(iv) Magnetic field lines always form closed curves.

13.3.3 Magnetic Field of an Electric current

As it is known that all electric currents produce magnetic fields.The size and shape of magnetic field depends on the size of the current and the shape(configuration) of the conductor through which the current is travelling.

Magnetic field of a Straight Current Carrying Wire

The magnetic field due to a straight wire may be plotted using the apparatus shown in the figure below.Iron fillings are sprinkled on a horizontal board and current is passed through the wire as a result of which a magnetic field will be produced.Iron fillings will be in the indicated pattern showing the magnetic field around a straight current carrying wire will be in the form of concentric circles. The separation of lines increases with the distance from the wire,indicating the field is decreasing in strength as we move away from the wire.The field also increases as the current is increased in the wire.The direction of field can be found by placing magnetic compasses or using right hand rule which states that:

"Imagine hold the conductor in the right hand with the thumb pointing in the direction of the current,the curled fingers will point in the direction of field."

13.4 Force on a current carrying conductor

The interaction of magnetic fields produced by two magnets causes force of attraction or repulsion between the two.If a conductor is placed between the poles and a current is passed through the conductor,the magnetic fields of the current-carrying conductor and the magnet may interact,causing forces between them.In order to explain,we will demonstrate it by the following experiment:

Place a straight wire between the poles of a magnet. When a current flows in the wire,a force is exerted on the wire. In first demonstration,the current flows inward direction (into the page),the wire experiences a downward push.This force is neither parallel to magnetic field nor parallel to the wire.Instead this force is directed at right angle to the magnetic field and wire.Now,if the current is reversed(out of the page),the direction of push will also be reversed i.e. upward.It is found that the direction of force is always perpendicular to the wire and also perpendicular to the direction of field.

These demonstrations lead us to define a rule for the direction of force,i.e.

"Outstrech the fingers of your right hand in the direction of current,then bend the fingers in the direction of magnetic field,the extend thumb will indicate the direction of the force in the current carrying wire"

13.4.1 Magnitude of Force

It is found experimentally that the magnitude of the force is directly proportional to :

(i) Current in the wire

(ii) Length of the wire inside the magnetic field

(iii) Strength of the field

i.e.

$$F \propto ILB$$

Secondly,it was also found that :

- When the wire was perpendicular to the field,the force was maximum
- When the wire as parallel to to the field,there was no force at all
- At any angle,it varies with the sine of the angle between $\vec{L}$ and $\vec{B}$.

So,

$$F \propto sin\theta$$

$$F \propto ILBsin\theta$$

Here constant of proportionality is 1,so

$$F = ILBsin\theta$$

Generalized Form

The magnitude as well as the direction of the magnetic force on a current carrying wire can be described in vector notations by the following cross product:

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\vec{F} = ILBsin\theta\hat{n}$$

Where, $\vec{L}$ is a vector whose magnitude is the length of the wire and whose direction is along the wire(assumed straight) in the direction of current.The unit vector $\hat{n}$ is along the direction of $\vec{F}$ and is perpendicular to $\vec{L}$, $\vec{B}$ and plane determined by $\vec{L}$ and $\vec{B}$.

Fleming's Left-Hand Rule

The rule we discussed earlier is known to us already if we have a knowledge of cross product (extending fingers in the direction of first vector in the cross product and curl them towards other vector, thumb will indicate the direction of the resultant of this cross product i.e. the direction of $\hat{n}$. An alternate rule for the direction of force is the Fleming's left hand rule which states that :

"If the forefinger,central finger and thumb of left hand are held mutually perpendicular with the Forefinger pointing in the direction of Field,Central finger in the direction of Current,the thumb would indicate the Motion of conductor(the direction of force on conductor)."

Definition of $\vec{B}$

$\vec{B}$ being a vector has magnitude as well as direction.

Direction of $\vec{B}$

The direction of $\vec{B}$ at any point of the magnetic field is the direction in which the force acting on a straight current carrying wire,placed at that point,is zero, i.e. $\vec{L}$ —— $\vec{B}$.

As we know that when the wire is in the direction of field,it experiences no force,so we move wire so that the point comes when it experiences no force,we say that the direction of the wire at that point will be the direction of field.

Magnitude of $\vec{B}$

The magnitude of $\vec{B}$ is defined when angle between $\vec{B}$ and $\vec{L}$ is 90° and force is maximum,so,

$$B = F_{max}/IL$$

So, ***"It is the maximum force acting on a conductor of unit length when one ampere current passes through it"***

13.4.2 Unit of Magnetic Field

The SI unit of magnetic field is tesla (T) and :

$$1T = INA^{-1}m^{-1}$$

One tesla

"Magnetic field at any point is said to be one tesla if it exerts a force of 1 N on one metre length of the conductor placed at right angles to the field when a current of 1 A passes through it."

Other units

An older name of tesla is weber per metre squared i.e. Wb/m^2. Another commonly used unit is gauss (G). And,

13.5 Magnetic Flux

"The number of magnetic field lines passing through a surface is known as magnetic flux"

OR

"The dot product of magnetic induction $\vec{B}$ and vector area element $\vec{A}$ is known as magnetic flux."

Symbol: Its symbol is ϕ_B.

13.5.1 Mathematical Form

If $\vec{B}$ is the magnetic induction and $\vec{A}$ is the area vector (a vector having magnitude equal to the area of surface and direction normal to the area element), then the flux would be:

$$\phi_B = BAcos\theta$$

where θ is the smaller angle between $\vec{B}$ and $\vec{A}$.

13.5.2 Explanation

As magnetic flux for magnetic induction $\vec{B}$ and element of area $\Delta\vec{A}$ is given by:

$$\phi_B = B\Delta Acos\theta$$

We know that area vector is normal to the plane of area.If the area is not a flat surface i.e. the angle between area vector and magnetic induction is different at different points, thus we divide area into smaller 'n' elements. So, the total magnetic flux through the whole area placed in a field of magnetic induction $\vec{B}$ is the sum of the contributions from the individual area elements is given by:

$$\phi_T = \sum_{i=1}^{n} \Delta\phi_i$$

$$\phi_T = \sum_{i=1}^{n} B_i \Delta A_i cos\theta_i$$

In a uniform field,

$$\phi_T = B \sum_{i=1}^{n} \Delta A_i cos\theta_i$$

13.5.3 Maximum Flux

If the surface area is held normal to the field lines such that area vector $\vec{A}$ is parallel to the field,then the maximum lines of force will pass and flux will be maximum i.e.

$$\phi_B = BAcos0^\circ$$

$$\phi_B = BA$$

13.5.4 Minimum Flux

If the surface area is placed such that it is parallel to the lines of force, so that area vector $\vec{A}$ is normal to the field, no lines will pass,ϑ will be 90 degrees,flux will be zero.

$$\phi_B = BAcos90^\circ$$

$$\phi_B = 0$$

13.5.5 Unit

Unit of magnetic flux is Tm^2 called as weber(Wb).

13.5.6 Magnetic Flux Density

Using equation

$$B = \frac{\phi}{A}$$

So magnetic induction B can also be defined as:

"Magnetic flux per unit area.". Hence it is also called magnetic flux density. It has unit Wb/m^2 (T).

13.6 Ampere's Circuital Law

13.6.1 Background

We know that a current carrying wire has a magnetic field around it. The direction of the field can be determined by right hand rule. The magnitude of the field can be determined by a relation called Ampere's circuital law.

13.6.2 Statement

"The sum of the dot products 'B' and 'L' around a closed path in the magnetic field of a current is equal to μo times the current enclosed by the path."

13.6.3 Mathematically

$$\sum \vec{B} \cdot \Delta\vec{L} = \mu_0 I$$

13.6.4 Explanation

Let us first consider a special case of the magnetic field of a long straight-current carrying wire as shown: From experiments as well as from the cylindrical symmetry of the wire, it is obvious that the magnitude of magnetic induction is constant on a circle of radius 'r' centred on wire.It is further observed that 'B' around a long straight current-carrying wire is directly proportional to the current 'I' and inversely proportional to the distance 'r' from the wire i.e.

$$B \propto I$$

$$B \propto 1/r$$

$$B \propto I/r$$

$$B \propto \mu_0 I/2\pi r$$