The quantum Heisenberg-Ising spin-1/2 chain on a 1-dimensional ring

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Abstract

We numerically solve the quantum Heisenberg-Ising spin-1/2 chain, aka. the XXZ model, on a 1-dimensional Ring. The numerical solution is powered by the Bethe ansatz. In particular, via the Bethe ansatz, we find a numerical solution by (numerically) solving a system of algebraic equation and checking an initial condition identity. We developed an open-access Python program to implement the numerical solution and numerically verify the necessary initial condition. Additionally, we used the numerical solution to compute some statistics of the XXZ model on a ring, such as the 1-point function and the gap probability. This project is a stepping stone in developing numerical evidence to determine if the XXZ model in the Kardar-Parisi-Zhang (KPZ) universality class, a conjecture put forward in [STW22] ?.

1 Introduction

1.1 The Model

The quantum Heisenberg-Ising spin-1/2 chain is a finite dimensional closed quantum system. The evolution of the system is determined by the Schrodinger equation

$$i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle \tag{1.1}$$

where $|\Psi(t)\rangle$ is the wave function and H is Hermitean matrix.

1.2 Main result

Theorem 1.1.

$$|\Psi(t)\rangle = \sum_{x}\left(\sum_{z}\frac{1}{\det(I+M)}\sum_{\sigma\epsilon S_N}A_{\sigma(z)}\prod_{i=1}^{N}z_{\sigma(i)}^{x_i-y_{\sigma(i)-1}}e^{-it\epsilon(z_i)}\right)|x\rangle \tag{1.2}$$