A list of (possibly) open problems for the discussion section

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1. **Reaction-diffusion systems on a ring.** The single time marginal for either coalescing or annihilating random walks on $\mathbb{Z}$ is a Pfaffian point process. On a ring, the Pfaffian structure survives in the annihilating case, but disappears in the coalescing case. There is still a complete set of duality functions as well as closed equations for their expectations. What is the structure of the solution? Annihilating systems are equivalent to free fermions. In the context of coalescence, are we dealing with an example of a non-free fermion system, which is dual to free fermions in some special cases?

2. **Analyse the ternary reaction** $3A \to A(\emptyset)$ **on** $\mathbb{Z}$**.** It can be modelled by a continuous time Markov chain on $\{0,1,2\}^{\mathbb{Z}}$ with the generator

$$\begin{aligned} Lf(\eta) &= \sum_{x\in\mathbb{Z}} \frac{\eta_x}{2}\Big[f(\ldots,\eta_{x-1},\eta_x-1,\eta_{x+1}\boxplus 1,\eta_{x+2},\ldots) \\ &+ f(\ldots,\eta_{x-2},\eta_{x-1}\boxplus 1,\eta_x-1,\eta_{x+1},\ldots) - 2f(\eta)\Big], \end{aligned}$$

where $\boxplus$ denotes the addition modulo 3 (the annihilating case). In the limit $t\to\infty$ the system approaches the trivial state of zero particle density, but the approach itself is interesting. For example, for the translation-invariant initial condition, and under the mean field assumption the mean density of particles satisfies $\partial_t\rho = -g\rho^3$, which gives

$$\rho_t \sim (2gt)^{-\frac{1}{2}},\ t\to\infty,$$

but the conjectured correct answer is $C\left(\frac{\log t}{t}\right)^{1/2}$ for some constant $C > 0$. Moreover, the conjectured time dependence of the n-point correlation function is

$$\left(\frac{\log t}{t}\right)^{n/2} (\log t)^{n(n-1)(n-2)/6}$$

Can any of these conjectures be proved rigorously? Here is some background: ternary annihilation is an example of a model with critical dimension equal to 1. It is therefore expected that mean field answers for correlation functions are correct up to factors of $\log t$. It is also expected that the logarithmic corrections can be computed exactly by estimating the probabilities of triple collisions. The main question is to characterise the large scale-time universal stochastic process describing the evolution of our particle system or even prove that it exists. The answer will contribute to understanding of the theory of regularity structures at criticality. The fact that the model is one-dimensional is likely to make analysis easier.

3. **Inhomogeneous SEP**. What can be said about the inhomogeneous SEP (particles attempt to hop from even sites with rate $1/2$ and odd sites with rate $\alpha \neq 1/2$? This model is equivalent to the hardest model on the classification list of idempotent stochastic matrices corresponding to IPS's of the type considered in the lectures.