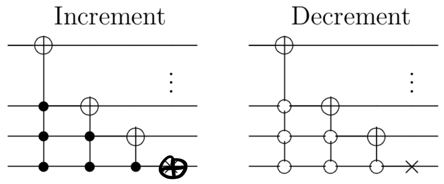
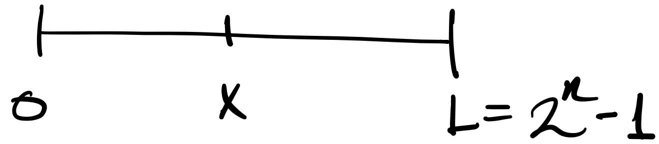
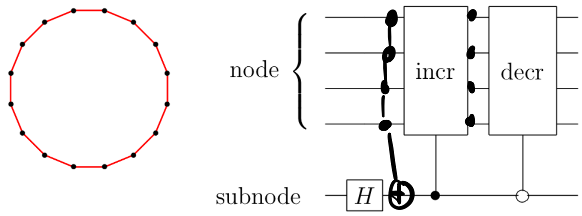


FIG. 1: Increment and decrement gates on n qubits, producing cyclic permutations in the 2^n bit-string states.



$$x = q_0 2^0 + q_1 2^1 + \cdots + q_{n-1} 2^{n-1}$$

$$|x\rangle \longleftrightarrow |q_0 q_1 \cdots q_{n-1}\rangle$$



$$(0,1,0) = 2 \to 3 = (1,1,0)$$

$$(1,0,1) = 5 \to 4 = (0,1,0)$$

$$x \longmapsto x \pm 1$$

$$q \longmapsto q^{\pm} = (q_0^{\pm}, \ldots, q_{n-1}^{\pm})$$

w/ $$q_i^{+} = \begin{cases} q_i \oplus 1 & \text{if } q_0 = \cdots = q_{i-1} = 1 \\ q_i & \text{otherwise} \end{cases}$$

Notation

$\oplus$ is mod 2 addition

$$q_i^{-} = \begin{cases} q_i \oplus 1 & \text{if } q_0 = \cdots = q_{i-1} = 0 \\ q_i & \text{otherwise} \end{cases}$$

Rem

addition by 1 and subtraction by 1 is the same mod 2.

Eg ($n=4$)

$$x = 11 = 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3$$

$$0 \cdot q_0 + \cdots + 0 \cdot q_{n-2} + 0 q_{n-1}$$

$$0$$

$$\to q = (1,1,0,1)$$

$$\to q^{+} = (0,0,1,1) \quad \& \quad q^{-} = (0,1,0,1)$$

Eg: $(n=4)$

$$X = 8 = 0\cdot 2^0 + 0\cdot 2^1 + 0\cdot 2^2 + 1\cdot 2^3$$

$\to q = (0,0,0,1)$

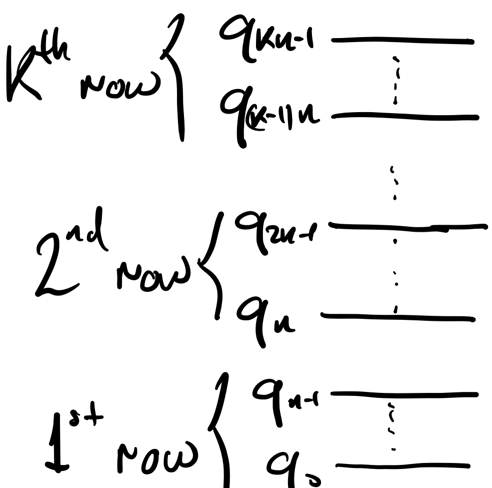
$\to q^+ = (1,0,0,1)$ & $q^- = (1,1,1,0)$

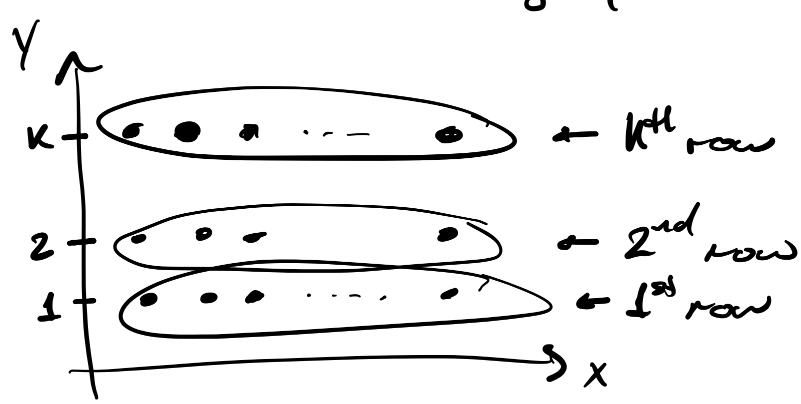
Q: How do we map $(X,Y) \mapsto q = (q_0, \dots, q_{n+m-1})$?

- $0 \le X \le 2^n - 1$ & $0 \le Y \le 2^m - 1$
- $N := 2^n$ & $M = 2^m$
- $(X,Y) \mapsto Y\cdot 2^n + X = q_0 \cdot 2^0 + q_1 \cdot 2^1 + \dots + q_{n+m-1}\cdot 2^{n+m-1}$

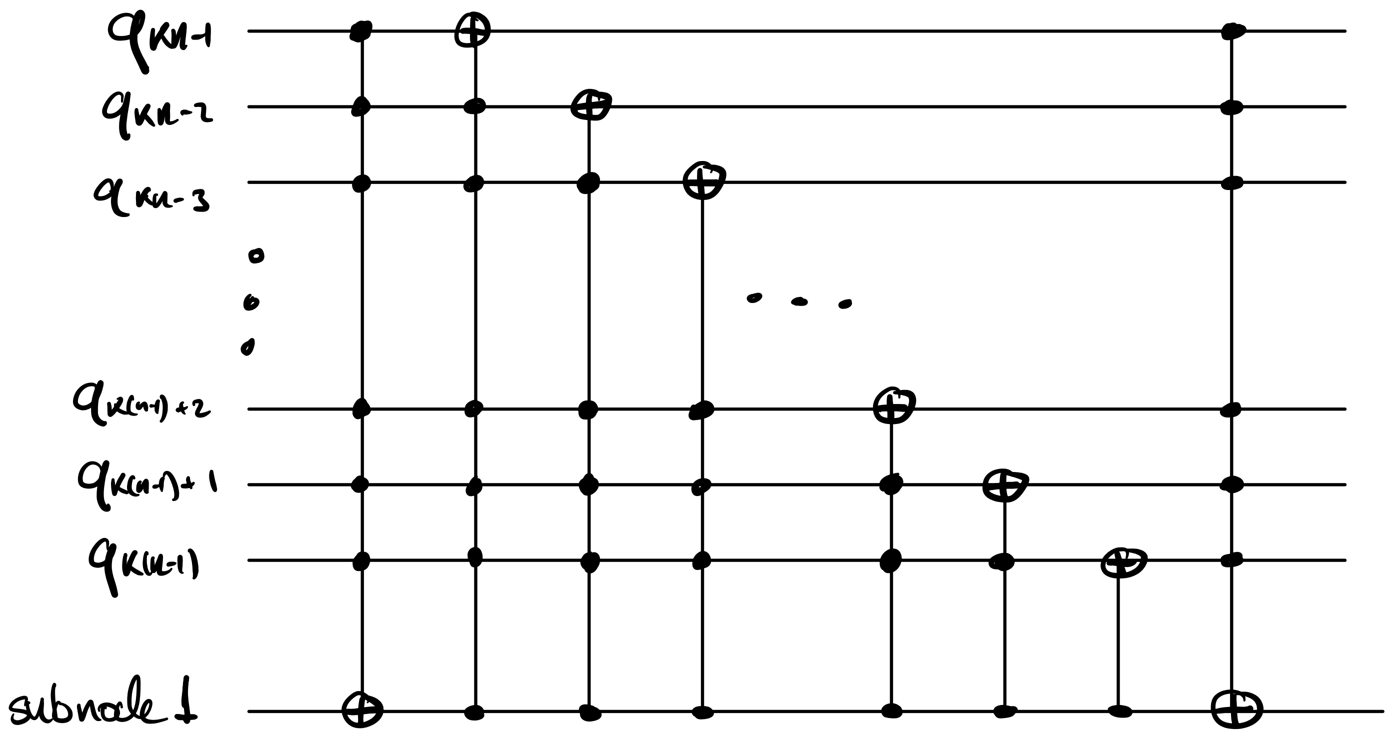
$$\mapsto (q_0, q_1, \dots, q_{n+m-1})$$

- The K^{th} row corresponds to the K^{th} n-block of of q bits.





• Increment in the x-direction along the K^{th} row



Rem: Note that, compared to Fig 1, we've added two additional controlled gates, at the beginning and at the end. These gates block the move

$$L-1 \rightarrow 0$$

• Decrement in the x-direction along the K^{th} row