

Analysis of Mile per Gallon vs. Transmission via Regression Models

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1 Overview

In this report, we explore the relationship between a set of variables and *miles per gallon* (MPG). The dataset of interest in this report is **mtcars** from the dataset package.

The data includes the following variables:

- **cyl**: Number of cylinders
- **disp**: Displacement (cu.in.)
- **hp**: Gross horsepower
- **drat**: Rear axle ratio
- **wt**: Weight (1000 lbs)
- **qsec**: 1/4 mile time
- **vs**: Engine shape, i.e., V-shaped or straight
- **am**: Automatic (0) or manual (1) transmission
- **gear**: Number of forward gears
- **carb**: Number of carburetors

We are particularly interested in the following two questions:

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- “Is an automatic or manual transmission better for MPG”
- “Quantify the MPG difference between automatic and manual transmissions”

In [Sect.2](#), we do some preliminary analysis, including loading, transformation, and some summary and exploratory analysis. We show that, in general, we expect that manual transmission works better than automatic transmission with respect to fuel economy. We deeply investigate this in the subsequent sections, considering many other factors.

In [Sect.3](#), for each numeric variable *var*, we build a linear model with **mpg** as the output and *var* as the regressor considering its interaction with the transmission type. We first select linear models which are worth considering. Then, we address our main questions using by these linear models.

In [Sect.4](#), we consider multivariate regression models. Again, we select the best fitting models, and then study them to address our analysis questions.

The R scripts of [Sect.2](#), [Sect.3](#), and [Sect.4](#) can be found in [Appendix.A](#), [Appendix.B](#), and [Appendix.C](#). [Appendix.D](#) include the diagnosis plots for the fitting models.

We consider 0.05 as the significance rate in all statistical analyses in this report.

2 Preliminary Analysis

Let us first take a look at the structure of the data: (We transform **cyl** (number of cylinders), **vs** (engine shape), **am** (transmission), **gear**, and **carb** (number of carburetors) to their equivalent factor variables. Also, we have renamed the levels of the **am** and **vs** variables. Moreover, we have changed the unit of **wt** from 1000lb to tonne.)

```
'data.frame': 32 obs. of 11 variables:
 $ mpg : num 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
 $ cyl : Factor w/ 3 levels "4","6","8": 2 2 1 2 3 2 3 1 1 2 ...
 $ disp: num 160 160 108 258 360 ...
 $ hp : num 110 110 93 110 175 105 245 62 95 123 ...
 $ drat: num 3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
 $ wt : num 1.19 1.3 1.05 1.46 1.56 ...
 $ qsec: num 16.5 17 18.6 19.4 17 ...
 $ vs : Factor w/ 2 levels "v","s": 1 1 2 2 1 2 1 2 2 2 ...
 $ am : Factor w/ 2 levels "automatic","manual": 2 2 2 1 1 1 1 1 1 1 ...
 $ gear: Factor w/ 3 levels "3","4","5": 2 2 2 1 1 1 1 2 2 2 ...
 $ carb: Factor w/ 6 levels "1","2","3","4",...: 4 4 1 1 2 1 4 2 2 4 ...
```

As we see in the boxplot in [Fig. 1](#), in general, the manual transmission is better than the automatic transmission in the sense of fuel economy. The mean of MPG for automatic transmission and manual transmission are 17.15 and 24.39, respectively.

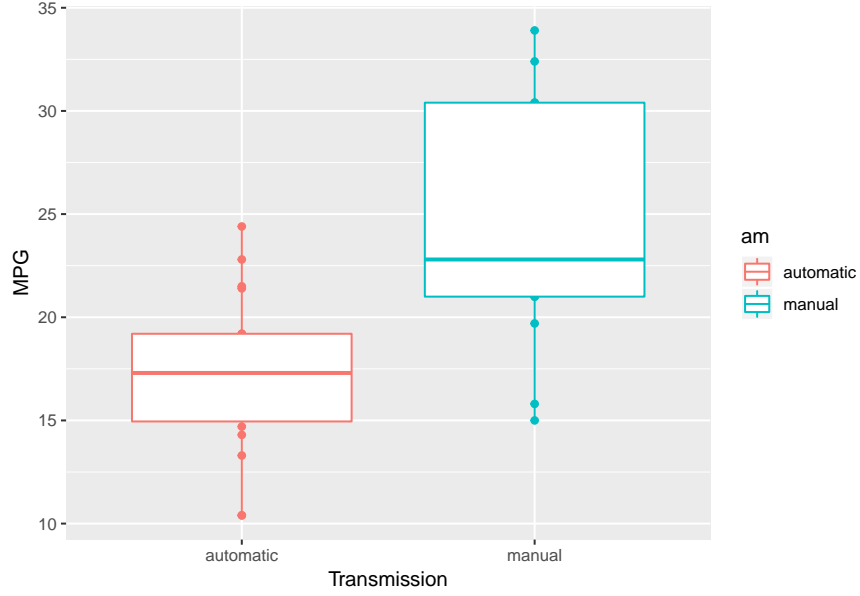


Figure 1: The Box Plot for MPG per Transmission Type

3 Single Variate Regression Models

In this section, for each numeric variable var , we build a linear model with MPG as the output and var as the regressor considering its interaction with the transmission type, i.e., the model $mpg \sim var * factor(am)$.

In Sect.3.1, we select the linear models which are worth considering. In Sect.3.2, Sect.3.3, and Sect.3.4, we address our main analysis question on the selected models.

3.1 Model Selection

We show that

Theorem 1 *The best three single variate models with mpg as the output are $mpg \sim wt*am$, $mpg \sim disp*am$, and $mpg \sim hp*am$. \square*

The rest of this section is devoted to the proof of the theorem.

The correlation between the numeric variables in the dataset is shown in the chart in Fig. 2. As we see in the chart, the correlation between **mpg** and **qsec**, i.e., 0.42, is not that high. Therefore, we ignore the model fitting of **mpg** vs. **qsec**.

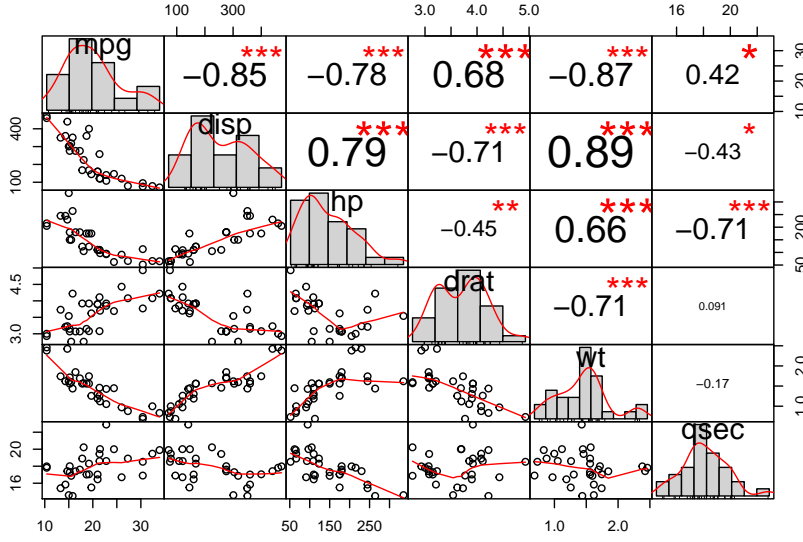


Figure 2: The Correlation Chart

Our models are as follow. Note that the weight unit in the dataset is 1000lb; however, we change it to tonne.

```
fit_wt <- lm(mpg ~ wt * factor(am), data = mtcars)
fit_disp <- lm(mpg ~ disp * factor(am), data = mtcars)
fit_hp <- lm(mpg ~ hp * factor(am), data = mtcars)
fit_drat <- lm(mpg ~ drat * factor(am), data = mtcars)
```

Since the above models are non-nested models, we take advantage of the *coxtest* function from the *lmtree* package to test them. In Table. 1, each row and column belongs to a fitting model. For any $1 \leq i, j \leq 4$, the cell in the position $[i, j]$ represents the P-value of comparing m_i against n_j , where $(\forall t) m_t$ denotes the fitted model in row t , and n_t denotes the model represented in column j .

Table 1: P-values of Comparing Single Variate Models by *coxtest*

	fit_wt	fit_disp	fit_hp	fit_drat
fit_wt	0	0.01	0.00	0.38
fit_disp	0	0.00	0.03	0.11
fit_hp	0	0.01	0.00	0.15
fit_drat	0	0.00	0.00	0.00

Considering 0.05 as our significance rate, the results are as follow:

- All other fitting models are preferred over the model `fit_drat`.
- The models other than `fit_drat` have no preference over each other, though the best among all the models is `fit_wt`.

Therefore, we discard the model `fit_drat`.

One could find the corresponding diagnosis plots in [Appendix. 1](#). As we see in the diagnostic plots, everything (including normality of errors and residuals) looks more less ok for the fitting models.

In the rest of this section, we investigate the fitting models excluding `fit_drat`.

3.2 MPG vs. Weight

The first model that we study is `fit_wt`, i.e., $\text{mpg} \sim \text{wt} * \text{am}$. Let us take a look at its coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	31.42	3.02	10.40	0
wt	-8.35	1.73	-4.82	0
factor(am)manual	14.88	4.26	3.49	0
wt:factor(am)manual	-11.68	3.19	-3.67	0

The 95% confidence intervals of the coefficients are represented in the following table.

	Intercept	wt	factor.am.manual	wt.factor.am.manual
Lower	25.23	-11.89	6.15	-18.21
Upper	37.61	-4.81	23.61	-5.15

The estimated intercept in automatic transmission is about 31.42, and 14.88 is the estimated change in the intercept of the linear relationship between weight and MPG going from automatic transmission to manual transmission. The estimated slope in automatic transmission is -8.35 while the estimated change in the slope switching from automatic to manual is -11.68. In other words:

- The estimated MPG for automatic and manual vehicles with 0 weight are 31.42 and 46.3, respectively.
- The expected change in MPG per 1 tonne change in weight for automatic and manual vehicles are -8.35 and -20.03, respectively.

Fig. 3 represents the corresponding plot, where the regression lines for automatic transmission and manual transmission are shown in red and blue, respectively.

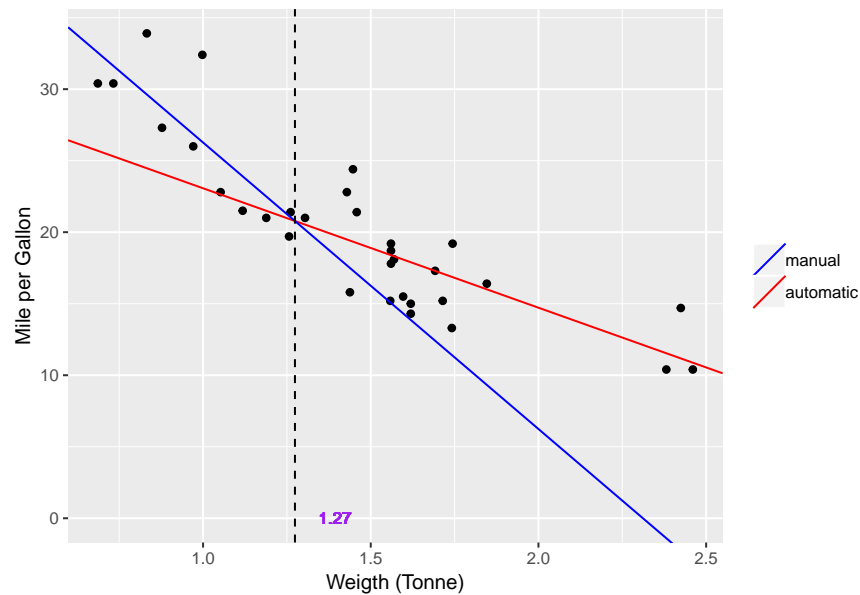


Figure 3: MPG vs. Weight

As it is clear heavier vehicle results in more fuel consumption. The regression lines meet at point **1.27** tonne. As seen, we predict that for vehicles with weight less (more, respectively) than 1.27 tonne, the manual (automatic, respectively) transmission is a better for fuel economy.

3.3 MPG vs. Displacement

The next model is `fit_disp`, i.e., $\text{mpg} \sim \text{disp} * \text{am}$ whose coefficients are as follow:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	25.16	1.93	13.07	0.00
disp	-0.03	0.01	-4.44	0.00
factor(am)manual	7.71	2.50	3.08	0.00
disp:factor(am)manual	-0.03	0.01	-2.75	0.01

The 95% confidence intervals of the coefficients are represented in the following table.

	Intercept	disp	factor.am.manual	disp.factor.am.manual
Lower	21.21	-0.05	2.59	-0.05
Upper	29.11	-0.01	12.83	-0.01

The estimated intercept in automatic transmission is about 25.16, while 7.71 is the estimated change in the intercept of the linear relationship between displacement and MPG going from automatic transmission to manual transmission. The estimated slope in automatic transmission is -0.03 while the estimated change in the slope switching from automatic to manual is -0.03. In other words:

- The estimated MPG for automatic transmission and manual transmission vehicles with 0 cu.in. displacement are 25.16 and 32.87, respectively.
- The expected change in MPG per 1 cu.in. change in displacement for automatic and manual vehicles are -0.03 and -0.06, respectively.

Fig. 4 the corresponding plot, where the regression lines for automatic transmission and manual transmission are shown in blue and red, respectively.

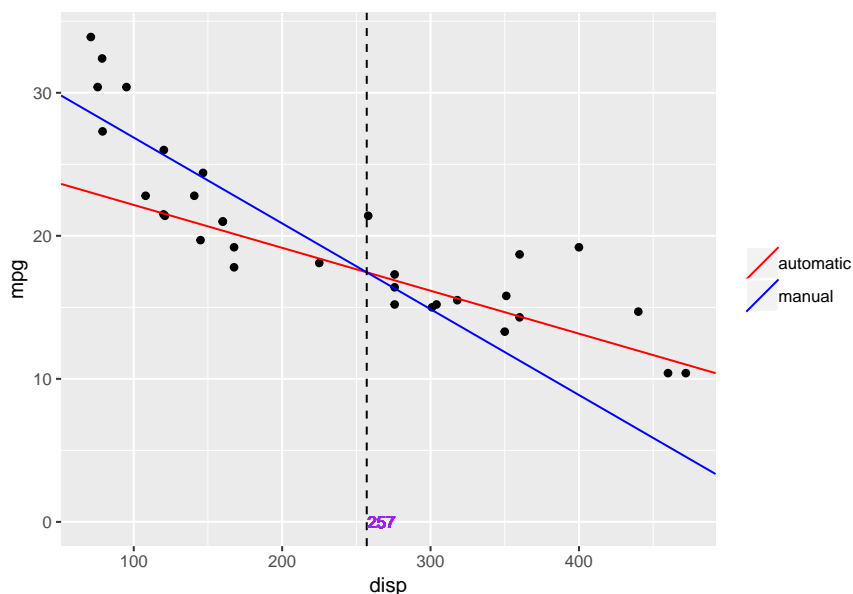


Figure 4: MPG vs. Displacement

As it is clear, higher displacement results in more fuel consumption. The regression lines meet at point **257** cu.in. As seen in the plot, we predict that for vehicles with displacement less (more, respectively) than 257 (cu.in.), the manual (automatic, respectively) transmission works better w.r.t fuel economy.

3.4 MPG vs. Horsepower

The last model to study is `fit_hp`, i.e., $\text{mpg} \sim \text{hp} * \text{am}$ with the following coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	26.62	2.18	12.20	0.00
hp	-0.06	0.01	-4.57	0.00
factor(am)manual	5.22	2.67	1.96	0.06
hp:factor(am)manual	0.00	0.02	0.02	0.98

The 95% confidence intervals of the coefficients are represented in the following table.

	Intercept	hp	factor.am.manual	hp.factor.am.manual
Lower	22.15	-0.08	-0.25	-0.04
Upper	31.09	-0.04	10.69	0.04

The estimated intercept in automatic transmission is about 26.62. 5.22 is the estimated change in the intercept of the linear relationship between horsepower and MPG going from automatic transmission to manual transmission. The estimated slope in automatic transmission is -0.06 while the estimated change in the slope switching from automatic to manual is about 0. This shows that there is no significant interaction between **am** and **hp**. In other words:

- The expected MPG for automatic transmission and manual transmission vehicles with horsepower 0 are 26.62 and 31.84, respectively.
- The expected change in MPG per unit change in horsepower for both automatic and manual vehicles is about -0.06.

Note that the second bullet implies that the corresponding regression lines for automatic and manual would be parallel. Fig. 5 represents the corresponding plot, where the regression lines for automatic transmission and manual transmission are shown in red and blue, respectively.

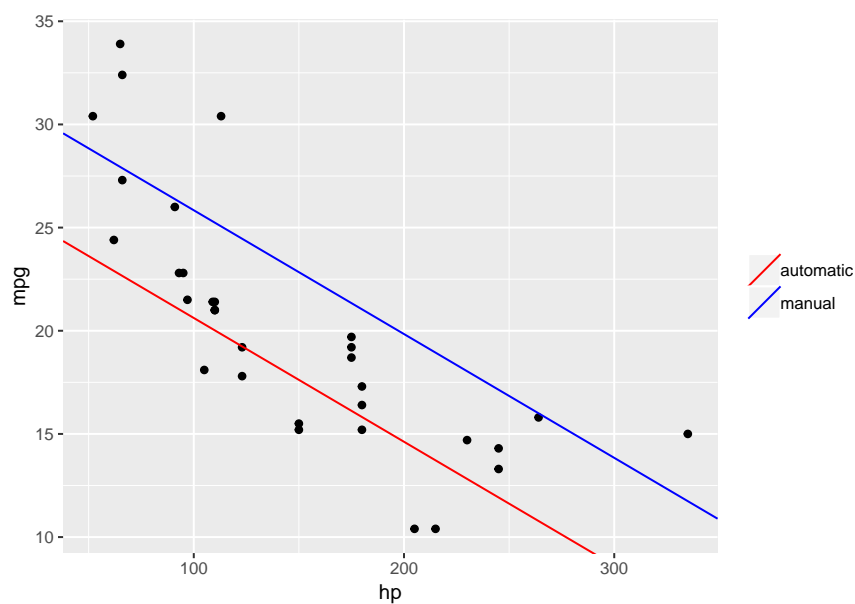


Figure 5: MPG vs. Horsepower

Clearly, higher horsepower results in worse fuel economy. As seen in the above plot, we predict that manual transmission is always better than automatic transmission with respect to fuel economy for a given horsepower.

4 Multivariate Regression Models

In this section, we consider more complicated models, i.e., multivariate regression models.

The structure of this section is as follows: In [Sect.4.1](#), we select the multivariate linear regression models which are worth considering. In [Sect.4.2](#) and [Sect.4.3](#), we address our main analysis question on the selected models.

4.1 Model Selection

We show that:

Theorem 2 *The best two fitting linear models with mpg as the output are $mpg \sim wt+hp$ and $mpg \sim wt+qsec$.* \square

As we already saw, among the single variate models, the models worth to consider are **Model1**: $mpg \sim wt$, **Model2**: $mpg \sim hp$, and **Model3**: $mpg \sim disp$. Now, we want to see if adding some new regressors to these models make sense. We show that

Lemma 1 *The best models including wt as a regressor are $mpg \sim wt+hp$ and $mpg \sim wt+qsec$.* \square

Lemma 2 *The best model including hp as a must regressor is $mpg \sim hp+wt$.* \square

Lemma 3 *Considering disp as a must regressor in our models, the best linear model is $mpg \sim disp+wt$.* \square

Note that these three lemmas together prove our main theorem.

4.1.1 Proof of Lemma. 1

Let us first see if adding some regressors to $mpg \sim wt$ makes senses. We take advantage of the *anova* function to address this question. The P-values for comparing the model $mpg \sim wt$ vs. $mpg \sim wt + var$, where $var \in \{disp, hp, drat, qsec\}$ are represented in [Table. 2](#).

Table 2: P-values of Comparing 2-variate Models with wt as the Regressor				
	$mpg \sim wt+disp$	$mpg \sim wt+hp$	$mpg \sim wt+drat$	$mpg \sim wt+qsec$
$mpg \sim wt$	0.06	0	0.33	0

Considering the significance rate 0.05, we see that only the two models “ $mpg \sim wt + hp$ ” and “ $mpg \sim wt + qsec$ ” are preferred over $mpg \sim wt$. Now, let us see which of these two models works better:

`Cox test`

Model 1: $mpg \sim wt + hp$

Model 2: $mpg \sim wt + qsec$

	Estimate	Std. Error	z value	Pr(> z)
fitted(M1) ~ M2	-2.25	1.54	-1.46	0.14
fitted(M2) ~ M1	-2.30	1.53	-1.50	0.13

Therefore, considering 0.05 as the significance rate, none of them are preferred over the other.

Now, let us see if their combination, i.e., $mpg \sim wt + hp + qsec$, works better. In the following, we show that $mpg \sim wt + hp + qsec$ is NOT preferred over $mpg \sim wt + hp$.

Analysis of Variance Table

```
Model 1: mpg ~ wt + hp
Model 2: mpg ~ wt + hp + qsec
      Res.Df    RSS Df Sum of Sq    F Pr(>F)
1         29 195.05
2         28 186.06  1      8.9885 1.3527 0.2546
```

Now, we show that $\text{mpg} \sim \text{wt} + \text{hp} + \text{qsec}$ is NOT preferred over $\text{mpg} \sim \text{wt} + \text{qsec}$:

Analysis of Variance Table

```
Model 1: mpg ~ wt + qsec
Model 2: mpg ~ wt + hp + qsec
      Res.Df    RSS Df Sum of Sq    F Pr(>F)
1         29 195.46
2         28 186.06  1      9.4043 1.4153 0.2442
```

Therefore, we showed that the best models including **wt** as a must regressor is $\text{mpg} \sim \text{wt} + \text{hp}$ or $\text{mpg} \sim \text{wt} + \text{qsec}$. Lemma. 1 was proven.

4.1.2 Proof of Lemma. 2

Let us now see if adding some regressors to $\text{mpg} \sim \text{hp}$ makes our fitting model any better. Again, we apply the *anova* function to address this question. The P-value for comparing the model $\text{mpg} \sim \text{hp}$ vs. $\text{mpg} \sim \text{hp} + \text{var}$, where $\text{var} \in \{\text{disp}, \text{wt}, \text{drat}, \text{qsec}\}$ are represented in Table. 3.

Table 3: P-values of Comparing 2-variate Models with hp as the Regressor				
	$\text{mpg} \sim \text{hp} + \text{disp}$	$\text{mpg} \sim \text{hp} + \text{wt}$	$\text{mpg} \sim \text{hp} + \text{drat}$	$\text{mpg} \sim \text{hp} + \text{qsec}$
$\text{mpg} \sim \text{hp}$	0	0	0	0.11

Considering the significance rate 0.05, all the models excluding $\text{mpg} \sim \text{hp} + \text{qsec}$ are preferred over $\text{mpg} \sim \text{hp}$. Now, using the *covtest* command, we want to see which of them works the best:

The following tables show that $\text{mpg} \sim \text{hp} + \text{wt}$ has preference over $\text{mpg} \sim \text{hp} + \text{disp}$ and $\text{mpg} \sim \text{hp} + \text{drat}$:

Cox test

```
Model 1: mpg ~ hp + disp
Model 2: mpg ~ hp + wt
      Estimate Std. Error z value Pr(>|z|)
fitted(M1) ~ M2 -9.0487      1.7108 -5.2892 1.229e-07 ***
fitted(M2) ~ M1 -0.2062      2.1047 -0.0980  0.922
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Cox test

```
Model 1: mpg ~ hp + wt
Model 2: mpg ~ hp + drat
      Estimate Std. Error z value Pr(>|z|)
fitted(M1) ~ M2 -2.9554      1.7423 -1.6963  0.08983 .
fitted(M2) ~ M1 -10.9891      1.5147 -7.2548 4.023e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Therefore, we showed that the best model including **hp** as a must regressor is $mpg \sim hp + wt$. Lemma. 2 is proven.

4.1.3 Proof of Lemma. 3

The last model to be considered is $mpg \sim disp$. Applying the *anova* function, the P-values for comparing the model $mpg \sim disp$ vs. $mpg \sim disp + var$, where $var \in \{hp, wt, drat, qsec\}$ are represented in Table. 4.

As we see in the table, $mpg \sim disp + wt$ is the only model which is preferred over $mpg \sim disp$. In other words, considering $disp$ as a must regressor in our models, the best linear model is $mpg \sim disp + wt$. Lemma. 3 is proven.

Table 4: P-values of Comparing 2-variate Models with $disp$ as the Regressor

	$mpg \sim disp + hp$	$mpg \sim disp + wt$	$mpg \sim disp + drat$	$mpg \sim disp + qsec$
$mpg \sim disp$	0.07	0.01	0.25	0.57

4.2 MPG vs. Weight plus Horsepower

In this section, we study the model with **wt** and **hp** as regressors. Let's first consider the full interaction between transmission type and the regressors, i.e., the following model:

```
fit_wt_hp <- lm(mpg ~ (wt + hp) * factor(am), data = mtcars)
```

The coefficients of the model are as follow:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	30.70	2.68	11.48	0.00
wt	-4.09	2.08	-1.96	0.06
hp	-0.04	0.01	-3.00	0.01
factor(am)manual	13.74	4.22	3.25	0.00
wt:factor(am)manual	-12.72	4.57	-2.78	0.01
hp:factor(am)manual	0.03	0.02	1.45	0.16

As we see, the P-value for the interaction of **hp** and **am** is not significant. Moreover, the P-value for the coefficient **wt** is a little higher than the significance rate (0.05). Therefore, we modify the model as follows:

```
fit_wt_hp <- lm(mpg ~ (wt * factor(am)) + hp, data = mtcars)
```

The coefficients of the model are as follow:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	30.95	2.72	11.36	0.00
wt	-5.55	1.86	-2.98	0.01
factor(am)manual	11.55	4.02	2.87	0.01
hp	-0.03	0.01	-2.75	0.01
wt:factor(am)manual	-7.89	3.18	-2.48	0.02

The 95% confidence intervals of the coefficients are represented in the following table.

	Intercept	wt	factor.am.manual	hp	wt.factor.am.manual
Lower	25.37	-9.37	3.3	-0.05	-14.41
Upper	36.53	-1.73	19.8	-0.01	-1.37

The estimated intercept in automatic transmission is about 30.95, and 11.55 is the estimated change in the intercept of the linear relationship going from automatic transmission to manual transmission. The estimated coefficient of weight in automatic transmission is -5.55 while the estimated change in the weight coefficient switching from automatic to manual is -7.89. The estimated coefficient of horsepower is -0.03.

In other words:

- The estimated MPG is 30.95 and 42.5 for the automatic transmission and the manual transmission vehicle with weight 0 and horsepower 0, respectively.
- The expected change in MPG for an automatic transmission and manual transmission per tonne change in weight are -5.55 and -13.44, respectively, by holding the horsepower constant.
- The expected change in MPG for both automatic and manual vehicle per unit change in horsepower is -0.03, by holding weight constant.

We have represented the diagnosis plots of this model in Fig. 9.

4.3 MPG vs. Weight plus 1/4-Mile-Time

In this section, we study the model with **wt** and **qsec** as regressors. We first consider the full interaction between transmission type and the regressors, i.e., the following model:

```
fit_wt_qsec <- lm(mpg ~ (wt + qsec) * factor(am), data = mtcars)
```

The coefficients of the model are as follow:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	11.25	6.99	1.61	0.12
wt	-6.61	1.52	-4.34	0.00
qsec	0.95	0.31	3.08	0.00
factor(am)manual	8.93	12.67	0.70	0.49
wt:factor(am)manual	-8.29	3.34	-2.48	0.02
qsec:factor(am)manual	0.24	0.56	0.42	0.68

As we see, the P-value for the interaction of **qsec** and **am** is not significant. Therefore, we modify the model as follows:

```
fit_wt_qsec <- lm(mpg ~ (wt * factor(am)) + qsec, data = mtcars)
```

The coefficients of the new model are as follow:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.72	5.90	1.65	0.11
wt	-6.47	1.47	-4.41	0.00
factor(am)manual	14.08	3.44	4.10	0.00
qsec	1.02	0.25	4.04	0.00
wt:factor(am)manual	-9.13	2.64	-3.46	0.00

The 95% confidence intervals of the coefficients are represented in the following table.

	Intercept	wt	factor.am.manual	qsec	wt.factor.am.manual
Lower	-2.39	-9.49	7.02	0.51	-14.55
Upper	21.83	-3.45	21.14	1.53	-3.71

Since the P-value associated with intercept is high, our estimated intercept in automatic transmission is 0. 14.08 is the estimated change in the intercept of the linear relationship going from automatic transmission

to manual transmission. The estimated coefficient of weight in automatic transmission is -6.47 while the estimated change in the weight coefficient switching from automatic to manual is -9.13. The estimated coefficient of qsec is 1.02.

In other words:

- The estimated MPG is 0 for an automatic transmission vehicle with weight 0 and qsec 0.
- The estimated MPG is 14.08 for a manual transmission vehicle with weight 0 and qsec 0.
- The expected change in MPG for an automatic transmission vehicle per tonne change in weight is -6.47, by holding the qsec constant.
- The expected change in MPG for an automatic manual vehicle per tonne change in weight is -15.6, by holding the qsec constant.
- The expected change in MPG for both automatic and manual vehicle per unit change in qsec is 1.02, by holding weight constant

The diagnosis plot of this models can be found in Fig. 10.

Appendix A: R Scripts of Sect. 2

Loading and transforming the data:

```
library(datasets)
data("mtcars")
mtcars$am <- as.factor(mtcars$am)
levels(mtcars$am) <- c("automatic", "manual")
mtcars$cyl <- as.factor(mtcars$cyl)
mtcars$vs <- as.factor(mtcars$vs)
levels(mtcars$vs) <- c("v", "s")
mtcars$gear <- as.factor(mtcars$gear)
mtcars$carb <- as.factor(mtcars$carb)
mtcars$wt <- mtcars$wt/2.20462
str(mtcars)
```

The boxplot for MPG per each transmission type:

```
library(ggplot2)
qplot(am, mpg, data = mtcars, colour = am) + geom_boxplot() +
  xlab("Transmission") + ylab("MPG")
```

Appendix B: R Scripts of Sect. 3

The correlation chart:

```
library(corrplot)
library(PerformanceAnalytics)
chart.Correlation(mtcars[, c(1, 3:7)], histogram=TRUE, pch=19)
```

Code of Table. 1 (P-values of Comparing Single Variate Models)

```
library(lmtest)
wt_disp <- round(coxtest(fit_wt, fit_disp)$`Pr(>|z|)` , 2)
wt_hp <- round(coxtest(fit_wt, fit_hp)$`Pr(>|z|)` , 2)
```

```

wt_drat <- round(coxtest(fit_wt, fit_drat)$`Pr(>|z|)` , 2)
disp_hp <- round(coxtest(fit_disp, fit_hp)$`Pr(>|z|)` , 2)
disp_drat <- round(coxtest(fit_disp, fit_drat)$`Pr(>|z|)` , 2)
hp_drat <- round(coxtest(fit_hp, fit_drat)$`Pr(>|z|)` , 2)
wt_c <- c(0, wt_disp[2], wt_hp[2], wt_drat[2])
disp_c <- c(wt_disp[1], 0, disp_hp[2], disp_drat[2])
hp_c <- c(wt_hp[1], disp_hp[1], 0, hp_drat[2])
drat_c <- c(wt_drat[1], disp_drat[1], hp_drat[1], 0)
tests_pval <- as.data.frame(cbind(wt_c, disp_c, hp_c, drat_c))
row.names(tests_pval) <- c("fit_wt", "fit_disp", "fit_hp", "fit_drat")
colnames(tests_pval) <- paste(" ", row.names(tests_pval), sep = "")

library(knitr)
library(kableExtra)
kable(tests_pval, "latex", booktabs = T,
      caption = "P-values of Comparing Single Variate Models by coxtest") %>%
  kable_styling(latex_options = "hold_position")

```

The coefficients of fit_wt:

```

cff_wt <- round(summary(fit_wt)$coefficient, 2)
cff_wt

```

The 95% confidence interval for the fit_wt coefficients:

```

q <- c(-1,1) * qt(.975, df = fit_wt$df)
int_aut <- cff_wt[1,1] + q * cff_wt[1,2]
int_aut2man <- cff_wt[3,1] + q * cff_wt[3,2]
wt_aut <- cff_wt[2,1] + q * cff_wt[2,2]
wt_aut2man <- cff_wt[4,1] + q * cff_wt[4,2]

conf_wt <- data.frame(Intercept = int_aut,
                      wt = wt_aut,
                      `factor(am)manual` = int_aut2man,
                      `wt:factor(am)manual` = wt_aut2man)

row.names(conf_wt) <- c("Lower", "Upper")

kable(round(conf_wt, 2), "latex", booktabs = T) %>%
  kable_styling(position = "center")

```

The corresponding plot for fit_wt

```

line <- data.frame(intercept = c( cff_wt[1, 1], cff_wt[1, 1] + cff_wt[3, 1]),
                  slope = c( cff_wt[2, 1], cff_wt[2, 1] + cff_wt[4, 1]),
                  row.names = c("automatic", "manual") )
x_com_wt <- (line[1,1] - line[2,1]) / (line[2,2] - line[1, 2])
qplot(wt, mpg, data = mtcars) +
  xlab("Weighth (Tonne)") + ylab("Mile per Gallon") +
  geom_abline(aes(intercept=intercept, slope=slope,
                  colour=c("red", "blue")), data=line) +
  theme(legend.title=element_blank()) +
  scale_color_manual(labels = c("manual", "automatic"), values = c("blue", "red")) +
  geom_vline(xintercept = x_com_wt, linetype = "dashed") +
  geom_text(aes(x_com_wt+0.12,0,label = round(x_com_wt, 2)), size = 3,

```

```
color = "purple")
```

The coefficients of fit_disp:

```
cff_disp <- round(summary(fit_disp)$coefficient, 2)
cff_disp
```

The 95% confidence interval for coefficients of fit_disp:

```
q <- c(-1,1) * qt(.975, df = fit_disp$df)
int_aut <- cff_disp[1,1] + q * cff_disp[1,2]
int_aut2man <- cff_disp[3,1] + q * cff_disp[3,2]
disp_aut <- cff_disp[2,1] + q * cff_disp[2,2]
disp_aut2man <- cff_disp[4,1] + q * cff_disp[4,2]

conf_disp <- data.frame(Intercept = int_aut,
                        disp = disp_aut,
                        `factor(am)manual` = int_aut2man,
                        `disp:factor(am)manual` = disp_aut2man)

row.names(conf_disp) <- c("Lower", "Upper")

kable(round(conf_disp, 2), "latex", booktabs = T) %>%
  kable_styling(position = "center")
```

The corresponding plot for fit_disp:

```
line <- data.frame(intercept = c( cff_disp[1, 1], cff_disp[1, 1] + cff_disp[3, 1]),
                  slope = c( cff_disp[2, 1], cff_disp[2, 1] + cff_disp[4, 1]),
                  row.names = c("automatic", "manual") )
x_com_disp <- (line[1,1] - line[2,1]) / (line[2,2] - line[1, 2])
qplot(disp, mpg, data = mtcars) +
  geom_abline(aes(intercept=intercept, slope=slope,
                  colour=c("blue", "red")), data=line) +
  theme(legend.title=element_blank()) +
  scale_color_manual(labels = c("automatic", "manual"), values = c("red", "blue")) +
  geom_vline(xintercept = x_com_disp, linetype = "dashed") +
  geom_text(aes(x_com_disp+10,0,label = round(x_com_disp, 2)), size = 3,
            color = "purple")
```

The coefficients of fit_hp:

```
cff_hp <- round(summary(fit_hp)$coefficient, 2)
cff_hp
```

The 95% confidence interval for the coefficients of fit_hp:

```
q <- c(-1,1) * qt(.975, df = fit_hp$df)
int_aut <- cff_hp[1,1] + q * cff_hp[1,2]
int_aut2man <- cff_hp[3,1] + q * cff_hp[3,2]
hp_aut <- cff_hp[2,1] + q * cff_hp[2,2]
hp_aut2man <- cff_hp[4,1] + q * cff_hp[4,2]

conf_hp <- data.frame(Intercept = int_aut,
                     hp = hp_aut,
```

```

      `factor(am)manual` = int_aut2man,
      `hp:factor(am)manual` = hp_aut2man)

row.names(conf_hp) <- c("Lower", "Upper")

kable(round(conf_hp, 2), "latex", booktabs = T) %>%
  kable_styling(position = "center")

```

The corresponding plot for fit_hp:

```

line <- data.frame(intercept = c( cff_hp[1, 1], cff_hp[1, 1] + cff_hp[3, 1]),
  slope = c( cff_hp[2, 1], cff_hp[2, 1] + cff_hp[4, 1]),
  row.names = c("automatic", "manual") )

qplot(hp, mpg, data = mtcars) +
  geom_abline(aes(intercept=intercept, slope=slope,
    colour=c("blue", "red")), data=line) +
  theme(legend.title=element_blank()) +
  scale_color_manual(labels = c("automatic", "manual"), values = c("red", "blue"))

```

Appendix C: R Scripts of Sect. 4

The script for Table. 2

```

fit11 <- lm(mpg ~ wt, data = mtcars)
fit12 <- lm(mpg ~ wt + disp, data = mtcars)
fit13 <- lm(mpg ~ wt + hp, data = mtcars)
fit14 <- lm(mpg ~ wt + drat, data = mtcars)
fit15 <- lm(mpg ~ wt + qsec, data = mtcars)

disp_val <- round(anova(fit11, fit12)$`Pr(>F)`[2], 2)
hp_val <- round(anova(fit11, fit13)$`Pr(>F)`[2], 2)
drat_val <- round(anova(fit11, fit14)$`Pr(>F)`[2], 2)
qsec_val <- round(anova(fit11, fit15)$`Pr(>F)`[2], 2)

tests_pval <- as.data.frame(cbind(disp_val, hp_val, drat_val, qsec_val))
row.names(tests_pval) <- c("mpg~wt")
colnames(tests_pval) <- c("mpg~wt+disp", " mpg~wt+hp", " mpg~wt+drat", " mpg~wt+qsec")

kable(tests_pval, "latex", booktabs = T,
  caption = "P-values of Comparing 2-variate Models with wt as the Regressor") %>%
  kable_styling(latex_options = "hold_position")

```

Comparing two models mpg~wt+hp and mpg~wt+qsec:

```
round(coxtest(fit13, fit15), 2)
```

Comparing two models mpg~wt+hp+qsec and mpg~wt+hp:

```
fit <- lm(mpg ~ wt + hp + qsec, data = mtcars)
anova(fit13, fit)
```

Comparing two models mpg~wt+hp+qsec and mpg~wt+qsec:

```
anova(fit15, fit)
```

The script for Table. 3:

```
fit21 <- lm(mpg ~ hp, data = mtcars)
fit22 <- lm(mpg ~ hp + disp, data = mtcars)
fit23 <- lm(mpg ~ hp + wt, data = mtcars)
fit24 <- lm(mpg ~ hp + drat, data = mtcars)
fit25 <- lm(mpg ~ hp + qsec, data = mtcars)

disp_val <- round(anova(fit21, fit22)$`Pr(>F)`[2], 2)
wt_val <- round(anova(fit21, fit23)$`Pr(>F)`[2], 2)
drat_val <- round(anova(fit21, fit24)$`Pr(>F)`[2], 2)
qsec_val <- round(anova(fit21, fit25)$`Pr(>F)`[2], 2)

tests_pval <- as.data.frame(cbind(disp_val, wt_val, drat_val, qsec_val))
row.names(tests_pval) <- c("mpg~hp")
colnames(tests_pval) <- c("mpg~hp+disp", " mpg~hp+wt", " mpg~hp+drat", " mpg~hp+qsec")

kable(tests_pval, "latex", booktabs = T,
      caption = "P-values of Comparing 2-variate Models with hp as the Regressor") %>%
  kable_styling(latex_options = "hold_position")
```

Comparing mpg~hp+wt vs. mpg~hp+disp and mpg~hp+drat:

```
coxtest(fit22, fit23)
coxtest(fit23, fit24)
```

The script for Table. 4.

```
fit31 <- lm(mpg ~ disp, data = mtcars)
fit32 <- lm(mpg ~ disp + hp, data = mtcars)
fit33 <- lm(mpg ~ disp + wt, data = mtcars)
fit34 <- lm(mpg ~ disp + drat, data = mtcars)
fit35 <- lm(mpg ~ disp + qsec, data = mtcars)

hp_val <- round(anova(fit31, fit32)$`Pr(>F)`[2], 2)
wt_val <- round(anova(fit31, fit33)$`Pr(>F)`[2], 2)
drat_val <- round(anova(fit31, fit34)$`Pr(>F)`[2], 2)
qsec_val <- round(anova(fit31, fit35)$`Pr(>F)`[2], 2)

tests_pval <- as.data.frame(cbind(hp_val, wt_val, drat_val, qsec_val))
row.names(tests_pval) <- c("mpg~disp")
colnames(tests_pval) <-
  c("mpg~disp+hp", " mpg~disp+wt", " mpg~disp+drat", " mpg~disp+qsec")

kable(tests_pval, "latex", booktabs = T,
      caption = "P-values of Comparing 2-variate Models with disp as the Regressor") %>%
  kable_styling(latex_options = "hold_position")
```

The coefficients of the fit_wt_hp:

```
cff_wt_hp <- round(summary(fit_wt_hp)$coefficient, 2)
cff_wt_hp
```


The coefficients of the new fit_wt_hp:

```
cff_wt_hp <- round(summary(fit_wt_hp)$coefficient, 2)
cff_wt_hp
```

The confidence intervals for coefficients for fit_wt_hp:

```
q <- c(-1,1) * qt(.975, df = fit_wt_hp$df)
int_aut <- cff_wt_hp[1,1] + q * cff_wt_hp[1,2]
int_aut2man <- cff_wt_hp[3,1] + q * cff_wt_hp[3,2]
wt_aut <- cff_wt_hp[2,1] + q * cff_wt_hp[2,2]
wt_aut2man <- cff_wt_hp[5,1] + q * cff_wt_hp[5,2]
hpc <- cff_wt_hp[4,1] + q * cff_wt_hp[4,2]

conf_wthp <- data.frame(Intercept = int_aut,
                        wt = wt_aut,
                        `factor(am)manual` = int_aut2man,
                        hp = hpc,
                        `wt:factor(am)manual` = wt_aut2man)

row.names(conf_wthp) <- c("Lower", "Upper")

kable(round(conf_wthp, 2), "latex", booktabs = T) %>%
  kable_styling(position = "center")
```

The coefficients of the model fit_wt_qsec:

```
cff_wt_qsec <- round(summary(fit_wt_qsec)$coefficient, 2)
cff_wt_qsec
```

The coefficients of the new model fit_wt_qsec:

```
cff_wt_qsec <- round(summary(fit_wt_qsec)$coefficient, 2)
cff_wt_qsec
```

The confidence intervals for coefficients for fit_wt_qsec:

```
q <- c(-1,1) * qt(.975, df = fit_wt_qsec$df)
int_aut <- cff_wt_qsec[1,1] + q * cff_wt_qsec[1,2]
int_aut2man <- cff_wt_qsec[3,1] + q * cff_wt_qsec[3,2]
wt_aut <- cff_wt_qsec[2,1] + q * cff_wt_qsec[2,2]
wt_aut2man <- cff_wt_qsec[5,1] + q * cff_wt_qsec[5,2]
qsecc <- cff_wt_qsec[4,1] + q * cff_wt_qsec[4,2]

conf_wtqsec <- data.frame(Intercept = int_aut,
                        wt = wt_aut,
                        `factor(am)manual` = int_aut2man,
                        qsec = qsecc,
                        `wt:factor(am)manual` = wt_aut2man)

row.names(conf_wtqsec) <- c("Lower", "Upper")

kable(round(conf_wtqsec, 2), "latex", booktabs = T) %>%
  kable_styling(position = "center")
```

Appendix D: The Diagnosis Plots

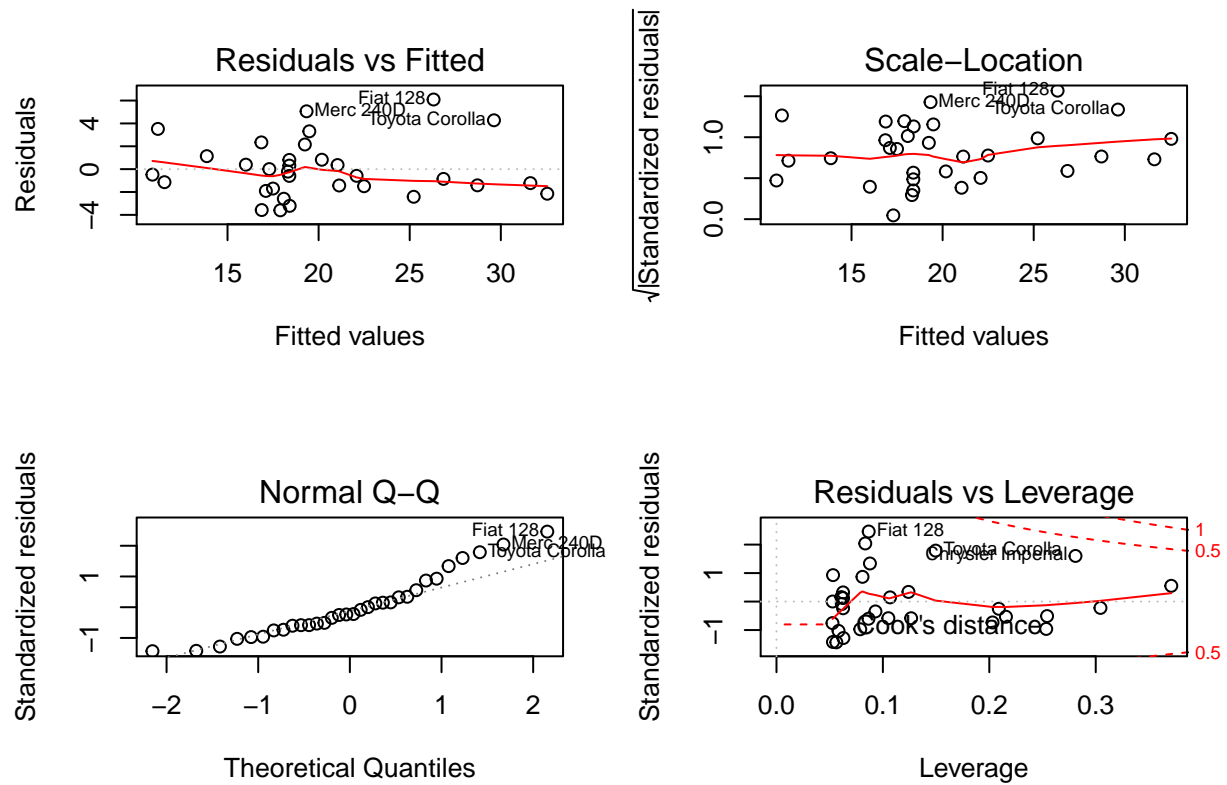


Figure 6: Diagnosis Plots for mpg vs. $\text{wt} * \text{factor}(\text{am})$

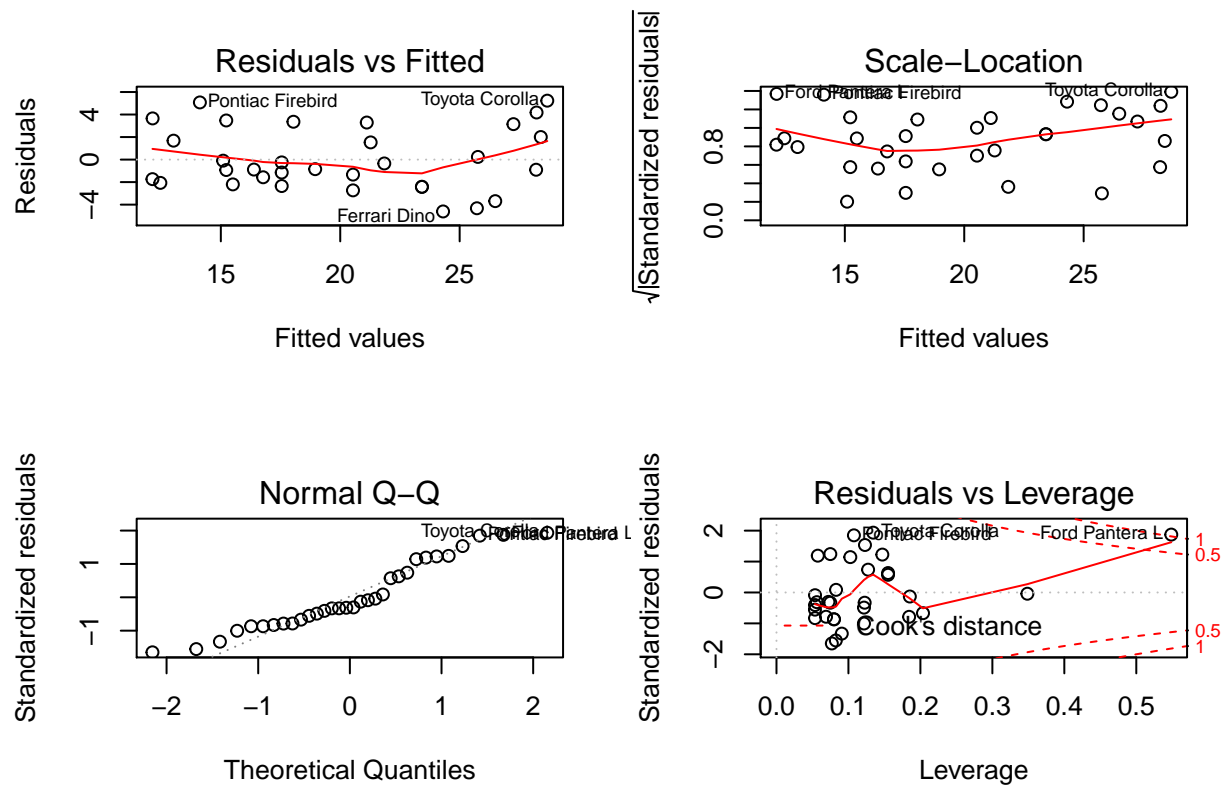


Figure 7: Diagnosis Plots for mpg vs. $\text{disp} * \text{factor}(\text{am})$

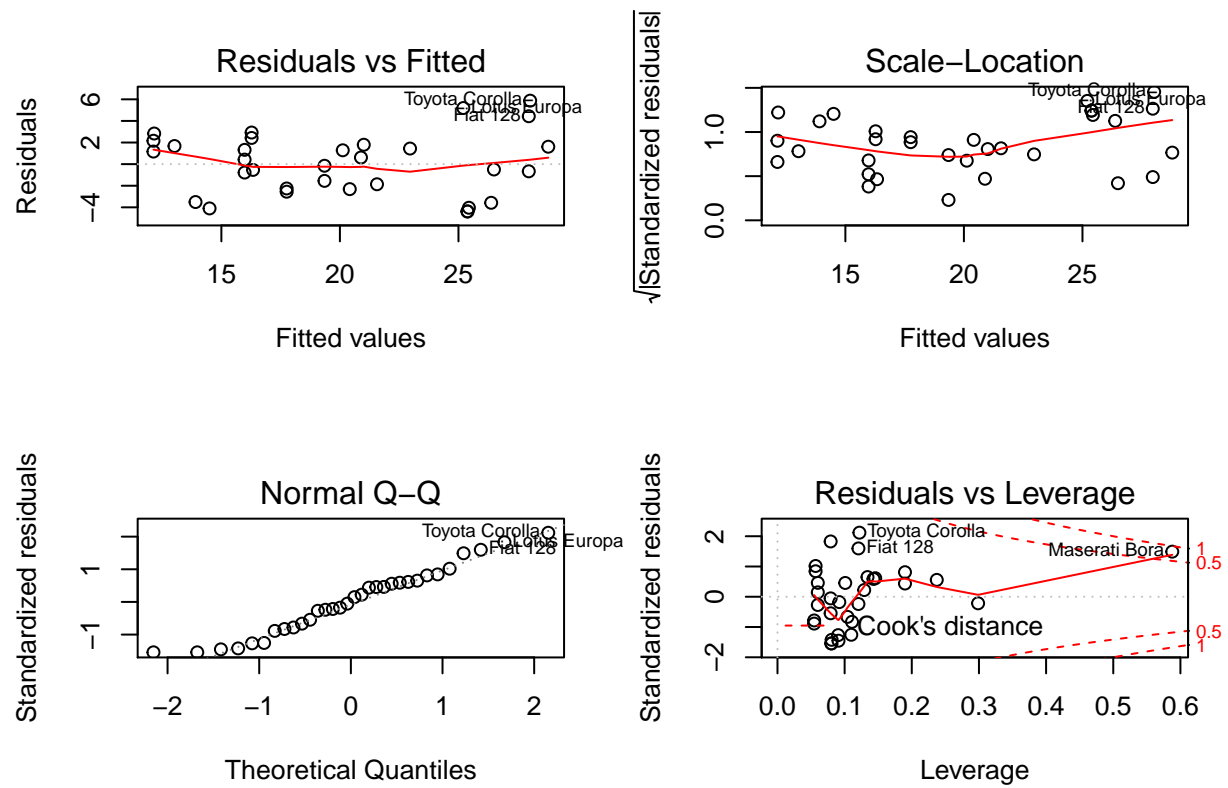


Figure 8: Diagnosis Plots for $\text{mpg} \sim \text{hp} * \text{factor(am)}$

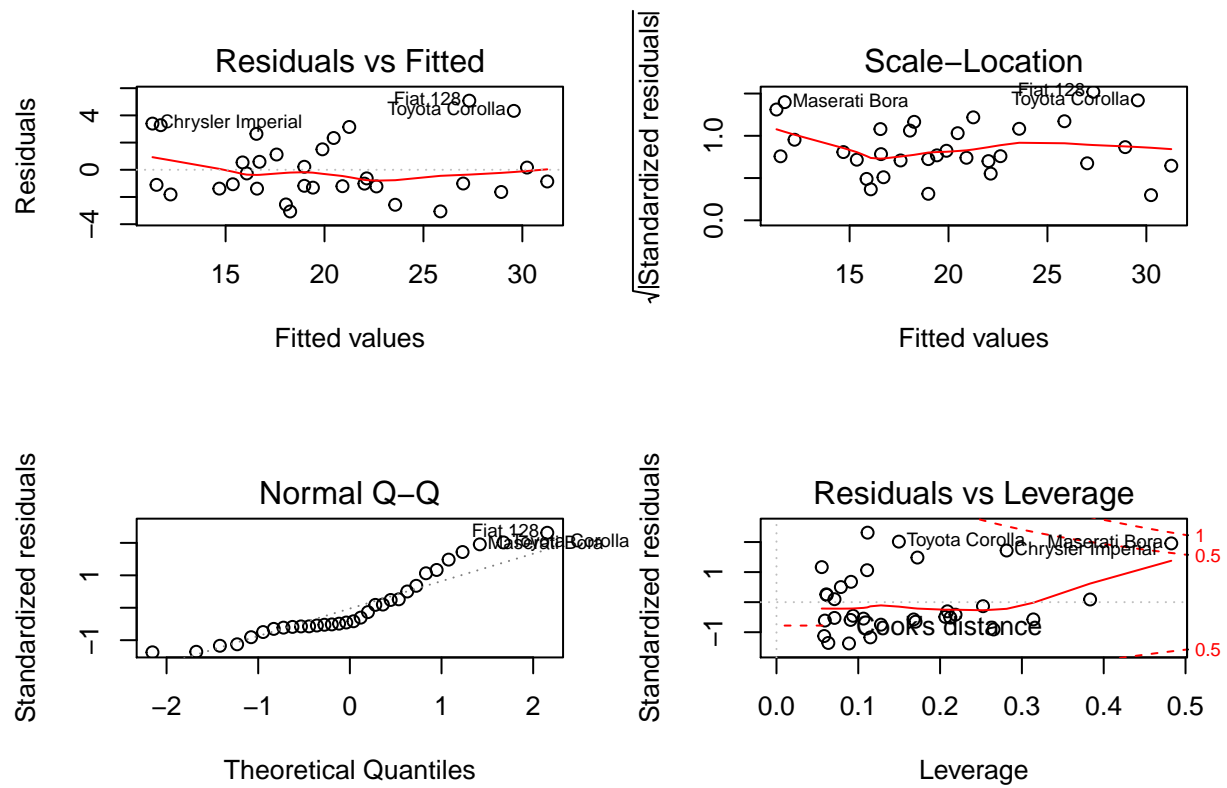


Figure 9: Diagnosis Plots for mpg vs. $(wt + hp) * \text{factor}(am)$

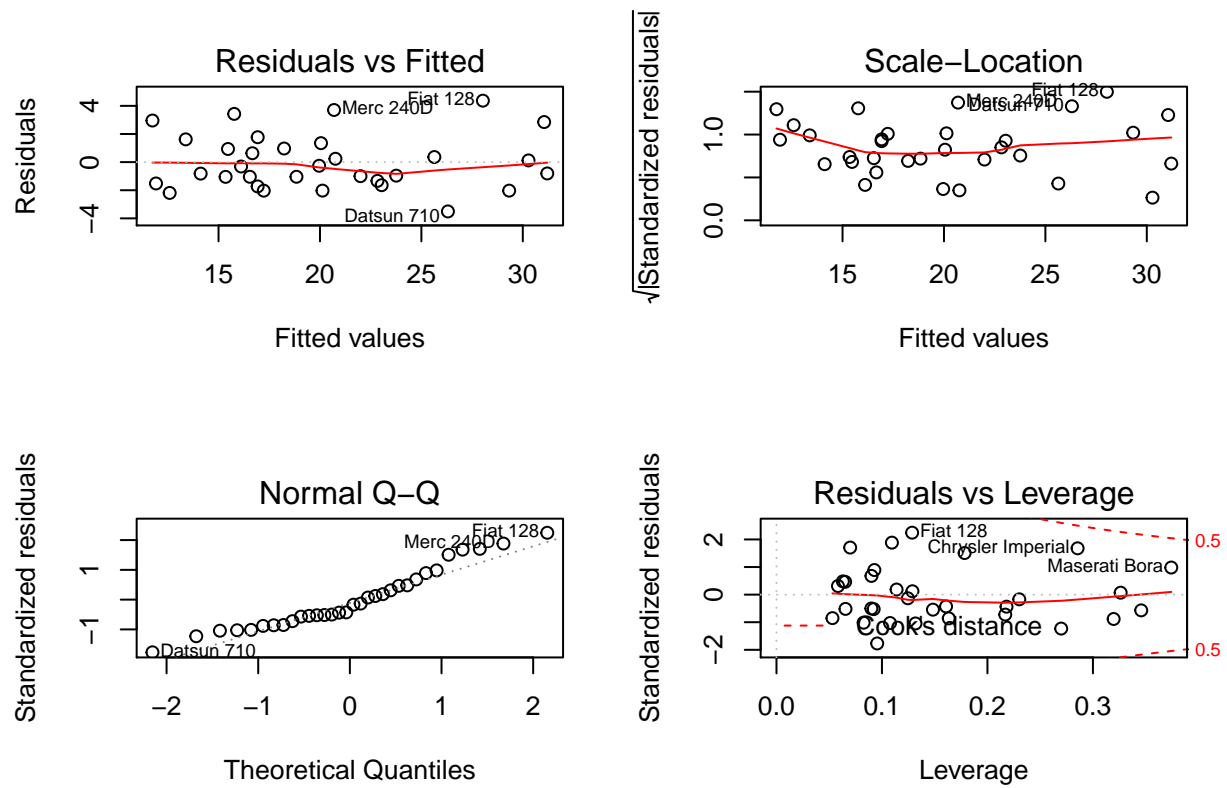


Figure 10: Diagnosis Plots for $\text{mpg} \sim (\text{wt} + \text{qsec}) * \text{factor}(\text{am})$