Geometry

From 3D to 2D

and

From 2D to 3D

Geometry

- The projection of 3D scenes into 2D images
- Recovering the 3D scene from 2D images
- Calibration
- Structure from motion
- Mathematical tools:
  - Algebra
  - Projective geometry

3D Reconstruction

- 3D shape of a scene:
  - Object recognition
  - Navigation
  - Graphics applications,...

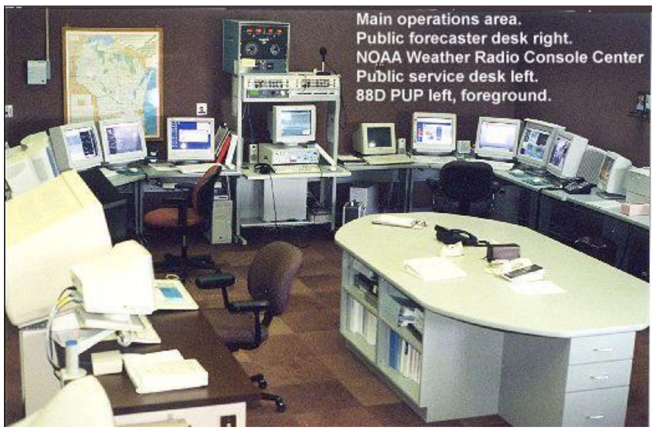


Image Formation

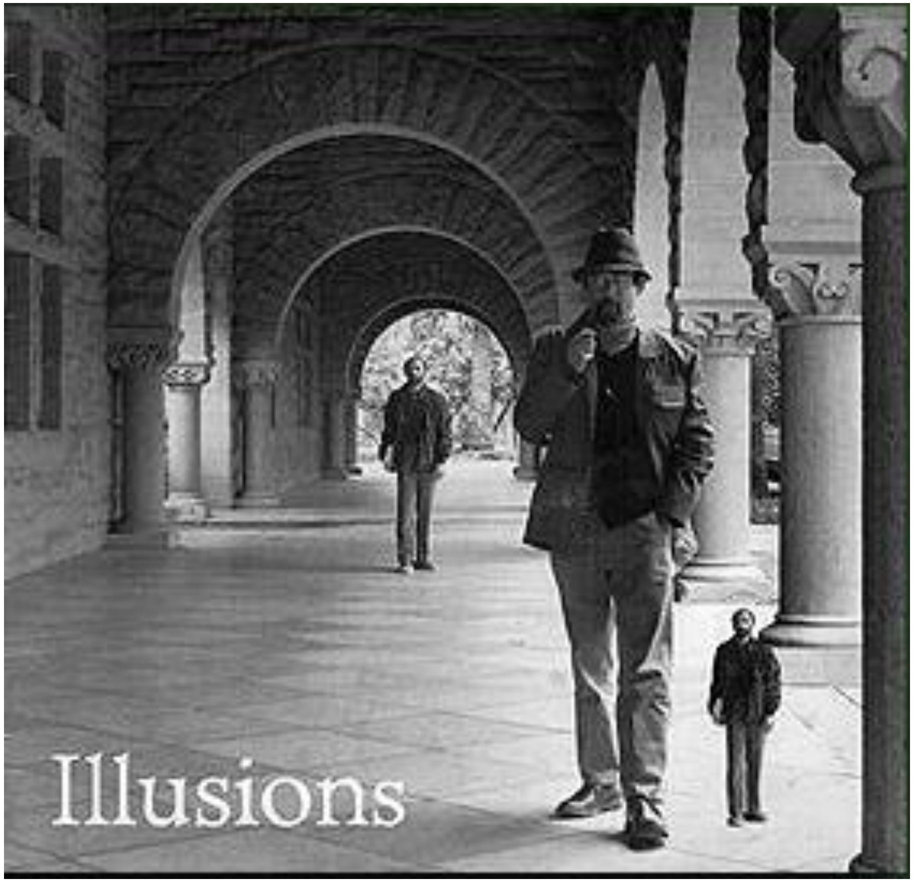
- Radiometric:
  The point color
- Geometric:
  The point location

Geometry:

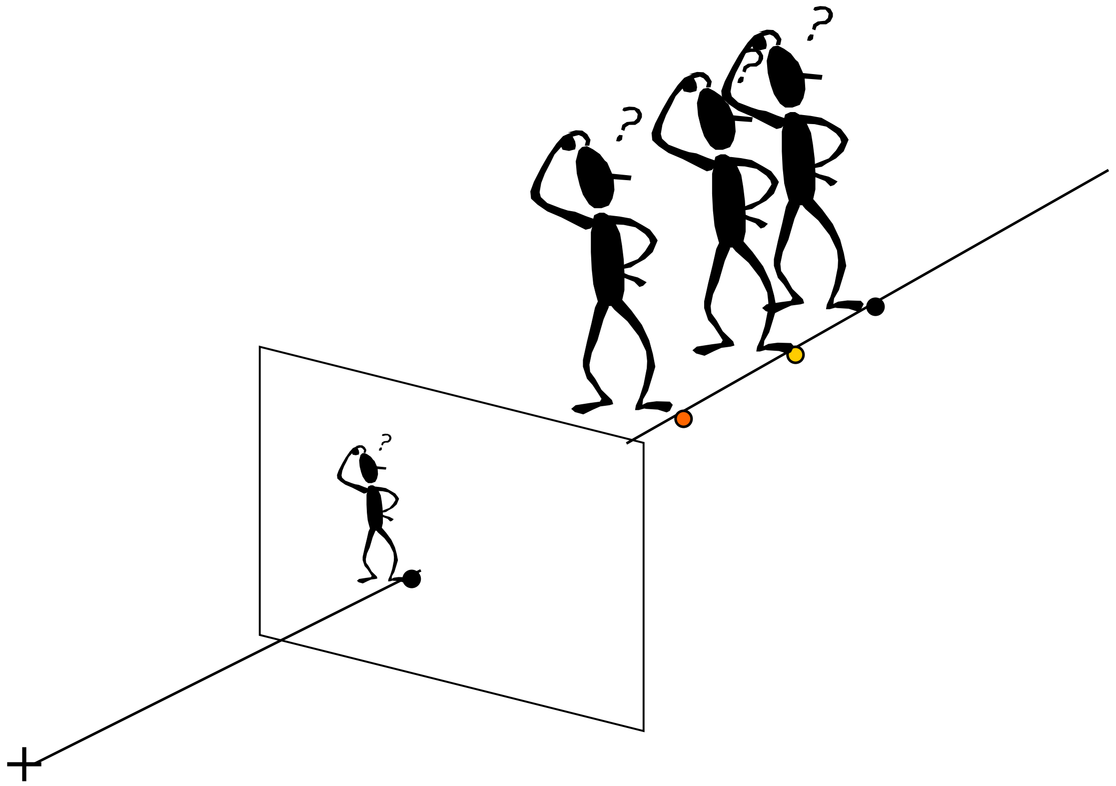
What determines the location of the projection of an object point in the image?

- The point location with respect to the camera
- The camera optics

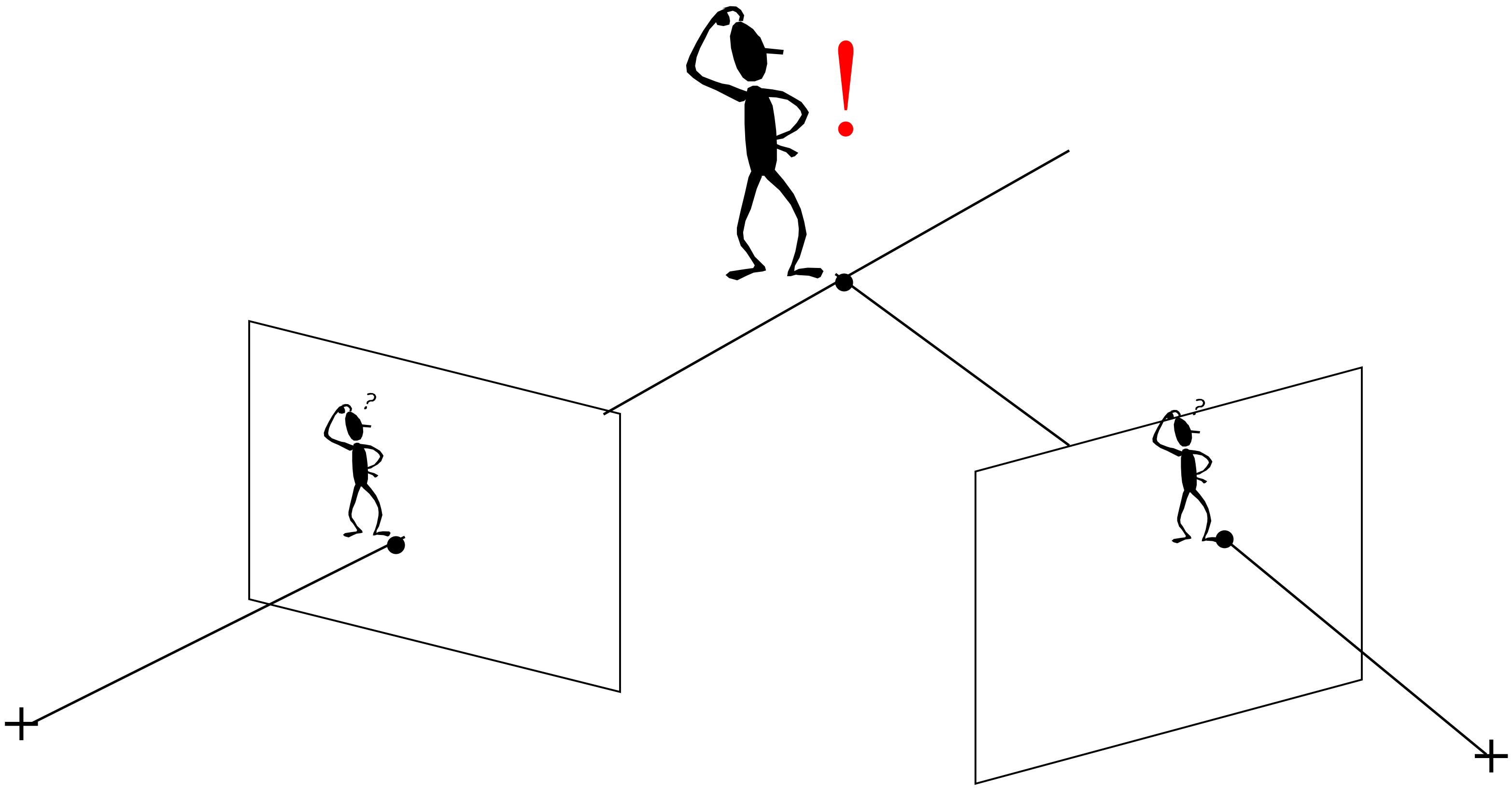
Ambiguity: 3D Shape from a Single Image:

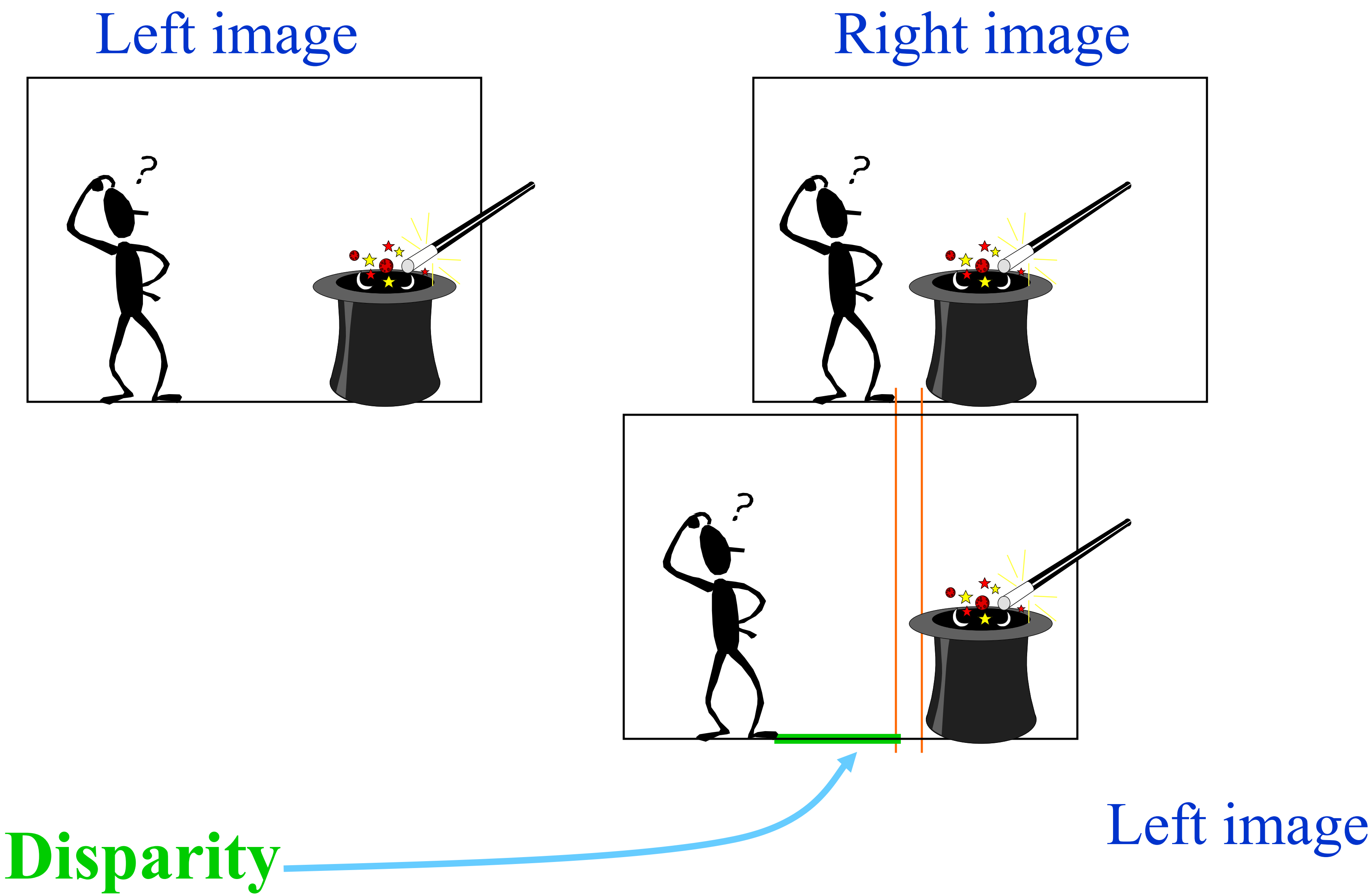


3D Shape from a Single Image



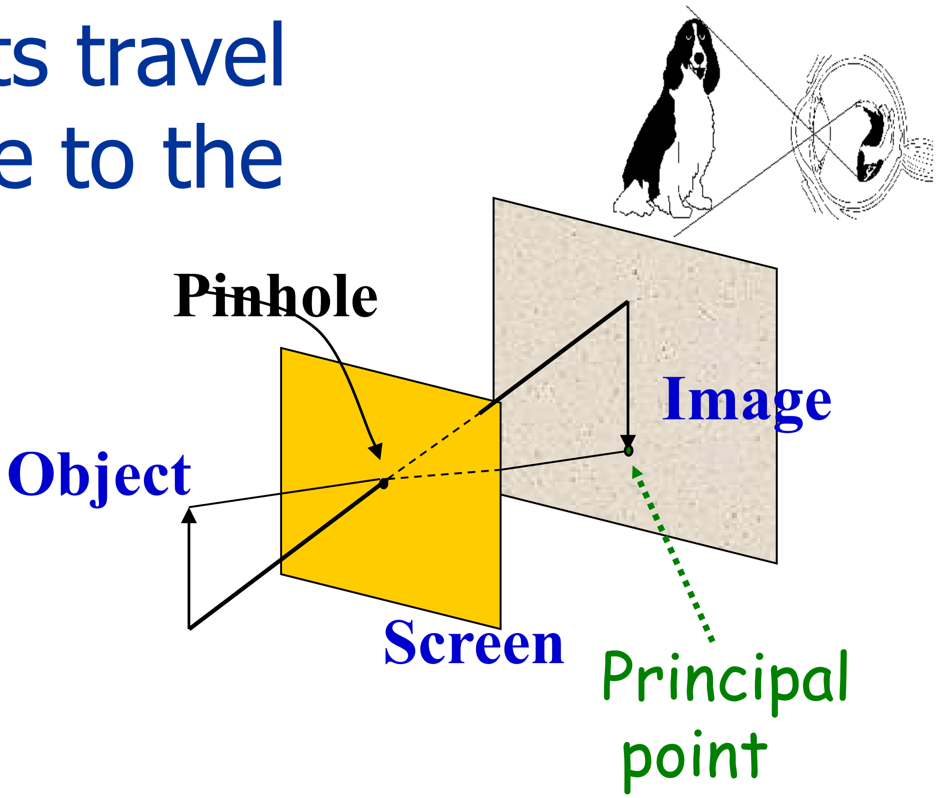
Two Images




Left image
Right image
Left image
Disparity

The Pinhole Camera

- Infinitesimally small aperture.
- Straight rays of lights travel through the aperture to the image plane
- Each scene point projects to a single image point



Pinhole Camera Images with Variable Aperture

2 mm

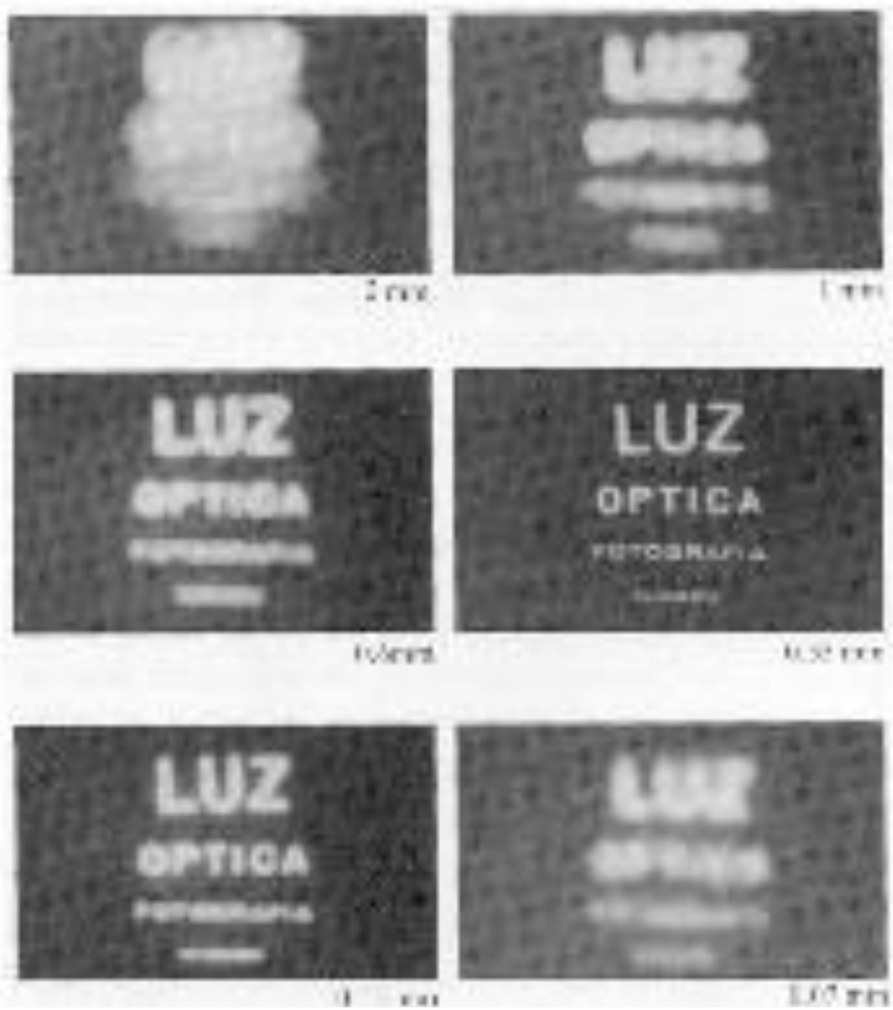
1mm

.6 mm

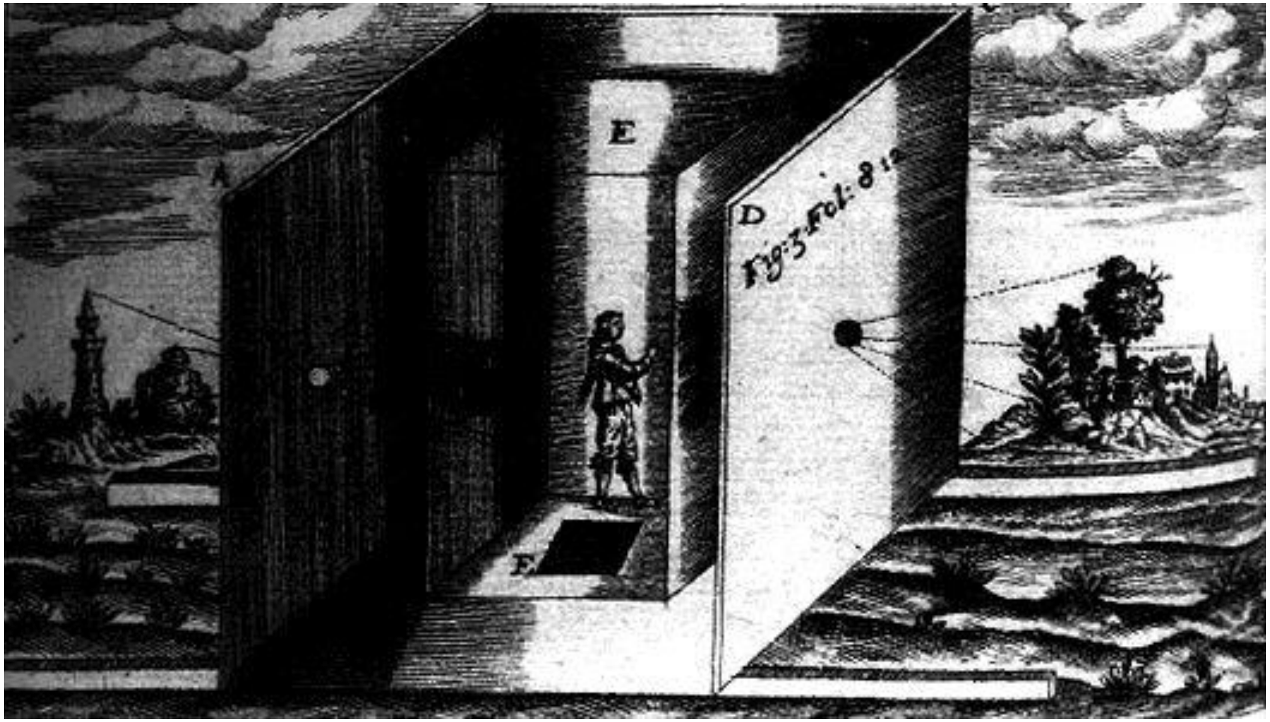
.35 mm

.15 mm

.07 mm



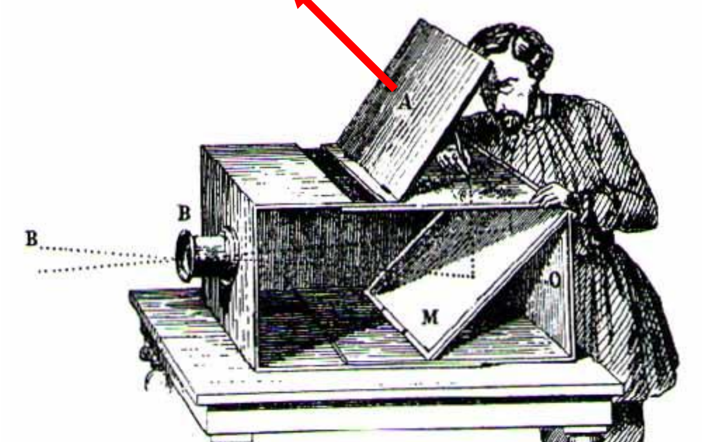
Camera Obscura



מצלמות

מצלמות של פעם

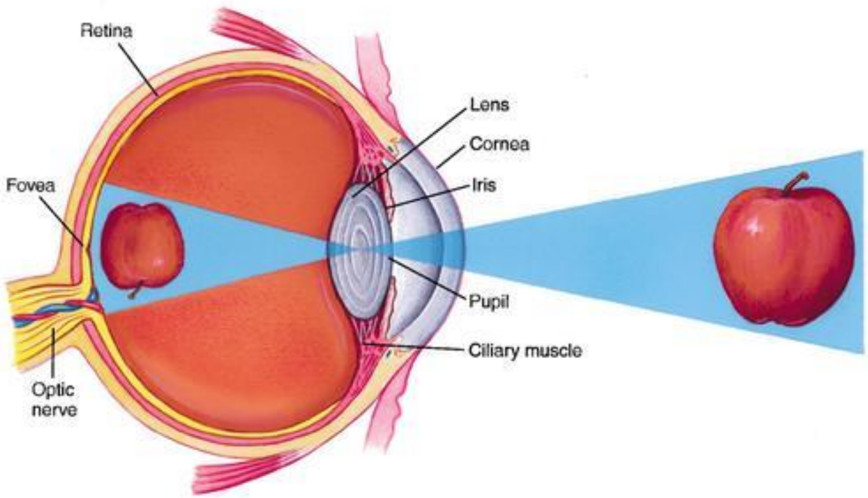
מצלמה דיגיטלית

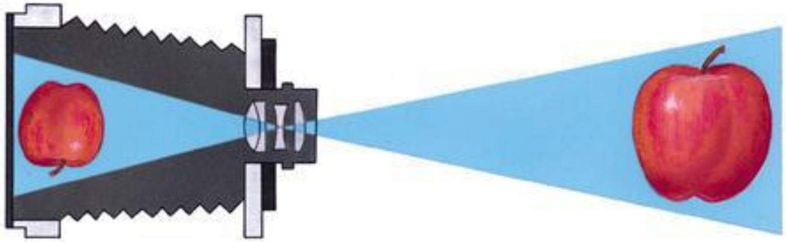


מצלמת רשת

קמרה אובסקורה

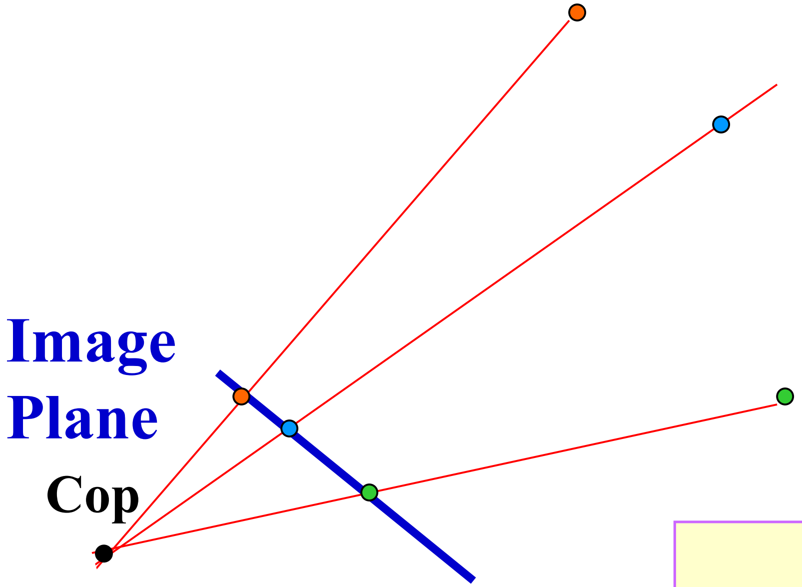
The Camera and the Eye

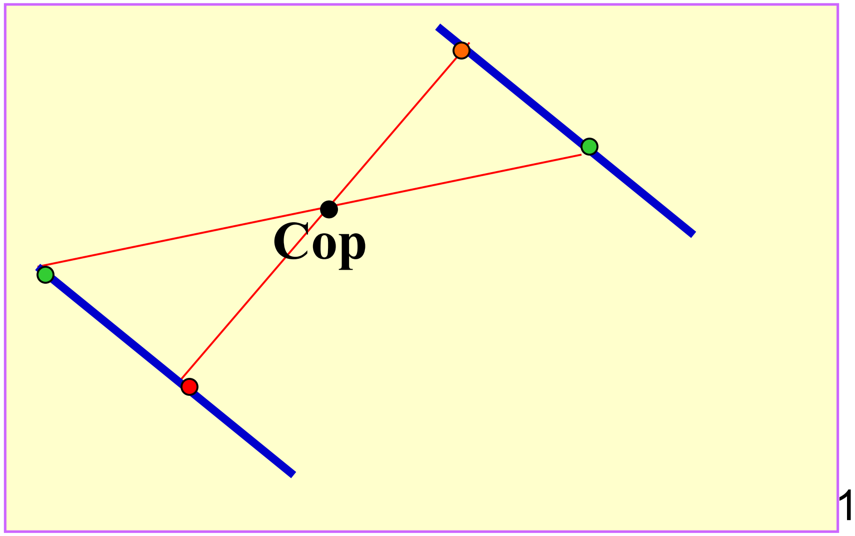




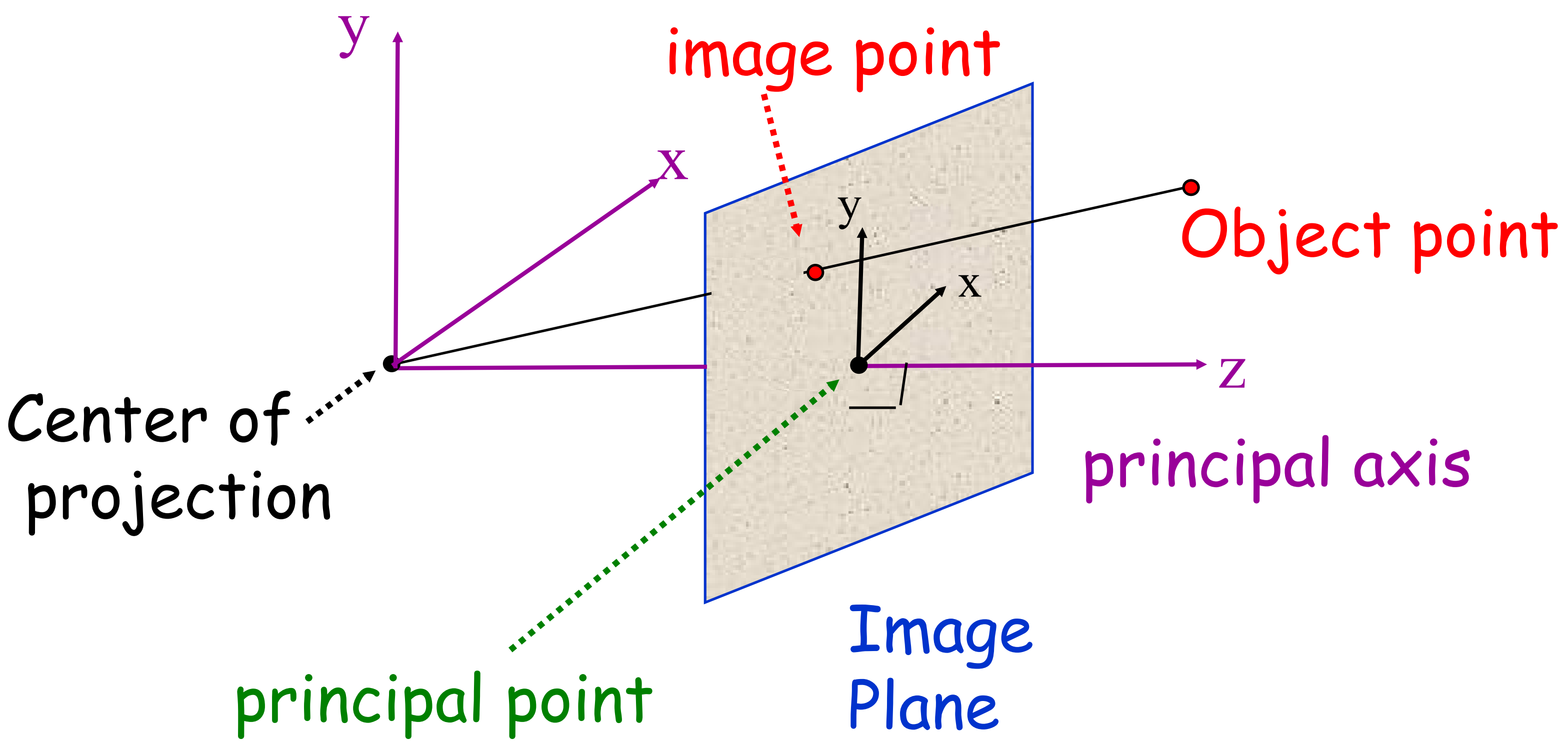
First photograph: 1829 J. Nicéphore Niépce

The Image Plane





Pinhole Camera Geometry

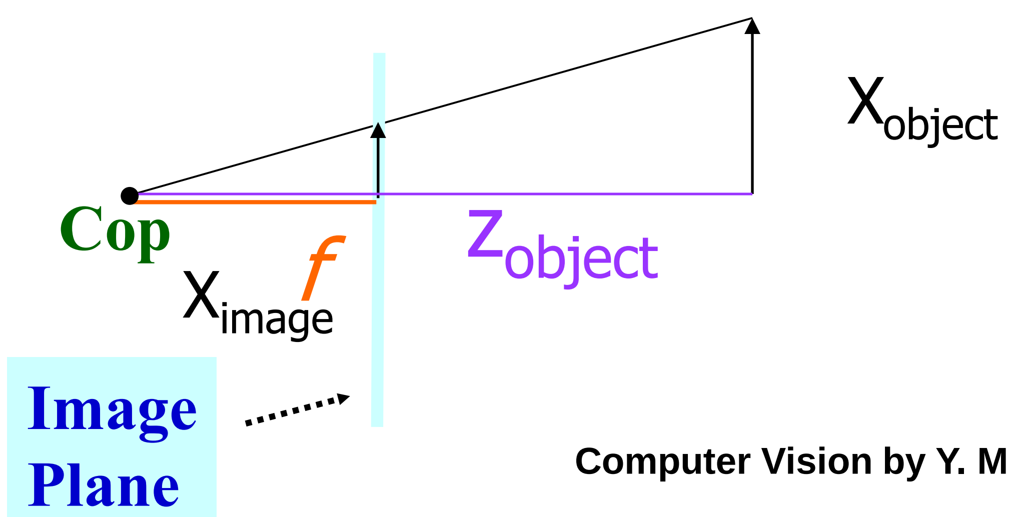


Perspective Projection

$$\frac{z_{object}}{x_{object}} = \frac{f}{x_{image}}$$

$$x_{image} = \frac{f}{z_{object}} x_{object}$$

f: the focal length



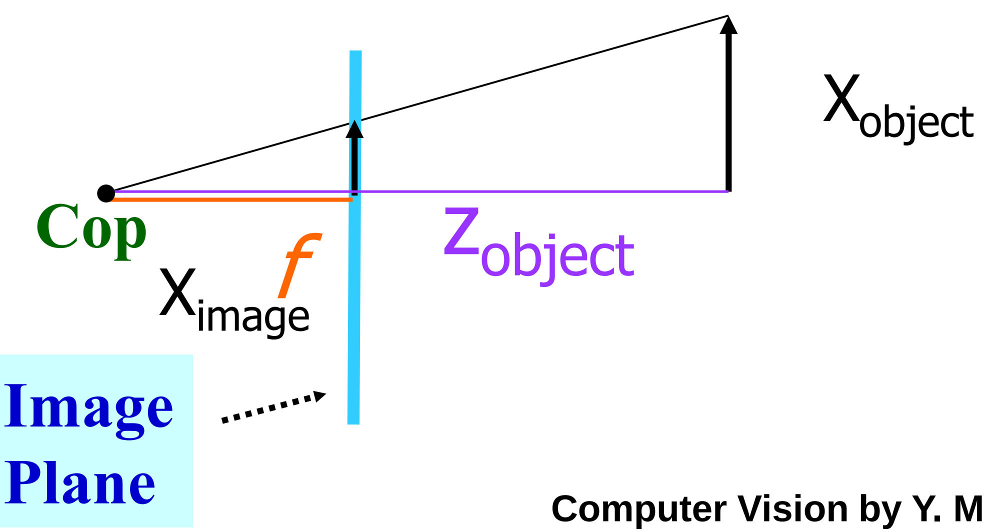
Perspective Projection

$$\frac{z_{object}}{x_{object}} = \frac{f}{x_{image}}$$

$$\frac{z_{object}}{y_{object}} = \frac{f}{y_{image}}$$

$$x_{image} = \frac{f}{z_{object}} x_{object}$$

$$y_{image} = \frac{f}{z_{object}} y_{object}$$

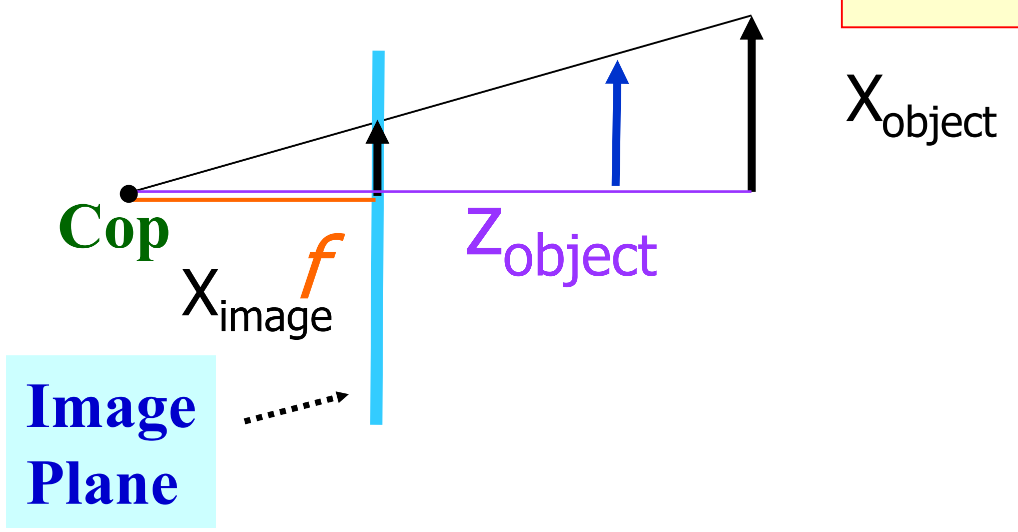


Perspective Projection

$$x_{image} = \frac{f}{z_{object}} x_{object}$$

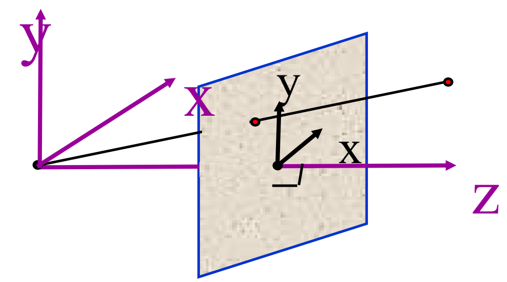
$$y_{image} = \frac{f}{z_{object}} y_{object}$$

No information about the distance



Pinhole Camera: Algebra

- Setting coordinate systems:
  - The camera coordinates system
  - The image coordinate system



- Euclidean projection:

$$x_{image} = \frac{f}{z_{object}} x_{object} \;;\; y_{image} = \frac{f}{z_{object}} y_{object}$$

- In world coordinate system (using projective algebra):

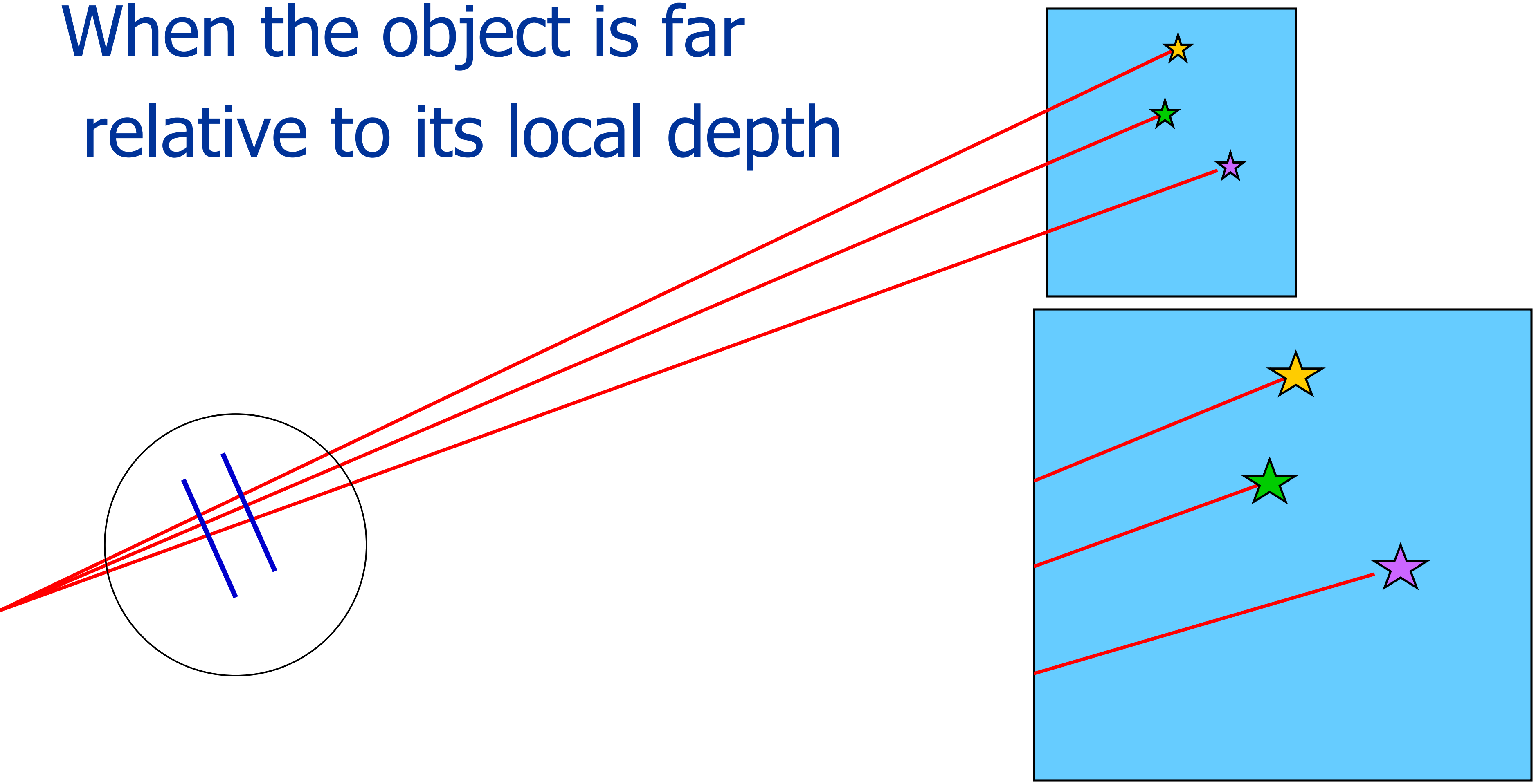
$$\tilde{p} = M\tilde{P}$$

Next class

where M is a $3x4$ matrix, $\tilde{p}$ and $\tilde{P}$

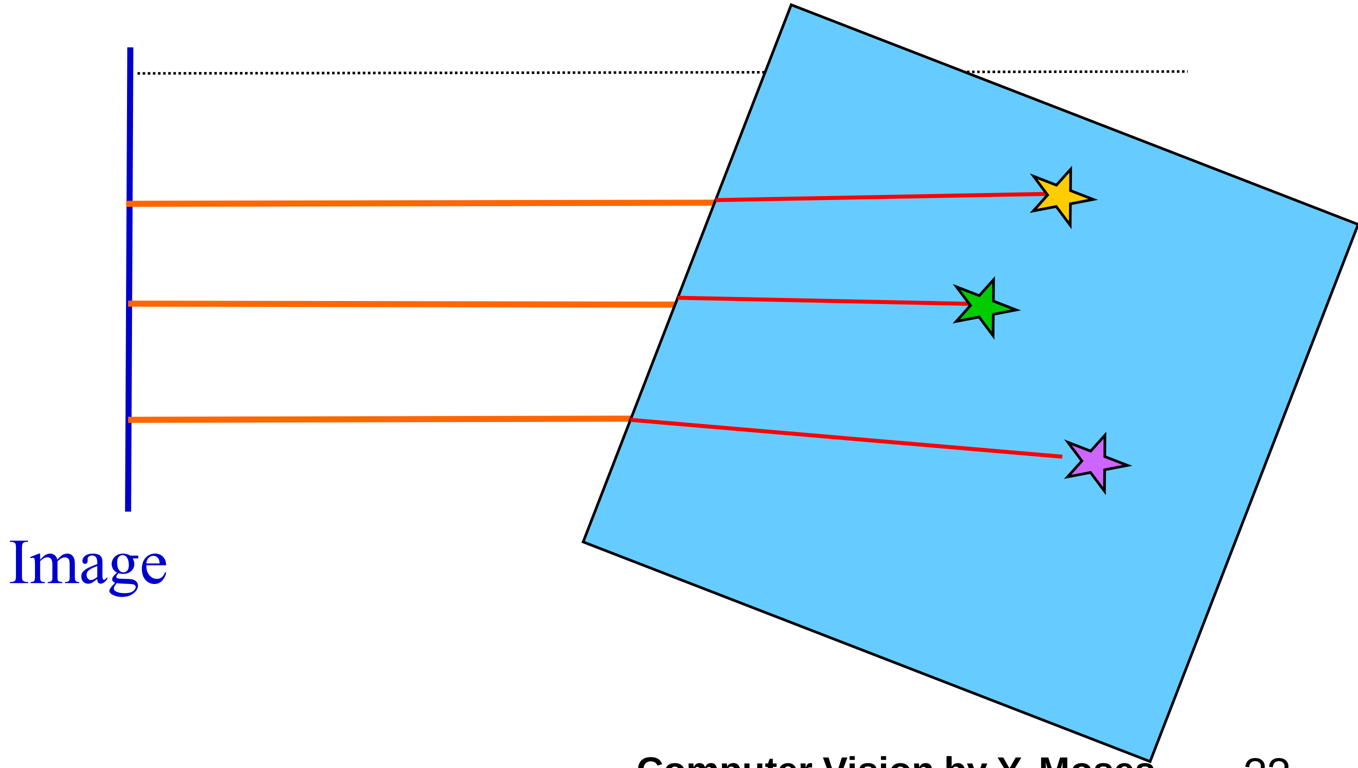
Orthographic:

When the object is far relative to its local depth



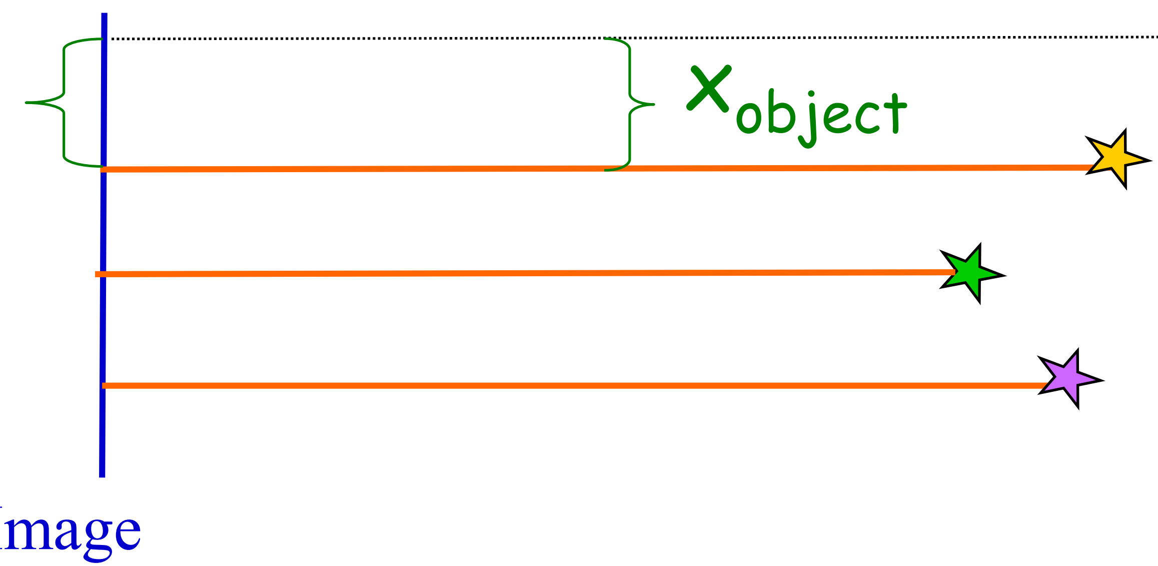
Orthographic:

When the object is far
relative to its local depth



Orthographic:

When the object is far
relative to its local depth



$x_{object}=x_{image}$

$y_{object}=y_{image}$

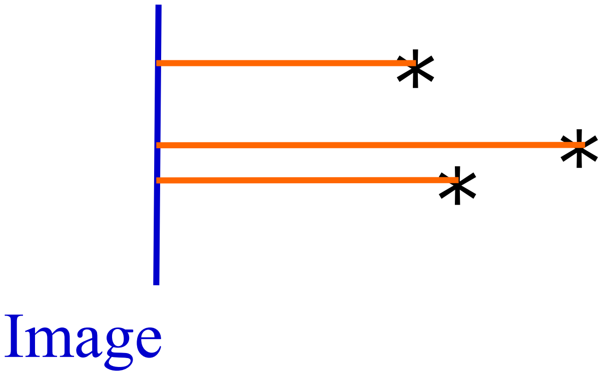
Paraperspective

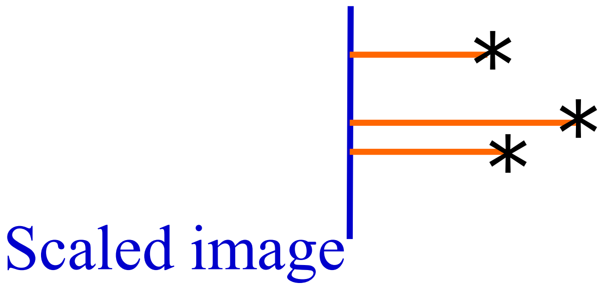
Orthographic + scaling s

$x_{object} = s x_{image}$

$y_{object} = s y_{image}$

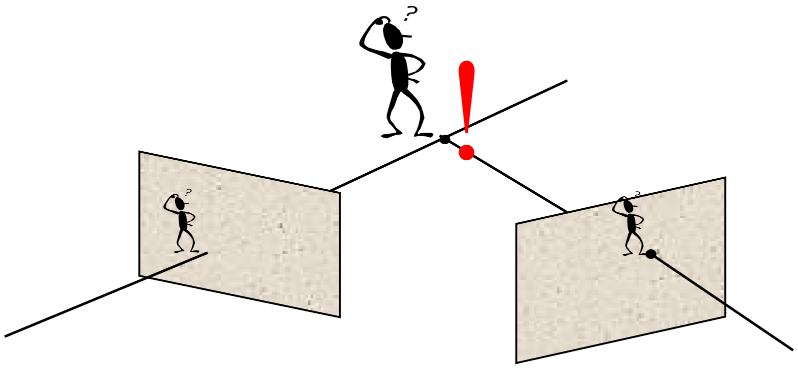
s is fixed for the whole image





Stereo Vision

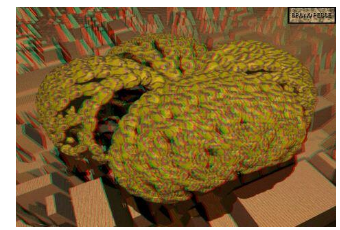
- Output: the 3D shape of the scene
- Input: two images
  from two viewpoints
- The human visual
  system uses stereo vision
- Many industrial applications

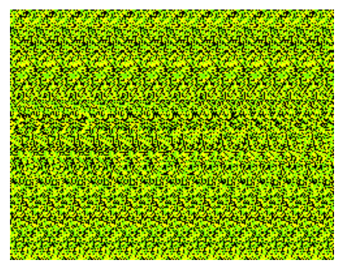


A Single Image Stereo Pair

Red/Green Image:

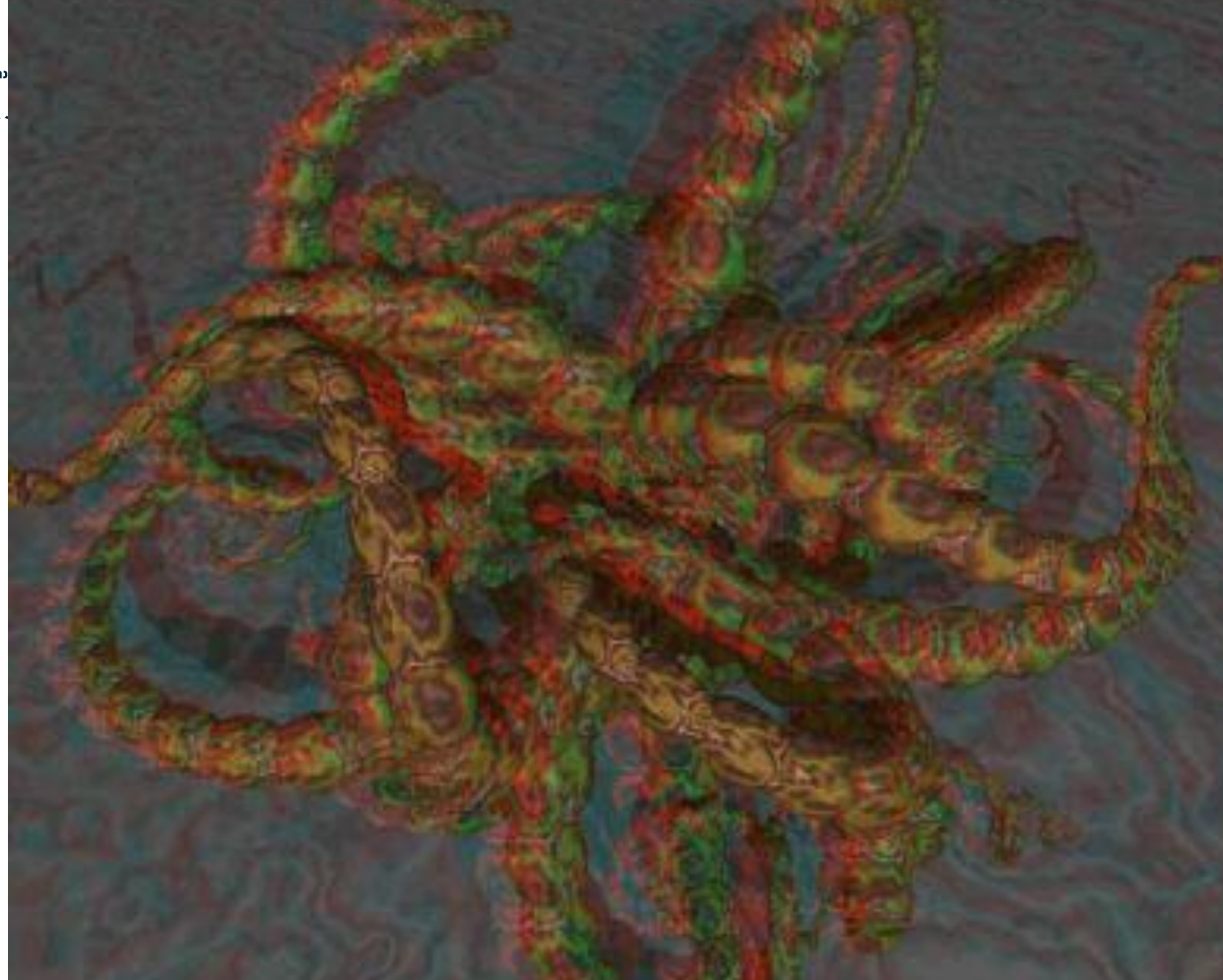
The images of the two eyes are separated by colors



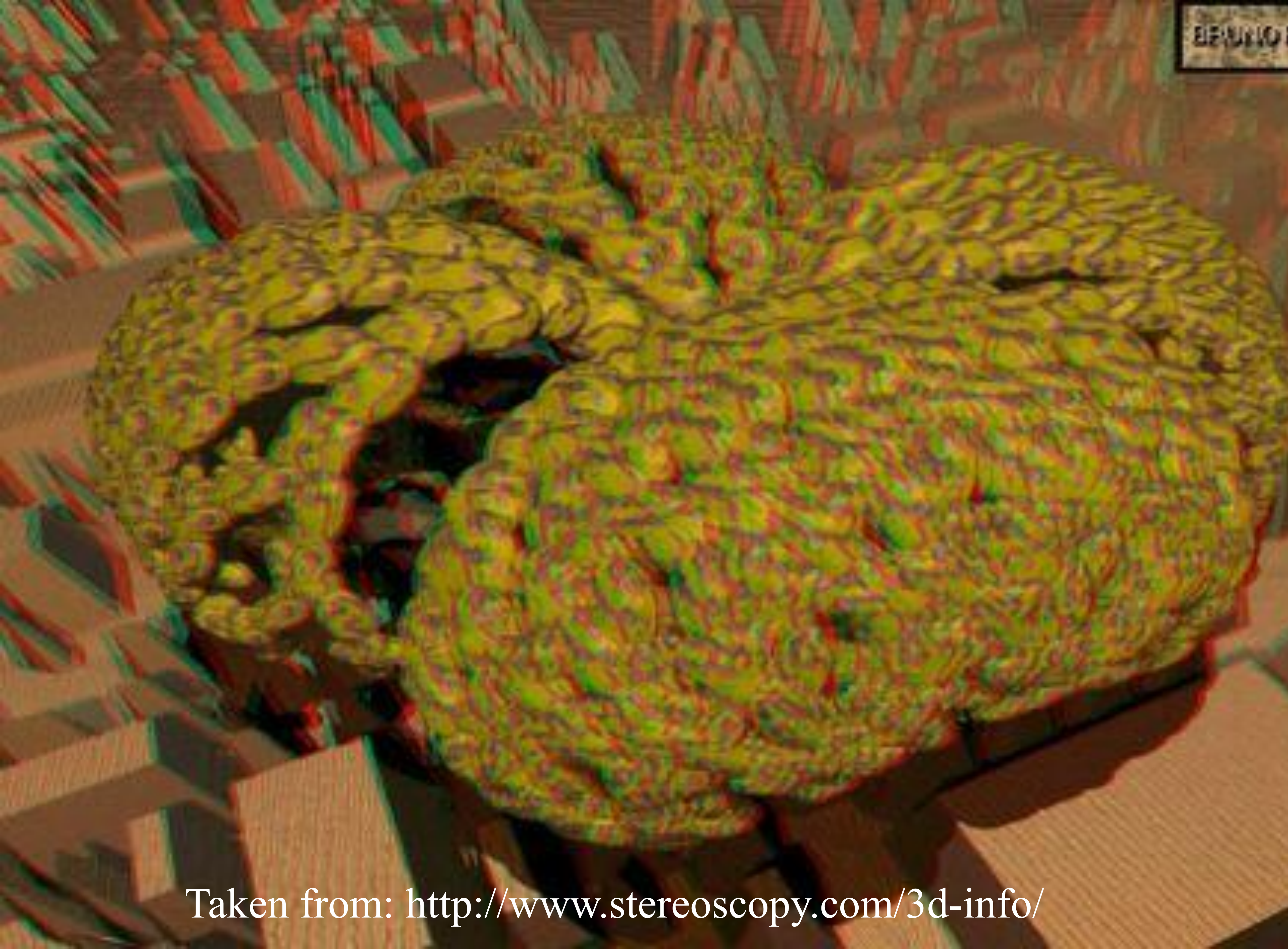


Magic Eye:

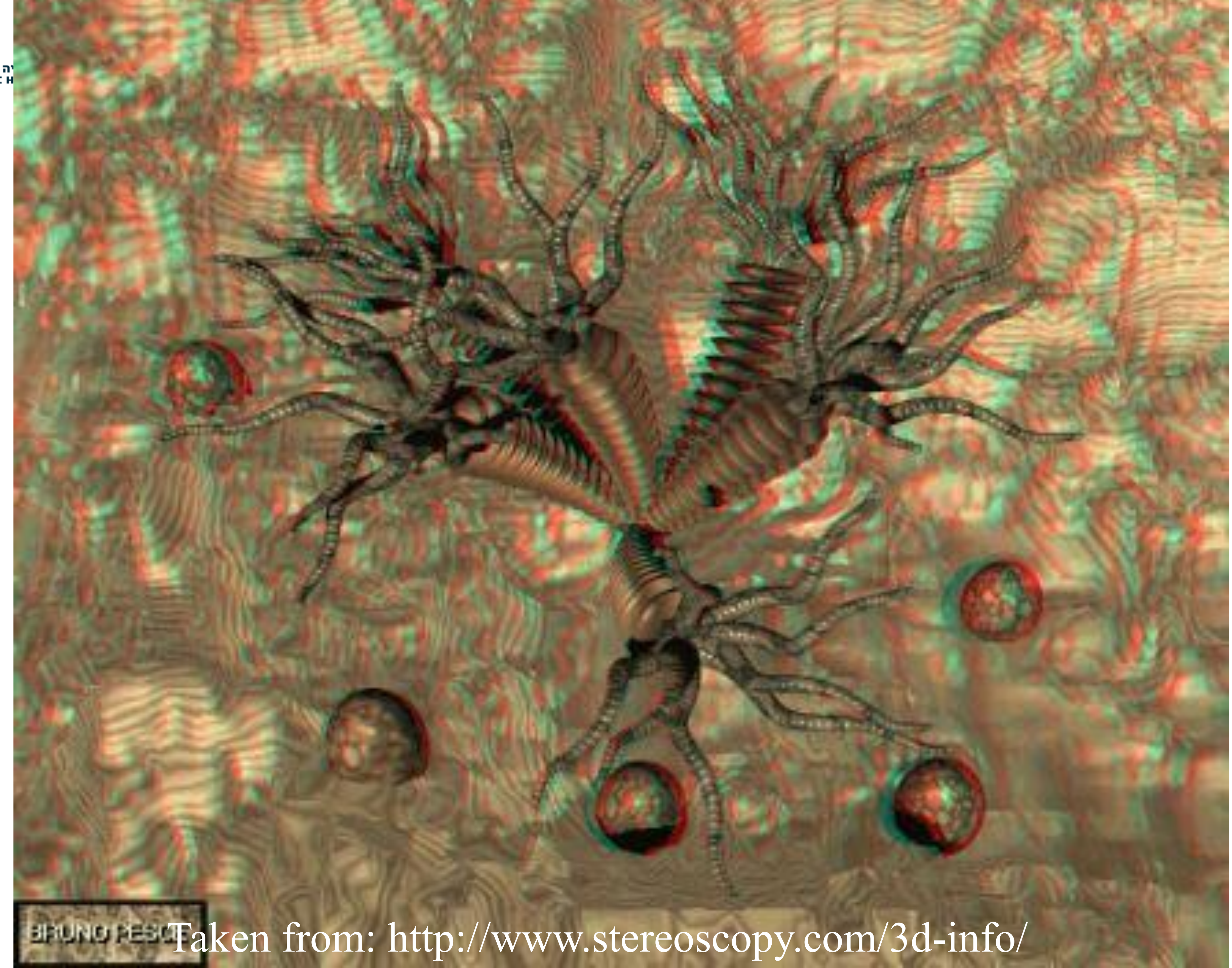
Using frequencies to separate the two eyes' images.



Taken from: http://www.stereoscopy.com/3d-info/



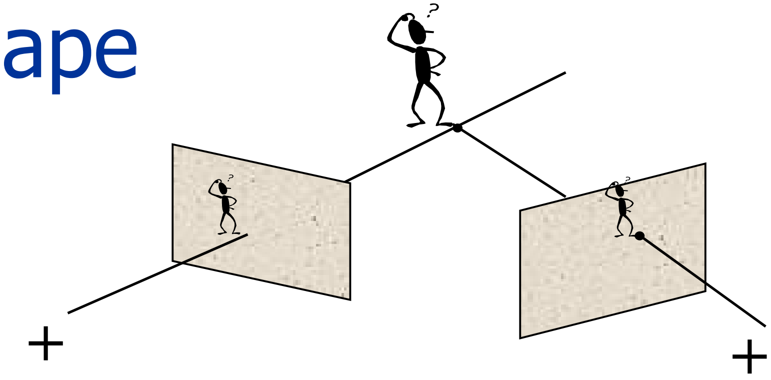
Taken from: http://www.stereoscopy.com/3d-info/



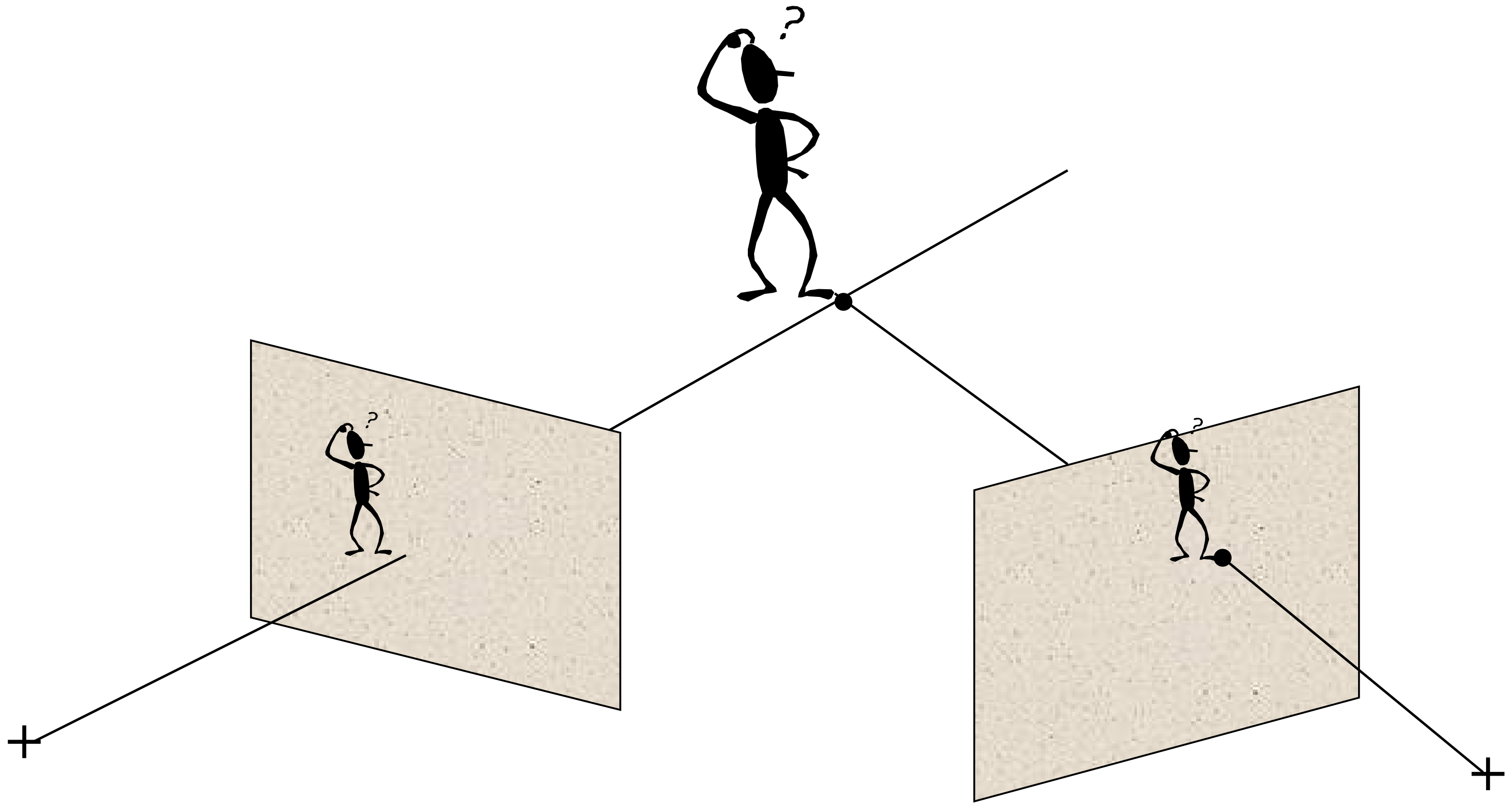
Taken from: http://www.stereoscopy.com/3d-info/

Stereo: Main Issues

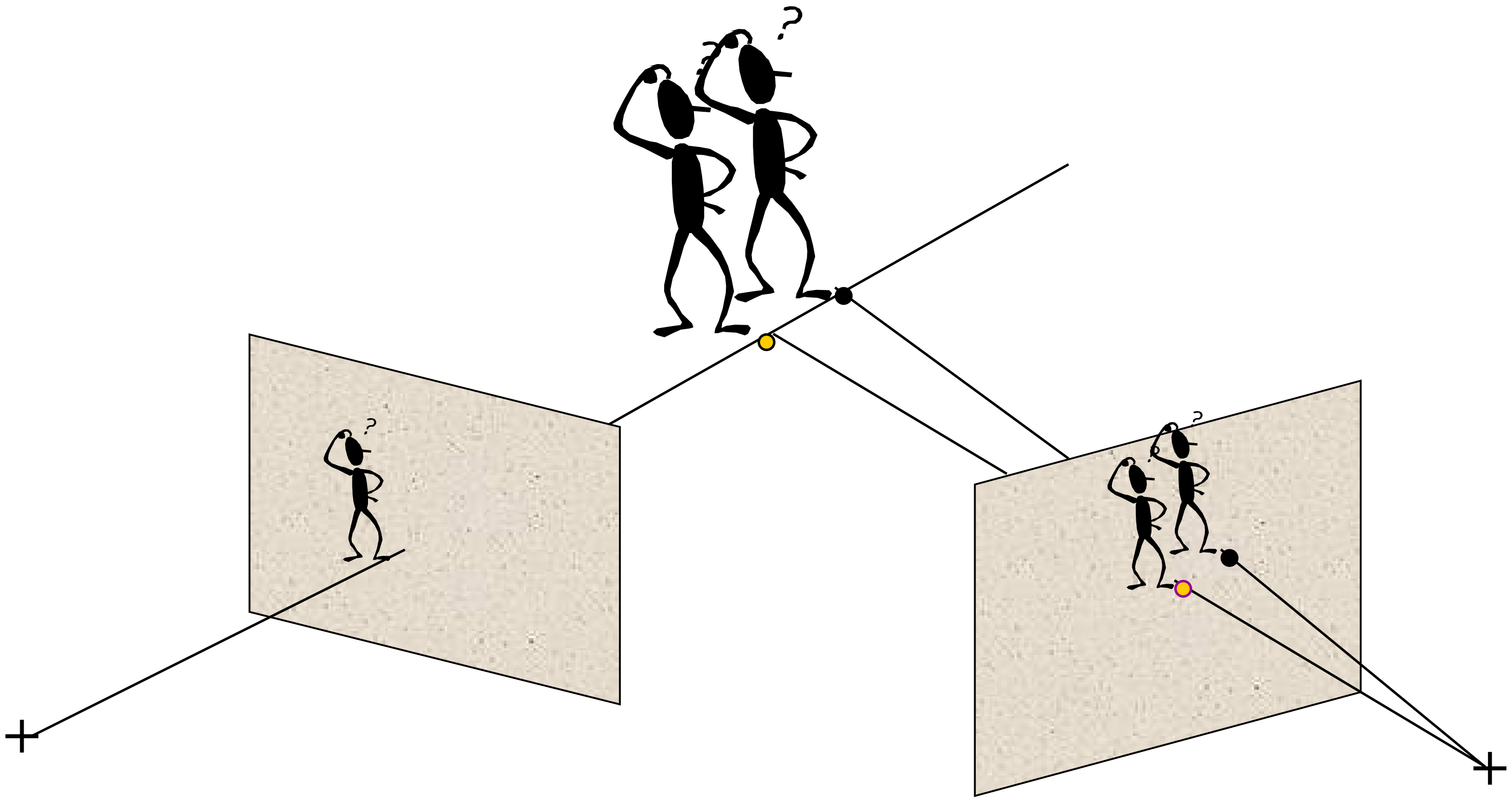
- Determine the point correspondence between the two images
- Based on point correspondence and the calibrated cameras: reconstruct the 3D geometry of the shape
- Camera calibration



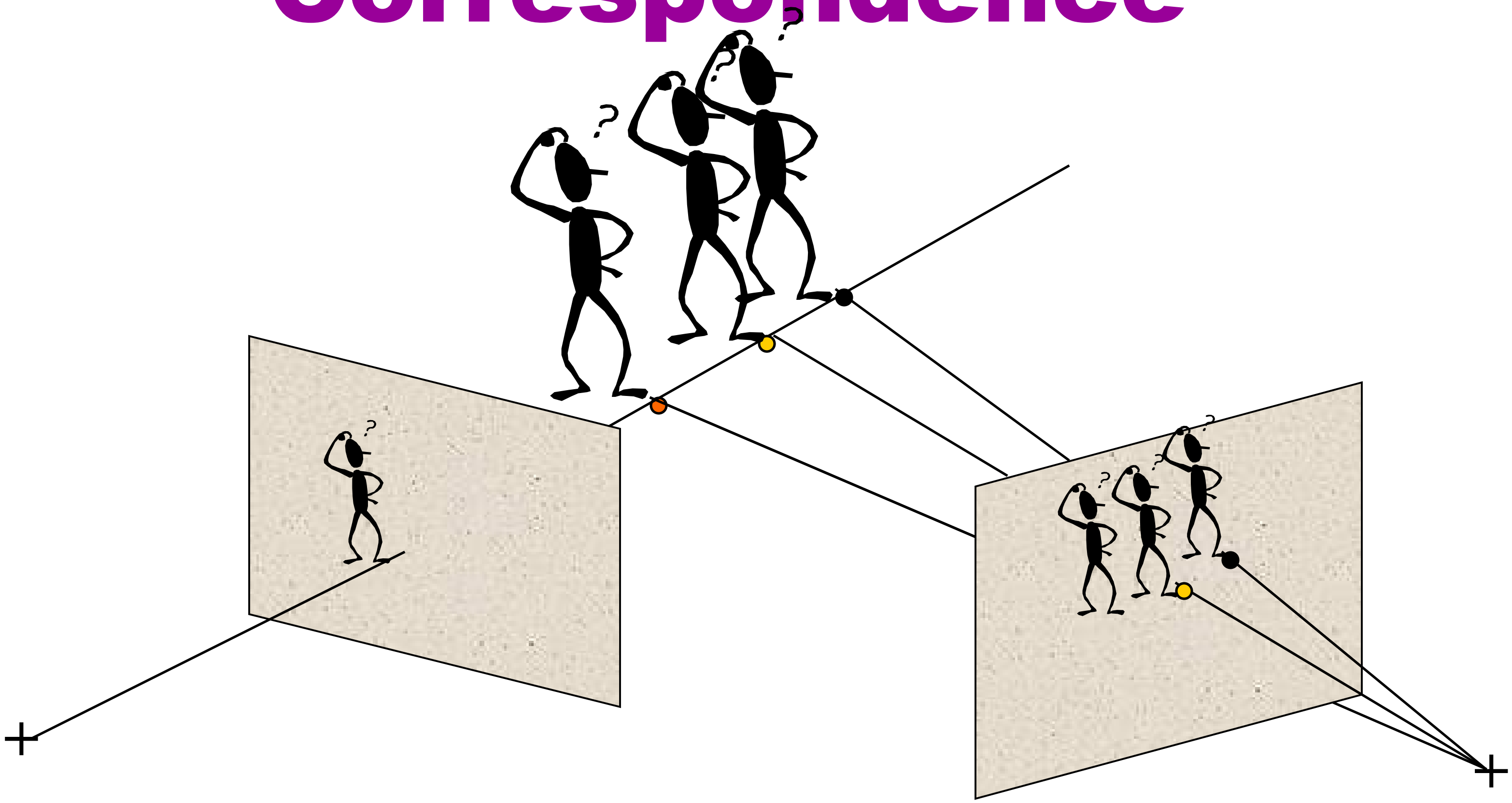
Correspondence



Correspondence

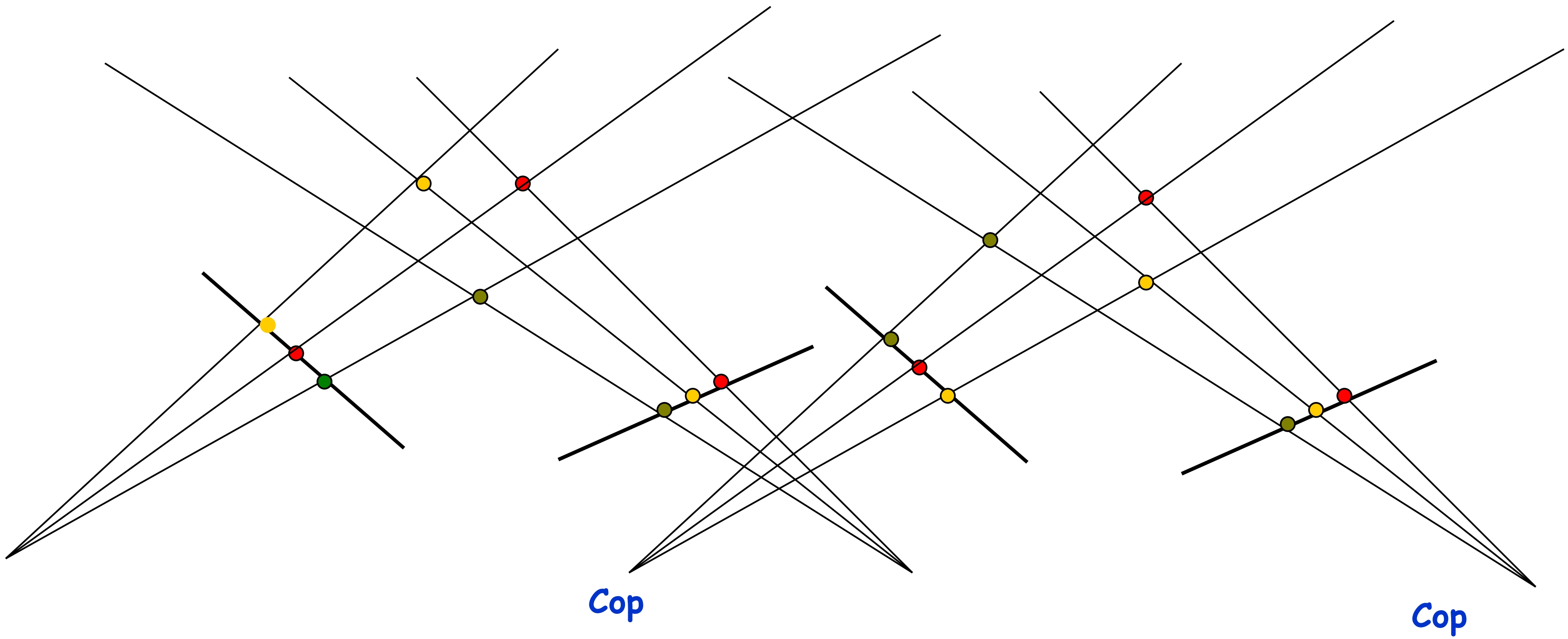


Correspondence

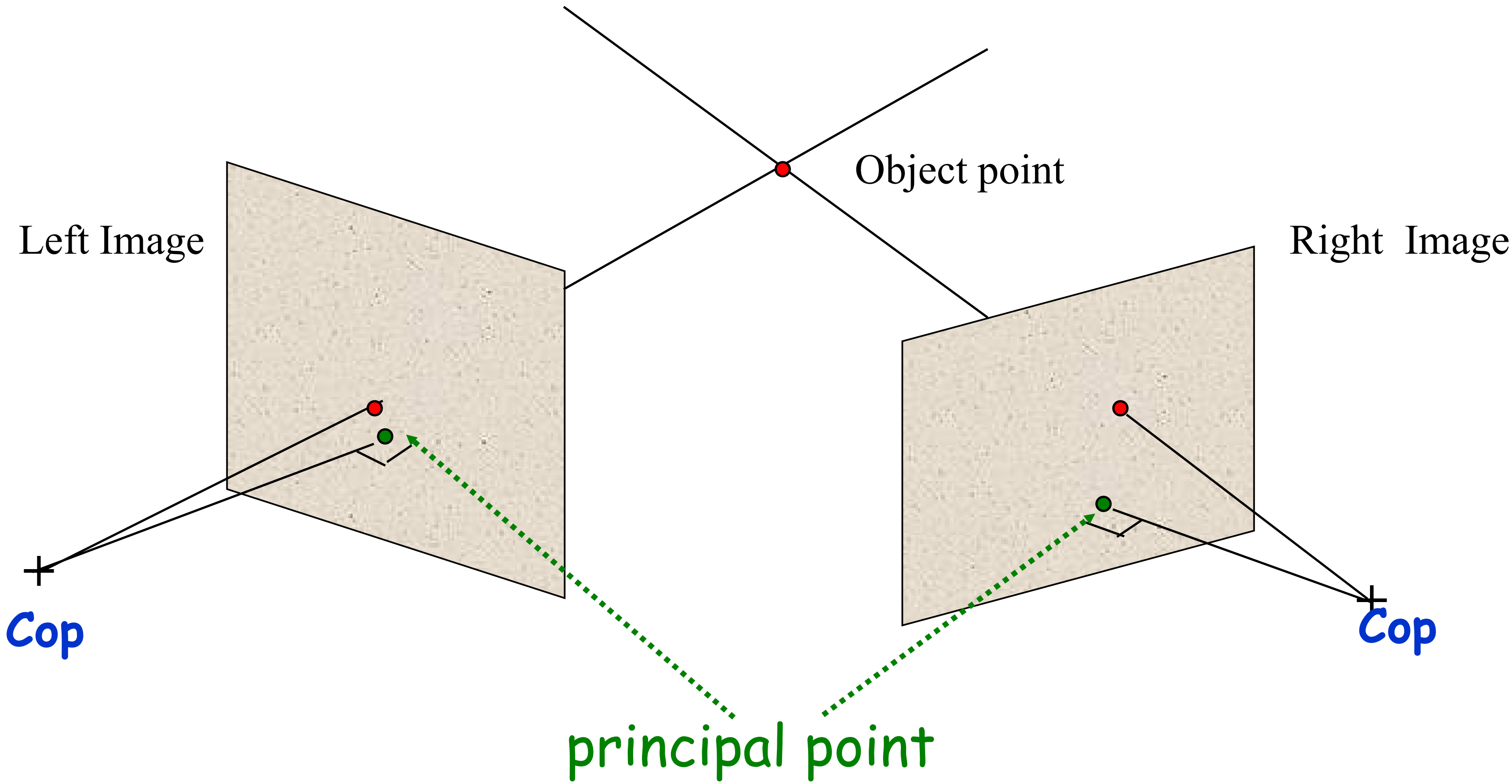


Correspondence

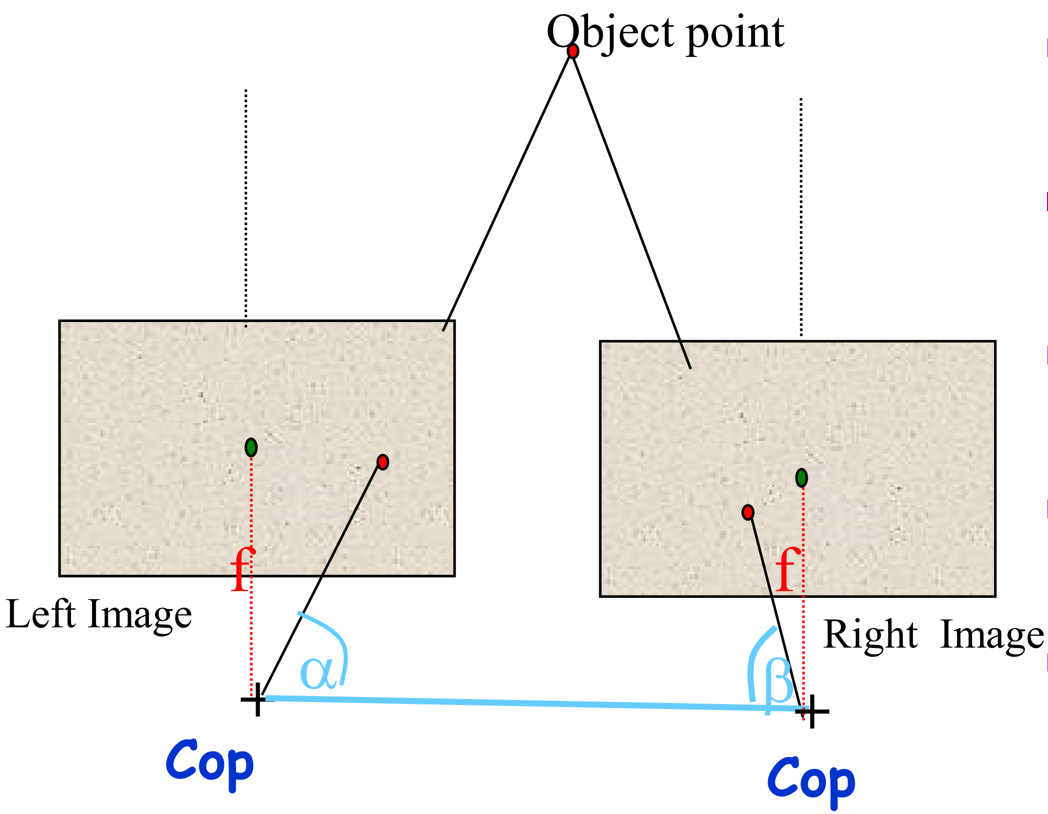
Ambiguous correspondence occurs often



Triangulation



A Simple Stereo System



- The two camera planes are coplanar
- The two optical axes are parallel
- The two focal lengths are identical
- The fixation point lies at infinity
- The correspondence is given

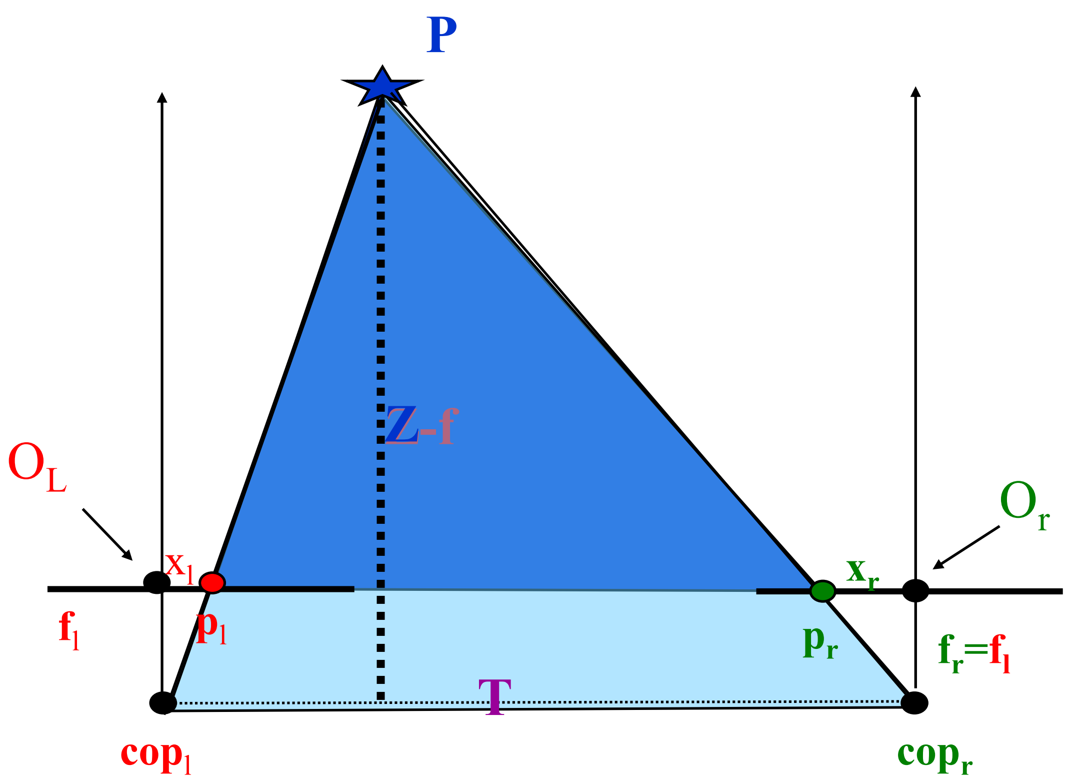
Computing the Depth from Disparity

$$\frac{T + x_r - x_l}{z - f} = \frac{T}{z}$$

The disparity:

$d = x_l - x_r$

$$z = f\,\frac{T}{d}$$

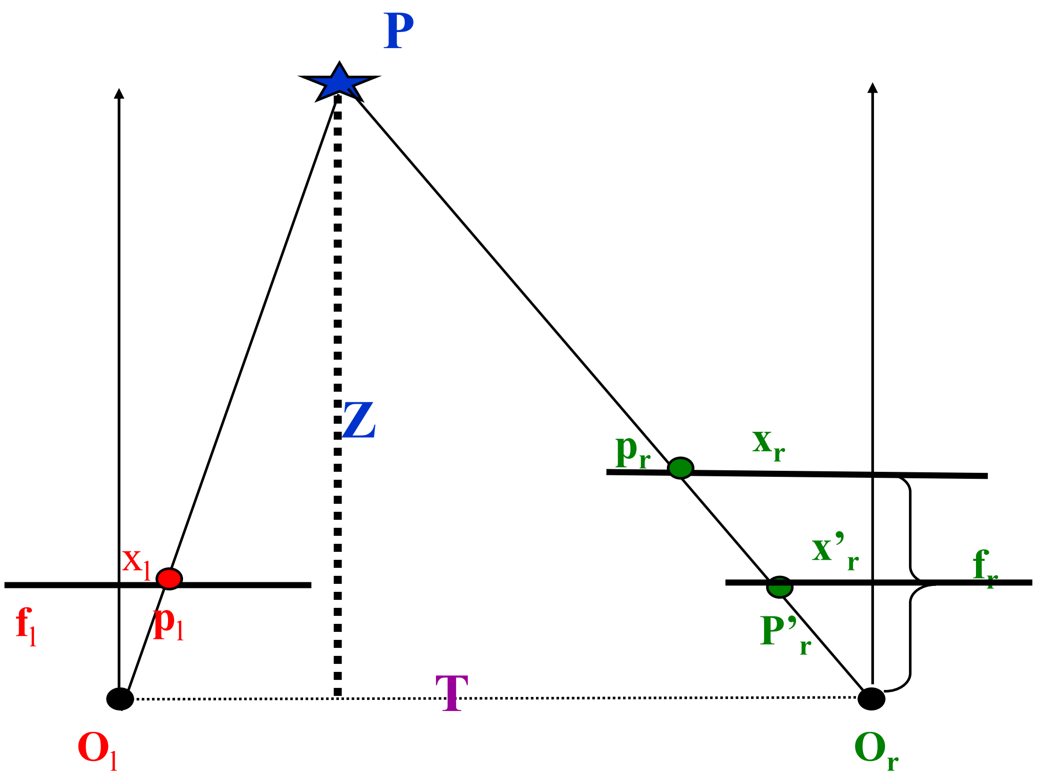


Different focal length

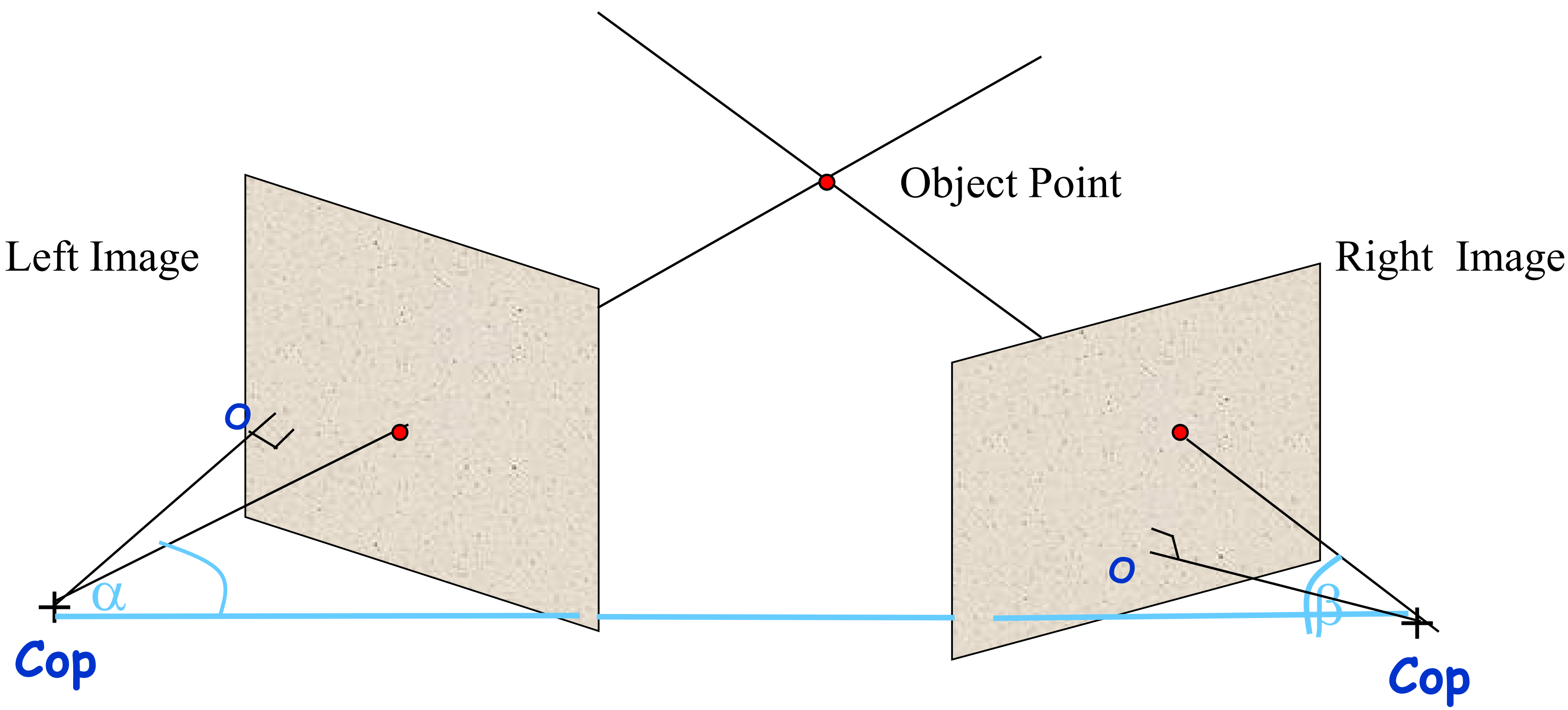
The disparity:

$$d = x'_l - x_r$$

$$z = f\,\frac{T}{d}$$

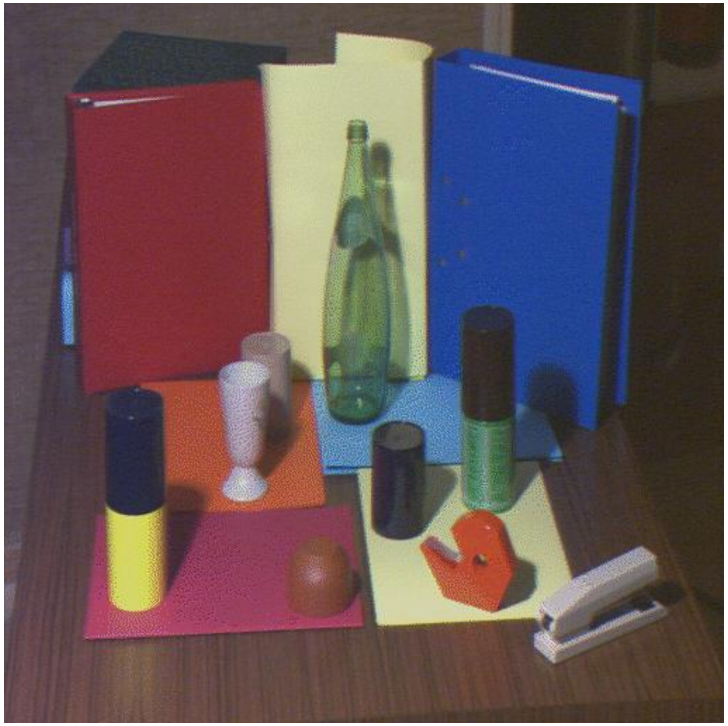


General Case Triangulation

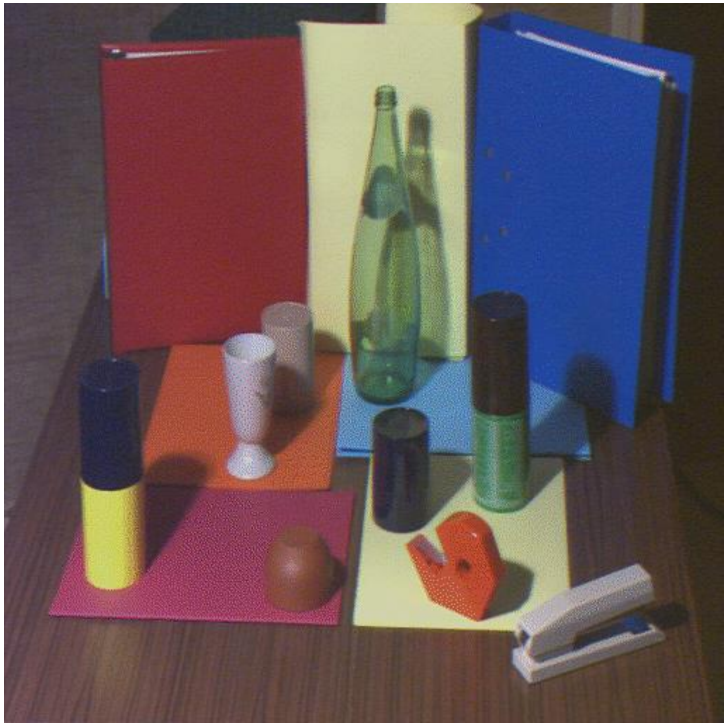


Possible to use trigonometry but next class we will use projective geometry

A Stereo Pair

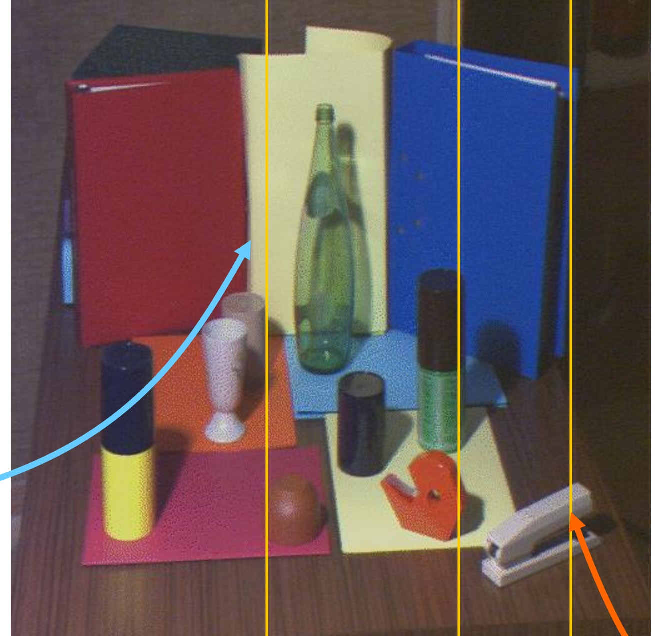


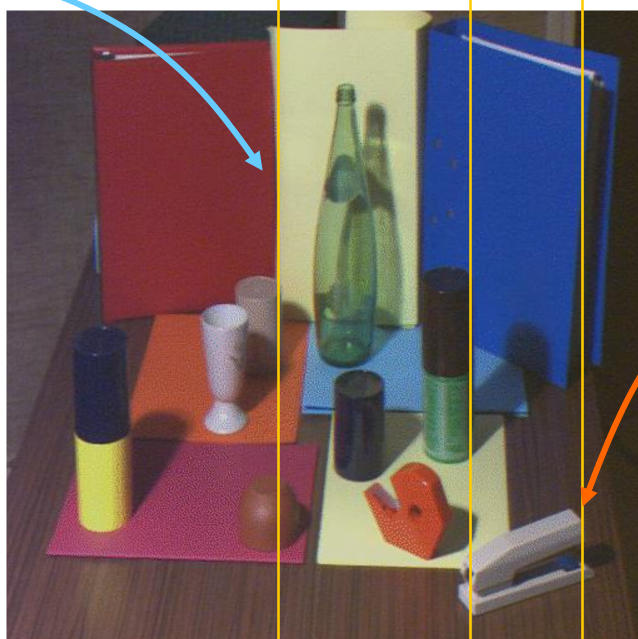
Left image



Right image

Left image





Right image

Disparity:

change of location between left and right images.

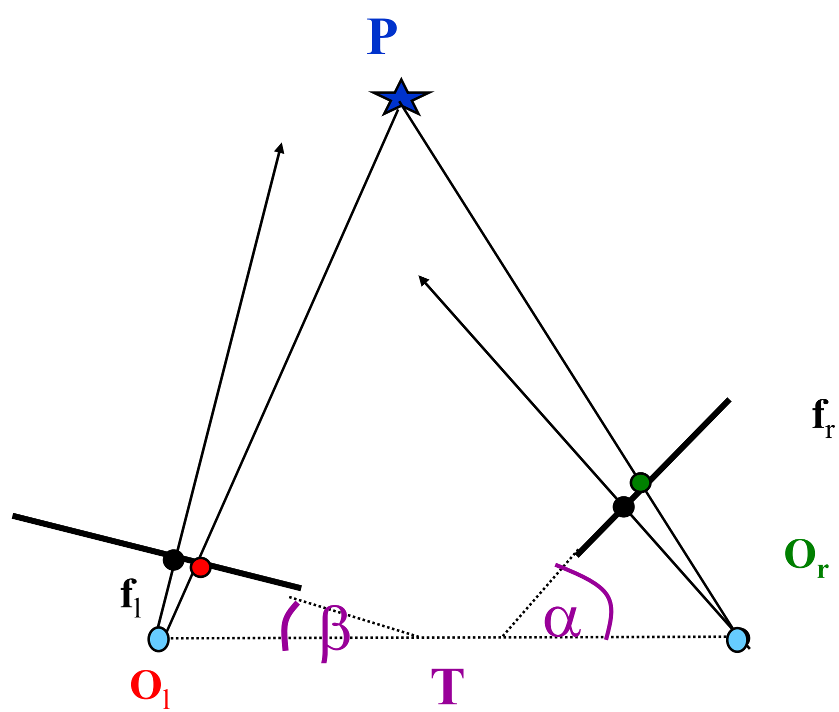
The Parameters of a Stereo System

Intrinsic parameters:

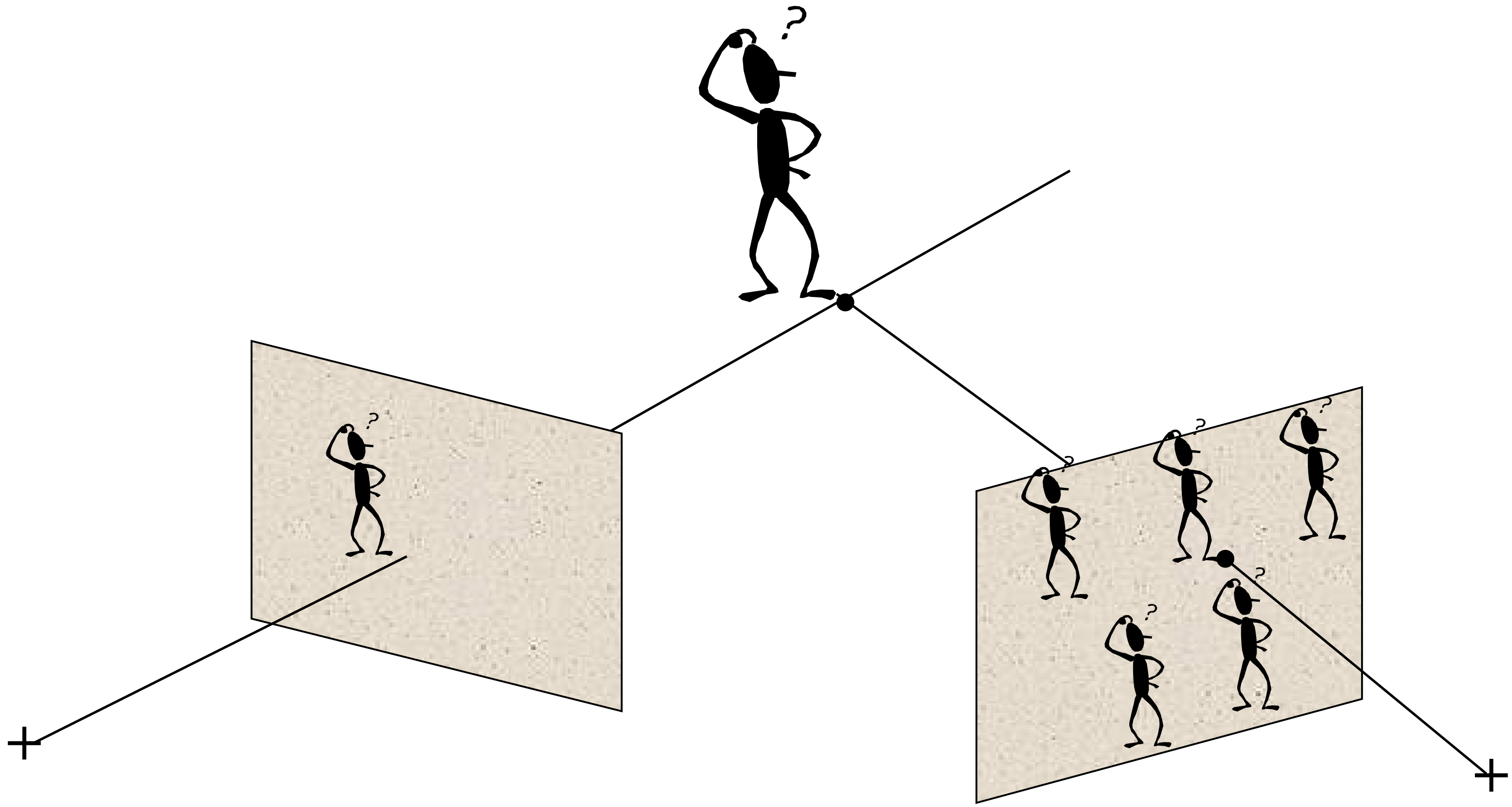
- Focal length
- Principal point
- Scaling

Extrinsic parameters

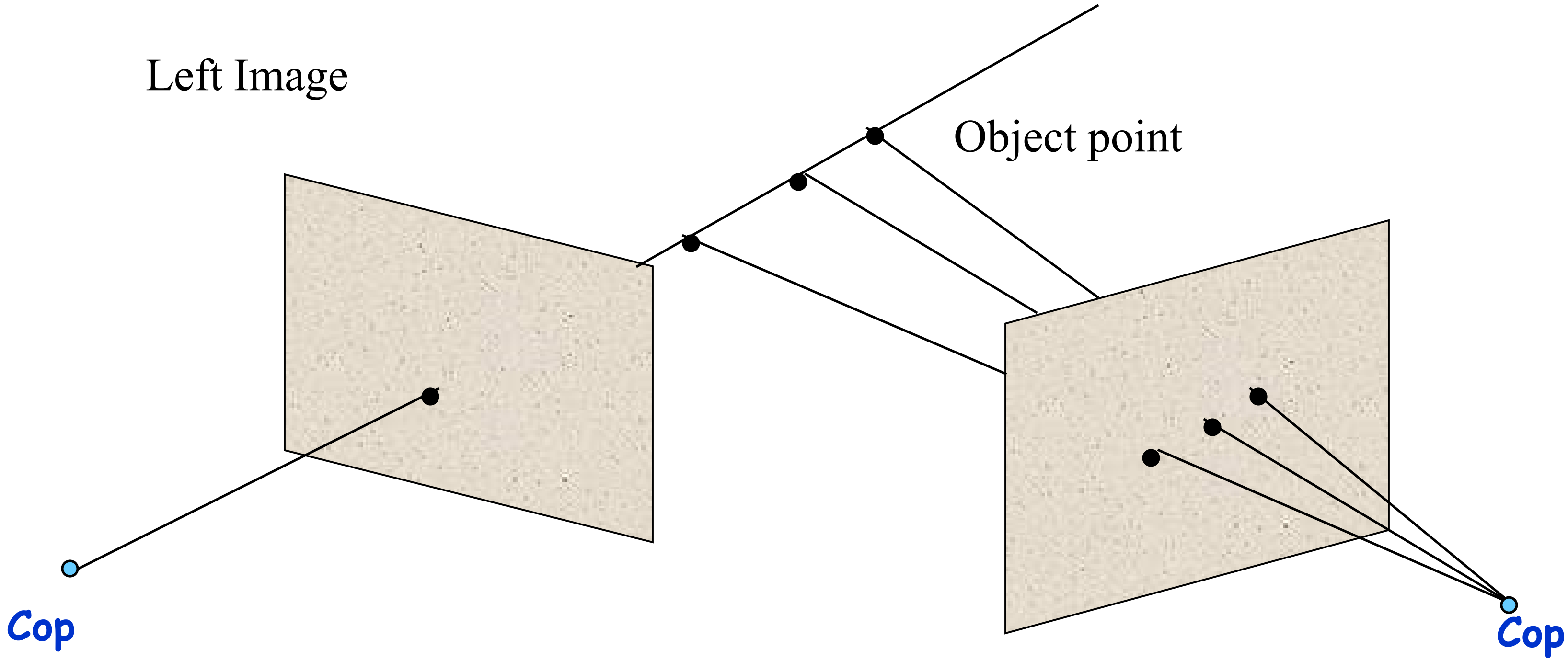
- The relative position and orientation of the two cameras



Back to Correspondence

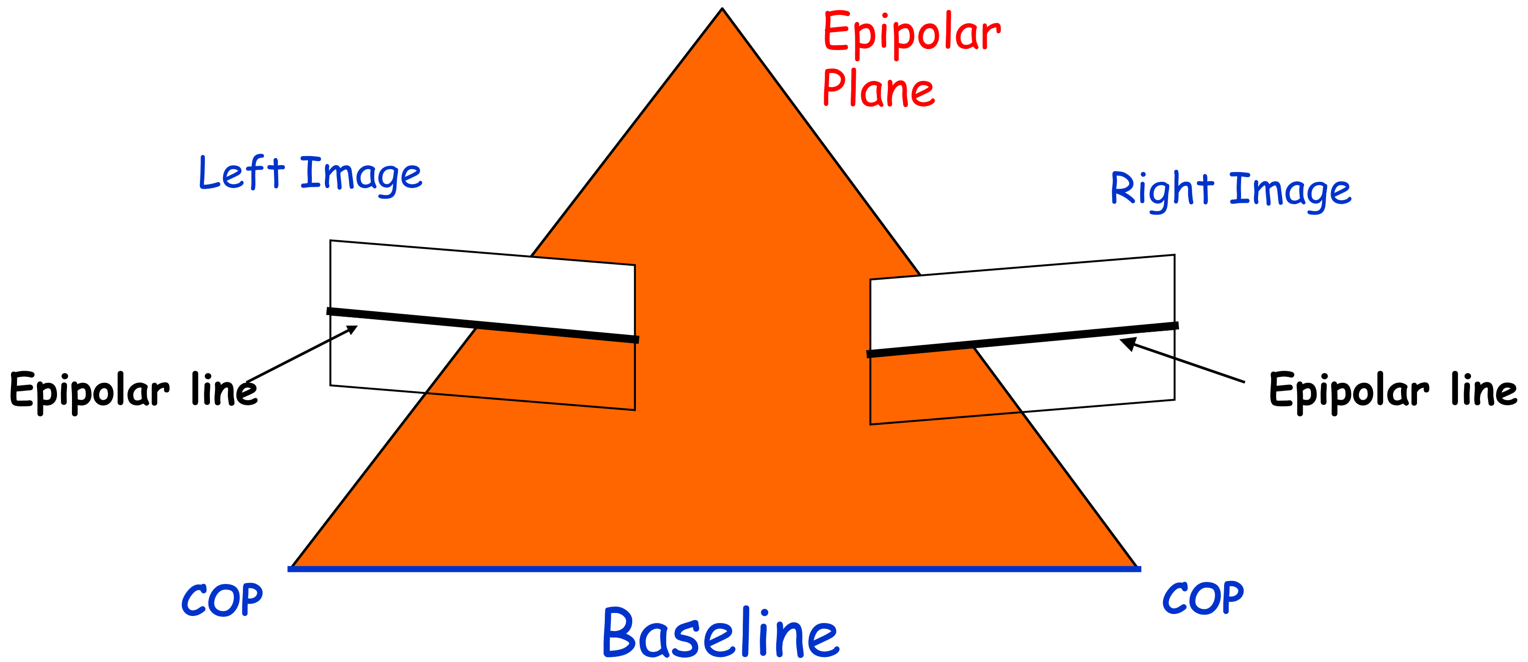


Epipolar Lines

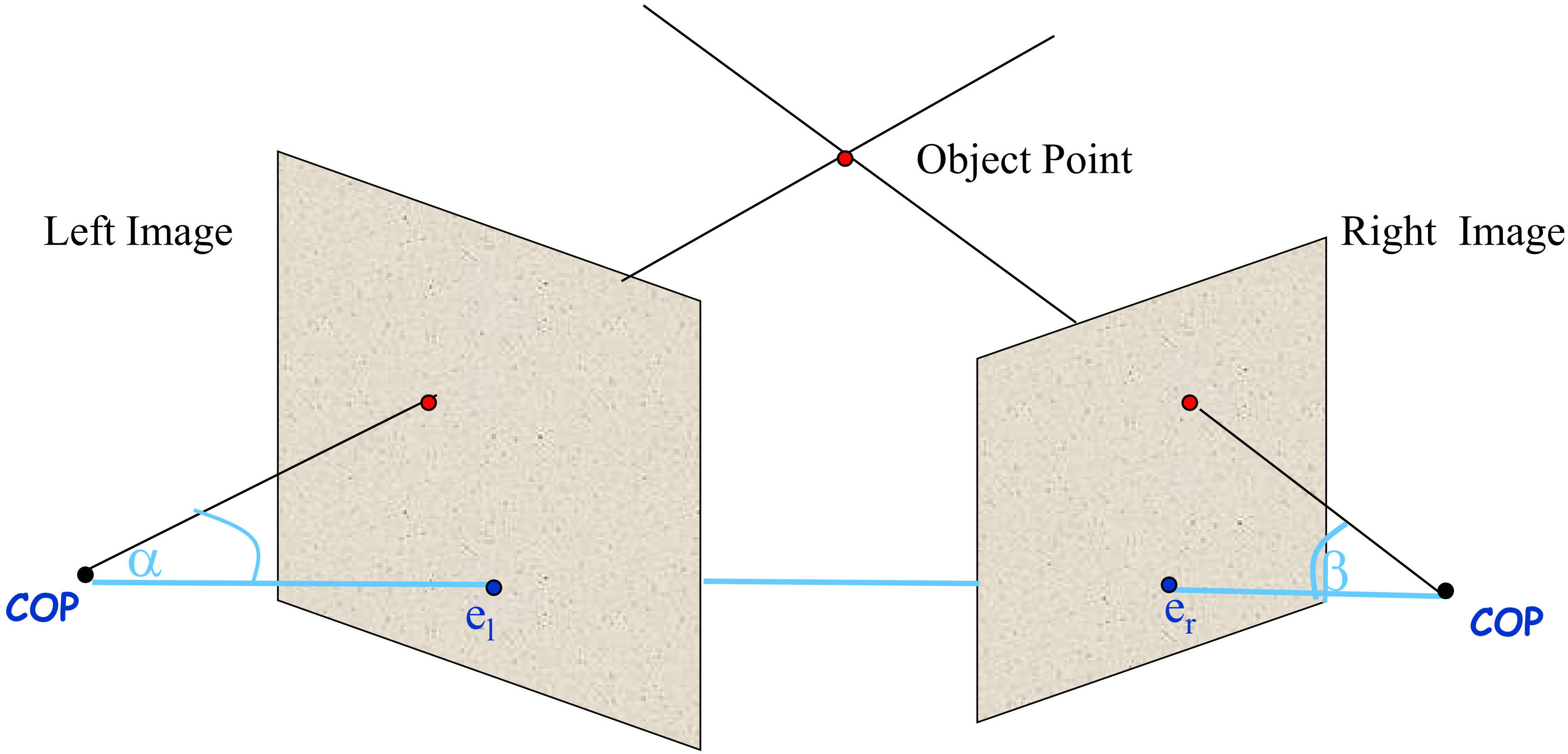


Epipolar Geometry

A pair of corresponding points must lay on corresponding epipolar lines



The epipole points are e_l and e_r



Epipole

- The Epipole is the projection of the *cop* of the other camera
- Except for the epipole, only one epipolar line goes through any image point
- All the epipolar lines go through the epipole

הבינתחומי הרצליה
IDC HERZLIYA

Epipole

- What happens when the two camera planes are parallel?

Next

- Formulation using projective geometry:
  - Stereo, triangulation, and calibration
- Other applications of Epipolar Geometry
- Homography
- More than 2 images