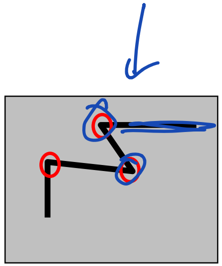
(7.4) Feature detectors



- An *edge detector* finds pixels where the magnitude of the gradient is large.

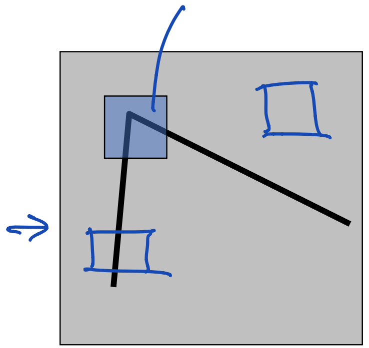
- A *feature point detector* (or interest operator) seeks pixels where the graylevel values vary locally in more than one direction.
- Such points are interesting becasuse they are unique and distingusihable from other pixels using information in the local neighborhood. Called **feature points, interest points or corner points**.

- Such points can be used to find/classify objects, to align multiple images, camera calibration, in stereo imaging, image registration etc.

Harris Corner Detector: Main idea

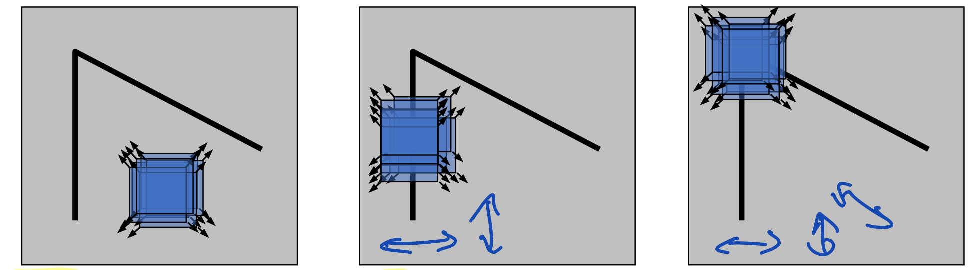
- The most famous ***corner detector*** is the *Harris corner detector* after a work of Chris Harris and Mike Stephens from 1988.

- We should easily recognize a corner point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



Some of the illustrations used on Harris corner detector is taken from a presentation based on slides from Rick Szeliski's lecture notes, CSE576, Spring 2005.

The Harris corner detector



"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

Harris Detector: Mathematics

I(x,y) is image, w(x,y) is local window.

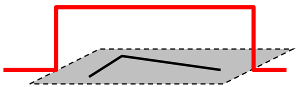
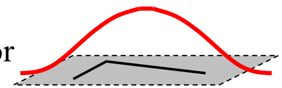
Change of intensity for the shift [u,v]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

Window function $w(x,y)$ =  or 

1 in window, 0 outside

Gaussian

I(x+u,y+v) can be approximated using a Taylor series.

$$\boxed{f(x) = f(a) + \frac{f'(a)}{1!}(x-a)} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Define image gradients :
These can be found by ex. Sobel

$$\boxed{I_x(x,y) = \frac{\partial I(x,y)}{\partial x}} \quad \boxed{I_y(x,y) = \frac{\partial I(x,y)}{\partial y}}$$

$$\boxed{I(x+u, y+v)} \approx I(x,y) + I_x(x,y)(\cancel{x} + u - \cancel{x}) + I_y(x,y)(\cancel{y} + v - \cancel{y}) + \cdots$$

$$I(x+u, y+v) \approx I(x,y) + I_x(x,y)u + I_y(x,y)v$$

Remember, I(x,y) is image, w(x,y) is local window. Change of intensity for the shift [*u,v*]

$$\boxed{E(u,v) = \sum_x \sum_y w(x,y)[I(x+u,y+v) - I(x,y)]^2}$$

Now, for small shifts (u,v) we use this bilinear approximation:

$$I(x+u,y+v) \approx I(x,y) + I_x(x,y)u + I_y(x,y)v$$

$$\Rightarrow E(u,v) \approx \sum_x \sum_y w(x,y)[\cancel{I(x,y)} + I_x(x,y)u + I_y(x,y)v - \cancel{I(x,y)}]^2$$

$$\boxed{E(u,v) \approx \sum_x \sum_y w(x,y)[I_x^2(x,y)u^2 + 2I_x(x,y)I_y(x,y)uv + I_y^2(x,y)v^2]}$$

Express in matrix form?

Harris Detector: Mathematics

For small shifts $[u,v]$ we have a bilinear approximation:

$$E(u,v) \cong [u,v] \cdot M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Measure for average change in image intensity for small displacements

$u = \begin{bmatrix} x \\ y \end{bmatrix}$

$d = \begin{bmatrix} u \\ v \end{bmatrix}$

$$I(\mathbf{u}+\mathbf{d}) \approx I(\mathbf{u}) + \mathbf{d}^T \nabla I \quad \text{for small displacements } \mathbf{d}$$

$$E(\mathbf{d}) = \sum_{\mathbf{u}\in\mathcal{W}} w(\mathbf{u})\left[I(\mathbf{u}+\mathbf{d}) - I(\mathbf{u})\right]^2 \approx \sum_{\mathbf{u}\in\mathcal{W}} w(\mathbf{u})\left[\mathbf{d}^T \nabla I\right]^2$$

$$\approx \sum_{\mathbf{u}\in\mathcal{W}} w(\mathbf{u})\mathbf{d}^T \nabla I \nabla I^T \mathbf{d} = \mathbf{d}^T \left[\sum_{\mathbf{u}\in\mathcal{W}} w(\mathbf{u}) \nabla I \nabla I^T\right]\mathbf{d}$$

$$\approx \mathbf{d}^T \mathcal{M} \mathbf{d} \quad \text{where} \quad \boxed{\mathcal{M} = \sum_{\mathbf{u}\in\mathcal{W}} w(\mathbf{u}) \nabla I \nabla I^T .}$$

Harris and Stephens used the properties of the matrix M in the detection of corner points. This matrix M is determined solely by first partial derivatives of the image in a spatial local window, and is called the **gradient covariance matrix.**

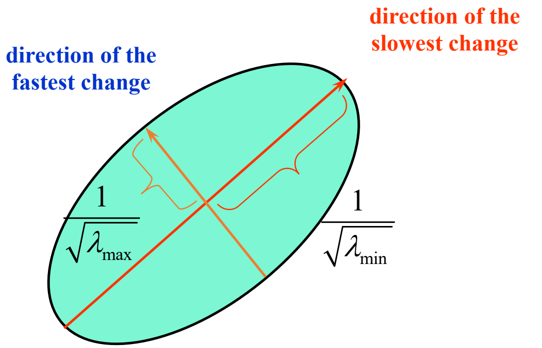
Properties of the gradient covariance matrix, M

$$\mathcal{M} = \sum_{\mathbf{u} \in \mathcal{W}} w(\mathbf{u}) \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} \quad \text{where} \quad I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}.$$

$$\boxed{E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}}$$

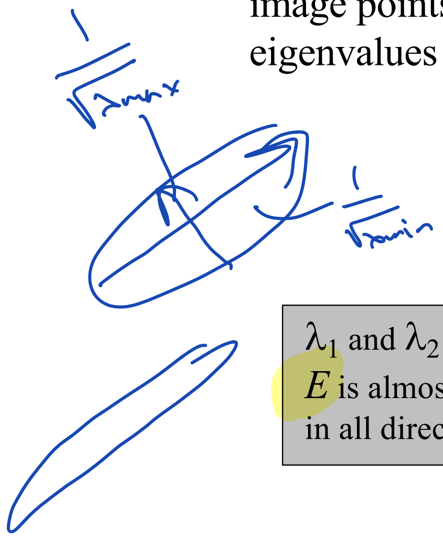
The properties of this matrix can be described by its eigenvalues: λ_1, λ_2.

- The first eigenvector (the one whose corresponding eigenvalue has the largest absolute value) is the direction of greatest curvature.
- The second eigenvector (the one whose corresponding eigenvalue has the smallest absolute value) is the direction of least curvature.
- The corresponding eigenvalues are the respective amounts of these curvatures.

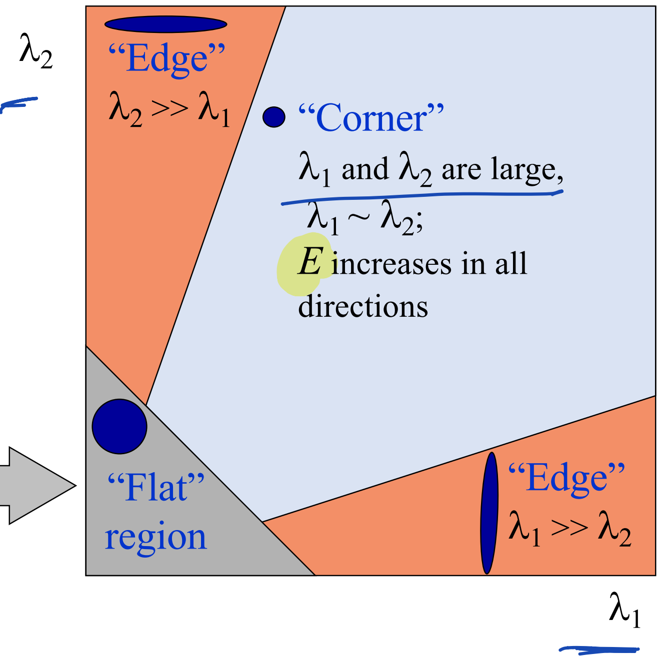


Harris Detector: Mathematics

Classification of image points using eigenvalues of M:



λ_1 and λ_2 are small;
E is almost constant in all directions



Harris Detector: Mathematics

Can we avoid doing a SVD decomposition to find eigenvalues explicitly?

Harris and Stephens used the following measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

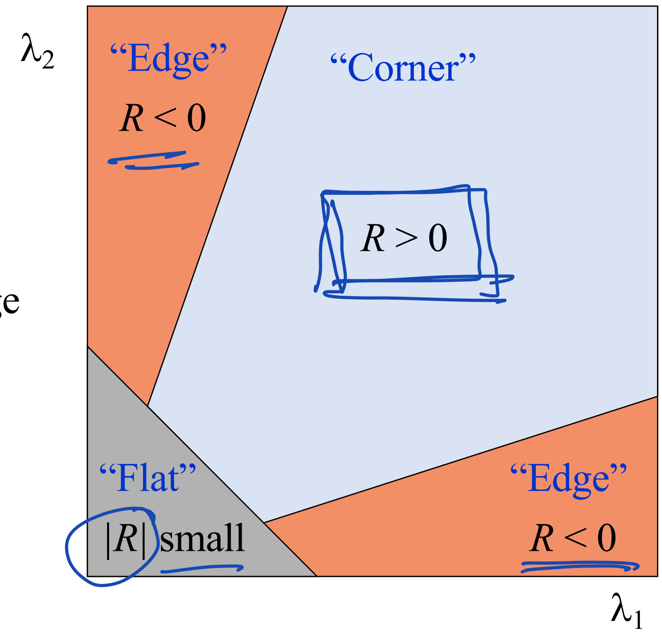
$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

k is an empirical constant for the “corner point” given by the matrix M and its eigenvalues. The value of *k* is balancing between edge-like and corner-like structures. Typical values for corners are in the range (0.04 – 0.06).

Harris Detector: Mathematics

- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



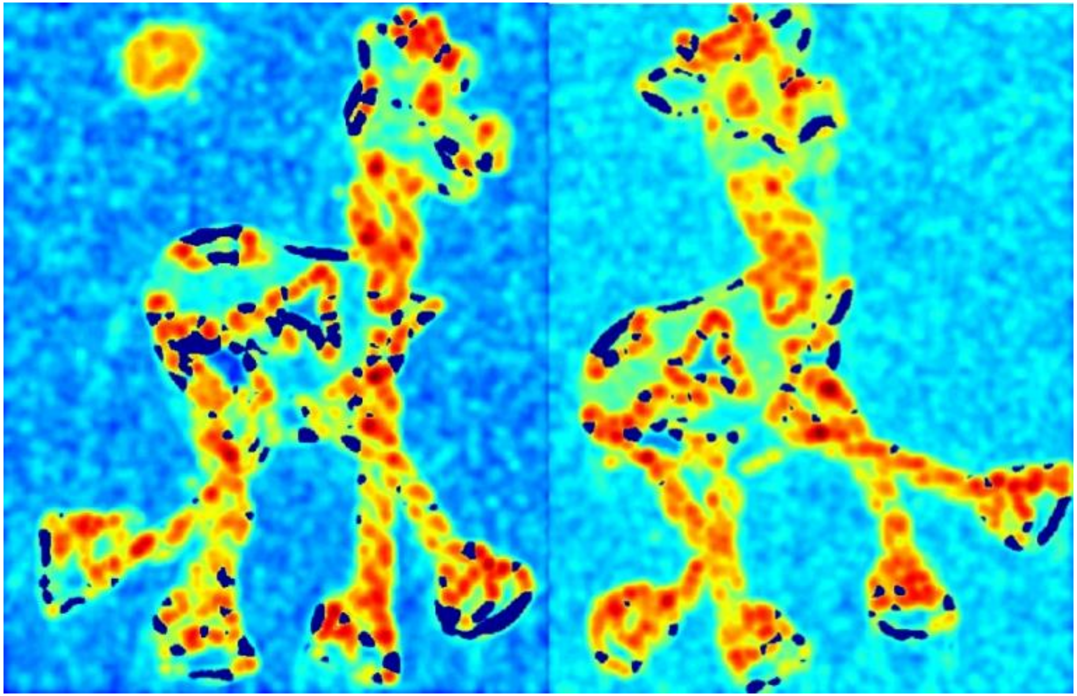
Harris detector

- The Algorithm:
  - Compute $R(x,y)$, corner respons, for all pixel positions.
  - Find points with large corner response function R ($R >$ threshold)
  - Take the points of local maxima of R *(and $R >$ threshold)*

Harris Detector: Workflow

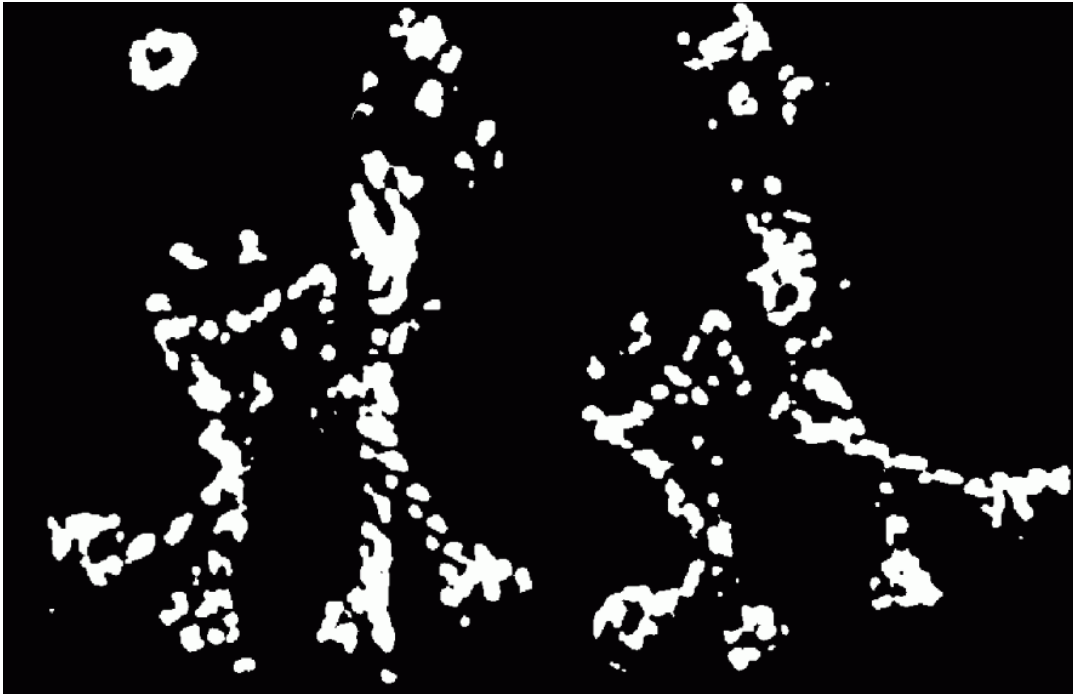
Harris Detector: Workflow

Compute corner response R



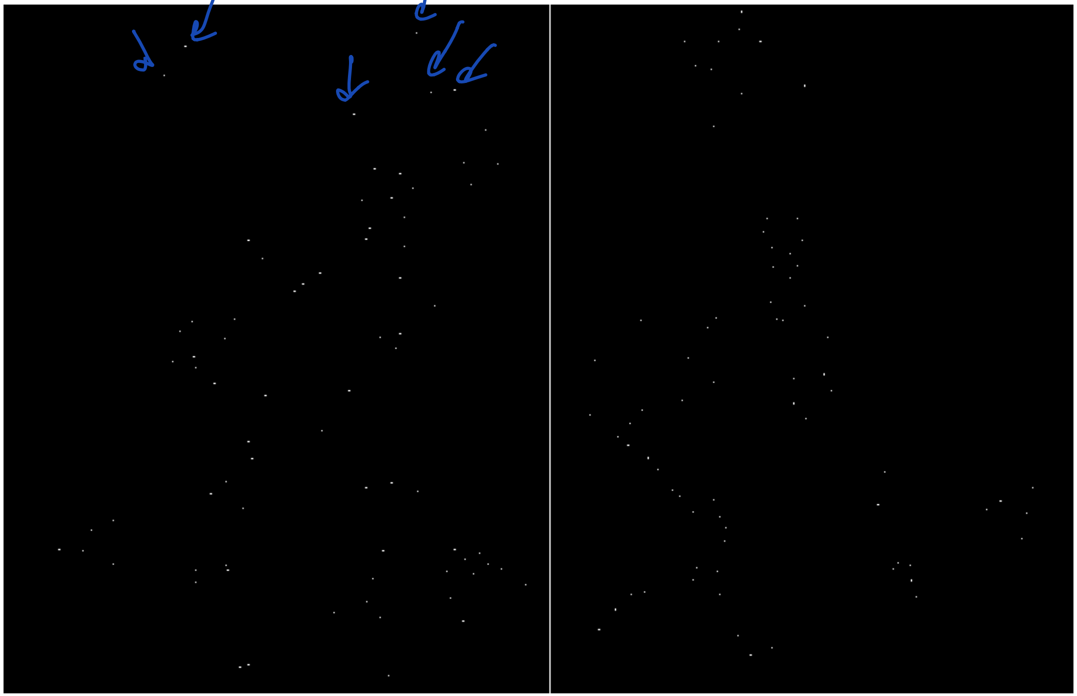
Harris Detector: Workflow

Find points with large corner response: $R >$ threshold

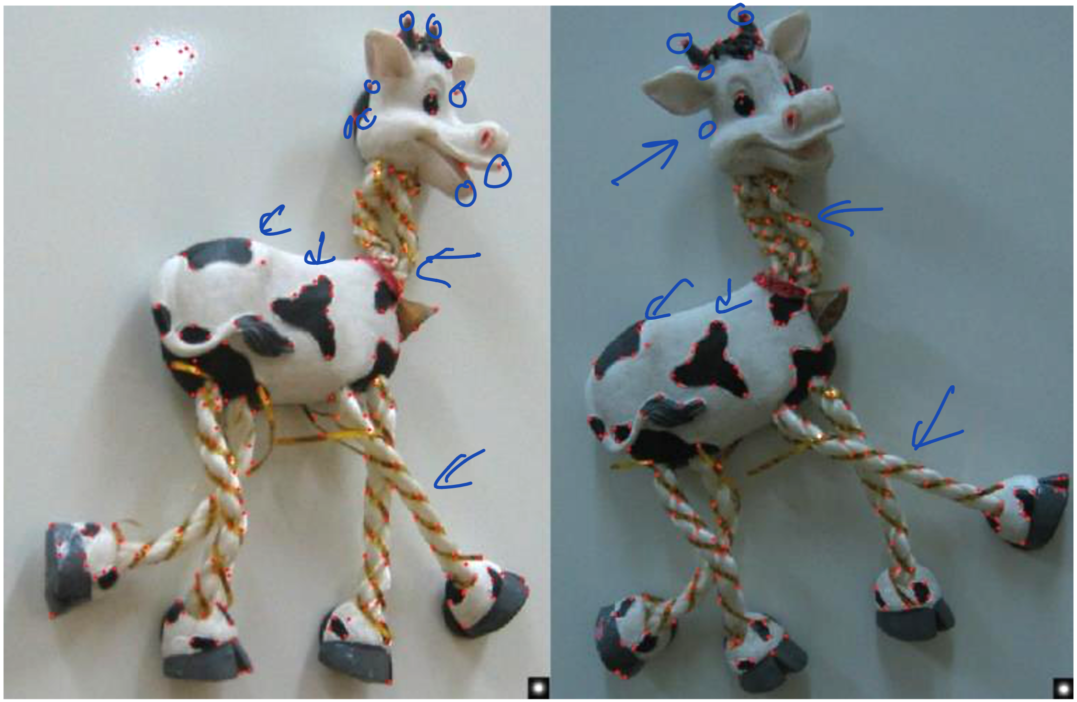


Harris Detector: Workflow

Take only the points of local maxima of R



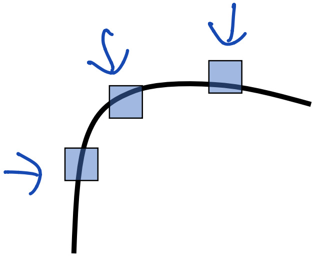
Harris Detector: Workflow



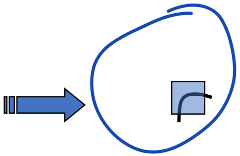
Harris detector, properties

The Harris and Stephens corner detector is

- Rotation invariant
- Translation invariant
- *Not scale invariant*



All points will be classified as edges



This is a "corner" !

Beaudet Detector

Harris: $M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$

- The **Hessian** of a function is a matrix containing the second-order partial derivatives of the function.
- Therefore, the Hessian of the region surrounding a pixel is given by:

$$H \equiv \sum_{\mathbf{x} \in \mathcal{R}} w(\mathbf{x}) \begin{bmatrix} I_{xx}(\mathbf{x}) & I_{xy}(\mathbf{x}) \\ I_{xy}(\mathbf{x}) & I_{yy}(\mathbf{x}) \end{bmatrix}$$

- The **Beaudet detector** uses these second derivatives and is defined as the determinant of the Hessian:

$$\text{cornerness} \equiv I_{xx} I_{yy} - I_{xy}^2 \qquad (\text{Beaudet})$$

Other feature detectors/ descriptors

- Some examples are

- SIFT, Scale Invariant Feature Transform
- FAST, Features from Accelerated Segment Test
- SURF, Speeded-Up Robust Features