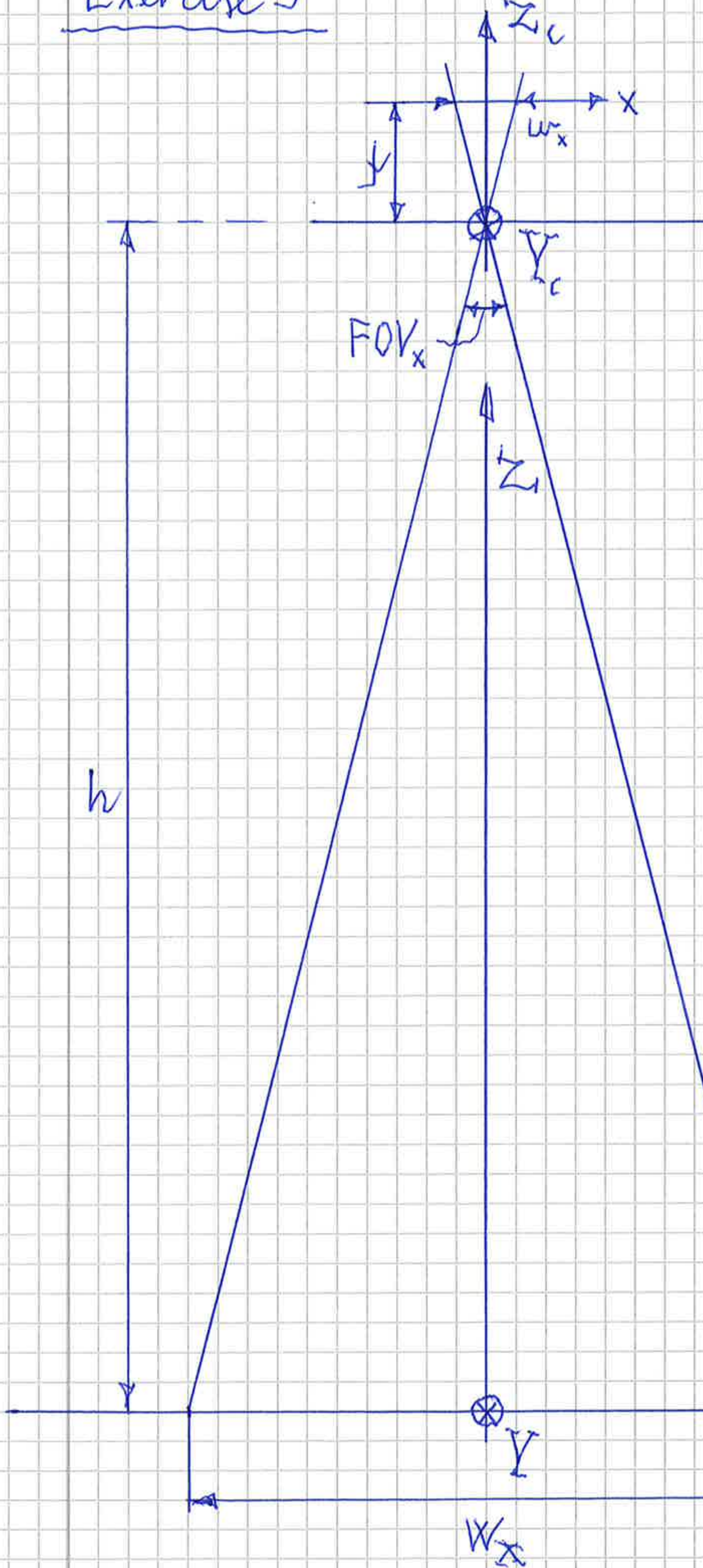
Exam, ELE510, solution to Exercise 3 — 4 Dec. 2017

a)



$$FOV = 2\,\mathrm{Arctan}\left(\frac{w/2}{f}\right)$$

Area on ground:

$$W_X \cdot W_Y$$

$$\frac{W}{w} = \frac{h}{f} \Rightarrow W = h \cdot \frac{w}{f}$$

b) $w_x = 20\text{ mm}$, $w_y = 10\text{ mm}$, $f = 40\text{ mm}$

$$FOV_x = 2\,\text{Arctan}\left(\frac{0.02/2}{0.04}\right) = 2\,\text{Atan}\left(\frac{1}{4}\right) \simeq \underline{\underline{28.07°}}$$

$$FOV_y = 2\,\text{Arctan}\left(\frac{0.01/2}{0.04}\right) = 2\,\text{Atan}\left(\frac{1}{8}\right) \simeq \underline{\underline{14.25°}}$$

$$\left.\begin{aligned} W_x &= h \cdot \frac{w_x}{f} = h \cdot \frac{20}{40} = \frac{1}{2}h \\ W_y &= h \cdot \frac{w_y}{f} = h \cdot \frac{10}{40} = \frac{1}{4}h \end{aligned}\right\}$$

Area on the ground:

$$W_x \cdot W_y = \frac{1}{2}h \cdot \frac{1}{4}h = \underline{\underline{\frac{1}{8}h^2}}$$

c) The camera coordinates, as defined in a), are parallell to the world coordinates. Therefore <u>no rotation is needed</u>.

$$\underline{\underline{\tilde{R} = \tilde{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}$$

For $t=0$ the translation along X and Y is 0 and $Z_c = Z - h$. Then:

$$\underline{\underline{\tilde{t} = \begin{bmatrix} 0 \\ 0 \\ -h \end{bmatrix}}} \quad , \quad X_c = X ,\ Y_c = Y$$

$$\tilde{RT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -h \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \Bigg\{ \text{Rotation-translation matrix}$$

d) $\Delta x = \frac{20 \cdot 10^{-3}}{4000} m = 5 \cdot 10^{-6} m = 5.0 \mu m$

$\Delta y = \frac{10 \cdot 10^{-3}}{2000} m = 5 \cdot 10^{-6} m = 5.0 \mu m$

$\frac{f}{\Delta x} = \frac{f}{\Delta y} = \frac{40 \cdot 10^{-3}}{5 \cdot 10^{-6}} = 8000 \Rightarrow \alpha = \beta = 8000$

No skew $\Rightarrow \theta = 90°, \sin\theta = 1, \cot\theta = 0$

$x_0 = \frac{4000}{2} = 2000 \; , \; y_0 = \frac{2000}{2} = 1000$

$$\underset{\sim}{K} = \begin{bmatrix} \alpha & 0 & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8000 & 0 & 2000 \\ 0 & 8000 & 1000 \\ 0 & 0 & 1 \end{bmatrix}$$

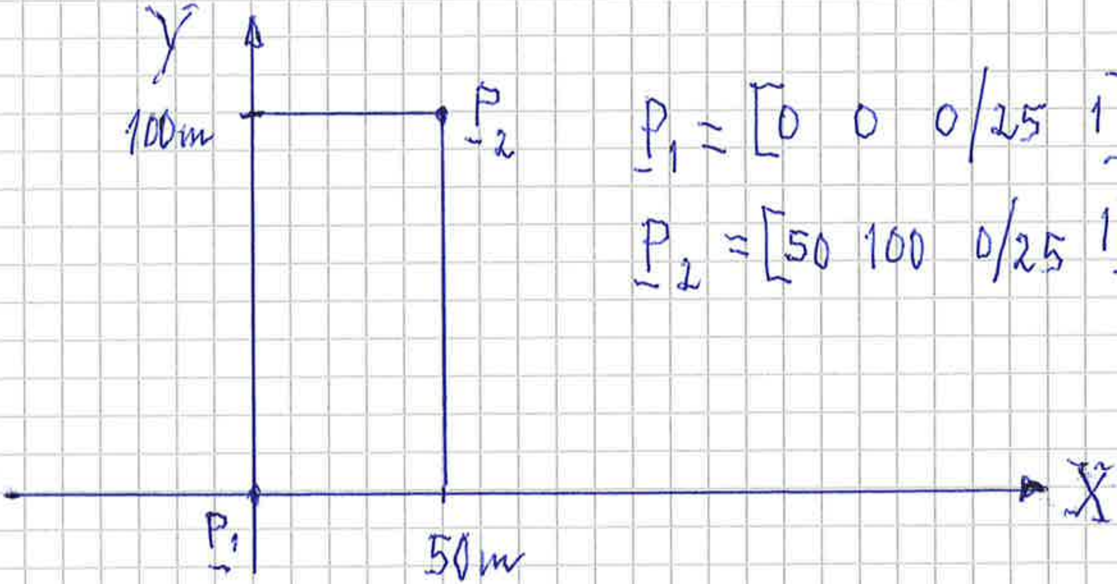
e) $\underset{\sim}{M}' = \underset{\sim}{K} [\underset{\sim}{R} \; \underline{t}]$

$$= \begin{bmatrix} 8000 & 0 & 2000 \\ 0 & 8000 & 1000 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -h \end{bmatrix}$$

$$= \begin{bmatrix} 8000 & 0 & 2000 & -h \cdot 2000 \\ 0 & 8000 & 1000 & -h \cdot 1000 \\ 0 & 0 & 1 & -h \end{bmatrix}$$

$$\underset{\sim}{M} = -\frac{1}{h} \underset{\sim}{M}' = \begin{bmatrix} -\frac{8000}{h} & 0 & -\frac{2000}{h} & 2000 \\ 0 & -\frac{8000}{h} & -\frac{1000}{h} & 1000 \\ 0 & 0 & -\frac{1}{h} & 1 \end{bmatrix}$$

f)



$$\underline{P}_1 = [0 \quad 0 \quad 0/25 \quad 1]^T$$

$$\underline{P}_2 = [50 \quad 100 \quad 0/25 \quad 1]^T$$

h = 1000 m give the same matrix:

$$\underline{M} = \begin{bmatrix} -8 & 0 & -2 & 2000 \\ 0 & -8 & -1 & 1000 \\ 0 & 0 & -0.001 & 1 \end{bmatrix} ; \quad \lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underline{M}\,\underline{P}$$

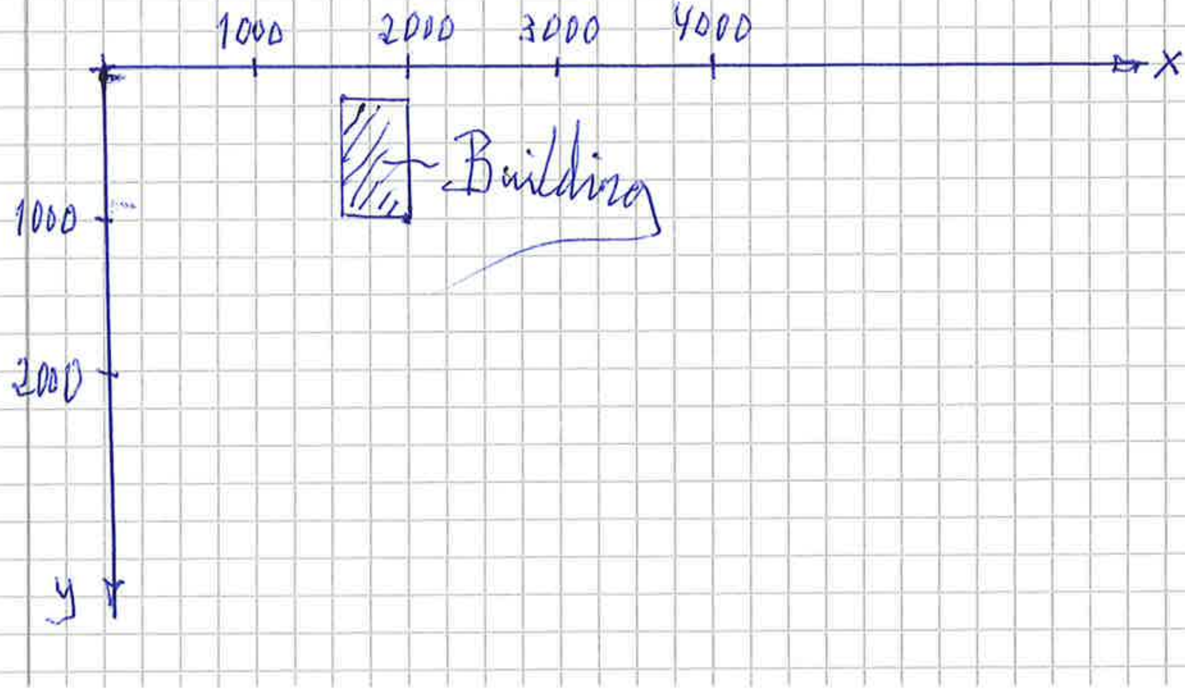
For $\underline{P}_1$ and $\underline{P}_2$ with $Z = 0$ we get:

$\lambda = 1$ and

$x_1 = 2000$, $y_1 = 1000$ (the center pixel)

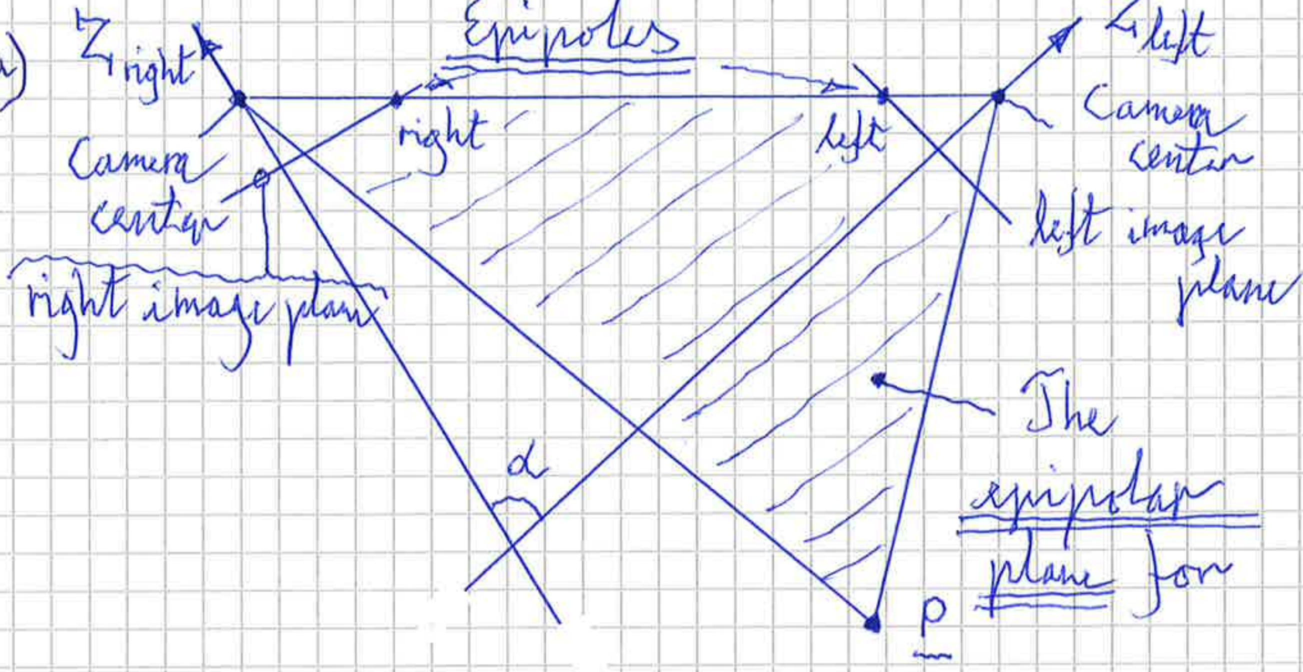
$x_2 = -8 \cdot 50 + 2000 = 1600$

$y_2 = -8 \cdot 100 + 1000 = 200$

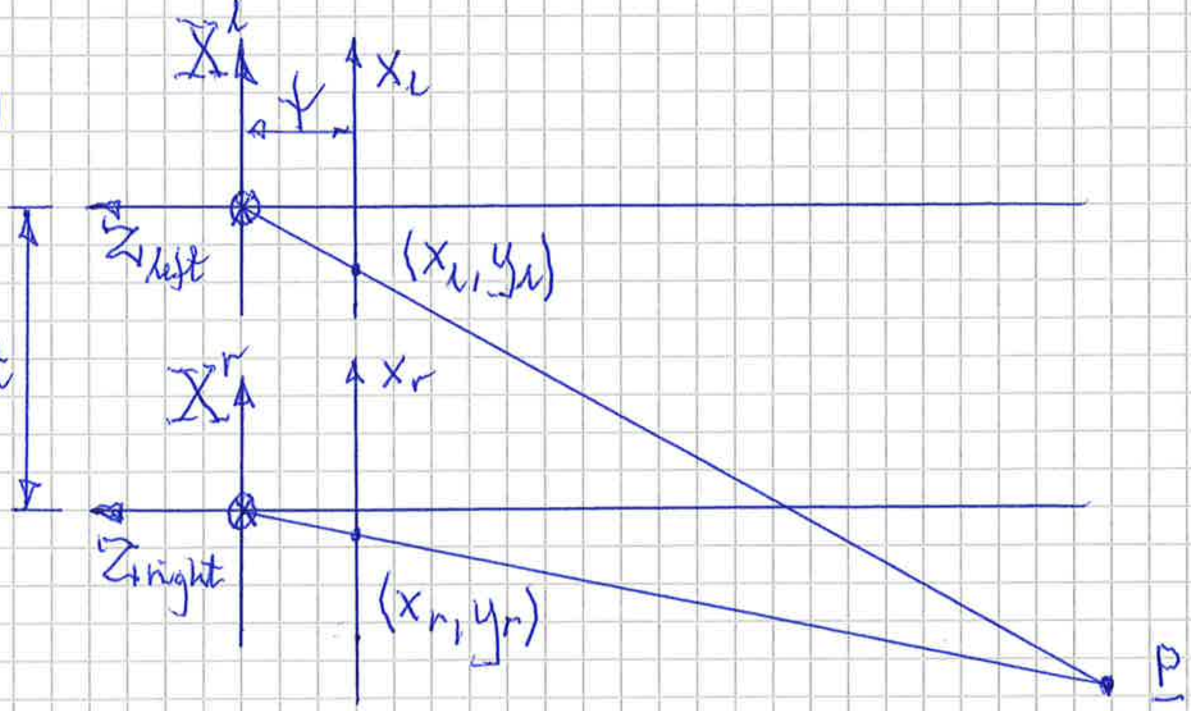


Exercise 4

a)



b)



Disparity:

$$\underline{d} = \begin{bmatrix} x_l - x_r \\ y_l - y_r \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

c) See figure above and let $Y_{left} = Y_{right} = 0$
i.e. $d_y = 0$.

let $Z_{left} = Z_{right} = -Z$

$$\left.\begin{aligned} \frac{Z}{f} &= \frac{X_l}{x_l} && \Rightarrow X_l = \frac{Z}{f} x_l \\ \frac{Z}{f} &= \frac{X_r}{x_r} = \frac{X_l - t}{x_r} && \Rightarrow X_l = \frac{Z}{f} x_r + t \end{aligned}\right\}$$

$$\Rightarrow \frac{Z}{f}(x_l - x_r) = t \Rightarrow \underline{\underline{Z}} = \frac{ft}{x_l - x_r} = \underline{\underline{\frac{ft}{d_x}}}$$

d) There is no motion in the direction of the optical axis, $T_Z = 0$, $(T_y = 0, T_x = -V_x)$, $\underline{\omega} = \underline{0}$.

Then the velocity field (optical flow) in the image plane is parallel.

$$\underline{v} = -\frac{f}{Z_c} \cdot \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \underline{\underline{\frac{f}{Z-h} \cdot \begin{bmatrix} V_x \\ 0 \end{bmatrix} = -\frac{f}{h} \cdot \begin{bmatrix} V_x \\ 0 \end{bmatrix}}}, \; Z = 0$$

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -8 & 0 & -2 & -8V_x t + 2000 \\ 0 & -8 & -1 & 1000 \\ 0 & 0 & -0.001 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \lambda x &= -8X - 8V_x t + 2000 &&= -8(V_x t + X) + 2000 \\ \lambda y &= -8Y + 1000 &&= -8Y + 1000 \\ \lambda &= 1 \end{aligned}$$

$$\underline{\underline{x(t) = -8(V_x t + X) + 2000}}$$

e) $\Delta t = \frac{1}{100}\,s = 10\,ms$, $t = k \cdot \Delta t = \frac{k}{100}$, $k = 0, 1, 2, \dots$

at $t = 0.1\,s$, $k = 10$

Computes the velocity as the displacement between frames, $k=11$ and $k=10$.

$$x(k=11) = 2000 - 8\cdot\left(100\cdot\frac{k}{100} + \underline{X}\right) = 2000 - 8\cdot(11 + \underline{X})$$
$$x(k=10) = 2000 - 8\cdot(10 + \underline{X})$$

$$\Rightarrow v_x = x(k=11) - x(k=10) = -8(11-10) = -8 \text{ pixels}$$
$$v_y = y(k=11) - y(k=10) = 0$$

$$\underline{v} = \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$ for all $\underline{X}$ and $\underline{Y}$ in the FOV