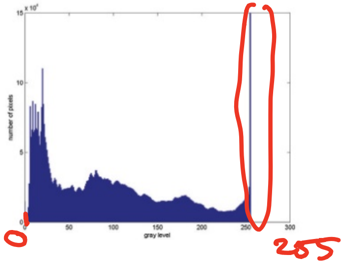
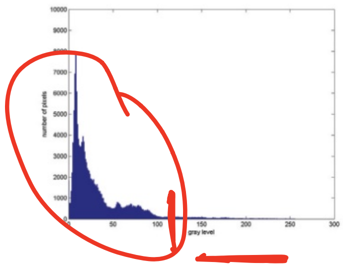
(3.3) Gray level histograms

Histograms give a simple but powerful technique for capturing the statistics of any type of data.

- The histogram records the number of occurrences (pixels) in each bin (with a specific value). For a [0,255] , 8bpp image we have 256 bins, one for each gray level. Can also look at gray level intervals (fewer bins).
- Gives no information on where in the image the gray levels occur

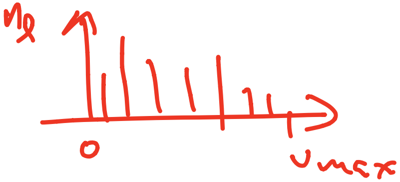




Normalized histogram: computed from the histogram by dividing each value by the total number of pixels in the image. This is an estimation of the **Probability density function (PDF)**

$h[l] = n_l$ $\quad l = 0, 1 \ldots \underbrace{v_{max}}_{255}$

w - width } image
h = height }



$\sum_l n_l = n = w \cdot h$

$\Rightarrow$ no of pixels in the image

$$\bar{h}[l] = \frac{h[l]}{n}$$

$\Rightarrow$ normalized histogram

$\sim$ pdf

Cumulative distribution function (CDF): $$\boxed{\bar{c}[\ell] = \sum_{k=0}^{\ell} \bar{h}[k], \quad \ell = 0, \ldots, 255}$$

Histogram equalization:

A pixel with gray level l gets new gray level l' according to:

$$\boxed{l' = round(255 * \bar{c}[l])}$$

v_{max}

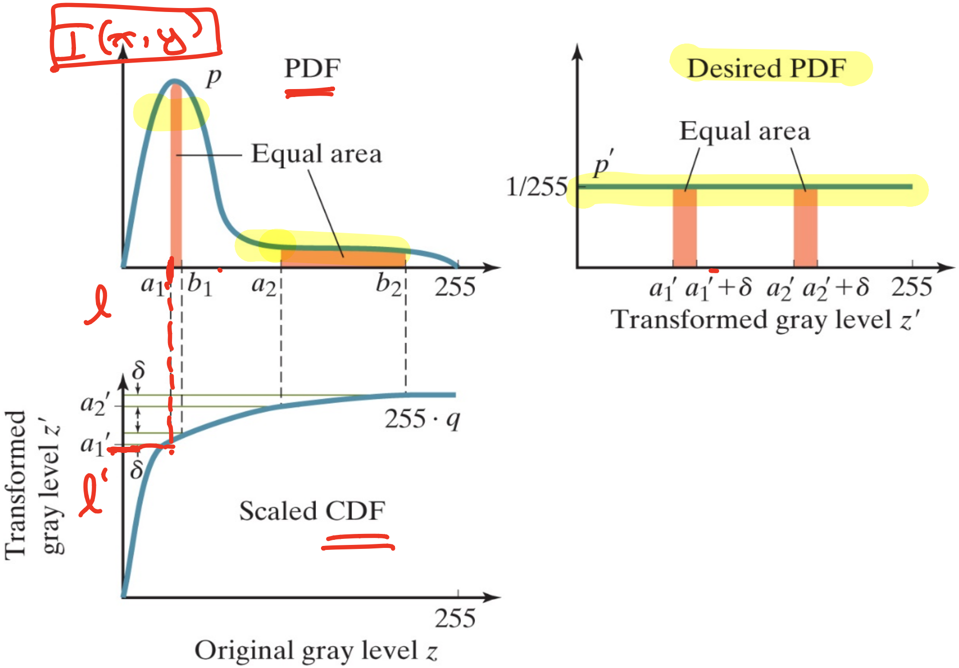
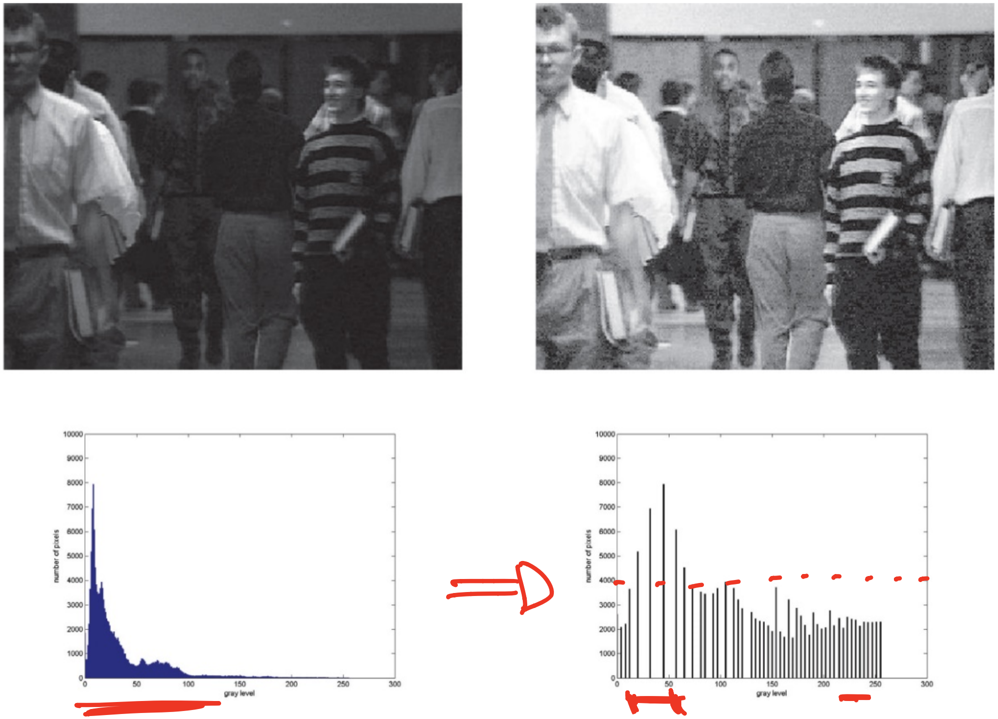


Figure 3.21 The result of histogram equalization applied to an image. The increase in contrast is noticeable. The normalized histogram of the result is much flatter than the original histogram, but it is not completely flat due to discretization effects. Source: Movie *Hoop Dreams*.

$I(x,y)$ $\qquad$ $I'(x,y)$



Try at home (for Q&A class)

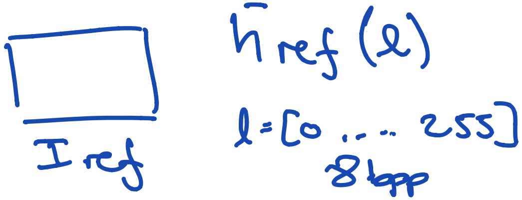
For the toy-image I given below :

1) Find histogram and normalized histogram
2) Perform histogram equalization

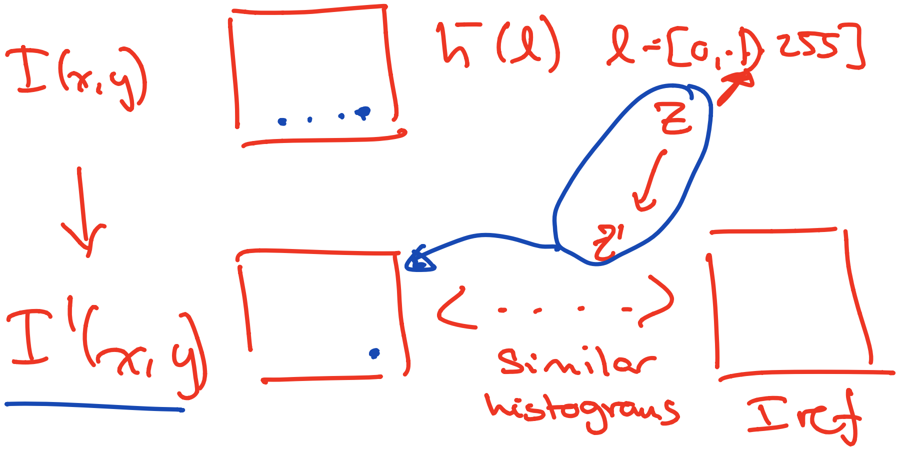
3 bpp $\rightarrow$ [0, ... 7]

$$I(x,y) = \begin{bmatrix} 7 & 4 & 2 & 0 \\ 4 & 2 & 4 & 5 \\ 3 & 3 & 5 & 6 \end{bmatrix}$$

Histogram Matching



- Histogram equalization flattens the histogram as much as possible. Sometimes we rather match it to another (reference) histogram to make images comparable. (to compensate for changing light conditions etc.) -> histogram matching.
- Again, the cumulative distribution function is used (CDF)



Histogram Matching

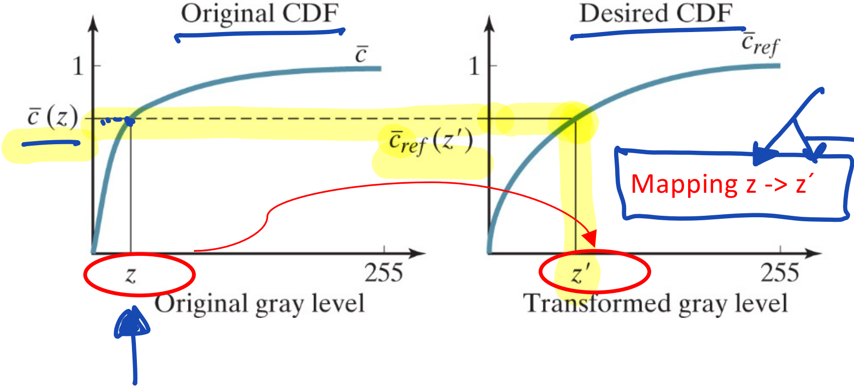
$$\bar{c}(z) = \int_0^{z} \bar{h}(\zeta)\, d\zeta$$

$$\bar{c}_{ref}(z') = \int_0^{z'} \bar{h}_{ref}(\zeta)\, d\zeta$$

want $\bar{c}(z) = \bar{c}_{ref}(z')$

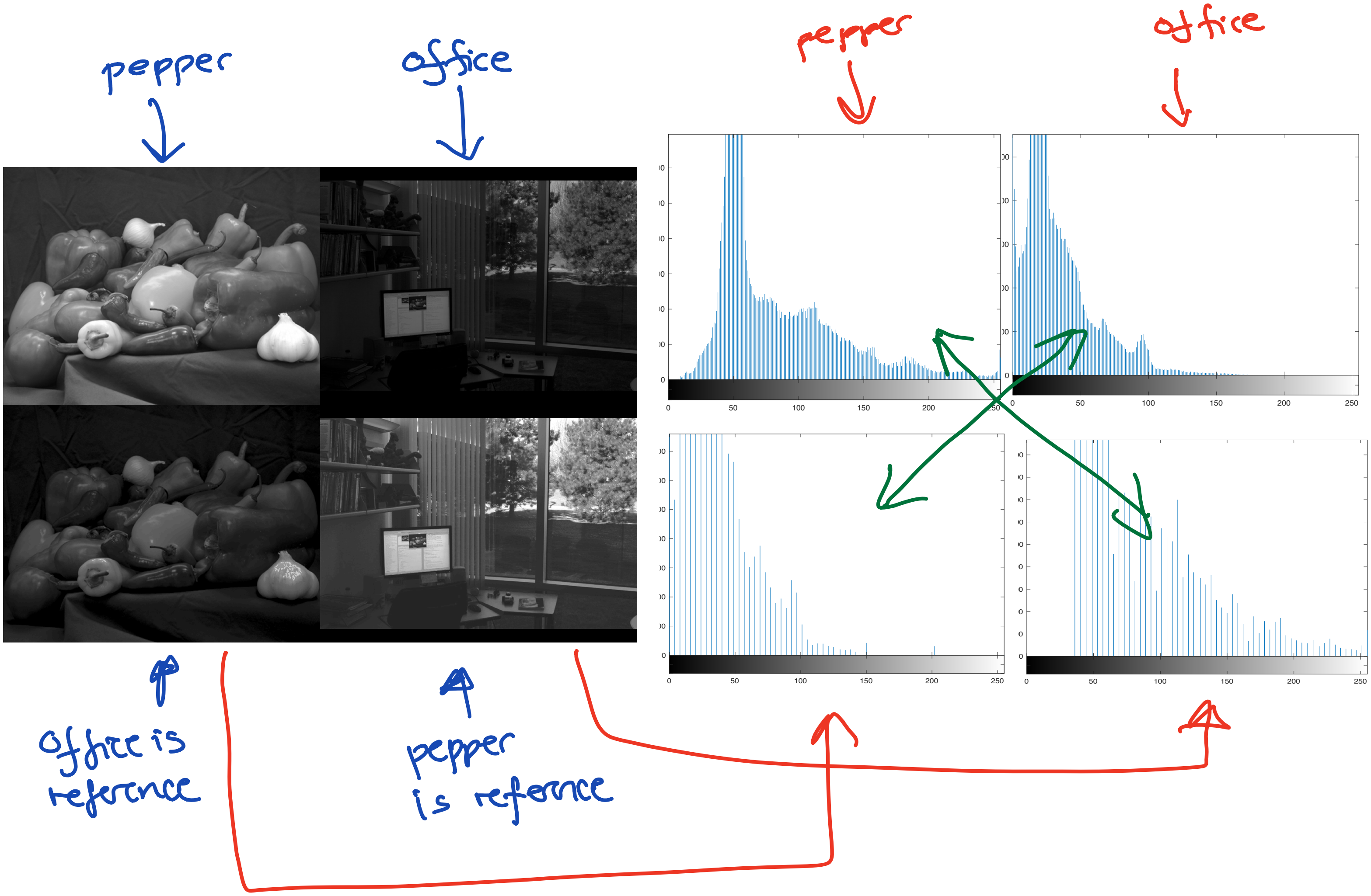
$$z' = \bar{c}_{ref}^{-1}(\bar{c}(z))$$

Figure 3.23 Histogram matching. Given the CDF $\bar{c}$ of the original image and the desired CDF $\bar{c}_{ref}$, histogram matching transforms an original gray level z to a new gray level z' by finding the value of z' such that $\bar{c}(z) = \bar{c}_{ref}(z')$. As before, discretization effects are ignored in this illustration.



100 $I(x,y)$

105 $I'(x,y)$


pepper
office
pepper
office
office is reference
pepper is reference
0
50
100
150
200
250

(3.4) Multispectral transformations

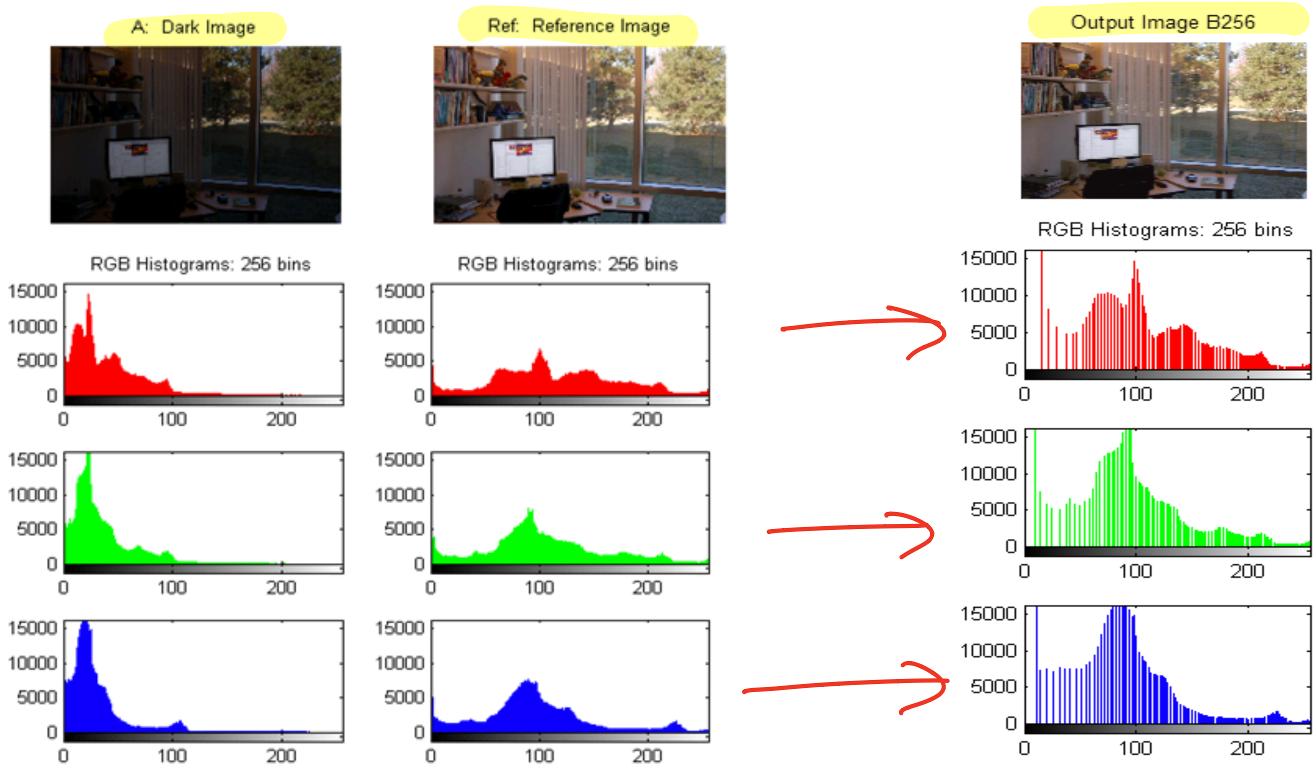
- Multiple values for each pixel (ex. color RGB, or other)
- A mapping where either the input, the output or both images are multispectral is called a multispectral transformatin.
- Simple RGB to gray:

$$I'(x,y) = \frac{1}{4}[I_R(x,y) + 2I_G(x,y) + I_B(x,y)]$$

- Simple gray to RGB:

$$I'_R(x,y) = I(x,y), \quad I'_G(x,y) = I(x,y), \quad I'_B(x,y) = I(x,y)$$

Multispectral histogram matching



Multispectral images – LANDSAT

$$I'_{NDVI}(x,y) = \frac{I_{IR}(x,y) - I_{red}(x,y)}{I_{IR}(x,y) + I_{red}(x,y)}$$

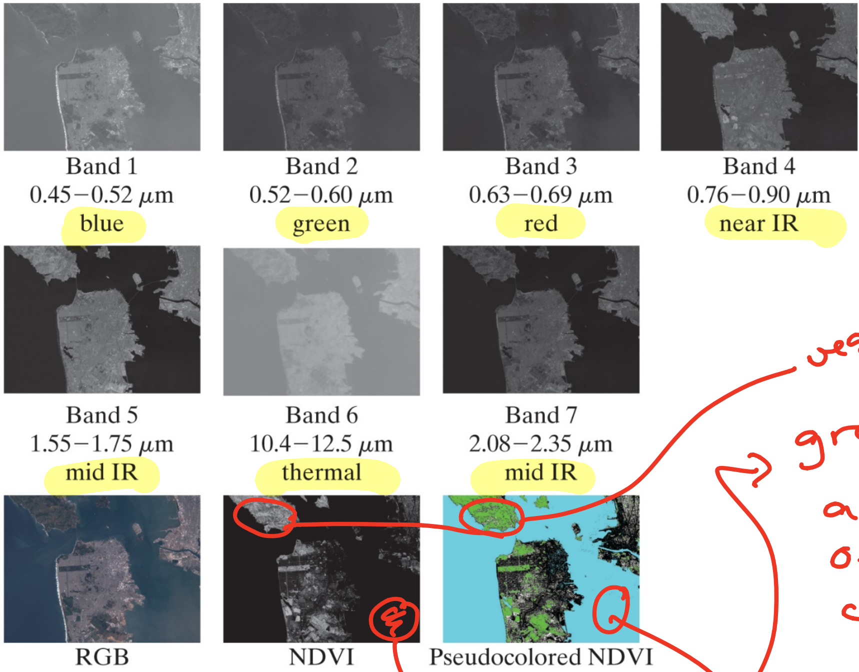


Figure 3.26 7 bands of a Landsat image of the San Francisco Bay Area. The bottom row shows the RGB image obtained by combining bands 1, 2, and 3; the NDVI calculated using bands 3 and 4; and the pseudocolored image obtained by density slicing on the NDVI (blue indicates water, green indicates vegetation, and tan indicates soil). Notice the vegetation occurs outside the city itself in Marin County (upper peninsula in the image) and several parks (namely, the Presidio, Golden Gate Park, and San Bruno Mountain State Park on the lower peninsula). Source: http://glcf.umd.edu, http://glcf.umd.edu/data/landsat/

vegetation = green

→ grayscale image

assign a range of values to a color.

band1,2,3

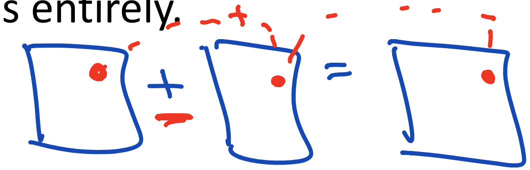
sea = blue

→ density slicing

(3.5) Multi-Image (point) Transformations

- **Multi-image transformation:** includes images taken of the same scene at different times, or of different scenes entirely.

$I'(x,y) = I_1(x,y) + I_2(x,y)$



- logical operations (and, or, not, xor)
- arithmetic operations; examples:
  - **Absolute difference:** computes for each pixel location the absolute value of the difference between the pixels:

$$I'(x, y) = |I_1(x, y) - I_2(x, y)|$$

  - **Linear interpolation:** produces a convex combination of the two inputs:

$$I'(x, y) = \eta I_1(x, y) + (1 - \eta)I_2(x, y)$$

(3.6) Change detection – frame differencing

- **Difference image:** to compare successive image frames in the video sequence.

$$I'(x, y) = |I_t(x, y) - I_{t-1}(x, y)| \boxed{> \tau}$$

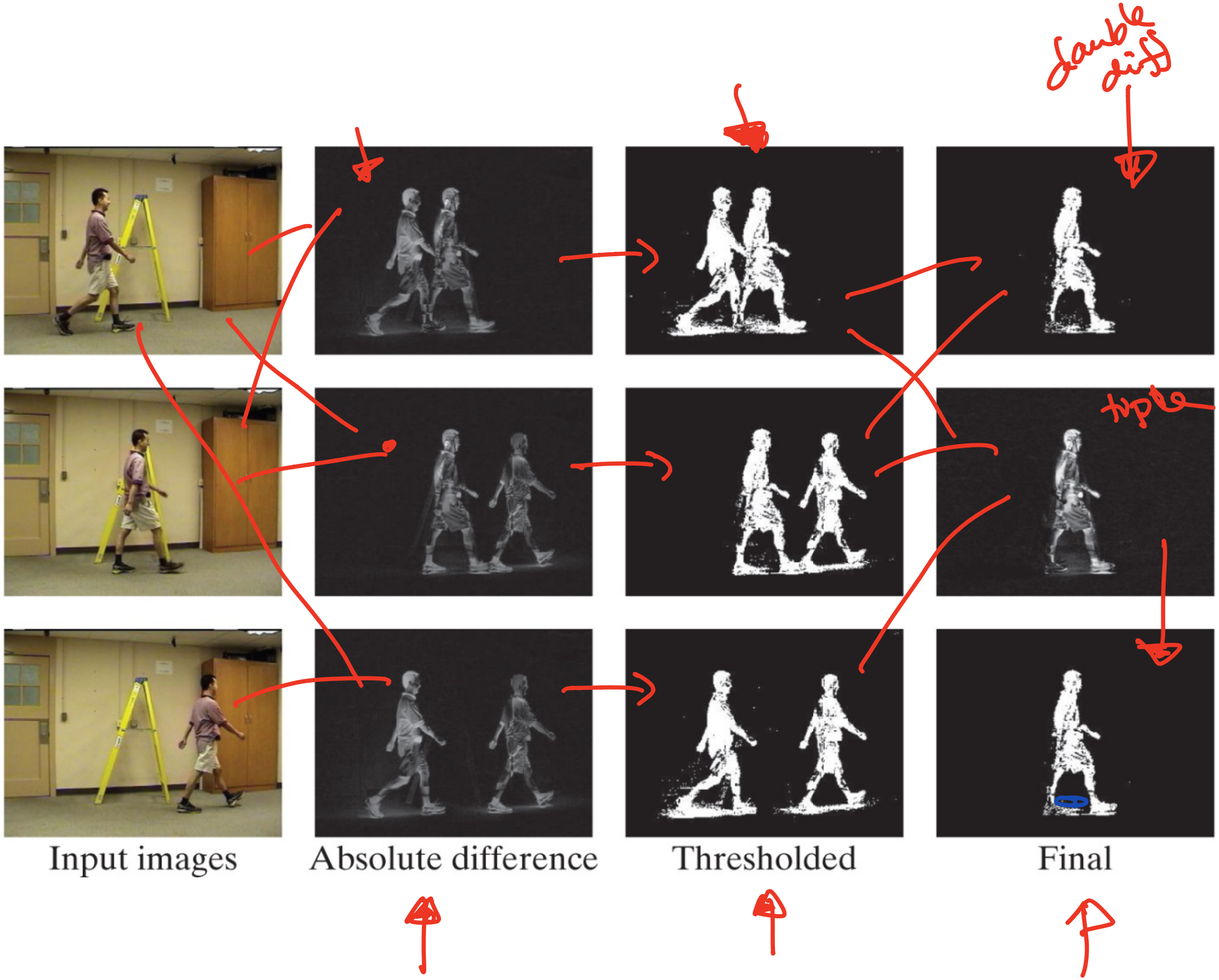
- **Double-difference image:** used to solve the double-image problem, that is, the difference image will contain foreground pixels not only where the foreground object is located in the current frame but also where it was in the previous frame.

$$I' = |I_t - I_{t-1}| > \tau \text{ AND } |I_{t+1} - I_t| > \tau$$

- **Triple-difference image:** combines the absolute differences from all three image pairs using addition and subtraction prior to thresholding:

$$I'(x, y) = (|I_{t-1} - I_t| + |I_{t+1} - I_t| - |I_{t-1} - I_{t+1}|) > \tau$$

Figure 3.27 Detecting a moving object by frame differencing. LEFT COLUMN: Three image frames from a video sequence. SECOND COLUMN: The absolute difference between pairs of frames. THIRD COLUMN: Thresholded absolute difference. RIGHT COLUMN: Final result using double difference (top), triple difference (middle), and thresholded triple difference (bottom) methods.



Background subtraction

$B(x,y) \approx I'_{mean}(x,y) = \frac{1}{n}\sum_{i=1}^{n} I_i(x,y)$

- **Background image:** a reference image that does not contain any foreground objects. If available;

$$I'(x,y) = |I(x,y) - B(x,y)|$$

- Can capture foreground objects even if they do not move for a while (not possible with frame difference)

- **Digital subtraction angiography (DSA)**: a reference image is captured of a blood vessel before injecting it with dye to increase contrast.

- Alternative; find **average image** over successive video frames

Figure 3.28 TOP: Five images from a video sequence. BOTTOM: Each column shows the mean image obtained using all the images up to and including the one above it. As time progresses the moving objects disappear, leaving only the background.