Prof. Kjersti Engan

ELE510 Image processing and computer vision

Camera and calibration, (chap 13.4, 13.5 Birchfield) 2020

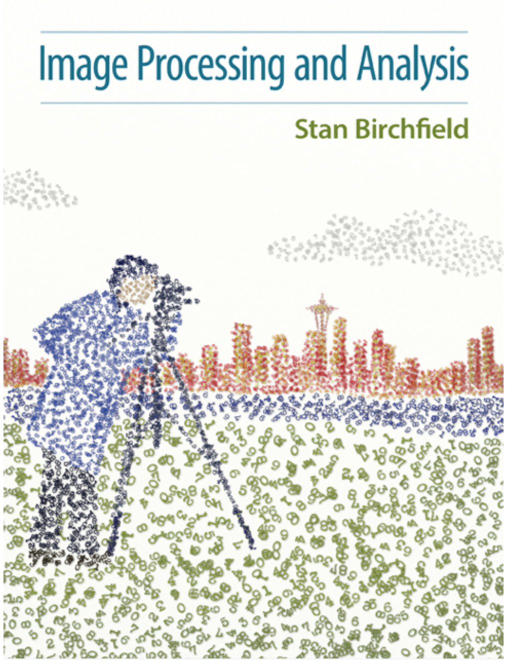

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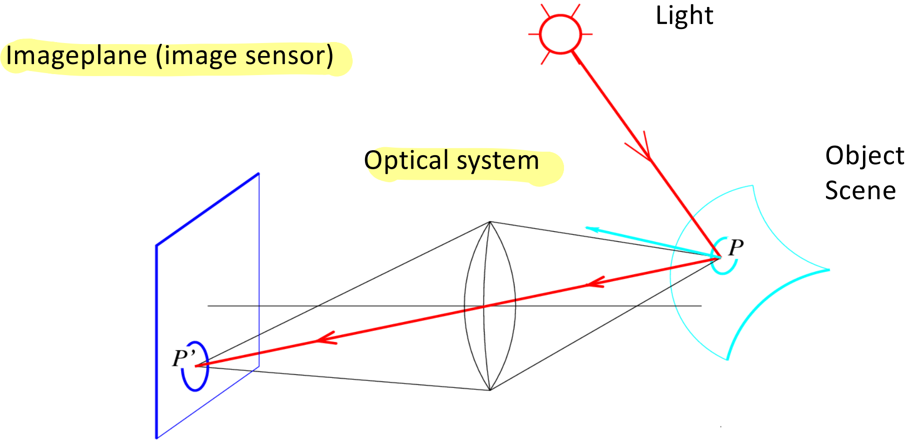
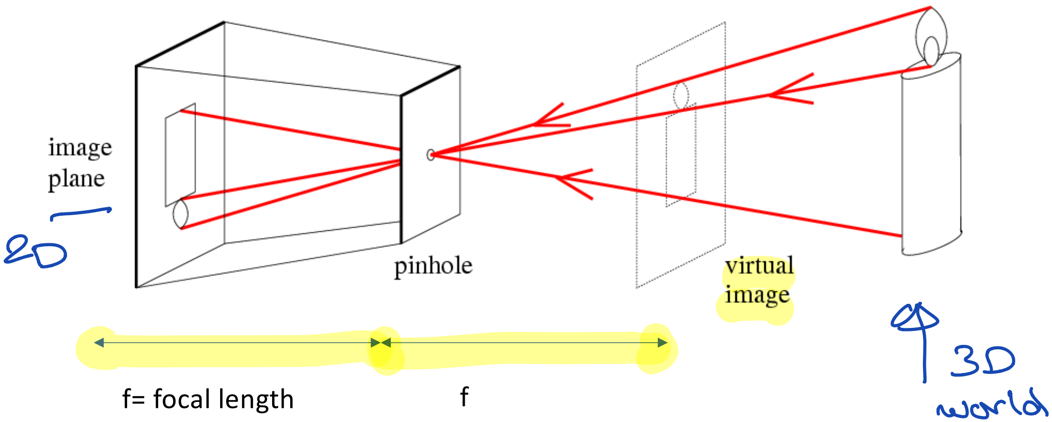
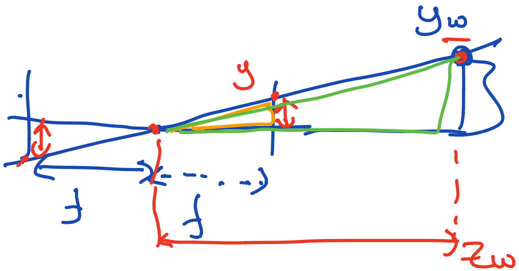
Image Formation



Geometric **Optical** **Photometric**

Pinhole camera - revisited





$$\frac{y}{f} = \frac{y_w}{z_w} \Rightarrow y = f \cdot \frac{y_w}{z_w}$$

$$x = f \cdot \frac{x_w}{z_w}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{f}{z_w} & 0 & 0 & 0 \\ 0 & \frac{f}{z_w} & 0 & 0 \\ 0 & 0 & \frac{1}{z_w} & 0 \end{bmatrix} \cdot \overbrace{\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}}^{P^w}$$

we dont know z_w!

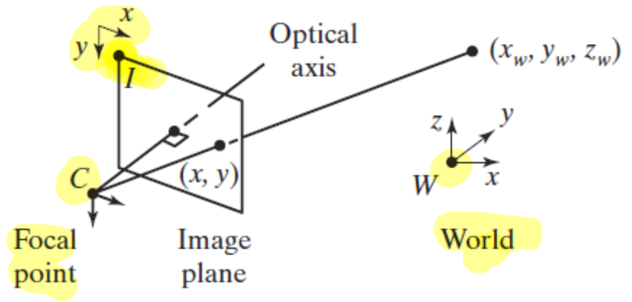
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underbrace{\frac{1}{z_w}}_{\text{dont know!}} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{m.} \underbrace{\propto}_{\text{proportional to}} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}}_{m}$$

Perspective Imaging

- Three coordinate systems are involved in the process of imaging:
  - one attached to the world
  - one attached to the camera
  - one associated with the image

- Points in the world are described in the *world coordinate system*, in which lengths are measured in meters.



- The camera coordinate system is centered at the focal point, x and y axes aligned with the rightward horizontal and downward vertical directions, respectively, of the image plane, and the positive z axis points along the optical axis toward the world.

- The *image coordinate system* is centered at the top left corner of the image, with the positive x and y axes pointing along the rows and columns, respectively, of the imaging sensor. In the image coordinate system, measurements are made in pixels.

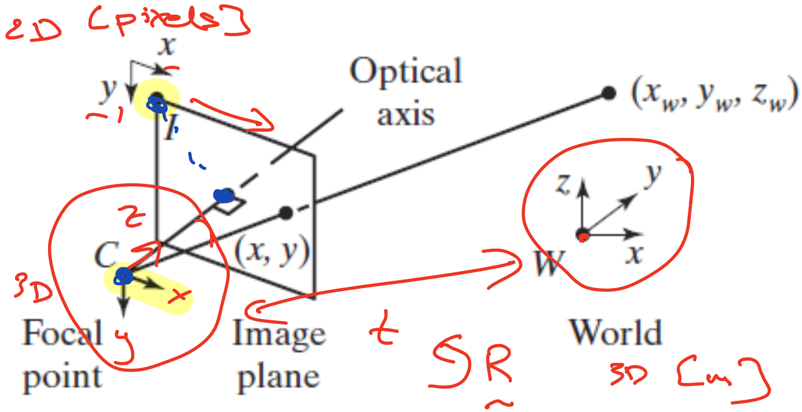


Figure 13.24 The projection of a world point (x_w, y_w, z_w) onto an image plane at point (x, y), assuming a pinhole camera model with no diffraction. The three coordinate systems are the world coordinate system (W), the camera coordinate system (C), and the image coordinate system (I).

Perspective Imaging

- Using homogeneous coordinates, the imaging process is captured mathematically as:

$$\underset{\substack{2D\\ [\text{pixels}]}}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}} \propto \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\mathbf{P}_{\{3\times 4\}}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad 3D \quad [m] \quad \boxed{z_w}$$

- Where **P** is a 3 X 4 projection matrix that itself is composed of two parts:

$$\mathbf{P}_{\{3\times 4\}} = \underbrace{\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}_{\{3\times 3\}}} [\mathbf{R}_{\{3\times 3\}} \ \ \mathbf{t}_{\{3\times 1\}}]$$

- **Intrinsic** (camera dependent) parameters are found in **K.** Converts meters to pixels, moves the origo in the image plane etc.
- **Extrinsic** (how do we describe the world relative to the camera) parameters in **R** and **t**

Intrinsic Camera parameters

$$\underbrace{\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}_{\{3\times 3\}}}$$

- u0 and v0 specifies the principal point, i.e. The intersection of the optical axis and the image plane.
- α, β, ɣ are related to the focal length in x direction (fx), in y direction (fy) and the skew θ between x and y axis in the image plane.

x

y

$\frac{\pi}{2}$

$$\boxed{\begin{aligned} \alpha &= f_x \\ \beta &= f_y/\sin(\theta) \\ \gamma &= -f_x/\tan(\theta) \end{aligned}}$$

$f_x = f/\Delta x$, $f_y = f/\Delta y$ Often: $f_x \approx f_y$ (tolerance of 5%)

$$\mathbf{K} = \begin{pmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Often we can assume θ≈(π/2) :
you would need to account for axis skew when calibrating unusual cameras or cameras taking photographs of photographs, else you can usually ignore the skew parameter.

Lens Distortion

- Real cameras have lenses. Light bends due to curvature in the lens.
- The dominant distortion of a typical lens is **radial distortion**

It is a function only of the radial distance from the center of the image.

- Let (x_u, y_u) be the undistorted coordinates of a pixel in the image:

$$x_u = x_d + \bar{x}_d f(r_d)$$

$$y_u = y_d + \bar{y}_d f(r_d)$$

Where (r is the radial distance between the pixel and the undistorted point):

$$f(r_d) = k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6 + \cdots$$

- If a more accurate model is desired, **tangential distortion** (or *decentering distortion*) can be included

- The coefficients have to be estimated, thereafter the image can be unwarped

Extrinsic parameters

- Rotation and translation of the World coordinates with respect to the Camera coordinates:

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3x3 3x1

$$\mathbf{TR} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathcal{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{pmatrix} \qquad \boxed{{}^{C}\mathbf{P} = \mathbf{TR}\,{}^{W}\mathbf{P}}$$

$${}^{w}P \quad \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$