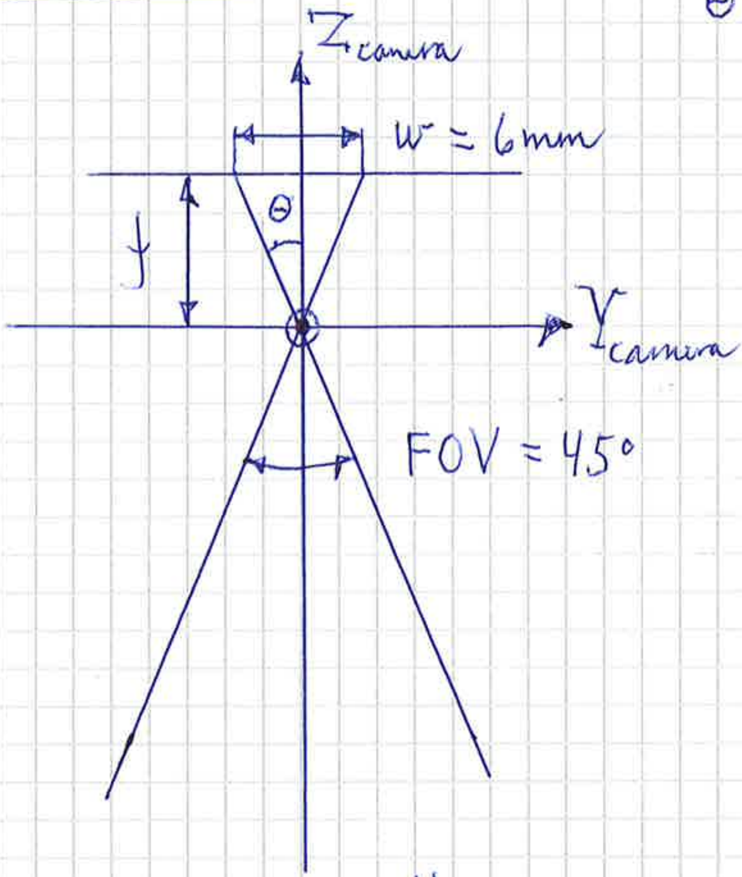
Solution, ELE 510 Image Processing with Robot Vision, Dec. 7th, 2016.

Exercise 3

a)



$$\Theta = \frac{45}{2} = 22.5°$$

$$\tan\Theta = \frac{(w/2)}{f}$$

$$f = \frac{w}{2\tan(\Theta)} = \frac{6 \cdot 10^{-3}}{2\tan(22.5°)} = 7.24 \cdot 10^{-3} m = \underline{\underline{7.24 \ mm}}$$

b) See figure on the next page.

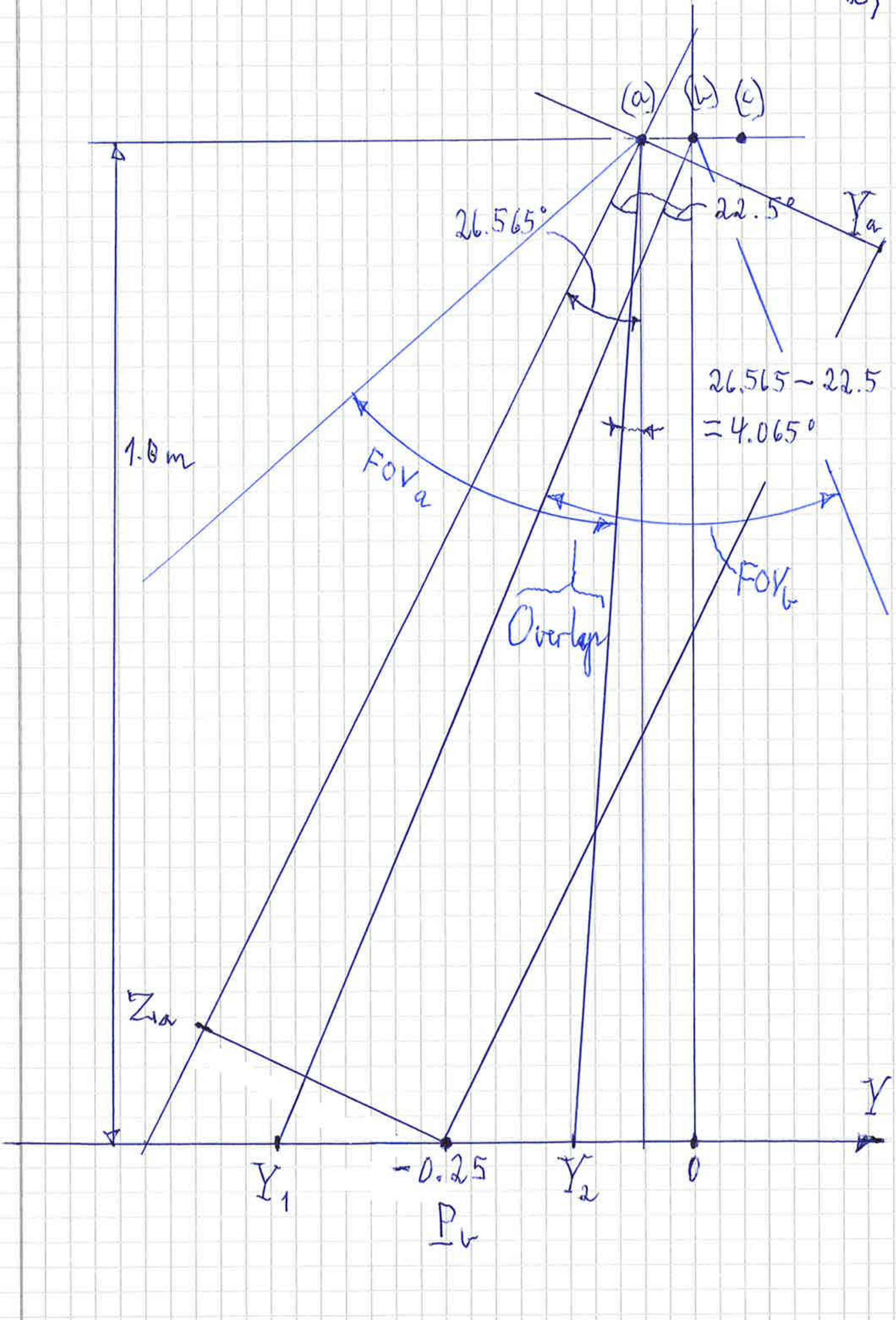
Overlap region $Y \in [Y_1, Y_2]$ where

$$Y_1 = -(1m) \cdot \tan(22.5°) = -0.4142 m$$

$$Y_2 = -(1m) \cdot \tan(4.065°) - 0.05m = -0.1211 m$$

$$\underline{\underline{Y \in [-0.414, -0.121] m}}$$

2)



(a)
(b)
(c)
22.5°
26.565°
Y_a
26.565 − 22.5
= 4.065°
1.8 m
FOV_a
FOV_b
Overlap
Z_a
Y
Y_1
−0.25
Y_2
0
$\underline{P}_b$

3)

c) The left camera ((b)) is used as reference (world coordinates). The rotation is done clockwise with angle φ on the camera (a) coordinates. The rotation matrix is

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8944 & 0.4472 \\ 0 & -0.4472 & 0.8944 \end{bmatrix}}}$$

The translation is along Y_b (for $\underline{P}_a = 0$, $\underline{P}_b = \underline{t}$)

$$\underline{t} = \underline{\underline{\begin{bmatrix} 0 \\ -0.05\,m \\ 0 \end{bmatrix}}}$$

Check of this result by using $\underline{P}_b = [0 \quad -0.25m \quad -1.0m]^T$

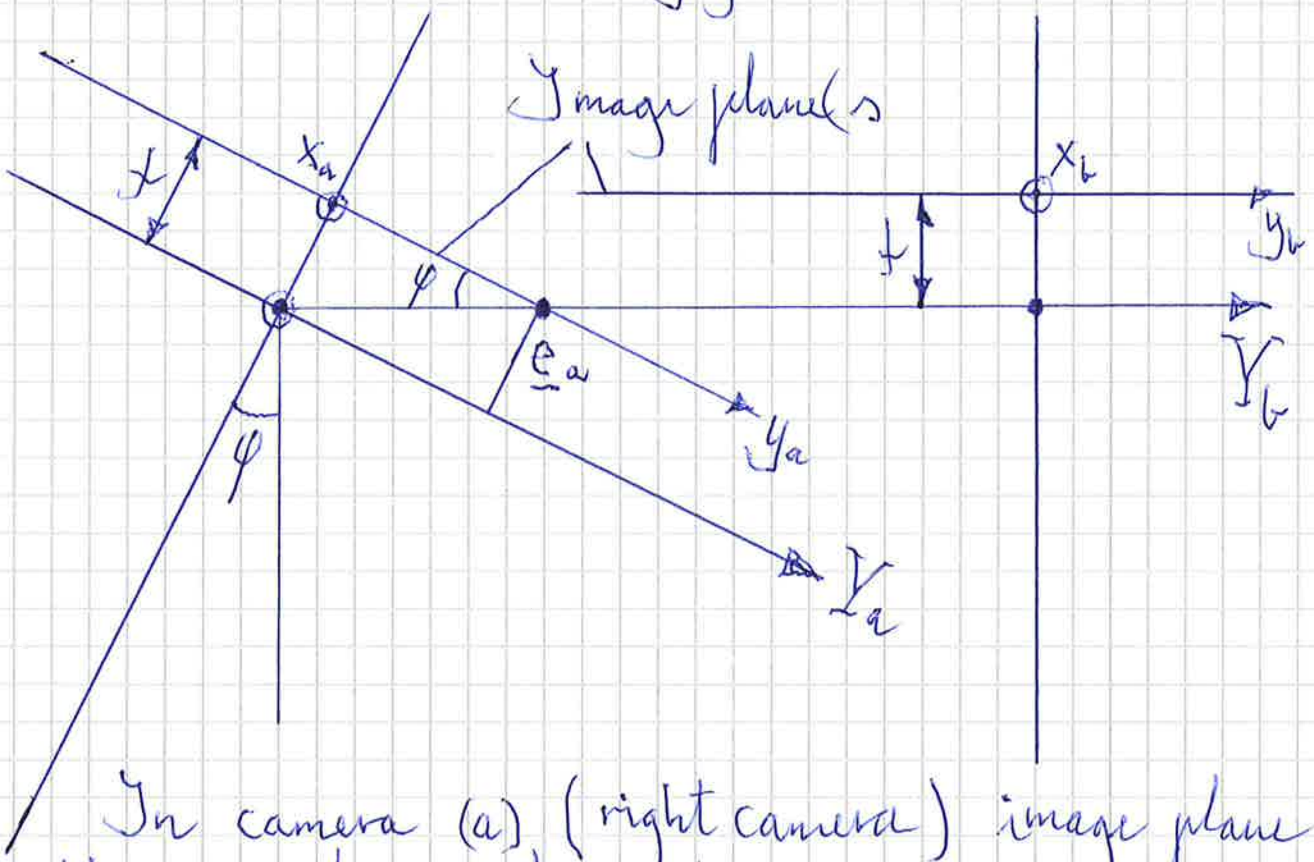
$$\underline{P}_a = R^T(\underline{P}_b - \underline{t}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8944 & -0.4472 \\ 0 & 0.4472 & 0.8944 \end{bmatrix} \begin{bmatrix} 0 - 0 \\ -0.25 + 0.05 \\ -1.0 - 0 \end{bmatrix}$$

$$= [0 \quad -0.2 \cdot 0.8944 \quad -0.2 \cdot 0.4472]^T$$

$$= \underline{\underline{[0 \quad 0.2683 \quad -0.9839]^T m}}$$

This is correct according to the figure on the previous page.

d) The position of the epipoles are found by the crossing point between the image plane and the base line (between the two camera centers). See figure below:



In camera (a) (right camera) image plane the epipole is at position:

$$\underline{e}_a = \begin{bmatrix} 0 \\ \frac{f}{\tan\varphi} \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ f/0.5 \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ 2f \\ f \end{bmatrix} \approx \begin{bmatrix} 0 \\ 14.5 \\ 7.24 \end{bmatrix} \text{mm}$$

For camera (b) (left camera) the epipole is at infinity.

$$\underline{e}_b = \lim_{\varphi \to 0} \begin{bmatrix} 0 \\ \frac{f}{\tan\varphi} \\ f \end{bmatrix} \to \begin{bmatrix} 0 \\ -\infty \\ f \end{bmatrix}$$

e) $\Delta x = \Delta y = \frac{w}{M} = \frac{6.0 \cdot 10^{-3}}{1000} = 6.0 \cdot 10^{-6} m = \underline{6.0 \mu m}$

$\alpha = \beta = \frac{f}{\Delta x} = \frac{f}{\Delta y} = \frac{7.24 \cdot 10^{-3}}{6.0 \cdot 10^{-6}} = \underline{\underline{1207.1}}$

$x_0 = y_0 = \frac{1000}{2} = \underline{\underline{500}}$

$$\underset{\sim}{K} = \begin{bmatrix} 1207.1 & 0 & 500 \\ 0 & 1207.1 & 500 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{\text{zero skew}}$$

f) For camera (b) we need no rotation and the translation is given as follows:

$$\underset{\sim}{R} = \underset{\sim}{I}d_{3\times3}, \quad \underline{t} = [0 \;\; 0 \;\; -1m]$$

The camera matrix is then:

$$\underset{\sim}{M} = \underset{\sim}{K}[\underset{\sim}{R} \;\; \underline{t}] = \begin{bmatrix} 1207.1 & 0 & 500 \\ 0 & 1207.1 & 500 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1207.1 & 0 & 500 & -500 \\ 0 & 1207.1 & 500 & -500 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Or normalize by multiplic. with -1.

Exercise 4 a)

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -1000 & 0 & -250 & 500 \\ 0 & -1000 & -250 & 500 \\ 0 & 0 & -0.5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -V_y t + 0.2 \cdot j \\ 0 \\ 1 \end{bmatrix}$$

The third row gives $\lambda = 1$, and we get

$x = 500$ (center column in the image)

$y = 1000\, V_y t - 200 j + 500$

$$V_y \cdot t = 0.5 \cdot 0.01 \cdot k \quad , \quad k = 0, 1, 2, \ldots$$
$$= 5 \cdot 10^{-3} \cdot k$$

$$y(k) = 1000 \cdot 5 \cdot 10^{-3} \cdot k - 200 j + 500$$
$$= 5 \cdot k - 200 j + 500$$

k is the time index and j the spatial position

The crossing points move 5 pixels in the image y-direction for each time step. The optical flow is then:

$$\underline{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

This is valid for all points in the ground plane.

b) We have a parallell motion of the camera with constant distance. Then the optical flow field is parallell with equal value in all points!

c) We use the same approach as in a).

The crossing points can now be described as follows:

$$\underline{P}_{ij}(t) = \begin{bmatrix} 0.2 \cdot i \\ 0.2 \cdot j \\ -V_z \cdot t \\ 1 \end{bmatrix} \qquad \begin{array}{l} t = 0.01 \cdot k, \; k \in \{0, 1, 2, \dots \\ i, j \in \{\dots -2, -1, 0, 1, 2 \dots\} \\ V_z = \end{array}$$

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -1000 & 0 & -250 & 500 \\ 0 & -1000 & -250 & 500 \\ 0 & 0 & -0.5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.2 \cdot i \\ 0.2 \cdot j \\ -V_z t \\ 1 \end{bmatrix}$$

$$\lambda x = -200 \cdot i + 250 V_z t + 500 = -200 \cdot i + 500(0.5 V_z t + 1)$$

$$\lambda y = -200 \cdot j + 250 V_z t + 500 = -200 \cdot j + 500(0.5 V_z t + 1)$$

$$\lambda = 0.5 V_z t + 1$$

$$\underline{\underline{x = 500 - \frac{200 \cdot i}{0.5 V_z t + 1}, \quad y = 500 - \frac{200 \cdot j}{0.5 V_z t + 1}}}$$

$$V_z t = 10 \cdot 0.01 \cdot k = 0.10 \cdot k$$

$$x(k) = 500 - \frac{200 \cdot i}{1 + 0.05 \cdot k}, \quad y(k) = 500 - \frac{200 \cdot j}{1 + 0.05 \cdot k}$$

d) $x(0) = 500 - 200i = \begin{cases} 700, & i=-1 \\ 500, & i=0 \\ 300, & i=1 \end{cases}$

$y(0) = 500 - 200j = \begin{cases} 700, & j=-1 \\ 500, & j=0 \\ 300, & j=1 \end{cases}$

$x(1) = 500 - \frac{200i}{1.1} = \begin{cases} 682, & i=-1 \\ 500, & i=0 \\ 318, & i=1 \end{cases}$

$y(1) = 500 - \frac{200j}{1.1} = \begin{cases} 682, & j=-1 \\ 500, & j=0 \\ 318, & j=1 \end{cases}$

$v_x = x(1) - x(0)$, $v_y = y(1) - y(0)$, $\underline{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$

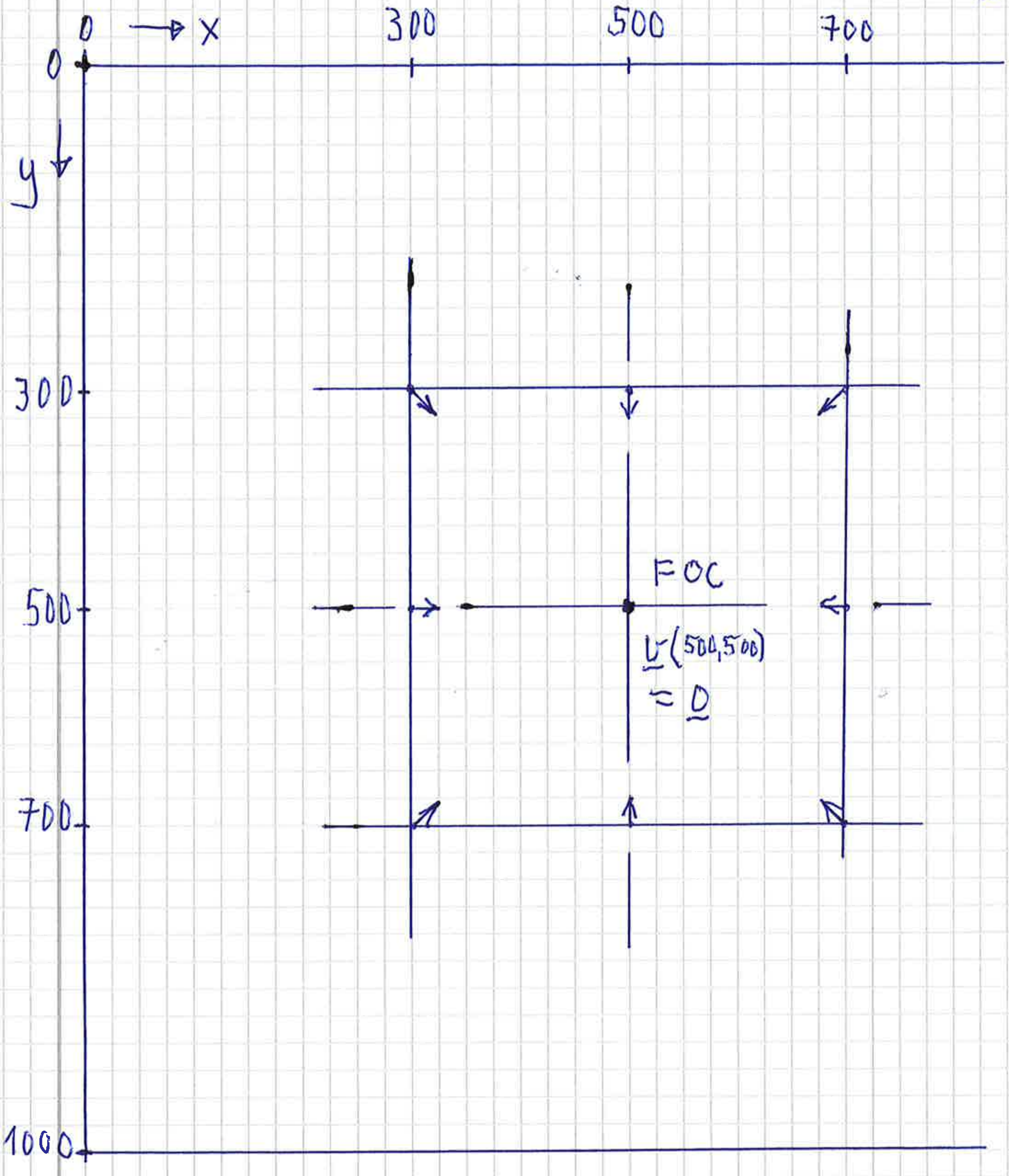
$\underbrace{\underline{v}(0,0)}_{500,500} = \begin{bmatrix} 500-500 \\ 500-500 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Focus of contraction

$\underbrace{\underline{v}(1,0)}_{300,500} = \begin{bmatrix} 318-300 \\ 500-500 \end{bmatrix} = \begin{bmatrix} 18 \\ 0 \end{bmatrix}$

$\underbrace{\underline{v}(1,1)}_{300,300} = \begin{bmatrix} 318-300 \\ 318-300 \end{bmatrix} = \begin{bmatrix} 18 \\ 18 \end{bmatrix}$

Contracting velocity field when we move away from the tiles.

See figure next page.



FOC = Focus of contraction