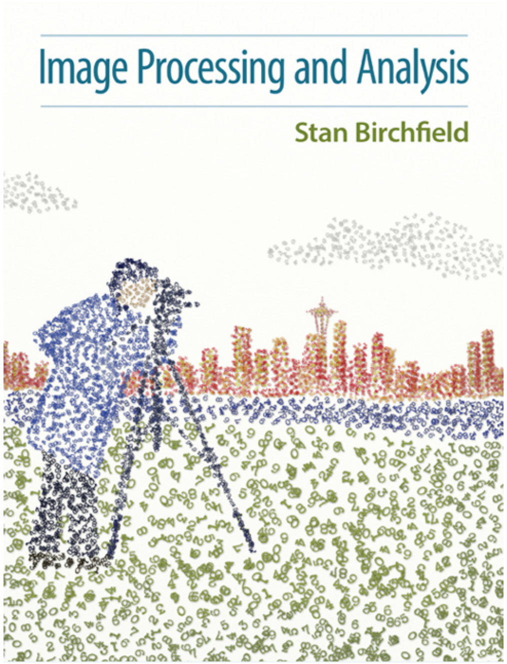
Prof. Kjersti Engan

ELE510 Image processing and computer vision

Frequency Domain processing, Frequency domain filtering (Chap6 Birchfield)
2020



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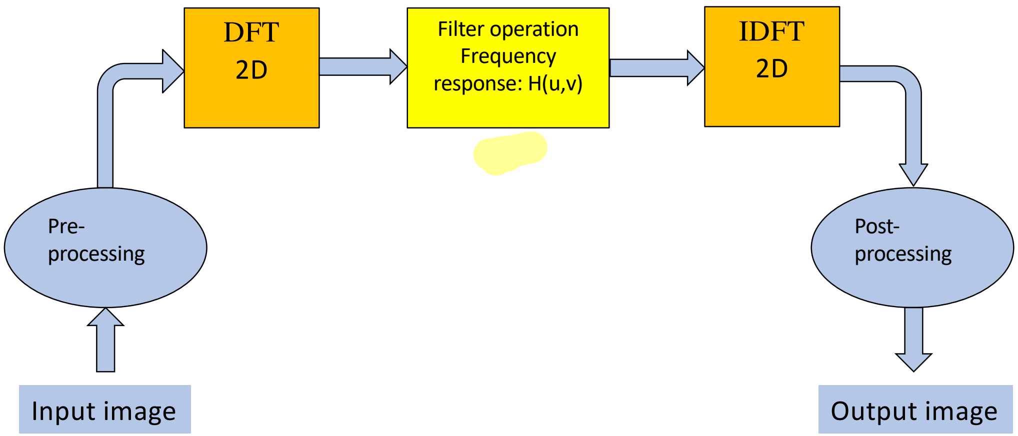
(6.4) Filtering in the Frequency Domain

By using the Convolution Theorem we can do linear filtering in the frequency domain. This is more Computational Efficient than spatial domain filtering when the Filter Kernel is larger than approximately (8x8) pixels.



Frequency domain filtering

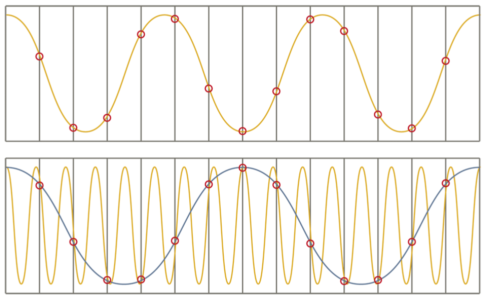
$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$y(n) = \sum_k h(k)x(n-k)$$

$$F(y(n)) = Y(w) = H(w)X(w)$$

- Linear filters : convolution in time (space) domain is mulitplicatioin in freq.domain. (1D and 2D)

Frequency domain filtering



Digital images/signals –> digital frequency

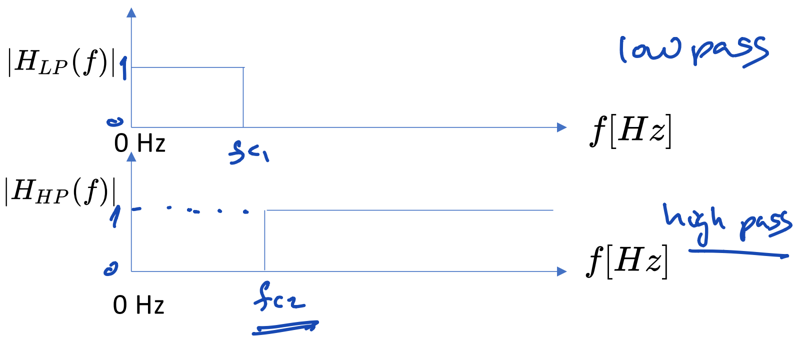
Remember the aliasing discussion.

Can only differentiate between [0, Fs] where Fs is sampling freq.

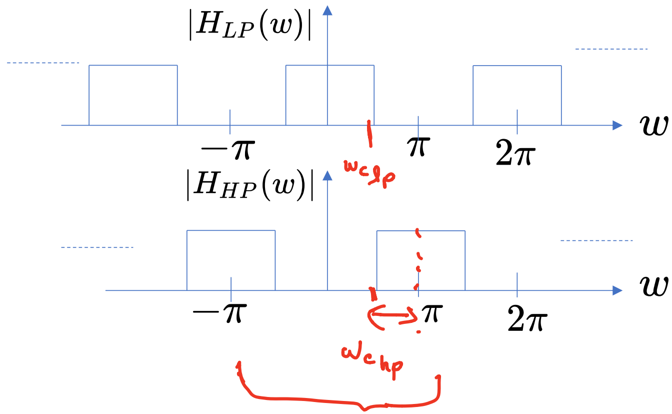
Equivalent to: [-Fs/2, Fs/2] (periodic)

We often describe this as the digital frequency, w: [0, 2π] or [-π, π].

Analog filters:



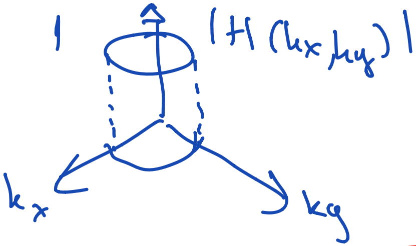
Digital filters:



Ideal filters

What does it mean in the space/time domain if we demand ideal filters in the frequency domain?

2D



1D

$$|H(\omega)| = \begin{cases} 1, & \text{if } -\omega_0 < \omega < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

<u>LP</u>

DTFT $\quad X(\omega) = \mathcal{F}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$

IDTFT $\quad x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} 1 \cdot e^{j\omega n}\, d\omega$$

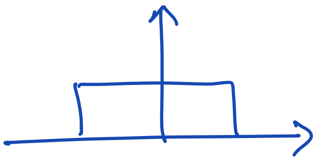
$$e^{j\omega n} = \cos(\omega n) + j\sin(\omega \cdot n)$$

$$h(n) = \frac{1}{2\pi}\left[\underbrace{\int_{-\omega_0}^{\omega_0} \cos \omega \cdot n\, d\omega}\; + j \underbrace{\int_{-\omega_0}^{\omega_0} \sin(\omega n)\, d\omega}_{=0 \text{ (odd symetric)}}\right]$$

$$h(n) = \frac{1}{2\pi}\left[\frac{\sin \omega \cdot n}{n}\right]_{-\omega_0}^{\omega_0} = \cancel{2} \cdot \frac{1}{\cancel{2}\pi}\,\frac{\sin \omega_0 n}{n}$$

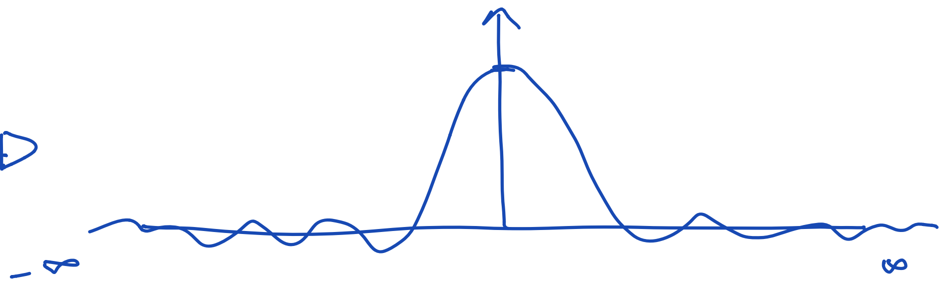
$$h(n) = \frac{\sin \omega_0 n}{\pi n} = \frac{\omega_0}{\pi} \cdot \mathrm{sinc}(\omega_0 \cdot n)$$

$$\boxed{\mathrm{sinc}\, x = \frac{\sin x}{x}}$$

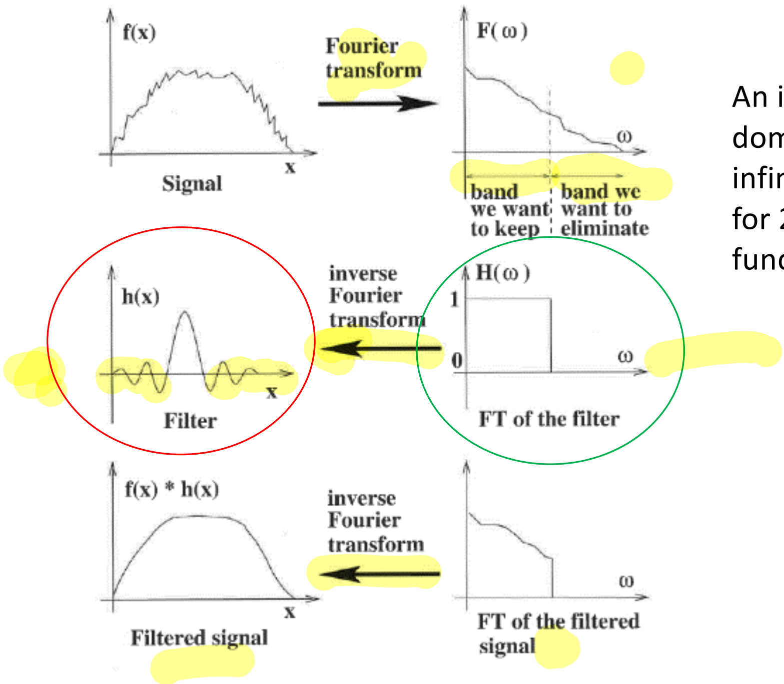


freq. dom.

$\Rightarrow$



Ideal filter, frequency domain

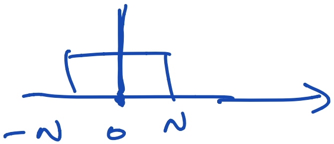


An ideal filter in the frequency domain corresponds to an infinite impuls respons (IIR).
for 2D: infinite point spread function

Simple filter in the space domain

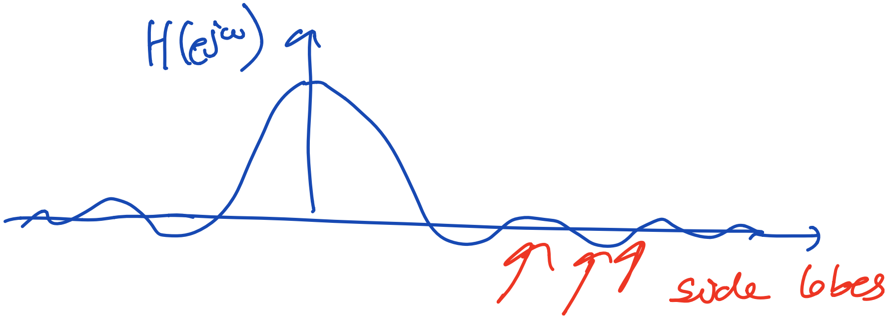
What does a box-filter in space domain mean for the frequency domain?

$$h(k) = \begin{cases} 1 & k = -N, \dots, -1, 0, 1, \dots N \\ 0 & \text{otherwise} \end{cases}$$



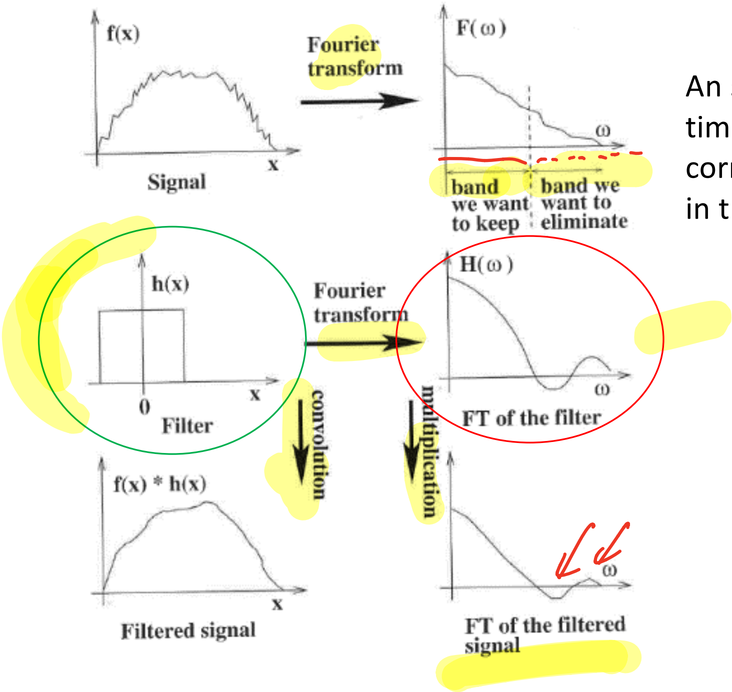
$\Downarrow$

sinc function in the freq. dom.



can create high freq. artifacts, ringing.

Simple filter in time/space domain



An simple mean filter in the time/space domain corresponds to high sidelobes in the frequency domain

No filter can have finite extension in both spatial and frequency domain.

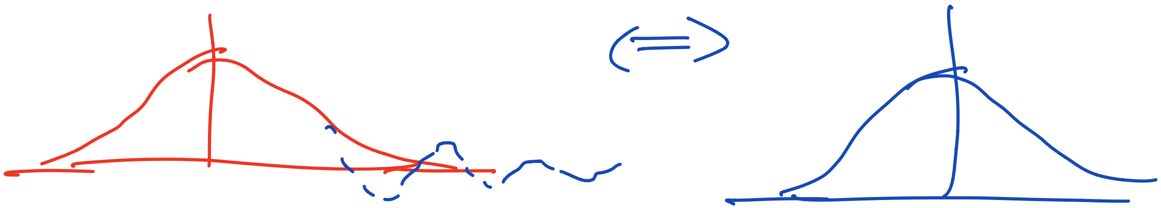
Any filter:

- infinite in space domain
- infinite in frequency domain
- or both.

What is a good trade off?

Fourier transform of a Gaussian i still a Gaussian.

For image processing Gaussian filters turns out to be a good balance, but not very steep cut-off.



Low pass filtering for noise reduction

White noise affects all frequencies.

Information of the image is often in low freq.

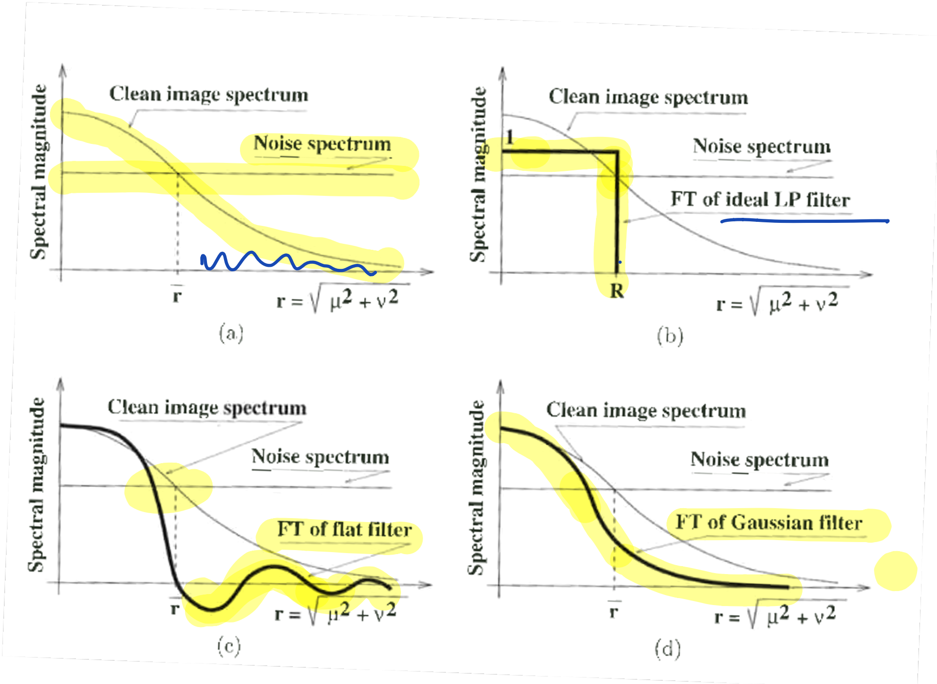
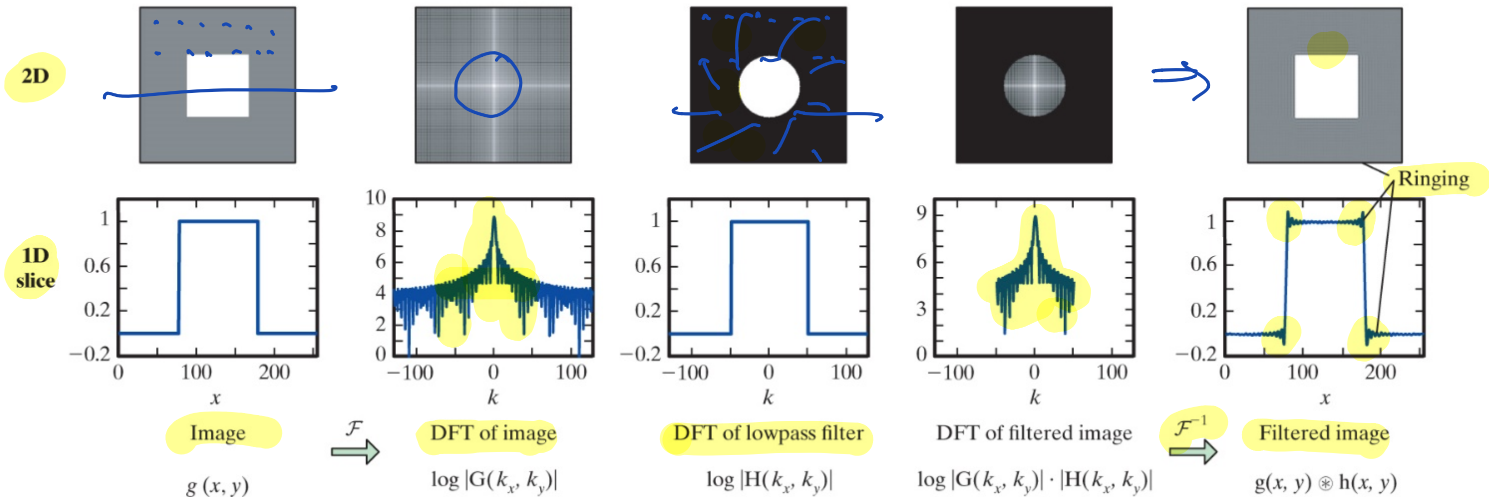
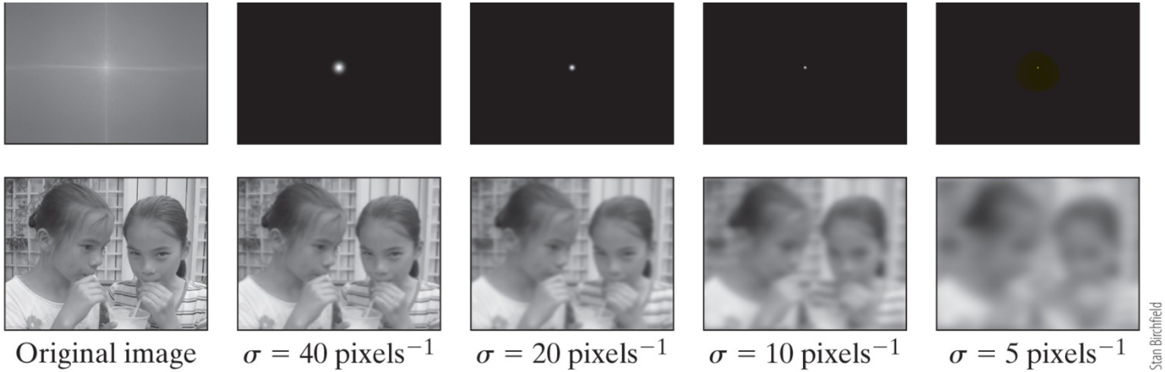
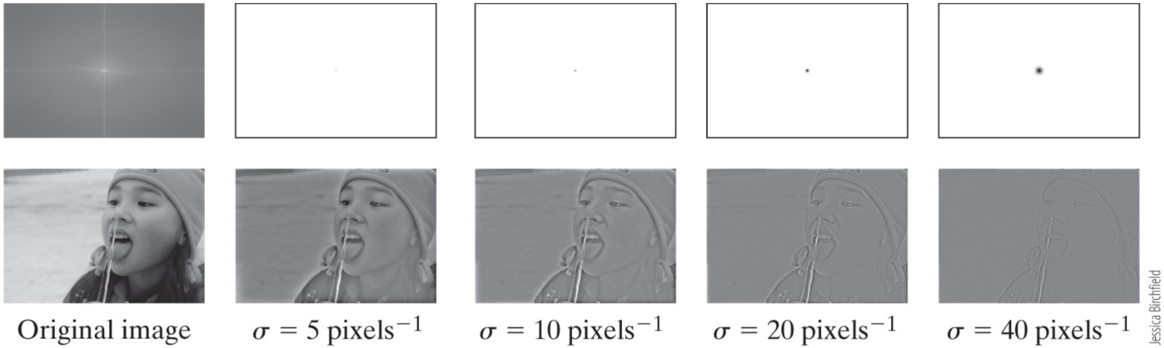


Figure 6.14 The process of frequency-domain filtering. From left to right: The DFT of the image is computed and multiplied by the frequency-domain filter, followed by the inverse DFT to yield the filtered image. Notice in this example that the ideal lowpass filter causes significant ringing in the output.





Above: Gaussian LP filtering in frequency domain, below: Gaussian HP filtering in frequency domain.
Top rows: DFT of image and filters, where dark values corresponds to attenuated frequencies.



$$|H_{HP}(w)| = 1 - |H_{LP}(w)|$$