Solution to exam: ELE510, December 4th 2017.

The two first exercises are given a brief suggestion for solution with reference to the textbook and the compendium.

Exercise 1

a) 1) Image resolution: "***Spatial resolution** is the ability of the image formation process to distinguish or separate close details in the scene."* Usually given by the number of pixels or the size of each pixel. However, the actual resolution also depends on the optical system, (see lecture presentations). Quantization is the process of representing each intensity value with a finite number of bits (see Compendium, part I, appendix A).
2) A 2D separable operator is a product of two 1D operators $h(i,j) = h_1(i) \cdot h_2(j)$ (see "ConvolutionNotesLinFilters").
3) The stacking operator stacks the image columns into a single column vector (similar to the Matlab (:)-operator imcol = Im(:); See textbook Box 1.3).

b) If the number of non-zero eigenvalues is *r*, then we can write:

$$\mathbf{F} = \sum_{i=1}^{r} \lambda_i^{\frac{1}{2}} \mathbf{u}_i \mathbf{v}_i^T \quad \text{where} \quad \mathbf{u}_i \mathbf{v}_i^T \; i \in \{1, 2, \cdots, r\} \text{ are the eigenimages}$$

We can find an approximation by keeping k<r terms:

$$\mathbf{F}_k = \sum_{i=1}^{k} \lambda_i^{\frac{1}{2}} \mathbf{u}_i \mathbf{v}_i^T$$

See textbook and lecture presentation.

c) The DFT is separable. If we consider image (a) and (c) they are either zero or constant along columns. This results in a contribution only at frequency 0 in the vertical direction; we get a 1D response along a horizontal line in the center of the image representing the frequency spectrum. For image (a) we have a periodic sine-wave that give three impulses in the spectrum, one at the negative actual frequency and one at the positive frequency in addition to the "DC" component (frequency (0,0)) caused by the positive image mean value. The result in (c) depends on the width of the white stripe. If the width is one pixel the result is a flat (allpass) spectrum. When the width increases, the result is a sinc-function with increasing number of zero-crossings and decreasing main-lobe width. For image (b) we get a sinc-function representing each of the directions, giving a "cross"-like pattern. See Example 1 and 2 in the lecture presentation.

d) Equation (7) cannot give a flat result. The number of pixels in each bin cannot change; it can only be moved to another position or added to another bin. See textbook and LAB 3.

Exercise 2

a) See textbook. A linear mean or Gaussian filter can be used for Gaussian noise, but a very poor for salt and pepper noise. For salt and pepper noise we use a non-linear filter, a median filter works very well!

b) The Prewitt filter mask, derivative along rows is

$$\mathcal{H}_{\mathrm{Prewitt}-x} = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

The output image is the convolution between this filter and the input image in (1). The result for the central 2x2 pixels is (not use of zero-padding)

$$I_x = \begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$$

The image gradient is found as a vector where the first component is the result above, and the second is the result from using the y-derivative Prewitt filter:

$$\mathcal{H}_{\mathrm{Prewitt}-y} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix}$$

See textbook, lecture presentation and LAB 4.

c) i)

$$\begin{aligned} f''(n) &= f'(n+1) - f'(n) = f(n+2) - f(n+1) - f(n+1) + f(n) \\ &= f(n+2) - 2f(n+1) + f(n) \end{aligned}$$

This represents the 1D-filter [1 -2 1]. The Laplacian is the sum of the second derivative in the x-direction and the second derivative in the y-direction. We therefore get

$$\mathcal{H}_{Laplacian} = \mathcal{H}_{xx} + \mathcal{H}_{yy} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This is the negative result of equation (2). For the use of the Laplacian the sign is not important, it is common practice to use the form with a positive peak in the center (Mexican hat).

ii) The differentiation amplifies high frequency content (noise) in the image. Therefore, it is useful to smooth (LP-filter) the image before the differentiation. A Gaussian filter is often used. Then by combining the filter mask for the Gaussian and the Laplacian we get the Laplacian of Gaussian filter.

d) The solution to this problem is found by using equation (11) in the formulas at the end of the exam. The answer is then t = 5.25. However, the probability functions given make no sense since their values are negative for most t-values. They does not look like probability functions. See textbook, chapter 6, Figure 6.4.

<u>Exercise 3</u> and **<u>Exercise 4</u>**

Solution is given in a separate document.