University
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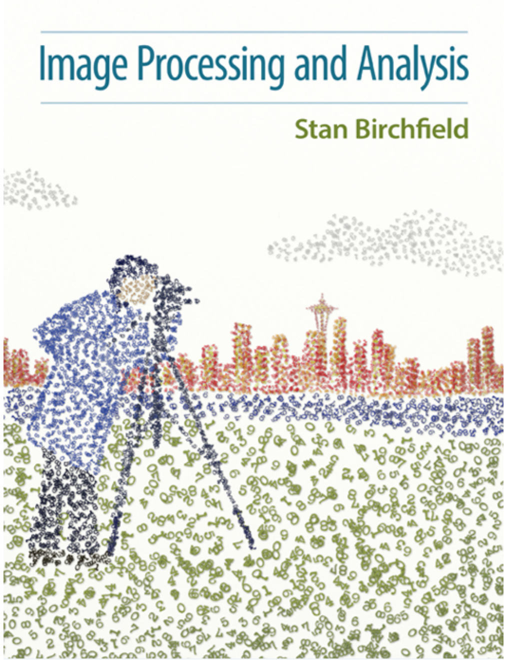
ELE510 Image processing and computer vision

Geometry of multiple views, (chap 13.6 Birchfield) 2020



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13.3 non-rectified cam.

$$\underset{\sim}{F} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 3 & 3 & 1 \end{bmatrix}$$

can $\underline{x} \Rightarrow (x, y) = (100, 50)$

$\underline{x}' \Rightarrow (x', y') = (78, 40)$

can these be corresponding points?

$$\underline{l}' = \underset{\sim}{F} \cdot \underline{x} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 3 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 100 \\ 50 \\ 1 \end{bmatrix} = \begin{bmatrix} 201 \\ -402 \\ 451 \end{bmatrix}$$

know $\underline{x}'^T \cdot \underline{l}' = 0$ point have to be on the line

test $\begin{bmatrix} 78 & 40 & 1 \end{bmatrix} \cdot \begin{bmatrix} 201 \\ -402 \\ 451 \end{bmatrix} = 49 \neq 0$

does not look like corresponding points

BUT need to normalize first

The Essential Matrix

- Like the fundamental matrix **F**, the essential matrix, **E**, captures the geometric relationship between the cameras in a compact matrix form.

- The fundamental matrix relates *uncalibrated* cameras (pixel values), while the essential matrix relates *calibrated* cameras (meter).
- If we know **F** or **E** we can find the other with the help of K and K' (the matrices of intrinsic camera parameters for the two cameras)

- The relationship between the two matrices is: $\boxed{\mathbf{F} = \mathbf{K}'^{-\mathsf{T}}\mathbf{E}\mathbf{K}^{-1}, \text{ or } \mathbf{E} = \mathbf{K}'^{\mathsf{T}}\mathbf{F}\mathbf{K}}$
- E has 5 parameters: 3 for rotation and 2 for translation in direction between the cameras.

$$\underset{\sim}{K} = \begin{bmatrix} \alpha & \boxed{\gamma} & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$=0$

The Essential Matrix

K and K' are the intrinsic calibration matrices for the two cameras

$\bar{\mathbf{x}}$ and $\bar{\mathbf{x}}'$ are metric coordinates $\quad$ $\mathbf{x}$ and $\mathbf{x}'$ are pixel coordinates

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

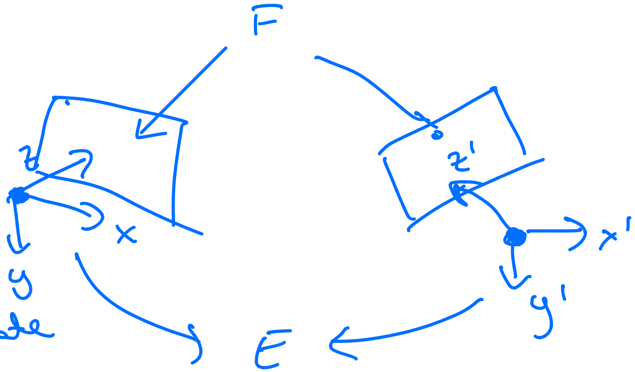
* $\underline{x} = \underset{\sim}{K} \cdot \underline{\bar{x}}$ $\qquad$ $\underline{x}' = \underset{\sim}{K}' \cdot \underline{\bar{x}}'$ $\Rightarrow$ $\underline{\bar{x}}' = \underset{\sim}{K}'^{-1} \cdot \underline{x}'$

Fundamental matrix $\quad \boxed{\underline{x}'^T \underset{\sim}{F}\, \underline{x} = 0}$

Essential matrix $\quad \underline{\bar{x}}'^T \underset{\sim}{E}\, \underline{\bar{x}} = 0$ $\qquad$ $\underline{\bar{x}} = \underset{\sim}{K}^{-1} \cdot \underline{x}$

$$\Rightarrow (\underset{\sim}{K}'^{-1} \underline{x}')^T \underset{\sim}{E}\, \underset{\sim}{K}^{-1} \underline{x} = \underline{x}'^T \underbrace{\underset{\sim}{K}'^{-T} \underset{\sim}{E}\, \underset{\sim}{K}^{-1}}_{\underset{\sim}{F}} \underline{x} = 0$$

$\underset{\sim}{F} = \underset{\sim}{K}'^{-T} \underset{\sim}{E}\, \underset{\sim}{K}^{-1}$ or $\underset{\sim}{E} = \underset{\sim}{K}'^{T} \underset{\sim}{F}\, \underset{\sim}{K}$



world coordinate system

is not world

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{t} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\left.\begin{matrix} \underset{\sim}{R} \\ \underline{t} \end{matrix}\right\}$ describing right camera syst. relative to world. $\Rightarrow$ left camera.

$$E = \begin{bmatrix} \underset{\sim}{R} & \underline{t} \end{bmatrix}$$

Calibration and Reconstruction 3D world

- The general procedure to reconstruct the metric geometry of a scene from point correspondences taken by a pair of calibrated cameras:

calibrate stereo setup

  - (1) estimate the fundamental matrix from the correspondences; F
  - (2) construct the essential matrix using the intrinsic camera calibration parameters and the fundamental matrix; K, K' $\Rightarrow E$
  - (3) decompose the essential matrix into rotation and translation (up to scale); and
  - (4) estimate the 3D coordinates of world points. $\mapsto$

Reconstruction

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \propto \underset{3\times 4}{\mathbf{P}} \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$

P is camera projection matrix

$$\underset{\sim}{P} = \underset{\sim}{K} \cdot [\underset{\sim}{R} \quad \underline{t}]$$

intrinsic

m → pixels

extrinsic.
camera coords coord. syst. relative to the world.

$$\boxed{\begin{aligned} \underset{\sim}{P} &= \underline{\underset{\sim}{K}} \cdot [\underset{\sim}{I} \quad 0] \\ \underset{\sim}{P}' &= \underline{\underset{\sim}{K}}' \cdot \underbrace{[\underset{\sim}{R} \quad \underline{t}]}_{\text{cam 2 rel. to cam 1.}} \end{aligned}}$$

$$\underline{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

cam 2 rel. to
cam 1.

Estimating the fundamental matrix

- If we let **x** and **x'** be points in the two images, then we can rewrite the fundamental matrix equation by explicitly listing the individual elements of the matrix and vectors as follows:

$$\boxed{\mathbf{x}'^{T}\mathbf{F}\mathbf{x} = 0} \qquad \mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = [x' \;\; y' \;\; 1] \underbrace{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

- Rearranged we can collect the nine unknowns (f_{ij}) in a vector:

$$\mathbf{A}_{\{n\times 9\}}\mathbf{f}_{\{9\times 1\}} = \mathbf{0}_{\{9\times 1\}}$$

- Where $n \geq 8$ is the number of corresponding pairs $\boxed{(x_i\,, y_i) \Leftrightarrow (x'_i\,, y'_i)}$
- Can solve for F, also need to make sure is a valid fundamental matrix

Decomposing the Essential Matrix

- From **F** and **K, K'** we can find **E**: $\boxed{\mathbf{F} = \mathbf{K}'^{-\top}\mathbf{E}\mathbf{K}^{-1}, \text{ or } \mathbf{E} = \mathbf{K}'^{\top}\mathbf{F}\mathbf{K}}$
- Given an essential matrix **E**, we would like to be able to extract the translation vector **t** and rotation matrix **R**.
- This describes the rotation and translation between the two cameras.
- If we have the intrinsic parameters (**K**, and **K´**) and **R** and **t**, we can find the camera projection matrices **P**, and **P´**

$$\boxed{\mathbf{P} = \mathbf{K}\,[\,\mathbf{R} \;\; \mathbf{t}]}$$

- Decomposing **E** is can be done by computing its SVD and applying some properties. We will not dig into that – some details in the book.

Finding the Camera projection Matrices

- The Essential matrix, E, can be decomposed into $[\mathbf{R}_{\{3\times3\}}\mathbf{t}_{\{3\times1\}}]$

describing rotation and translation between the two cameras.

- When that is done we can estimate the camera matrices. The left camera coordinate system is the world system, so no rotation and translation for P:

$$\mathbf{P} = \mathbf{K}[\mathbf{I}_{\{3\times3\}}\mathbf{0}_{\{3\times1\}}]$$

$$\mathbf{P}' = \mathbf{K}'[\mathbf{R}_{\{3\times3\}}\mathbf{t}_{\{3\times1\}}]$$

ALGORITHM 13.15 Estimate the projection matrices of a pair of internally calibrated cameras

ESTIMATECAMERAPROJECTIONMATRICES$(\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}_{i=1}^n, \mathbf{K}, \mathbf{K}')$

Input: n corresponding pairs of points $(x_i, y_i) \leftrightarrow (x'_i, y'_i)$ between two images intrinsic camera parameters $\mathbf{K}$ and $\mathbf{K}'$

Output: the 3×4 camera projection matrices $\mathbf{P}$ and $\mathbf{P}'$

```
F ← EightPointFundamental({x_i ↔ x'_i}_{i=1}^n)
E ← K'^T F K
R, t ← DecomposeEssentialMatrix(E)
P ← K [I_{3×3}  0_{3×1}]
P' ← K' [R  t]
return P, P'
```

Calibration stereo - summary

Calibration for stereo setup means finding P and P’ camera projection matrices:

1) use Zhangs calibration algorithm to find K and K’ (each camera alone)
2) Need at least 5, typically 8 correspondence points x,x’ to be able to estimate F
   (need to search a larger space. Dont know the epipolar lines yet)
3) Find E from F, K and K’
4) Decompose E to get R and t

Doing 3D reconstruction

- When we have estimated the camera projection matrices P and P', i.e. calibrated our stereo vision setup, we can use it to reconstruct 3D world:
- Find corresponding points (**x, x'**). We know the fundamental matrix **F**, so this is a 1D search on the epipolar lines.
- Estimate the 3D world point **w** from (**x, x', P** and **P')**.

Computing 3D Point Coordinates

$$\begin{array}{c} \underline{x} \\ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \end{array} \propto \underset{\sim}{P} \begin{array}{c} w \\ \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \end{array}$$

- Given the camera projection matrices $\mathbf{P}_{\{3x4\}}$ and $\mathbf{P'}_{\{3x4\}}$, the 3D coordinates of a world point $\mathbf{w}$ can be estimated from its projections onto the two images.

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1^\top \\ \mathbf{p}_2^\top \\ \mathbf{p}_3^\top \end{bmatrix} \quad \text{and} \quad \mathbf{P}' = \begin{bmatrix} \mathbf{p}_1'^\top \\ \mathbf{p}_2'^\top \\ \mathbf{p}_3'^\top \end{bmatrix}$$

- Let $\mathbf{w}_{\{4\times1\}}$ be the homogeneous coordinates of the world point. coordinates of the image point (in pixels) are then given by:

$$(x, y) = \left(\frac{\mathbf{p}_1^\top \mathbf{w}}{\mathbf{p}_3^\top \mathbf{w}}, \frac{\mathbf{p}_2^\top \mathbf{w}}{\mathbf{p}_3^\top \mathbf{w}} \right) \quad \text{and} \quad (x', y') = \left(\frac{\mathbf{p}_1'^\top \mathbf{w}}{\mathbf{p}_3'^\top \mathbf{w}}, \frac{\mathbf{p}_2'^\top \mathbf{w}}{\mathbf{p}_3'^\top \mathbf{w}} \right)$$

$x \qquad y \qquad\qquad\qquad x' , y'$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \propto \begin{bmatrix} - p_1^T - \\ p_2^T \\ - p_3^T - \end{bmatrix} \begin{bmatrix} xw \\ yw \\ zw \\ 1 \end{bmatrix}$$

$$\underline{x} \quad \propto \quad \underset{\sim}{p} \quad \underline{w}$$

$$\Rightarrow \quad \underline{x} \propto p_1^T \cdot \underline{w}$$

$$x = \frac{p_1^T \underline{w}}{p_3^T \cdot \underline{w}}$$

$$y \propto p_2^T \cdot \underline{w} \qquad y = \frac{p_2^T \cdot \underline{w}}{p_3^T \cdot \underline{w}}$$

$$1 = \frac{p_3^T \cdot w}{p_3^T \cdot w} = 1$$

$$x\, \underline{p}_3^T \cdot \underline{w} = p_1^T \cdot \underline{w} \quad \Rightarrow \quad x\, p_3^T \underline{w} - \underline{p}_1^T \underline{w} = 0$$

$$y\, \underline{p}_3^T \underline{w} = p_2^T \cdot \underline{w} \quad \Rightarrow \quad y\, p_3^T \underline{w} - p_2^T \underline{w} = 0$$

x', y' do the same using P'

$$(x\, \underline{p}_3^T - \underline{p}_1^T) \cdot \underline{w} = 0$$

$$(y\, p_3^T - p_2^T) \cdot \underline{w} = 0$$

Put together

$$\begin{bmatrix} x\underline{p}_3^T - \underline{p}_1^T \\ y\underline{p}_3^T - \underline{p}_2^T \\ x'\underline{p}_3'^T - \underline{p}_1'^T \\ y'\underline{p}_3'^T - \underline{p}_2'^T \end{bmatrix} \cdot \underline{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{w} = \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$

Want to find.

using $(x, y) \leftrightarrow (x', y')$

P, P'

can estimate $\underline{w} \Rightarrow$ 3D, includes depth!

3D Reconstruction from Image Pairs - summary

- Find interest points
- Match interest points
- Compute fundamental matrix F
- Calibrate cameras (Zhangs), find K, K´
- Estimate E from F, K and K´
- Compute camera matrices P and P’ from E

Calibrating stereo setup

[R t]

- For each matching image points x and x’, compute world point in scene using (x,y), (x´,y´) P and P´ (Use F to find matching pairs)

We can use the fundamental matrix **F** to limit correspondence to 1D search for general stereo camera positions in the same way as is possible for simple stereo