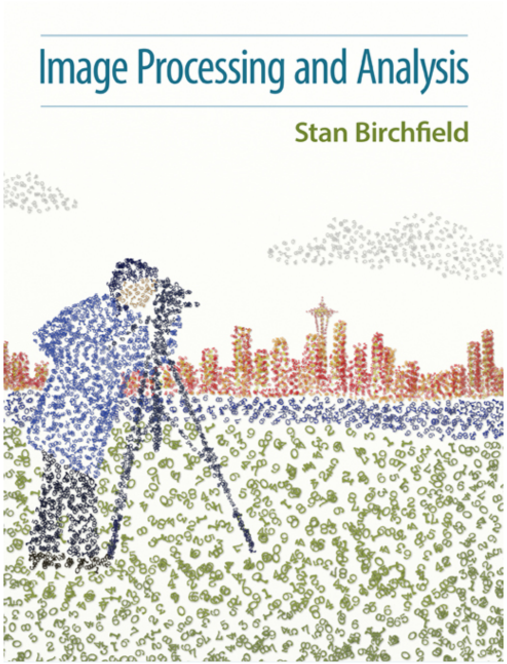
University of Stavanger

Prof. Kjersti Engan

ELE510 Image processing and computer vision

Frequency Domain processing, 2D DFT (Chap 6.3 Birchfield) 2020



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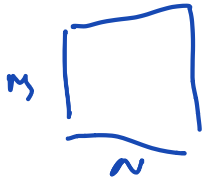
(6.3) Discrete Fourier Transform – 2D

The 2D DFT is a natural extension of the 1D case:

$\hat{g}$



DFT of image $\boxed{g(k,l)}$

$$\mathcal{F}(g(m,n)) = \hat{g}(m,n) = \boxed{\frac{1}{MN}} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} g(k,l) e^{-j2\pi\left(\frac{km}{M} + \frac{ln}{N}\right)}$$

IDFT
(inverse DFT)

$$g(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \hat{g}(m,n) e^{j2\pi\left(\frac{km}{M} + \frac{ln}{N}\right)}$$

Discrete Fourier Transform

$$\hat{g}(m,n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} g(k,l) e^{-j2\pi\left(\frac{km}{M} + \frac{ln}{N}\right)}$$

$$\hat{g}(m,n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} g(k,l) e^{-j2\pi\frac{km}{M}} e^{-j2\pi\frac{ln}{N}}$$

Discrete Fourier Transform

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\frac{km}{M}}\,e^{-j2\pi\frac{ln}{N}}$$

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)\,\frac{1}{M}e^{-j2\pi\frac{km}{M}}\;\frac{1}{N}\,e^{-j2\pi\frac{ln}{N}}$$

Discrete Fourier Transform

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\left(\boxed{\frac{km}{M}}+\boxed{\frac{ln}{N}}\right)}$$

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\frac{km}{M}}\, e^{-j2\pi\frac{ln}{N}}$$

Not dependent on l

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)\boxed{\frac{1}{M}e^{-j2\pi\frac{km}{M}}}\frac{1}{N}\, e^{-j2\pi\frac{ln}{N}}$$

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\frac{1}{M}\, e^{-j2\pi\frac{km}{M}}\underbrace{\sum_{l=0}^{N-1} g(k,l)\,\frac{1}{N}\, e^{-j2\pi\frac{ln}{N}}}$$

Discrete Fourier Transform

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\frac{km}{M}}\,e^{-j2\pi\frac{ln}{N}}$$

Not dependent on *l*

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)\frac{1}{M}e^{-j2\pi\frac{km}{M}}\frac{1}{N}\,e^{-j2\pi\frac{ln}{N}}$$

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\frac{1}{M}\,e^{-j2\pi\frac{km}{M}}\boxed{\sum_{l=0}^{N-1} g(k,l)\,\frac{1}{N}\,e^{-j2\pi\frac{ln}{N}}}$$

1D: DFT

Discrete Fourier Transform

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

$$\hat{g}(m,n) = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)e^{-j2\pi\frac{km}{M}}\, e^{-j2\pi\frac{ln}{N}}$$

Not dependent on l

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(k,l)\boxed{\frac{1}{M}e^{-j2\pi\frac{km}{M}}}\frac{1}{N}\, e^{-j2\pi\frac{ln}{N}}$$

$$\hat{g}(m,n) = \sum_{k=0}^{M-1}\frac{1}{M}\, e^{-j2\pi\frac{km}{M}}\boxed{\sum_{l=0}^{N-1} g(k,l)\frac{1}{N}\, e^{-j2\pi\frac{ln}{N}}}$$

1D: DFT

DFT is linear and separable!!

2D DFT – book notation

- The 2D DFT is a natural extension of the 1D case:

Replace the single frequency *k* with two frequencies in the two directions, k_x and k_y, so that kx/w becomes $k_x x/w + k_y y/h$.

$$G(k) = \mathcal{F}\{g(x)\} = \sum_{x=0}^{w-1} g(x) e^{-j2\pi kx/w}$$

$$g(x) = \mathcal{F}^{-1}\{G(k)\} = \frac{1}{w}\sum_{k=0}^{w-1} G(k) e^{j2\pi kx/w}$$

$\underline{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ position in image

$x \in [0, w-1]$

$y \in [0, h-1]$

$\underline{f} = \begin{bmatrix} \frac{k_x}{w} \\ \frac{k_y}{h} \end{bmatrix}$

$\underline{x}^T \underline{f} = [x \ \ y] \cdot \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \frac{k_x \cdot x}{w} + \frac{k_y \cdot y}{h}$

$$G(k_x, k_y) = \sum_{x=0}^{w-1}\sum_{y=0}^{h-1} g(x,y) e^{-j2\pi \mathbf{x}^T\mathbf{f}} \qquad \text{(forward DFT)}$$

$$g(x,y) = \frac{1}{wh}\sum_{k_x=0}^{w-1}\sum_{k_y=0}^{h-1} G(k_x, k_y) e^{j2\pi \mathbf{x}^T\mathbf{f}} \qquad \text{(inverse DFT)}$$

Scaling factor. Sometimes (some notations) placed in forward equation, sometimes in invers equaiton, and sometimes as 1/sqrt(*) in both forward and invers. The latter gives a unitary transform ($U^{*T} \cdot U = I$)

DC component in 2d FFT

$$G(k_x, k_y) = \sum \sum g(x,y) e^{-j2\pi \left(\frac{k_x \cdot x}{w} + \frac{k_y \cdot y}{h} \right)}$$

DC comp? freq. of 0. $k_x = 0, k_y = 0$

$$G(0,0) = \sum \sum g(x,y) \underbrace{e^{-j2\pi \left(\frac{0 \cdot x}{w} + \frac{0 \cdot y}{h} \right)}}_{e^{-j2\pi \cdot 0} = e^0 = 1}$$

$$G(0,0) = \sum_x \sum_y g(x,y)$$

if scaling is done in forward transform

$\rightarrow G(0,0) = \frac{1}{w \cdot h} \cdot \sum \sum g(x,y)$, average pix. value or mean value.

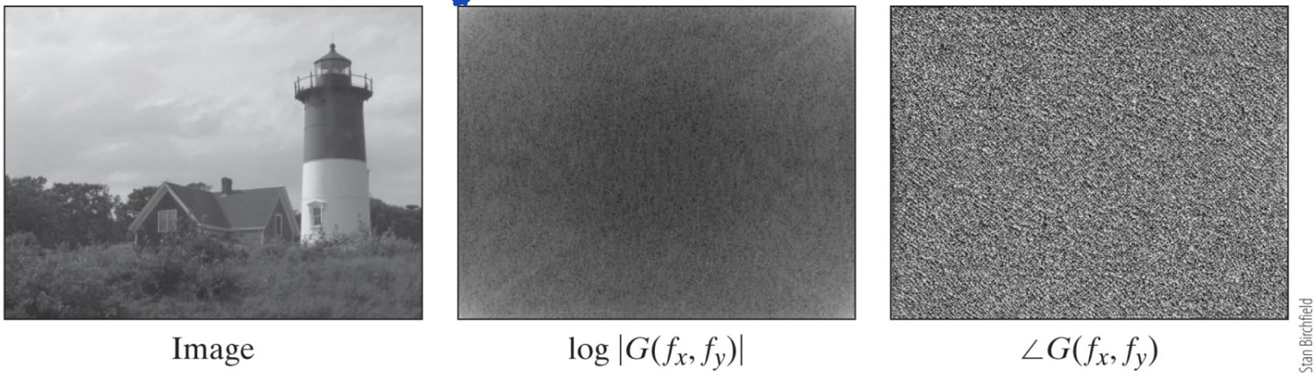
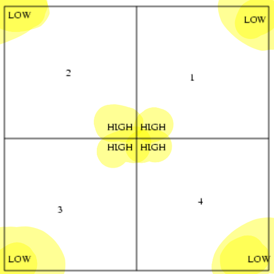
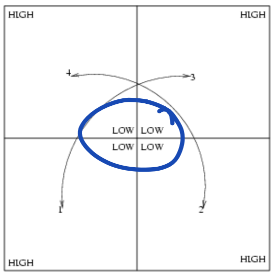


Figure 6.10 An image and its 2D DFT shown as magnitude and phase. (To increase the dynamic range of the display, the log of the magnitude is shown.) The DC component, which is the top-left corner of the magnitude, is difficult to see.





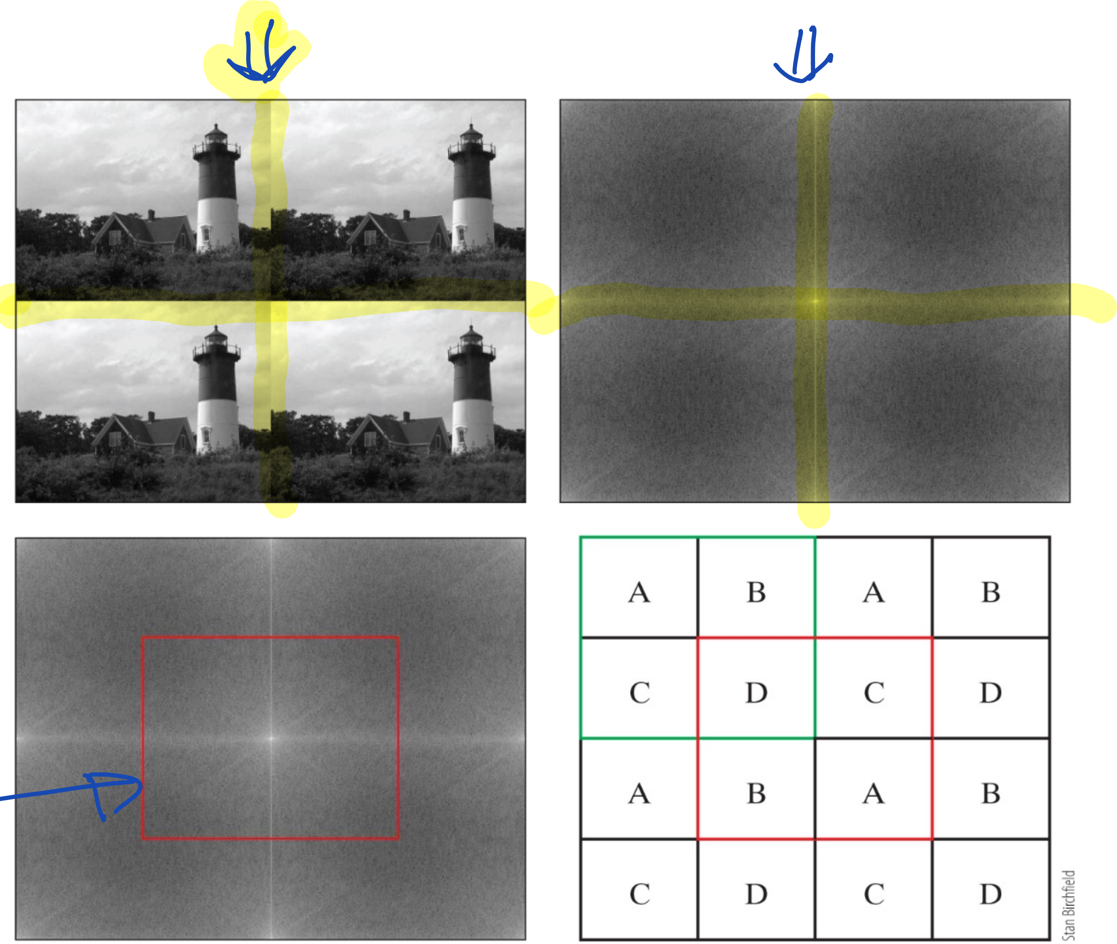
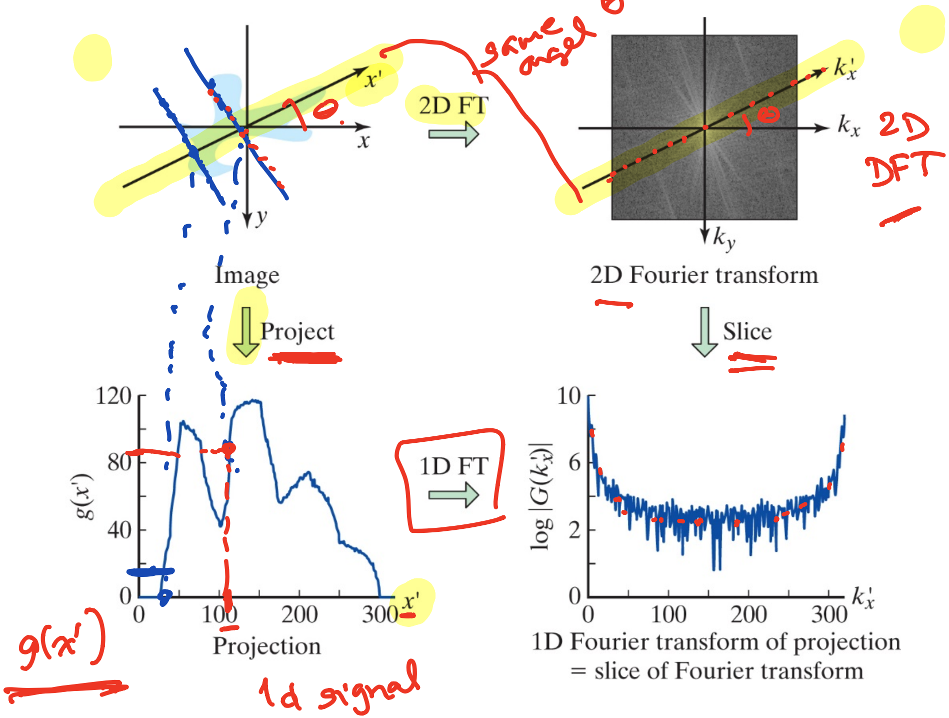


Figure 6.12 Top: The DFT treats the input as a replicated input, and produces a replicated output. Bottom: It is easier to visualize the DFT by shifting it so that the DC component is in the center, which causes no loss of information. The quadrants A, B, C, and D present in the original DFT output are also present in the shifted output, just in a different order.

Projection-slice theorem



Fourier transform – more on properties

- Linear
- Separable
- Invertable
- Convolution theorem (*convolution in space/time domain is multiplication in frequency domain*)

- Rotation (*if g is rotated by an angle, the FT of the rotated image equals the FT of the original image rotated by the same angle.*)