Solution, ELE510 Image Processing with Robot Vision
26. Feb. 2016

Exercise 3

See Figure 3.1 next page.

a) $FOV = 2 \cdot \text{Atan}\left(\frac{w/2}{f}\right) = 2 \cdot \text{Atan}\left(\frac{3}{10}\right) = \underline{\underline{33.4°}}$

b) Rotation around X with angle $\beta = -135°$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-135°) & -\sin(-135°) \\ 0 & \sin(-135°) & \cos(-135°) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -a & a \\ 0 & -a & -a \end{bmatrix}, \quad a = \frac{1}{2}\sqrt{2}$$

From Figure 2 in the exam text we find the translation vector:

$t_z = 0.5 \cdot \cos 45° = 0.5 \cdot \frac{1}{2}\sqrt{2} = \underline{0.5 \cdot a}, \quad a = \frac{1}{2}\sqrt{2}$

$t_y = -\left(1.5 \cdot \sqrt{2} - 0.5 \cdot \frac{1}{2}\sqrt{2}\right) = -2.5 \cdot \frac{1}{2}\sqrt{2} = \underline{-2.5a}$

$t_x = 0$

$$\underline{t} = \begin{bmatrix} 0 \\ -2.5a \\ 0.5a \end{bmatrix}, \quad \underset{\sim}{RT} = \begin{bmatrix} \underset{\sim}{R} & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -a & a & -2.5a \\ 0 & -a & -a & 0.5a \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

$$FOV = 2\cdot \mathrm{Atan}\left(\frac{w/2}{f}\right) = 2\mathrm{Atan}\left(\frac{3}{10}\right) = 33.4^\circ$$

$$\frac{FOV}{2} = \beta = 16.7^\circ \quad , \tan\beta = 0.3$$

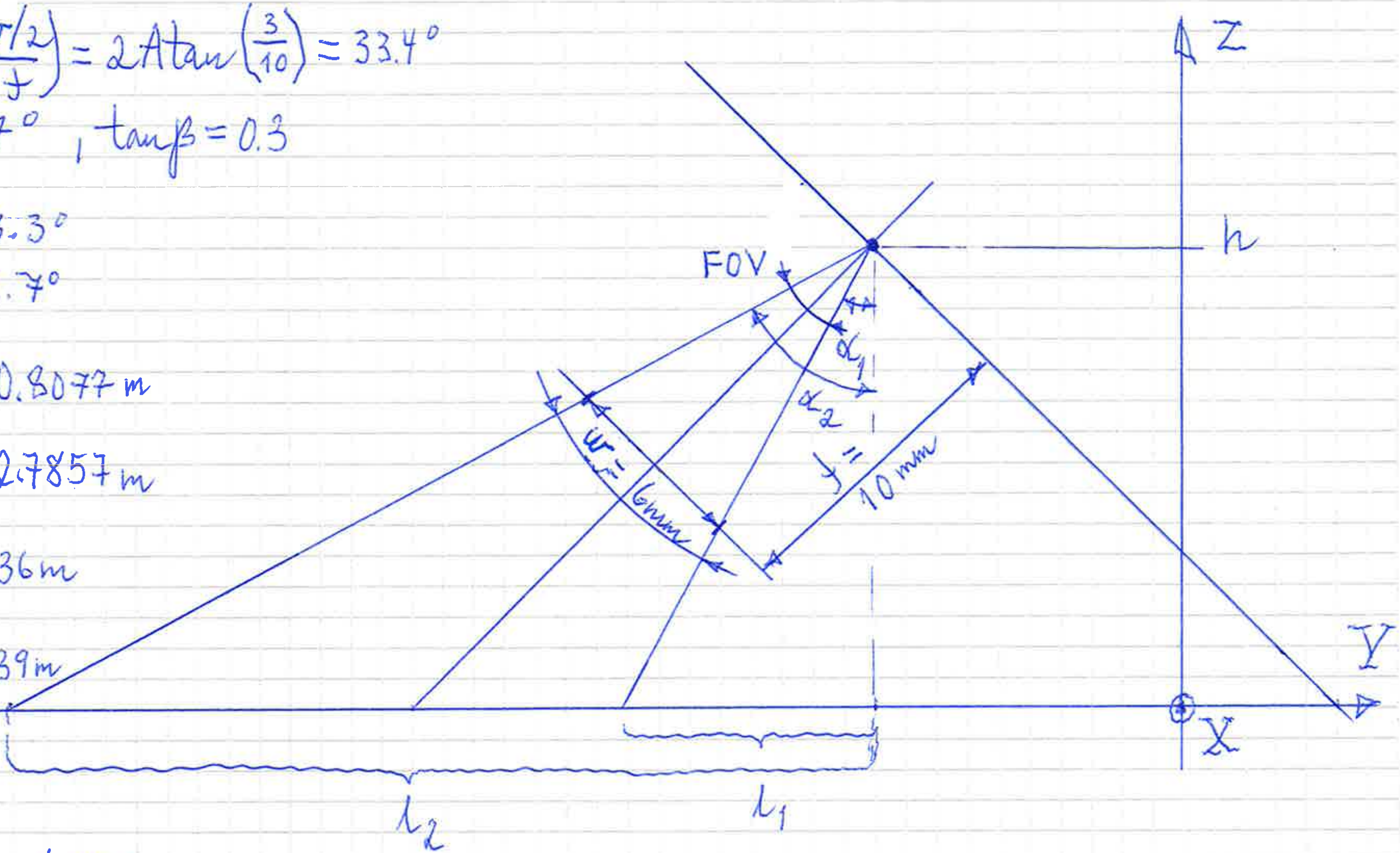
$$\alpha_1 = 45^\circ - \beta = 28.3^\circ$$
$$\alpha_2 = 45^\circ + \beta = 61.7^\circ$$

$$l_1 = 1.5 \tan\alpha_1 = 0.8077\,m$$

$$l_2 = 1.5 \tan\alpha_2 = 2.7857\,m$$

$$r_1 = \frac{1.5}{\cos\alpha_1} = 1.7036\,m$$

$$r_2 = \frac{1.5}{\cos\alpha_2} = 3.1639\,m$$



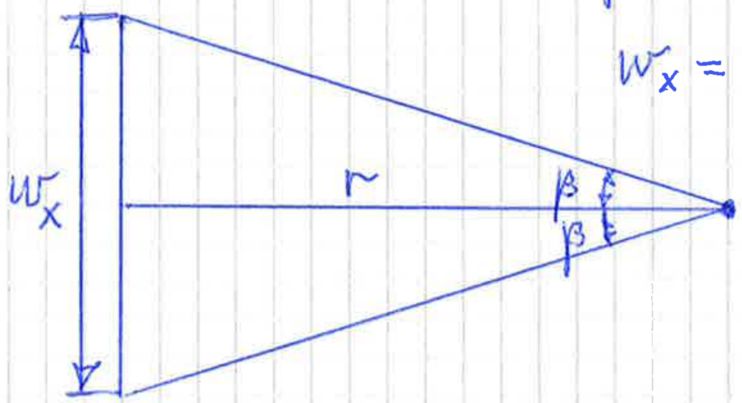
$$w_{x1} = 2r_1 \tan\beta = 1.022\,m$$
$$w_{x2} = 2r_2 \tan\beta = 1.898\,m$$

Figure 3.1

c) From the figure on the previous page we see that the region in the ground plane extends from $Y_1 = -(1.0m + l_1)$ to $Y_2 = -(1.0m + l_2)$ along the Y-axis.

The width along X can be found from the following triangles. Let r be the distance from the camera center to the ground

$$\tan\beta = \frac{w_x/2}{r} \Rightarrow$$



$$w_x = 2r \cdot \tan\beta$$

$$\beta = \frac{FOV}{2} = 16.7°$$

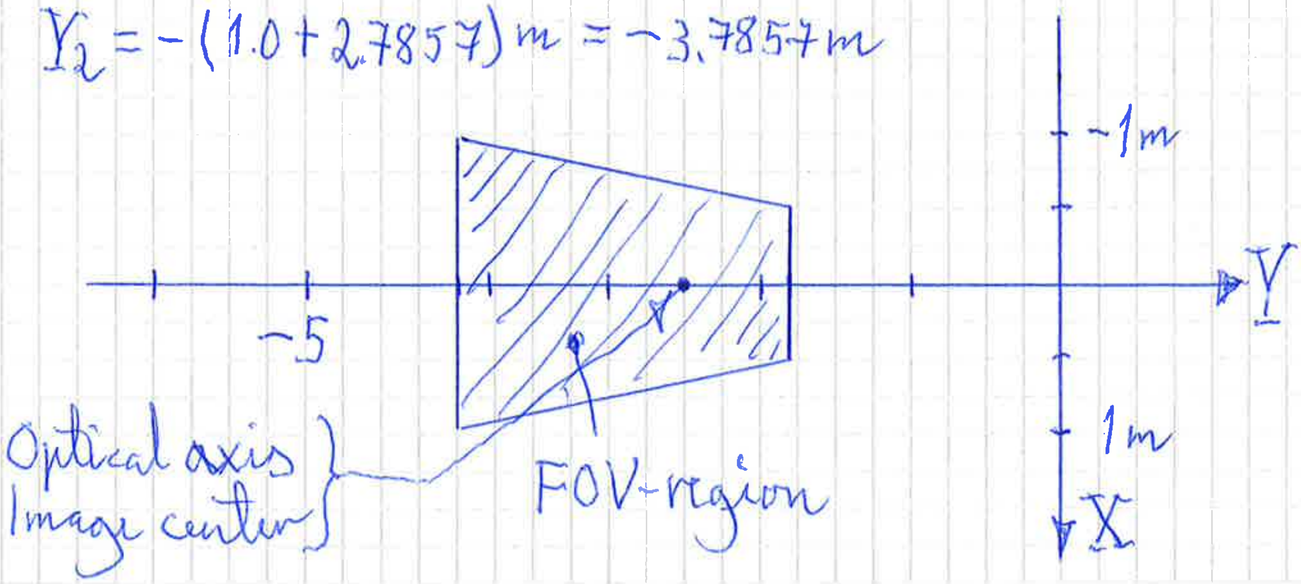
See previous page: $r_1 = 1.7036\,m$, $r_2 = 3.1639\,m$

$$w_{x_1} = 2 \cdot r_1 \cdot \tan\beta = 2 \cdot 1.7036 \cdot \tan(16.7°) = 1.022\,m$$

$$w_{x_2} = 2 \cdot r_2 \cdot \tan\beta = 2 \cdot 3.1639 \cdot \tan(16.7°) = 1.898\,m$$

$$Y_1 = -(1.0 + 0.8077)\,m = -1.8077\,m$$

$$Y_2 = -(1.0 + 2.7857)\,m = -3.7857\,m$$



d) $\Delta x = \Delta y = \frac{w}{M} = \frac{w}{N} = \frac{6 \cdot 10^{-3}}{4000} m = 1.5 \cdot 10^{-6} m = \underline{1.5 \mu m}$

$$\alpha = \beta = \frac{f}{\Delta x} = \frac{f}{\Delta y} = \frac{10 \cdot 10^{-3}}{1.5 \cdot 10^{-6}} = \underline{6667}$$

No skew $\Rightarrow$ $\theta = 90°$, $\sin\theta = 1$, $\cot\theta = 0$

$$x_0 = y_0 = \frac{M}{2} = \frac{N}{2} = \frac{4000}{2} = \underline{2000}$$

Then we have the internal calibration matrix:

$$\underset{\sim}{K} = \begin{bmatrix} \alpha & 0 & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 6667 & 0 & 2000 \\ 0 & 6667 & 2000 \\ 0 & 0 & 1 \end{bmatrix}}}$$

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underset{\sim}{K} \, \underset{\sim}{\Pi_0} \, \underline{P}_c = \begin{bmatrix} 6667 & 0 & 2000 & 0 \\ 0 & 6667 & 2000 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

From the third row we get $\underline{\underline{\lambda = Z_c}}$

e) The condition for weak perspective is that $Z_c \approx$ const for all image points.
This is not the case here as can clearly be seen from Figure 3.1. The distance along Z_c almost doubles from one side w_{x1} of the image to the opposite side, w_{x2}.

f)

Exercise 4 a)

$$\lambda \underline{p} = \begin{bmatrix} -2000 & -800 & 0 & \left\{\begin{matrix}1800\\-200\end{matrix}\right\} \\ 0 & -800 & 2000 & -200 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ Y \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} -800 \cdot Y + \left\{\begin{matrix}1800\\-200\end{matrix}\right\} \\ -800 \cdot Y - 200 \\ -Y + 1 \end{bmatrix} \rightarrow \lambda = 1 - Y$$

From the third row:

We devide the first two rows by $\lambda = 1 - Y$ and get:

$$\underline{\underline{x_A}} = \frac{-800Y + 1800}{1 - Y} = \underline{\underline{800 - \frac{1000}{Y-1}}}, \quad Y > 1m$$

$$\underline{\underline{x_B}} = \frac{-800Y - 200}{1 - Y} = \underline{\underline{800 + \frac{1000}{Y-1}}}, \quad Y > 1m$$

$$\underline{\underline{y_{A,B}}} = \frac{-800Y - 200}{1 - Y} = \frac{-800Y + 800 - 1000}{1 - Y}$$

$$= \underline{\underline{800 + \frac{1000}{Y-1}}}, \quad Y > 1m$$

Inside the camera sensor: $x, y \in (0, 1600]$

This gives $\frac{1000}{Y_{min} - 1} = 800, \quad Y_{min} > 1.0m$

$$\underline{\underline{Y_{min}}} = \frac{1000}{800} + 1 = \underline{\underline{2.25m}}$$

b) Disparity

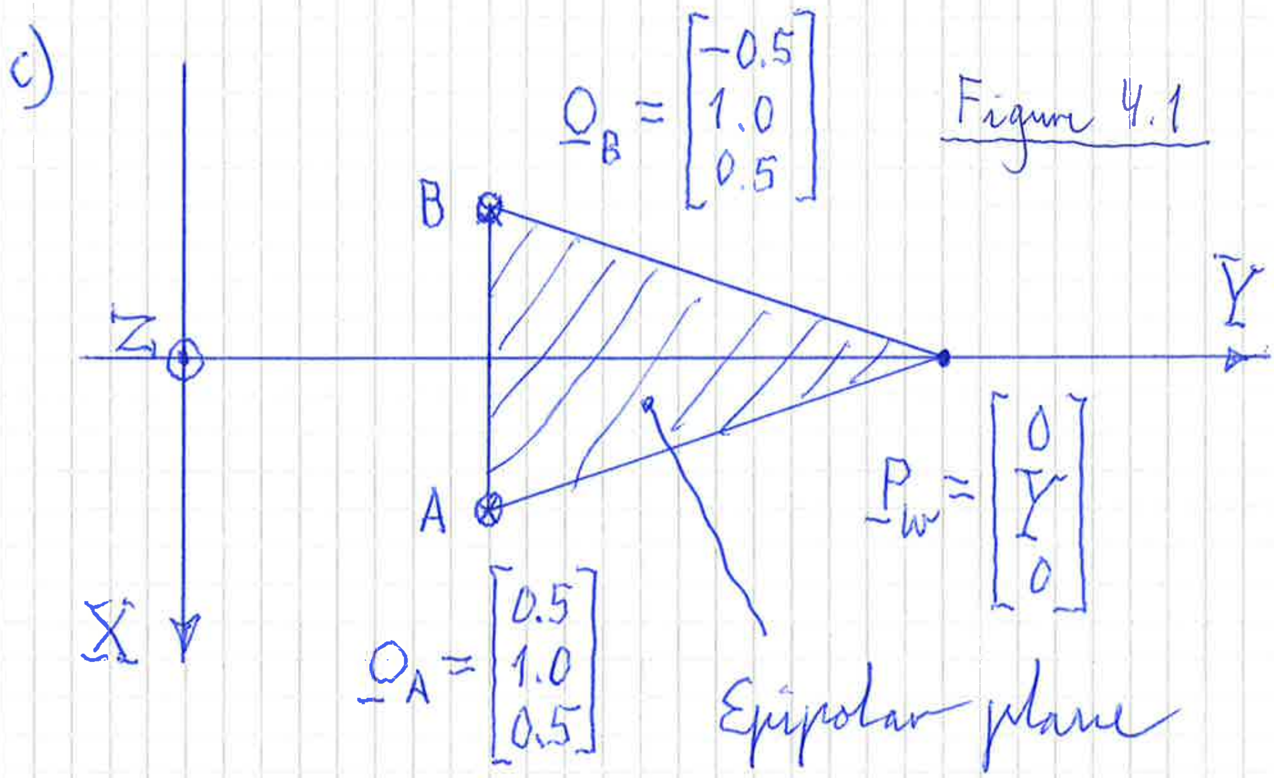
$$\underline{d} = \begin{bmatrix} x_A - x_B \\ y_A - y_B \end{bmatrix} = \begin{bmatrix} 800 - \frac{1000}{Y-1} - \left(800 + \frac{1000}{Y-1}\right) \\ 800 + \frac{1000}{Y-1} - \left(800 + \frac{1000}{Y-1}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2000}{Y-1} \\ 0 \end{bmatrix}$$

We solve for Y from the first row and get

$$Y - 1 = -\frac{2000}{d_x} \Rightarrow Y = 1 - \frac{2000}{d_x} = 1 + \frac{2000}{x_B - x_A}$$

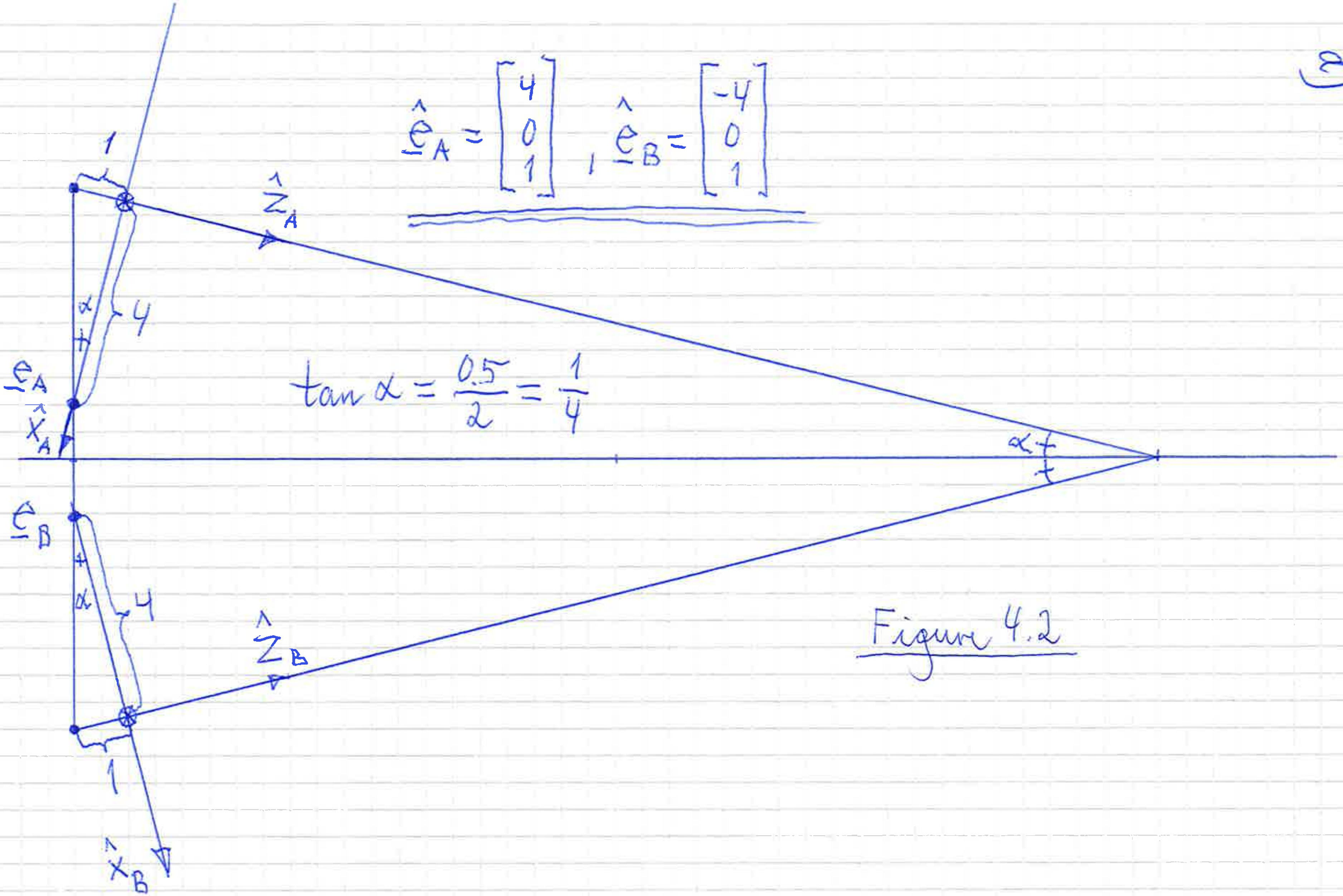
c)



Note that the world point $\underline{P}_w$ is in the ground plane $(Z=0)$, while the two camera centers, $\underline{O}_A$ and $\underline{O}_B$, are at a height $Z = 0.5\,\text{m}$.

d)

$$\hat{\underline{e}}_A = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{\underline{e}}_B = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$



$$\tan \alpha = \frac{0.5}{2} = \frac{1}{4}$$

Figure 4.2