Highpass filter

$$|H_{hp}(f)| = 1 - |H_{lp}(f)|$$

$$|H_{i\,hp}(f)| = \begin{cases} 1 & \text{if } f \geq f_c \\ 0 & \text{otherwise} \end{cases}$$

ideal

$$\boxed{\text{if the filter is zero phase} \quad H(f) = |H(f)|}$$

let $\boxed{g'(x) = g(x) \circledast h_{hp}(x)}$

$$= \mathcal{F}^{-1}\{G(f) \cdot \boxed{H_{hp}(f)}\}$$

$$= \mathcal{F}^{-1}\{G(f)[1 - H_{lp}(f)]\}$$

$$= \mathcal{F}^{-1}\{G(f)\} - \mathcal{F}^{-1}\{G(f) H_{lp}(f)\}$$

$$g'(x) = g(x) - g(x) \circledast h_{lp}(x)$$

Bandpass filter

$$|H(f)| = \begin{cases} 1 & \text{if } f_{lo} \leq f \leq f_{hi} \\ 0 & \text{otherwise} \end{cases}$$

most common : Laplacian of Gaussian
LoG

$$|H(f)| = -f^2 e^{-f/2f_c^2}$$

$$\mathcal{F}\left\{ \frac{d^n g(x)}{dx^n} \right\} = (jf)^n G(f)$$

let $g(x)$ be gaussian and $n=2$

Given a lowpass kernel:

$$h_{lp}(x,y) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} \circledast \frac{1}{3}\begin{bmatrix}1&1&1\end{bmatrix}$$

$$g(x) \circledast h_{hp} = g(x) - g(x) \circledast h_{lp}(x)$$

find the highpass kernel.

$$= g(x) \circledast \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} - g(x) \circledast h_{lp}(x)$$

$$= g(x) \circledast \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} - g(x) \circledast \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$= g(x) \circledast \left[\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\right]$$

$$= g(x) \circledast \frac{1}{9}\left[\begin{bmatrix}0&0&0\\0&9&0\\0&0&0\end{bmatrix} - \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\right]$$

$$I(x,y) \circledast (-LoG) = \eta \left[I - I \circledast h_{lp} \right]$$

$$= \eta \left[I \circledast \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - I \circledast \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right]$$

$$= \eta \left(I \circledast \left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right] \right)$$

$$= I(x,y) \circledast \underbrace{\frac{\eta}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}}_{LoG_{0.33}}$$