

Highpass filter

$$|H_{hp}(f)| = 1 - |H_{lp}(f)|$$

ideal \nearrow

$$|H_{c, hp}(f)| = \begin{cases} 1 & \text{if } f \geq f_c \\ 0 & \text{otherwise} \end{cases}$$

if the filter is zero phase \rightarrow

$$H(f) = |H(f)|$$

let

$$g'(x) = g(x) \otimes h_{hp}(x)$$

$$= \mathcal{F}^{-1} \{ G(f) \cdot H_{hp}(f) \}$$

$$= \mathcal{F}^{-1} \{ G(f) [1 - H_{lp}(f)] \}$$

$$= \mathcal{F}^{-1} \{ G(f) \} - \mathcal{F}^{-1} \{ G(f) H_{lp}(f) \}$$

$$\underline{g'(x)} = \underline{g(x)} - \underline{g(x) \otimes h_{lp}(x)}$$

Bandpass filter

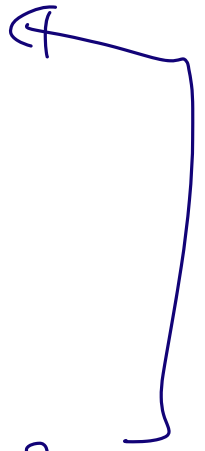
$$|H(f)| = \begin{cases} 1 & \text{if } f_{l0} \leq f \leq f_{h1} \\ 0 & \text{otherwise} \end{cases}$$

most common : Laplacian of Gaussian
LoG

$$|H(f)| = -f^2 e^{-f^2 / 2f_c^2}$$

$$\mathcal{F} \left\{ \frac{d^n g(x)}{dx^n} \right\} = (jf)^n G(f)$$

let $g(x)$ be gaussian and $n=2$



Given a lowpass kernel:

$$h_{lp}(x,y) = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \otimes \frac{1}{3} [1 \ 1 \ 1]$$

$$g(x) \otimes h_{hp} = g(x) - g(x) \otimes h_{lp}(x)$$

find the highpass kernel.

$$= g(x) \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - g(x) \otimes h_{lp}(x)$$

$$= g(x) \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - g(x) \otimes \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= g(x) \otimes \left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right]$$

$$= g(x) \otimes \frac{1}{9} \left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right]$$

$$I(x, y) \otimes (-\text{LoG}) = \gamma [I - I \otimes h_{lp}]$$

$$= \gamma \left[I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - I \otimes \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right]$$

$$= \gamma \left(I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

$$= I(x, y) \otimes \underbrace{\frac{\gamma}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}}_{\text{LoG}_{0.33}}$$