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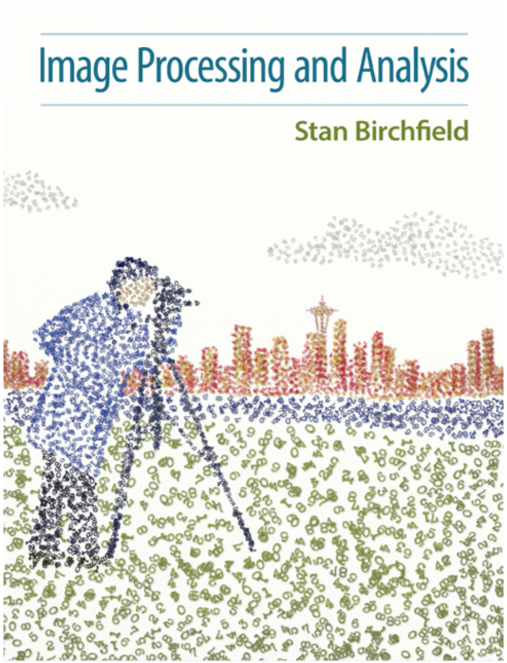
ELE510 Image processing and computer vision

Geometry of multiple views, (chap 13.6 Birchfield) 2020



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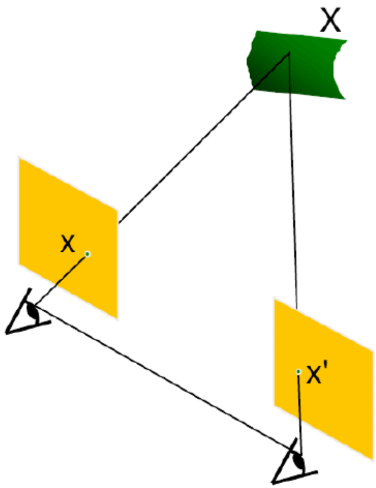
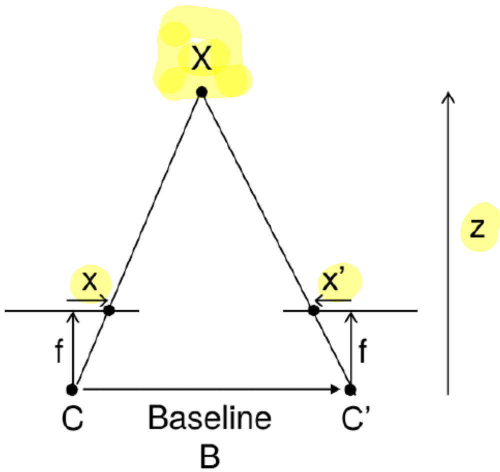
Depth from Stereo

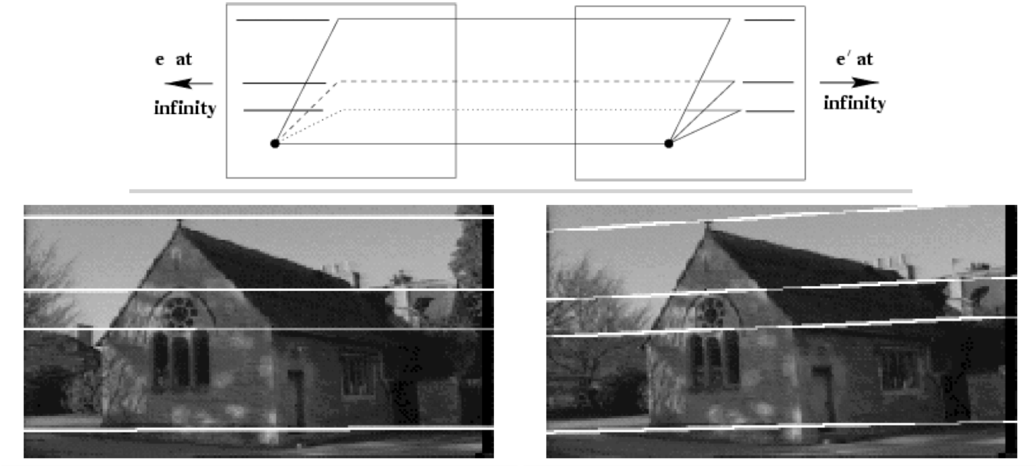
Goal: recover depth by finding image coordinate x′ that corresponds to x

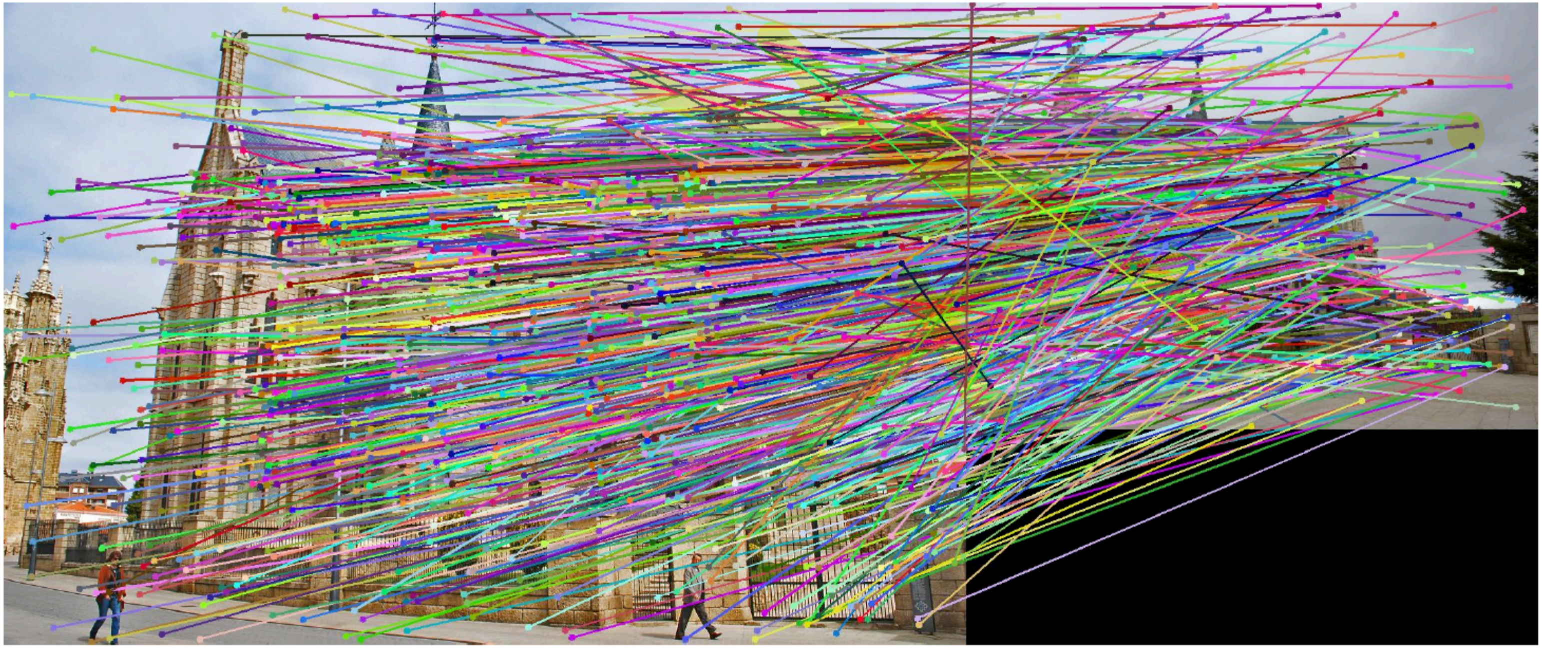
Subproblems:
Calibration
Correspondence
Reconstruction

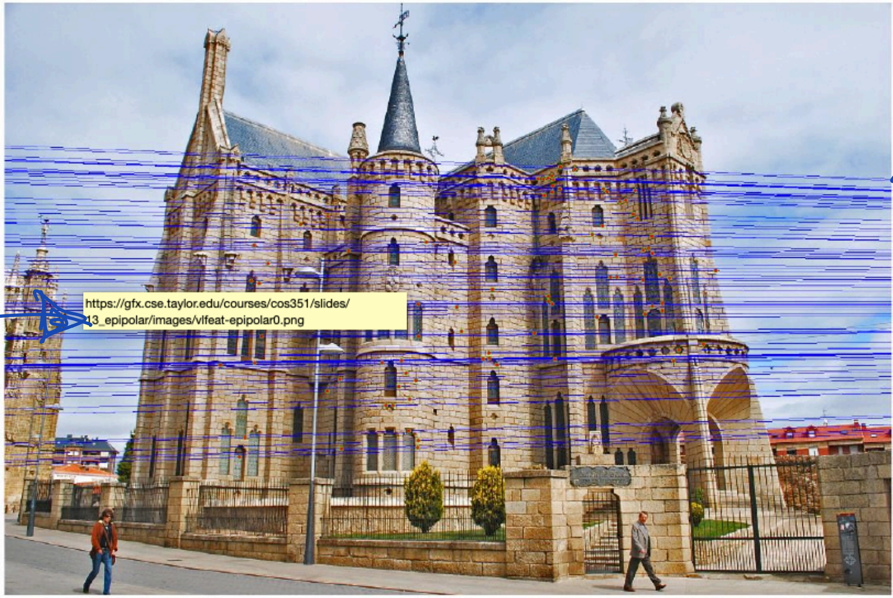


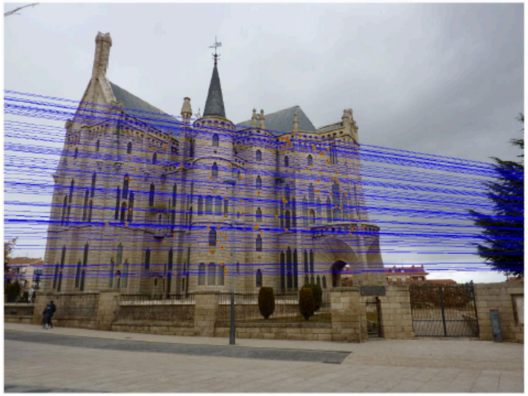


- Wouldn't it be nice to know where matches can live?
- Constrain our 2D search to 1D to find corresponding points
- Rectified cameras: corresponding points are on the scan lines (same y coordinate).
- This 1D line of possible corresponding points is called the epipolar line





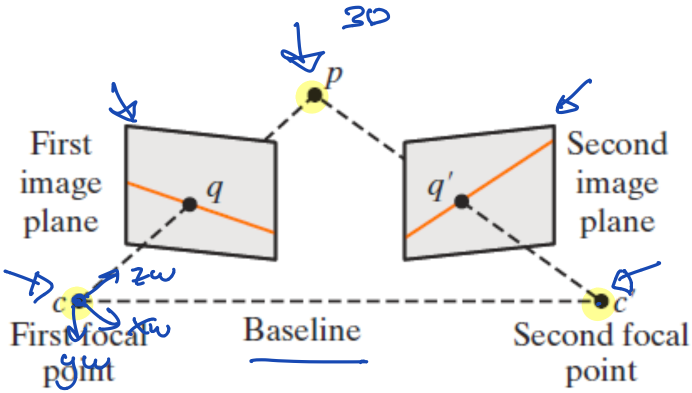

https://gfx.cse.taylor.edu/courses/cos351/slides/
13_epipolar/images/vlfeat-epipolar0.png



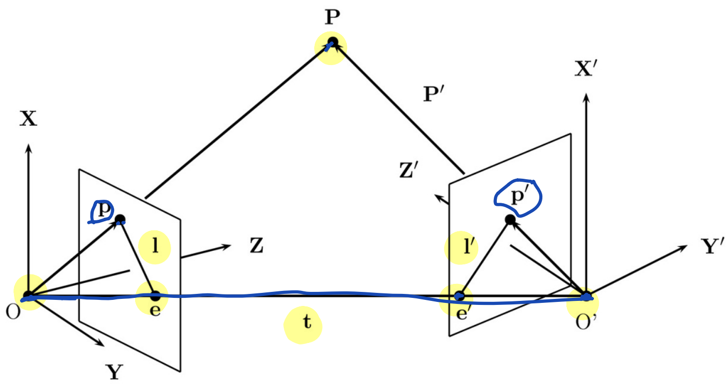
Epipolar Geometry

- The **epipolar plane** is attached to the two poles (centers of projection).
- The epipolar plane intersects the two image planes along two **epipolar lines**.

Figure 13.30 Epipolar geometry of two cameras viewing the same scene. The epipolar lines are shown on each of the image planes, while the epipoles, in this example, are outside the finite extent of the image plane.



Epipolar Geometry



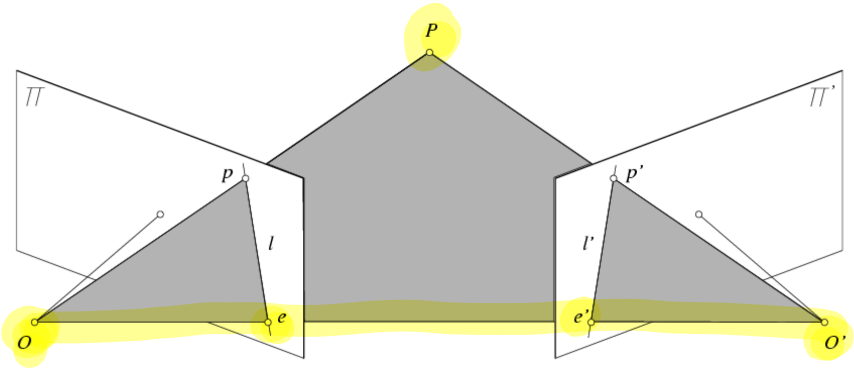
The *left camera coordinates are used as reference*.

Right camera coordinates are denoted by primed symbols ‘

The *Epipolar Plane*: OO’P
Epipoles: e, e’
Epipolar lines: l, l’

Baseline: OO’ = t (here)
Camera centers: O, O’
Optical axises: Z, Z’
Image points: p, p’

Epipolar Geometry



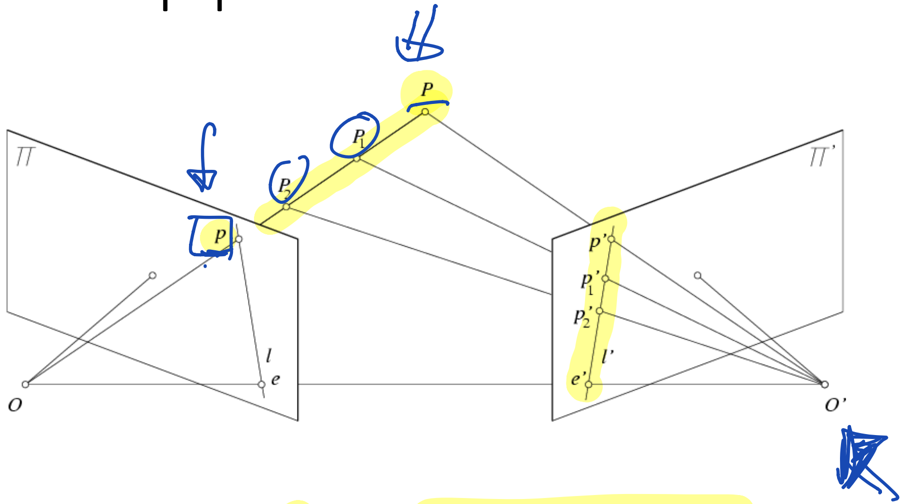
The *Epipolar Plane*: OO'P
Epipoles: e, e'
Epipolar lines: l, l'
Image points: p, p'

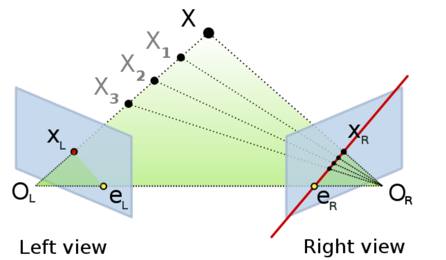
Note! There is a separate *epipolar plane* for each point in the scene.

The optical centers of the cameras lenses are distinct, thus each center projects onto a distinct point into the other camera's image plane. These two image points, here denoted by **e** and **e´**, are called *epipoles or epipolar points*.

Both epipoles **e** and **e´** in their respective image planes and both optical centers **O** and **O´** lie on a single 3D line.

Epipolar Constraint

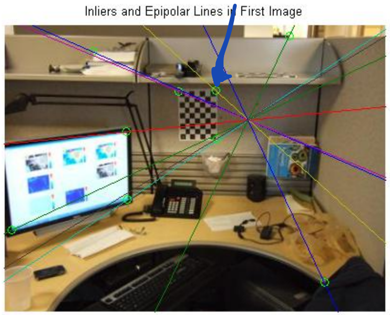


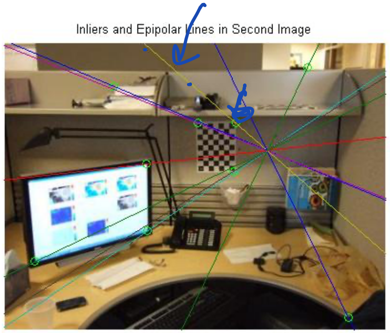


Source: https://commons.wikimedia.org/wiki/File:Epipolar_geometry.svg#/media/File:Epipolar_geometry.svg, Licensed under: CC BY-SA 3.0

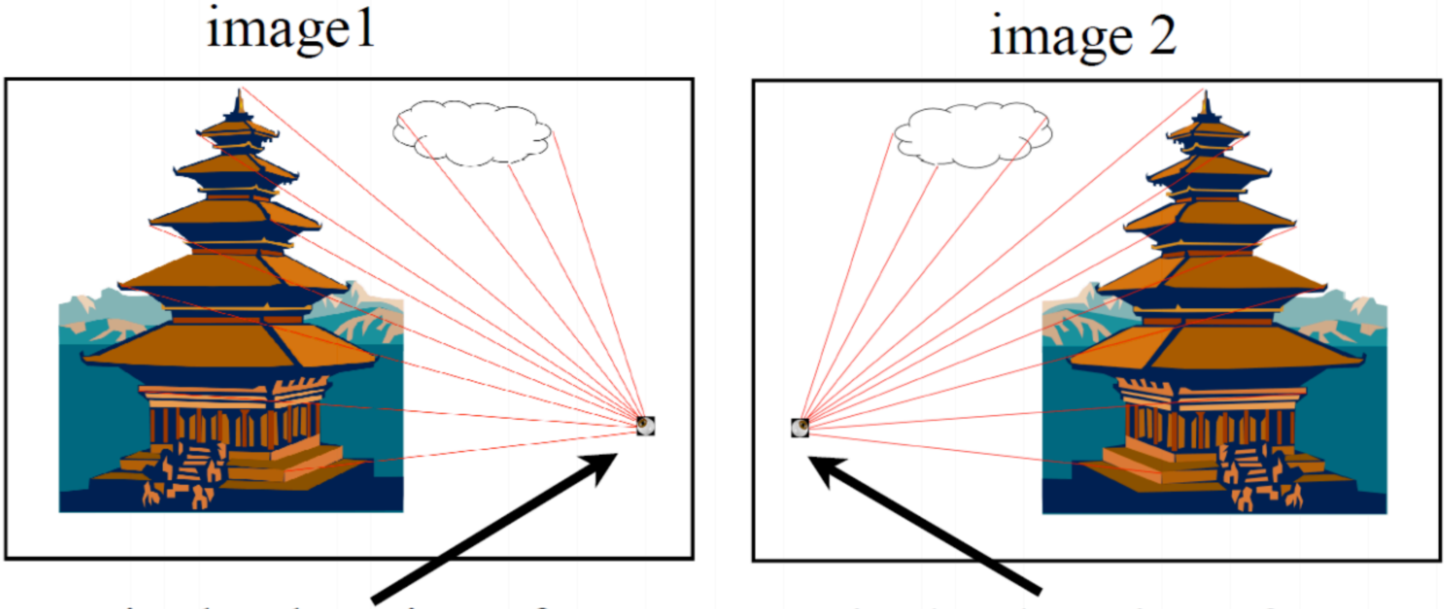
- Potential matches for ***p*** have to lie on the corresponding epipolar line ***l'***.
- Potential matches for ***p'*** have to lie on the corresponding epipolar line ***l***.

Epipolar Lines





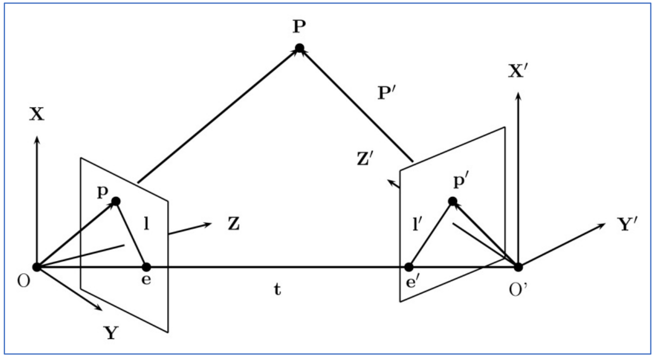
- Potential matches for ***p*** have to lie on the corresponding epipolar line ***l'***.
- Potential matches for ***p'*** have to lie on the corresponding epipolar line ***l***.



Epipole : location of cam2 as seen by cam1.

Epipole : location of cam1 as seen by cam2.

Relationship between camera coordinates for the two cameras



The *left camera coordinates are used as reference*.

Right camera coordinates are denoted by primed symbols ‘

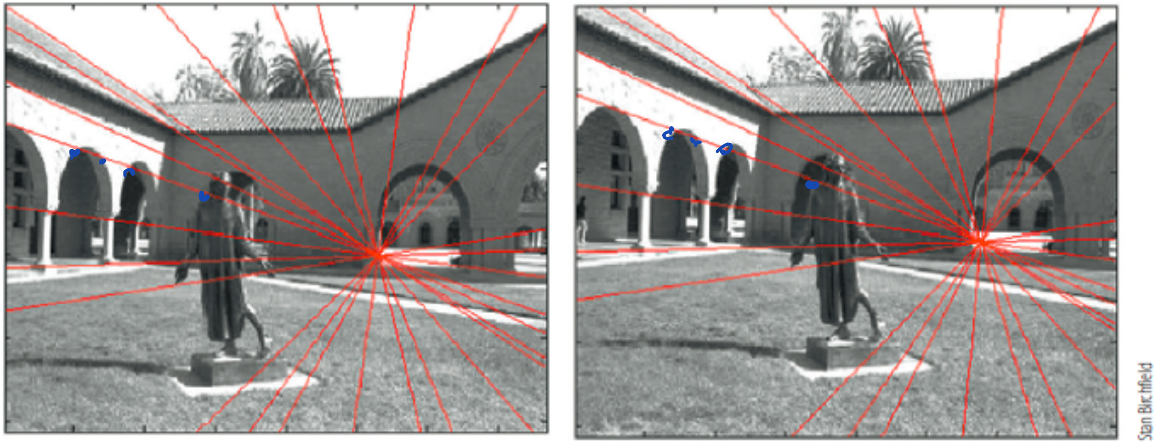
There is a rotation, $\mathcal{R}$, and a translation, **t**, between the right and left coordinate systems. $\mathcal{R}$ is defined such that $\mathcal{R}$**P**’ is adjusted to left coordinates. Baseline translation **(t)** is given in left coordinates.

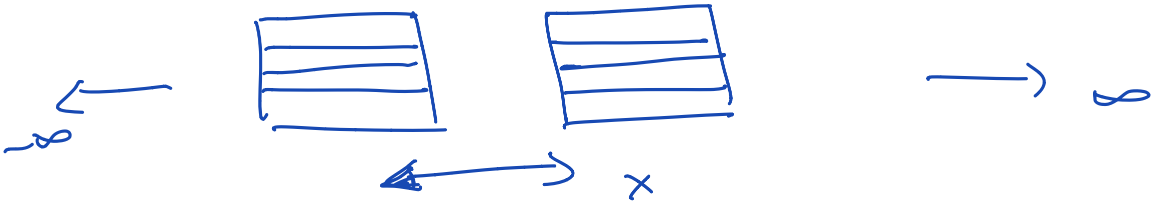
We get:

$$\mathbf{P} = \mathcal{R}\mathbf{P}' + \mathbf{t} \quad \Leftrightarrow \quad \mathbf{P}' = \mathcal{R}^{-1}\left(\mathbf{P} - \mathbf{t}\right) = \mathcal{R}^{T}\mathbf{P} - \mathcal{R}^{T}\mathbf{t}$$

Here P and P’ refers to the 3D world point described with left and right camera coordinate systems respectively.

Figure 13.31 Two images of a static scene, with some of the epipolar lines overlaid, whose intersection reveals the epipoles. Note that corresponding points on the two images lie on corresponding epipolar lines.





The Fundamental Matrix

- The fundamental matrix **F** capture the relative geometry between two cameras. Let x and x' be corresponding points: $(x_i, y_i) \Leftrightarrow (x'_i, y'_i)$

$$x = \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

- Given the fundamental matrix **F**, the epipolar line l' in the second image associated with the point **x** in the first image is given by:

$$\boxed{l' = \mathbf{F}\mathbf{x}}$$

- Similarly, the epipolar line l in the first image associated with the point **x'** in the second image is given by

$$\boxed{l = \mathbf{F}^T\mathbf{x}'}$$

$$\mathbf{x}^T\mathbf{l} = 0$$
$$\mathbf{x}'^T\mathbf{l}' = 0$$

Because the point lie on the epipolar lines

$$\mathbf{x}'^T\mathbf{F}\mathbf{x} = 0$$
$$\mathbf{x}^T\mathbf{F}^T\mathbf{x}' = 0$$

Fundamental matrix – rectified cameras

For rectified cameras: y - y' = 0

$x \leftrightarrow x'$

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = \begin{pmatrix} x' & y' & 1 \end{pmatrix} \underset{3\times 3}{\mathbf{F}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

$$\underset{\sim}{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$y - y' = [0 \quad 1 \quad -y'] \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = y - y' = 0$$

$$[x' \quad y' \quad 1] \cdot \underset{\sim}{F} = [0 \quad 1 \quad -y'] \;:\; [x' \; y' \; 1] \cdot \begin{bmatrix} 0, & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= [0 \quad 1 \quad -y']$$

13.3 non-rectified cam.

$$\underset{\sim}{F} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 3 & 3 & 1 \end{bmatrix}$$

cam $\underline{x} \Rightarrow (x, y) = (100, 50)$

$\underline{x}' \Rightarrow (x', y') = (78, 40)$

can these be corresponding points?

$$\underline{l}' = \underset{\sim}{F} \cdot \underline{x} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 3 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 100 \\ 50 \\ 1 \end{bmatrix} = \begin{bmatrix} 201 \\ -402 \\ 451 \end{bmatrix}$$

know $\underline{x}'^T \cdot \underline{l}' = 0$ point have to be on the line

test $$\begin{bmatrix} 78 & 40 & 1 \end{bmatrix} \cdot \begin{bmatrix} 201 \\ -402 \\ 451 \end{bmatrix} = 49 \neq 0$$

does not look like corresponding points

BUT need to normalize first

$\underline{l}' = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ need $\begin{bmatrix} a \\ b \end{bmatrix}$ have unit norm

$$\underline{l}'_n = \frac{[201 \quad -402 \quad 451]^T}{\sqrt{201^2 + (-402)^2}} = \begin{bmatrix} 0.497 \\ -0.894 \\ 1.003 \end{bmatrix}$$

new test: $\underline{x}'^T \underline{l}'_n = [78 \quad 40 \quad 1] \cdot \underline{l}'_n = 0.109$

This is within range of real world noise $\approx$ 1/10 pixel

acceptable

Conclusion: $\underline{x} = (100, 50)$ $\underline{x}' = (78, 40)$

<u>can</u> be corresponding points.

Fundamental matrix and epipoles

The fundamental matrix also encodes the epipoles **e, e'**

$$\left.\begin{aligned} \mathbf{l}^T \mathbf{e} &= 0 \\ \mathbf{l} &= \mathbf{F}^T \mathbf{x}' \end{aligned}\right\} \quad \mathbf{x}'^T \mathbf{F} \mathbf{e} = 0$$

This must be true for $\boxed{\text{any } \mathbf{x}'.}$ This means:

$$\mathbf{F}\mathbf{e} = 0 \qquad \mathbf{F}^T \mathbf{e}' = 0$$

The Essential Matrix

- Like the fundamental matrix **F**, the essential matrix, **E**, captures the geometric relationship between the cameras in a compact matrix form.

- The fundamental matrix relates *uncalibrated* cameras (pixel values), while the essential matrix relates *calibrated* cameras (meter).
- If we know **F** or **E** we can find the other with the help of K and K' (the matrices of intrinsic camera parameters for the two cameras)

- The relationship between the two matrices is: $\boxed{\mathbf{F} = \mathbf{K}'^{-\mathsf{T}}\mathbf{E}\mathbf{K}^{-1}, \text{ or } \mathbf{E} = \mathbf{K}'^{\mathsf{T}}\mathbf{F}\mathbf{K}}$
- E has 5 parameters: 3 for rotation and 2 for translation in direction between the cameras.