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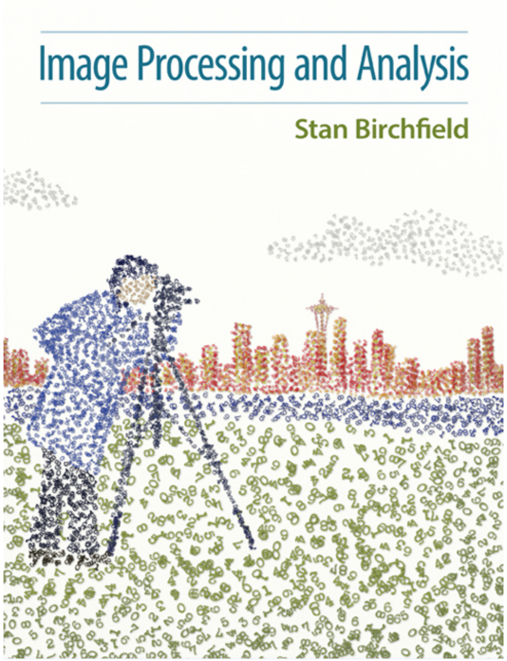
ELE510 Image processing and computer vision

Frequency Domain processing, 2D DFT (Chap 6.3 Birchfield) 2020



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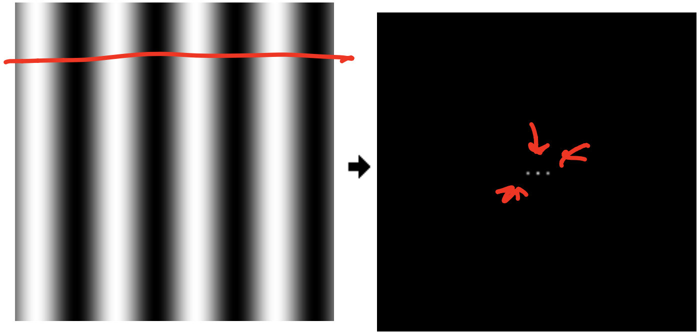
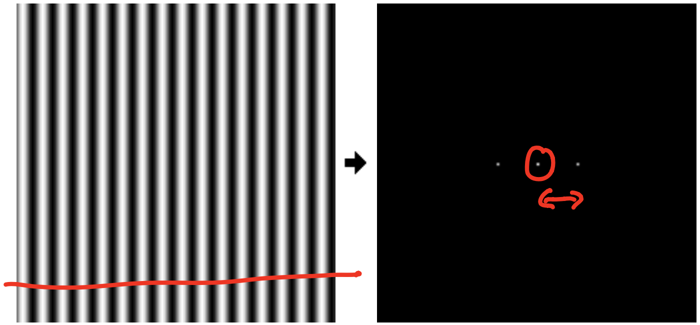
Fourier examples



The center dot is the average DC value. The other two dots represent the perfect sine wave that the Fourier Operator found in the image. There are exactly 4 cycles across the image, and as a result two frequency points are 4 pixels away from the center DC value.
The two points represent the single wave with two different descriptions; one with a negative direction and phase compared to the other.

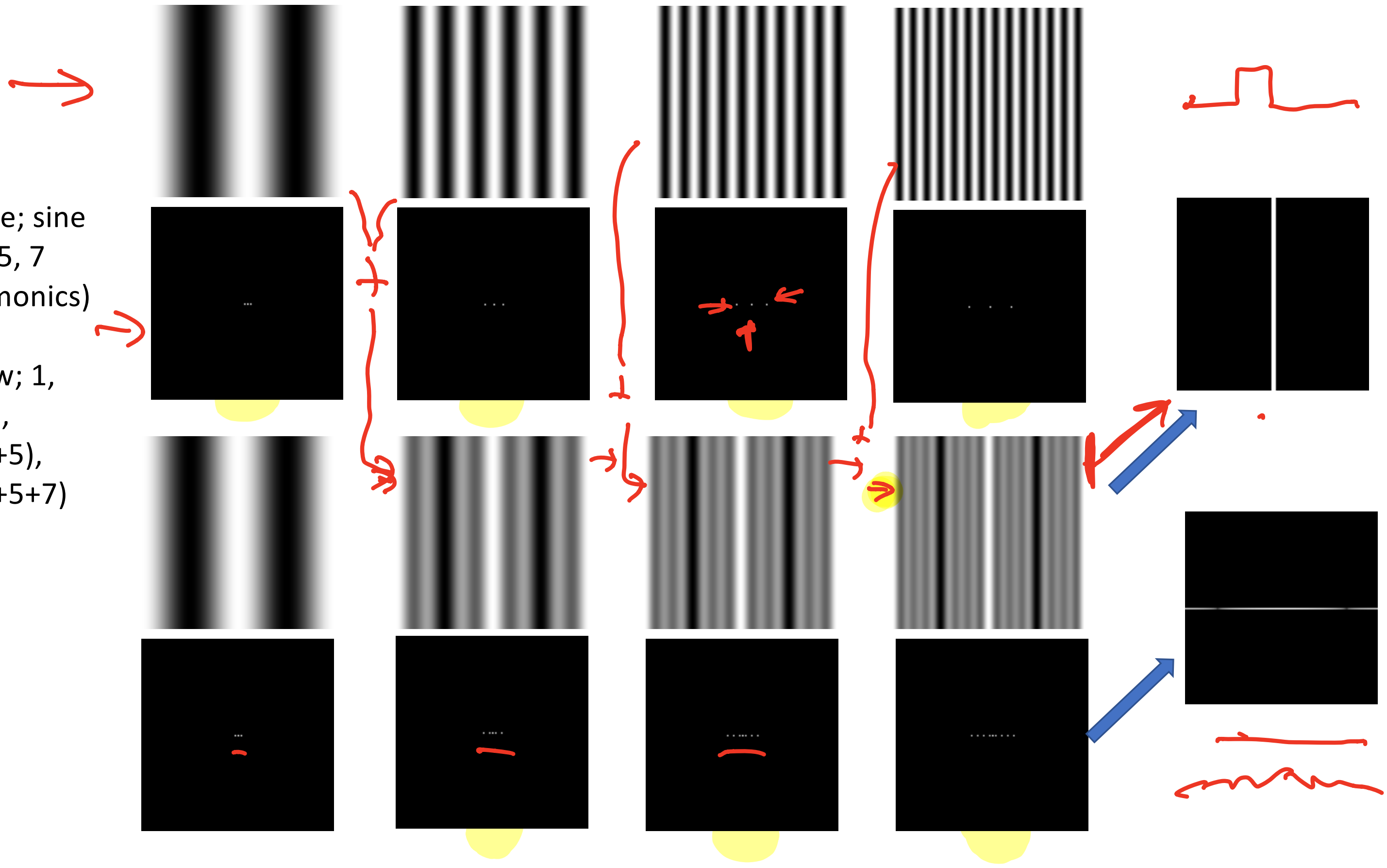


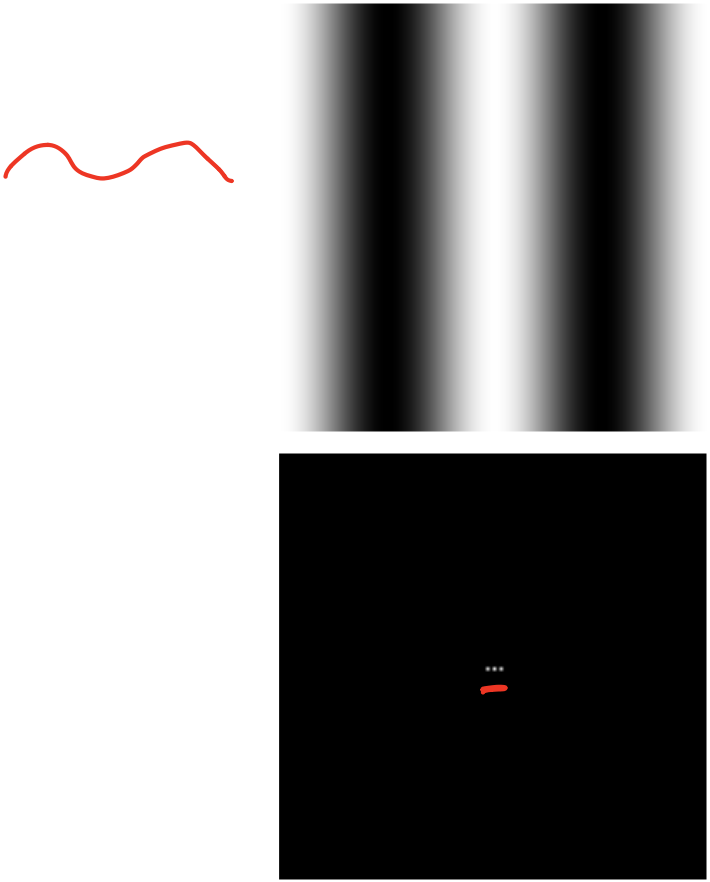
Higher frequency = smaller wavelength.
In the Fourier specter the frequency points will be further away from the DC value.

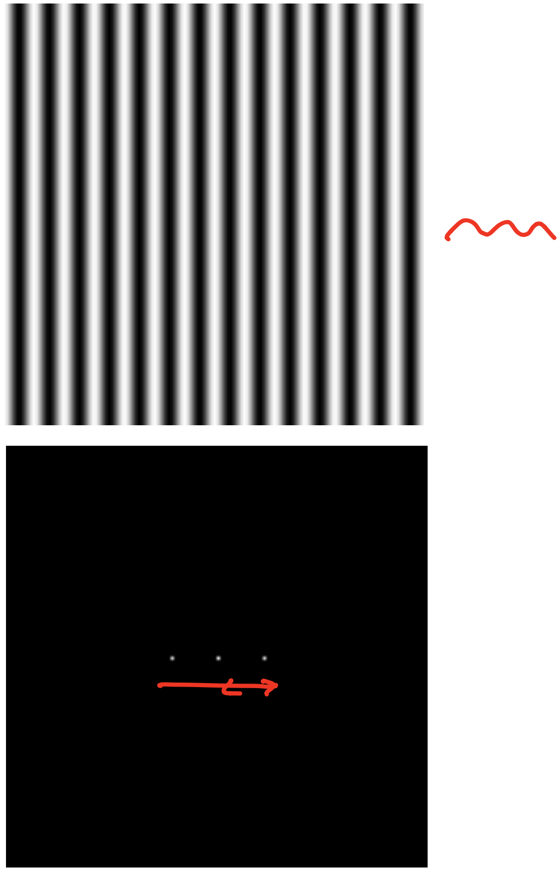
http://www.imagemagick.org/Usage/fourier/#fft
http://cns-alumni.bu.edu/~slehar/fourier/fourier.html

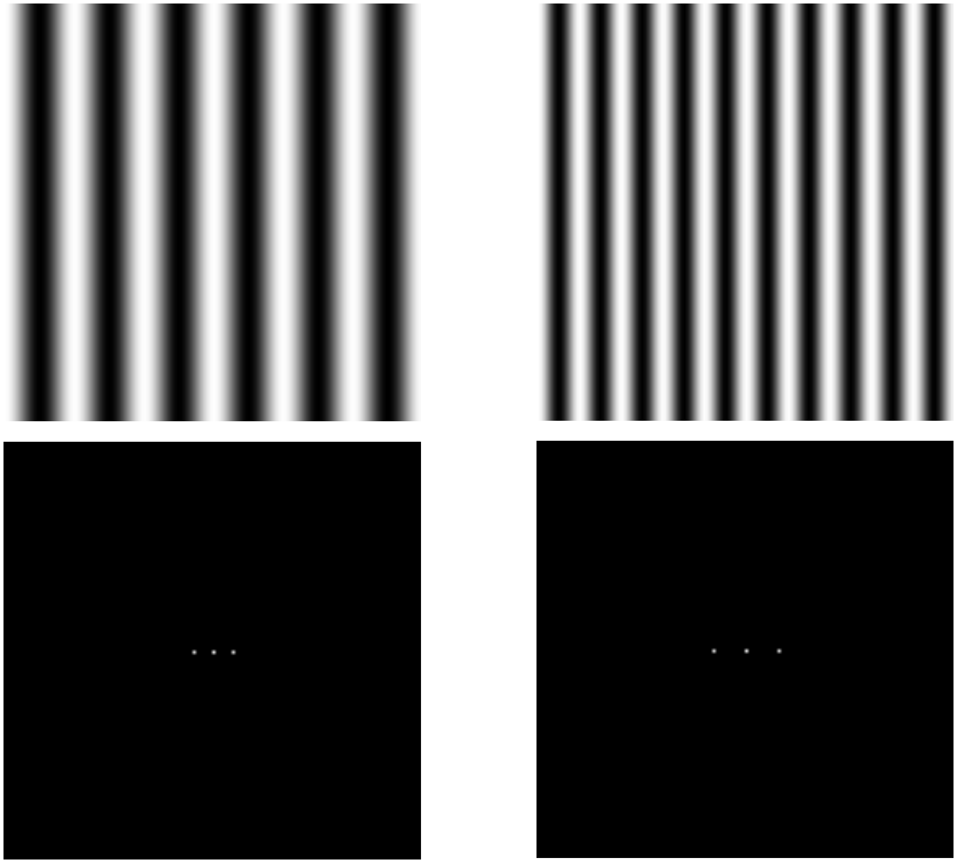
Above; sine
1, 3, 5, 7
(harmonics)

Below; 1,
(1+3),
(1+3+5),
(1+3+5+7)

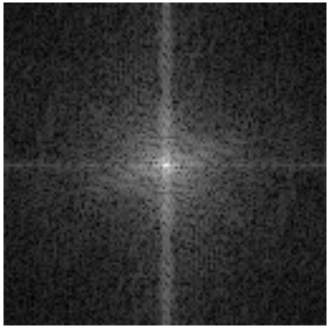
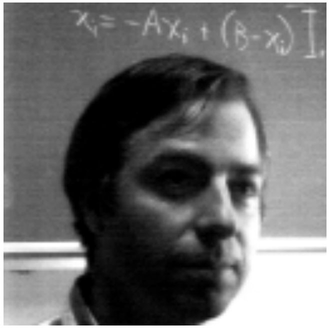
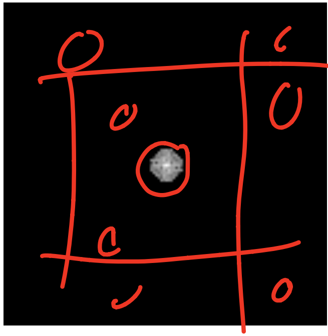




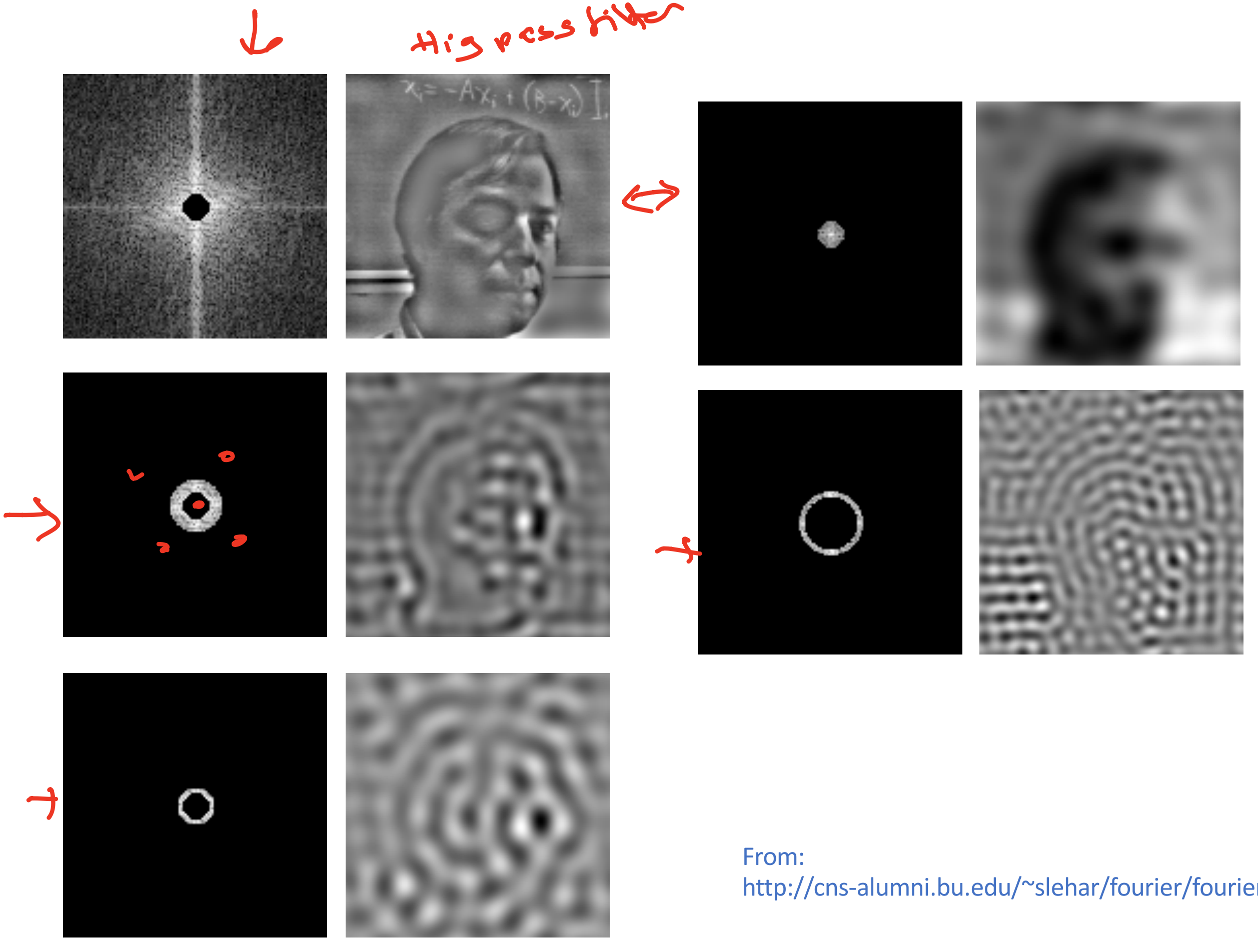




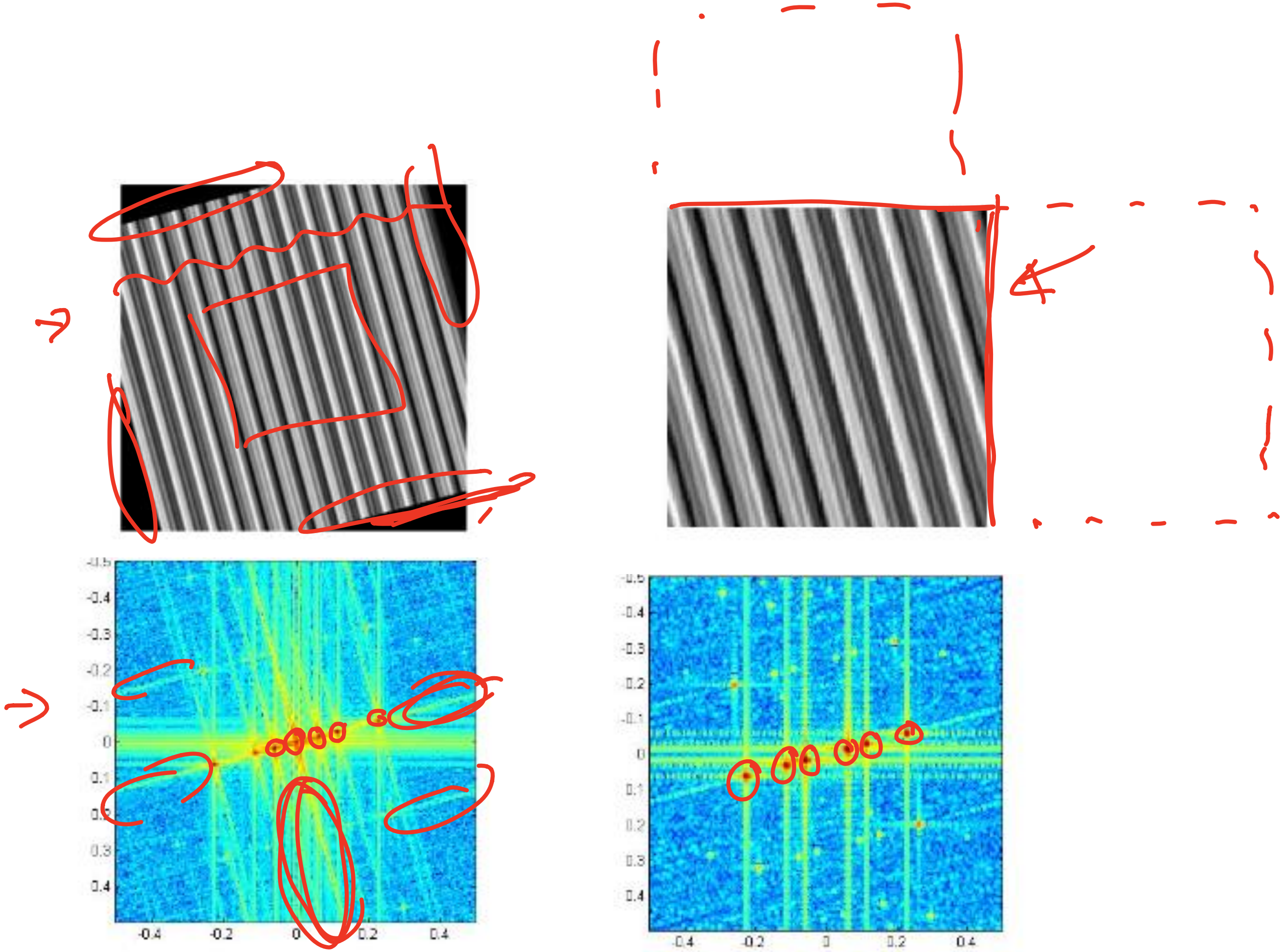
DFT magnitude (+ phase)

Image	Fourier transformed	invers Fourier transformed
		
Lowpass filtered and invers transformed.		

From:
http://cns-alumni.bu.edu/~slehar/fourier/fourier.html



From:
http://cns-alumni.bu.edu/~slehar/fourier/fourier.html


-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
-0.4
-0.2
0
0.2
0.4
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
-0.4
-0.2
0
0.2
0.4

Fourier transform is generally complex

- We need to consider both magnitude and phase!
- In general for a complex number:

$$c = a + jb = |c|e^{j\theta}$$

$$|c| = \sqrt{a^2 + b^2} \qquad \theta = \tan^{-1}(\frac{b}{a})$$

DFT is complex

DFT of image

image

$$\hat{g}(k,l) = \mathcal{F}[g(x,y)] = real\{\mathcal{F}[g(x,y)]\} + j \cdot im\{\mathcal{F}[g(x,y)]\}$$

$$Phase : arg(\mathcal{F}[g(x,y)) = \tan^{-1}\left(\frac{im\{\mathcal{F}[g(x,y)]\}}{real\{\mathcal{F}[g(x,y)]\}}\right)$$

Phase

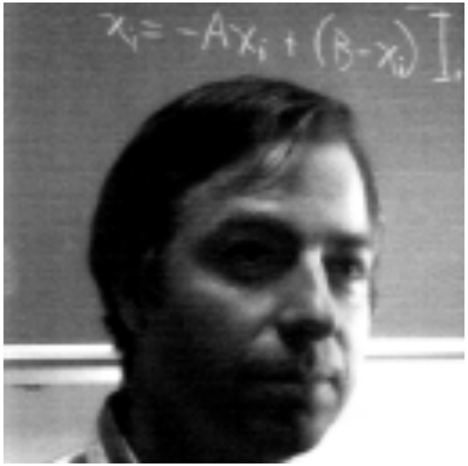
Magnitude

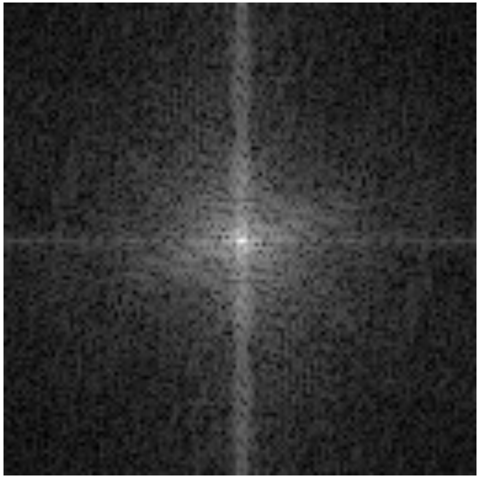
$$|\mathcal{F}[g(x,y)| = \sqrt{im\{\mathcal{F}[g(x,y)]\}^2 + real\{\mathcal{F}[g(x,y)]\}^2}$$

$$|\mathcal{F}[g(x,y)| = \sqrt{\mathcal{F}[g(x,y)]\mathcal{F}[g(x,y)]^*}$$

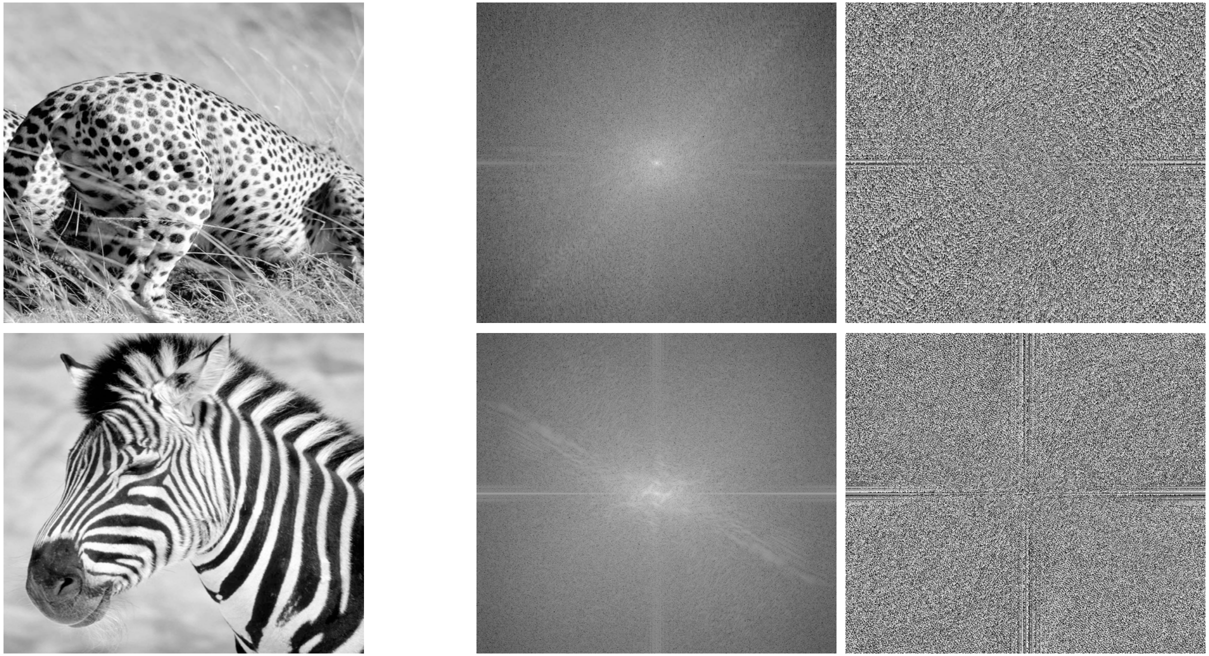
DFT is complex

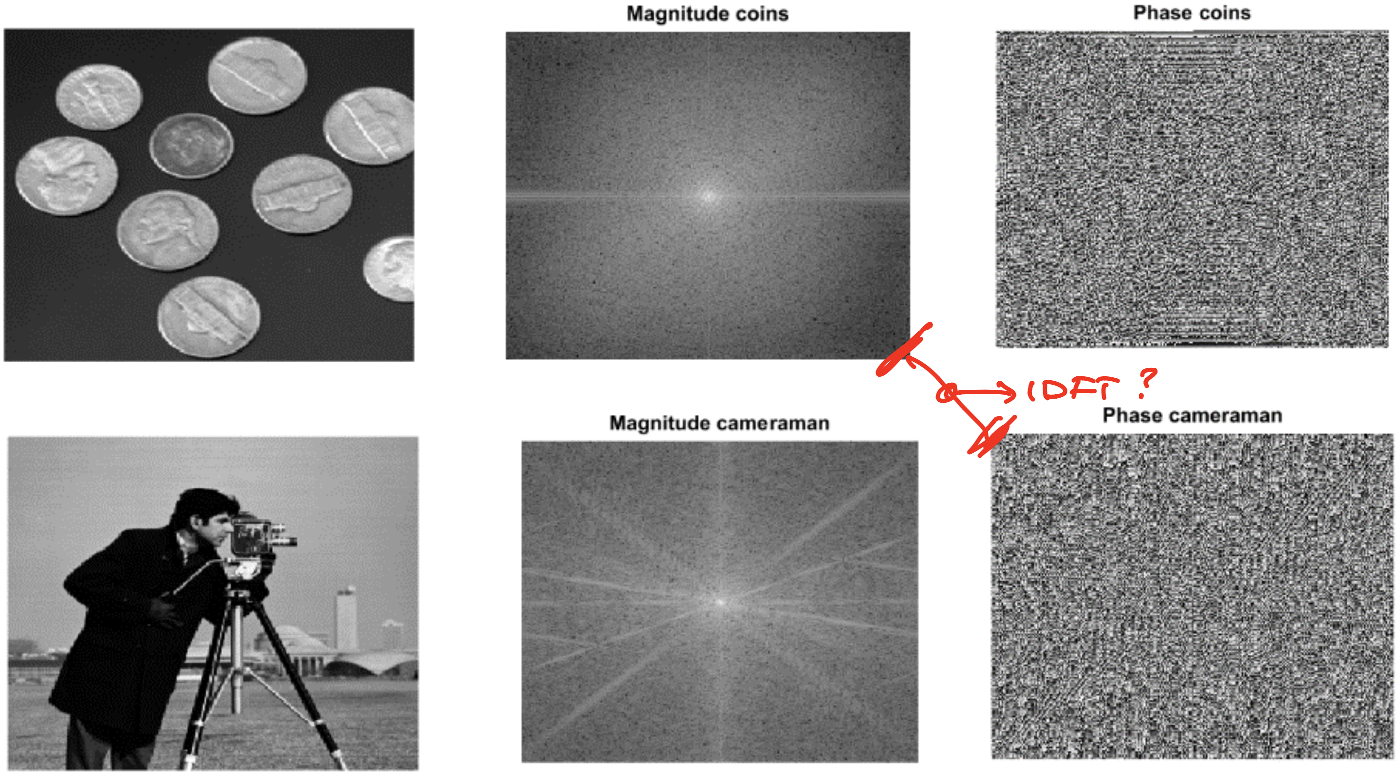
- In previous examples we looked at the DFT magnitude





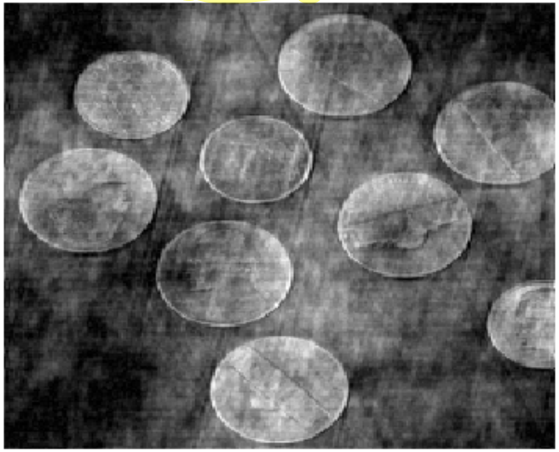
Examples, magnitude and phase




Magnitude coins
Phase coins
IDFT ?
Magnitude cameraman
Phase cameraman

The phase is surpringsingly important!

magnitude from cameraman, phase from coins

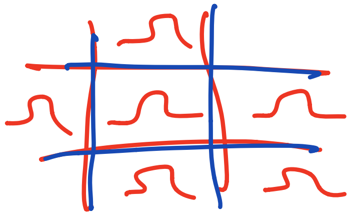


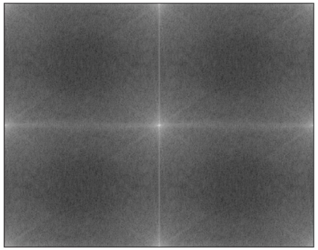
magnitude from coins, phase from cameraman

Power spectral density - Periodogram

- How is the power (or energy) of the signal/image distributed over the different frequencies?
- Often seen as magnitude of DFT squared, called Periodogram:

$$P(i,j) = |G(k_x, k_y)|^2$$



DFT believes that the input signal/image is periodic. This gives false edges

Often better to use a window / bellshape to smooth the effect of edges before looking at the power spectrum density P(i,j)

This is called windowed Periodogram

There are many other methods to estimate power spectrum – not covered here.

2D DFT – some observations

- Offers a rich representation of an image
- It is a global transform
- Can apply the DFT in local windows. This leads to what is called Gabor functions

- DFT is in gerneral complex – which is negative.
- If the image is symmetric about both axes, the DFT values are real (not complex). This leads to the definition of Discrete Cosine Transform (DCT)

Discrete Cosine Transform (DCT)

- DFT is generelly complex
  - Symmetric extension would be real
  - This idea is used when defining the DCT

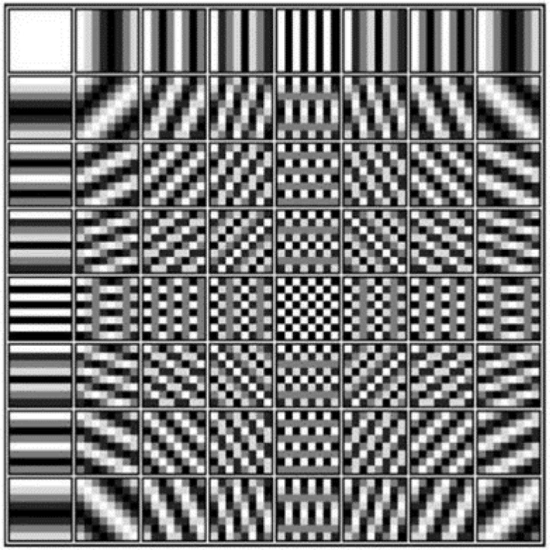
- DFT implicitly treat the signal/image as if it was one period of a periodic signal/image

- DCT implicitly treat the signal/image as if it was one period of an mirrowed/symmetric extension of the signal/image

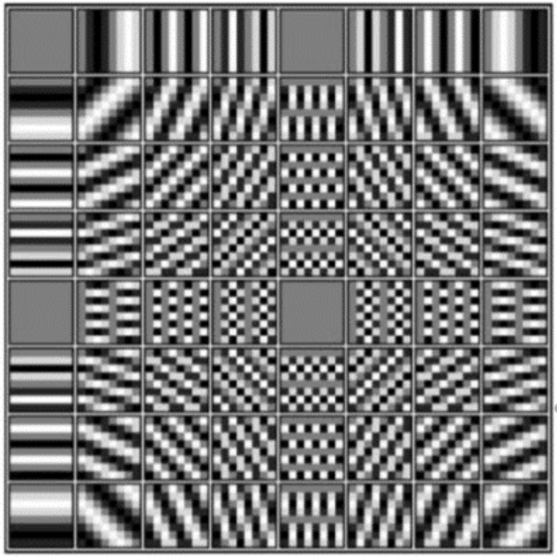
(Remember eigenimages, SVD)

Basis images for 8x8 image, DFT

$$\hat{g} = \sum_{i=1}^{N} \sum_{j=1}^{M} g_{ij} \mathbf{u}_i \mathbf{u}_j^T$$



Real part of basis images



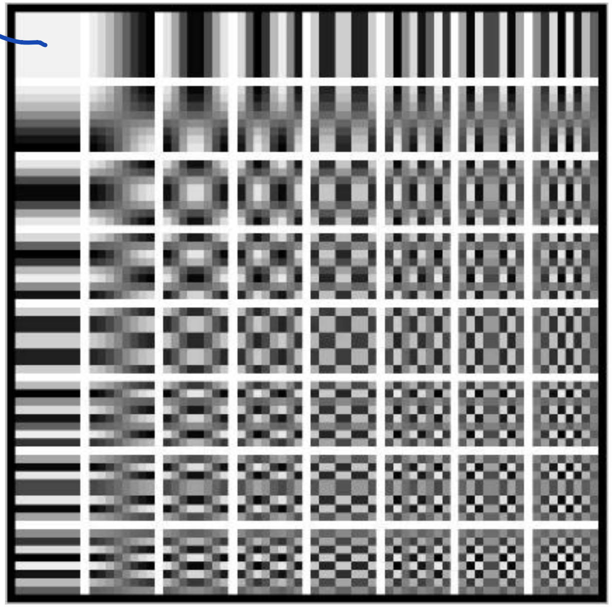
Imaginary part of basis images

Discrete Cosine Transform (DCT) - Basis images for 8x8 image (patch)

Can build any image patch as a linear combination of these basis images.

DCT is a unitary transform. It is not complex, thus it is a orthogonal transform.

DC



$$\hat{g} = \sum_{i=1}^{N} \sum_{j=1}^{M} g_{ij} \mathbf{u}_i \mathbf{u}_j^T$$

Image patch

Basis images

coefficients

Filtering in the Frequency Domain

- Why filter?
  - Noise reduction (denoising)
  - Image enhancement
  - Effective representation
    - Compression
    - Feature extraction – classification
  - Etc..

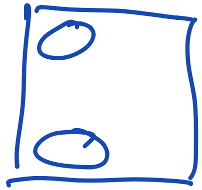
Noise

- Additive $g(x,y) = f(x,y) + n(x,y)$
- Multiplicative $g(x,y) = f(x,y) \cdot n(x,y)$

- **Gaussian**
  - Each pixel has an additive noise component drawn from a Gaussian distribution

$$n(i,j) \in N(\mu(i,j), \sigma(i,j))$$

  - If homogeneous: $\mu(i,j) = \mu \;\text{ and }\; \sigma(x,y) = \sigma \;\text{ for all }\; i,j$



- **Salt and pepper / impulse**
  - Some pixels have noise. These are white or black.
  - Probability for a pixel of being influenced by noise is often modelled as a Poisson distribution

More on noise

- Homogeneous – noise model/parameteres are the same for all pixels.
- Zero-mean = unbiased noise
- White noise = flat power spectrum = all frequencies has equal noise power.
- Uncorrelated and zero-mean = white noise
  - Can be dependent even if it is uncorrelated
- iid – independent identically distributed (If the noise is iid and zero-mean it is white noise)
  - Independent noise: if the value of the noise at one pixel position is independent from the noise value at other pixel positions, the noise is independent.