University
of Stavanger

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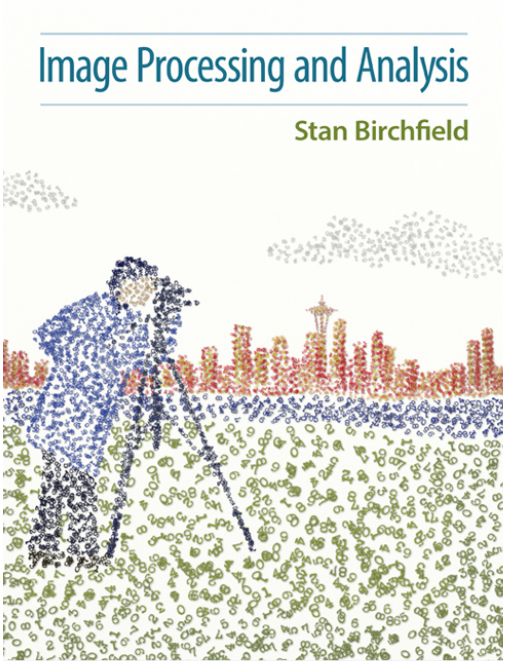
ELE510 Image processing and computer vision

Camera and calibration, (chap 13.4, 13.5 Birchfield) 2020



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(13.5) Camera calibration

- Goal of calibration is to estimate intrinsic and extrinsic parameters from one or several images.
- Calibrating a pinhole camera involves estimating 11 parameters because the matrix $\mathbf{P_M}$ contains 12 elements, but it is unique only up to an unknown nonzero scaling factor. 3D-> 2D means we loose depth, therefore one free parameter.
- It is common to set Pm34 = 1 as a normalization of $\mathbf{P_M}$

2D image [pixels]

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \propto \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\mathbf{P}_{\{3\times 4\}}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

3D world [m]

11 param.

$p_{34} = 1$

Camera calibration

$$\mathbf{P}_{\{3\times 4\}} = \underbrace{\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}_{\{3\times 3\}}} [\mathbf{R}_{\{3\times 3\}} \ \mathbf{t}_{\{3\times 1\}}]$$

- 6 values for the Euclidean rotation and translation (3 values each), and 5 values for the internal calibration matrix **K.**

- These are what we called the **extrinsic** and **intrinsic parameters**, respectively.

Camera calibration

- Camera calibration is the operation to *measure* and *compute* an estimate of the

1. **Intrinsic camera parameters**, and
2. **Extrinsic camera parameters**

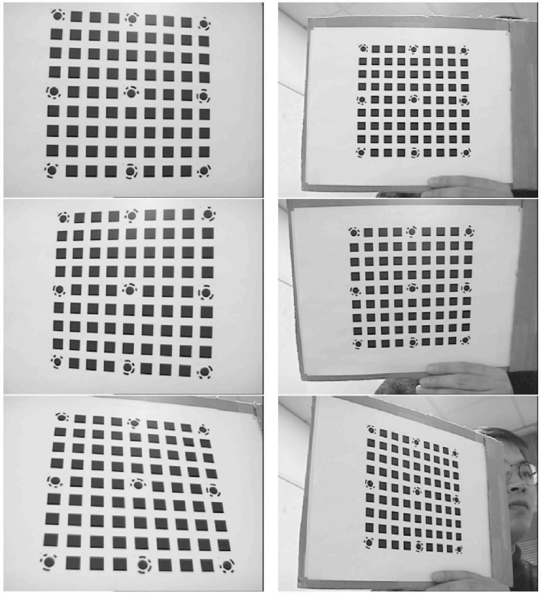
- Camera calibration is *necessary in machine-* and *robot vision* because we want to
- **Reconstruct** a world model of the scene or objects in the scene, and or
- **Measure** physical distances in the scene, and or
- **Interact** with objects in the scene (robot, hand-eye coordination).

- The most popular technique for calibrating the intrinsic camera parameters is **Zhang's algorithm**.

Zhangs algorithm - overview

- Require a planar calibration target with known coordinates /dimensions in meter. Typically used checkerboard
- Require 6-10 (more than 3) different images captured of the target at different positions.
- Find feature points (corners in the chekerboard, use Harris/SIFT) in the images (x,y)coordinates in the image, along with knowing the physical distance of the corner points in meters.
- From this we can estimate the intrinsic parameters K. (will look at some more details)
- Can also find extrinsic parameters. This is not always intersting

Checkerboard calibration pattern



(a) (b)

“Checkerboard” pattern suggested by Zhengyou Zhang (Microsoft Research) 1999

A plane viewed from different unknown orientations.

Zhangs Calibration Algorithm

- The projection of a target point onto the image plane is given by:

planar object

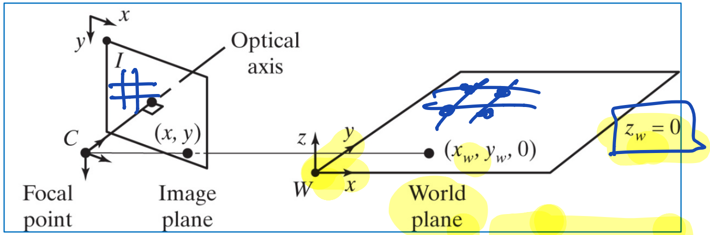
$$\begin{bmatrix}x\\y\\1\end{bmatrix} \propto \underbrace{\begin{bmatrix}\alpha&\gamma&u_0\\0&\beta&v_0\\0&0&1\end{bmatrix}}_{\mathbf{K}_{\{3\times3\}}} \underbrace{[\mathbf{r}_1\quad \mathbf{r}_2\quad \mathbf{r}_3\quad \mathbf{t}]}_{[\mathbf{R}_{\{3\times3\}}\quad \mathbf{t}_{\{3\times1\}}]} \begin{bmatrix}x_w\\y_w\\0\\1\end{bmatrix} = \underbrace{\mathbf{K}\,[\mathbf{r}_1\quad \mathbf{r}_2\quad \mathbf{t}]}_{\mathbf{H}_{\{3\times3\}}} \begin{bmatrix}x_w\\y_w\\1\end{bmatrix}$$

$[h_1\quad h_2\quad h_3]$

$$\boxed{\mathbf{K}\mathbf{r}_i \propto \mathbf{h}_i \implies \mathbf{r}_i \propto \mathbf{K}^{-1}\mathbf{h}_i}$$

$$\Rightarrow \boxed{\mathbf{r}_1^T\mathbf{r}_2 = 0}$$

$$\Rightarrow \boxed{\mathbf{r}_1^T\mathbf{r}_1 = \mathbf{r}_2^T\mathbf{r}_2} = 1$$



Define image of the absolute conic IAC: $\mathbf{w}_\infty = \mathbf{K}^{-T}\mathbf{K}^{-1} = (\mathbf{K}\mathbf{K}^T)^{-1}$

Combine with equations in red frame from prev. slide:

$r_1^T \cdot r_2 = 0$

$$\mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 = 0 \Rightarrow \boxed{\Longrightarrow \mathbf{h}_1^T\mathbf{w}_\infty\mathbf{h}_2 = 0}.$$

$$\mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 \quad \boxed{\Longrightarrow \mathbf{h}_1^T\mathbf{w}_\infty\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{w}_\infty\mathbf{h}_2}$$

$r_1^T \cdot r_1 = r_2^T \cdot r_2$

$r_i = K^{-1} h_i$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \propto \underbrace{\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}_{\{3\times3\}}} \underbrace{[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3 \quad \mathbf{t}]}_{[\mathbf{R}_{\{3\times3\}} \quad \mathbf{t}_{\{3\times1\}}]} \begin{bmatrix} x_w \\ y_w \\ 0 \\ 1 \end{bmatrix} = \underbrace{\mathbf{K}[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}]}_{\mathbf{H}_{\{3\times3\}}} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}$$

$$[h_1 \quad h_2 \quad h_3]$$

Define image of the absolute conic IAC: $\mathbf{w}_\infty = \mathbf{K}^{-T}\mathbf{K}^{-1} = (\mathbf{K}\mathbf{K}^T)^{-1}$

Combine with equations in red frame from prev. slide:

1x3 3x3 3x1

$$\mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 = 0 \qquad \boxed{\Longrightarrow \ \mathbf{h}_1^T\mathbf{w}_\infty\mathbf{h}_2 = 0} \quad (1)$$

$$\mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 \quad \boxed{\Longrightarrow \ \mathbf{h}_1^T\mathbf{w}_\infty\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{w}_\infty\mathbf{h}_2} \quad (2)$$

$a_{11}^T \cdot w$ $a_{22}^T \cdot w$

Rewrite! **aij** are known from **H** matrix, **wi** contains unknown from $\mathbf{w}_\infty$

$$\mathbf{a}_{12}^T\mathbf{w} = 0 \quad (1)$$

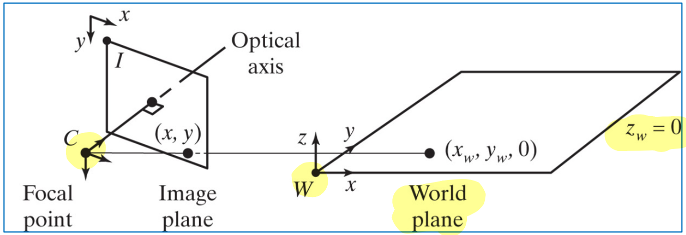
$$(\mathbf{a}_{11} - \mathbf{a}_{22})^T\mathbf{w} = 0 \quad (2)$$

From these we can estimate $\mathbf{w}_\infty$ when we have a number of different H´s. We have as many H´s as we have images.

From $\mathbf{w}_\infty$ we can find **K**

How to estimate the homography matrix H?

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \propto \mathbf{H} \begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix} = \boxed{[\mathbf{h}_1 \mathbf{h}_2 \mathbf{h}_3]} \begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix}$$



Using SIFT / Harris on calibration images we find corresponding pairs of points:

$$\boxed{(x_i\,, y_i) \leftrightarrow (x_{w_i}\,, y_{w_i})}$$

From these we can estimate **H**.
Normalized Direct Linear Troansfomration (DLT) algorithm can be used.
We will not look at those details, but with a number of points this can be done.

Have as many **H** estimate as we have images.

When we have K and Hj, we can find Rj and tj for each image j

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \propto \underbrace{\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}_{\{3\times 3\}}} \underbrace{[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3 \quad \mathbf{t}]}_{[\mathbf{R}_{\{3\times 3\}} \quad \mathbf{t}_{\{3\times 1\}}]} \begin{bmatrix} x_w \\ y_w \\ 0 \\ 1 \end{bmatrix} = \underbrace{\mathbf{K}\,[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}]}_{\mathbf{H}_{\{3\times 3\}}} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}$$

$$\underbrace{K \cdot [r_1 \quad r_2 \quad t] \cdot \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}}$$

$\Downarrow$

[

Thereafter we can do a reprojection , i.e. With these parameters where would the physical point be on the image plane? Is that far from the true image point? -> Reprojection error:

reprojection

$$RepErr = \sum_i \sum_j ||\mathbf{x}_{ij} - g(\mathbf{K}, \mathbf{R}_i, \mathbf{t}_i, \mathbf{x}_w^j, \phi)||^2$$

-> lens distortion

⇒ Can update iteratively the intrinsic and extrinsic parameters minimizing the reprojection error -> bundle adjustment

ALGORITHM 13.12 Zhang's camera calibration routine

CALIBRATECAMERAZHANG

Input: $n \geq 3$ images of a known planar target at different orientations and directions
Output: intrinsic parameters $\mathbf{K}$, lens distortion parameters ϕ, and extrinsic parameters

for each image **do**
    Detect image features corresponding to known points on target.
    Compute the homography using the normalized DLT algorithm.

$\underset{\sim}{H_j}$ imag j

Stack the entries from the homographies into matrix $\mathbf{A}$.
Solve $\mathbf{A}\mathbf{w} = \mathbf{0}$ for $\mathbf{w}$, then reshape into ω_∞.
Compute the five intrinsic parameters of $\mathbf{K}$ from $\boxed{\omega_\infty}$ using either
    Cholesky decomposition, or
    Equation (13.143) for v_0, then Equation (13.142) for λ, then Equations (13.144)–(13.147).
Compute extrinsic parameters $\mathbf{R}_i$ and $\mathbf{t}_i$ for each image $i = 1, \ldots, n$ using Equations (13.148)–(13.151).
Using these results as a starting point, minimize Equation (13.152) to perform bundle adjustment.

reprojection error