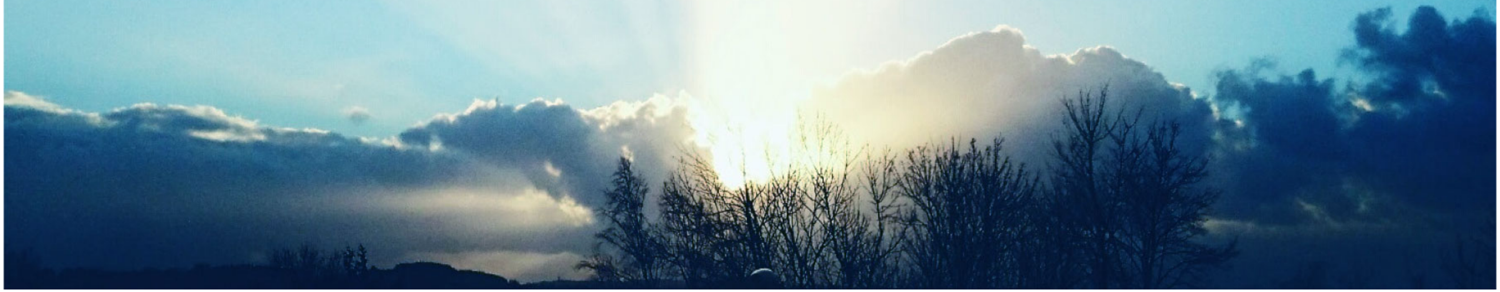
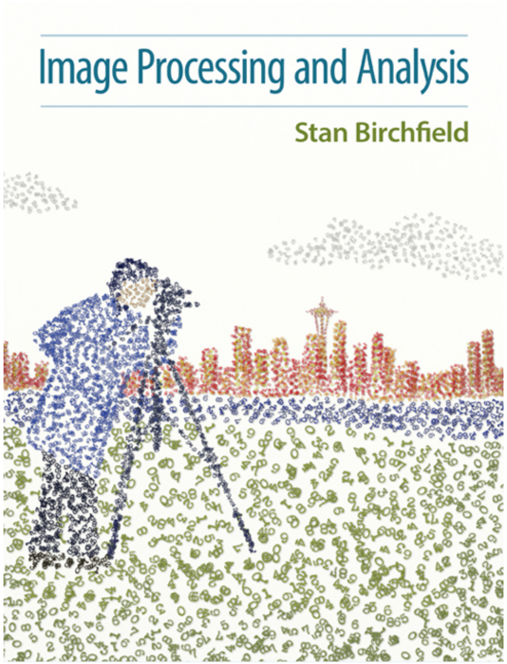
University of Stavanger

Prof. Kjersti Engan

ELE510 Image processing and computer vision

Frequency Domain processing, Fourier transform (Chap 6.1-6.2 Birchfield) 2020





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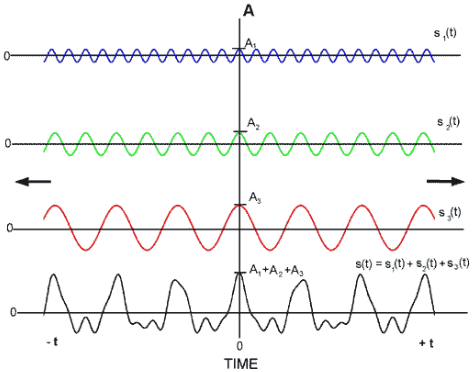
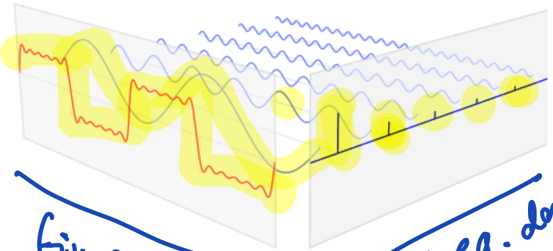
(6.1) Fourier transform – frequency content



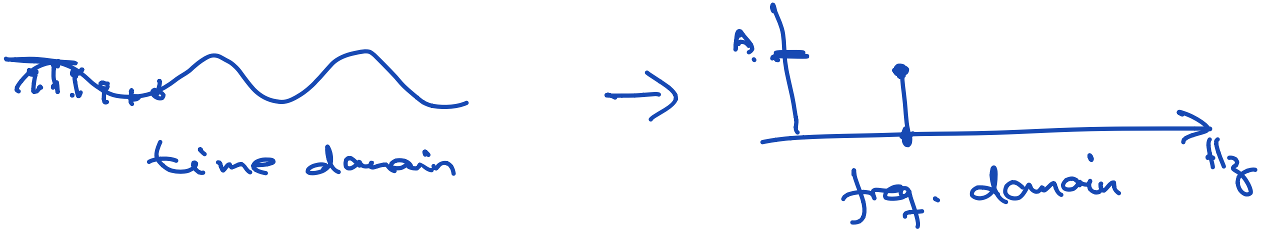
A signal can be regarded as a weighted sum of different frequency components.

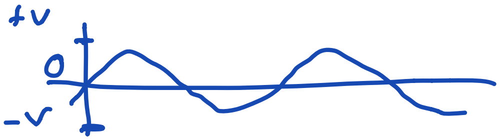


time

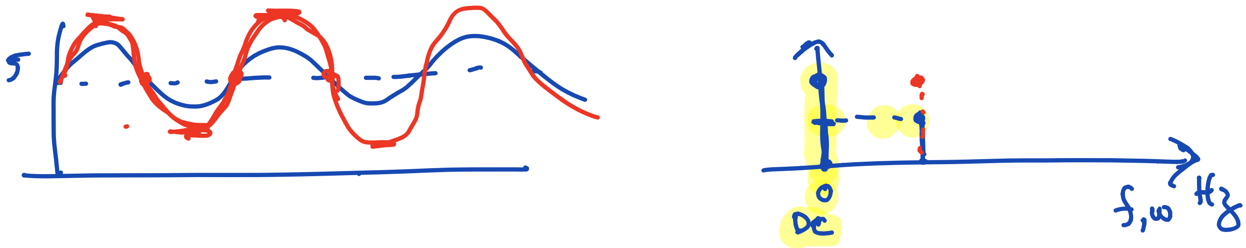
freq. domain

Time (space) vs. Freq. domain – main idea





mean = 0, DC-level is 0



Fourier transform

- Fourier series: any periodic function can be represented as a weighted sum of sines and cosines.
- Can be extended to aperiodic signals -> Fourier transform
- Discrete signals –> discrete time Fourier transform
- Can we extend to 2D (images)? -> Yes!
- We look at complex sinusoids: $\boxed{e^{jw}}$

$$e^{jw} = cosw + j \cdot sinw$$

Complex

Eulers formula

Fourier transform (continous)

- **Fourier transform *G(f)*:** the integration of the signal after first multiplying by a certain complex exponential:

$$\boxed{G(f) \equiv \mathcal{F}\{g\} \equiv \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}\,dt} \quad \Leftrightarrow \quad \boxed{g(t) = \mathcal{F}^{-1}\{G\} \equiv \int_{-\infty}^{\infty} G(f)e^{j2\pi ft}\,df}$$

ω

- If *t* is measured in seconds, then *f* is measured in inverse seconds, also known as hertz.
- By applying **Euler's formula:**

$$G(f) = \underbrace{\int_{-\infty}^{\infty} g(t)\cos 2\pi ft\,dt}_{G_{even}} + j\underbrace{\int_{-\infty}^{\infty} -g(t)\sin 2\pi ft\,dt}_{G_{odd}}$$

Discrete time Fourier transform (1D)

$$X(e^{jw}) = \mathcal{F}(x(n)) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$$

$$x(n) = \boxed{\frac{1}{2\pi}} \int_{-\pi}^{\pi} X(e^{jw})e^{jwn}dw$$

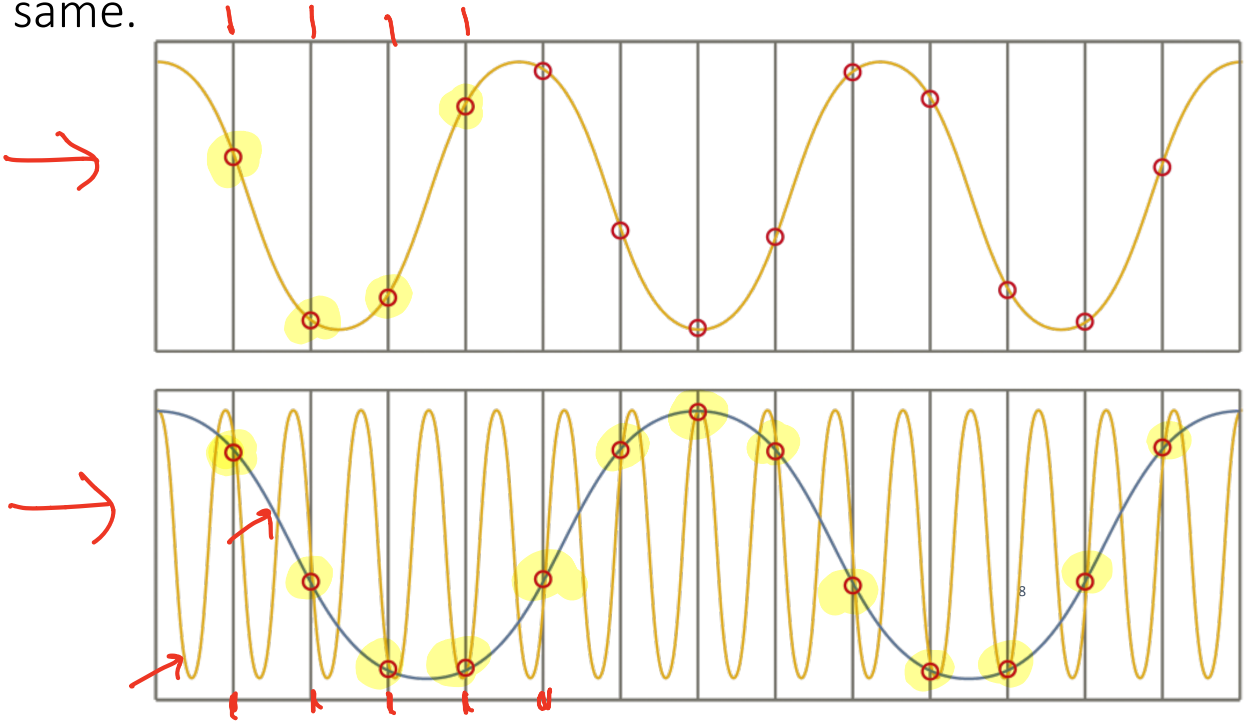
Eulers formula:

$$e^{jx} = \cos x + j \sin x$$

Dealing with a discrete time signal (or image) we can only represent frequencies between [-Fs/2 , Fs/2]. Fs is the samplingfrequency.

The frequency is normalized to [-pi, pi] rad. Outside this frequency area, the specter repeats itself, i.e. the Discrete time fourier transform is periodic. WHY? - aliasing

Aliasing – for discrete signals, multiple frequencies look the same.



Nyquist rate

- **Nyquist-Shannon sampling theorem**: if a certain condition holds true, then the discrete samples contain just as much information as the original signal, so that the original signal can be reconstructed *exactly* from the discrete samples.
- The sampling rate must be greater than the **Nyquist rate**, which is twice the highest frequency in the signal.

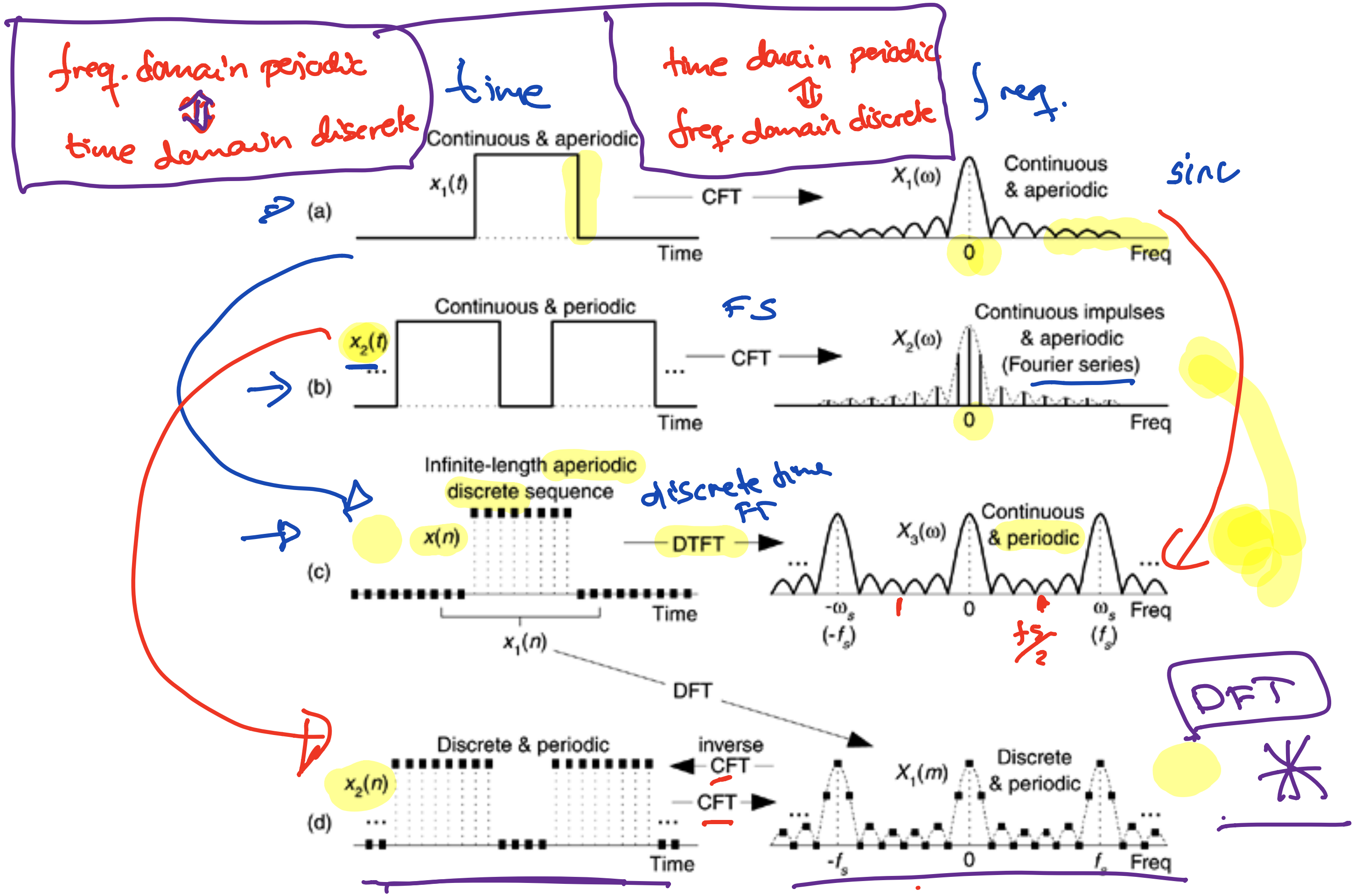
- **Oversampled:** when the sampling frequency is greater than the Nyquist rate.
  - Perfect reconstruction is possible.

- **Undersampled:** when the sampling frequency is lower than the Nyquist rate.
  - Important information about the signal is irrecoverably lost.
  - When a signal is undersampled, **aliasing** occurs.

- **Critically sampled:** when the sampling frequency is exactly the Nyquist rate.
  - The original signal is also unrecoverable.

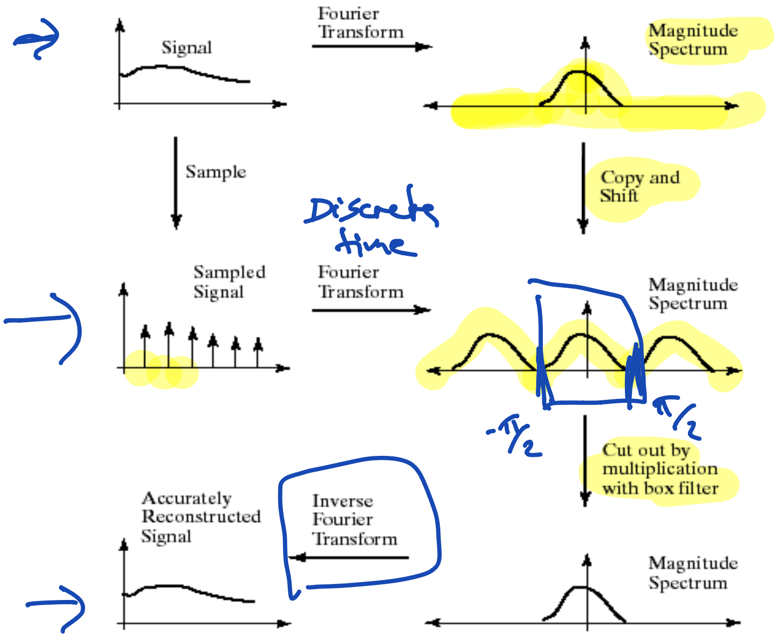
Fourier series and transform

	Continuous signals (images)	discrete signal
aperiodic signals	Fourier transform (FT) aperiodic	discrete time fourier transform (DTFT) periodic
periodic signals	Fourier series (FS) aperiodic, discrete output.	Discrete Fourier transform (DFT, FFT) periodic, discrete



freq. domain periodic ⇕ time domain discrete
time
time domain periodic ⇕ freq. domain discrete
freq.
sinc
Continuous & aperiodic
x1(t)
(a)
CFT
X1(ω)
Continuous & aperiodic
Time
0
Freq
FS
Continuous & periodic
x2(t)
(b)
CFT
X2(ω)
Continuous impulses & aperiodic (Fourier series)
Time
0
Freq
Infinite-length aperiodic discrete sequence
discrete time FT
x(n)
(c)
DTFT
X3(ω)
Continuous & periodic
-ωs (-fs)
0
fs/2
ωs (fs)
Freq
Time
x1(n)
DFT
Discrete & periodic
inverse CFT
CFT
x2(n)
(d)
X1(m)
Discrete & periodic
-fs
0
fs
Freq
Time
DFT

Sampling without aliasing



Sampling with aliasing