Example Euclidean transformation

Rotation
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\underset{\sim}{x}' = \underset{\sim}{R} \cdot \underset{\sim}{x}$$

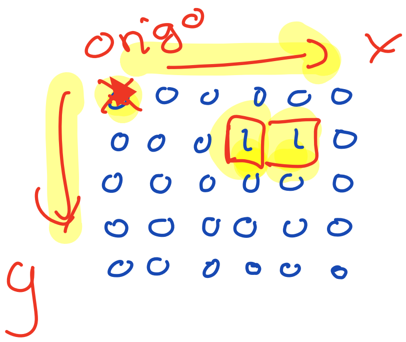
$\theta = \pi/2$

$\cos(\pi/2) = 0$

$\sin(\pi/2) = 1$

$$\underset{\sim}{R} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\underset{\sim}{I}_0$

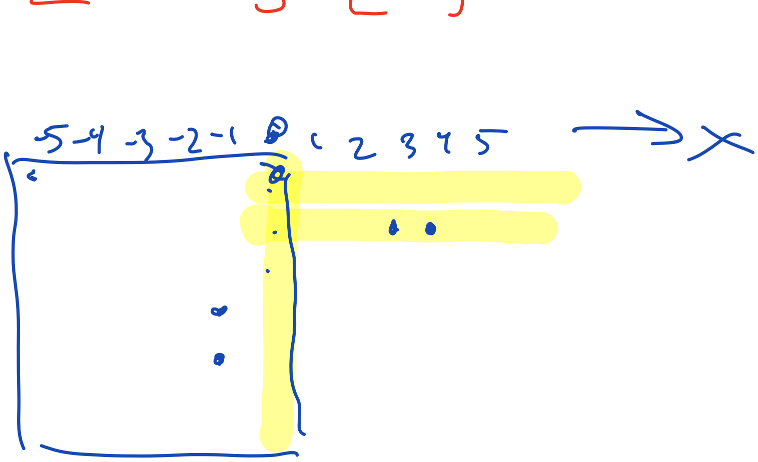


$$\underset{\sim}{x}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \underset{\sim}{x}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\underset{\sim}{x}_1' = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \underset{\sim}{x}_2' = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

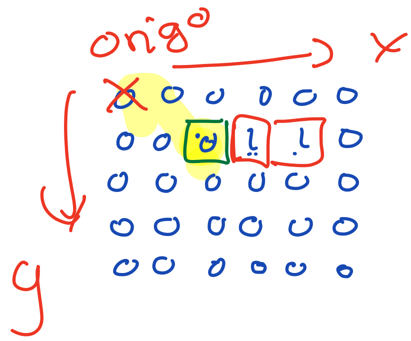
$$\underset{\sim}{x}_1' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\underset{\sim}{x}_2' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$



We do not want the negative index. Lets rotate around $\underline{c}$ instead. $\boxed{\underline{c} = \begin{pmatrix}2\\1\end{pmatrix}}$

$\underset{\sim}{I}_0$



$$\underline{x}' = \underset{\sim}{R}(\underline{x}-\underline{c}) + \underline{c}$$

move before rotation.

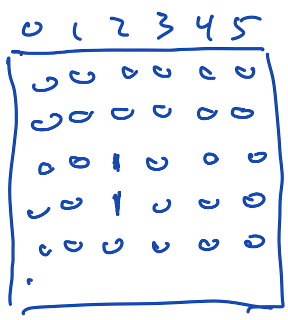
move back

$\underline{}\; x_1 \begin{pmatrix}3\\1\end{pmatrix} \quad x_2 \begin{pmatrix}4\\1\end{pmatrix}$

$$x_1 - c = \begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\underline{x}_1' = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$$

$$\underline{x}_2' = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\left[\begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}\right] = \begin{bmatrix}0\\2\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$



Rotated around $\underline{c}$

Rotation + translation

$$x' = \underset{\sim}{R}(\underline{x} - \underline{c}) + \underline{c} + \underline{t} \qquad \underline{t} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \underset{\sim}{R}x + \underbrace{(-\underset{\sim}{R}c + \underline{c} + \underline{t})}_{\underline{\tilde{t}}}$$

$\Rightarrow$ Rotation about $\underline{c}$ followed by translation $\underline{t}$

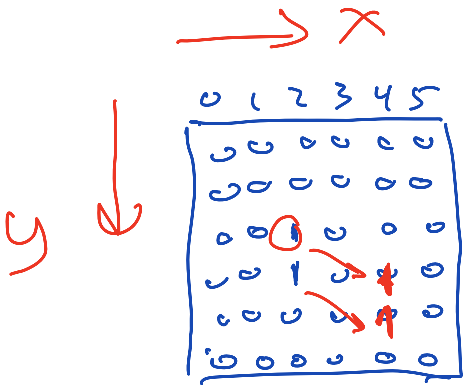
OR (equivalent to)

$\Rightarrow$ Rotation about $\underline{0}$ (origin) followed by translation $\underline{\tilde{t}}$

$$-\underset{\sim}{R}\underline{c} + \underline{c} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

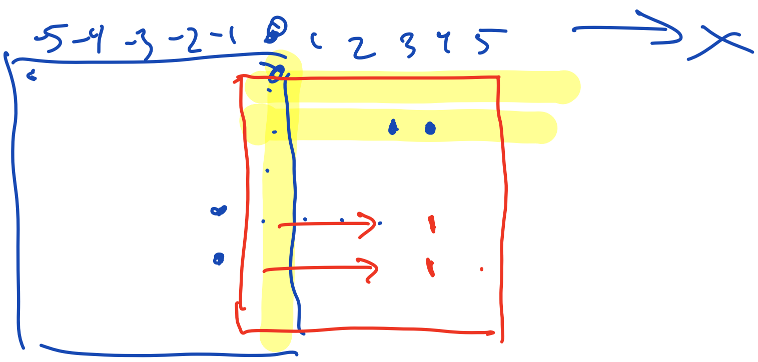
$$= -\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\tilde{t} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

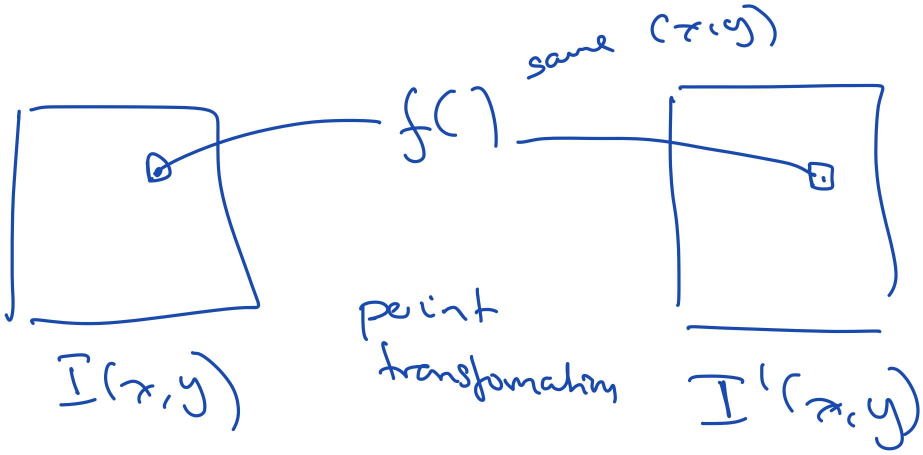


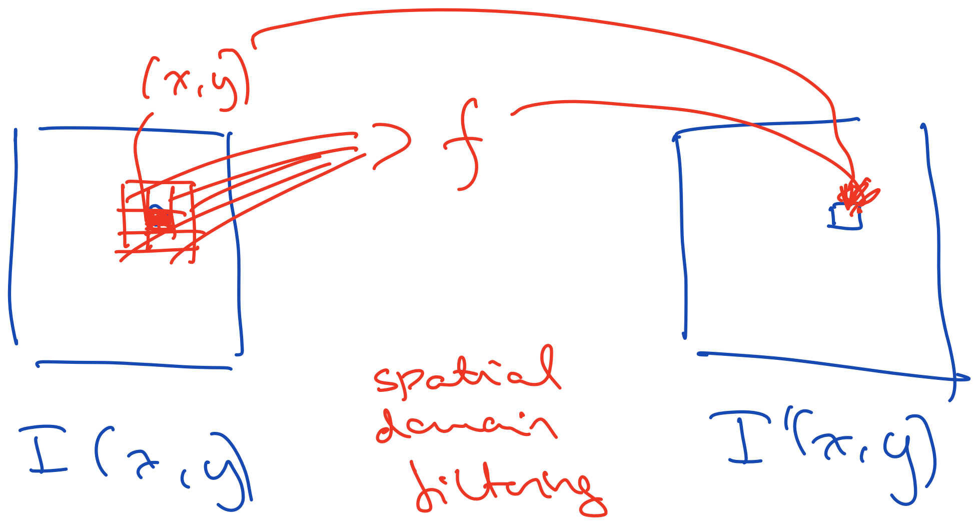
$$\underline{t} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

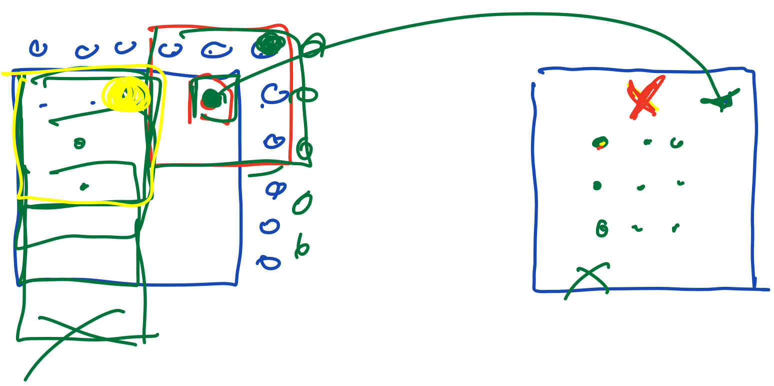
Rotated around $\underline{c}$ + translate with $\underline{t}$

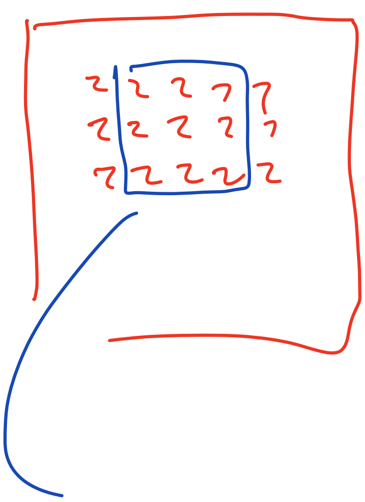


rotated around $\underline{0}$ add $\underline{\tilde{t}} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$





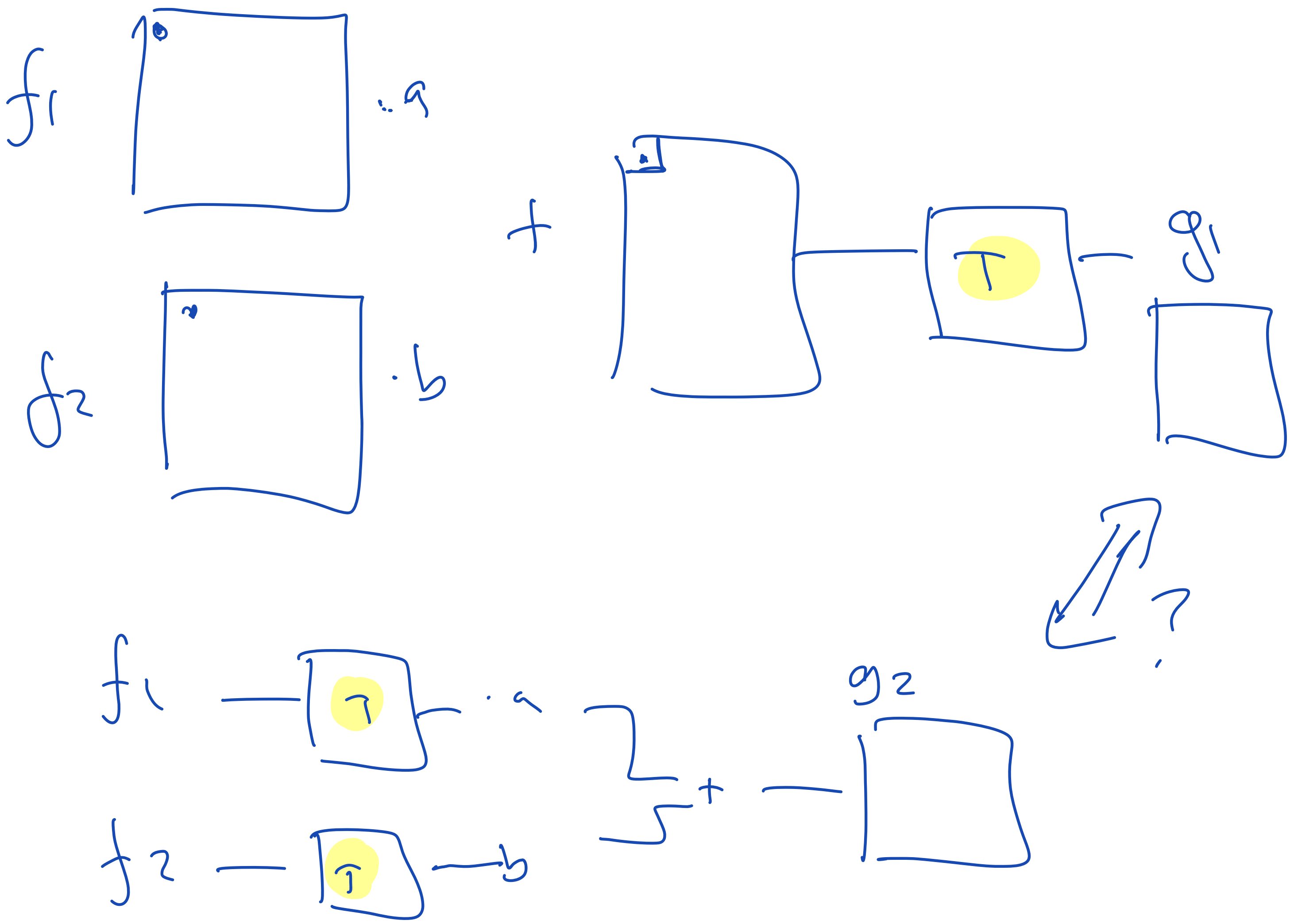


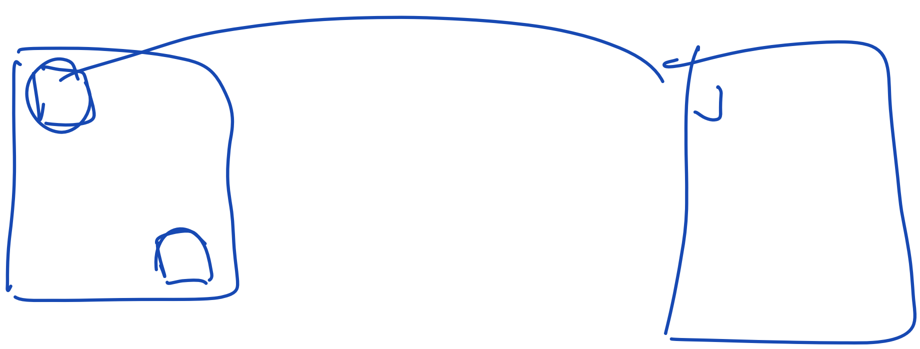


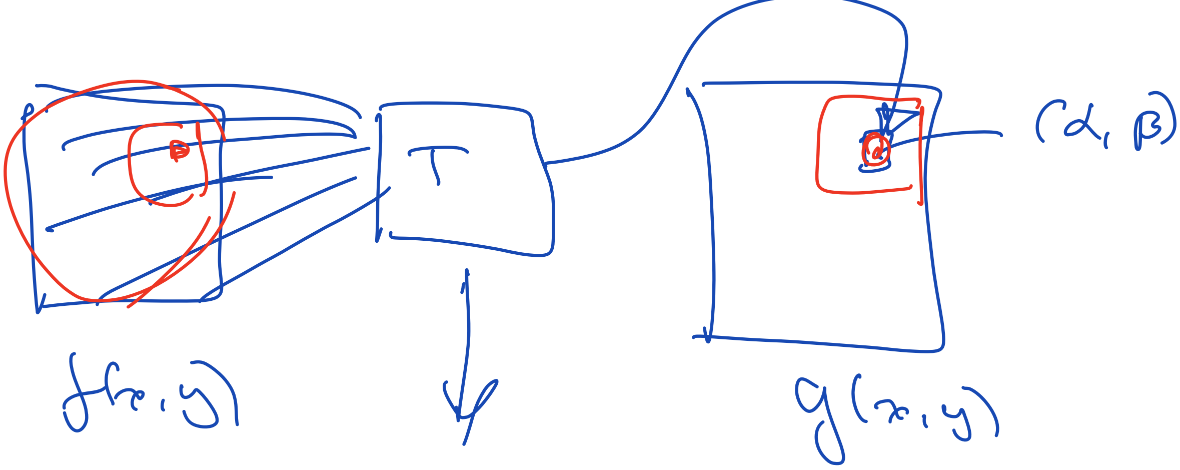
$$2\cdot a_{11} + 2\cdot a_{12} + 2\cdot a_{13} + 2\cdot a_{31} \cdots$$

$$2\cdot \underbrace{\sum_{ij} a_{ij}}_{=0} = 0$$

$$2 \cdot \underbrace{\sum_{ij} a_{ij}}_{=1} = 2$$







$$h(x, \alpha, y, \beta)$$

$$h(x-\alpha, y-\beta)$$