$$f'(x) = T(f(x)) = (f(x))^2$$

$$\alpha \cdot f_1(x) + \beta \cdot f_2(x) \longrightarrow \boxed{T} \longrightarrow T[\alpha f_1(x) + \beta f_2(x)]$$

$$\neq ?$$

$$\alpha \cdot T[f_1(x)] + \beta\, T[f_2(x)]$$

linear? NO

$$f'(x) = T_2[f(x)] = \frac{1}{4} f(x-1) + \frac{1}{2} f(x) + \frac{1}{4} f(x+1)$$

linear ? yes.

$f_1(x)+f_2(x)$

$$\boxed{g = \frac{1}{4}[1\ \ 2\ \ 1]}$$

before or after should be the same

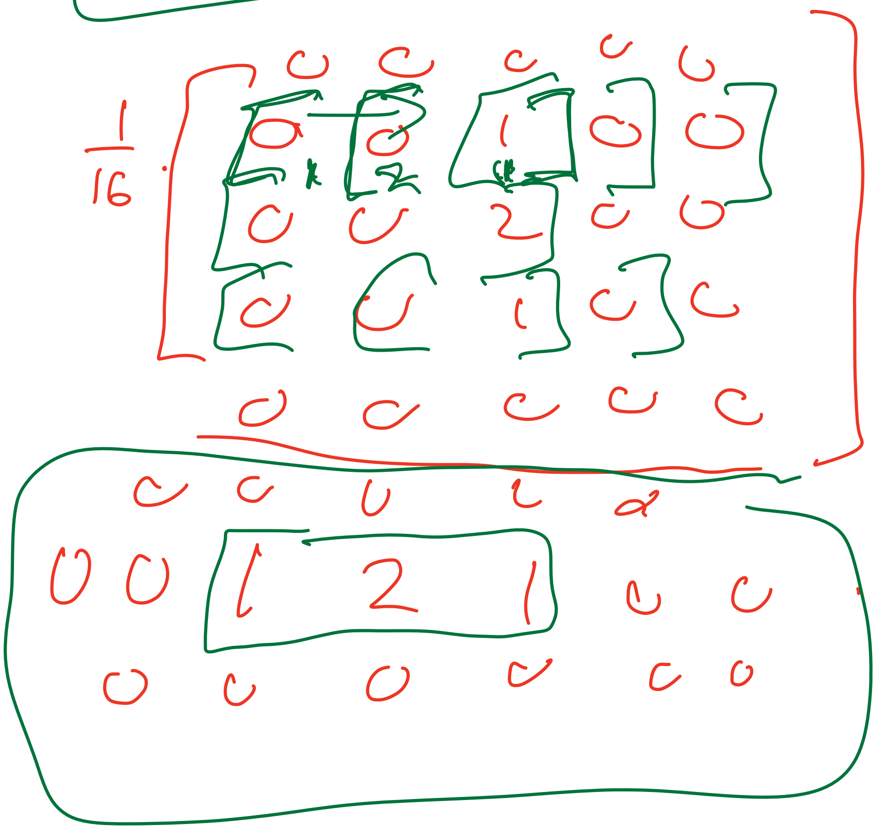
test yourself

Separable filter

Gauss is separable

$$I(x,y) \circledast \left[\frac{1}{16} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}\right].$$

$$\frac{1}{4}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \circledast \frac{1}{4}\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \frac{1}{16}$$

$$\left[I(x,y) \circledast \frac{1}{4}\begin{bmatrix}1\\2\\1\end{bmatrix} \right] \circledast \underbrace{\frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}}$$

$$[\,1 \quad -1\,]$$

Smoothing $[\,1 \quad 1\,] \cdot \frac{1}{2}$

$$\left(f(x) \circledast \frac{1}{2}[\,1 \quad 1\,] \right) \circledast [\,1 \quad -1\,]$$

$$= f(x) \circledast \underbrace{\left(\frac{1}{2}[\,1 \quad 1\,] \circledast [\,1 \quad -1\,] \right)}$$

Smooth

derivative approx.

$$\left.\begin{matrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & \end{matrix}\right\} \circledast \Rightarrow \boxed{1 \quad 0 \quad -1} \quad 0 \quad 0$$

$$= f(x) \circledast \frac{1}{2} \cdot [\,1 \quad 0 \quad -1\,]$$

$$\frac{d}{dx}\left[f(x) \circledast g(x)\right] = f(x) \circledast \left[\frac{d}{dx} g(x)\right]$$

smoothing first because derivative is noise sensitive

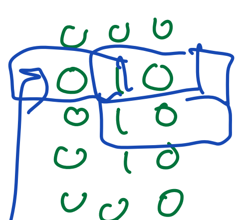
Prewott :

$$\frac{d}{dx}\text{gauss} \rightarrow \frac{1}{2}\begin{bmatrix}1 & 0 & -1\end{bmatrix}$$

$$\left(= \begin{bmatrix}1 & 1\end{bmatrix} \circledast \begin{bmatrix}1 & -1\end{bmatrix} \cdot \frac{1}{2} \right)$$

2D kernel

$$\frac{1}{3} \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} \circledast \frac{1}{2}\begin{bmatrix}1 & 0 & -1\end{bmatrix}$$



$$\Rightarrow \begin{bmatrix}1 & 0 & -1\\1 & 0 & -1\\1 & 0 & -1\end{bmatrix} \cdot \frac{1}{6} \qquad \text{Prewitt}_x$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{-1 \quad 0 \quad 1} & & & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

What if

1. → derivative in both direct.
   → smooth in both directions

2. larger sobel kernels.