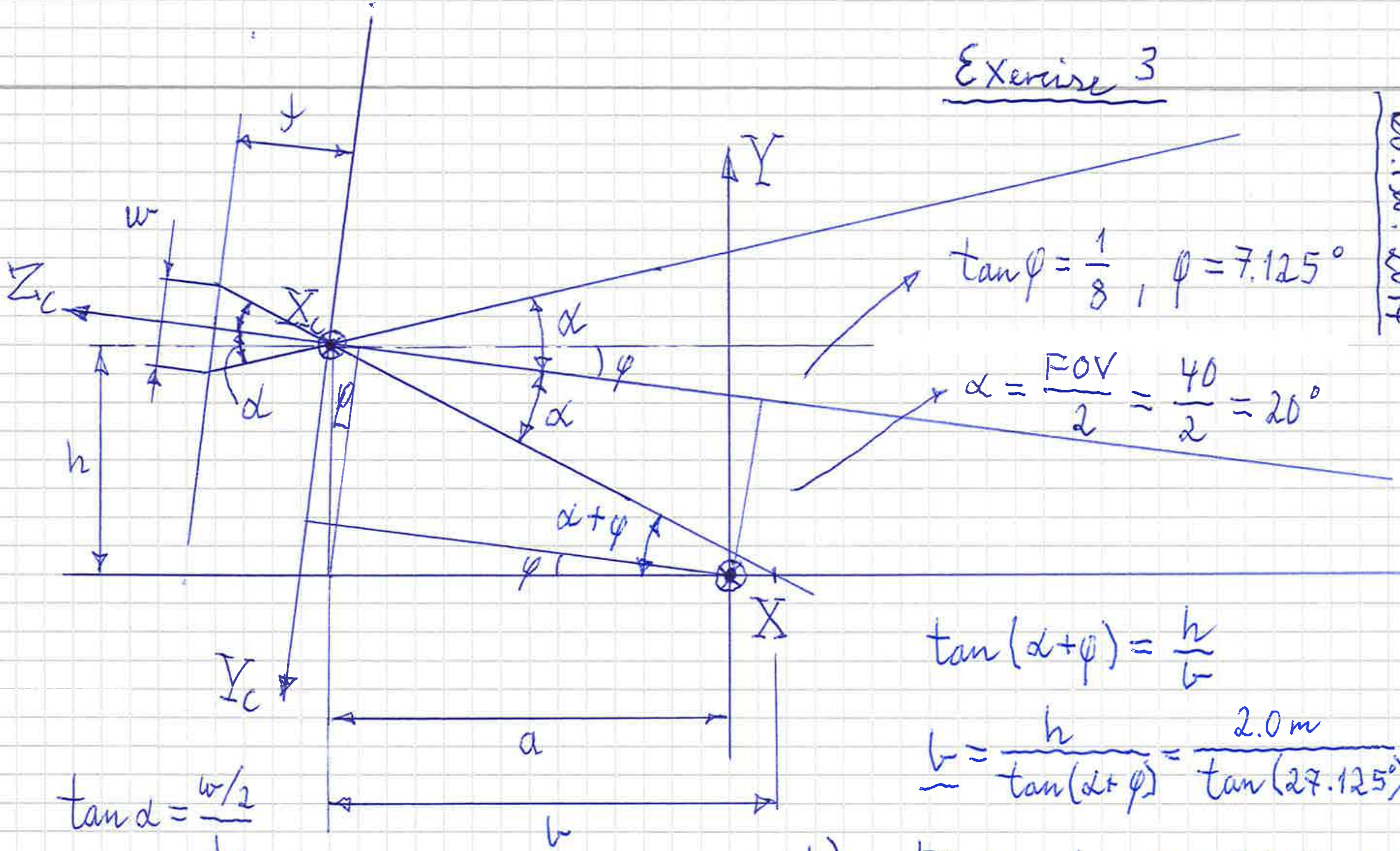
Løsningsforslag

Solution ELE510

28. Feb. 2017

Exercise 3



$\tan\varphi = \frac{1}{8}$, $\varphi = 7.125°$

$\alpha = \frac{FOV}{2} = \frac{40}{2} = 20°$

a) $\tan\alpha = \frac{w/2}{f}$

$\underline{\underline{f}} = \frac{w}{2\tan\alpha} = \frac{6 \cdot 10^{-3}\,m}{2 \cdot \tan 20°} = 8.24 \cdot 10^{-3}\,m$
$\underline{\underline{= 8.24\ mm}}$

b) $\tan(\alpha+\varphi) = \frac{h}{b}$

$\underset{\sim}{b} = \frac{h}{\tan(\alpha+\varphi)} = \frac{2.0\,m}{\tan(27.125°)} = \underline{3{,}904\,m}$

$\underline{\underline{Z_{min}}} = b - a = 3.904 - 3.5 = \underline{\underline{0.404\,m}}$

$a = 3.5\,m$, $h = 2.0\,m$

c) The internal calibration matrix:

$$\underline{\alpha = \beta} = \frac{f}{\Delta X} = \frac{f}{\Delta y} = \frac{f \cdot N}{w} = \frac{f \cdot M}{w} = \frac{8.24 \cdot 10^{-3} \cdot 4000}{6.0 \cdot 10^{-3}}$$

$$\underline{= 5495}$$

$$\underline{x_0 = y_0} = \frac{M}{2} = \frac{N}{2} = \frac{4000}{2} = \underline{2000}$$

$$\underset{\sim}{K} = \begin{bmatrix} \alpha & 0 & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5495 & 0 & 2000 \\ 0 & 5495 & 2000 \\ 0 & 0 & 1 \end{bmatrix}$$ no skew

d) The extrinsic parameters:

Rotation around X: $-(180° + \varphi) = -187.125°$

$c = \cos(-187.125°)$, $s = \sin(-187.125°)$
$= -0.9923$ $\quad$ $= 0.1240$

$$\underset{\sim}{R}_{3D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix}$$

Translations: (See figure on the previous page)

$$t_z = -(h \cdot \sin\varphi + a \cos\varphi) = -3.721\, m$$

$$t_y = h \cdot \cos\varphi - a \cdot \sin\varphi = 1.5504\, m$$

$$t_x = 0 \qquad \underline{t} = [t_x \; t_y \; t_z]^T$$

The translation-rotation matrix:

$$\underset{\sim}{TR} = \begin{bmatrix} \underset{\sim}{R}_{3D} & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.9923 & -0.1240 & 1.5504 \\ 0 & 0.1240 & -0.9923 & -3.721 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) The camera matrix:

$$\underset{\sim}{M} = \underset{\sim}{K} \left[\underset{\sim}{R}_{3D} \; \underline{t} \right]$$

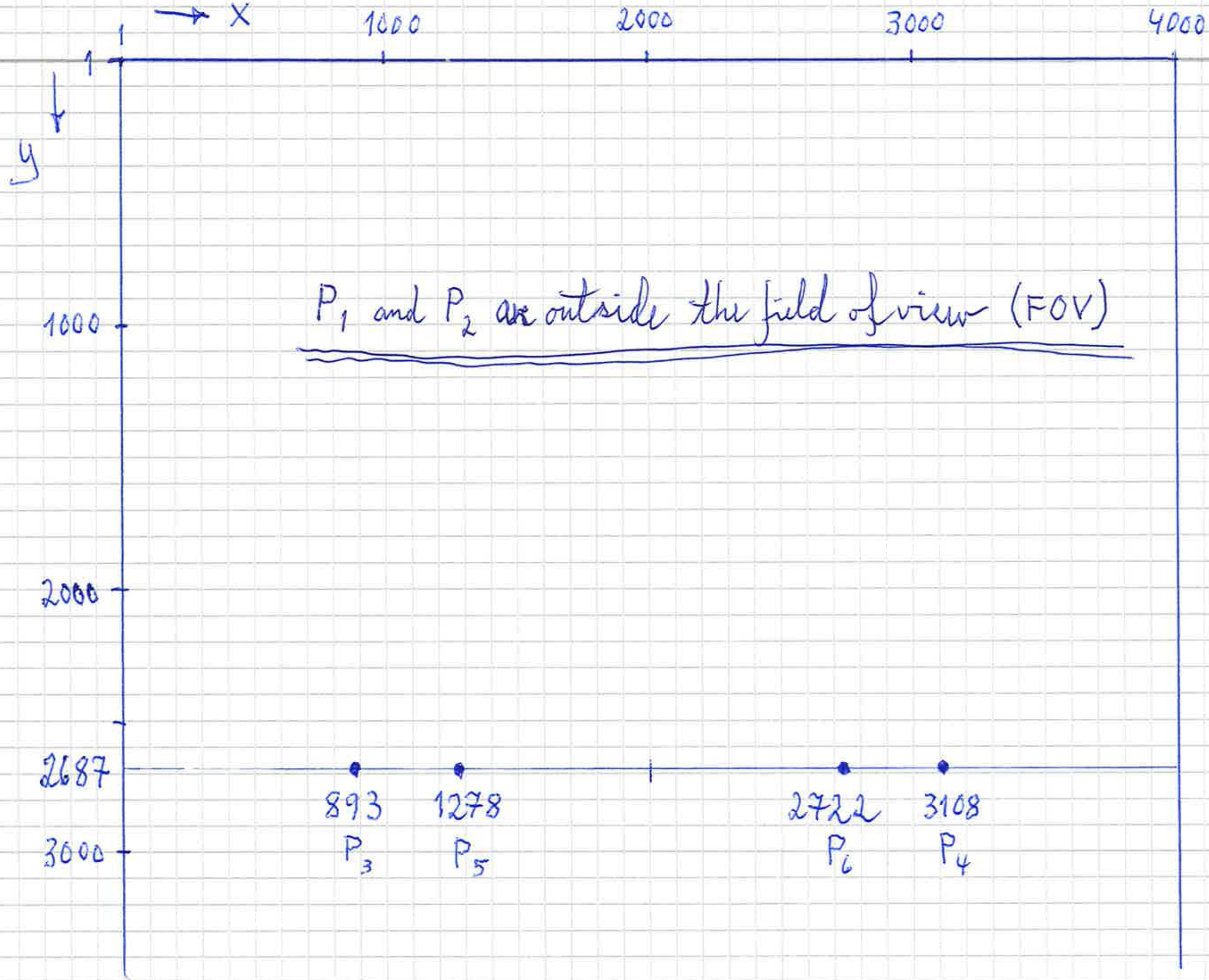
$$= \begin{bmatrix} 5495 & 0 & 2000 \\ 0 & 5495 & 2000 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.9923 & -0.1240 & 1.5504 \\ 0 & 0.1240 & -0.9923 & -3.721 \end{bmatrix}$$

$$= \begin{bmatrix} 5495 & 248.1 & -1984.6 & -7442.1 \\ 0 & -5204.5 & -2666.1 & 1077.5 \\ 0 & 0.124 & 0.992 & 3.721 \end{bmatrix}$$

Normalized, dividing by $m_{34} = 3.721$:

$$\underset{\sim}{M}_{norm.} = \begin{bmatrix} -1476.7 & 66.67 & 533.33 & 2000 \\ 0 & 1398.65 & 716.498 & -289.564 \\ 0 & 0.0333 & 0.26667 & 1 \end{bmatrix}$$

f)



P_1 and P_2 are outside the field of view (FOV)

Exercise 4 a)

$$\lambda \cdot \underline{p}A = \lambda \cdot \begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} = \begin{bmatrix} 6000 & 0 & -2000 & -13000 \\ 0 & -6000 & -2000 & -1000 \\ 0 & 0 & -1 & -3.5 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.0 \\ Z \\ 1 \end{bmatrix}$$

3. rad gir : $\lambda = -(Z + 3.5)$

$$\lambda \cdot x_A = 1.5 \cdot 6000 - 2000 \cdot Z - 13000 = -2000 \cdot (Z + 3.5) + 3000$$
$$\lambda \cdot y_A = -2.0 \cdot 6000 - 2000 \cdot Z - 1000 = -2000 \cdot (Z + 3.5) - 6000$$

$$\underline{\underline{x_A = 2000 - \frac{3000}{Z + 3.5} \quad , \quad y_A = 2000 + \frac{6000}{Z + 3.5}}}$$

$$\lambda \cdot \underline{p}B = \lambda \cdot \begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = \begin{bmatrix} 6000 & 0 & -2000 & -1000 \\ 0 & -6000 & -2000 & -1000 \\ 0 & 0 & -1 & -3.5 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.0 \\ Z \\ 1 \end{bmatrix}$$

3. rad gir : $\lambda = -(Z + 3.5)$

$$\lambda x_B = 1.5 \cdot 6000 - 2000 Z - 1000 = -2000 \cdot (Z + 3.5) + 15000$$
$$\lambda y_B = -2.0 \cdot 6000 - 2000 Z - 1000 = -2000 \cdot (Z + 3.5) - 6000$$

$$\underline{\underline{x_B = 2000 - \frac{15000}{Z + 3.5} \quad , \quad y_B = 2000 + \frac{6000}{Z + 3.5}}}$$

b) The disparity for $\underline{P}^1$ and $\underline{P}^3$

$$\underline{P}^1 = [1.5 \quad 2 \quad 0 \quad 1]^T, \quad \underline{P}^3 = [1.5 \quad 2 \quad 4 \quad 1]^T$$

Both points are on the line given in a)

with: $\underline{P}^1 \to Z_1 = 0$, $\underline{P}^3 \to Z_3 = 4$

Using the expressions (3) and (4) then give

$$\underline{d} = \underline{p}_A - \underline{p}_B = \begin{bmatrix} -\frac{3000}{Z+3.5} - \left(-\frac{15000}{Z+3.5}\right) \\ \frac{6000}{Z+3.5} - \frac{6000}{Z+3.5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{12000}{Z+3.5} \\ 0 \end{bmatrix} \quad \underline{d}_1 = \underline{d}(Z=0) = \underline{\underline{\begin{bmatrix} 3429 \\ 0 \end{bmatrix}}}$$

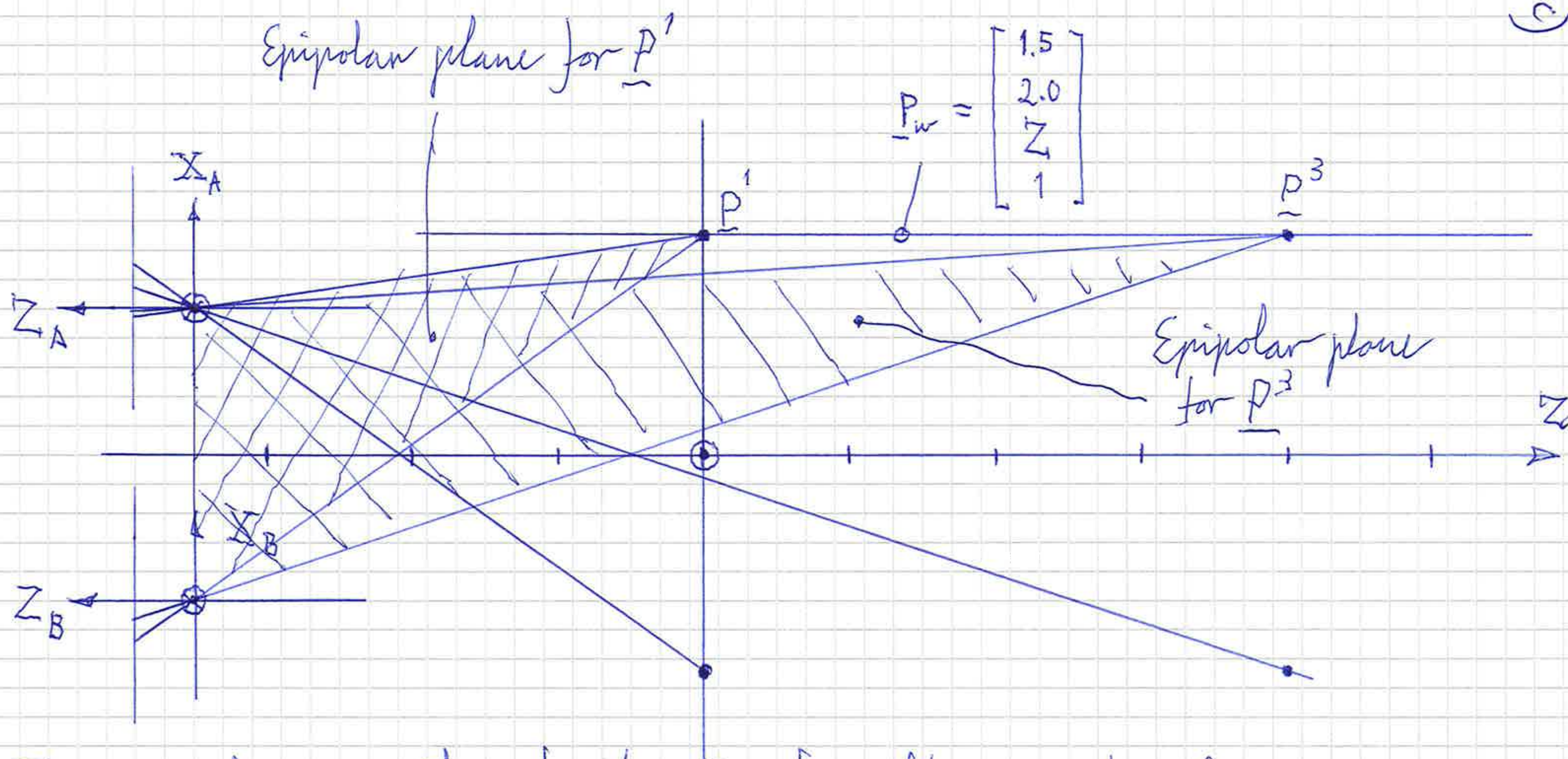
Obs! For this point:

$$\underline{p}_A^1 = \begin{bmatrix} 1143 \\ 3714 \end{bmatrix} \text{ and } \underbrace{\underline{p}_B^1 = \begin{bmatrix} -2286 \\ 3714 \end{bmatrix}}_{\text{Outside the FOV for camera B}}$$

$$\underline{\underline{\underline{d}_3 = \underline{d}(Z=4) = \begin{bmatrix} 1600 \\ 0 \end{bmatrix}}}$$ Inside the FOV for both cameras.

$$\underline{p}_A^3 = \begin{bmatrix} 1600 \\ 2800 \end{bmatrix} \text{ and } \underline{p}_B^3 = \begin{bmatrix} 0 \\ 2800 \end{bmatrix}$$

c)



The epipoles are at infinity therefore the epipolar lines are <u>horizontal along rows</u> (x-axis).

d) $Z = 10 - V_z t = 10 - 20 \cdot \frac{1}{40} \cdot k = 10 - 0.5 \cdot k$ (8

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} 6000 & 250 & -2000 & -7400 \\ 0 & -5700 & -2700 & 1860 \\ 0 & 0.124 & -1 & -3.7 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 10 - 0.5 \cdot k \\ 1 \end{bmatrix}$$

$$\lambda = -\left[10 - 0.5 \cdot k + 3.7\right] = -13.7 + 0.5 \cdot k$$

$$\lambda x = -2000 \cdot (10 - 0.5k) - \overbrace{7400}^{2000 \cdot 3.7} = 2000 \cdot \left[-13.7 + 0.5 \cdot k\right]$$

$$\lambda y = -2700 \cdot (10 - 0.5k) + 1860 = 2700 \cdot \left[-13.7 + 0.5k\right] + 2700 \cdot 3.7 + 1860$$

This give: $\underline{x = 2000}$

$$y = 2700 - \frac{11850}{13.7 - 0.5k} \qquad \underline{p}(k) = \begin{bmatrix} 2000 \\ 2700 - \frac{11850}{13.7 - 0.5k} \end{bmatrix}$$

$$\left(t = \frac{1}{40} \cdot k \,, \quad k = 40t \right)$$

<u>Optical flow at $t = 0$</u>: We use the image frames $k = 0$ and $k = 1$ and get:

$$\underline{v}(0) = \underline{p}(1) - \underline{p}(0) = \begin{bmatrix} 2000 \\ 2700 - \frac{11850}{13.2} \end{bmatrix} - \begin{bmatrix} 2000 \\ 2700 - \frac{11850}{13.7} \end{bmatrix}$$

$$= \begin{bmatrix} 2000 \\ 1802 \end{bmatrix} - \begin{bmatrix} 2000 \\ 1835 \end{bmatrix} = - \begin{bmatrix} 0 \\ 33 \end{bmatrix}$$

The motion is <u>-33 pixels along the y-axis</u>.