ДЗ 10

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1. (a) Сделаем замену $w = \frac{y^2}{x}$ $v = \frac{x^2}{y}$. Тогда

$$2p \leq w \leq 2q \;\; 2r \leq v \leq 2s$$

$$x = w^{\frac{1}{3}} v^{\frac{2}{3}} \;\; x = w^{\frac{2}{3}} v^{\frac{1}{3}}$$

Якобиан

$$\det \begin{pmatrix} \frac{1}{3} w^{-\frac{2}{3}} v^{\frac{2}{3}} & \frac{2}{3} w^{\frac{1}{3}} v^{-\frac{1}{3}} \\ \frac{2}{3} w^{-\frac{1}{3}} v^{\frac{1}{3}} & \frac{1}{3} w^{\frac{2}{3}} v^{-\frac{2}{3}} \end{pmatrix} = \frac{1}{9} - \frac{4}{9} = -\frac{1}{3} \Rightarrow \text{ модуль якобиана } \frac{1}{3}$$

$$S = \int_{2p}^{2q} du \int_{2r}^{2s} \frac{1}{3} dv = \frac{4}{3}(q-p)(s-r)$$

(b) Сделаем замену $x = ar(\cos^4 \phi)$ $y = br(\sin^4 \phi) \Rightarrow 0 \leq \sqrt{r} \leq 1, \;\; 0 \leq \phi \leq \frac{\pi}{2}$ Якобиан

$$\det \begin{pmatrix} a(\cos^4 \phi) & -ar \cos^3 \phi \sin \phi \\ b(\sin^4 \phi) & 4br \sin^3 \phi \cos \phi \frac{1}{3} w^{\frac{2}{3}} v^{-\frac{2}{3}} \end{pmatrix} = 4abr \cos^3 \phi \sin^3 \phi$$

$$S = \int_{r \leq 1, 0 \leq \phi \leq \frac{\pi}{2}} \int 4abr \cos^3 \phi \sin^3 \phi dx dy = \int_0^{\frac{\pi}{2}} 2ab \cos^3 \phi \sin^3 \phi \frac{ab}{6}$$

2. (a) $z \leq x^2 + y^2, y \geq x^2, y < 1, z \geq 0$

$$V = \int_{-1} 1dx \int_{x^2} 1dy \int_0 x^2 + y^2 dz = \int_{-1} 1dx \int_{x^2} 1(x^2 + y^2) dy =$$

$$= \int_{-1}^{1} dx \left(x^2 y + \frac{y^3}{3} \right) |_{x^2}^{1} = \int_{-1}^{1} \left(x^2 + \frac{1}{3} \right) - \left(x^4 + \frac{x^6}{3} \right) dx =$$

$$= \left(\frac{x^3}{3} + \frac{x}{3} - \frac{x^5}{5} - \frac{x^7}{21} \right) |_{-1}^{1} = \frac{88}{105}$$

(b) Делаем $x = ar \cos \phi$ $y = br \sin \phi$. Тогда $r^2 - \frac{z^2}{c^2} \geq -1 \quad -c\sqrt{r^2+1} \leq z \leq c\sqrt{r^2+1}$

$$det \begin{pmatrix} a \cos \phi & -ra \sin \phi & 0 \\ b \sin \phi & rb \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = abr$$

$$V = \int_0^{2\pi} dy \int_0^1 dr \int_{-c\sqrt{r^2+1}}^{c\sqrt{r^2+1}} abrdz = \int_0^{2\pi} dy \int_0^1 2abcr\sqrt{r^2+1}dr =$$
$$= abc \int_0^{2\pi} dy \int_0^1 d2r\sqrt{r^2+1} = abc \int_0^{2\pi} dy \int_1^2 \sqrt{t}dt =$$
$$= abc \int_0^{2\pi} dy (\frac{\sqrt[2]{t^3}}{3}|_1^2) = \frac{4abc\pi(\sqrt{2}-1)}{3}$$

3.

$$\int_0^1 dx \int_0^1 dy \int_0^{x^2+y^2} dz$$
$$0 \le x \le 1 \quad 0 \le y \le 1 \quad 0 \le z \le 2$$

Есть 2 случая (фиксируем z):

$$0 \le z \le 1$$
$$1 \le z \le \sqrt{2}$$

Тогда

$$\int_0^1 dz \left(\int_0^{\sqrt{z}} dx \int_{\sqrt{z-x^2}}^1 f(x,y,z)dy + \int_{\sqrt{z}}^1 dx \int_0^1 f(x,y,z)dy \right) + \int_1^2 dz \int_{\sqrt{z-1}}^1 dx \int_{\sqrt{z-x^2}}^1 f(x,y,z)dy$$