Topic: **Energy compaction**

Energy Compaction Property

In signal compression applications, it is desirable to use orthogonal transforms with high-energy compaction properties. In such transforms dominant samples of the transform with high values are usually in the low frequency range and contain most of the energy. In addition, transform samples with very small values tend to be in the high frequency range and can be set to zero values. An inverse transform of the modified transform samples with zero-valued high frequency samples in thus an approximate representations in the time domain of the original time-domain sequence.

If L samples of the transform are set to zero with $L << N$ and if $x^m[n]$ denotes the inverse of the modified transform, then a measure of the energy compaction property caused by the removal of samples is given by the mean-square error approximation error:

$$E[L] = \frac{1}{N}\sum_{n=0}^{N-1} |x[n] - x^m[n]|^2$$

DFT

In the case of N- point DFT, the high frequency samples have indices around $\frac{N}{2}$. Here the modified DFT is given by

$$X_{DFT}^m[k] = \begin{cases} X_{DFT}[k], & 0 \le k \le \dfrac{N-1-L}{2} \\ 0, & \dfrac{N+1-L}{2} \le k \le \dfrac{N-1+L}{2} \\ X_{DFT}[k], & \dfrac{N+1+L}{2} \le k \le N-1. \end{cases}$$

Where, in this case, $X_{DFT}[k]$ is the N-point DFT of $x[n]$ and If $L = 1$ the term $X[\frac{N}{2}]$ is removed. If $L = 3$ then the terms $X[\frac{N}{2}]$ and $X[\frac{N}{2} - 1]$ and its corresponding complex conjugate $X\left[\frac{N}{2} + 1\right]$ are removed, and so forth; i.e., $x_{DFT}^m[n]$ for $L = 1,3,5 \ldots . N-1$ is the sequence that is synthesized by symmetrically omitting L DFT coefficients.

DCT

In this case the high frequency samples have high indices, and thus the modified DCT obtained by setting to zero the L samples with high indices is given by

$$X_{DCT}^{m}[K] = \begin{cases} X_{DCT}[k], & 0 \le k \le N - L - 1 \\ 0, & otherwise \end{cases}$$

And $x_{DCT}^{m}[n]$ is the inverse DCT of $X_{DCT}^{m}[k]$.
To show how the approximation errors depend on L for the DFT and DCT-2 we define,

$$E^{DFT}[L] = \frac{1}{N}\sum_{n=0}^{N-1}|x[n] - x_{DFT}^{m}[n]|^2$$

And

$$E^{DCT}[L] = \frac{1}{N}\sum_{n=0}^{N-1}|x[n] - x_{DCT}^{m}[n]|^2$$

Energy Compaction Efficiency Comparison b/w DFT and DCT

1. The one-dimensional signal used for the comparison is any row/column of the 512×512 image called "Goldhill" (given by TA). Use the keyword "imread" to read an image in MATLAB.

2. Then use the previous lab codes to compute DCT/DFT coefficients of the signal. While calculating the DCT, use the DCT matrix which is **not satisfying the unitary property**

3. Now vary the L values form $101\ to\ 500$, for each value of L reconstruct the signal from the newly obtained DCT/DFT Coefficients. Then calculate the mean squared error between the original signal and the reconstructed signal.

4. Plot mean square error vs. L, and write your observations on the graph obtained by using DFT and DCT.

*__Note:__

1. Generalize your program as much as possible, which will be helpful for further labs
2. Zip all your files (includes soft copy and '.m' files) and submit to respective lab TA.