Shiv Nadar University

Department of Electrical Engineering-(SoE)

EED364: Graph Signal Processing **Lab-8** **Instructor:** Prof. Vijay Kumar Chakka

Topic: Graph, Spectral Spread and Plotting Frequency Response of Graph Filters

I. Graph and Spectral Spread:

Definition: Consider a simple, undirected graph $G = (V(G), E(G))$. Then the Graph spread of a finite-energy signal x, (i.e., $x \in l^2(G)$) defined on a graph G is

$$\Delta_g^2 = \frac{1}{||x||^2} \min_{u_0 \in V} \sum_{u \in V} d(u, u_0)^2 |x(u)|^2 .$$

Where $d(u, v)$ is known as Geodesic distance between vertex u and vertex v, which is the smallest number of edges that need to be traversed to get from vertex u to vertex v. Similarly, the spectral spread is defined as,

$$\Delta_s^2 = \frac{1}{||x||^2} x^T L x = \sum_{(u,v) \in E} \left(\frac{x(u)}{\sqrt{\delta(u)}} - \frac{x(v)}{\sqrt{\delta(v)}} \right)^2 .$$

Where $\delta(v)$ is the degree of the vertex v and L is normalized Laplacian matrix

1. Calculate the product of graph spread and spectral spread for the following signals defined on Bucky ball graph G
   a. A finite-energy random signal x on G using uniform distribution having a range from -2 to 2 (**Hint:** use 'rand' function in MATLAB)
   b. Consider $x(n) = e^{-2n}$, where n is the node index.
2. Repeat the question 1 for an unweighted, undirected path graph of node size 60

II. Plotting Frequency Response of Graph Filters:

3. Consider a system (filter) defined by its coefficients

$$C = [c_0, c_1, \ldots, c_l, \ldots, c_{L-1}] .$$

   On a Minnesota graph $G = (V, E)$. Now plot the frequency response for the following systems (filters)
   a. $L = 10, c_l = 1 \; \forall \, l.$
   b. $L = 25, c_l = 0.8^{-l} \; \forall \, l.$