Sum of exponentials and gating time constants

$$\frac{dm}{dt} = \frac{m_\infty - m}{\tau_m} \; ; \quad \frac{dh}{dt} = \frac{h_\infty - h}{\tau_h} \qquad (1)$$

$$I(t) = g\, m^p h\,(V - E) \qquad (2)$$

$$\left.\begin{matrix} m(0) = h(0) = 0 \\ p = 3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} m(t) = m_\infty\left(1 - e^{-t/\tau_m}\right) \\ h(t) = h_\infty\left(1 - e^{-t/\tau_h}\right) \\ (2) \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow I(t) = g\, m_\infty^3 \left(1 - e^{-t/\tau_m}\right)^3 h_\infty \left(1 - e^{-t/\tau_h}\right)(V - E) =$$

$$= g\, m_\infty^3 \left(1 - \cancel{e^{-3t/\tau_m}} - 3e^{-t/\tau_m} + \cancel{3e^{-2t/\tau_m}}\right) h_\infty \left(1 - e^{-t/\tau_h}\right)(V - E) \simeq$$

$$\simeq g\, m_\infty^3 \left(1 - 3e^{-t/\tau_m}\right) h_\infty \left(1 - e^{-t/\tau_h}\right)(V - E) =$$

$$= g\, m_\infty^3 h_\infty \left(1 - e^{-t/\tau_h} - 3e^{-t/\tau_m} + \cancel{e^{-t/\tau_m - t/\tau_h}}\right)(V - E) \simeq$$

$$\simeq g\, m_\infty^3 h_\infty \left(1 - e^{-t/\tau_h} - 3e^{-t/\tau_m}\right)(V - E) =$$

$$= \underbrace{g\, m_\infty^3 h_\infty (V - E)}_{\text{Steady state term}} \underbrace{- g\, m_\infty^3 h_\infty (V - E)\, e^{-t/\tau_h} - 3 g\, m_\infty^3 h_\infty (V - E)\, e^{-t/\tau_m}}_{\text{Transient term}}$$

$$\boxed{\begin{aligned} I_{transient}(t) &= A_1 e^{-t/\tau_m} + A_2 e^{-t/\tau_h} \\ A_1 &= -3 g\, m_\infty^3 h_\infty (V - E) \\ A_2 &= -g\, m_\infty^3 h_\infty (V - E) \end{aligned}}$$

← Thus, τ_m and τ_h can be directly fitted by recorded transient currents $I_{transient}$