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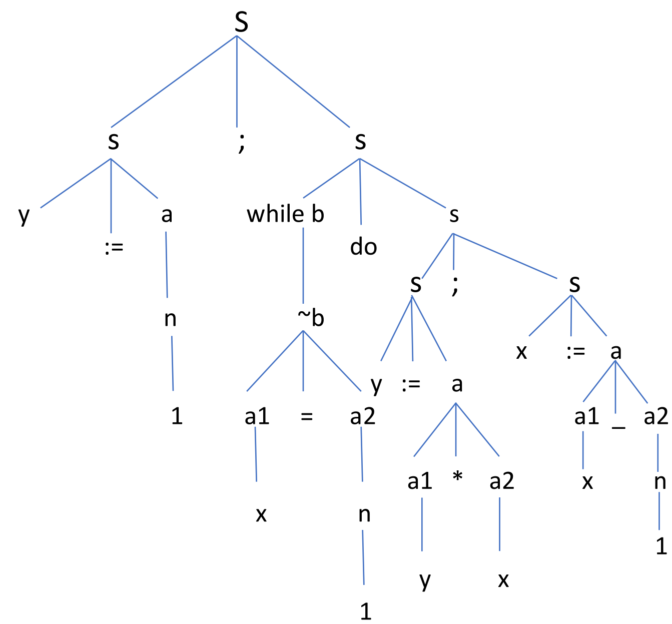
Homework 4

Exercise 1.1 The following statement is in While: y:=1; while ¬(x=1) do (y:=y⋆x; x:=x−1)

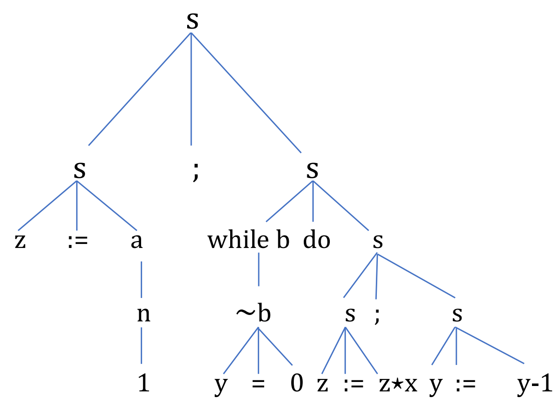


Exercise 1.2 Assume that the initial value of the variable x is n and that the initial value of y is m. Write a statement in While that assigns z the value of n to the power of m, that is

n⋆⋯⋆n

m times
Give a linear as well as a graphical representation of the abstract syntax.

z:=1; while ~(y=0) do (z:= z⋆x; y:=y-1)



Exercise 1.8 Assume that s x = 3 and determine B[¬(x = 1)]s.

→ B [[~ (x=1)]] s

→B [[~ (A[[a1]] s = A[[a2]] s)]] s

→B [[~(A[[x]] s = A [[n]] s)]] s

→B [[~ ((s x) = N [[1]])]] s

→B [[~ (3=1)]] s

→ ~ (ff)

→ tt

Exercise 2.3 Consider the statement
z:=0; while y≤x do (z:=z+1; x:=x−y)

Construct a derivation tree for this statement when executed in a state where x has the value 17 and y has the value 5.

$$\frac{\langle z:=0, s\rangle \rightarrow s\ 0\ 17\ 5 \qquad\qquad T1}{\langle z:=0;\ \text{while } y\le x \text{ do } (z:=z+1;\ x:=x-y), s\rangle \rightarrow s\ 325}$$

T2 T3

< while y≤x do (z: = z+1; x: = x-y), s 0 17 5>→ s 3 2 5

T2 is a derivation tree with root

<z: = z+1; x: = x-y, s 0 17 5> → s 1 12 5

and T3 is a derivation tree with root

< while y≤x do (z: = z+1; x: = x-y), s 1 12 5>→ s 2 7 5

< while y≤x do (z: = z+1; x: = x-y), s 2 7 5>→ s 3 2 5

< z: = z+1, s 2 7 5 > → s 3 7 5 < x: = x-y> → s 3 2 5

<z: = z+1; x: = x-y, s 2 7 5 > → s 3 2 5 T4

< while y≤x do (z: = z+1; x: = x-y), s 2 7 5 >→ s 3 2 5

Where s 3 7 5 = s[z→3][x→7][y→5], s 3 2 5 = s[z→3][x→2][y→5] and T4 is a derivation tree with root with

while y≤x do (z: = z+1; x: = x-y), s 3 2 5 >→ s 3 2 5

Exercise 2.4 Consider the following statements

• while ¬(x=1) do (y:=y⋆x; x:=x−1)

It terminates when only x >= 1. If x is less than one it will loop forever.

• while 1≤x do (y:=y⋆x; x:=x−1)

It always terminates. While x is greater or equal than one it will eventually terminates.

• while true do skip

It always loops, because while true it will skip and never terminates.

Exercise 2.6 Prove that the two statements $S_1;(S_2;S_3)$ and $(S_1;S_2);S_3$ are semantically equivalent. Construct a statement showing that $S_1;S_2$ is not, in general, semantically equivalent to $S_2;S_1$.

S1; S2

<s1, s> → s’, <s2, s’> → s’’

<s1; s2>→s’’

For example: x:= x-y; y:= x⋆y

S1 → x: = x-y

S2 → y: = x⋆y

x s = 3

y s = 1

<x: =x-y, s31> → s21, <y: = x⋆y, s21>→ s22

<x: = x-y; y: = x⋆y, s31> → s22

S2; S1

<x: =x-y, s31> → s33, <y: = x⋆y, s21>→ s03

<x: = x-y; y: = x⋆y, s31> → s03

Asala Kensara

Exercise 2.11 The semantics of arithmetic expressions is given by the function A. We can also use an operational approach and define a natural semantics for the arithmetic expressions. It will have two kinds of configurations:

⟨a, s⟩ denoting that a has to be evaluated in state s, and z denoting the final value (an element of Z).

The transition relation $\to_{\text{Aexp}}$ has the form $\langle a, s\rangle \to_{\text{Aexp}} z$

where the idea is that a evaluates to z in state s. Some example axioms and rules are

$\langle n, s\rangle \to_{\text{Aexp}} \mathcal{N}[n]$ $\langle x, s\rangle \to_{\text{Aexp}} s\ x$

$$\frac{\langle a_1, s\rangle \to_{\text{Aexp}} z_1,\ \langle a_2, s\rangle \to_{\text{Aexp}} z_2}{\langle a_1 + a_2, s\rangle \to_{\text{Aexp}} z} \qquad \text{where } z = z_1 + z_2$$

Complete the specification of the transition system.

$$\frac{\langle a_1, s\rangle \to_{\text{Aexp}} z_1,\ \langle a_2, s\rangle \to_{\text{Aexp}} z_2}{\langle a_1 \star a_2, s\rangle \to_{\text{Aexp}} z}$$

$$\frac{\langle a_1, s\rangle \to_{\text{Aexp}} z_1,\ \langle a_2, s\rangle \to_{\text{Aexp}} z_2}{\langle a_1 - a_2, s\rangle \to_{\text{Aexp}} z}$$

Exercise 2.17 Extend While with the construct repeat S until b and spec- ify the structural operational semantics for it. (The semantics for the repeat- construct is not allowed to rely on the existence of a while-construct.)

$[\text{repeat}_{\text{sos}}]$ < repeat s until b > ⇒

< S; if b then skip else (repeat S until b), s >