Blood Flow Analysis in the Carotid Artery based on Finite Element Method

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Research practice 1
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Abstract

The purpose of this paper is to present a simulation of blood flow through the carotid artery in two dimensions. The simulation is performed on a section of the artery extracted from an angiography, on which modifications are made to mimic the presence of stenosis in the artery. Finite element method is used to solve the linear simplification of the Navier-Stokes equations, seeking to identify variations in the pressure and velocity fields of the fluid and to analyze the impact of mild and severe cases of stenosis caused by atherosclerosis on human hemodynamics.

Keywords: Finite element method, Computational fluid dynamics, Hemodynamics, Ischemia, Stroke

1 Introduction

The brain is by far the most metabolically active organ in the human body, accounting for about 25% of the total metabolic activity of a person while being only around 2.5% of his/her total weight[1]. This is why the brain is also one of the most sensitive organs to decreases in the supply of oxygen delivered to it and therefore has the most complex circulatory self-regulation system of the body, having multiple mechanisms to keep the blood flow as stable as possible and to compensate for the oscillations caused by body processes[2]. When these normal levels of blood flow are compromised, ischemia, or in more general terms, a stroke is caused.

A stroke is a medical condition where brain cells die due to oxygen starvation, originated from a disruption in the blood supply to the brain. It can be classified into two main types, hemorrhagic, when the cause is internal bleeding on the brain and ischemic when it is caused by the lack of blood flow, being the latter the most common, accounting for 87% of all strokes[3]. They are considered one of the leading causes of death and a major cause of disability worldwide. In 2019 Stroke's global prevalence was 101.5 million people, of which 6.6 million were fatal[4], and even though this shows a decrease in the number of incidents in comparison to previous years, Strokes keep being a matter of concern for the medical scientific community.

It's because of this and because of many other reasons that in recent years not only the medical community, but the scientific community in general, has focused great efforts on blood flow analysis, and this is where computational fluid dynamics (CFD) becomes important. CFD is a branch of applied mathematics, where Numerical Methods and computational resources are used in order to solve fluid dynamics problems, an area in which due to the great complexity of the dominant partial differential equations (Navier-Stokes Equations, Euler Equations, etc) it has been impossible to give an analytical answer to its problems.

This work focuses on the application of CFD and in specific of the finite element method (FEM) in the analysis of blood flow in healthy patients, in patients with mild atherosclerosis, and patients with severe atherosclerosis in the carotid artery to observe and compare the flow behavior in different cases and to understand the variations that eventually lead to the formation of cerebral ischemias.

This project involves, first of all, a challenge that has been developed for years by hundreds of researchers and that is to really understand mathematically how the human body works and although it is clear that this work focuses on the functioning of a specific system (the circulatory system), it is still a step towards the great goal of really understanding what's going on inside our body. Now, it is true that many of these processes have already been explained biologically in a complete way, but this cannot be further from the true goal of being able to understand the event so perfectly to the point of being able to recreate it accurately and study it without the need for real patients, an achievement for which we still have much to discover.

Now, with this project, we are looking for more than just the theoretical implementation of the FEM, but to continue advancing with what has been obtained in recent years in the research and application of CFD in the study of body hemodynamics. This is because we really believe that the development of mathematical techniques such as CFD in medicine can imply great progress for the scientific community, and could even facilitate the prevention of multiple diseases that today are considered complex and even dangerous.

This paper begins by presenting a literature review related to the use of CFD in the study of hemodynamics. Once this review is done, the methodology followed in this research is presented, in which a brief explanation of the finite element method is presented along with an analysis of the mathematical model to be simulated and the process how the meshing of the artery was developed. Finally, the theoretical validation of the model is presented as well as the results obtained by performing the simulation in the cases of interest for the research, to close the article with a brief discussion of these results.

2 State of the art

As mentioned above, computational fluid dynamics has become a widely used tool in the study of hemodynamics in recent years, which, of course, implies the existence of a large body of literature on the subject, among which there are multiple articles where they focus on the study of cerebral hemodynamics [5] [6] [7] and several others where they center the study on the flow-through specific vessels with and without atherosclerosis or in a more theoretical approach to CFD applications [8] [9] [10] [11] [12] [13] .

In [5] Berndorf and Wang begin by giving a medical contextualization of the subject matter of the article: aneurysms and their most common forms of treatment, as well as clarifying why computational fluid dynamics is useful in this field. Then, they proceed to explain some aspects of the method they will use (Lattice Boltzmann method) to finish with the results of the study. In [6], the area of study is, as in [5], cerebral aneurysms; however, in this case, the model developed will be implemented using the finite element method. In addition to this, Oshima *et al.* are much more explicit in the mathematics of the model, greatly clarifying the mathematical development of the method and inviting the reader to understand not only the study but the way in which it was developed.

Fatahillah *et al.* develop a model by means of FEM for the analysis of blood flow in stenotic intracranial arteries affected by ischemic stroke in [7]. Here the authors begin by giving a brief definition of stenosis and discussing the serious risks it can pose, including, of course, ischemic strokes. They then go on to cite multiple authors including Shi *et al.*[14] for their work in CFD in the area of ischemia, and Roy *et al.* [15] for their application of CFD models in arteries with stenosis; they do this to show the viability of CFD for the development of their research. Finally, they explain in general terms the mathematical model and proceed by showing the results obtained in the different cases considered.

In [8] [9] Boujema *et al.* present two-dimensional and three-dimensional approaches to the analysis of blood flow through arteries with atherosclerosis under non-standard conditions. Here, although they do not rely on any complex geometry, they achieve interesting results on the use of rigid walls for hemodynamic simulation and the existence of a solution to the Navier-Stokes equations under the given conditions.

Quateroni and Formaggia in [10], Doost *et al.* in [11] and Byoung-Kwon in [12] show us a very general approach of CFD on the study of hemodynamics, showing techniques, examples of common applications in the field and explaining in a good way how CFD methods work when applied in this area. And Naranjo *et al.* show us in [13] a specific application of CFD and specifically of FEM, performing a simulation of the blood flow through the femoral artery. The numerical solution of the problem is found by dividing the analysis into three parts: first the analysis of the linearization of the Navier-Stokes equations, then adding the nonlinear part of the problem and solving it by Newton-Raphson and finally performing the temporal analysis using the backward Euler method.

As well as the texts mentioned above, there are many articles in the literature that relate CFD with hemodynamics, which shows the growing use of these numerical techniques for the study of blood flow in the body.

3 Methodology

The methodology used for this research is based on two main stages, in the first one an image analysis is performed to extract the section of the artery in which the simulation is going to be developed, and in the second Stage a Finite Element implementation is made in Python using fenicsproject, A specialized library for the finite element method, and then, the simulation is run on three different meshes imitating a healthy artery, an artery with mild atherosclerosis and an artery with severe atherosclerosis, in order to obtain meaningful results that can be compared to analyze the variation in the pressure and velocity fields of the fluid. Let's now explain each of the stages in more detail.

For image analysis (Figure 1) an angiography of the carotid artery is extracted from a medical database [16]. Then it is turned into a binary image in order for the program to be able to read it, it is smothened and finaly it is converted to a serie of points from its outline. With this set of points the mesh is finally generated with the library mshr from python.

Let's talk now a little bit about the finite element method. The finite element method is a numerical method highly used in computational fluid dynamics to solve problems whose domain is defined by irregular geometries. The method makes it possible to find the numerical solution to these problems by dividing the initial domain of the same (a continuous element) into a large number of non-intersecting subdomains, called finite elements. Each element will have a set of representative points called nodes, which will make up the complete grid of the problem, on which

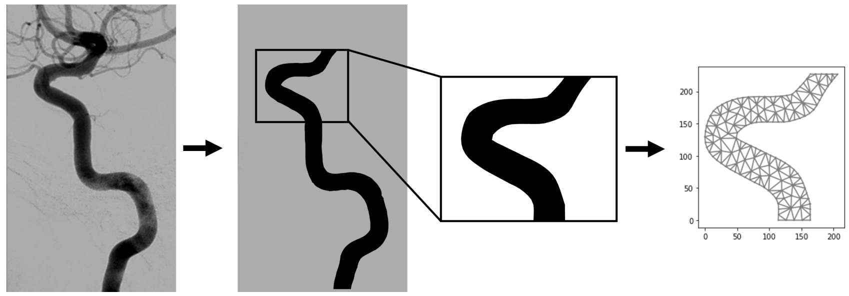


Figure 1: Angiography processing for mesh generation

the calculations will be performed. At last, the formulation of a boundary value problem ends up giving a system of algebraic equations, which are then solved using variational methods in order to approximate a solution, minimizing an associated error function

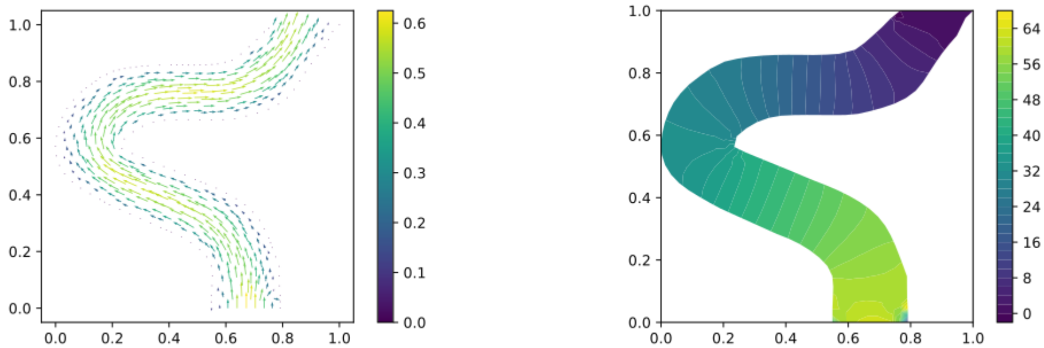
For this project the equation used was the stokes equation Eq.1 which is a linear version of the navier stokes equations, this was possible because assuming that the blood flow is a Newtonian flow and considering its usual laminar behavior, the nonlinearity of the equation can be eliminated and leaves us with a flow independent of time and represented by a linear differential equation.

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 & \mathbf{x} \in \Omega \\ \nu \Delta \mathbf{u} - \nabla p = \mathbf{f} & \mathbf{x} \in \Omega \end{cases} \tag{1}$$

To solve the PDE with the software used (fenicsproject) it was then necessary to transform this equation into its variational form, in which the equation is multiplied by a test function of an appropriate function field and integrated over its spatial domain, by doing this the equation 2 is obtained.

$$\begin{cases} \int_\Omega q \nabla \cdot \mathbf{u} = 0 & \forall v, q \\ \int_\Omega \nu \nabla \mathbf{u} \cdot \nabla \mathbf{v} - \int_\Omega p \nabla \cdot \mathbf{v} = \int_\Omega \mathbf{f} \cdot \mathbf{v} & \forall v, q \end{cases} \tag{2}$$

With this equation ready, it is now time to apply the method, the first step is to define on the mesh created in the image interpretation, the elements of both fields, element that in this case will be Lagrange-1 elements for the pressure field and Lagrange-2 elements for the velocity field, this looking for an adequate stability of the method, then we define our variational equation using the symbolic language offered by fenics and later we define a preconditioning matrix that allows our iterative method to work properly. After this, it only remains to run the method and obtain the results that will be shown below.



(a) Velocity field (m/s)

(b) Pressure field (mmHg)

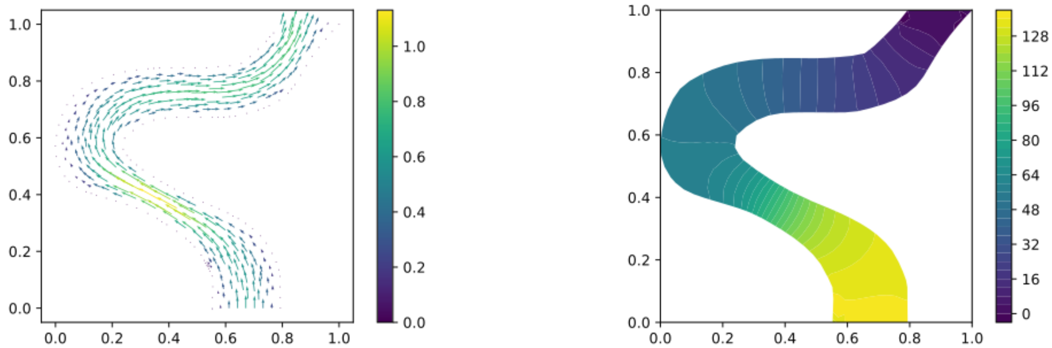
Figure 2: Simulation in a healthy artery

4 Results

First, the simulation was run in a mesh representing a healthy artery, not only to validate the model and see if both pressure and velocity field gives values inside the normal ranges, but to have a base to compare the alteration on both these fields with the mild and severe stenosis cases; these results can be shown in Figure 2, in the subfigure a we can see the results of the velocity field in which is easy to see the representative laminar flow of the blood flow with the highest velcity in the center of the vassel and null velocity in the walls; another interesting thingh to see in the velocity field is how th evelocity of out fluid does not exceed $0.6 m/s$ which is approximately the normal velocity field through the carotid artery. In subfigure b, we can see the pressure field, in which it can be interesting to see how this pressure doesn't event rise to its normal value of 120, but this is due to the linealization of the equation, and because it is assumed that this section of artery is isolated from the rest of the body, therefore it is normal for the model to give this low value.

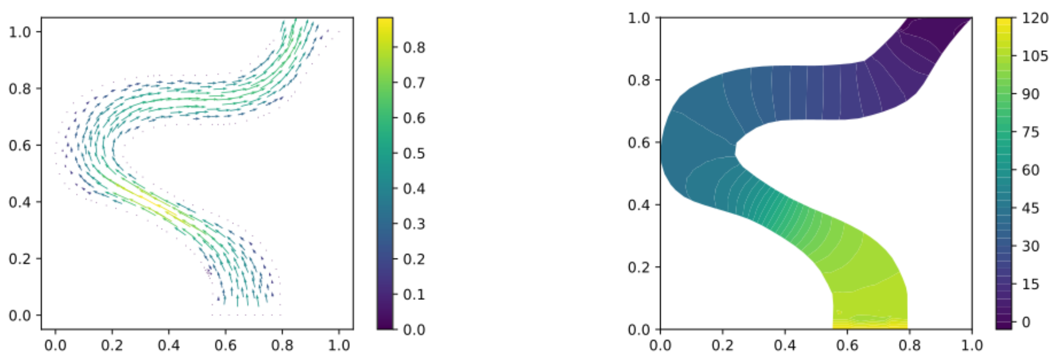
Now in Figure 3 and Figure 4 the simulation is made over an arthery with mild atherosclerosis. For Figure 3 is made controlling the inflow velocity and setting it in normal values of velocity for the artery, as we can see here the pressure is rised, showing how with just this minimum disminution on the diameter of the vassel in a small section, the pressure starts beng affected. Figure 4 show the result of controlling the pressure field for the inflow of the atery, as it can be easily seen, with this level of pressure (120 mmHg), the velcity of out fluid starts decreasing and therefore taking into account the high sensibility of the brain to changes in the blood suply, whit this level of affectation, symptoms may begin to occur.

Finally Figures 5 and 6 show the simulation made on a severe stenosis case. As it can be seen in both figures, whethere we control the velocity field or the pressure field, we obtain inhuman conditions with whethere completely absurd pressure levels of over 800 mmHg or we obtain a flow close to null and therefore in both cases no human would be alive. Now ignoring this, it is interesting to see how the increase in stenosis generated such an absurd variation, since when comparing with the previously exposed case, we can see how without necessarily being very different, the values



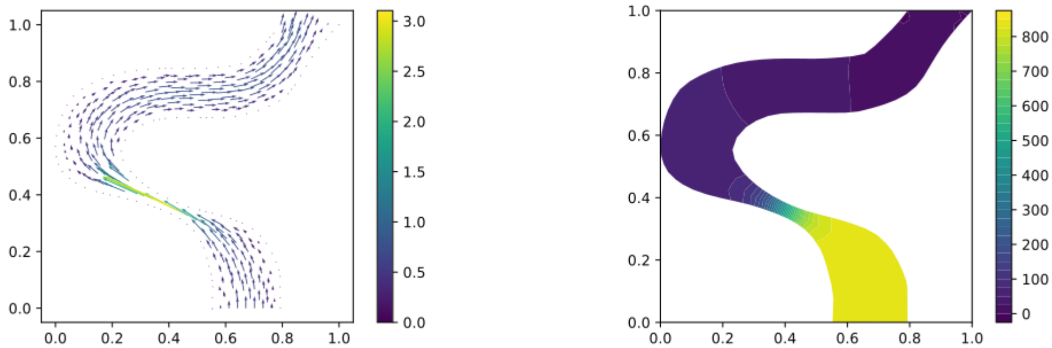
(a) Velocity field (m/s) (b) Pressure field (mmHg)

Figure 3: Simulation in mild atherosclerotic artery with boundary conditions of inflow velocity



(a) Velocity field (m/s) (b) Pressure field (mmHg)

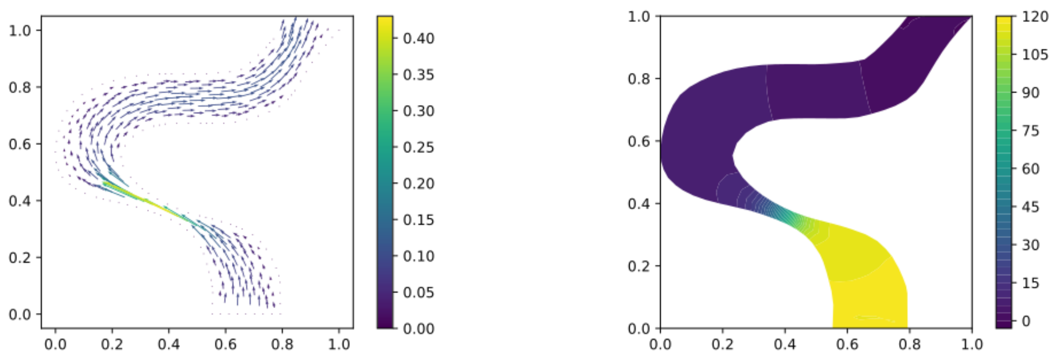
Figure 4: Simulation in mild atherosclerotic artery with boundary conditions of inflow pressure



(a) Velocity field (m/s)

(b) Pressure field (mmHg)

Figure 5: Simulation in severe atherosclerotic artery with boundary conditions of inflow velocity



(a) Velocity field (m/s)

(b) Pressure field (mmHg)

Figure 6: Simulation in severe atherosclerotic artery with boundary conditions of inflow pressure

obtained do change dramatically, showing us why in mild cases of atherosclerosis it is so difficult to identify the presence of the disease.

5 Conclusions and future research

Mainly this study has allowed us to observe how even with minimal variations in the diameter of the artery, both the flow and the arterial pressure will show significant variations, however, the results obtained present inconsistencies with reality and therefore it is necessary to reparameterize the model in the company of an expert in the subject, and to obtain real data on which a correct validation can be made and finally give significant conclusions for the research.

Now then, leaving aside the need for revalidation, the graphs shown in the results section show that mild cases of stenosis do not present as great changes in our pressure and velocity fields as do severe cases and that is why the real question that has been confusing the medical scientific community for years is now being asked: is it possible to identify changes in mild cases of atherosclerosis in order to prevent them from reaching mild cases? This remains a problem for future research and presents a real challenge that we hope will be solved.

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