Title

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Research practice 3
Research proposal
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1 Introduction

The brain is by far the most metabolically active organ in the human body, accounting for about 25% of the total metabolic activity of a person while being only around 2.5% of his/her total weight [1]. This is why the brain is also one of the most sensitive organs to decreases in the supply of oxygen delivered to it and therefore has the most complex circulatory self-regulation system of the body, having multiple mechanisms to keep the blood flow as stable as possible and to compensate for the oscillations caused by body processes [2]. When these normal levels of blood flow are compromised, ischemia, or in more general terms, a stroke is caused.

A stroke is a medical condition where brain cells die due to oxygen starvation, originated from a disruption in the blood supply to the brain. It can be classified into two main types, hemorrhagic, when the cause is internal bleeding on the brain and ischemic when it is caused by the lack of blood flow, being the latter the most common, accounting for 87% of all strokes [3]. They are considered one of the leading causes of death and a major cause of disability worldwide. In 2019 Stroke's global prevalence was 101.5 million people, of which 6.6 million were fatal [4], and even though this shows a decrease in the number of incidents in comparison to previous years, Strokes keep being a matter of concern for the medical scientific community.

It's because of this and because of many other reasons that in recent years not only the medical community, but the scientific community in general, has focused great efforts on blood flow analysis, and this is where computational fluid dynamics (CFD) becomes important. CFD is a branch of applied mathematics, where Numerical Methods and computational resources are used in order to solve fluid dynamics problems, an area in which due to the great complexity of the dominant partial differential equations (Navier-Stokes Equations, Euler Equations, etc) it has been impossible to give an analytical answer to its problems.

One of the most used methods of CFD is the Finite Element Method (FEM), a numerical method consisting on the discretization of the problem's domain, to later solve a variational formulation of the problem with determined test functions, to obtain an algebraic system of equations, which, when solved, gives the approximated solution of the problem. However, due to the large amount of degrees of freedom it needs in order to correctly solve the CFD problem, it remains as an extremely expensive method both in terms of CPU and memory demand, therefore not being too useful for real-time contexts [5].

As a consequence of the later, this project focuses on the implementation of models with reduced computational cost, in specific Physical Informed Neural Networks (PINNs) and

Neural Network asisted Reduced Basis Models (NN-RBM), in a simulated *real-time* context in the analysis of blood flow in both healthy and atherosclerotic patients, using the carotid artery as a study object, to observe and compare the flow behavior in different cases and to understand the variations that eventually lead to the formation of cerebral ischemias

2 Statement of the problem

En este apartado debemos ampliar la descripción del problema.

2.1 Statement of the problem (4 a 5 párrafos)

En esta subsección se hace una ampliación del problema descrito en la introducción. Acá hay algunas aspectos que se pueden abordar acá.

- En algunos casos, un problema tiene diferentes nombres en la literatura. En estos casos es bueno contarle al lector con qué otros nombre se conoce el problema en diferentes áreas de conocimiento.
- Si es una investigación aplicada, es conveniente pensar en que matices o particularidades adquiere el problema al aplicarse en un contexto específico. Puede que lo que no sea un problema en un lugar si lo sea en otro.
- ¿A quienes afecta el problema?¿a qué escala opera? (grupos poblacionales, zonas geográficas, período temporal: pasado, actual o futuro).
- ¿Qué desencadena o genera el problema?
- ¿Qué repercusiones o efectos tiene el problema?
- ¿Se han postulado soluciones a este problema pero no lo suficientemente satisfactorias?

2.2 Formalization of the problem

En esta subsección debén escribir una descripción más formal, muy al grano, del problema. Se deben incluir los principales elementos matemáticos que tiene el problema. En la siguiente figura hay un ejemplo de formalización del problema tomado de Duque, J. C., Anselin, L., & Rey, S. J. (2012). The max-p-regions problem. Journal of Regional Science, 52(3), 397-419.

2. PROBLEM STATEMENT

Areas

Let $A = \{A_1, A_2, \ldots, A_n\}$ denote a set $n = |A|$ areas.

Attributes

Let A_{iy} denote the attribute y of area A_i, where $y \in Y = \{1, 2, \ldots, m\}$ with $m \geq 1$; and l_i denote a spatially extensive attribute of area A_i.

Relationship

Let $d : A \times A \to \mathbb{R}^+ \cup \{0\}$ be the dissimilarity between areas based on the set of attributes Y such that $d_{ij} \equiv d(A_i, A_j)$ satisfies the conditions $d_{ij} \geq 0, d_{ij} = d_{ij}$ and $d_{ij} = 0$ for $i, j = 1, 2, \ldots, n$. Distance funtions can also be utilized; i.e., d_{ij} can also satify the subadditivity, or triangle inequality, condition: $d_{ij} \leq d_{ik} + d_{kj}$ for $i, j, k = 1, 2, \ldots, n$.

Let $W = (V, E)$ denote the contiguity graph associated with A such that vertices $v_i \in V$ correspond to areas $A_i \in A$ and edges $\{v_i, v_j\} \in E$ if and only if areas A_i and A_j share a common border. For the max-p-regions model W must be a connected graph.

Feasible Partitions of A

Let $P_p = \{R_1, R_2, \ldots, R_p\}$ denote a partition of areas A into p regions with $1 \leq p \leq n$ such that:

$$|R_k| > 0 \quad \text{for} \quad k = 1, 2, \ldots, p;$$

$$R_k \cap R_{k'} = \emptyset \quad \text{for} \quad k, k' = 1, 2, \ldots, p \ \wedge \ k \neq k';$$

$$\bigcup_{k=1}^{p} R_k = A;$$

$$\sum_{A_i \in R_k} l_i \geq \mathit{threshold} \begin{cases} \text{for } k = 1, 2, \ldots, p, \text{ and} \\ \mathit{threshold} \in \mathbb{R}^+ \cup \{0\} | 0 \leq \mathit{threshold} \leq \sum_{A_i \in A} l_i; \end{cases}$$

3 Objectives

3.1 General objective (sólo uno)

To identify variations in the hemodynamics of blood flow through the carotid artery in patients with mild and severe atherosclerosis compared to patients with healthy carotid arteries, by applying NN-RBM and PINN like models to the flow over a section obtained from an angiography.

3.2 Specific objectives (3 o 4 objetivos)

- Achieve a correct recreation of a two-dimensional representation of the carotid artery, in which the most significant irregularities of the artery are adequately observed
- To show in a simple way the complex behavior of blood flow in healthy arteries and arteries with atherosclerosis
- Analize the hemodynamics of blood flow through the carotid artery in multiple scenarios

- Understand the formation of ischemic strokes from the hemodynamics of blood flow through the Carotid artery

4 Justification (4 o 5 párrafos)

En esta sección se argumenta el por qué es importante resolver el problema o contestar la pregunta de investigación. Las argumentaciones pueden ser de tipo teórico o práctico y soportadas en literatura.

- Destaca los beneficios derivados del aporte (¿para qué servirá esta investigación?, ¿qué aporta de nuevo esta investigación?, ¿cuáles son los beneficios?, ¿quiénes serán los beneficiados y de qué modo?, ¿qué se prevé cambiar con la investigación?, ¿cuál es la utilidad?, ¿resolverá algún problema práctico?, ¿se cubrirá algún gap de conocimiento?, ¿los resultados se podrán generalizar?, ¿sirve para apoyar alguna teoría?, ¿permite un mejor estudio de una población o fenómeno?, ¿se pueden establecer plazos para los beneficios?). OJO: las preguntas no se incluyen en el cuerpo del texto, son sólo una guía para encontrar los argumentos.

- Las respuestas a estas preguntas deben considerar tres aspectos: teórico, práctico y metodológico.

5 Scope (2 o 3 párrafos)

Describe las principales barreras o limitaciones de la investigación, así como las principales herramientas y otros recursos que esperaba utilizar durante la ejecución del proyecto. También describe los principales resultados esperados de la investigación

6 State of the art (5 a 6 párrafos)

Describe las principales referencias relacionadas con el problema. Este estado del arte puede referirse al problema o aplicación específica, o bien a los métodos aplicados para solucionarlo. No debes olvidar incluir los trabajos más importantes y los más recientes.

7 Proposed methodology (5 o 6 párrafos)

Describe en este apartado los métodos, técnicas, algoritmos, etc. que se utilizarán durante la ejecución del proyecto.

Incluye aspectos como:

- ¿Qué métodos se suelen usar para responder la pregunta de investigación?

- ¿Por qué seleccionaste el método que usarás?

- Describe el método: Supuestos básicos, ventajas y desventajas del método.

8 Schedule, commitments and deliverables

- Cronograma de las actividades a realizar durante la PI.
- Compromisos entre el tutor y el estudiante (e.g., periodicidad de reuniones, entrega de datos, etc.).
- Lista clara de entregables que se esperan de la práctica.

Adapta es siguiente cronograma a tu PI.

Table 1: Schedule

Activity	*Weeks*																	
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
Literature Review	■	■	■	■	■													
Research proposal writting					■	■												
Activity 3									■	■	■	■						
Results Analysis and comparison													■	■	■			
Final Report writting																■	■	■

9 Intellectual property

According to the internal regulation on intellectual property within Universidad EAFIT, the results of this research practice are product of *Alejandro Salazar Arango* and *Cristhian David Zambrano Mora*.

In case further products, beside academic articles, that could be generated from this work, the intellectual property distribution related to them will be directed under the current regulation of this matter determined by [6].

References

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