Ejercicios de Cálculo

Temas: Derivadas en n variables: Extremos relativos
Titulaciones: Todas

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Estudiar los extremos relativos de la función $f(x,y) = x^2y + \dfrac{1}{x} + \dfrac{1}{y}$.

Estudiar los extremos relativos de la función f.

Datos

$$f(x,y) = x^2y + \frac{1}{x} + \frac{1}{y}$$

$$\nabla f(x,y) = \left(2xy - \frac{1}{x^2},\ x^2 - \frac{1}{y^2}\right) = (0,0)$$

$$\begin{cases} 2xy - \dfrac{1}{x^2} = 0 \\ x^2 - \dfrac{1}{y^2} = 0 \Rightarrow x^2 = \dfrac{1}{y^2} \Rightarrow y^2 = \dfrac{1}{x^2} \Rightarrow y = \pm\dfrac{1}{x} \end{cases}$$

$$2xy - \frac{1}{x^2} \underset{y=\frac{1}{x}}{=} 2\cancel{x}\frac{1}{\cancel{x}} - \frac{1}{x^2} = 2 - \frac{1}{x^2} = 0 \Rightarrow \frac{1}{x^2} = 2 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$$

$$2xy - \frac{1}{x^2} \underset{y=-\frac{1}{x}}{=} 2\cancel{x}\left(-\frac{1}{\cancel{x}}\right) - \frac{1}{x^2} = -2 - \frac{1}{x^2} = 0 \Rightarrow \frac{1}{x^2} = -2 \Rightarrow x^2 = -\frac{1}{2} \Rightarrow$$ No tiene solución

Puntos críticos

$$x = -\sqrt{1/2} = \frac{-1}{\sqrt{2}} \quad y = \frac{1}{x} = \frac{1}{-1/\sqrt{2}} = -\sqrt{2} \Rightarrow \left(\frac{-1}{\sqrt{2}}, -\sqrt{2}\right)$$

$$x = +\sqrt{1/2} = \frac{1}{\sqrt{2}} \quad y = \frac{1}{x} = \frac{1}{1/\sqrt{2}} = \sqrt{2} \Rightarrow \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$$

Estudiar los extremos relativos de la función f.

Datos

$$f(x,y) = x^2y + \frac{1}{x} + \frac{1}{y}$$

$$\nabla f(x,y) = \left(2xy - \frac{1}{x^2}, x^2 - \frac{1}{y^2}\right)$$

Puntos críticos:

$(-1/\sqrt{2}, -\sqrt{2})$ y $(1/\sqrt{2}, \sqrt{2})$

$$\nabla^2 f(x,y) = \begin{pmatrix} 2y + 2x^{-3} & 2x \\ 2x & 2y^{-3} \end{pmatrix}$$

$$Hf\left(-\frac{1}{\sqrt{2}}, -\sqrt{2}\right) = \begin{vmatrix} 2\cdot(-\sqrt{2}) + 2\left(\frac{-1}{\sqrt{2}}\right)^{-3} & 2\left(\frac{-1}{\sqrt{2}}\right) \\ 2\left(-\frac{1}{\sqrt{2}}\right) & 2\cdot(-\sqrt{2})^{-3} \end{vmatrix} = \begin{vmatrix} -6\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & -\frac{\sqrt{2}}{2} \end{vmatrix} = \left.\begin{matrix} 6-2=4>0 \\ \frac{d^2f}{dx^2} = -6\sqrt{2}<0 \end{matrix}\right\} \text{Máx}$$

$$Hf\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right) = \begin{vmatrix} 2\sqrt{2} + 2\left(\frac{1}{\sqrt{2}}\right)^{-3} & 2\,\frac{1}{\sqrt{2}} \\ 2\,\frac{1}{\sqrt{2}} & 2\sqrt{2}^{-3} \end{vmatrix} = \begin{vmatrix} 6\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \frac{\sqrt{2}}{2} \end{vmatrix} = \left.\begin{matrix} 6-2=4>0 \\ \frac{d^2f}{dx^2} = 6\sqrt{2}>0 \end{matrix}\right\} \text{Mín}$$