EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

Pharmacy/Biotechnology 1st year — Version A — **November, 18 2019**

Duration: 1 hour and 15 minutes.

(2 pts.) 1. In a population where the prevalence of a disease is 10% we apply a diagnostic test with a sensitivity 85%. What must be the minimum specificity of the test to diagnose the disease when the outcome of the test is positive?

Solution

The specificity must be at least 0.9056.

(2.5 pts.) 2. In a stretch of a road there is an average of 2 accidents per day.

(a) Compute the probability of having more than 2 accidents a random day.

(b) Compute the probability of having more than 2 accidents a random day, knowing that there is at least one accident that day.

(c) Compute the probability of having 14 accidents a random week.

Solution

(a) Let X be the number of accidents in a day. $X \sim P(2)$ and $P(X > 2) = 0.3233$.

(b) $P(X > 2 | X \geq 1) = 0.3739$.

(c) Let Y be the number of accidents in a week. $X \sim P(14)$ and $P(X = 14) = 0.106$.

(2.5 pts.) 3. In a study about the efectiveness of two flu drugs A and B it has been observed in a clinical trial that in 12% of cases only drug A is effective, in 24% of cases only drug B is effective and in 80% of cases where drug A was effective, also was effective the drug B.

(a) What is the probability that both drugs are efective at the same time?

(b) What is the probability that only one of the drugs is effective?

(c) What is the probability that none of the drugs are effective?

(d) Are the effectiveness of the two drugs independent?

Solution

According to the problem statement, $P(A \cap \overline{B}) = 0.12$, $P(\overline{A} \cap B) = 0.24$ and $P(B|A) = 0.8$.

(a) $P(A \cap B) = 0.48$.

(b) $P(A \cap \overline{B}) + P(\overline{A} \cap B) = 0.36$.

(c) $P(\overline{A} \cap \overline{B}) = 0.16$.

(d) The events are dependent because $P(B) = 0.72 \neq P(B|A) = 0.8$.

(3 pts.) 4. It is known that the annual rainfall in a region follows a normal probability distribution. If the statistics show that 15% of the years the annual rainfall has been greater than 45 cm and 3% of the years less than 30 cm,

(a) Compute the mean and the stantdard deviation of the annual rainfall.
Remark: Use $\mu = 40$ cm y $\sigma = 3$ cm for the next part if you do not know how to compute them.

(b) What is the probability that in the next 5 years at least one year the annual rainfall was above 50 cm?

Solution

(a) Let X be the annual rainfall. $X \sim N(\mu, \sigma)$, and according to the statement $P(X > 45) = 0.15$ and $P(X < 30) = 0.03$.
$\mu = 39.6708$ cm and $\sigma = 5.1419$ cm.

(b) Let Y be the number of years in the next 5 years with annual rainfall above 50 cm. Then $Y \sim B(5, 0.0223)$, and $P(X \geq 1) = 0.1065$.
