# EXAM OF STATISTICS
# (PROBABILITY AND RANDOM VARIABLES)

**2nd Physiotherapy** — Version A — **June, 7 2021**

**Duration**: 1 hour.

(3 pts.) 1. In women, the shoulder circumference follows a normal distribution with mean 98 cm and standard deviation 5 cm.

(a) Compute the percentage of women in the population with a shoulder circumference between 95 and 105 cm.

(b) Above what value are the 5% of women with a highest shoulder circumference?

(c) Compute the probability that in a sample of 50 women there is at least 2 with a shoulder circumference less than 90 cm.

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**Solution**

Let $X$ be the shoulder circumference, then $X \sim N(98, 5)$.

(a) $P(95 \leq X \leq 105) = 0.645$, that is 6.45%.

(b) $P_{95} = 106.22$ cm.

(c) Let $Y$ be the number of women with a shoulder circumference less than 90 cm in a sample of 50 women. Then, $Y \sim B(50, 0.0548) \approx P(2.74)$, and $P(Y \geq 2) = 0.7585$.

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(3 pts.) 2. It has been observed that a company of components for physiotherapy machines produces 12 defective components every 300 hours on average.

(a) What is the probability of producing more than 2 defective components in 100 hours?

(b) What is the probability of producing at most one defective component in 50 hours?

(c) If there are 7 companies in Spain that produce these components, and assuming that all of them produce the same number of defective components on average, compute the probability that at least one company produces more than 3 defective components in 50 hours.

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**Solution**

(a) Let $X$ be the number of defective components in 100 hours, then $X \sim P(4)$, and $P(X > 2) = 0.7619$.

(b) Let $Y$ be the number of defective components in 50 hours, then $X \sim P(2)$, and $P(X \leq 1) = 0.406$.

(c) Let $Z$ be the number of companies that produce more than 3 defective components in 50 hours in a sample of 7 companies, then $Z \sim B(7, 0.1429)$, and $P(Y \geq 1) = 0.6601$.

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(4 pts.) 3. We want to study the risk for a new vaccine to cause thrombi compared with a traditional vaccine. After applying the new vaccine to 1000 persons and the traditional vaccine to 3000 persons, we observed 30 persons with thrombi in the new vaccine group and 42 persons with thrombi in the traditional vaccine group.

(a) Compute the relative risk of suffering thrombi with the new vaccine and interpret it.

(b) Compute the odds ratio of suffering thrombi with the new vaccine and interpret it.

(c) Which association measure is more reliable?

(d) In a random experiment we applied both vaccines (in different moments) to a sample and we observed that 4% of persons suffered some thrombi (due to the new vaccine or to the traditional vaccine). Compute the probability of suffering thrombi with the new vaccine and no with the traditional one.

(e) Are the events corresponding to suffering thrombi with the new vaccine and the traditional vaccine independent?

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**Solution**

Let $T$ be the event of suffering thrombi.

(a) $RR(T) = 2.1429$. Thus, the risk of suffering thrombi with the new vaccine is more than the double that with traditional vaccine.

(b) $OR(T) = 2.1782$. Thus, the odds of suffering thrombi with the new vaccine is more than the double that with traditional vaccine.

(c) Both measures are reliable because the study is prospective and we can estimate the incidence, but the relative risk is easier to interpret.

(d) Let $T_n$ and $T_t$ the events of suffering thrombi with the new and the traditional vaccines, respectively. $P(T_n \cap \overline{T_t}) = 0.026$.

(e) $P(T_t|T_n) = 0.1333 \neq P(T_t) = 0.014$, thus the events are dependent.

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