# EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

**Pharmacy/Biotechnology 1st year** Version A **January 18, 2021**

**Duration**: 1 hour.

(3.5 pts.) 1. A test to detect prostate cancer produces 1% of false positives and 0.2% false negatives. It is known that 1 in 400 males suffer this type of cancer.

(a) Compute the sensitivity and the specificity of the test.

(b) If a male got a positive outcome in the test, what is the chance of developing cancer?

(c) Compute and interpret the negative predictive value.

(d) Is this test better to predict or to rule out the cancer?

(e) To study wh ether there is an association between the practice of sports and this type of cancer, a sample of 1000 males was drawn, of which 700 practiced sports, and it was observed that there were 2 males with cancer in the group of males who practiced sports, and there were 3 males with cancer in the group of males who did not practice sports. Compute the relative risk and the odds ratio and interpret them.

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**Solution**

Let $D$ the event corresponding to suffering prostate cancer and $+$ and $-$ the events corresponding to get a positive and a negative outcome respectively.

(a) The sensitivity is $P(+|D) = 0.2$ and specificity $P(-|\overline{D}) = 0.99$.

(b) Positive predictive value $P(D|+) = 0.0476$.

(c) Negative predictive value $P(\overline{D}|-) = 0.998$.

(d) As the positive predictive value is smaller than the negative predictive value, this test is better to rule out the disease. In fact, we can not use this test to detect the prostate cancer because the positive predictive value is less than 0.5.

(e) $RR(D) = 0.2857$ and $OR(D) = 0.2837$. Thus, there is an association between the practice of sports and the prostate cancer and the risks and the odds of developing cancer is almost one fourth smaller if the male practice sports.

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(3 pts.) 2. The probability that a child of a mother with the color-blind gene and a father without the color-blind gene is a color-blind male is 0.25. It is also known that in a population there is one color-blind male for every 5000 males.

(a) If this couple has 5 children, what is the probability that at most 2 of them are color-blind males?

(b) If this couple has 5 children, and the gender of the children is equiprobable, what is the probability that 3 or more are females?

(c) In a random sample of 10000 males of this population, what is the probability that more than 3 are color-blind males?

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**Solution**

(a) Let $X$ be the number of color-blind sons in a sample of 5 children, then $X \sim B(5, 0.25)$ and $P(X \leq 2) = 0.8965$.

(b) Let $Y$ be the number of girls in a sample of 5 children, then $Y \sim B(5, 0.5)$ and $P(Y \geq 3) = 0.5$.

(c) Let $Z$ be the number of color-blind males in a sample of 10000 males, then $Z \sim B(10000, 2e-04) \approx P(2)$ and $P(Z > 3) = 0.1429$.

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(3.5 pts.) 3. The primate cranial capacity follows a normal distribution with mean 1200 cm$^3$ and standard deviation 140 cm$^3$.

(a) Compute the probability that the cranial capacity of a primate is greater than 1400 cm$^3$.

(b) Compute the probability that the cranial capacity of a primate is exactly than 1400 cm$^3$.

(c) Above what cranial capacity will 20% of primates be?

(d) Compute the interquartile range of the cranial capacity of primates and interpret it.

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**Solution**

Let $X$ be the primate cranial capacity. Then $X \sim N(1200, 140)$.

(a) $P(X > 1400) = 0.0766$.

(b) $P(X = 1400) = 0$.

(c) $P_{80} = 1317.827$ cm$^3$.

(d) $Q_1 = 1105.5714$ cm$^3$, $Q_3 = 1294.4286$ cm$^3$ and $IQR = 188.8571$ cm$^3$. Thus the 50% of central data will be concentranted in an interval of width 188.8571 cm$^3$, that is a small spread.

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