

EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

Pharmacy/Biotechnology 1st year

Version A

February 8, 2021

Duration: 1 hour.

- (5 pts.) 1. Two treatments are being tested against Covid-19. Treatment A has an effectiveness of 90 % while treatment B has an effectiveness of 70 %. It has also been found that in 5% of the patients affected by Covid-19 neither of the two treatments is effective.
- (a) Compute the probability that for a patient with Covid-19 only treatment A is effective.
 - (b) What is the probability that treatment B is effective if A was not effective for a Covid-19 patient?
 - (c) Are independent the two treatments?
 - (d) Compute the probability that in a sample of 6 Covid-19 patients the treatment A is effective in at least 4 of them.
 - (e) Compute the probability that in a sample of 100 Covid-19 patients there are less than 3 where none of the treatments are effective.
 - (f) Compute the probability that in a sample of 5 Covid-19 patients some of the treatments is effective in at least 3 of them.

Solution

- (a) $P(A \cap \overline{B}) = 0.25$.
- (b) $P(B|\overline{A}) = 0.5$.
- (c) The events are dependent, since $P(B|\overline{A}) \neq P(B)$.
- (d) Let X be the number of patients with treatment A effective in a sample of 6. Then $X \sim B(6, 0.9)$ and $P(X \geq 4) = 0.9842$.
- (e) Let Y be the number of patients with none of the treatments effective in a sample of 100. Then $X \sim B(100, 0.05) \approx P(5)$ and $P(Y < 3) = 0.1247$.
- (f) Let Z be the number of patients with some treatment effective in a sample of 5. Then $Z \sim B(5, 0.95)$ and $P(X \geq 3) = 0.9988$.

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- (5 pts.) 2. The Human chorionic gonadotropin (hCG) hormone is used to detect the pregnancy during the first weeks of the gestation. A test diagnose the pregnancy when the level of hCG is above 12 mUI/ml. Knowing that the concentration of hCG in non-pregnant females follows a normal probability distribution with mean 5 mUI/ml and standard deviation 2 mUI/ml, while in pregnant females follows a normal probability distribution with mean 25 mUI/ml and standard deviation 5 mUI/ml:
- (a) Compute the percentage of pregnant females with a level of hCG between 15 and 20 mUI/ml.
 - (b) Above what level of hCG are the 1% of non-pregnant females?
 - (c) Compute the sensitivity and the specificity of the test.
 - (d) If there are 0.5% of pregnant females in a population, what is the percentage of false positives for the test? And the percentage of false negatives?

Solution

Let X be the hCG concentration in non-pregnant females. Then $X \sim N(5, 2)$.
Let Y be the hCG concentration in pregnant females. Then $Y \sim N(25, 5)$.

- (a) $P(15 \leq Y \leq 20) = 0.1359$.
 - (b) $P_{99} = 9.6527$ mUI/ml.
 - (c) The sensitivity is $P(Y > 12) = 0.9953$ and the specificity is $P(X < 12) = 0.9998$.
 - (d) 0.0231% of false positives and 0.0023% of false negatives.
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