

EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

2nd Physiotherapy

Version A

June, 19 2020

Duration: 1 hour.

- (3 pts.) 1. Two symptoms of COVID-19 are fever and cough. We know that 30% of people with COVID-19 cough and 20% have fever and cough. Also, if somebody with COVID-19 have fever then the probability of coughing 0.5.
- (a) Construct the probability tree for the sample space of the random experiment consisting in picking a random person with COVID-19 and measuring the symptoms that he or she have.
 - (b) Calculate the probability of having any of the symptoms.
 - (c) Calculate the probability of having only cough.
 - (d) Calculate the probability of having only fever.
 - (e) Calculate the probability no fever nor cough.
 - (f) Are the symptoms dependent or independent?

Solution

Let C and F be the events of having cough and fever respectively. According to the statement $P(C) = 0.3$, $P(C \cap F) = 0.2$ and $P(C|F) = 0.5$.

- (a) $P(C \cup F) = 0.5$.
- (b) $P(C \cap \overline{F}) = 0.1$.
- (c) $P(\overline{C} \cap F) = 0.2$.
- (d) $P(\overline{C} \cap \overline{F}) = 0.5$.
- (e) The events are dependent since $P(C) \neq P(C|F)$.

- (4 pts.) 2. The sensitivity and specificity of a diagnostic test are 0.58 and 0.01, respectively, and the probability of a true positive is 0.02.
- (a) Calculate the prevalence of the disease.
 - (b) Calculate predictive values.
 - (c) Is the test more useful to rule out or confirm the disease?
 - (d) If we have 10 non-sick patients, what is the probability that more than 9 have a misdiagnosis?
 - (e) If we have 60 patients, what is the probability that at least two of them have a correct diagnosis?

Solution

- (a) $P(D) = 0.0345$.
- (b) $PPV = P(D|+) = 0.0205$ and $NPV = P(\overline{D}|-) = 0.4$.
- (c) The test is not helpful to confirm nor to rule out the disease, since both the positive and the negative predictive values are below 0.5.
- (d) Let X be the number non sick patients with a positive outcome, then $X \sim B(10, 0.99)$, and $P(X \geq 9) = 0.9957$.

- (e) Let Y be the number of patients with a right diagnose, then $Y \sim B(60, 0.0297) \approx P(1.7793)$, and $P(Y \geq 2) = 0.531$.
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- (3 pts.) 3. The time required to cure a basketball injury with a rehabilitation technique follows a normal distribution with quartiles $Q_1 = 22$ days and $Q_2 = 25$ days.
- (a) Calculate the mean and standard deviation of the curation time.
 - (b) If a player has just been injured and has to play a match in 30 days, what is the probability that he will miss it?
 - (c) Calculate the interquartile range of the curation time distribution.
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Solution

- (a) Let X be the time required to cure the injury, then $X \sim N(25, 4.4478)$.
 - (b) $P(X > 30) = 0.1305$.
 - (c) $IQR = 6$ days.
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