EXAM OF STATISTICS
(PROBABILITY AND RANDOM VARIABLES)

2nd Physiotherapy   Version A   **May, 31 2018**

Duration: 1 hour and 30 minutes.

(2.5 pts.) 1. An ultrasonic technique is used to diagnose a disease with a sensitivity of 91% and a specificity of 98%. The prevalence of the disease is 20%,

  (a) If we apply the technique to an individual and the outcome is positive, what is the probability of having the disease for that individual?

  (b) If the outcome was negative, what is the probability of not having the disease?

  (c) Is this technique more reliable to confirm or to rule out the disease? Justify the answer.

  (d) Compute the probability of having a correct diagnose with this technique.

Solution

Let D the event corresponding to have the disease and + and - the events corresponding to have a positive and negative outcome respectively in the test.

  (a) $PPV = 0.9192$.

  (b) $NPV = 0.9776$.

  (c) It is more reliable to rule out the disease since the NPV is greater than the PPV.

  (d) $P(D \cap +) + P(\overline{D} \cap -) = 0.966$.

(2.5 pts.) 2. It is known that the femur length of a fetus with 25 weeks of pregnancy follows a normal distribution with mean 44 mm and standard deviation 2 mm.

  (a) Compute the probability that the femur length of a fetus with 25 weeks is greater than 46 mm.

  (b) Compute the probability that the femur lenght of a fetus with 25 weeks is between 46 and 49 mm.

  (c) Compute an interval (a, b) centered at the mean, such that it contains 80% of the femur lengths of fetus with 25 weeks.

Solution

Let $X \sim N(44, 2)$ be the femur length of fetus with 25 weeks of pregnancy.

  (a) $P(X > 46) = 0.1587$.

  (b) $P(46 < X < 49)) = 0.1524$.

  (c) The interval centered at 44 that contains 80% of the femur lengths of fetus with 25 weeks is $(41.4369, 46.5631)$.

(2.5 pts.) 3. The probability that an injury A is repeated is 4/5, the probability that another injury B is repeated is 1/2, and the probability that none of them are repeated is 1/20. Compute the probability of the following events:

(a) At least one injury is repeated.

(b) Only injury B is repeated.

(c) Injury B is repeated if injury A has been repeated.

(d) Injury B is repeated if injury A has not been repeated.

Solution

(a) $P(A \cup B) = 19/20$.

(b) $P(B \cap \overline{A}) = 3/20$.

(c) $P(B/A) = 7/16$.

(d) $P(B/\overline{A}) = 3/4$.

(2.5 pts.) 4. A physical therapy clinic opens 6 hours a day and the average number of patients that arrive to the clinic is 12 a day.

(a) Compute the probability of arriving more than 4 patients in 1 hours.

(b) If the clinic has 4 physiotherapists and each of them can treat one patient per hour, what is the probability that a day there was some hour in which some patient can not be attended? How many physiotherapists must be in the clinic to guarantee that this probability is less than 10%?

Solution

(a) Let X be the number of patients that arrive in 1 hours. $X \sim P(2)$ and $P(X > 4) = 0.0527$.

(b) Let Y be the number of hours in a day in which some patient can not be treated. $Y \sim B(6, 0.0527)$ and $P(Y > 0) = 0.2771$.
The clinic requires 5 phisiotherapists, since $P(X > 5) = 0.0527$ and $P(Y > 0) = 0.0954$, with $Y \sim B(6, 0.0166)$ now.