

EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

2nd Physiotherapy

Version A

May, 27 2019

Duration: 1 hour and 15 minutes.

- (4 pts.) 1. It has been observed that the concentration of a metabolite in urine can be used as a diagnostic test for a disease. The concentration (in mg/dl) in healthy individuals follows a normal distribution with mean 90 and standard deviation 8, while in sick individuals follows a normal distribution with mean 120 and standard deviation 10.
- (a) If the cut-off point is set at 105 mg/dl (positive above and negative below), what is the sensitivity and the specificity of the test?
 - (b) If the cut-off point is set at 105 mg/dl and we assume a prevalence of 10%, what is the probability of a correct diagnostic?
 - (c) If we want a sensitivity of 95%, where must we set the cut-off point? What would the specificity of the test be ?

Solution

Let X and Y be the distributions of the concentration of metabolite in healthy and sick individuals respectively.

- (a) Sensitivity: $P(+|D) = P(Y > 105) = 0.9332$.
Specificity: $P(-|\bar{D}) = P(X < 105) = 0.9696$.
- (b) $P(\text{correct diagnostic}) = P(D \cap +) + P(\bar{D} \cap -) = 0.966$.
- (c) Cut-off point 103.5515 mg/dl.
Specificity: $P(-|\bar{D}) = P(X < 103.5515) = 0.9549$.

-
- (3 pts.) 2. Let A and B be two events of a random experiment, such that A is three times as likely as B , $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.2$.
- (a) Compute $P(A)$ and $P(B)$.
 - (b) Compute $P(A - B)$ and $P(B - A)$.
 - (c) Compute $P(\bar{A} \cup \bar{B})$ and $P(\bar{A} \cap \bar{B})$.
 - (d) Compute $P(A|B)$ and $P(B|A)$.
 - (e) Are A and B independent?

Solution

- (a) $P(A) = 0.75$ and $P(B) = 0.25$.
 - (b) $P(A - B) = 0.55$ and $P(B - A) = 0.05$.
 - (c) $P(\bar{A} \cup \bar{B}) = 0.8$ and $P(\bar{A} \cap \bar{B}) = 0.2$.
 - (d) $P(A|B) = 0.8$ and $P(B|A) = 0.2667$.
 - (e) No, they are dependent since $P(A|B) \neq P(A)$.
-

- (3 pts.) 3. The employees of a courier company send an average of 246.2 messages in a period of 12 hours. It is also known that the mean of messages sent by males is 256.2 and by females is 237.4 in the same period.
- (a) Compute the probability that a random person of the company sends 5 messages in a period of half an hour.
 - (b) If we draw randomly 10 women of this company, what is the probability that at least 3 of them sends more than one message in a period of one hour?
 - (c) If we draw randomly 100 men of this company, what is the probability that none of them sends less than 2 messages in a period of a quarter of an hour?

Solution

- (a) Let X be the number of messages sent in 1 hour. Then $X \sim P(10.2583)$ and $P(X = 5) = 0.0332$.
 - (b) Let Y be the number of women in a sample of 10 that sent more than 1 message in 1 hour. Then $Y \sim B(10, 1)$ and $P(Y \geq 3) = 1$.
 - (c) Let Z be the number of men in a sample of 100 that sent less than 2 messages in a quarter of hour. Then $Z \sim B(100, 0.0305)$ and $P(Z = 0) = 0.0166$.
-