EXAM OF STATISTICS
(PROBABILITY AND RANDOM VARIABLES)

Pharmacy/Biotechnology 1st year Version B **January 17, 2022**

Duration: 1 hour.

(3 pts.) 1. A diagnostic test for a disease with a prevalence of 10% has a positive predictive value of 40% and negative predictive value of 95%.

(a) Compute the sensitivity and the specificity of the test.

(b) Compute the probability of a right diagnose.

(c) What must be the minimum sensitivity of the test to be able to diagnose the disease?

Solution

(a) Sensitivity $P(+|D) = 0.571$ and specificity $P(-|\overline{D}) = 0.9048$.

(b) $P(\text{Right diagnose}) = P(D \cap +) + P(\overline{D} \cap -) = 0.8714$.

(c) Minimum sensitivity to diagnose the disease $P(+|D) = 0.857$.

(2 pts.) 2. To study the effectiveness of two antigen tests for the COVID both tests have been applied to a sample of 100 persons. The table below shows the results:

Test A	Test B	Num persons
$+$	$+$	8
$+$	$-$	2
$-$	$+$	3
$-$	$-$	87

Define the following events and compute its probabilities:

(a) Get a $+$ in the test A.

(b) Get a $+$ in the test A and a $-$ in the test B.

(c) Get a $+$ in some of the two tests.

(d) Get different results in the two tests.

(e) Get the same result in the two tests.

(f) Get a $+$ in the test B if we got a $+$ in the test A.

Are the outcomes of the two tests independent?

Solution

Let A and B the events of getting positive outcomes in the tests A and B respectively.

(a) $P(A) = 0.1$.

(b) $P(A \cap \overline{B}) = 0.02$.

(c) $P(A \cup B) = 0.13$.

(d) $P(A \cap \overline{B}) + P(\overline{A} \cap B) = 0.05$.

(e) $P(A \cap B) + p(\overline{A} \cap \overline{B}) = 0.95$.

(f) $P(B|A) = 0.8$.

As $P(B|A) \neq P(B)$ the events are dependent.

(5 pts.) 3. It is known that the life of a battery for a peacemaker follows a normal distribution. It has been observed that 20% of the batteries last more than 15 years, while 10% last less than 12 years.

(a) Compute the mean and the standard deviation of the battery life.
Remark: If you are not able to compute the mean and the standard deviation, use a mean of 14 years and a standard deviation of 1.5 years for the following parts.

(b) Compute the fourth decile of the battery life.

(c) If we take a sample of 5 batteries, what is the probability that more than half of them last between 13 and 14 years?

(d) If we take a sample of 100 batteries, what is the probability that some of them last less than 11 years?

Solution

Let X be the duration of a battery. Then $X \sim N(\mu, \sigma)$.

(a) $\mu = 13.8108$ years and $\sigma = 1.413$ years.

(b) $D_4 = 13.4528$ years.

(c) Let Y be the number of batteries lasting between 13 and 14 years in a sample of 5 batteries. Then $Y \sim B(5, 0.2702)$ and $P(Y > 2.5) = 0.0209$.

(d) Let U be the number of batteries lasting less than 11 years in a sample of 100 batteries. Then $U \sim B(100, 0.0233) \approx P(2.3335)$ and $P(U \geq 1) = 0.903$.
