CEU Universidad San Pablo

Facultad de Medicina

CARRERA PHYSIOTHERAPY	
APELLIDOS	NOMBRE
ASIGNATURA STATISTICS (PROBABILITY)	FECHA 2019/05/27 · GRUPO
CALIFICACIÓN	

1) $X \equiv$ CONCENTRATION OF METABOLITE IN HEALTHY INDIVIDUALS $\sim N(90, 8)$

$Y \equiv$ " " " " SICK " $\sim N(120, 10)$

a) SENSITIVITY: $P(+|D) = P(Y > 105) \underset{\uparrow}{=} P\left(\frac{Y-120}{10} > \frac{105-120}{10}\right) =$ 0.25

STANDARDIZATION 0.25

$= P(Z > -1.5) = 1 - P(Z \leq -1.5)$

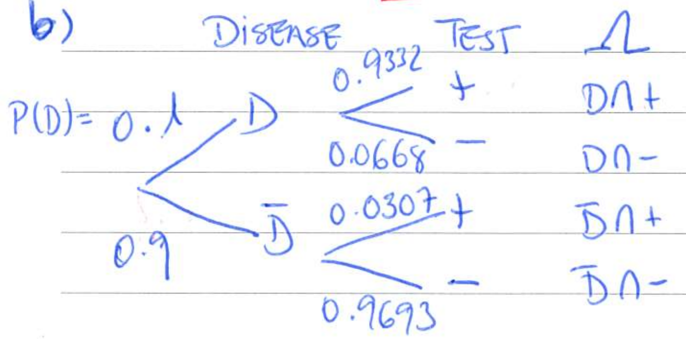
$= 1 - 0.0668 = \boxed{0.9332}$ 0.25

SPECIFICITY: $P(-|\bar{D}) = P(X < 105) = P\left(\frac{X-90}{8} < \frac{105-90}{8}\right) =$

0.25

$P(Z < 1.87) \overset{\text{TABLE}}{=} \boxed{0.9693}$ 0.25

b)



0.75

$P(\text{CORRECT DIAGNOSTIC}) = P(D \cap +) + P(\bar{D} \cap -) = 0.1 \cdot 0.9332 + 0.9 \cdot 0.9693 =$

$= \boxed{0.9657}$ 0.25 0.5

c) SENSITIVITY: $P(+|D) = 0.95$

$P(+|D) = P(Y > P_5) \underset{\uparrow}{=} P\left(\frac{Y-120}{10} > \frac{P_5 - 120}{10}\right) = P\left(Z > \frac{P_5-120}{10}\right) = 0.95 \Rightarrow$

0.25 0.25 0.25

$\Rightarrow P\left(Z \leq \frac{P_5-120}{10}\right) \overset{\text{STANDARDIZATION}}{=} 0.05 \underset{\text{TABLE}}{\Rightarrow} \frac{P_5-120}{10} = -1.645 \Rightarrow P_5 = 120 - 1.645 \cdot 10 = \boxed{103.55}$ mg/dL

SPECIFICITY: $P(-|\bar{D}) = P(X < 103.55) = P\left(\frac{X-90}{8} < \frac{103-90}{8}\right) =$

$= P(Z < 1.62) = \boxed{0.9474}$ 0.25 0.25

2) $P(A) = 3P(B)$, $P(A \cup B) = 0.8$, $P(A \cap B) = 0.2$

0.25

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow$

$\Rightarrow \quad 0.8 = 3P(B) + P(B) - 0.2 \Rightarrow$

$\Rightarrow \quad 4P(B) = 1 \Rightarrow P(B) = \frac{1}{4} = \boxed{0.25}$ 0.25

$P(A) = 3 \cdot P(B) = 3 \cdot \frac{1}{4} = \frac{3}{4} = \boxed{0.75}$ 0.25

b) $P(A - B) = P(A) - P(A \cap B) = \frac{3}{4} - 0.2 = \boxed{0.55}$ 0.25

$P(B - A) = P(B) - P(A \cap B) = 0.25 - 0.2 = \boxed{0.05}$ 0.25

0.25

c) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.2 = \boxed{0.8}$ 0.25

MORGAN'S LAWS

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.8 = \boxed{0.2}$ 0.25

d) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.25} = \boxed{0.8}$ 0.25

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.75} = \boxed{0.2667}$ 0.25

e) A and B are dependent events since $P(A|B) \neq P(A)$ 0.5

3) a) $X \equiv$ NUM OF MESSAGES SENT IN 1/2 HOUR $\sim P\left(246.2/24\right) = \boxed{P(10.26)}$ 0.5 0.25

$$P(X=5) = e^{-10.26}\,\frac{10.26^5}{5!} = \boxed{0.0332} \quad 0.25$$

b) $Y \equiv$ NUM OF WOMEN IN A SAMPLE OF 10 THAT SENT MORE THAN ONE MESSAGE IN 1 HOUR $\sim \boxed{B(10, 0.9999)}$ 0.25

$$p = P(X_1 > 1) = 1 - P(X \leq 1) = 1 - \left(f(0) + f(1)\right) = 1 - e^{-19.78}\,\frac{19.78^0}{0!} - e^{-19.78}\,\frac{19.78^1}{1!} \simeq \boxed{0.9999} \quad 0.75$$

$X_1 \equiv$ NUM OF MESSAGES SENT IN 1 HOUR BY A WOMAN $\sim P\left(237.4/12\right) = \boxed{P(19.78)}$ 0.25

$$P(Y \geq 3) = 1 - P(Y < 3) = 1 - \left(f(0) + f(1) + f(2)\right) =$$

$$= 1 - \left(\underbrace{\binom{10}{0} 0.9999^0 (1-0.9999)^{10-0}}_{\simeq 0} + \underbrace{\binom{10}{1} 0.9999^1 (1-0.9999)^{10-1}}_{\simeq 0} + \underbrace{\binom{10}{2} 0.9999^2 (1-0.9999)^{10-2}}_{\simeq 0} \right)$$

$$\simeq \boxed{1} \quad 0.25$$

c) $Z \equiv$ NUM OF MEN IN A SAMPLE OF 100 THAT SENT LESS THAN 2 MESSAGES IN A QUARTER OF HOUR $\sim \boxed{B(100, 0.0304)}$ 0.25

$$p = P(X_2 < 2) = f(0) + f(1) = e^{-5.34}\,\frac{5.34^0}{0!} + e^{-5.34}\,\frac{5.34^1}{1!} = \boxed{0.0304} \quad 0.25$$

$X_2 \equiv$ NUM OF MESSAGES SENT IN 1/4 HOUR BY A MAN $\sim P\left(256.2/48\right) = \boxed{P(5.34)}$ 0.25

$$Z \sim B(100, 0.0304) \underset{\text{LAW OF RARE EVENTS}}{\simeq} \boxed{P(3.04)} \quad 0.25$$

$$P(Z=0) = f(0) = e^{-3.04}\,\frac{3.04^0}{0!} = e^{-3.04} \simeq \boxed{0.0478}$$