

EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

2nd Physiotherapy

Version A

May, 27 2019

Duration: 1 hour and 15 minutes.

- (4 pts.) 1. It has been observed that the concentration of a metabolite in urine can be used as a diagnostic test for a disease. The concentration (in mg/dl) in healthy individuals follows a normal distribution with mean 90 and standard deviation 8, while in sick individuals follows a normal distribution with mean 120 and standard deviation 10.
- (a) If the cut-off point is set at 105 mg/dl (positive above and negative below), what is the sensitivity and the specificity of the test?
 - (b) If the cut-off point is set at 105 mg/dl and we assume a prevalence of 10%, what is the probability of a correct diagnostic?
 - (c) If we want a sensitivity of 95%, where must we set the cut-off point? What would the specificity of the test be ?

Solution

- (3 pts.) 2. Let A and B be two events of a random experiment, such that A is three times as likely as B , $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.2$.
- (a) Compute $P(A)$ and $P(B)$.
 - (b) Compute $P(A - B)$ and $P(B - A)$.
 - (c) Compute $P(\bar{A} \cup \bar{B})$ and $P(\bar{A} \cap \bar{B})$.
 - (d) Compute $P(A|B)$ and $P(B|A)$.
 - (e) Are A and B independent?

Solution

- (3 pts.) 3. The employees of a courier company send an average of 246.2 messages in a period of 12 hours. It is also known that the mean of messages sent by males is 256.2 and by females is 237.4 in the same period.
- (a) Compute the probability that a random person of the company sends 5 messages in a period of half an hour.
 - (b) If we draw randomly 10 women of this company, what is the probability that at least 3 of them sends more than one message in a period of one hour?
 - (c) If we draw randomly 100 men of this company, what is the probability that none of them sends less than 2 messages in a period of a quarter of an hour?

Solution

- (a) Sea X el número de pacientes que llegan en 1 horas. $X \sim P(2)$ y $P(X > 4) = 0.0527$.

- (b) Sea Y el número de horas en un día en las que algún paciente no puede ser atendido. $Y \sim B(6, 0.0527)$ y $P(Y > 0) = 0.2771$. Se necesitan 5 empleados para que esta probabilidad sea menor del 10%.

Se necesitan 5 empleados para que esta probabilidad sea menor del 10%, ya que $P(X > 5) = 0.0527$ y $P(Y > 0) = 0.0954$, siendo ahora $Y \sim B(6, 0.0166)$.
