

EXAM OF STATISTICS (PROBABILITY AND RANDOM VARIABLES)

2nd Physiotherapy

Version A

May, 5 2021

Duration: 1 hour.

- (3 pts.) 1. The average number of injuries in an international tennis tournament is 2.
- Compute the probability that in an international tennis tournament there are more than 2 injuries.
 - If a tennis circuit has 6 international tournaments, what is the probability that there are no injuries in some of them?

Solution

- Let X be the number of injuries in a tournament, then $X \sim P(2)$ and $P(X > 2) = 0.3233$.
- Let Y be the number of tournaments in the tennis circuit with no injuries, then $Y \sim B(6, 0.1353)$ and $P(Y > 0) = 0.5821$.

- (4 pts.) 2. The tables below corresponds to two tests A and B to detect an injury that have been applied to the same sample.

Test A	Injury	No injury	Test B	Injury	No injury
+	87	14	+	104	115
−	33	866	−	16	765

- Which test is more sensitive? Which one is more specific?
- According to the predictive values, which test is better to diagnose the injury? Which one is better to rule out the injury?
- Assuming that both tests are independent, what is the probability of getting a right diagnose with both tests if we apply both tests to a healthy person?
- Assuming that both tests are independent, what is the probability of getting at least a positive outcome if we apply both tests to a random person?

Solution

- Test A : $\text{sen} = 0.725$ and $\text{spe} = 0.9841$
 Test B : $\text{sen} = 0.8667$ and $\text{spe} = 0.8693$
 Thus, test A is more specific and test B is more sensitive.
- Test A : $\text{PPV} = 0.8614$ and $\text{NPV} = 0.9633$
 Test B : $\text{sen} = 0.4749$ and $\text{spe} = 0.9795$
 Thus, test A is better to diagnose the disease and test B is better to rule out the disease.
- $P(-_1 \cap -_2 | \overline{D}) = 0.8555$.
- $P(+_1 \cup +_2) = 0.2979$.

- (3 pts.) 3. A study tries to determine the effect of a low fat diet in the lifetime of rats. The rats were divided into two groups, one with a normal diet and another with a low fat diet. It is assumed that the lifetimes of both groups are normally distributed with the same variance but different mean. If 20% of rats with normal diet lived more than 12 months, 5% less than 8 months, and 85% of rats with low fat diet lived more than 11 months,
- (a) Compute the means and the standard deviation of the lifetime of rats following a normal diet and a low fat diet?
Remark: If you do not know how to compute them, for the next part use a mean of 11 months and a standard deviation of 1.5 months for normal diet rats, and a mean of 12.5 months for low fat diet rats.
 - (b) If 40% of the rats were under a normal diet, and 60% of rats under a low fat diet, what is the probability that a random rat die before 9 months?

Solution

Let X_1 and X_2 be the lifetime of rats with a normal diet and a low fat diet respectively,

- (a) $\mu_2 = 12.6673$ months and $s = 1.6087$ months.
 - (b) $P(X < 9) = 0.068$.
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