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Statistics Formulas

Descriptive Statistics

Frequencies

Sample size n num of individuals in the sample.

Absolute frequency n_i (num of x_i in the sample)

Relative frequency $f_i = n_i/n$

Cumulative absolute freq $N_i = \sum_{k=0}^{i} n_i$

Cumulative relative freq $F_i = N_i/n$

Central tendency statistics

Mean $\bar{x} = \dfrac{\sum x_i}{n}$

Median me The value with cum.rel.freq. $F_{me} = 0.5$.

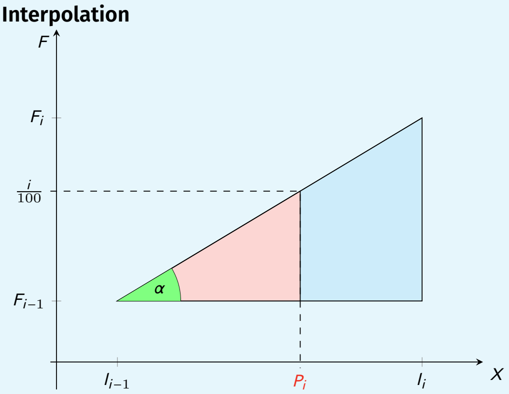
Mode mo The most frequent value.

Position statistics

Quartiles Q_1, Q_2, Q_3 divide the distribution into 4 equal parts. Their cum.rel.freqs. are $F_{Q_1} = 0.25$, $F_{Q_2} = 0.5$ and $F_{Q_3} = 0.75$.

Percentiles $P_1, P_2, \cdots, P_{99}$ divide the distribution into 100 equal parts.
The cum.rel.freq. is $F_{P_i} = i/100$.

Interpolation



$$P_i = l_i + \frac{\frac{i}{100} - F_{i-1}}{F_i - F_{i-1}}(l_i - l_{i-1})$$

Dispersion statistics

Interquartile range $IQR = Q_3 - Q_1$

Variance $s^2 = \dfrac{\sum(x_i - \bar{x})^2}{n} = \dfrac{\sum x_i^2}{n} - \bar{x}^2$

Standard deviation $s = +\sqrt{s^2}$

Coefficient of variation $cv = \dfrac{s}{|\bar{x}|}$

Shape statistics

Coefficient of skewness $g_1 = \dfrac{\sum(x_i - \bar{x})^3}{ns^3}$

Coefficient of kurtosis $g_2 = \dfrac{\sum(x_i - \bar{x})^4}{ns^4} - 3$

Linear transformations

Linear transformation $y = a + bx$

$$\bar{y} = a + b\bar{x}$$
$$s_y = bs_x$$

Standarization $z = \dfrac{x - \bar{x}}{s_x}$

Regression and correlation

Linear regression

Covariance $s_{xy} = \dfrac{\sum x_i y_j}{n} - \bar{x}\bar{y}$

Regression lines :

$$y \text{ on } x : y = \bar{y} + \frac{s_{xy}}{s_x^2}(x - \bar{x})$$
$$x \text{ on } y : x = \bar{x} + \frac{s_{xy}}{s_y^2}(y - \bar{y})$$

Regression coefficients

$$(y \text{ on } x)\ b_{yx} = \frac{s_{xy}}{s_x^2} \quad (x \text{ on } y)\ b_{xy} = \frac{s_{xy}}{s_y^2}$$

Coefficient of determination

$$r^2 = \frac{s_{xy}^2}{s_x^2 s_y^2} \qquad 0 \le r^2 \le 1$$

Correlation coefficient

$$r = \frac{s_{xy}}{s_x s_y}. \qquad -1 \le r \le 1$$

Non-linear regression

Exponential model $y = e^{a+bx}$
Apply the logarithm to the dependent variable and compute the line $\log y = a + bx$.

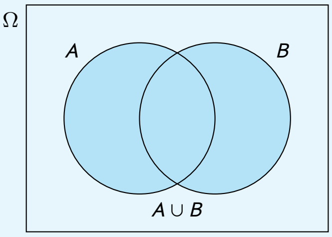
Logarithmic model $y = a + b\log x$
Apply the logarithm to the independent variable and compute the line $y = a + b\log x$.

Potential model $y = ax^b$
Apply the logarithm to both variables and compute the line $\log y = a + b\log x$.

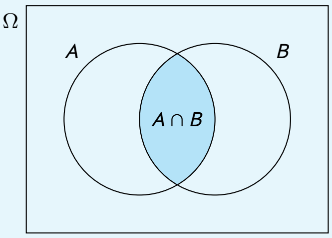
Probability

Event operations

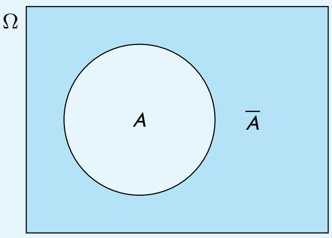
Union



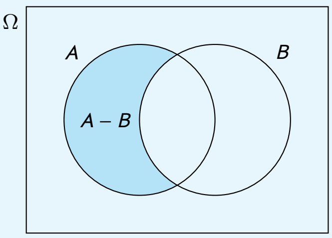
Intersection



Complement



Difference



Algebra of events

Idempotency $A \cup A = A, \quad A \cap A = A$

Commutative $A \cup B = B \cup A, \quad A \cap B = B \cap A$

Associative $(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$

Distributive $(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Neutral element $A \cup \varnothing = A, \quad A \cap \Omega = A$

Absorbing element $A \cup \Omega = \Omega, \quad A \cap \varnothing = \varnothing.$

Complementary symmetric element $A \cup \overline{A} = \Omega, \quad A \cap \overline{A} = \varnothing$

Double contrary $\overline{\overline{A}} = A$

Morgan's laws $\overline{A \cup B} = \overline{A} \cap \overline{B}, \quad \overline{A \cap B} = \overline{A} \cup \overline{B}$

Basic probability

Union $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Intersection $P(A \cap B) = P(A)P(B|A)$

Difference $P(A - B) = P(A) - P(A \cap B)$

Contrary $P(\overline{A}) = 1 - P(A)$

Conditional probability

Conditional probability $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Independent events $P(A|B) = P(A)$.

Total probability Theorem

$$P(B) = \sum_{i=1}^{n} P(A_i)P(B|A_i)$$

Bayes Theorem

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}$$

Risks

	E	$\overline{E}$
Treatment	a	b
Control	c	d

Prevalence Proportion of individuals with E: $P(E)$

Incidence rate or absolute risk $R(E) = \dfrac{a}{a+b}$

Odds $O(E) = \dfrac{a}{b}$

Relative risk $RR(E) = \dfrac{a/(a+b)}{c/(c+d)}$

Odds ratio $OR(E) = \dfrac{a/b}{c/d} = \dfrac{a \cdot d}{b \cdot c}$

Diagnostic tests

	Disease D	No disease $\overline{D}$
Test $+$	VP	FP
Test $-$	FN	VN

Sensitivity $P(+|D) = \dfrac{VP}{VP+FN}$

Specificity $P(-|\overline{D}) = \dfrac{VN}{FP+VN}$

Positive Predictive Value (PPV) $P(D|+) = \dfrac{VP}{VP+FP}$

Negative Predictive Value (NPV) $P(\overline{D}|-) = \dfrac{VN}{FN+VN}$

Positive Likelihood Ratio (LR+) $\dfrac{P(+|D)}{P(+|\overline{D})}$

Negative Likelihood Ratio (LR-) $\dfrac{P(-|D)}{P(-|\overline{D})}$

Random Variables

Discrete

Binomial probability function $B(n,p)$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Poisson probability function $P(\lambda)$

$$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

Law of rare events $B(n,p) \approx P(np)$ for $n \geq 30$ and $p \leq 0.1$.

Continuous

Normal $N(\mu,\sigma)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard normal $N(0,1)$

Chi-square $\chi^2(n)$

$$X = Z_1^2 + \cdots + Z_n^2,$$

where $Z_i \sim N(0,1)$.

Student's t $T(n)$

$$T = \frac{Z}{\sqrt{X/n}},$$

where $Z \sim N(0,1)$ and $X \sim \chi^2(n)$.

Fisher's F $F(n,m)$

$$F = \frac{X/m}{Y/n},$$

where $X \sim \chi^2(m)$ and $Y \sim \chi^2(n)$.