

# EXERCISES OF STATISTICS

**Subject:** Statistics Applied to the Health Sciences

**Course:** 2<sup>nd</sup>

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# 1 Descriptive Statistics

1. Classify the following variables

- (a) Daily hours of exercise.
- (b) Nationality.
- (c) Blood pressure.
- (d) Severity of illness.
- (e) Number of sport injuries in a year.
- (f) Daily calorie intake.
- (g) Size of clothing.
- (h) Subjects passed in a course.

2. The number of injuries suffered by the members of a soccer team in a league were

0	1	2	1	3	0	1	0	1	2	0	1
1	1	2	0	1	3	2	1	2	1	0	1

- (a) Construct the frequency distribution table of the sample.
- (b) Draw the bar chart of the sample and the polygon.
- (c) Draw the cumulative frequency bar chart and the polygon.

3. A survey about the daily number of medicines consumed by people over 70 years, shows the following results:

3	1	2	2	0	1	4	2	3	5	1	3	2	3	1	4	2	4	3	2
3	5	0	1	2	0	2	3	0	1	1	5	3	4	2	3	0	1	2	3

- (a) Construct the frequency distribution table of the sample.
- (b) Draw the bar chart of the sample and the polygon.
- (c) Draw the cumulative relative frequency bar chart and the polygon.

4. In a survey about the dependency of older people, 23 persons over 75 years were asked about the help they need in daily life. The answers were

B D A B C C B C D E A B C E A B C D B B A A B

where the meanings of letters are:

- A No help.
- B Help climbing stairs.
- C Help climbing stairs and getting up from a chair or bed.
- D Help climbing stairs, getting up and dressing.
- E Help for almost everything.

Construct the frequency distribution table and the suitable chart.

5. The number of people treated in the emergency service of a hospital every day of November was

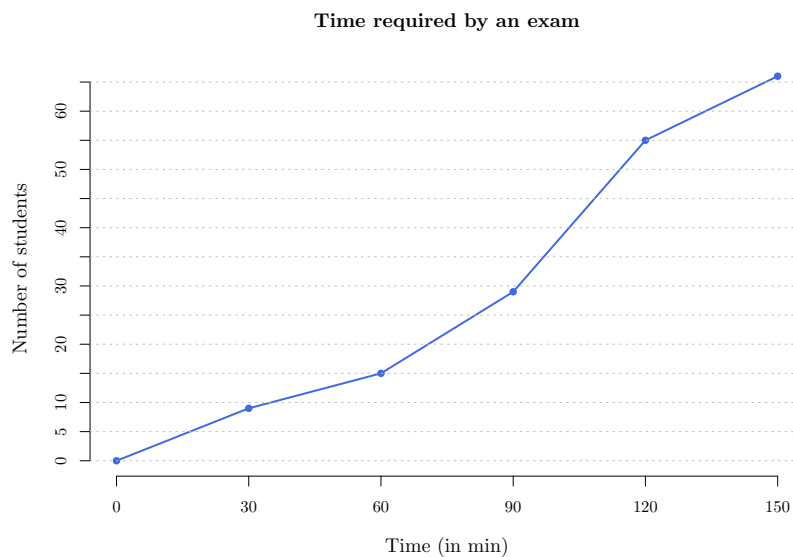
15	23	12	10	28	7	12	17	20	21	18	13	11	12	26
30	6	16	19	22	14	17	21	28	9	16	13	11	16	20

- (a) Construct the frequency distribution table of the sample.
- (b) Draw a suitable chart for the frequency distribution.

- (c) Draw a suitable chart for the cumulative frequency distribution.
6. The following frequency distribution table represents the distribution of time (in min) required by people attended in a medical dispensary.

Time	$n_i$	$f_i$	$N_i$	$F_i$
[0, 5)	2			
[5, 10)			8	
[10, 15)				0.7
[15, 20)	6			

- (a) Complete the table.
- (b) Draw the ogive.
7. Use the data of exercise 2 to calculate the following statistics and interpret them.
- (a) Mean.
- (b) Median.
- (c) Mode.
- (d) Quartiles.
- (e) Percentile 32.
8. The chart below shows the cumulative distribution of the time (in min) required by 66 students to do an exam.



- (a) At which time have finished half of the students? And 90% of students?
- (b) Which percentage of students have finished after 100 minutes?
- (c) Which is the time that best represent the time required by students in the sample to finish the exam? Is this value representative or not?
9. In a study about the children growth two samples were drawn, one for newborns and the other for one year old. The height in cm of children in both samples were

Newborn children: 51, 50, 51, 53, 49, 50, 53, 50, 47, 50  
 One year old children: 62, 65, 69, 71, 65, 66, 68, 69

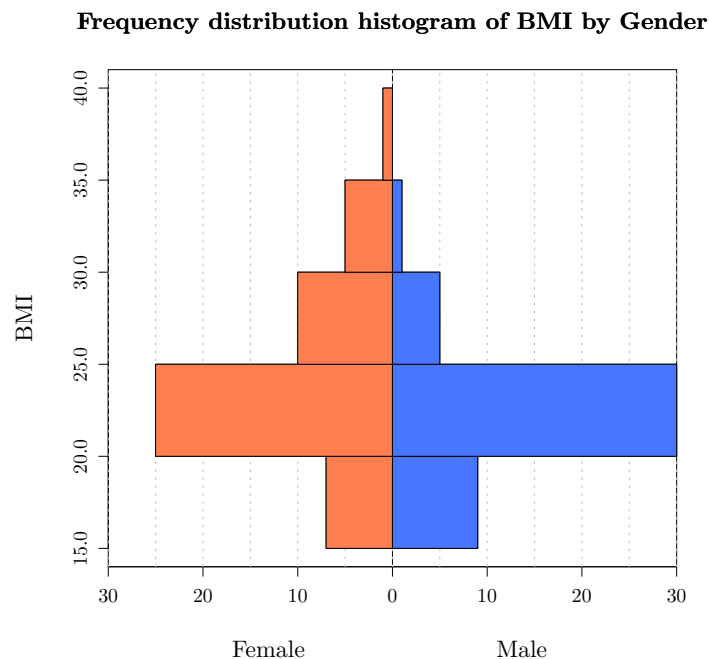
In which group is more representative the mean? Justify the answer.

10. To determine the accuracy of a method for measuring hematocrit in blood, the measurement was repeated 8 times on the same blood sample. The results in percentage of hematocrit in plasma were

42.2 42.1 41.9 41.8 42 42.1 41.9 42

What do you think about the accuracy of the method?

11. The histogram below shows the frequency distribution of the body mass index (BMI) of a group of people by gender.



- Draw the pie chart for the gender.
- In which group is more representative the mean of the BMI?
- Calculate the mean for the whole sample.

Use the following sums

$$\text{Males: } \sum x_i = 1002 \text{ kg/m}^2 \quad \sum x_i^2 = 22781 \text{ kg}^2/\text{m}^4$$

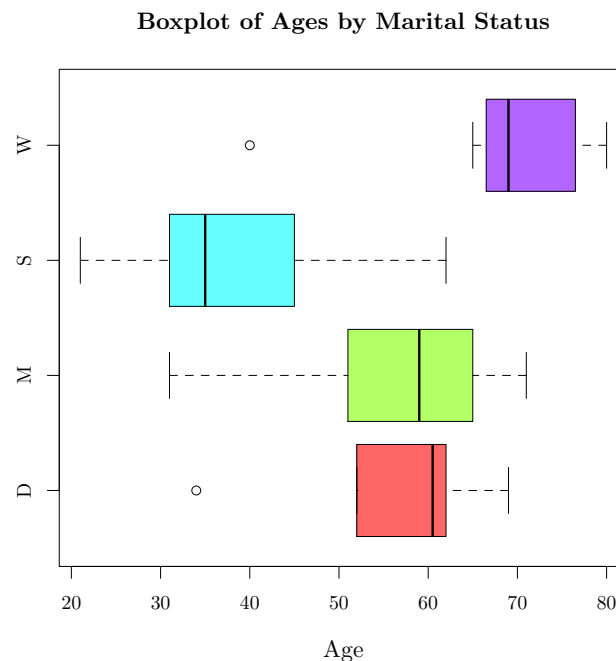
$$\text{Females: } \sum x_i = 1160 \text{ kg/m}^2 \quad \sum x_i^2 = 29050 \text{ kg}^2/\text{m}^4$$

12. The following table represents the frequency distribution of the yearly uses of a health insurance in a sample of clients of a insurance company.

Uses:	0	1	2	3	4	5	7
Clients:	4	8	6	3	2	1	1

Draw the box plot. How is the symmetry of the distribution?

13. The box plots below correspond to the age of a sample of people by marital status.



- (a) Which group has higher ages?  
 (b) Which group has lower central dispersion?  
 (c) Which groups have outliers?  
 (d) Which group has a distribution of ages more asymmetric?
14. The following table represents the frequency distribution of ages at which a group of people suffered a heart attack.

Age	[40-50)	[50-60)	[60-70)	[70-80)	[80-90)
Persons	6	12	23	19	5

Could we assume that the sample comes from a normal population?

Use the following sums:  $\sum x_i = 4275$  years,  $\sum (x_i - \bar{x})^2 = 7462$  years<sup>2</sup>,  $\sum (x_i - \bar{x})^3 = -18249$  years<sup>3</sup>,  $\sum (x_i - \bar{x})^4 = 2099636$  years<sup>4</sup>.

15. To compare two rehabilitation treatments  $A$  and  $B$  for an injury, every treatment was applied to a different group of people. The number of days required to cure the injury in every group is shown in the following table:

Days	$A$	$B$
20-40	5	8
40-60	20	15
60-80	18	20
80-100	7	7

- (a) In which treatment is more representative the mean?  
 (b) In which treatment the distribution of days is more skew?  
 (c) In which treatment the distribution is more peaked?

Use the following sums:

$A$ :  $\sum x_i = 3040$  days,  $\sum (x_i - \bar{x})^2 = 14568$  days<sup>2</sup>,  $\sum (x_i - \bar{x})^3 = 17011.2$  days<sup>3</sup>,  $\sum (x_i - \bar{x})^4 = 9989603$

days<sup>4</sup>  
 $B: \sum x_i = 3020 \text{ days}, \sum (x_i - \bar{x})^2 = 16992 \text{ days}^2, \sum (x_i - \bar{x})^3 = -42393.6 \text{ days}^3,$   
 $\sum (x_i - \bar{x})^4 = 12551516 \text{ days}^4$

16. The systolic blood pressure (in mmHg) of a sample of persons is

135 128 137 110 154 142 121 127 114 103

- Calculate the central tendency statistics.
- How is the relative dispersion with respect to the mean?
- How is the skewness of the sample distribution?
- How is the kurtosis of the sample distribution?
- If we know that the method used for measuring the blood pressure is biased, and, in order to get the right values, we have to apply the linear transformation  $y = 1.2x - 5$ , which are values of the statistics required to answer the previous questions for the corrected values of the blood pressure?

Use the following sums:  $\sum x_i = 1271 \text{ mmHg}$ ,  $\sum (x_i - \bar{x})^2 = 2188.9 \text{ mmHg}^2$ ,  $\sum (x_i - \bar{x})^3 = 2764.32 \text{ mmHg}^3$ ,  $\sum (x_i - \bar{x})^4 = 1040080 \text{ mmHg}^4$ .

17. The table below contains the frequency of pregnancies, abortions and births of a sample of 999 women in a city.

Num	Pregnancies	Abortions	Births
0	61	751	67
1	64	183	80
2	328	51	400
3	301	10	300
4	122	2	90
5	81	2	62
6	29	0	0
7	11	0	0
8	2	0	0

- How many birth outliers are in the sample?
- Which variable has lower spread with respect to the mean?
- Which value is relatively higher, 7 pregnancies or 4 abortions? Justify the answer.

Use the following sums:

Pregnancies:  $\sum x_i = 2783$ ,  $\sum x_i^2 = 9773$ .

Abortions:  $\sum x_i = 333$ ,  $\sum x_i^2 = 559$ .

Births:  $\sum x_i = 2450$ ,  $\sum x_i^2 = 7370$ .

## 2 Regression and correlation

18. Give some examples of:

- (a) Non related variables.
- (b) Variables that are increasingly related.
- (c) Variables that are decreasingly related.

19. In an study about the effect of different doses of a medicament, 2 patients got 2 mg and took 5 days to cure, 4 patients got 2 mg and took 6 days to cure, 2 patients got 3 mg and took 3 days to cure, 4 patients got 3 mg and took 5 days to cure, 1 patient got 3 mg and took 6 days to cure, 5 patients got 4 mg and took 3 days to cure and 2 patients got 4 mg and took 5 days to cure.

- (a) Construct the joint frequency table.
- (b) Get the marginal frequency distributions and compute the main statistics for every variable.
- (c) Compute the covariance and interpret it.

20. The table below shows the two-dimensional frequency distribution of a sample of 80 persons in a study about the relation between the blood cholesterol ( $X$ ) in mg/dl and the high blood pressure ( $Y$ ).

$X \setminus Y$	[110, 130)	[130, 150)	[150, 170)	$n_x$
[170, 190)		4		12
[190, 210)	10	12	4	
[210, 230)	7		8	
[230, 250)	1			18
$n_y$		30	24	

- (a) Complete the table.
- (b) Construct the linear regression model of cholesterol on pressure.
- (c) Use the linear model to calculate the expected cholesterol for a person with pressure 160 mmHg.
- (d) According to the linear model, what is the expected pressure for a person with cholesterol 270 mg/dl?

Use the following sums:  $\sum x_i = 16960$  mg/dl,  $\sum y_j = 11160$  mmHg,  $\sum x_i^2 = 3627200$  (mg/dl)<sup>2</sup>,  $\sum y_j^2 = 1576800$  mmHg<sup>2</sup> y  $\sum x_i y_j = 2378800$  mg/dl·mmHg.

21. A research study has been conducted to determine the loss of activity of a drug. The table below shows the results of the experiment.

Time (in years)	1	2	3	4	5
Activity (%)	96	84	70	58	52

- (a) Construct the linear regression model of activity on time.
- (b) According to the linear model, when will the activity be 80%? When will the drug have lost all activity?

22. A basketball team is testing a new stretching program to reduce the injuries during the league. The data below show the daily number of minutes doing stretching exercises and the number of injuries along the league.

Stretching minutes	0	30	10	15	5	25	35	40
Injuries	4	1	2	2	3	1	0	1

- (a) Construct the regression line of the number of injuries on the time of stretching.
- (b) What is the reduction of injuries for every minute of stretching?



(c) How many minutes of stretching are required for having no injuries? Is reliable this prediction?

Use the following sums ( $X$ =Number of minutes stretching, and  $Y$ =Number of injuries):  $\sum x_i = 160$  min,  $\sum y_j = 14$  injuries,  $\sum x_i^2 = 4700$  min<sup>2</sup>,  $\sum y_j^2 = 36$  injuries<sup>2</sup> and  $\sum x_i y_j = 160$  min·injuries.

23. For two variables  $X$  and  $Y$  we have

- The regression line of  $Y$  on  $X$  is  $y - x - 2 = 0$ .
- The regression line of  $X$  on  $Y$  is  $y - 4x + 22 = 0$ .

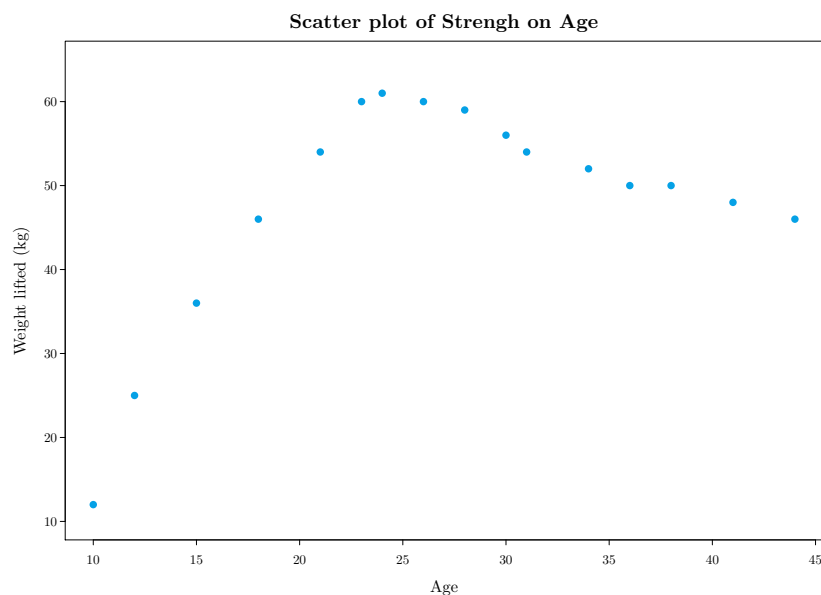
Calculate:

- (a) The means  $\bar{x}$  and  $\bar{y}$ .
- (b) The correlation coefficient.

24. The means of two variables  $X$  and  $Y$  are  $\bar{x} = 2$  and  $\bar{y} = 1$ , and the correlation coefficient is 0.

- (a) Predict the value of  $Y$  for  $x = 10$ .
- (b) Predict the value of  $X$  for  $y = 5$ .
- (c) Plot both regression lines.

25. A study to determine the relation between the age and the physical strength gave the scatter plot below.



- (a) Calculate the linear coefficient of determination for the whole sample.
- (b) Calculate the linear coefficient of determination for the sample of people younger than 25 years old.
- (c) Calculate the linear coefficient of determination for the sample of people older than 25 years old.
- (d) Which model explains better the relation between the age and the strength?

Use the following sums ( $X$ =Age and  $Y$ =Weight lifted).

- Whole sample:  $\sum x_i = 431$  years,  $\sum y_j = 769$  Kg,  $\sum x_i^2 = 13173$  years<sup>2</sup>,  $\sum y_j^2 = 39675$  Kg<sup>2</sup> and  $\sum x_i y_j = 21792$  years·Kg.
- Young people:  $\sum x_i = 123$  years,  $\sum y_j = 294$  Kg,  $\sum x_i^2 = 2339$  years<sup>2</sup>,  $\sum y_j^2 = 14418$  Kg<sup>2</sup> and  $\sum x_i y_j = 5766$  years·Kg.

- Old people:  $\sum x_i = 308$  years,  $\sum y_j = 475$  Kg,  $\sum x_i^2 = 10834$  years<sup>2</sup>,  $\sum y_j^2 = 25257$  Kg<sup>2</sup> and  $\sum x_i y_j = 16026$  years·Kg.

26. A dietary center is testing a new diet in sample of 12 persons. The data below are the number of days of diet and the weight loss (in Kg) until them for every person.

(33 , 3.9), (51 , 5.9), (30 , 3.2), (55 , 6.0), (38 , 4.9), (62 , 6.2),  
(35 , 4.5), (60 , 6.1), (44 , 5.6), (69 , 6.2), (47 , 5.8), (40 , 5.3)

- Draw the scatter plot. According to the point cloud, what type of regression model explains better the relation between the weight loss and the days of diet?
- Construct the linear regression model and the logarithmic regression model of the weight loss on the number of days of diet.
- Use the best model to predict the weight that will lose a person after 100 days of diet. Is this prediction reliable?

Use the following sums ( $X$ =days of diet and  $Y$ =weight loss):  $\sum x_i = 564$  days,  $\sum \log(x_i) = 45.8086$  log(days),  $\sum y_j = 63.6$  Kg,  $\sum x_i^2 = 28234$  days<sup>2</sup>,  $\sum \log(x_i)^2 = 175.6603$  log(days)<sup>2</sup>,  $\sum y_j^2 = 347.7$  Kg<sup>2</sup>,  $\sum x_i y_j = 3108.5$  days·Kg,  $\sum \log(x_i) y_j = 245.4738$  log(days)·Kg.

27. The concentration of a drug in blood, in mg/dl, depends on time, in hours, according to the data below.

Drug concentration	2	3	4	5	6	7	8
Hours	25	36	48	64	86	114	168

- Construct the linear regression model of drug concentration on time.
- Construct the exponential regression model of drug concentration on time.
- Use the best regression model to predict the drug concentration after 4.8 hours? Is this prediction reliable? Justify the answer.

Use the following sums ( $C$ =Drug concentration and  $T$ =time):  $\sum c_i = 35$  mg/dl,  $\sum \log(c_i) = 10.6046$  log(mg/dl),  $\sum t_j = 541$  hours,  $\sum \log(t_j) = 29.147$  log(hours),  $\sum c_i^2 = 203$  (mg/dl)<sup>2</sup>,  $\sum \log(c_i)^2 = 17.5206$  log(mg/dl)<sup>2</sup>,  $\sum t_j^2 = 56937$  hours<sup>2</sup>,  $\sum \log(t_j)^2 = 124.0131$  log(hours)<sup>2</sup>,  $\sum c_i t_j = 3328$  mg/dl·hours,  $\sum c_i \log(t_j) = 154.3387$  mg/dl·log(hours),  $\sum \log(c_i) t_j = 951.6961$  log(mg/dl)·hours,  $\sum \log(c_i) \log(t_j) = 46.08046$  log(mg/dl) · log(hours).

28. A researcher is studying the relation between the obesity and the response to pain. Te obesity is measured as the percentage over the ideal weight, and the response to pain as the nociceptive flexion pain threshold. The results of the study appears in the table below.

Obesity	89	90	75	30	51	75	62	45	90	20
Pain threshold	10	12	4	4.5	5.5	7	9	8	15	3

- According to the scatter plot, what model explains better the relation of the response to pain on the obesity?
- According to the best regression model, what is the response to pain expected for a person with an obesity of 50%? Is this prection reliable?
- According to the best regression model, what is the expected obesity for a person with a pain threshold of 10? Is this prediction reliable?

Use the following sums ( $X$ =Obesity and  $Y$ =Pain threshold):  $\sum x_i = 627$ ,  $\sum \log(x_i) = 40.3858$ ,  $\sum y_j = 78$ ,  $\sum \log(y_j) = 19.4119$ ,  $\sum x_i^2 = 45141$ ,  $\sum \log(x_i)^2 = 165.4516$ ,  $\sum y_j^2 = 738.5$ ,  $\sum \log(y_j)^2 = 40.0458$ ,  $\sum x_i y_j = 5538.5$ ,  $\sum x_i \log(y_j) = 1306.051$ ,  $\sum \log(x_i) y_j = 327.3887$ ,  $\sum \log(x_i) \log(y_j) = 80.1831$ .

### 3 Probability

29. Construct the sample space of the following random experiments:
- (a) Pick a random person and measure the gender and whether she or he is smoker or not.
  - (b) Pick a random person and measure the blood type and whether she or he is smoker or not.
  - (c) Pick a random person and measure the gender, the blood type and whether she or he is smoker or not.
30. There are two boxes with balls of different colors. The first box contains 3 white balls and 2 black balls, and the second one contains 2 blue balls, 1 red ball and 1 green ball. Construct the sample space of the following random experiments:
- (a) Pick a random ball from every box.
  - (b) Pick two random balls from every box.
31. The Morgan's laws state that given two events  $A$  and  $B$  from the same sample space,  $\overline{A \cup B} = \bar{A} \cap \bar{B}$  and  $\overline{A \cap B} = \bar{A} \cup \bar{B}$ . Proof both statements graphically using Venn diagrams.
32. Calculate the probability of the following events of the random experiment consisting in tossing 3 coins:
- (a) Get exactly 1 heads.
  - (b) Get exactly 2 tails.
  - (c) Get two or more heads.
  - (d) Get some tails.
33. In a laboratory there are 4 flasks with sulfuric acid and 2 with nitric acid, and in another laboratory there are 1 flask with sulfuric acid and 3 with nitric acid. A random experiment consist in picking two flask, one from every laboratory. Calculate the probability of the following events:
- (a) The two picked flasks are of sulfuric acid.
  - (b) The two picked flasks are of nitric acid.
  - (c) The tow picked flasks contains different acids.
- Calculate the same probabilities if the flask picked in the first laboratory is put in the second laboratory before picking the flask from it.
34. Let  $A$  and  $B$  be events of the same sample space, such that  $P(A) = 3/8$ ,  $P(B) = 1/2$ ,  $P(A \cap B) = 1/4$ . Calculate the following probabilities:
- (a)  $P(A \cup B)$ .
  - (b)  $P(\bar{A})$  y  $P(\bar{B})$ .
  - (c)  $P(\bar{A} \cap \bar{B})$ .
  - (d)  $P(A \cap \bar{B})$ .
  - (e)  $P(A|B)$ .
  - (f)  $P(A|\bar{B})$ .
35. In a hospital the probability of getting hepatitis in a blood transfusion from a unit of blood is 0.01. A patient gets two units of blood while staying at the hospital. What is the probability of getting hepatitis?
36. Let  $A$  and  $B$  be two events of the same sample space, such that  $P(A) = 0.6$  and  $P(A \cup B) = 0.9$ . Calculate  $P(B)$  with the following assumptions:
- (a)  $A$  and  $B$  are incompatible.
  - (b)  $A$  and  $B$  are independent.

37. A study about smoking has published that 40% of smokers have a smoker father, 25% have a smoker mother and 52% have at least one of the parents smoker. We pick a random person from this population. Answer the following questions:
- What is the probability of having a smoker mother if the father smokes?
  - What is the probability of having a smoker mother if the father doesn't smoke?
  - Are independent the events having a smoker father and having a smoker mother?
38. The probability that an injury  $A$  is repeated is  $4/5$ , the probability that another injury  $B$  is repeated is  $1/2$ , and the probability that both injuries are repeated is  $1/3$ . Calculate the probability of the following events:
- Only injury  $B$  is repeated.
  - At least one injury is repeated.
  - Injury  $B$  is repeated if injury  $A$  has been repeated.
  - Injury  $B$  is repeated if injury  $A$  hasn't been repeated.
39. In a digestive clinic from every 1000 patients that arrive with stomach pain, 700 have gastritis, 200 have an ulcer and 100 have cancer. After analyzing the gastric symptoms, it is known that the probability of having vomiting is 0.3 in case of gastritis, 0.6 in case of ulcer and 0.9 in case of cancer. What is the diagnosis for a new patient with stomach pain that has vomiting?
- Note: Assume that the only diseases are gastritis, ulcer and cancer and that are incompatible among them.
40. To evaluate the effectiveness of a diagnosis test, the test was applied to a sample of 6900 people with the following results:

	Test +	Test -
Sick	4680	120
Healthy	120	2020

Calculate for this test:

- The sensibility and the specificity.
  - The positive and negative predictive value.
  - The probability of a correct diagnosis.
41. We know, from a research study, that 10% of people over 50 years suffer a particular type of arthritis. We have developed a new method to detect the disease and after clinical trials we observe that if we apply the method to people with arthritis we get a positive result in 85% of cases, while if we apply the method to people without arthritis, we get a positive result in 4% of cases. Answer the following questions:
- What is the probability of getting a positive result after applying the method to a random person?
  - If the result of applying the method to one person has been positive, what is the probability of having arthritis?
42. A severe pain without effusion in a particular zone of the knee joint is a symptom of sprained lateral collateral ligament (SLCL). If the sprains in that ligament are classified into grade 1, when there is only distension (60% of cases), grade 2 when there is a partial tearing (30% of cases) and grade 3 when there is a complete tearing (10% of cases). Taking into account that the symptom appears in 80% of cases of grade 1 sprains, 90% of cases of grade 2 and 100% of cases of grade 3, answer the following questions:
- If a person has SLCL what is the probability that he or she present severe pain without effusion?
  - What is the diagnosis for a person with severe pain without effusion?
  - From a total of 10000 people with severe pain without effusion, how many are expected to have a grade 1 sprain? How many are expected to have a grade 2 sprain? And a grade 3 sprain?