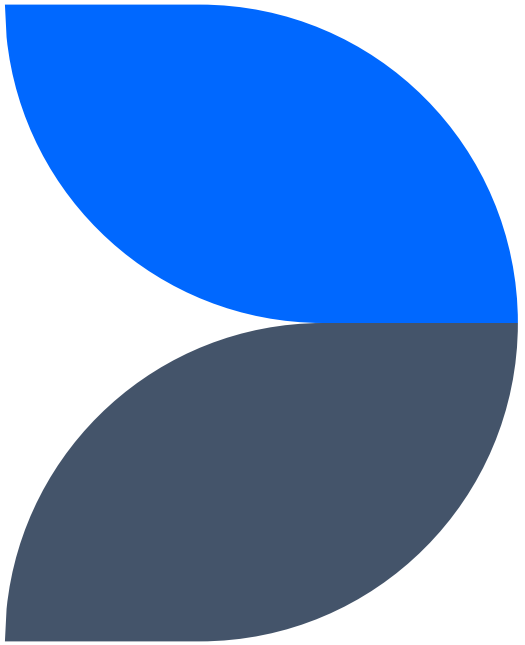
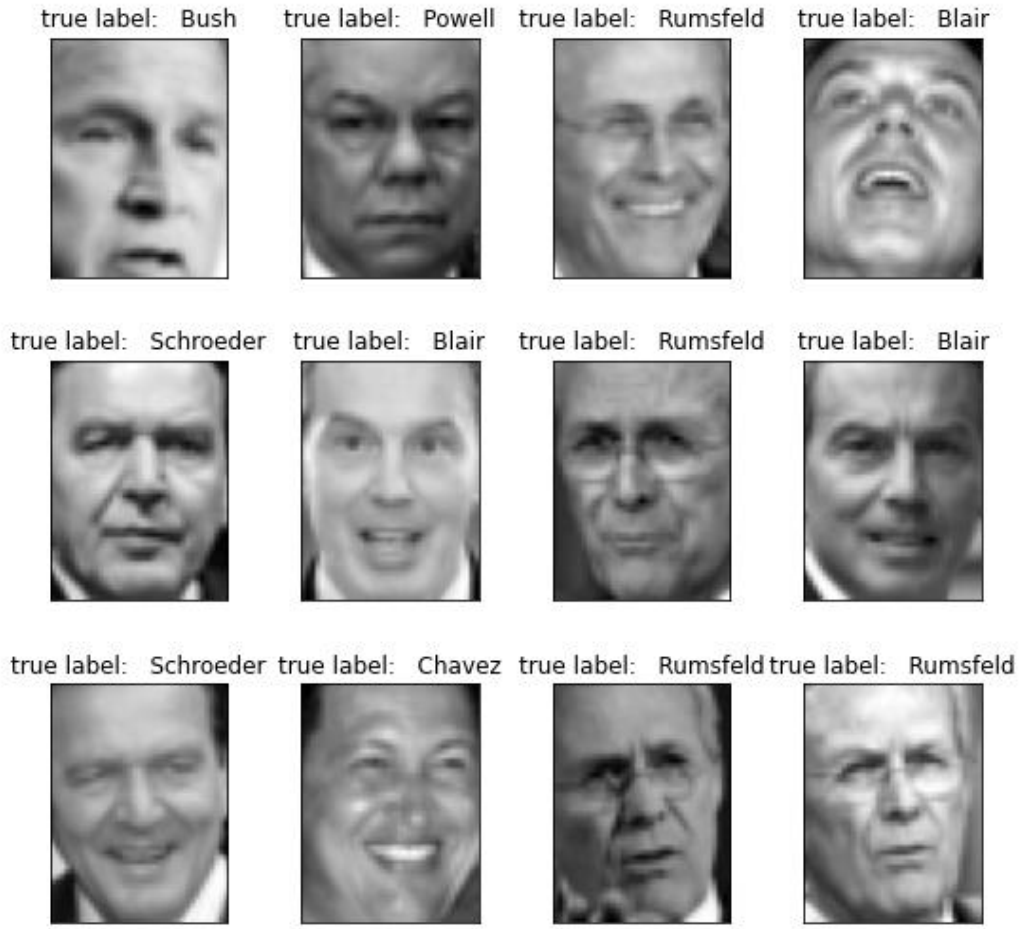
Face Recognition Using Eigenfaces
(PCA Algorithm)

Training Algorithm



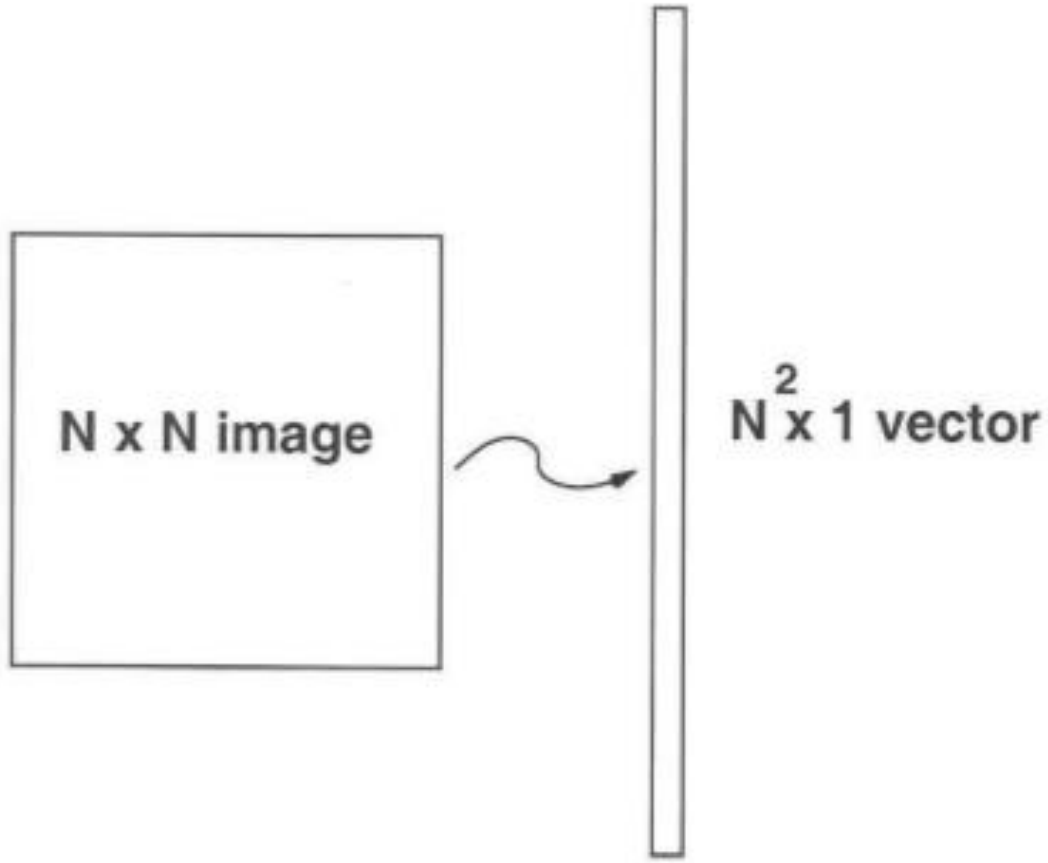
Let’s Consider a set of *m* images of dimension *N*N* (training images).



Training Image with True Label (people’s dataset)

We first convert these images into vectors of size N2 such that:

$$x_1, x_2, x_3, \ldots, x_m$$



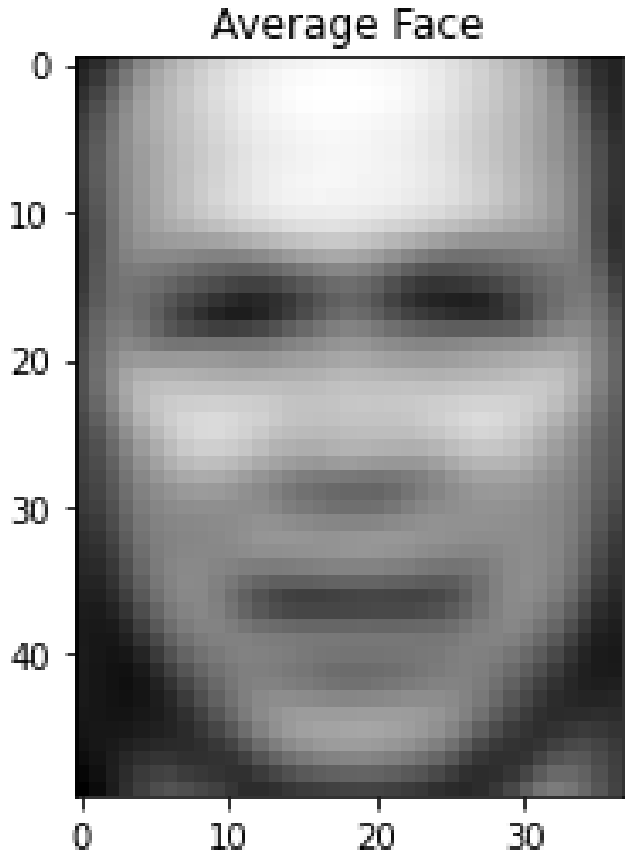
Now we calculate the average of all these face vectors and subtract it from each vector.

$$\psi = \frac{1}{m} \sum_{i=1}^{m} x_i$$

$$\mathfrak{a}_i = x_i - \psi$$

We take all face vectors so that we get a matrix of size of N2 * M.

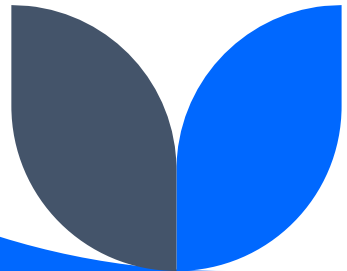
$$A = \begin{bmatrix} a_1 & a_2 & a_3 & .... & a_m \end{bmatrix}$$



Now, we find Covariance matrix by multiplying A with A^T.
A has dimensions $N^2 * M$, thus A^T has dimensions $M * N^2$.

We calculate our covariance matrix by **multiplying A^T and A**. This gives us $M * M$ matrix which has M *(assuming* **$M << N^2$***)* eigenvectors of size M.

$$Cov = A^T A$$



In this step we calculate eigen values and eigenvectors of covariance matrix.

$C' = AA^T \qquad u_i = A\nu_i$

$$A^T A\nu_i = \lambda_i \nu_i$$

$$AA^T A\nu_i = \lambda_i A\nu_i$$

$$C'u_i = \lambda_i u_i$$

C' and C have same eigenvalues and their eigenvectors are related by the equation: $u_i = A\nu_i$

Thus, the M eigenvalues (and eigenvectors) of covariance matrix gives the **M** largest eigenvalues(and eigenvectors) of C'.

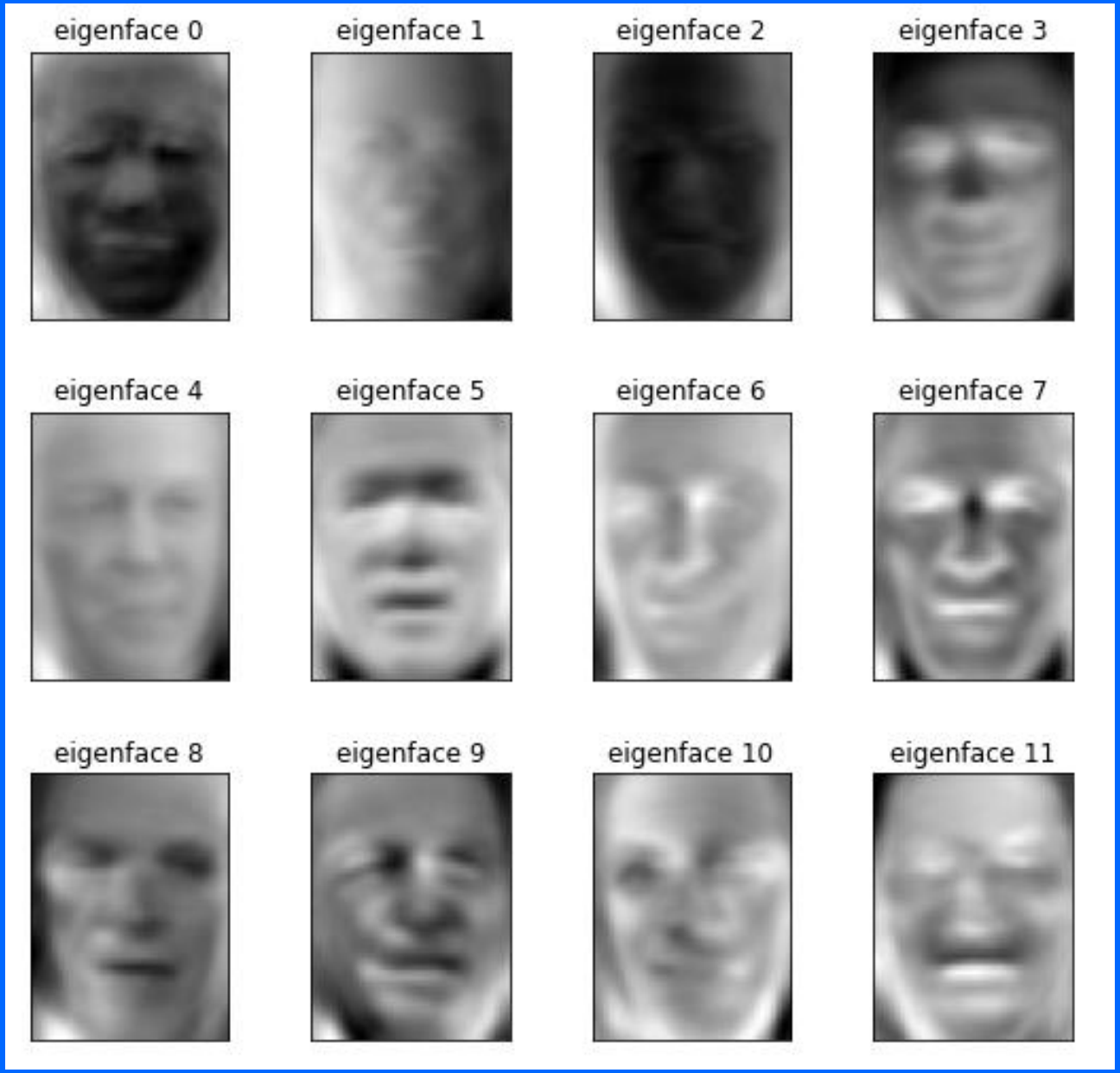
Now we calculate Eigenvector and Eigenvalues of this reduced covariance matrix and map them into the C' by using the formula $u_i = A\nu_i$

We select the K eigenvectors of C' corresponding to the K largest eigenvalues (where K < M). These eigenvectors has size N^2.

In this step we used the eigenvectors that we got in previous step. We take the normalized training faces (face – average face) x_i and represent each face vectors in the linear of combination of the best K.

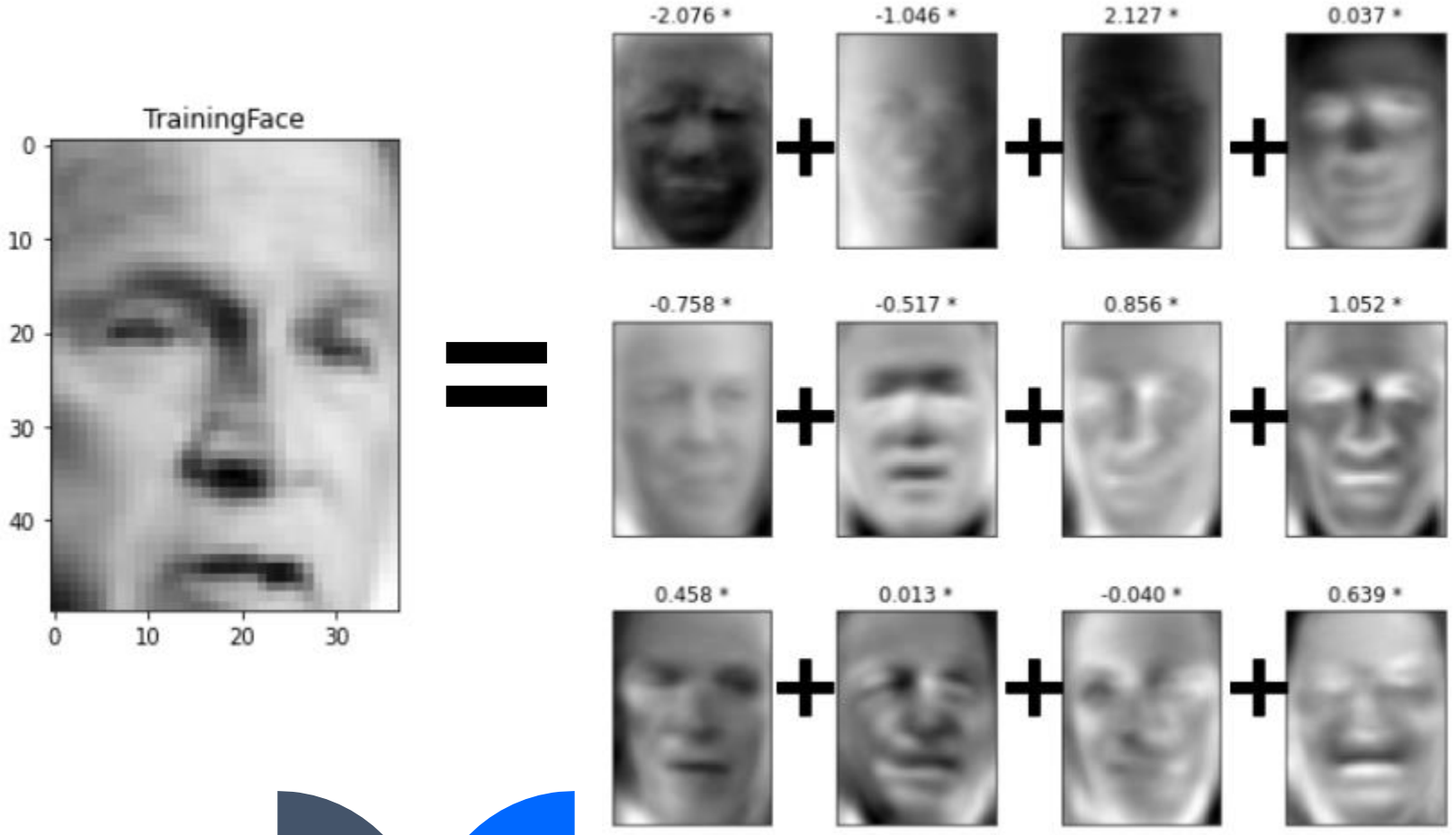
$$\mathrm{x}_i - \psi = \sum_{j=1}^{K} w_j u_j$$

These $\mathrm{u_j}$ are called **EigenFaces**.


eigenface 0
eigenface 1
eigenface 2
eigenface 3
eigenface 4
eigenface 5
eigenface 6
eigenface 7
eigenface 8
eigenface 9
eigenface 10
eigenface 11

EigenFaces

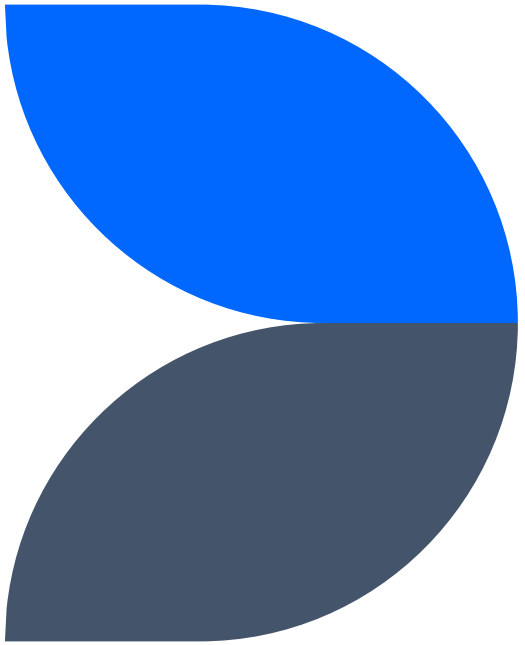
In this step, we take the coefficient of eigenfaces and represent the training faces in the form of a vector of those coefficients.

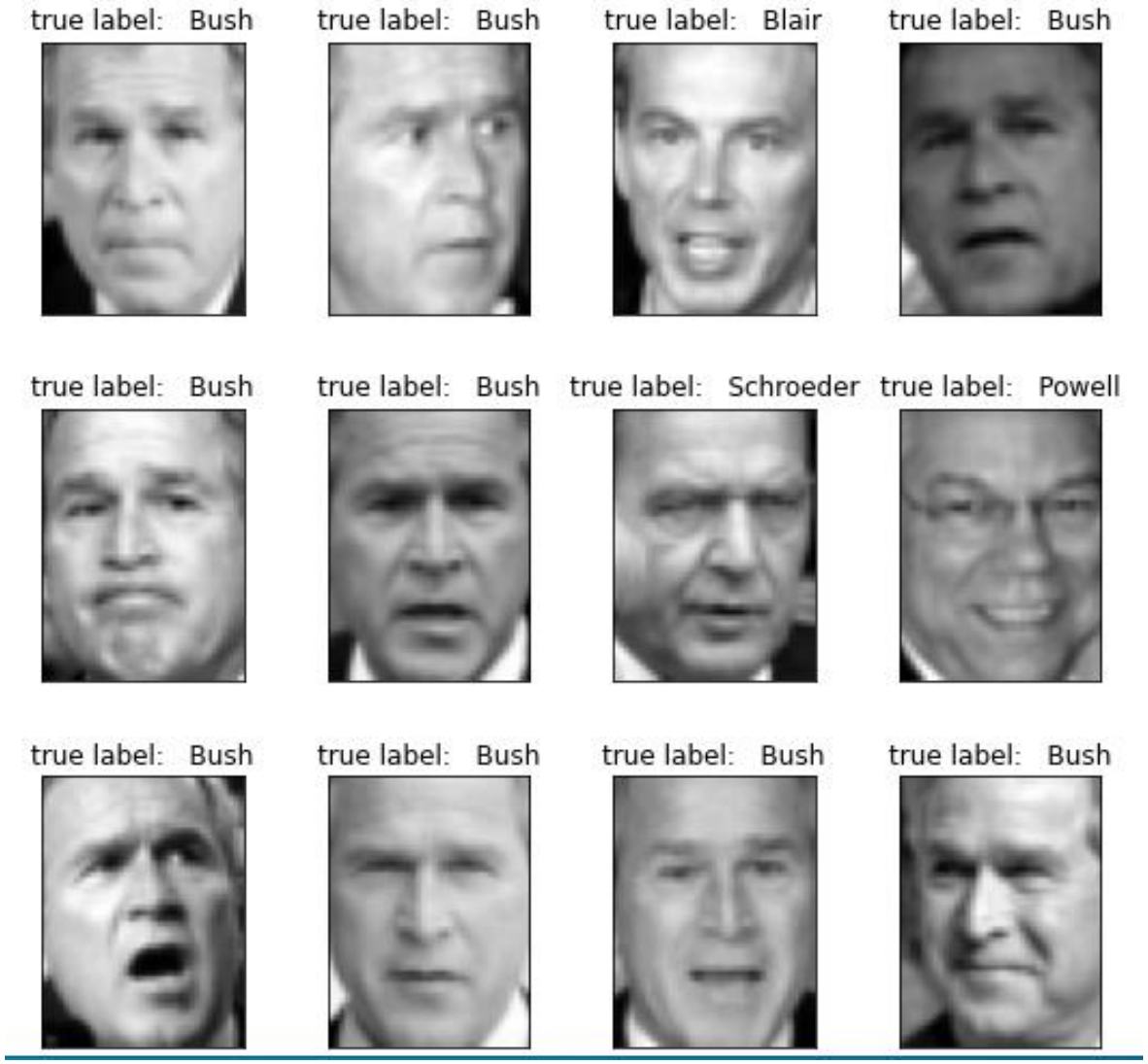


$$x_i = \begin{bmatrix} w_1^i \\ w_2^i \\ w_3^i \\ . \\ . \\ w_k^i \end{bmatrix}$$

Linear Combination of EigenFaces

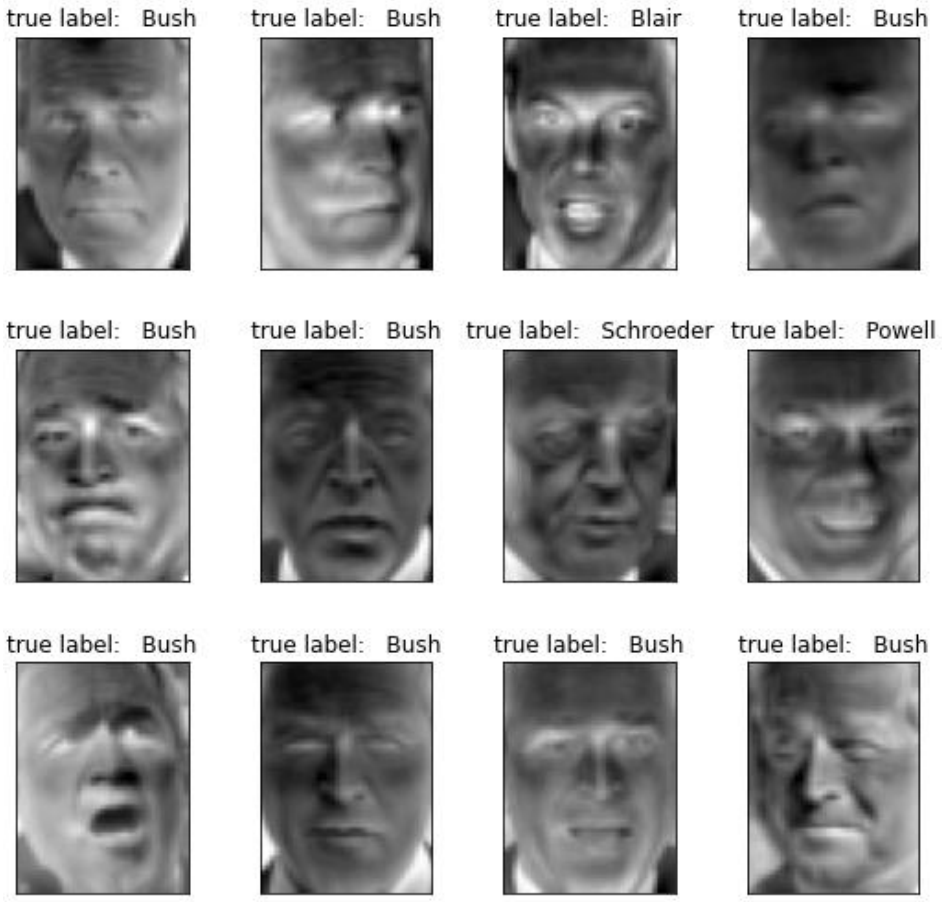
Testing Algorithm





Test Images
With true labels

Given an unknown face *y*, we need to first preprocess the face to make it centered in the image and have the same dimensions as the training face.
Now, we subtract the face from the average face ψ



$$\phi = y - \psi$$

Now, we project the normalized vector into eigenspace to obtain the linear combination of eigenfaces.

$$\phi = \sum_{i=1}^{k} w_i u_i$$

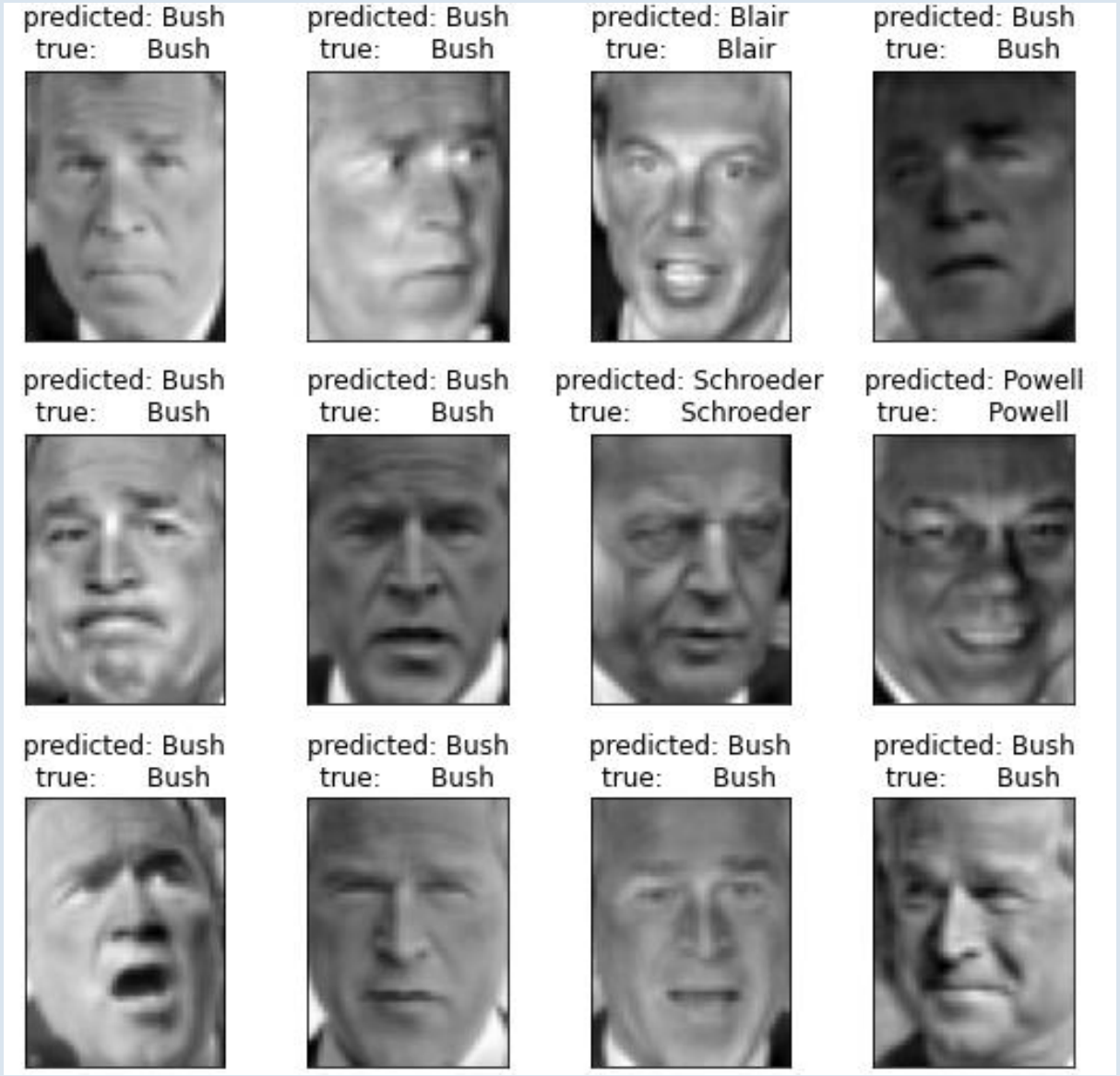
From the above projection, we generate the vector of the coefficient such that

$$\Omega = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ w_k \end{bmatrix}$$

We take the vector generated in the above step and subtract it from the training image to get the minimum distance between the training vectors and testing vectors.

$$e_r = min_l \, ||\Omega - \Omega_l||$$

If this e_r is below tolerance level T_r, then it is recognised with I face from training image else the face is not matched from any faces in training set.



Test images With prediction

Advantages:

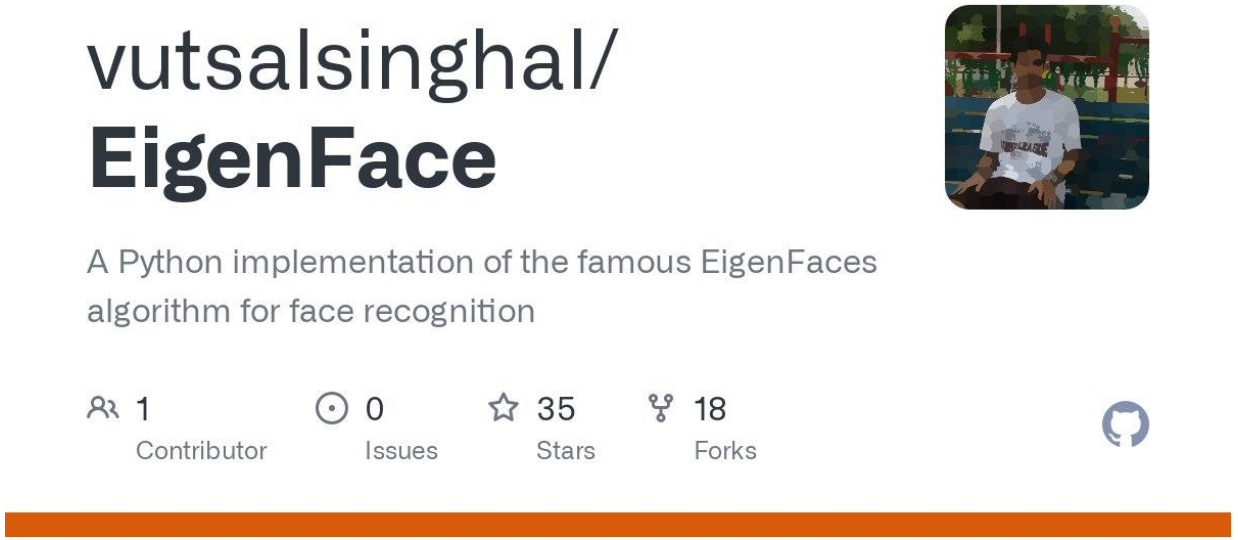
- Easy to implement and computationally less expensive.
- No knowledge (such as facial feature) of the image required (except id).

Limitations:

- Proper centered face is required for training/testing.
- The algorithm is sensitive to lightining, shadows and also scale of face in the image .
- Front view of the face is required for this algorithm to work properly.

Implementation

https://github.com/vutsalsinghal/EigenFace/blob/master/Face%20Recognition.ipynb



References

Matthew Turk and Alex Pentland. 1991. Eigenfaces for recognition. J. Cognitive Neuroscience 3, 1 (Winter 1991), 71–86. https://doi.org/10.1162/jocn.1991.3.1.71

https://www.geeksforgeeks.org/ml-face-recognition-using-eigenfaces-pca-algorithm/

https://www.youtube.com/

Thank you

Asal Mohammadjafari