

# Equilibrium Implications of Delegated Asset Management under Benchmarking\*

MARKUS LEIPPOLD and PHILIPPE ROHNER

*Swiss Banking Institute, University of Zurich*

**Abstract.** Despite the enormous growth of the asset management industry during the past decades, little is known about the asset pricing implications of investment intermediaries. Standard models of investment theory neither address the distinction between individual and institutional investors nor the potential implications of direct investing and delegated investing. In a model with endogenous delegation, the authors find that delegation leads to a more informative price system and lower equity premia. In the presence of relative return objectives, stocks exhibiting high correlations with the benchmark have significantly lower returns than stocks with low correlations. The authors' empirical results support the model's predictions.

*JEL Classification:* G11, G14, D83

## 1. Introduction

In this paper, we study the joint equilibrium implications of benchmarking and delegated portfolio management. The terms “delegated” and “delegation” refer to a situation in which the investor mandates a professional asset manager to make investment decisions on her behalf on a discretionary basis. Since the demand for risky assets of institutional investors differs from that of individuals, it raises the question of how asset prices are affected in equilibrium. In a model with two groups of investors, individual investors and professional asset managers, we provide

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a formal framework of delegation under benchmarking and we illustrate how endogenous delegation optimally arises in general equilibrium. Furthermore, we establish an expected return–beta relation to study the cross-sectional implications of portfolio management delegation and benchmarking. Our analysis highlights that delegation and benchmarking are two related issues that need to be examined simultaneously. Our empirical results provide evidence that both the growing importance of delegation and the practice of benchmarking have cross-sectional implications on stock returns. Both factors contribute to a decreasing equity risk premium. When controlling for size, we find that stocks with high institutional ownership (IO) exhibit lower returns than do stocks with low IO. Furthermore, we find evidence that holding non-benchmark stocks or stocks with a low correlation to the benchmark is rewarded with a premium.

Over the past decades, the asset management industry has faced rapid growth. Nowadays, a substantial part of wealth is invested through investment intermediaries such as banks, investment funds, pension funds, and other institutional investors. For example, back in 1952, individuals in the USA directly held over 90% of corporate equities. By 2008, this proportion was down to 36%. At the same time, the fraction of equities held by investment funds (including mutual, closed-end, and exchange-traded funds) rose from 2.9% to 22.4%. The share of equities held by pension funds grew from 0.9% to 19.6%.<sup>1</sup> This structural shift of investment discretion from private households to institutional investors may have implications for equilibrium asset prices and returns since the investment objectives of institutional investors and private investors differ in many respects. Along this line, Gompers and Metrick (2001) argue that the presence of institutional investors tends to increase the demand for large and liquid stocks, which leads to changes in equity risk premia. Furthermore, many institutional asset managers are bound to performance objectives relative to a benchmark portfolio, which increases the demand for those stocks included in the benchmark.

Despite the growing importance of institutional investors, the resulting asset pricing implications are poorly understood. Because traditional financial market theory is based on the representative investor paradigm, it does not consider the impact of financial institutions on asset pricing. Given the tremendous growth of the asset management industry, it is obvious that if we neglect the impact of portfolio delegation, then we may miss some important aspects of asset pricing.

One question we address in this paper is that of why investment decisions are delegated to professional asset managers in the first place. Apart from reasons related to market frictions and economies of scale, the main justification for the employment of asset managers is their investment skill and their superior capacity to observe and process information. Since professional asset management may be

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<sup>1</sup>) The Federal Reserve Board, [www.federalreserve.gov/releases/z1/](http://www.federalreserve.gov/releases/z1/).

seen as an indirect way of selling information,<sup>2</sup> we introduce information asymmetry between managers and investors as a potential trigger for delegation. We endow the investors with a certain amount of information capacity that allows them to observe a noisy but unbiased signal about the true risky asset payoff. In contrast, the managers face costly information capacity and need to decide how much additional capacity is optimal to acquire. Thus, we model information asymmetry in terms of different uncertainties about the risky assets' payoffs. This setup leads to a two-step investment process in which agents solve an information allocation problem in the first stage and a portfolio choice problem in the second stage. To keep our analysis straightforward, we do not consider conflicts of interest between the investor and the manager. Rather, we assume that the manager's investment strategy is fully transparent for the investor.

Another important issue we address is that of the role of benchmarking. Active managers are primarily concerned with attracting new money.<sup>3</sup> Given that the manager is compensated on the basis of total assets under management, which is generally the case for institutional investors such as mutual funds, the manager has an implicit incentive to focus on relative performance. Therefore, we model managers as relative return investors who try to outperform a passive benchmark portfolio such as the S&P 500 index. The composition of the benchmark affects the portfolio holdings of managers and the utility of the delegation, which in turn determines the number of investors who delegate their portfolio decision. At the same time, the number of delegating investors is a crucial determinant for the magnitude of equilibrium effects due to benchmarking. Therefore, to consistently model the impact of delegation in an equilibrium model, we need to address delegation and benchmarking simultaneously. We do so by modeling the manager as a strategic agent who takes into account the actions of the investor. Hence, there is a two-way relation between delegation and benchmarking.

For the general equilibrium analysis, we use the concept of a noisy rational expectations equilibrium as proposed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). In our model, which we develop along the lines of the multiple risky asset framework of Admati (1985), we show that the expected return of an asset is a linear function of the asset's covariance with the market portfolio and its covariance with the benchmark portfolio. In equilibrium, the risk premium is proportional to the covariance with a residual portfolio orthogonal to the benchmark. From the previous literature, it is well known that portfolio delegation leads to a more informative price system and a lower equity risk premium. However, due to the endogenous modeling of delegation, we can show that the equilibrium risk premium may decrease or increase with respect to the fraction of delegation depending

<sup>2</sup> See Admati and Pfleiderer (1990).

<sup>3</sup> See Gruber (1996), Chevalier and Ellison (1997), and Sirri and Tufano (1998).

on the underlying force driving the change in delegation. All these equilibrium effects are amplified by the fraction of delegating agents in the economy.

The paper is organized as follows. In Section 2, we briefly discuss related studies. In Section 3, we introduce the basic setup and discuss the sequence of events in the model. Section 4 presents the equilibrium implications of our model. We also derive an expected return–beta relation and analyze the cross-sectional implications of delegated portfolio management and benchmarking. In Section 5, we provide empirical evidence for the main predictions of the model. Section 6 concludes. All proofs are provided in the Appendix.

## 2. Background

Our paper is related to different streams of the delegated asset management literature. Brennan (1993) studies a static mean–variance model with two types of investors, private investors who hold the standard market portfolio and benchmark investors with performance objectives relative to a benchmark portfolio. He shows that equilibrium expected returns are characterized by a two-factor model, the two factors being the assets' covariance with market returns and the assets' covariance with the returns on the benchmark portfolio. Stutzer (2003) and Cornell and Roll (2005) perform a similar analysis. Brennan (1995), Gomez and Zapatero (2003), and Brennan and Li (2008) find empirical support for those types of models.

In the same vein, Cuoco and Kaniel (2010) examine the equilibrium impact of symmetric and asymmetric delegation contracts with relative performance objectives. These authors find that symmetric contracts tilt portfolio choices toward stocks that are part of the benchmark portfolio, but asymmetric contracts lead fund managers to choose portfolios that maximize tracking error. Ross (2005) demonstrates that the noisy rational expectations equilibrium in which investors have information of varying degrees of precision is unstable since better-informed agents may choose to sell their information by establishing asset management services for less-informed agents. In equilibrium, managers of a range of precisions can survive even when their precisions are known to be inferior because they first provide a mechanism for diversifying among funds and second charge different fees in line with their precisions. Ross also shows that markets will exhibit lower equity premia if dominated by institutional investors. Kapur and Timmermann (2005) study a similar problem but explicitly model the delegation decision process by introducing a decision rule based on an expected utility argument. In contrast to these static models, Vayanos and Wolley (2008) use a dynamic framework to analyze the equilibrium implications of fund flows and the flows' relation to fund performance in a multiple asset framework. In an economy with delegated asset management, the manager's demand for risky assets is determined by the information available and by fund flows among different managers and a risk-free asset.

The studies cited above impose delegation or the agents' information precision exogenously and hence are not able to explain why delegation occurs. In contrast, we explicitly model the agents' information choice problem, which allows the endogenous modeling of delegation. Similar to our paper, Garcia and Vanden (2009) study the endogenous formation of mutual funds. In a single-asset setting, they show that informed agents are always better off establishing mutual funds rather than using their private information to trade on their own accounts. Furthermore, they show that a sufficiently efficient mutual fund sector yields more informative prices and lower equity premia. Although some of our results are similar to those in Garcia and Vanden (2009), there are some fundamental differences between the two papers. While the managers in our model choose an optimal level of information capacity in order to maximize management fees, Garcia and Vanden first determine the fraction of informed and uninformed agents and then the group of informed agents sets management fees. This means that agents pay a fixed cost up front for a given degree of information precision, while our information acquisition stage is different in the sense that managers individually choose the optimal level of information capacity. Furthermore, the compensation contract in Garcia and Vanden (2009) is endogenously determined within a given class of contracts that are restricted to include proportional and fixed fees, while we assume a compensation contract relative to a benchmark portfolio. Another paper related to ours is that of Petajisto (2009), who demonstrates that the existence of active managers has implications on the cross-sectional pricing of assets. However, Petajisto does not explicitly specify the benchmark portfolio or the information allocation process.

Also related to our paper are the studies on relative performance objectives in asset management. Roll (1992) shows how relative performance objectives can be incorporated into the traditional mean–variance framework. He states that benchmarking is generally not mean–variance efficient, unless the benchmark corresponds to a portfolio on the efficient frontier. Other papers in this area include those by Basak, Shapiro, and Tepla (2006) and Basak, Pavlova, and Shapiro (2007, 2008). Garcia and Strobl (2009) and Gomez, Pristley, and Zapatero (2009) derive equilibrium implications of relative wealth concerns with a single risky asset and multiple risky assets, respectively. Both studies find that relative performance objectives lower equilibrium risk premia.

### 3. Basic Setup

To derive a noisy rational expectations equilibrium in an economy with endogenous delegation and benchmarking, we assume two types of agents, private investors (“investors”) and professional portfolio managers (“managers”). Investors face the decision problem of whether to delegate the management of their wealth to

a manager or to invest directly. We trigger delegation in our model by assuming that investors and managers have different posterior beliefs. Delegation results from the investors' quest for better information and can be rational when managers have superior private information. Once an investor has decided to delegate, the manager has the final decision about which assets to hold in the portfolio. Accordingly, if the investor uses direct investment, then the investor has the final decision about the portfolio composition.

### 3.1 ASSETS

The investment universe consists of  $N$  risky assets with price vector  $\mathbf{p} \in \mathcal{R}^N$  and one risk-free asset with a constant rate of return  $r$ . Final payoffs of the risky assets are captured by the normally distributed random vector  $\mathbf{f} \in \mathcal{R}^N$  with a mean  $\boldsymbol{\mu} \in \mathcal{R}^N$  and a covariance matrix  $\Sigma \in \mathcal{R}^{N \times N}$ . Asset payoffs are driven by  $L$  orthogonal risk factors or principal components. We decompose the covariance matrix  $\Sigma$  into a diagonal eigenvalue matrix  $\Lambda \in \mathcal{R}^{L \times L}$  and an eigenvector matrix  $\Gamma \in \mathcal{R}^{N \times L}$ ,

$$\Sigma = \Gamma \Lambda \Gamma'. \quad (1)$$

The eigenvector matrix  $\Gamma$  captures the factor loadings and may include both systematic risk factors and firm-specific risks. By  $\Gamma_l$  we denote the  $l$ -th column of  $\Gamma$  that contains the loading of each asset on the  $l$ -th risk factor. The  $i$ -th row of  $\Gamma$  captures asset  $i$ 's factor loadings on each factor. The eigenvalue matrix  $\Lambda = (\lambda_l)_{l=1, \dots, L}$  captures the variance of the  $l$ -th risk factor  $\lambda_l$  on its diagonal. Basically, we could reduce the dimensionality of the problem by considering only the  $L$ -th largest eigenvalues ( $L < N$ ). However, without loss of generality and for notational simplicity, we can assume that the number of orthogonal risk factors equals the number of assets, that is,  $L = N$ . Using this correlation structure, we can write the virtual payoff of a risk factor as a linear combination of asset payoffs. Hence, the vector of factor payoffs reads  $\Gamma' \mathbf{f}$ . We note that the correlation structure of assets  $\Gamma$  is constant and known to all agents. The variance of the risk factors  $\Lambda$  is what drives the uncertainty of asset payoffs.

### 3.2 PREFERENCES AND BELIEFS

Agents have some initial information about the distribution of  $\mathbf{f}$ . Both the investor and the manager start with the same set of prior beliefs

$$p(\mathbf{f} | \mathcal{F}_0) \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma). \quad (2)$$

We use  $\mathcal{F}_0$  to denote the set of prior information. In the subsequent period, all agents  $k$  observe a noisy but unbiased signal  $\Gamma' s_k$  about the true realization of factor payoffs  $\Gamma' \mathbf{f}$ ,

$$\Gamma' s_k := \Gamma' f + \eta_k, \quad (3)$$

where  $\eta_k \sim \mathcal{N}(0, V_k)$ ,  $s_k \in \mathcal{R}^N$  is the vector of signals about asset payoffs, and  $k \in \{j, m\}$  denotes investor  $j$  and manager  $m$ , respectively.  $V_k$  is a covariance matrix that determines the precision of the signal. We assume that  $V_k$  is diagonal. Therefore, the signal about one risk factor does not contain information about another risk factor. Since factor loadings are known,  $L$  signals about risk factors translate into  $N$  signals about asset payoffs. Therefore, we define the covariance matrix of the signals about assets as  $\Sigma_{s,k} := \Gamma V_k \Gamma'$ .

In a general equilibrium setting, the price is an additional source of information. By observing the equilibrium price, agents can infer part of other agents' private information. The price signal is normally distributed,  $f|p \sim \mathcal{N}(\mu_p, \Sigma_p)$ , where  $\Sigma_p := \Gamma V_p \Gamma'$ . Agents combine the prior belief, the private signal, and the price signal using Bayes' theorem. The posterior belief of agent  $k$  about the realization of the asset payoffs is

$$p_k(f|\mathcal{F}_k) \sim \mathcal{N}(\hat{\mu}_k, \hat{\Sigma}_k), \quad (4)$$

with posterior mean

$$\hat{\mu}_k := (\Sigma^{-1} + \Gamma V_k^{-1} \Gamma' + \Gamma V_p^{-1} \Gamma')^{-1} (\Sigma^{-1} \mu + \Gamma V_k^{-1} \Gamma' s_k + \Gamma V_p^{-1} \Gamma' \mu_p) \quad (5)$$

and posterior variance

$$\hat{\Sigma}_k := (\Sigma^{-1} + \Gamma V_k^{-1} \Gamma' + \Gamma V_p^{-1} \Gamma')^{-1}, \quad (6)$$

where  $\mathcal{F}_k$  is the information set conditional on observing the private signal. Given that  $V_k$  and  $V_p$  are positive semidefinite, the posterior variance is always lower than (or equal to) the prior variance, reflecting the fact that the signals generates additional information.

Following Van Nieuwerburgh and Veldkamp (2010), we define utility in terms of posterior beliefs. For the investors, we assume a standard mean–variance utility  $U_j$  with a constant absolute risk-aversion parameter  $\rho_j$ , that is, for a given arbitrary set of beliefs  $(\tilde{\mu}_j, \tilde{\Sigma}_j)$ , we define

$$U_j(\tilde{\mu}_j, \tilde{\Sigma}_j) = q_j'(\tilde{\mu}_j - pr) - \frac{1}{2} \rho_j q_j' \tilde{\Sigma}_j q_j, \quad (7)$$

where  $q_j$  is the vector of portfolio weights. As we explain later,  $q_j$  takes different values, depending on whether the investor delegates or not.

The managers derive utility from relative return between portfolio and benchmark minus the tracking error of the portfolio, which we define as follows:

$$U_m(\phi, \tilde{\mu}_m, \tilde{\Sigma}_m) = \underbrace{(\mathbf{q}_{m,\phi} - \phi)'(\tilde{\mu}_m - \mathbf{p}r)}_{\text{relative return}} - \frac{1}{2} \underbrace{\rho_m (\mathbf{q}_{m,\phi} - \phi)' \tilde{\Sigma}_m (\mathbf{q}_{m,\phi} - \phi)}_{\text{tracking error}}, \quad (8)$$

where  $\rho_m$  is the manager's aversion to the risk of deviating from the benchmark and  $\mathbf{q}_{m,\phi}$  is the manager's portfolio given the performance objective relative to the benchmark portfolio  $\phi \in \mathcal{R}^N$ . The benchmark portfolio  $\phi$  has the same dimension as  $\mathbf{q}_{m,\phi}$ , but not necessarily all entries of the benchmark weight vector must be nonzero. Note that  $\rho_j$  measures the investor's aversion to absolute risk.  $\rho_m$  measures the manager's aversion of deviating from the benchmark and is often referred to as "regret aversion."<sup>4</sup> Equation (8) implies that the manager can increase utility by holding non-benchmark assets with greater expected returns than the benchmark assets when accepting a certain level of tracking error risk. The manager's optimal degree of deviation from the benchmark is determined by the posterior covariance matrix  $\tilde{\Sigma}_m$ . The higher the manager's confidence in his beliefs, that is, the smaller posterior variances, the higher are his active positions. However, the manager maintains active positions only if the risk of deviating from the benchmark is rewarded with a premium. This feature of the model may, in part, explain some of the recent findings that managers that deviate more from the benchmark or have larger active positions tend to perform better (see Cremers and Petajisto, 2009). The choice of the benchmark and hence the form of the delegation contract is exogenously given. Active managers are primarily concerned about attracting new money. Empirical evidence suggests that money tends to flow into an investment fund if it performs well relative to a benchmark, see, for example, Gruber (1996), Chevalier and Ellison (1997), Admati and Pfleiderer (1997), and Sirri and Tufano (1998). The fact that the manager is compensated on the basis of assets under management (which is generally the case for mutual funds) gives the manager an implicit incentive to focus on relative performance.

### 3.3 INFORMATION ACQUISITION

Agents must not only solve for the optimal portfolio weights, but first, they must decide which assets they want to learn about prior to observing the signal. We refer to this phase as the "information acquisition" stage.

Borrowing from Van Nieuwerburgh and Veldkamp (2009), we simplify the information acquisition problem by letting agents learn about orthogonal risk factors

<sup>4</sup> We note that while Equation (8) is essential to study the cross-sectional implications of benchmarking, it is not crucial for modeling endogenous delegation. Instead, manager utility can also be defined in terms of absolute returns.



or principal components. The variance of the signal of risk factors is  $V_k = (v_{l,k})_{l=1, \dots, L}$  and serves as the variable of choice in agent  $k$ 's information acquisition problem. Information acquisition refers to the choice of optimal signal variance  $V_k$  that helps to reduce uncertainty about the risk factors and the realization of final payoffs. With  $V_k$  being positive semidefinite, we implicitly assume that agents cannot forget what they already know about factors. We then write the eigendecomposition for the posterior covariance matrix as  $\hat{\Sigma} = \Gamma \hat{\Lambda} \Gamma$ , where  $\hat{\Lambda}$  captures the posterior variance of the risk factors. Since the prior variance  $\Lambda$  is given, the choice of a specific signal variance  $V_k$  is associated with a unique posterior variance  $\hat{\Lambda}$ . Therefore, we can formulate the agent's information allocation problem for the choice of optimal posterior variance  $(\hat{\lambda}_{k,l})_{l=1, \dots, L}$ .

To operationalize the process in terms of how agents acquire information, we introduce two additional concepts, information capacity and the cost of information. First, to become informed, agents need information capacity, which we measure as the increase in total signal precision. We define information capacity  $K_k$  for agent  $k$  as the sum of the differences between the posterior and the prior precision of each risk factor,

$$K_k := \sum_{l=1}^L (\hat{\lambda}_{k,l}^{-1} - \lambda_l^{-1}), \quad k \in \{j, m\}. \quad (9)$$

For investor  $j$ , we assume a fixed amount of information capacity  $K_j$ . We argue that portfolio management is neither the investor's profession nor is it her main activity. Hence, the private investor needs only to decide how to optimally allocate the available capacity among the different assets; she does not have to decide how much information capacity to acquire. In contrast, the manager can acquire further information by increasing his information capacity, for example, by hiring new agents. However, information capacity comes at a cost. Each manager faces a trade-off between the monetary cost of capacity and the benefits of more accurate information.<sup>5</sup> We define the monetary cost of capacity thus as

$$C_{K_m} := c \sum_{l=1}^L \left( \hat{\lambda}_{m,l}^{-1} - \lambda_l^{-1} \right)^\psi, \quad (10)$$

with constants  $c > 0$  and  $\psi \geq 1$ . The constant  $c$  translates capacity expressed in terms of differences between posterior and prior precision into monetary

<sup>5</sup> Alternatively, one could assume that the investor faces costly (but unlimited) information capacity, where the information cost for the investor is higher than the information cost for the manager. While such a setup further complicates the modeling of endogenous delegation, the qualitative predictions of the model do not change.

costs and  $\psi$  specifies the curvature of the cost function. For  $\psi = 1$ ,  $C_{K_m}$  is linear in capacity. For values  $\psi > 1$ , the marginal cost for information is increasing.

We especially note two properties of Equation (10). First, no costs occur if the agent does not receive any private information, that is, information capacity is zero. From Equation (6) and the specification of the correlation structure, we know that  $\hat{\Lambda}_m = \Lambda_m$  if  $(\lambda_{\eta,l}^m)^{-1} = 0, \forall l$ . Second, if  $\psi > 1$ , the cost function ensures that the signal can never reveal the true asset payoffs because  $\lim_{\hat{\lambda}_{m,l} \rightarrow 0} c(\hat{\lambda}_{m,l}^{-1} - \lambda_l)^\psi = \infty$ .

In addition, due to the decreasing returns of information acquisition implied by  $\psi > 1$ , the convexity of the cost function makes it unattractive for the manager to acquire too much information about one single risk factor.

### 3.4 SEQUENCE OF EVENTS

Our framework is a static model with four time periods. At the information acquisition stage, the manager and the investor solve the information acquisition problem. The investor maximizes expected utility of final wealth, while the manager maximizes management fees. The manager acquires and allocates information with full anticipation of the investor's response in Period 2, but the investor chooses information independently. In the second period, the investor learns the manager's ability by observing his posterior variance. Based on this information, she decides whether to delegate. We refer to this period as the delegation stage of the model. In the first two stages of the model, we have a strategic Stackelberg game between the manager and the investor in which the manager moves first (Period 1) and the investor moves second (Period 2). In the third period, which we refer to as the trading stage, optimal asset demand is established either through the manager in case of delegation or through the investor in case of direct investment. Terminal wealth is realized in Period 4.

Figure 1 summarizes the sequence of events. The distinction between the first two periods is for illustrative purposes only and merely shows the chronology of information allocation and the delegation decision. However, since the signal is received in the third period, there is no difference between the first two periods in terms of information sets. In our setting, since the delegation contract is exogenously given and cannot be chosen by the investors, as, for example, in Kapur and Timmermann (2005), endogenous delegation rests on informational differences among agents. This property of the model makes the formulation of an information acquisition problem at the first stage necessary to induce endogeneity in the delegation decision.

The manager's posterior covariance matrix  $\hat{\Sigma}_m$  is a key variable in the model that indicates managerial ability and determines the investor's delegation decision. By

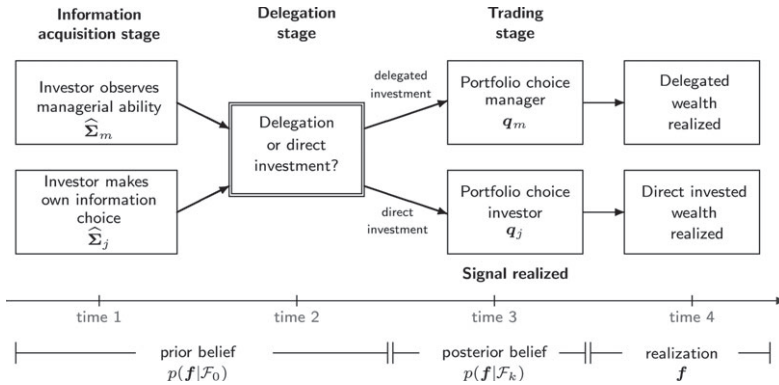


Figure 1. Sequence of events in the model with endogenous delegation. The chart shows the three steps of the investment process for the individual investor with (1) information choice, (2) delegation choice, and (3) portfolio choice. It also exhibits the different information sets that investors have. Before the signal is observed at the third stage, investors take the information acquisition and delegation decisions under the prior belief  $p(f|\mathcal{F}_0)$ . The portfolio choice is done under the posterior belief  $p(f|\mathcal{F}_k)$  that is unique to each investor  $k$ . The explicit formulation of the information acquisition problem at the first stage is necessary to induce endogenous delegation.

comparing the manager's posterior variance, the investor can assess the quality of the manager's information. Furthermore, since we can decompose the covariance matrix into a diagonal matrix of risk factor variances, the posterior covariance matrix reveals how much capacity the manager devotes to learning about specific risk factors and is thus also a measure of relative learning. Therefore, the manager's posterior covariance matrix determines both his total information capacity and the allocation of capacity among assets. We assert that managerial ability is determined by total information capacity and optimal information allocation.

### 3.5 SOLVING THE MODEL

We solve the model by using backward induction. First, we solve the portfolio choice problem at the trading stage. Given an arbitrary set of posterior beliefs  $\mathcal{F}_k = (\hat{\mu}_k, \hat{\Sigma}_k)$ , agents maximize their expected mean–variance utility in Equations (7) and (8), respectively, given their set of information:

$$\max_{q_k} \mathbb{E}[U_k | \mathcal{F}_k]. \quad (11)$$

At the delegation stage, the investor decides whether to delegate or not. To operationalize the delegation problem, we establish a decision rule for optimal delegation that we base on a certainty equivalent argument. In the context of portfolio management delegation, the certainty equivalent is the maximum fee an investor is willing to pay to get access to the manager's private

information. We will refer to this measure as the certainty equivalent of delegation (CED) and define it as

$$\text{CED}_j := \sup \left[ \xi | \mathbb{E}[U_{\text{dir}}^j(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j) | \mathcal{F}_0] \leq \mathbb{E}[U_{\text{del}}^j(\hat{\boldsymbol{\mu}}_m - \boldsymbol{\delta}, \hat{\boldsymbol{\Sigma}}_m) | \mathcal{F}_0] \right], \quad (12)$$

where  $\mathbb{E}[U_{\text{dir}}^j(\hat{\boldsymbol{\mu}}_j, \hat{\boldsymbol{\Sigma}}_j) | \mathcal{F}_0]$  is the investor's expected utility from direct investment and the expected utility from delegation in the second period, given the belief of the manager is  $\mathbb{E}[U_{\text{del}}^j(\hat{\boldsymbol{\mu}}_m, \hat{\boldsymbol{\Sigma}}_m) | \mathcal{F}_0]$ . The definition in Equation (12) allows a direct comparison with the fee effectively charged for active management. A rational investor delegates investment decisions to the portfolio manager if and only if the fee for active management  $\alpha_j$  is smaller (or equal) than her CED.

The posterior covariance matrices of investor  $j$  and the manager  $m$  are determined at the information acquisition stage. The investor chooses information independently of the manager's action. In contrast, the manager acts strategically and anticipates the investor's information choice at the information acquisition stage. Given the mean-variance utility in Equation (7) and the optimal portfolio choice  $\mathbf{q}_j$  from Period 3, we can write the investor's problem in the first period as

$$\max_{\hat{\Lambda}_j} \mathbb{E} \left[ \mathbf{q}_j' (\hat{\boldsymbol{\mu}}_j - \mathbf{p}r) - \frac{1}{2} \mathbf{p}_j \mathbf{q}_j' \hat{\boldsymbol{\Sigma}}_j \mathbf{q}_j | \mathcal{F}_0 \right], \quad (13)$$

subject to the capacity constraint

$$\sum_{l=1}^L (\hat{\lambda}_{j,l}^{-1} - \lambda_l^{-1}) \leq K_j \quad (14)$$

and the nonnegative learning constraint

$$\hat{\lambda}_{j,l}^{-1} \geq \lambda_l^{-1}, \quad \forall l. \quad (15)$$

The capacity constraint (14) limits the signal precision to a certain level of information capacity  $K_j$  and ensures that the investor cannot reduce her posterior variance beyond that level. The nonnegative learning constraint (15) prevents the investor from forgetting what she already knows at the expense of a more precise signal about another risk factor.

For the manager, information capacity  $K_m$  is costly, but he can acquire an arbitrary level of it. He faces a cost-benefit trade-off between the cost  $C_{K_m}$  of becoming informed and the management fees  $\alpha_j$  earned from delegation. We assume that the manager anticipates the investor's CED. Since the CED is the maximum fee the investor is willing to pay, the manager chooses information capacity such that the CED is maximized, given the constraint on information costs. Hence, the manager solves

$$\max_{\Lambda_m} \mathbb{E}[\text{CED}_j - C_{K_m} | \mathcal{F}_0], \quad \text{subject to } \hat{\lambda}_{m,l}^{-1} \geq \lambda_l^{-1}, \quad \forall l. \quad (16)$$

As long as the investor's certainty equivalent and the manager's net income are positive, both agents are better off with delegation. However, when information costs exceed the income generated by fees, the manager incurs a negative net income and has no incentive to open an investment fund. Since we assume that the manager's reservation utility is zero, a natural participation constraint for the manager is  $\alpha_j = \text{CED}_j \geq C(K_m)$ . To open an investment fund, the manager requires that at least the information costs be covered by management fees. When  $\text{CED}_j$  is negative, the manager's information costs are not covered and the manager does not engage in information acquisition.

It is important to note that managers act as local monopolists and thus do not face competition from other managers. We assume that each investor is associated exactly with one manager. The outcome of problem (16) for manager  $m$  is thus not affected by the fees  $\alpha_h$  set by other managers for their investors  $h \neq j$ .

#### 4. Equilibrium Impact of Delegation under Benchmarking

In this section, we derive the general equilibrium implications of delegation under benchmarking.<sup>6</sup> We denote by  $n$  the fraction of delegating investors. If  $j \in [0, n]$ , then investor  $j$  delegates the portfolio allocation. If  $j \in [n, 1]$ , then the investor invests directly in the market. The fraction of delegating investors is determined endogenously in equilibrium by a certainty equivalent rule. We assume that each investor is associated with one single manager. Each manager charges an individual fee  $\alpha_j$ . Asset prices  $\mathbf{p}$  are determined by market clearing. The *per capita* risky asset supply is  $\bar{\mathbf{x}} + \mathbf{x}$ , with  $\mathbf{x} \sim \mathcal{N}(0, \sigma_x^2 \mathbf{I})$ . This extra source of randomness can be, for example, due to liquidity traders and prevents the price system from fully revealing all private information. We characterize the equilibrium by splitting it into three stages: the trading stage, the delegation stage, and the information acquisition stage.

##### 4.1 EQUILIBRIUM AT THE TRADING STAGE

*At the trading stage, the equilibrium is defined as follows.*

**Definition 1.** *For a given fraction of delegation  $n \in [0, 1]$ , posterior variances of delegating investors  $\hat{\Sigma}_m^*$ ,  $j \in [0, n]$ , and direct investors  $\hat{\Sigma}_j$ ,  $j \in (n, 1]$ , a rational*

<sup>6</sup> We provide an extensive partial equilibrium analysis of our model in a technical supplement, which can be obtained by the authors on request.

expectations equilibrium at the trading stage is a collection of manager demands  $\mathbf{q}_{m,\phi}$ , a collection of investor demands  $\mathbf{q}_j$ , and a vector of risky asset prices  $\mathbf{p}$  such that:

- (a)  $\mathbf{q}_{m,\phi}$  solves Equation (11) for manager  $m$ ;
- (b)  $\mathbf{q}_j$  solves Equation (11) for investor  $j$ ;
- (c) asset demand equals supply such that the market clears:

$$\int_0^n \mathbf{q}_{\text{del}}^j dj + \int_n^1 \mathbf{q}_{\text{dir}}^j dj = \bar{\mathbf{x}} + \mathbf{x}.$$

#### Equilibrium prices

At the trading stage, we take the fraction of delegation  $n$  as well as the posterior variance  $\widehat{\Sigma}_k$  as given. In the following proposition, we derive the equilibrium price function in the presence of delegated portfolio management with benchmarking.

**Proposition 1.** *In the presence of delegation and benchmarking, the equilibrium price is a function of the “average” investor’s posterior mean and the covariance with the residual portfolio, where the residual portfolio is the difference between the market portfolio and the benchmark portfolio,*

$$\mathbf{p} = \frac{1}{r} \left( \widehat{\boldsymbol{\mu}}_a - \boldsymbol{\rho}_a \widehat{\Sigma}_a (\mathbf{x}_{\text{mkt}} - n\boldsymbol{\phi}) \right). \quad (17)$$

We present the expressions for  $\widehat{\boldsymbol{\mu}}_a$ ,  $\widehat{\Sigma}_a$ , and  $\boldsymbol{\rho}_a$  in the Appendix. We define the market portfolio as  $\mathbf{x}_{\text{mkt}} := \bar{\mathbf{x}} + \mathbf{x}$ .

Proposition 1 shows that the beliefs of the average investor are driven by both the individual investors and the managers. The higher the number of delegating investors, the more the average investor reflects manager beliefs and vice versa. As long as there is no benchmark ( $\phi_i = 0$ ,  $\forall i$ ), prices are determined by their expected payoff minus their covariances with the market portfolio. This result corresponds to the classic result of the capital asset pricing model (CAPM). When there is a benchmark, asset prices are determined by their covariance with the residual portfolio, that is, the difference between the market portfolio and the weighted benchmark portfolio. Therefore, benchmarking leads to a positive shift in asset demand for those assets included in the benchmark portfolio. A high weight of an asset in the benchmark creates a steady demand for this asset, independent of any beliefs. A high positive weight in the benchmark portfolio decreases the residual portfolio and leads to a higher price. This effect increases with increasing delegation  $n$ .

In our economy, agents act based on their individual signals and the information disclosed by equilibrium prices, but they do not observe the true market portfolio, due to the additional noise in total supply,  $\mathbf{x}$ . Therefore, in our framework, agents cannot observe the market portfolio *ex ante*. Biais, Bossaerts, and Spatt (2010) note that such a setup conforms with the critique of Roll (1977), who emphasizes that generally the market portfolio is not observable. Following Hellwig (1980), we can define the informational content of prices as the inverse of the conditional variance of asset payoffs absent any private information,

$$\text{Var}(\mathbf{f}|\mathbf{p})^{-1} = \Sigma^{-1} + \frac{1}{\rho_a^2 \sigma_x^2} \Psi \Psi'. \quad (18)$$

Equation (18) is the inverse of the posterior variance of an agent who has sufficient information capacity to observe the price signal but no capacity left to observe a private signal.  $\Psi$  is the signal precision of the average investor. From Equation (18), we also see that price informativeness is strictly increasing in  $\Psi$ . Since delegation takes place only if the manager has better information,  $\Psi$  is strictly increasing in  $n$  and  $\partial \Psi / \partial n > 0$ . Therefore, we have  $\partial \text{Var}(\mathbf{f}|\mathbf{p})^{-1} / \partial n > 0$  and we can state the following proposition.

**Proposition 2.** *Delegation leads to a more informative price system.*

We now analyze the cross-sectional implications of delegation. Rearranging Equation (17), we write the expected dollar excess return from the viewpoint of the average investor as

$$\mathbb{E}[\mathbf{f} - \mathbf{p}r | \mathcal{F}_a] = \rho_a \left( \widehat{\Sigma}_a \bar{\mathbf{x}} - n \widehat{\Sigma}_a \boldsymbol{\phi} \right). \quad (19)$$

In Equation (19), we use  $\mathcal{F}_a$  to indicate the information set of the average investor. The average information set across all agents cannot be observed. It deserves emphasis that  $\mathcal{F}_a$  is an average quantity and is not equal to the union of the information sets of all agents. Equation (19) expresses the expected excess return on an asset as a linear function of the asset's covariance with the portfolio  $\bar{\mathbf{x}}$  and its covariance with the benchmark portfolio  $\boldsymbol{\phi}$ . We note that the expected return is increasing in the covariance with the market portfolio and decreasing with the benchmark portfolio. The higher the asset's correlation with the benchmark, the stronger is this effect. The negative impact due to benchmarking is driven, first, by the asset's weight in the benchmark portfolio and, second, by the fraction of delegating investors  $n$ . When  $n = 0$ , Equation (19) collapses to the standard CAPM. Furthermore, the equilibrium expected return is increasing with the risk aversion  $\rho_a$  of the average investor.

From the viewpoint of the benchmarked manager, the benchmark portfolio serves as his risk-free asset. Therefore, the manager does not demand a premium for systematic risk included in the benchmark portfolio. However, holding some assets in proportions other than those provided in the benchmark, the manager would require a risk premium for the fraction that deviates from the benchmark weight. The higher the fraction of benchmarked managers in an economy, that is, the higher is  $n$ , the stronger this effect will be. Thus, benchmarking directly reduces the risk that must be borne and reflected in security prices. As a consequence, benchmarking lowers the expected returns of assets included in the benchmark portfolio.<sup>7</sup> We summarize this finding in Proposition 3.

**Proposition 3.** *In an economy with delegation under benchmarking, only the risk arising of deviating from the benchmark is priced in equilibrium and it is priced proportional to the covariance with the residual portfolio  $\hat{\Sigma}_a(\bar{\mathbf{x}} - n\boldsymbol{\phi})$ .*

*A two-factor CAPM*

Based on the equilibrium expected return given in Equation (19), we derive an expected return–beta relation similar as in the standard CAPM. For a single asset  $i$  with expected excess return  $R_i$ , we have

$$\begin{aligned}\mathbb{E}[f_i - p_i r | \mathcal{F}_a] &= \rho_a \sum_{k=1}^N \text{Cov}(f_i, f_k | \mathcal{F}_a) \bar{x}^k - \rho_a n \sum_{k=1}^N \text{Cov}(f_i, f_k | \mathcal{F}_a) \phi^k \\ &=: \mathbb{E}[R_i | \mathcal{F}_a],\end{aligned}\tag{20}$$

where  $\bar{x}^k$  and  $\phi^k$  are the weights of asset  $k$  in the market portfolio and the benchmark portfolio, respectively. Defining the payoff of the market portfolio as  $f_{\text{mkt}} := \sum_{i=1}^N f_i \bar{x}^i$  and the payoff of the benchmark portfolio as  $f_\phi := \sum_{i=1}^N f_i \phi^i$ , we can rewrite Equation (20) as

$$\mathbb{E}[R_i | \mathcal{F}_a] = \rho_a \text{Cov}(f_i, f_{\text{mkt}} | \mathcal{F}_a) - \rho_a n \text{Cov}(f_i, f_\phi | \mathcal{F}_a).\tag{21}$$

Furthermore, defining the market beta as  $\beta_{\text{mkt}}^i = \text{Cov}(f_i, f_{\text{mkt}} | \mathcal{F}_a) / \text{Var}(f_{\text{mkt}} | \mathcal{F}_a)$  and the benchmark beta as  $\beta_\phi^i = \text{Cov}(f_i, f_\phi | \mathcal{F}_a) / \text{Var}(f_\phi | \mathcal{F}_a)$ , where  $\text{Var}(f_{\text{mkt}} | \mathcal{F}_a)$  and  $\text{Var}(f_\phi | \mathcal{F}_a)$  denote the volatility of the market and benchmark portfolios, respectively, we get

<sup>7</sup> We note that when  $n$  is exogenously given, the expected return under benchmarking can theoretically approach zero when  $n \rightarrow 1$  and  $\boldsymbol{\phi} \rightarrow \bar{\mathbf{x}}$ . However, when  $n$  is endogenized at the delegation stage, it will become clear in Section 4.3 that when  $\boldsymbol{\phi} = \bar{\mathbf{x}}$  we cannot have  $n = 1$ . Therefore, the risk premium does not degenerate to zero in an economy with delegation and benchmarking.



$$\mathbb{E}[R_i|\mathcal{F}_a] = \rho_a \text{Var}(f_{\text{mkt}}|\mathcal{F}_a) \beta_{\text{mkt}}^i - \rho_a n \text{Var}(f_\phi|\mathcal{F}_a) \beta_\phi^i. \quad (22)$$

Following Brennan (1993), the payoff of the benchmark portfolio  $f_\phi$  is a linear combination of the payoff on the market portfolio and a residual payoff component orthogonal to the market payoff, that is,  $f_\phi = \beta_{\text{mkt}}^\phi f_{\text{mkt}} + f_\epsilon$ . Hence, we get a two-factor CAPM:

$$\mathbb{E}[R_i|\mathcal{F}_a] = \underbrace{\rho_a \text{Var}(f_{\text{mkt}}|\mathcal{F}_a) (1 - n\beta_{\text{mkt}}^\phi)}_{\pi_{\text{mkt}}} \beta_{\text{mkt}}^i - \underbrace{\rho_a n \text{Var}(f_\epsilon|\mathcal{F}_a)}_{\pi_\epsilon} \beta_\epsilon^i, \quad (23)$$

where  $\text{Var}(f_\epsilon|\mathcal{F}_a)$  is the variance of the residual portfolio and  $\beta_\epsilon^i = \text{Cov}(f_i, f_\epsilon|\mathcal{F}_a) / \text{Var}(f_\epsilon|\mathcal{F}_a)$  is the beta of the residual portfolio for asset  $i$ . The new risk factor captures the risk of deviating from the benchmark. This new factor represents a risk from the viewpoint of a relative return investor and is compensated with a premium. The higher the fraction of relative return investors, the higher is this premium. We interpret  $\pi_\epsilon$  as the price of benchmark risk and  $\pi_{\text{mkt}}$  as the market price of risk. For  $\beta_{\text{mkt}}^\phi > 0$  and  $\beta_{\text{mkt}}^i > 0$ , the market price of risk is decreasing in the number of delegating investors and in the beta between the market and the benchmark portfolio. The higher the (positive) correlation between the market portfolio and the benchmark, the stronger is this effect. Furthermore,  $\pi_{\text{mkt}}$  increases with a higher average risk aversion  $\rho_a$ . If the fraction of delegation is equal to one and the benchmark perfectly matches the market portfolio, then the price of market risk degenerates to zero.

#### *The adjusted beta*

As an alternative to the two-factor model, we can express expected returns as a function of the excess return of the market portfolio using an adjusted beta. We note that if Equation (21) holds true for asset  $i$ , then it also holds true for the market portfolio. Thus,

$$\mathbb{E}[R_{\text{mkt}}|\mathcal{F}_a] := \rho_a (\text{Var}(f_{\text{mkt}}|\mathcal{F}_a) - n \text{Cov}(f_{\text{mkt}}, f_\phi|\mathcal{F}_a)), \quad (24)$$

where  $\mathbb{E}[R_{\text{mkt}}|\mathcal{F}_a]$  is the expected excess return on the market portfolio. Solving for  $\rho_a$  in Equation (24) and substituting into Equation (21), we get<sup>8</sup>

$$\mathbb{E}[R_i|\mathcal{F}_a] = \beta_{\text{adj}}^i \mathbb{E}[R_{\text{mkt}}|\mathcal{F}_a], \quad (25)$$

where

$$\beta_{\text{adj}}^i = \frac{\text{Cov}(f_i, f_{\text{mkt}}|\mathcal{F}_a) - n \text{Cov}(f_i, f_\phi|\mathcal{F}_a)}{\text{Var}(f_{\text{mkt}}|\mathcal{F}_a) - n \text{Cov}(f_{\text{mkt}}, f_\phi|\mathcal{F}_a)}. \quad (26)$$

<sup>8</sup> Note that Equation (25) does not represent a single-factor model and  $\beta_{\text{adj}}^i$  is not a regression coefficient.

The adjusted beta,  $\beta_{\text{adj}}^i$ , differs from the standard CAPM beta  $\beta_{\text{CAPM}}^i$  in two respects. First,  $\beta_{\text{adj}}^i$  accounts for the number of delegated agents  $n$  and the covariance of the benchmark portfolio with the market and asset  $i$ . A high positive covariance between the market and the benchmark leads to a high beta for asset  $i$ . Also, a strong positive covariance between the asset and the benchmark tends to lower the adjusted beta. Second, we note that the beta without delegation under benchmarking but with information acquisition, say  $\beta_{\text{adj}}^i$ , is not equal to the standard CAPM beta  $\beta_{\text{CAPM}}^i$ . Although the correlation structure of the assets is not changed through learning, the standard deviation of the assets is what agents in our model can reduce through learning. From the definition of  $\beta_{\text{adj}}^i$ , we see that  $\beta_{\text{adj}}^i$  tends to be lower than the standard CAPM beta for those assets for which agents acquire additional information. Since learning decreases the uncertainty about the future asset payoff, assets held by agents with high information capacity tend to exhibit a lower expected return than do assets held by less sophisticated investors. Therefore, our model predicts a lower risk premium for those assets held by the asset managers.

**Proposition 4.** *Assets with a large fraction of well-informed investors exhibit lower expected returns than assets with a small fraction of well-informed investors.*

Table I summarizes the different cases for the beta adjustment under the assumption  $\text{Var}(f_{\text{mkt}}|\mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi|\mathcal{F}_a) > 0$ , which covers most of the practical cases. In some situations, it is not possible to make a general statement about the direction of the adjustment. However, we note that the correlation between the market and the benchmark usually takes values close to one for most widely used benchmarks such as, for example, the S&P 500 index. Such a practice implies that for assets with

Table I. Beta adjustments

Impact of beta adjustment for different cases of benchmark correlation of asset  $i$  and the market portfolio. The coefficient  $\beta_a^i$  is defined as  $\beta_a^i = \text{Cov}(f_i, f_{\text{mkt}}|\mathcal{F}_a) / \text{Var}(f_{\text{mkt}}|\mathcal{F}_a)$ , that is, the beta without delegation but with information acquisition. “Ambiguous” means that the effect of the adjustment can either be positive, negative, or neutral. Furthermore, we assume that  $\text{Var}(f_{\text{mkt}}|\mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi|\mathcal{F}_a)$ .

$\beta_a^i$	Benchmark effect	Impact on $\beta_{\text{adj}}^i$
$\beta_a^i > 1$	$\text{Cov}(f_i, f_\phi \mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) = \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) < \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$	Ambiguous Positive Positive
$\beta_a^i = 1$	$\text{Cov}(f_i, f_\phi \mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) = \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) < \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$	Negative No impact Positive
$\beta_a^i < 1$	$\text{Cov}(f_i, f_\phi \mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) = \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$ $\text{Cov}(f_i, f_\phi \mathcal{F}_a) < \text{Cov}(f_{\text{mkt}}, f_\phi \mathcal{F}_a)$	Negative Negative Ambiguous

moderate variance, we generally have  $\text{Cov}(f_i, f_\phi | \mathcal{F}_a) < \text{Cov}(f_{\text{mkt}}, f_\phi | \mathcal{F}_a)$ . But for assets with a high variance, it is possible to have  $\text{Cov}(f_i, f_\phi | \mathcal{F}_a) > \text{Cov}(f_{\text{mkt}}, f_\phi | \mathcal{F}_a)$ . Furthermore, assets with a big weight in the benchmark tend to have a higher covariance with the benchmark than do other assets. When  $\beta_a^i$  takes a value greater than one, then the adjustment due to the presence of delegated agents and benchmarking is positive. This statement will hold true as long as the covariance of asset  $i$  with the benchmark is lower or equal to the covariance between the market and the benchmark, that is, as long as  $\text{Cov}(f_i, f_\phi | \mathcal{F}_a) \leq \text{Cov}(f_{\text{mkt}}, f_\phi | \mathcal{F}_a)$ . However, when the standard beta  $\beta_a^i$  takes a value smaller than one, then the adjustment is negative as long as  $\text{Cov}(f_i, f_\phi | \mathcal{F}_a) \geq \text{Cov}(f_{\text{mkt}}, f_\phi | \mathcal{F}_a)$ .

#### 4.2 EQUILIBRIUM AT THE DELEGATION STAGE

At the delegation stage of the model, we determine the optimal equilibrium fraction of delegating investors  $n^*$  based on the certainty equivalent argument introduced in Section 3. When making this decision, each investor anticipates correctly the equilibrium at the trading stage. We define the equilibrium at the delegation stage as follows.

**Definition 2.** For given asset demands  $\mathbf{q}_k$ , asset prices  $\mathbf{p}$ , and manager's posterior covariance  $\hat{\Sigma}_m$ , an equilibrium at the delegation stage is a set of investors  $j \in [0, 1]$  such that (i) none of the delegating investors is better off by switching to direct investment, that is, if  $\text{CED}_j \geq \alpha_j$ , then investor  $j$  is a delegating investor and  $j \in [0, n]$ , and (ii) none of the direct investors is better off by switching to delegation, that is, if  $\text{CED}_j < \alpha_j$ , then investor  $j$  is a direct investor and  $j \in [n, 1]$ . The sum of all delegating investors determines  $n^*$ .

Investors observe managerial ability  $\hat{\Sigma}_m$  and decide whether or not to delegate by comparing the CED with the management fees. Investor  $j$  delegates if and only if  $\text{CED}_j \geq \alpha_j$ .

**Proposition 5.** In general equilibrium, the CED for investor  $j$  with risk aversion  $\rho_j$  is given as

$$\begin{aligned} \text{CED}_j := \sup \left[ \xi | \xi \leq \sum_{l=1}^L \left( \frac{a_j}{\hat{\lambda}_{m,l}^j} - \frac{b_j}{\hat{\lambda}_{j,l}^j} \right) X_l + L(b_j - a_j) \right. \\ \left. + \left( 1 - \frac{\rho_j}{\rho_m} \right) \mu_\phi - \frac{1}{2} \rho_j \left( \hat{\sigma}_\phi^j \right)^2 \right], \end{aligned} \quad (27)$$

where

$$X_l := \lambda_l (1 - \lambda_{B,l})^2 + \sigma_x^2 \lambda_{C,l}^2 + [((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})' \boldsymbol{\Gamma}_l]^2 \quad (28)$$

with  $a_j := (1 - \frac{1}{2\rho_j})\frac{1}{\rho_m}$ ,  $b_j := \frac{1}{2\rho_j}$ ,  $\mu_\phi := \phi'(\boldsymbol{\mu} - \mathbf{p}r)$ ,  $(\hat{\sigma}_\phi^j)^2 := \phi'\hat{\Sigma}_m^j\phi$ , and  $\lambda_{B,l}$  and  $\lambda_{C,l}$  denote the eigenvalues of  $\mathbf{B}$  and  $\mathbf{C}$ , respectively.

The coefficient  $X_l$  is independent from any private information, that is, every investor faces the same  $X_l$ . However, we note that the private information of each agent enters the coefficients  $\mathbf{a}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and its eigenvalues through the average precision matrix  $\Psi$ . Thus,  $X_l$  and hence the CED, is also driven by the information efficiency of prices. A high precision of price signals lowers  $X_l$  and the CED. Since we deal with a continuum of agents, we assume without loss of generality that  $X_l$  is a constant in the investors delegation problem.

The matrix  $\mathbf{B}$  in the linear price function captures the dependence of the price with the true payoff  $\mathbf{f}$ . A high eigenvalue  $\lambda_{B,l}$  indicates a strong dependence between the  $l$ -th risk factor's price and its true payoff. Equation (27) predicts a negative relation between  $\lambda_{B,l}$ ,  $\mathbf{B}$ , and CED. When the price reflects future asset payoffs more accurately, CED takes a lower value and delegation is less attractive. The matrix  $\mathbf{C}$  captures the correlation between prices and supply shocks  $\mathbf{x}$ . The higher  $\lambda_{C,l}$ , the noisier the  $l$ -th risk factor's price and the less information the price will contain about future payoffs. In this situation, it is more difficult to predict asset payoffs and delegation becomes more valuable. Thus, the information transmitted by equilibrium prices decreases the CED and makes delegation less attractive. This is an important property of our equilibrium. Proposition 2 shows that delegation only takes place when the manager has higher information capacity than the investor and thus delegation always increases price informativeness. When the fraction of delegating agents  $n$  is low, a relatively small amount of private information is revealed by equilibrium prices and the CED is high. As  $n$  increases and more investors delegate, the informational content of prices also increases. In return, the CED decreases with increased price informativeness  $\text{Var}(\mathbf{f}|\mathbf{p})^{-1}$ , making delegation less attractive. Investors stop delegating when CED reaches zero. Figure 2 plots the CED as a function of  $n$  for three different levels of information cost  $c$ . We observe that the CED is strictly decreasing in  $n$ . Furthermore, high information costs lower the manager's incentive to acquire information capacity and hence lower the CED.

**Proposition 6.** *The CED is a decreasing function of delegating agents  $n$ . All else being equal, the CED takes its highest value when  $n = 0$  and its lowest value when  $n = 1$ .*

We use the scaling factor  $h$  to specify the *per capita* market capitalization of the benchmark portfolio  $\phi$ . The capitalization of the benchmark can take any value between zero and the total market capitalization  $\mathbf{x}_{\text{mkt}}$ . If the benchmark capitalization is equal to that of the market portfolio, then a passive manager (with risk aversion

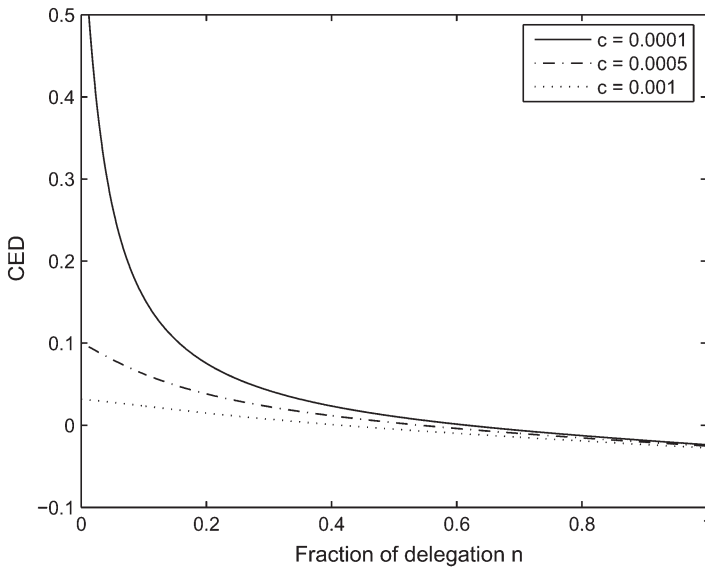


Figure 2. The CED as a function of  $n$  for three different levels of information cost  $c$  in the single-asset case. The exogenous parameters are  $r = 0.02$ ,  $\mu = 0.1$ ,  $\sigma = 0.3$ ,  $\sigma_x = 0.5$ ,  $\sigma_{s,j} = 0.5$ ,  $\rho_j = \rho_m = 5$ ,  $\psi = 2$ .

$\rho_m = \infty$ ) will hold exactly the market portfolio. However, we also consider a benchmark with constituents that have lower market capitalization than do their counterparts in the market portfolio. In this case, a passive manager only invests a fraction of his wealth in the benchmark portfolio and invests the remainder in the risk-free asset. If we denote the fully invested benchmark portfolio as  $\phi_{\text{full}}$ , then we can control for the market capitalization of the actual benchmark portfolio by defining  $\phi = h\phi_{\text{full}}$ , with  $h \in [0, 1]$  the benchmark scaling factor. Figure 3 shows three equilibria for different benchmark scaling factors  $h$ . In equilibrium,  $\text{CED}_j = 0$  for the marginal investor. For delegating investors  $\text{CED}_j \geq 0$  and for direct investors  $\text{CED}_j < 0$ .  $h = 0$  represents the case without benchmarking. As the  $h$  increases, both  $U_{\text{del}}$  and  $U_{\text{dir}}$  are lowered. Since the reduction in utility is always stronger for  $U_{\text{del}}$ , the interception is moved to the left side in Figure 3, resulting in a lower equilibrium fraction of delegation. However, we note that the impact of benchmarking on  $n^*$  is not substantial.

#### 4.3 EQUILIBRIUM AT THE INFORMATION ACQUISITION STAGE

At the information acquisition stage, the agents choose optimal information capacity. Managers act strategically and anticipate the outcome at the delegation stage, that is, manager  $m$  knows the CED of “his” investor  $j$  at the information acquisition stage. At the same time, manager  $m$  sets the optimal fee  $\alpha_j$  as a result of optimal information allocation. Thus, although the fund managers behave strategically when choosing the

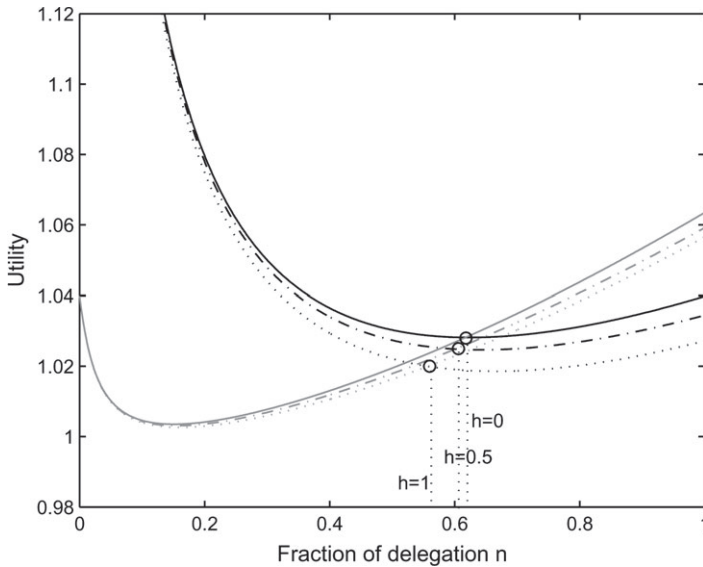


Figure 3. The figure shows equilibria for three different benchmark scaling factors  $h$ . The equilibrium point is the interception between utility from delegation (black) and utility from direct investment (grey). The solid line represents the case without benchmarking where  $h = 0$ . The dash-dotted line represents a benchmark scenario with a scaling factor  $h = 0.5$ . The dotted line represents a benchmark scenario with a scaling factor  $h = 1$ . The parameter values are  $r = 0.02$ ,  $\mu = 0.1$ ,  $\sigma = 0.3$ ,  $\sigma_x = 0.5$ ,  $\sigma_{s,j} = 0.5$ ,  $\rho_j = \rho_m = 5$ ,  $\psi = 2$ .

best information capacity, they are price takers in the risky asset market. We define the equilibrium at the information acquisition as follows.

**Definition 3.** For a given fraction of delegation  $n$ , asset demands  $\mathbf{q}_k$ , and prices  $\mathbf{p}$ , an equilibrium at the information acquisition stage is a set of optimal risk factor variances  $\hat{\Lambda}_m^j, j \in [0, n]$ , and  $\hat{\Lambda}_j, j \in (n, 1]$ , such that  $\hat{\Lambda}_m^j$  solves Equation (16) and  $\hat{\Lambda}_j$  solves Equation (13).

The investor's and manager's information acquisition problem does not allow a closed-form solution. The circular reference between the optimal fraction of delegation  $n^*$  and posterior variance  $\hat{\Lambda}$  further complicates the analysis. In equilibrium, the optimal fraction of delegation depends on the information revealed by prices, which in turn is a function of average signal precision,  $\Psi$ . On the other hand, the manager's information acquisition problem is driven by the average signal precision, which is a function of  $n$ . Therefore, we will present numerical results that illustrate optimal information acquisition.<sup>9</sup>

<sup>9</sup> For the single-asset case, we can derive some formal and analytical arguments for the existence of an interior solution  $n \in (0, 1)$ . This report can be requested by the authors.

An important feature of an equilibrium with delegated agents is the “feedback effect” through informative prices. The CED rule states that a necessary (but not sufficient) condition for delegation to take place is that the manager has better information than the investor. Hence, with an increasing number of delegating agents, the price system becomes more informative. However, with a more informative price system, investors can deduce more information from equilibrium prices, so the relative value of private information decreases. When  $n = 0$ , the equilibrium price contains no managerial private information since all marginal investors are direct investors. As  $n$  increases, the average private information  $\Psi$  and the informational advantage of the manager decreases, lowering the attractiveness of delegation. Panel (a) of Figure 4 shows the manager’s optimal posterior precision, which is a decreasing function of  $n$ . The feedback effect of the informative equilibrium price leads to the fact that the manager lowers information acquisition with increasing  $n$ . Panel (b) of Figure 4 shows the precision of the price signal as a function of  $n$ . The precision of the price signal  $\Sigma_p^{-1}$  is determined by two opposing effects: (i) the decreasing quality of managerial information and (ii) the increasing quantity of managerial information as  $n$  increases. For low values of  $n$ , the second effect dominates and leads to an increase in  $\Sigma_p^{-1}$ , as we move to higher levels of  $n$ . However, there is a point where the higher quantity does not compensate for the decreasing quality of managerial information. As we further increase  $n$ , the first effect starts dominating the second effect and lowers  $\Sigma_p^{-1}$ . There is a point on the interval  $n \in [0, 1]$  where  $\Sigma_p^{-1}$  has a maximum.

#### 4.4 CHARACTERIZATION OF EQUILIBRIUM

An important prerequisite for the model setup was to endogenise as many variables as possible. We therefore explore the characteristics of our equilibrium model with respect to the exogenous variables. The exogenous key variables are the information cost  $c$ , the supply shock volatility  $\sigma_x$ , and the riskiness of the benchmark portfolio  $h$ . The other exogenous variables of the model are of minor importance regarding the qualitative predictions of the model.

Figure 5 illustrates the relation between the equilibrium-implied equity premium  $\mathbb{E}[f - pr|\mathcal{F}_a]$  and the equilibrium fraction of delegation  $n^*$  by varying the exogenous model parameters  $c$ ,  $\sigma_x$ , and  $h$ . Panel (a) plots the equity premium for a range of equilibria under different information cost parameters. An increasing cost for information makes delegation less attractive and lowers the fraction of delegation in equilibrium. The resulting less informative price system leads to an increase in the equity premium.

Panel (b) of Figure 5 plots the equity premium for a range of equilibria under different levels of supply shock volatility. With a lower  $\sigma_x$ , more private information is revealed through the equilibrium price, lowering the equilibrium fraction of

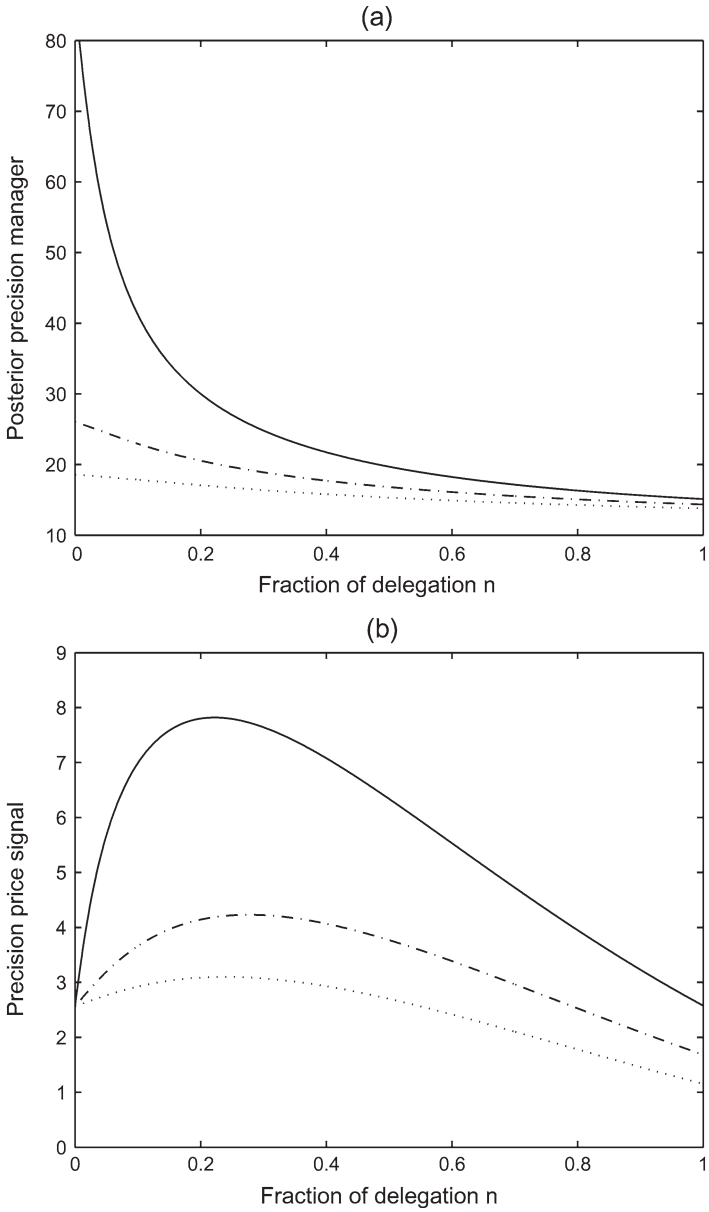
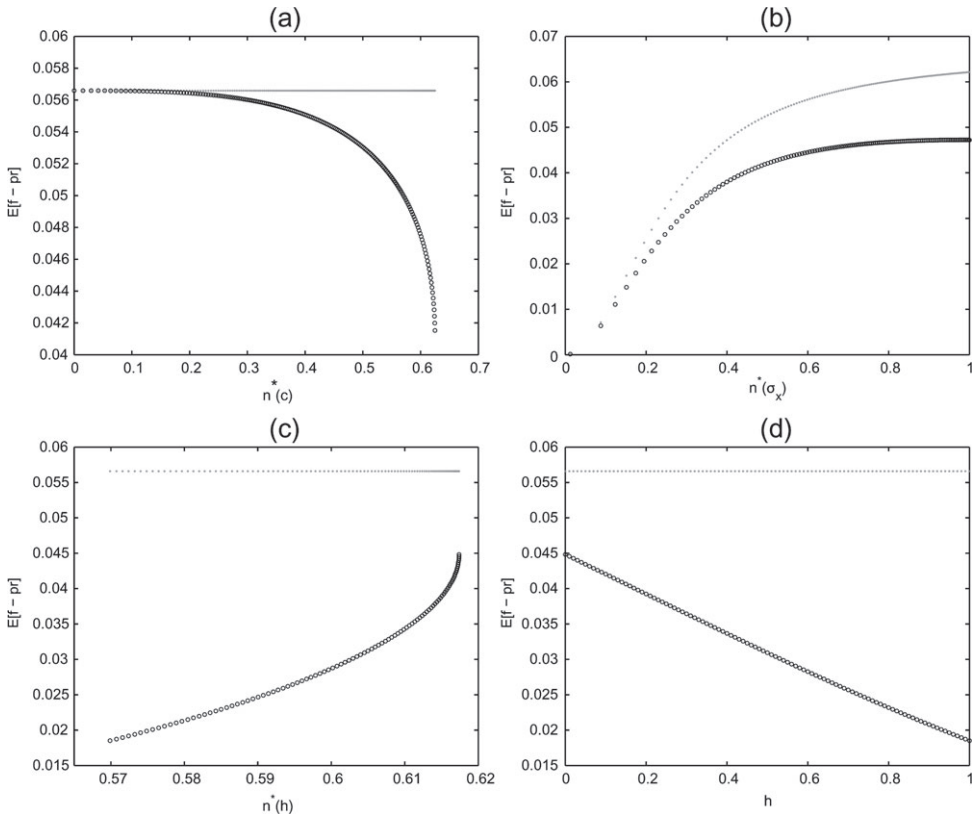


Figure 4. Posterior precisions for the manager and the price signal for three different levels of information costs  $c$ . The solid line is  $c = 0.0001$ ; the dash-dotted line is  $c = 0.0005$ ; the dotted line is  $c = 0.001$ . The parameter values are  $r = 0.02$ ,  $\mu = 0.1$ ,  $\sigma_I = 0.3$ ,  $\sigma_x = 0.5$ ,  $\sigma_{sj} = 0.5$ ,  $\rho_j = \rho_m = 5$ ,  $\psi = 2$ . Panel (a) plots the manager's posterior precision  $\hat{\Sigma}_m$  as a function of  $n$ . Panel (b) plots the precision of the price signal  $\Sigma_p^{-1}$  as a function of  $n$ .





*Figure 5.* Equilibrium implications on the equity risk premium. The figure shows the equilibrium implied risk premium  $\pi^*$  for equilibria with delegation (black marks) and equilibria with direct investment only (grey marks). Panel (a) plots  $\pi^*$  for a range of equilibria under different information cost parameters  $c \in (0, 0.0025)$  such that the equilibrium fraction of delegation  $n^*(c)$  is a function of  $c$ . Panel (b) plots  $\pi^*$  for a range of equilibria under different supply shock volatilities  $\sigma_x \in [0, 1]$ , such that  $n^* = f(\sigma_x)$ . Panel (c) plots  $\pi^*$  for a range of equilibria under different benchmark risk exposures  $h \in [0, 1]$  such that  $n^* = f(h)$ . Panel (d) plots  $\pi^*$  as a function of the benchmark risk exposure  $h$ . The other parameter values are  $r = 0.02$ ,  $\mu = 0.1$ ,  $\sigma = 0.3$ ,  $\sigma_x = 0.5$ ,  $\sigma_{s,j} = 0.5$ ,  $\rho_j = \rho_m = 5$ ,  $\psi = 2$ .

delegation. If the equilibrium price is fully revealing, no investor will have an incentive to delegate in order to have access to the manager's information, and the equilibrium fraction of delegation equals zero. Therefore, since delegation increases in  $\sigma_x$  due to a less informative price, we have a higher equity premium as  $n^*(\sigma_x)$  rises.

The findings of Figure 5 support the results of the two-factor CAPM. The equity premium is always lower in an equilibrium with delegation and benchmarking than in an equilibrium without delegated agents. However, it is not generally true that a higher level of delegation decreases the equity premium. To answer this question, one has to determine the source of the shift in delegation. Higher delegation as a result of lower information costs lowers the

equity premium, whereas higher delegation as a result of an increased supply shock volatility boosts the equity premium. In the same way, a positive shift in delegation due to a less risky benchmark raises the equity premium. The bottom line of this story is that the endogenous modeling of delegation is the key to determine the implications of delegated portfolio management on the equity premium. Previous studies such as Kapur and Timmermann (2005) conclude that an increase in delegation lowers the equity premium. When the fraction of delegation is an exogenous variable of the model, more delegation leads always to a more informative equilibrium price and a lower equity premium because the investor only delegates when the manager is better informed. However, in a setting with endogenous delegation, we show that a higher equilibrium fraction of delegation does not always lead to more informative prices and a lower equity premium.

## 5. Empirical Evidence

Based on our theoretical model from the previous section, we can make two separate predictions about the cross-sectional implications of delegation and benchmarking. Delegation refers to the situation where a professional asset manager holds discretionary control over delegated wealth and buys stocks on behalf of individual investor. The higher the delegation, the higher the proportion of indirect stock holdings through institutional investors. A stock's fraction of institutional investors is therefore a proxy for the degree of delegation and breaks down an economy's overall proportion of delegated wealth to a single stock level.<sup>10</sup> In our model, cross-sectional differences in IO arise due to the fact that each delegated manager selects a portfolio based on his unique posterior belief. Note that Equation (23) is formulated from the point of view of the average investor, which represents a weighted mean of delegating and direct investors. Hence, IO is not directly observable from Equation (23). In Hypothesis 1, we claim that IO is a determinant of asset returns. Thus, we have the following.

***Hypothesis 1.*** *The fraction of IO impacts the cross section of returns. Stocks with high IO exhibit lower expected returns and lower volatility than do stocks with low IO.*

The above hypothesis follows from Propositions 2 and 4. Institutions hold those stocks in their portfolios for which they have private information. Since most institutions have similar preferences regarding firm characteristics, many institutional investors learn about (and hold) similar stocks. In accordance with the Price

<sup>10</sup> Stock holdings managed through brokerage accounts, for which the broker has full discretion over the investment decision, are treated as stocks that are held by institutional investors. Therefore, these holdings increase a stock's IO as well as the fraction of delegated wealth.

Pressure Hypothesis of Scholes (1972), large block trades of these institutional investors lead to (temporary) price pressure and lower expected returns. The impact of benchmarking is captured by Hypothesis 2 and follows directly from the two-factor model derived in Section 4.

***Hypothesis 2.*** *There exists an additional risk factor that accounts for benchmark risk. The price of benchmark risk is negative. A positive loading on benchmark risk (a positive residual beta) is associated with lower returns.*

## 5.1 DATA AND METHOD

For our empirical analysis, we use a comprehensive sample of US stock market data from January 1935 to December 2008. We obtain data on stock returns, prices, and shares outstanding from the Center for Research in Security Prices (CRSP). We use returns including dividends and distributions. As an overall stock market index, we select the CRSP value-weighted total return index, which represents the performance of the entire universe of stocks included in the CRSP database. As a benchmark, we use the S&P 500 composite index. The S&P 500 index is the most widely used benchmark for the US stock market. Like the CRSP market index, the S&P 500 index is a value-weighted index that weights stocks according to their market capitalization. We obtain the S&P 500 Total Return Index via Bloomberg.

We use data on IO from the CDA/Spectrum database. In the USA, large investment managers such as banks, insurance companies, mutual funds, pension funds, and other investment advisors with assets under discretionary management larger than \$100 million are required to report their equity holdings to the U.S. Securities and Exchange Commission (SEC) by filing the Form 13F. Management must disclose quarterly all stock positions greater than 10,000 shares or \$200,000 over which the institution exercises investment discretion. This database gives us a comprehensive sample of the total number of shares held by institutional investors for all US stocks. Institutional holdings are available on a quarterly basis from March 1980 through December 2008. In the following, we refer to institutional investors as investment intermediaries or managers who are obligated to report their equity holdings to the SEC. We note that as long as they do not file the 13F reports, small investment intermediaries are classified as private investors. To construct our final sample of US common stock data, we exclude all nonequity issues such as certificates, American Depositary Receipts, primes, scores, closed-end funds, Real Estate Investment Trusts, and Units. Furthermore, we exclude all companies incorporated outside the USA and all stocks that do not have a CRSP share type code of 10 or 11. We exclude foreign stocks from the analysis because CDA/Spectrum data do not reflect the true number of institutionally held shares for those

stocks. We expect to find that US institutional investors only account for a small fraction of foreign stocks' institutional owners. Hence, the IOs reported to the SEC are far below the true number of institutionally held stocks, which creates a significant underestimation of foreign stocks' IOs listed in the USA.

The CDA/Spectrum database provides the exact number of shares of all stocks held in institutional portfolios. Based on this information, we calculate the total number of shares held by institutional investors (SHRIO) for each stock as of the end of each quarter from March 1980 through December 2008.<sup>11</sup> To calculate the institutional ownership  $IO_{i,t}$  for stock  $i$  at time  $t$ , we divide the total number of shares held by institutions  $SHRIO_{i,t}$  by total shares outstanding  $SHROUT_{i,t}$ ,

$$IO_{i,t} := \frac{\int_0^n q_{\text{del}}^i(j) dj}{\bar{x}_i + x_i} = \frac{SHRIO_{i,t}}{SHROUT_{i,t}}. \quad (29)$$

Note that  $IO_i$  is a function of all managers' portfolio decisions  $q_{\text{del}}^i$  in stock  $i$  and of the overall fraction of delegation  $n$ . Table II reports the summary statistics on IO as of the end of December 2008. The table shows that the cross-sectional mean of IO has steadily increased since 1980 from about 9% to 50% in 2008. These numbers confirm the growing importance of institutional investors. We further note that the shape of the IO distribution has changed significantly over time.<sup>12</sup>

IO is strongly related to firm size. Institutional investors prefer large-cap stocks that have a high degree of market liquidity and prudence characteristics such as S&P 500 membership, firm age, or dividend yield. All these characteristics are generally more pronounced in large-cap stocks. Thus, we expect to find a strong relation between firm size and IO. Since our theory does not make any predictions about the influence of size, we test whether IO can explain the cross section of stock

<sup>11</sup> We note that after screening for late filers and stale data, for some stocks, the number of institutionally held shares still exceeds total shares outstanding in certain quarters, which is obviously a problem in data quality. Officials from Thomson Reuters told us that this problem is due to wrong filings by institutions and cannot be corrected by the database vendor. Therefore, we delete all observations for which the number of institutionally held shares exceeds total shares outstanding. To merge the information on SHRIO with stock market data for the time period January 1980 through December 2007, we follow Nagel (2005) and assume that stocks that are in the CRSP universe but without any data on institutional holdings have a zero SHRIO. On the other hand, we exclude from the analysis stocks that are in the CDA/Spectrum database but cannot be matched to a stock in the CRSP universe.

<sup>12</sup> Since small institutions and foreign investment intermediaries are not required to file 13F forms but are considered as individual investors, the reported numbers on IO underestimate the true numbers. Some of the growth in IO is due to small institutions that crossed the cutoff level for institutional investors only because an increase in market value pushed their assets under management above the threshold of \$100 million. Hence, a small portion of growth in IO does not represent a true change in institutional holdings since the nominal threshold of \$100 million did not change with the growth in market value of total equity. However, as pointed out by Gompers and Metrick (2001), this bias is small.

Table II. Summary statistics of IO data

The table reports descriptive statistics on IO data as of the end of December. All numbers are cross-sectional statistics.  $N$  denotes the total number of stocks. SZ is cross-sectional mean of the natural logarithm of market capitalizations in \$. INST is the average number of institutional investors per stock. IO is the cross-sectional mean of the fraction of institutional ownership per stock  $IO_i$ , where  $IO_i$  is calculated as the sum of shares held by all institutional investors divided by total shares outstanding for stock  $i$ . CV denotes the coefficient of variation and Q1 and Q3 denote, respectively, the 25%-percentile and the 75%-percentile. The data on institutional ownership are from the CDA/Spectrum database, and the data on market capitalizations are from CRSP. Stocks that are on CRSP but without any data on institutional holdings are assumed to have zero IO. The sample includes only stocks with a CRSP share type code of 10 or 11.

	$N$	SZ	INST	Mean	Median	IO					
						Standard deviation	Skew	Kurt	CV	Q1	Q3
Dec-80	4,631	17.55	13.80	0.09	0.01	0.15	3.58	1.99	167.5	0.00	0.12
Dec-85	5,666	17.62	20.04	0.13	0.04	0.18	1.10	1.41	134.8	0.00	0.22
Dec-90	5,685	17.30	26.99	0.16	0.06	0.20	0.60	1.27	128.8	0.00	0.27
Dec-95	6,847	18.46	36.86	0.22	0.14	0.24	-0.30	0.90	108.2	0.00	0.39
Dec-00	6,295	18.69	60.86	0.27	0.19	0.27	-0.74	0.72	99.8	0.01	0.48
Dec-05	4,532	19.72	102.84	0.45	0.45	0.32	-1.38	0.02	70.6	0.14	0.75
Dec-08	4,155	19.03	115.15	0.50	0.52	0.30	-1.31	-0.10	60.5	0.22	0.77

returns when we hold size fixed. To control for size, we follow Hong, Him, and Stein (2000) and Nagel (2005) and sort stocks based on residual institutional ownership (RO). We obtain RO as the residual of a cross-sectional regression of IO on size.

## 5.2 PORTFOLIO RETURN ANALYSIS

To test our theory, we perform a return analysis based on portfolio sorts. By doing so, we can explore the relation between the expected returns and (i) IO and (ii) the residual benchmark beta of a stock.

First, we test Hypothesis 1. At the beginning of each quarter  $t$ , we sort stocks into five portfolios according to RO as of quarter  $t$ . As argued above, we use RO as sorting variable to control for size. We then equal-weight the stocks and hold them in these portfolios for 3 months until the end of quarter  $t$ . In order to be included in the sort, we require that a stock has reported returns on all 3 months of the given quarter  $t$ . The time period is from January 1980 through December 2008.

Table III reports characteristics (Panel A) and monthly mean excess returns (Panel B) on these RO quintile portfolios. P1 denotes the bottom RO quintile and P5 the top RO quintile. In Panel A, size is relatively constant among all

Table III. Characteristics and returns of RO-sorted portfolios

At the beginning of each quarter  $t$  from January 1980 to December 2008, stocks are ranked by RO as of quarter  $t$ , obtained from the cross-sectional regressions

$$\text{logit}(\text{IO}_{i,t}) = \delta_0 + \delta_1 \log(\text{SZ}_{i,t}) + \delta_2 \log(\text{SZ}_{i,t})^2 + \epsilon_{i,t},$$

where SZ denotes size,  $\text{logit}(\text{IO}_{i,t}) = \log(\text{IO}_{i,t}/(1 - \text{IO}_{i,t}))$ , and  $\epsilon_{i,t}$  is residual ownership for stock  $i$  in quarter  $t$ ,  $\text{RO}_{i,t}$ . Stocks are then sorted into equal-weighted quintile portfolios and held for the next 3 months until the end of quarter  $t$ . P1 denotes the bottom RO quintile and P5 the top RO quintile portfolio. Panel A exhibits characteristics on these portfolios. SZ denotes the natural logarithm of size, IO denotes institutional ownership, RO is residual ownership, INST is the average number of institutional investors per stock, and  $N$  is the average number of stocks in each portfolio. Panel B presents monthly mean portfolio returns in excess of the 1-month T-Bill rate. It also reports excess returns adjusted for market risk ( $\alpha_{\text{CAPM}}$ ), the three Fama–French factors ( $\alpha_{\text{FF3F}}$ ), and the Carhart four-factor model ( $\alpha_{4\text{F}}$ ). Returns in each portfolio are equally weighted and they are reported in percent per month. P1–P5 is the premium on a zero-cost portfolio that is long in the portfolio with the lowest RO and short in the portfolio with the highest RO. The  $t$ -statistics of the P1–P5 premium is reported using Newey–West heteroskedasticity and autocorrelation consistent standard errors. \*\* denote significance at the 5% and 1% levels, respectively. The portfolio returns are reported for three size subsamples. The first sample includes all US common stock. The second sample excludes the small-cap stocks below the 20th size percentile. The third sample includes only large-cap stocks and excludes all stocks below the 66th size percentile. Stock data are from CRSP and data on institutional ownership are from the CDA/Spectrum database.

Panel A: Portfolio characteristics

	P1	P2	P3	P4	P5	All
SZ	20.83	19.85	21.01	20.93	19.96	20.52
IO	0.02	0.13	0.27	0.39	0.45	0.25
RO	−5.18	−0.85	1.31	2.16	3.27	0.14
INST	10.34	51.52	77.52	75.47	47.92	52.56
$N$	949	949	949	949	949	4,746

Panel B: Portfolio returns

	P1	P2	P3	P4	P5	All	P1–P5	$t$ -stat
Full sample								
Raw	1.20	1.04	1.09	0.70	−0.71	0.66	1.92	11.01**
$\alpha_{\text{CAPM}}$	0.76	0.66	0.66	0.25	−1.17	0.23	1.93	11.13**
$\alpha_{\text{FF3F}}$	0.68	0.47	0.45	0.00	−1.48	0.02	2.17	15.00**
$\alpha_{4\text{F}}$	0.89	0.62	0.61	0.19	−1.18	0.23	2.07	14.76**
Stocks above the 20th size percentile								
Raw	1.06	0.75	1.09	0.68	−0.64	0.59	1.69	10.00**
$\alpha_{\text{CAPM}}$	0.60	0.35	0.65	0.21	−1.12	0.14	1.72	10.32**
$\alpha_{\text{FF3F}}$	0.54	0.15	0.43	−0.04	−1.43	−0.07	1.97	14.20**
$\alpha_{4\text{F}}$	0.76	0.31	0.60	0.15	−1.12	0.14	1.88	13.95**
Stocks above the 66th size percentile								
Raw	0.82	0.51	0.78	0.63	0.22	0.59	0.60	4.79**
$\alpha_{\text{CAPM}}$	0.38	0.10	0.32	0.14	−0.31	0.13	0.69	6.21**
$\alpha_{\text{FF3F}}$	0.26	−0.11	0.11	−0.08	−0.54	−0.07	0.80	7.93**
$\alpha_{4\text{F}}$	0.50	0.10	0.28	0.12	−0.28	0.14	0.78	7.80**

RO quintiles. Hence, sorting on RO is effective in creating variation in IO and holding size fixed. The number of institutional investors per stock does not appear to be related to RO. In fact, in results not reported here, we find that the number of institutions is closely related to IO but not to RO. Panel B exhibits raw and risk-adjusted mean excess returns. We observe a clear trend in the cross section. The returns tend to decrease with increasing RO. The spread between the bottom- and the top-ranked portfolio P1–P5 is positive and statistically significant for all observation periods. This result confirms Hypothesis 1. Stocks that have high IO relative to their size exhibit lower returns than do stocks with low relative IO. Our model predicts that the introduction of more precise information due to institutional investors drives this observation. Furthermore, we note that high returns in low RO quintiles are not a compensation for market, size, value, or momentum risk. When we consider risk-adjusted returns, we find that the *t*-statistics remains highly significant. These findings are robust when using different time subperiods. To further test the robustness of our results, we have recalculated the portfolio returns excluding all small-cap stocks below the 20th size percentile. The decreasing pattern in excess returns of RO-sorted portfolios does not change materially and the *t*-statistics of P1–P5 premium remains highly significant. The same result holds true for the large-cap stocks above the 66th size percentile.

Table IV reports stock volatility in dependence of IO controlling for market risk, size, style, and momentum. Each month from January 1980 to December 2008, we have calculated stock volatility as well as residual volatility for all stocks in the sample. Volatility is calculated as the stock's annualized standard deviation over the previous 24 months. Only stocks that have at least twenty observations over the 24-month estimation period are included. Residual volatility has been calculated as the annualized standard deviation of the stock's CAPM, Fama–French, and Carhart residuals, again for an estimation period of 24 months. The portfolio volatility reported in Table IV is the time series and cross-sectional average volatility of all stocks in the respective IO-ranked portfolio. P1 denotes the bottom IO quintile and P4 the top IO quintile.<sup>13</sup> We observe that the average volatility of stocks is clearly decreasing in the fraction of IO. Stocks with high IO tend to have a lower volatility than do stocks with low IO. Stocks ranked in the bottom IO group have an annualized volatility of 17.0% on average, whereas stocks ranked in the top IO group have a volatility of 11.2% on average. This observation is robust to the influence of market risk, size, style, and momentum factors. The average volatility of Carhart residuals is 15.8% in the bottom IO quintile and 9.7% in the top IO quintile.

<sup>13</sup> For IO sorts, we rank stocks into four portfolios since at the beginning of the sample, the number of stocks with IO equal to zero exceeds the number of stocks necessary for each quintile.

*Table IV.* Residual volatility of IO-sorted portfolios

The table shows average stock volatility for IO-sorted portfolios. Each month from January 1980 to December 2008, volatility and residual volatility have been calculated for all stocks in the sample. Volatility at any given month is the annualized standard deviation, measured over the past 24 months. Residual volatility is the annualized standard deviation of CAPM, Fama–French, and Carhart residuals, measured over the past 24 months. Portfolio volatility of portfolios P1–P4 reported in the table is the time series and cross-sectional average volatility of all stocks in the respective IO group. Volatility figures are reported in percent per annum. MR *p*-value is the *p*-value of the monotonicity test of Patton and Timmermann (2010). \* and \*\* denote significance at the 5% and 1 levels, respectively. Stock data are from CRSP and data on institutional ownership are from the CDA/Spectrum database.

	P1	P2	P3	P4	MR <i>p</i> -value
IO (%)	1.20	10.57	30.12	59.21	—
RO	−4.07	1.03	1.91	2.57	—
Volatility (%)	16.96	16.36	12.94	11.23	0.012*
CAPM residual volatility (%)	16.23	15.70	12.05	10.03	0.025*
FF residual volatility (%)	15.87	15.33	11.68	9.75	0.013*
CAR residual volatility (%)	15.78	15.22	11.61	9.69	0.008**

Indeed, we observe a monotonic relationship across the volatilities of the four IO-sorted portfolios.<sup>14</sup>

Hypothesis 2 states that a shift in the riskless point of reference from the risk-free asset to the benchmark portfolio results in an additional (negative) risk factor. Brennan and Li (2008) demonstrate that there is such a risk factor and that it is primarily significant for the time period starting in the mid 1970s. Anecdotal evidence suggests that this is a time period during which relative return objectives began to be extremely popular in the asset management industry. Since we use a somewhat different data sample, we first replicate the results of Brennan and Li to ensure that we have a statistically significant benchmark risk factor in our data sample. Our results, not reported here for sake of brevity, confirm the findings of Brennan and Li.

Given the fact that the benchmark risk factor exists, we complement the results of Brennan and Li (2008) and explore the cross-sectional properties of residual betas. Furthermore, we include in our study of excess returns also a momentum factor. To construct our portfolio sorts, we use residual betas as the sorting variable. To be consistent with the analysis on RO-sorted portfolios, we use again holding periods of 3 months. First, we calculate residual betas  $\beta_{\text{res}}$  for each stock. At the end of each quarter from 1939Q4 to 2008Q3, we regress the monthly returns on each security that has at least twenty-four observations over the past 5 years against the market returns  $R_{\text{mkt}}$  and the benchmark residual returns  $e_{\text{S\&P}}$ . We obtain the benchmark

<sup>14</sup> We have performed a monotonicity test outlined in Patton and Timmermann (2010) to test for a monotonic relationship between the four portfolios. The test statistic is significant at a 5% confidence level.



residual  $e_{S\&P,t}$  from a time-series regression over the full sample period of benchmark (S&P 500 index) returns  $R_\phi$  against the market returns (CRSP value-weighted index)  $R_{mkt}$ ,

$$R_{\phi,t} = \beta_0 + \beta_{mkt}^\phi R_{mkt} + e_{S\&P,t}. \quad (30)$$

The residual beta  $\beta_{res}^i$  for each stock  $i$  are calculated by running individual time-series regressions of monthly stock returns on the returns of the CRSP value-weighted index  $R_{mkt}$  and the S&P 500 index residual returns  $e_{S\&P,t}$ ,

$$R_{i,t} = \alpha + \beta_{mkt}^i R_{mkt,t} + \beta_{res}^i e_{S\&P,t}. \quad (31)$$

Then, at the beginning of the following quarter, we rank all stocks into value-weighted quintile portfolios according to their residual betas as of the end of the previous year. To be included in the portfolio, we require a stock to have reported returns for all 3 months. We then calculate the monthly returns on these portfolios. Thus, we arrive at a time series of portfolio returns, from which we calculate the test statistics.

Table V reports the characteristics and mean excess returns for the quintile portfolios. We measure excess returns in terms of the standard CAPM alpha, a three-factor Fama–French model, and the Carhart four-factor model including a momentum factor. P1 denotes the bottom residual beta quintile and P5 the top quintile. The benchmark effect is most pronounced in large stocks. With exception of the CAPM alpha, all  $t$ -statistics for the spread between top and bottom quintile portfolios are highly significant. Since benchmark risk is associated with a negative premium, a high loading on this risk factor decreases expected returns. The results for small-cap stocks are weak due to portfolio P5. While we observe a pattern of decreasing returns for P1–P4, P5 disrupts this trend. P5 exhibits very low RO compared to the other portfolios. In line with Hypothesis 1, this observation might explain the higher returns of portfolio P5 despite the fact that it has the largest residual benchmark beta. In addition, we note that the results for small-cap stocks are more volatile than for the other samples. Finally, effects such as, for example, illiquidity, which characterize small-cap stocks and are not inherent in medium- and large-cap stocks, might further distort the results.

To see how the benchmark effect evolves over time, we examine different subperiods. In Table VI, we divide the sample into three subperiods and find a strong benchmark effect from 1960 on, while the risk-adjusted returns for the 1940–59 period are not significant. Hence, the benchmark effect seems to become stronger over time, equivalent with the increasing importance of institutional asset management. We note that for each quintile portfolio, none of the CAPM alpha is significantly different from zero, which means that the CAPM does not predict a significant benchmark effect. The insignificance of the CAPM alphas might be due to the fact that the CAPM has no predictive power in explaining the cross

Table V. Monthly returns of residual beta-sorted portfolios

At the beginning of each quarter  $t$  from January 1940 to January 2008, stocks are ranked by residual beta as of the previous quarter  $t - 1$ . The residual betas  $\beta_{\text{res}}^i$  for each stocks  $i$  are calculated by running individual time series regressions of monthly stock returns on the returns of the CRSP value-weighted index  $R_{\text{mkt}}$  and the S&P 500 residual returns  $e_{\text{S\&P}}$ ,

$$R_{i,t} = \alpha + \beta_{\text{mkt}}^i R_{\text{mkt},t} + \beta_{\text{res}}^i e_{\text{S\&P},t}.$$

Stocks are then sorted into five value-weighted portfolios and held for the next 3 months until the end of quarter  $t$ . The table presents monthly excess returns on these portfolios for the full sample and three different size subsamples. It also reports excess returns adjusted for market risk ( $\alpha_{\text{CAPM}}$ ), the three Fama–French factors ( $\alpha_{\text{FF3F}}$ ), and the Carhart four-factor model ( $\alpha_{4F}$ ). P1 denotes the bottom and P5 the top residual beta quintile. P1–P5 is the premium on a zero-cost portfolio that is long in the portfolio with the lowest residual beta and short in the portfolio with the highest residual beta. The  $t$ -statistics of the P1–P5 premium is reported using Newey–West heteroskedasticity and autocorrelation consistent standard errors. \* and \*\* denote significance at the 5% and 1% levels, respectively. The stock data are from CRSP database.

	P1	P2	P3	P4	P5	All	P1–P5	$t$ -stat
Full sample								
Raw	2.24	1.87	1.62	1.43	1.26	1.68	0.99	4.55**
$\alpha_{\text{CAPM}}$	0.85	0.70	0.60	0.54	0.41	0.62	0.44	2.36*
$\alpha_{\text{FF3F}}$	1.04	0.77	0.64	0.51	0.35	0.66	0.68	4.83**
$\alpha_{4F}$	1.04	0.78	0.68	0.53	0.38	0.69	0.66	4.68**
Small-cap stocks (<33th size percentile)								
Raw	3.68	3.24	2.83	2.63	3.27	3.13	0.41	2.27**
$\alpha_{\text{CAPM}}$	2.35	2.14	1.85	1.77	2.33	2.09	0.02	0.09
$\alpha_{\text{FF3F}}$	2.21	1.91	1.60	1.49	2.11	1.87	0.10	0.65
$\alpha_{4F}$	2.20	1.90	1.56	1.47	2.01	1.83	0.19	1.20
Mid-cap stocks (33th–66th size percentile)								
Raw	2.75	2.43	2.26	2.17	2.21	2.36	0.54	2.68**
$\alpha_{\text{CAPM}}$	1.33	1.27	1.25	1.27	1.26	1.28	0.07	0.40
$\alpha_{\text{FF3F}}$	1.47	1.23	1.07	1.09	1.08	1.19	0.39	2.51*
$\alpha_{4F}$	1.41	1.16	1.06	1.07	1.05	1.15	0.35	2.25**
Large-cap stocks (>66th size percentile)								
Raw	1.93	1.64	1.46	1.36	1.26	1.53	0.70	3.91**
$\alpha_{\text{CAPM}}$	0.64	0.58	0.52	0.48	0.70	0.59	0.26	1.66
$\alpha_{\text{FF3F}}$	0.86	0.66	0.55	0.45	0.63	0.63	0.54	4.29**
$\alpha_{4F}$	0.89	0.70	0.58	0.51	0.69	0.67	0.55	4.39**

section of returns, in contrast to the Fama–French three-factor and the Carhart four-factor models that produce high  $R^2$  in empirical applications.

Figure 6 plots the 10-year sliding average spreads between the bottom (P1) and the top (P5) portfolios over time. On average, the spreads are increasing over time. For RO-sorted portfolios, depicted in Panel (a), the monthly spread increased from 1.5% to 2.5%. In the same period of time (1980–2008), IO in the USA increased from 10% to about 50%. The 10-year average spread for residual beta-sorted portfolios, depicted in Panel (b), is increasing as well. The fraction of delegation based on numbers of the Federal Reserve Board has increased since 1950 from 9% to more than 60%. These findings confirm the conjecture that RO and benchmark

Table VI. Monthly returns of residual beta-sorted portfolios for subperiods

At the beginning of each quarter  $t$  from January 1940 to January 2008, stocks are ranked by residual beta as of the previous quarter  $t - 1$ . Stocks are then sorted into five value-weighted portfolios and held for the next 3 months until the end of quarter  $t$ . The table presents monthly excess returns on these portfolios for different time periods. It also reports excess returns adjusted for market risk ( $\alpha_{CAPM}$ ), the three Fama–French factors ( $\alpha_{FF3F}$ ), and the Carhart four-factor model ( $\alpha_{4F}$ ). P1 denotes the bottom and P5 the top residual beta quintile. P1–P5 is the premium on a zero-cost portfolio that is long in the portfolio with the lowest residual beta and short in the portfolio with the highest residual beta. The  $t$ -statistics of the P1–P5 premium is reported using Newey–West heteroskedasticity and autocorrelation consistent standard errors. \* and \*\* denote significance at the 5% and 1% levels, respectively. The stock data are from CRSP database.

	P1	P2	P3	P4	P5	All	P1–P5	$t$ -stat
Subperiod 1940–59								
Raw	1.78	1.61	1.28	1.33	1.31	1.46	0.47	2.50*
$\alpha_{CAPM}$	0.43	0.47	0.23	0.26	0.13	0.30	0.30	1.75
$\alpha_{FF3F}$	0.38	0.42	0.23	0.26	0.14	0.29	0.24	1.69
$\alpha_{4F}$	0.32	0.39	0.22	0.26	0.18	0.28	0.13	0.94
Subperiod 1960–79								
Raw	2.04	1.84	1.53	1.24	0.95	1.52	1.09	2.77**
$\alpha_{CAPM}$	0.83	0.82	0.64	0.49	0.25	0.60	0.58	1.86
$\alpha_{FF3F}$	0.69	0.71	0.58	0.46	0.26	0.54	0.43	2.68**
$\alpha_{4F}$	0.63	0.71	0.59	0.47	0.30	0.54	0.33	2.04*
Subperiod 1980–2008								
Raw	2.80	2.12	1.99	1.67	1.46	2.01	1.34	3.25**
$\alpha_{CAPM}$	1.28	0.86	0.92	0.78	0.67	0.90	0.62	1.82
$\alpha_{FF3F}$	1.79	1.05	0.97	0.72	0.59	1.02	1.20	4.72**
$\alpha_{4F}$	1.85	1.07	1.02	0.77	0.62	1.07	1.23	4.84**

effects have become more important with increasing delegation and IO over the past decades.

### 5.3 ASSET PRICING TESTS

We adopt a two-step regression procedure of Fama and MacBeth (1973) to explore the predictive power of the two-factor CAPM. We contrast the results of our model with the three-factor model of Fama and French (1993) and the Carhart (1997) four-factor model. The two-factor CAPM is tested against a set of 100 value-weighted testing portfolios that are based on a  $10 \times 10$  sort on size and book-to-market values. First, the multivariate factor betas for each testing portfolio  $p$  are obtained from a time-series regression of portfolio excess returns on the factors,

$$R_{p,t} = \hat{\beta}_{0,p} + \hat{\beta}_{m,p}R_{m,t} + \hat{\beta}_{\epsilon,p}R_{\epsilon,t} + e_{p,t}, \quad p = 1, \dots, 100, \quad (32)$$

where  $R_{p,t}$  is the excess return of portfolio  $p$  at time  $t$ ,  $R_{m,t}$  is the excess return of the market,  $R_{\epsilon,t}$  is the excess return of the benchmark residual, defined in

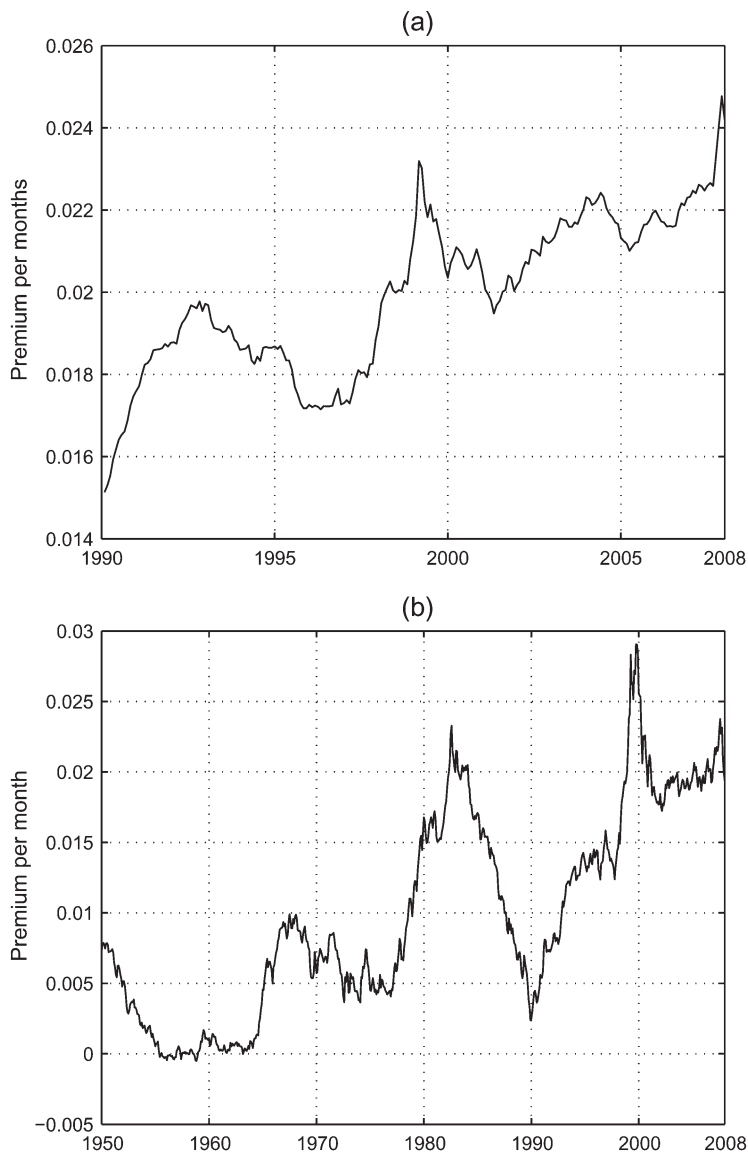


Figure 6. Portfolio spreads over time. The figure shows the average 10-year sliding spread between bottom (P1) and top (P5) portfolios. Panel (a) plots the spreads of bottom minus top RO-sorted portfolios. Panel (b) plots the spreads of bottom minus top residual beta-sorted portfolios. The numbers are based on the calculations given in Tables III and V, respectively.

Equation (30), and  $e_{p,t}$  is the error term. Second, the mean excess returns on the testing portfolios  $\bar{R}_p$  are cross-sectionally regressed on their estimated factor betas,

$$\bar{R}_p = \hat{\delta}_0 + \hat{\delta}_m \hat{\beta}_{m,p} + \hat{\delta}_\epsilon \hat{\beta}_{\epsilon,p} + \eta_{p,t}, \quad (33)$$

where  $\hat{\delta}_m$  and  $\hat{\delta}_\epsilon$  denote the estimated market and benchmark premia.

The results of the portfolio return analysis in the previous section show that stocks with low RO significantly outperform stocks with high RO. Based on this insight, we augment the two-factor CAPM with an additional risk factor that accounts for residual ownership risk. The residual ownership factor, referred to as residual ownership premium (ROP), is the premium on a zero-cost portfolio that is long in the bottom RO quintile portfolio and short in the top RO quintile. Furthermore, we include the standard Fama–French size and value factors, the Carhart momentum factor, and the liquidity factor of Pastor and Stambaugh (2003) into the analysis. The time period is from 1980 to 2008. We start in 1980 because the IO factor ROP—calculated in the portfolio analysis from the previous section—is not available before 1980. We want all results to be directly comparable with an identical time period.

We report the results of our asset pricing test in Table VII. Model (1) is the CAPM single-factor model, where MKT denotes the excess return of the CRSP value-weighted index. MKT is negative and statistically significant with a  $t$ -statistics of  $-6.95$ . The adj  $R^2$  for the market model is 33.25%. Model (2) is the two-factor CAPM derived in Section 4.1, where the second factor is the S&P 500 residual return, denoted as BMR. We observe that the inclusion of the benchmark risk factor significantly increases the explanatory power of the model (adj  $R^2 = 41.65\%$ ). The coefficient for BMR is negative and statistically significant ( $t = -4.05$ ). In Model (3)–Model (7), the two-factor CAPM is augmented by various risk factors. The inclusion of ROP in Model (3) leads to a significant increase in explanatory (adj  $R^2 = 46.94\%$ ). ROP is negative and statistically significant ( $t = -2.49$ ), as implied by the portfolio return analysis. To see the economic importance of the IO effect, we can multiply the estimate of the ROP coefficient of 0.25 with the spread between the average ROP beta of the top book-to-market group and the bottom book-to-market group which is  $(0.40 - (-0.42)) = 0.82$ . This calculation results in a premium of 0.20%, approximately 2.50% per annum. Similarly, multiplying the estimate of the ROP coefficient with the spread between the average ROP beta of the small-size group and the large-size group,  $(0.10 - (-0.17)) = 0.26$ , results in a premium of 0.06%, approximately 0.79% per annum. These results indicate that the economic implications of IO are substantial.

Model (4) includes the Fama–French size factor SMB. In an unreported correlation analysis, we find that BMR exhibits a strong negative correlation of  $-0.813$  with the size factor SMB. This observation implies that the benchmark residuals tend to be positive when large-cap stocks outperform small-cap stocks (negative size premium) and negative when small-cap stocks outperform large-cap stocks (positive size premium). Thus, Model (4) runs into the problem of multicollinearity.

Table VII. Asset pricing results 1980–2008

The table presents the results for various cross-sectional regressions, where the mean excess returns of 100 Fama–French size- and book-to-market-sorted testing portfolios are regressed on their factor betas. The multivariate factor betas for each testing portfolio are obtained from time series regressions of portfolio returns on the factors.  $k_0$  denotes the intercept of the regressions. MKT denotes the excess return of the CRSP value-weighted market index. BMR is the excess return of the S&P 500 residual. ROP is the excess return of a zero-cost portfolio that is long in the bottom RO quintile portfolio and short in the top RO quintile portfolio. SMB is the Fama–French size factor that captures the premium of small-cap relative to large-cap stocks. HML is the Fama–French value factor that captures the relative performance of value versus growth stocks. MOM is the momentum factor of Carhart. LIQ is a market-traded liquidity factor based on the methodology outlined in Pastor and Stambaugh (2003). All coefficients are multiplied by 100. The  $t$ -statistics is reported in parentheses. For each regression, we also report the adjusted  $R^2$ . The estimation period is January 1980 through December 2008. The returns of the testing portfolios and the SMB, HML, and MOM factors are obtained from the Web site of Kenneth French, <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>. The liquidity factor is obtained from CRSP. \* and \*\* denote significance at the 5% and 1% levels, respectively.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$k_0$	1.55 (11.91)**	1.86 (12.94)**	1.44 (7.93)**	1.72 (11.19)**	1.48 (6.16)**	1.88 (12.75)**	1.82 (11.01)**	1.46 (6.14)**	1.05 (4.04)**	0.98 (3.95)**
MKT	-0.88 (-6.95)**	-1.27 (-8.33)**	-0.94 (-5.47)**	-1.15 (-7.26)**	-0.94 (-4.18)**	-1.27 (-8.35)**	-1.23 (-7.27)**	-0.93 (-4.17)**	-0.51 (-2.01)**	-0.45 (-1.84)**
BMR		-0.06 (-4.05)**	-0.08 (-4.94)**	0.00 (0.10)	-0.07 (-4.29)**	-0.06 (-4.01)**	-0.07 (-3.91)**			0.02 (0.50)
ROP			-0.25 (-2.49)*							-0.33 (-2.87)**
SMB				0.03 (0.73)				0.07 (1.59)	0.08 (1.77)	0.10 (2.28)*
HML					0.40 (7.73)**			0.39 (7.96)**	0.42 (8.73)**	0.33 (6.65)**
MOM						0.32 (1.24)			1.02 (3.20)**	1.06 (4.05)**
LIQ							0.20 (0.50)			0.50 (1.18)
Adj $R^2$	32.57	41.65	46.94	44.29	42.82	41.43	41.21	44.37	49.12	56.74

Table VIII. Index constituents of global benchmark

The table shows the composition of the Global Benchmark. The number of funds denotes the number of US all-equity mutual funds reporting the index as their primary benchmark as of January 2007. Mutual fund assets denotes the total number of assets under management related to a given benchmark index. The Global Benchmark consists of the six largest benchmark indices for the US equity market in terms of mutual fund assets where the index constituents are weighted by the size of mutual fund assets. The data source is Morningstar. Parts of the table are reproduced from Cremers, Petajisto, and Zitzewitz (2008).

Index	Category	Number of mutual funds	Mutual fund assets (million US\$)	Weight (%)
S&P 500	Large cap	1,318	2,130,000	72
Russell 1000 value	Large-cap value	177	249,539	9
Russell 2000	Small cap	251	214,712	7
Russell 1000 growth	Large-cap growth	180	162,710	5
Wilshire 5000	All	20	114,092	4
S&P 400	Mid cap	74	102,241	3
Global Benchmark		2,020	2,973,294	100

Multicollinearity does not affect the predictive power of the model (adj  $R^2 = 44.29\%$ ) but affects the calculation of the individual coefficients. Both predictors are not significantly different from zero. The inclusion of the HML factor in Model (5) cannot increase the predictive power, but the coefficient of HML is positive and significantly different from zero. The inclusion of momentum MOM in Model (6) and liquidity LIQ in Model (7) cannot increase the adjusted  $R^2$  as well. Both coefficients are not statistically significant. Model (8) represents the standard Fama–French three-factor model. SML is positive but not statistically significant ( $t = 1.59$ ) and HML is positive and highly significant ( $t = 7.96$ ). The predictive power of the Fama–French model is only slightly above that of the two-factor CAPM. The inclusion of a momentum factor in the Carhart four-factor model, labeled as Model (9), increases the adjusted  $R^2$  to 49.12%. In contrast to Model (6), MOM is positive and statistically significant ( $t = 3.20$ ). Model (10) contains all seven risk factors. The predictive power is high (adjusted  $R^2 = 56.74\%$ ). Again, in conjunction with SMB, the benchmark risk factor BMR loses its significance. The coefficient of SMB turns out to be statistically significant in Model (10). The  $p$ -values of the  $F$ -tests for all model specifications are below 0.001. Thus, the null hypothesis that the model has no predictive capability is rejected for all models.

Overall the table shows strong evidence in favor of the two-factor CAPM. The benchmark risk factor is negative and statistically significant for all models, except when SMB is included. The residual ownership factor ROP is also negative and statistically significant as predicted by the theory. This finding implies that active managers are rewarded with a premium for bearing tracking error risk and holding non-benchmark assets. In terms of predictive power, the two-factor CAPM is only slightly behind the widely used Fama–French and Carhart factor models.

#### 5.4 MULTIPLE BENCHMARKS

In the previous analysis, we have taken the S&P 500 index as a proxy benchmark for the US equity market. In practice, however, different managers use different benchmarks. The use of multiple benchmarks gives rise to a so-called “Global Benchmark,” which is a weighted average of benchmark indices used by various mutual fund managers. The weighting factor is the size of mutual fund assets benchmarked to the respective index, that is, the size of delegated assets. The more assets that are benchmarked against a certain index, the higher the index’s weight in the Global Benchmark. For  $Z$  different benchmarks with market capitalizations  $h_z$ , where  $\sum_{z=1}^Z h_z < 1$ , the Global Benchmark  $\phi_{GB}$  can be expressed as

$$\phi_{GB} = h_1\phi_1 + \dots + h_Z\phi_Z. \quad (34)$$

To approximate the Global Benchmark empirically, we have selected the six most important all-equity benchmark indices for the US stock market, shown in Table VIII. The Global Benchmark covers 2,020 US all-equity mutual funds with assets totaling almost US\$ 3 trillion. Based on both number of mutual funds and mutual fund assets, the S&P 500 index is by far the most important benchmark index, underlining the results shown in the previous section that S&P 500 residual risk is indeed priced by the market.

Table IX shows the results of the residual beta portfolio sorts for the Global Benchmark. Again, we observe a clear trend in portfolio returns. The premium of a zero-cost portfolio that is long in the portfolio with the lowest residual beta and short in the portfolio with the highest residual beta is positive and significant for most of the raw and risk-adjusted returns, indicating that a high sensitivity with the Global Benchmark decreases expected returns, on average. In contrast to the S&P 500 portfolio sorts, the Carhart four-factor alpha of Global Benchmark portfolio sorts is positive and significant for the full sample as well as for the three size subsamples. Overall, the Global Benchmark leads to more significant results for small- and mid-cap stocks but less significant results for large-cap stocks, compared to the S&P 500 benchmark.

Table X exhibits asset pricing results for the Global Benchmark. In essence, the results are similar to those obtained for the S&P 500 index. The benchmark residual factor (BMR) is most significant in conjunction with the residual ownership factor ROP. The two-factor model augmented by the ROP factor (Model (3),  $\text{adj } R^2 = 46\%$ ) exhibits higher explanatory power than the classical Fama–French three-factor model (Model (8),  $\text{adj } R^2 = 44\%$ ) and is only slightly behind the Carhart four-factor model (Model (9),  $\text{adj } R^2 = 49\%$ ).

## 6. Conclusions

We analyze the equilibrium implications of delegated portfolio management under benchmarking. Delegation arises endogenously in equilibrium as a result of a given



Table IX. Monthly returns of residual beta-sorted portfolios for Global Benchmark

At the beginning of each quarter  $t$  from January 1980 to January 2008, stocks are ranked by residual beta as of the previous quarter  $t - 1$ . The residual betas  $\beta_{\text{res}}^i$  for each stocks  $i$  are calculated by running individual time series regressions of monthly stock returns on the returns of the CRSP value-weighted index  $R_{\text{mkt}}$  and the Global Benchmark (GB) residual returns  $e_{\text{GB}}$ ,

$$R_{i,t} = \alpha + \beta_{\text{mkt}}^i R_{\text{mkt},t} + \beta_{\text{res}}^i e_{\text{GB},t}.$$

Stocks are then sorted into five value-weighted portfolios and held for the next 3 months until the end of quarter  $t$ . The table presents monthly excess returns on these portfolios for the full sample and three different size subsamples. It also reports excess returns adjusted for market risk ( $\alpha_{\text{CAPM}}$ ), the three Fama–French factors ( $\alpha_{\text{FF3F}}$ ), and the Carhart four-factor model ( $\alpha_{4\text{F}}$ ). P1 denotes the bottom and P5 the top residual beta quintile. P1–P5 is the premium on a zero-cost portfolio that is long in the portfolio with the lowest residual beta and short in the portfolio with the highest residual beta. The  $t$ -statistics of the P1–P5 premium is reported using Newey–West heteroskedasticity and autocorrelation consistent standard errors. \* and \*\* denote significance at the 5% and 1% levels, respectively. The stock data are from CRSP database.

	P1	P2	P3	P4	P5	All	P1–P5	$t$ -stat
Full sample								
Raw	2.21	1.59	1.30	1.20	1.03	1.47	1.18	2.97**
$\alpha_{\text{CAPM}}$	0.72	0.37	0.32	0.27	0.23	0.38	0.49	1.41
$\alpha_{\text{FF3F}}$	1.28	0.62	0.37	0.24	0.09	0.52	1.19	4.60**
$\alpha_{4\text{F}}$	1.33	0.75	0.48	0.35	0.08	0.60	1.26	4.74**
Small-cap stocks (<33th size percentile)								
Raw	4.27	3.43	2.91	2.63	3.70	3.39	0.57	1.56
$\alpha_{\text{CAPM}}$	2.97	2.47	2.16	1.95	2.89	2.49	0.08	0.23
$\alpha_{\text{FF3F}}$	3.19	2.42	1.95	1.68	2.75	2.40	0.44	1.50
$\alpha_{4\text{F}}$	3.32	2.36	1.93	1.60	2.66	2.37	0.66	2.21*
Mid-cap stocks (33th–66th size percentile)								
Raw	3.03	2.58	2.18	2.06	2.28	2.43	0.75	1.62
$\alpha_{\text{CAPM}}$	1.53	1.45	1.30	1.30	1.43	1.40	0.10	0.27
$\alpha_{\text{FF3F}}$	2.01	1.53	1.10	1.01	1.16	1.36	0.85	3.02**
$\alpha_{4\text{F}}$	1.99	1.51	1.07	1.00	1.13	1.34	0.87	2.99**
Large-cap stocks (>66th size percentile)								
Raw	1.82	1.45	1.14	1.18	1.00	1.32	0.82	2.33*
$\alpha_{\text{CAPM}}$	0.43	0.32	0.19	0.33	0.22	0.30	0.20	0.65
$\alpha_{\text{FF3F}}$	0.89	0.50	0.19	0.26	0.06	0.38	0.84	3.45**
$\alpha_{4\text{F}}$	0.99	0.61	0.35	0.22	0.06	0.45	0.93	3.71**

benchmarking policy. First, we find that an equilibrium with delegated agents always exhibit a more informative price and a lower equity premium. The model also highlights that a higher level of delegation does not generally decreases the equity premium but depends on what causes the shift in delegation. This is an important finding and emphasizes the importance of modeling delegation endogenously. Second, for a benchmarking investor, the riskless point of reference is the benchmark portfolio. For assets that exhibit high correlations with the benchmark portfolio, relative return investors require a smaller risk premium than do absolute return investors. Hence, in equilibrium, a stock’s expected return is decreasing, the higher

Table X. Asset pricing results for Global Benchmark 1980–2008

The table presents the results for various cross-sectional regressions, where the mean excess returns of 100 Fama–French size- and book-to-market-sorted testing portfolios are regressed on their factor betas. The multivariate factor betas for each testing portfolio are obtained from time series regressions of portfolio returns on the factors.  $k_0$  denotes the intercept of the regressions. MKT denotes the excess return of the CRSP value-weighted market index. BMR is the excess return of the Global Benchmark residual. ROP is the excess return of a zero-cost portfolio that is long in the bottom RO quintile portfolio and short in the top RO quintile portfolio. SMB is the Fama–French size factor that captures the premium of small-cap relative to large-cap stocks. HML is the Fama–French value factor that captures the relative performance of value versus growth stocks. MOM is the momentum factor of Carhart. LIQ is a market-traded liquidity factor based on the methodology outlined in Pastor and Stambaugh (2003). All coefficients are multiplied by 100. The  $t$ -statistics are reported in parentheses. For each regression, we also report the adjusted  $R^2$  values. The estimation period is January 1980 through December 2008. The returns of the testing portfolios and the SMB, HML, and MOM factors are obtained from the Web site of Kenneth French, <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>. The liquidity factor is obtained from CRSP.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$k_0$	1.55 (11.91)**	1.88 (11.50)**	1.47 (8.12)**	1.47 (8.13)**	1.47 (5.98)**	1.89 (11.54)**	1.87 (10.54)**	1.46 (6.14)**	1.05 (4.04)**	1.03 (4.15)**
MKT	-0.88 (-6.95)**	-1.25 (-7.42)**	-0.95 (-5.50)**	-0.97 (-5.67)**	-0.90 (-3.91)**	-1.24 (-7.32)**	-1.24 (-6.86)**	-0.93 (-4.17)**	-0.51 (-2.01)*	-0.52 (-2.18)*
BMR		-0.04 (-3.21)**	-0.05 (-4.36)**	0.03 (1.37)	-0.04 (-3.49)**	-0.04 (-3.23)**	-0.04 (-2.80)**			0.02 (0.75)
ROP			-0.32 (-2.96)**							-0.21 (-1.75)
SMB				0.08 (1.84)				0.07 (1.59)	0.08 (1.77)	0.11 (2.60)**
HML					0.39 (7.24)**			0.39 (7.96)**	0.42 (8.73)**	0.34 (6.89)**
MOM						0.42 (1.52)			1.02 (3.20)**	1.03 (3.90)**
LIQ							0.09 (0.21)			0.45 (1.05)
Adj $R^2$	32.57	38.17	46.25	47.36	40.14	38.25	37.54	44.37	49.12	56.02

its correlation with the benchmark and the higher the fraction of relative return investors in the economy.

Our model provides us with two testable implications. Our first hypothesis claims that stocks with high IO exhibit lower returns and volatility. When we control for size, we find empirical evidence in favor of this prediction. Our second hypothesis claims the existence of benchmarking risk and that a positive loading on this risk factor has cross-sectional implications. Using the S&P 500 index as benchmark, we find that stocks with a high residual benchmark beta exhibit significantly lower returns. This effect is most pronounced in large capitalization stocks and in the period after 1980. Standard asset pricing tests reveal that the two-factor CAPM has predictive capability explaining the cross section of 100 size- and book-to-market-sorted testing portfolios. An extension of the analysis to the six most important US all-equity benchmarks confirms our empirical findings using multiple benchmarks.

A direction for further research is to bring the model into a dynamic setting. This would allow the inclusion of implicit incentives for managers based on dynamic fund flows as in Basak, Pavlova, and Shapiro (2008). Furthermore, in a dynamic setting, investors are able to learn about managerial skill instead of simply observing it.

## Appendix

### PROOF OF PROPOSITION 1

Following Admati (1985), we make the claim that the price is a linear function of the asset payoff and the unexpected component of the asset supply,

$$p = \frac{1}{r}(a + Bf + Cx). \quad (\text{A.1})$$

In general equilibrium, the posterior belief about the asset payoff  $f$  is conditional on prior information,  $\mu \sim \mathcal{N}(f, \Sigma)$ , information observed from the signal,  $s_j \sim \mathcal{N}(f, \Sigma_{s,j})$ , and information inferred from equilibrium prices,  $f|p \sim \mathcal{N}(\mu_p, \Sigma_p)$ . We note that all agents start with the same set of prior beliefs. This assumption is not crucial to the analysis but simplifies the analytical solution. From the conjecture of the linear price function, it follows directly that

$$\mu_p := B^{-1}(pr - a) \quad (\text{A.2})$$

and

$$\Sigma_p := \sigma_x^2 B^{-1} C C' (B^{-1})'. \quad (\text{A.3})$$

Standard Bayesian updating implies that the posterior about  $f$  is normally distributed with mean

$$\mathbb{E}[f|\mathcal{F}_j] = (-1 + \Sigma_{s,j}^{-1} + \Sigma_p^{-1})^{-1}(\Sigma^{-1}\mu + \Sigma_{s,j}^{-1}s_j + \Sigma_p^{-1}B - 1(pr - a)) =: \hat{\mu}_j \quad (\text{A.4})$$

and variance

$$\text{Var}(f|\mathcal{F}_j) = (\Sigma^{-1} + \Sigma_{s,j}^{-1} + \Sigma_p^{-1})^{-1} =: \hat{\Sigma}_j. \quad (\text{A.5})$$

Note that all investors (and managers)  $j \in [0, 1]$  start with identical priors. Information inferred from observed prices is equal for all investors as well. Only the signal  $s_j$  and the signal precision matrix  $\Sigma_{s,j}^{-1}$  are investor specific. Therefore, heterogeneity of investors in the model enters only through the signal and signal precision. The market clearing condition implies that the total demand for risky assets of delegating investors,  $q_{\text{del}}^j$ , and direct investing investors,  $q_{\text{dir}}^j$ , equals total supply of risky assets,  $\bar{x} + x$ ,

$$\int_0^n q_{\text{del}}^j dj + \int_n^1 q_{\text{div}}^j dj = \bar{x} + x, \quad (\text{A.6})$$

where  $n \in [0, 1]$ . The optimal portfolio demand is

$$q_{\text{dir}}^j = \frac{1}{\rho_j} \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) \quad (\text{A.7})$$

for direct investment and

$$q_{\text{del}}^j = \phi + \frac{1}{\rho_m^j} (\hat{\Sigma}_m^j)^{-1} (\hat{\mu}_m^j - pr). \quad (\text{A.8})$$

Therefore,

$$\int_0^n \left( \frac{1}{\rho_m^j} (\hat{\Sigma}_m^j)^{-1} (\hat{\mu}_m^j - pr) + \phi \right) dj + \int_n^1 \frac{1}{\rho_j} \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) dj = \bar{x} + x, \quad (\text{A.9})$$

where  $(\hat{\mu}_m^j, \hat{\Sigma}_m^j)$  denotes the set of posterior beliefs of the manager of the  $j$ -th investor and  $(\hat{\mu}_j, \hat{\Sigma}_j)$  the set of posterior beliefs of the  $j$ -th investor. Substituting Equations (A4) and (A5) into the market clearing condition, we get

$$\begin{aligned} & \int_0^n \left( \frac{1}{\rho_m^j} (\hat{\Sigma}_m^j)^{-1} (\hat{\Sigma}_m^j (\Sigma^{-1}\mu + (V_m^j)^{-1}s_m^j + \Sigma_p^{-1}B^{-1}(pr - a)) - rp) + \phi \right) dj \\ & + \int_n^1 \frac{1}{\rho_j} \hat{\Sigma}_j^{-1} (\hat{\Sigma}_j (\Sigma^{-1}\mu + V_j^{-1}s_j + \Sigma_p^{-1}B^{-1}(pr - a)) - rp) dj = \bar{x} + x. \end{aligned}$$

Define the following two average quantities. The first is the average signal precision matrix,

$$\Psi := \int_0^n (\Sigma_{s,m}^j)^{-1} dj + \int_n^1 \Sigma_{s,j}^{-1} dj. \quad (\text{A.10})$$

The second average quantity is the average risk tolerance of direct and delegating investors,

$$\rho_a^{-1} := \int_0^n \frac{1}{\rho_m^j} dj + \int_n^1 \frac{1}{\rho_j} dj. \quad (\text{A.11})$$

With these two definitions, the market clearing condition becomes

$$\begin{aligned} \frac{1}{\rho_a} \Sigma^{-1} \mu + \frac{1}{\rho_a} \Psi f + \frac{1}{\rho_a} \Sigma_p^{-1} B^{-1} (pr - a) - \frac{1}{\rho_a} \Sigma^{-1} pr - \frac{1}{\rho_a} \Psi pr - \frac{1}{\rho_a} \Sigma_p^{-1} pr \\ + n\phi = \bar{x} + x, \end{aligned} \quad (\text{A.12})$$

where we used the fact that signals  $s$  are iid around the true payoffs  $f$ . Solving for  $pr$  yields

$$\begin{aligned} pr = \rho_a (\Sigma^{-1} + \Psi - \Sigma_p^{-1} (B^{-1} - I))^{-1} \\ \times \left( \frac{1}{\rho_a} \Sigma^{-1} \mu - \frac{1}{\rho_a} \Sigma_p^{-1} B^{-1} a - \bar{x} + n\phi - \frac{1}{\rho_a} \Psi f + x \right) \end{aligned} \quad (\text{A.13})$$

The above expression is linear in  $f$  and  $x$  and confirms Equation (A1). Next step is to solve for the constants in Equation (A1). From Equations (A1) and (A13), we know that  $B$  takes the following form:

$$B = (\Sigma^{-1} + \Psi - \Sigma_p^{-1} (B^{-1} - I))^{-1}. \quad (\text{A.14})$$

Solving for  $B$  yields

$$B = (\Sigma^{-1} + \Psi + \Sigma_p^{-1})^{-1} (\Psi + \Sigma_p^{-1}). \quad (\text{A.15})$$

In the same manner, we can solve for the other constants.  $C$  can be expressed as

$$C = -\rho_a (\Sigma^{-1} + \Psi - \Sigma_p^{-1} (B^{-1} - I))^{-1}. \quad (\text{A.16})$$

Solving for  $C$  yields

$$C = -\rho_a (\Sigma^{-1} + \Psi + \Sigma_p^{-1})^{-1} (\Sigma_p^{-1} \Psi^{-1} + I). \quad (\text{A.17})$$

Last step is to solve for the constant  $a$ ,

$$\mathbf{a} = (\mathbf{\Sigma}^{-1} + \mathbf{\Psi} - \mathbf{\Sigma}_p^{-1}(\mathbf{B}^{-1} - \mathbf{I}))^{-1}(\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \mathbf{\Sigma}_p^{-1}\mathbf{B}^{-1}\mathbf{a} - \rho_a\bar{x} + \rho_an\boldsymbol{\phi}). \quad (\text{A.18})$$

Solving for  $\mathbf{a}$  yields

$$\mathbf{a} = (\mathbf{\Sigma}^{-1} + \mathbf{\Psi} + \mathbf{\Sigma}_p^{-1})^{-1}(\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \rho_a\bar{x} + \rho_an\boldsymbol{\phi}). \quad (\text{A.19})$$

Note that  $\mathbf{\Sigma}_p^{-1} = \frac{1}{\sigma_x^2}\mathbf{B}\mathbf{C}^{-1}(\mathbf{C}^{-1})'\mathbf{B}' = \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}\mathbf{\Psi}'$ . Substituting for  $\mathbf{\Sigma}_p$  in  $\mathbf{a}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$ , we can make the following claim.

**Corollary 1** (*Admati, 1985*). Asset prices are linear functions of the asset payoffs and the unexpected component of asset supply:

$$\mathbf{p} = \frac{1}{r}(\mathbf{a} + \mathbf{B}\mathbf{f} + \mathbf{C}\mathbf{x}), \quad (\text{A.20})$$

with constants

$$\mathbf{a} = \left( \mathbf{\Sigma}^{-1} + \mathbf{\Psi} \left( \mathbf{I} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}' \right) \right)^{-1} (\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \rho_a(\bar{x} - n\boldsymbol{\phi})), \quad (\text{A.21})$$

$$\mathbf{B} = \left( \mathbf{\Sigma}^{-1} + \mathbf{\Psi} \left( \mathbf{I} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}' \right) \right)^{-1} \left( \mathbf{\Psi} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}\mathbf{\Psi}' \right), \quad (\text{A.22})$$

$$\mathbf{C} = \left( \mathbf{\Sigma}^{-1} + \mathbf{\Psi} \left( \mathbf{I} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}' \right) \right)^{-1} \left( \mathbf{I} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi} \right) (-\rho_a), \quad (\text{A.23})$$

where  $\mathbf{\Psi} := \int_0^n (\mathbf{\Sigma}_{s,m}^j)^{-1} \mathrm{d}j + \int_n^1 \mathbf{\Sigma}_{s,j}^{-1} \mathrm{d}j$  and  $\rho_a^{-1} := \int_0^n \frac{1}{\rho_m'} \mathrm{d}j + \int_n^1 \frac{1}{\rho_j'} \mathrm{d}j$ .

The first term on the right-hand side of Equations (A21), (A22), and (A23) is the posterior covariance matrix of the average investor  $\widehat{\Sigma}_a$ ,

$$\widehat{\Sigma}_a := \left( \mathbf{\Sigma}^{-1} + \mathbf{\Psi} \left( \mathbf{I} + \frac{1}{\rho_a^2\sigma_x^2}\mathbf{\Psi}' \right) \right) \quad (\text{A.24})$$

Like the individual investor, the average investor has a posterior covariance matrix that is based on the prior and signal variance and the variance observed from the price level. Using the expression for the average posterior variance, the price function can be written as

$$pr = \hat{\Sigma}_a \left( \Sigma^{-1} \mu - \rho_a \left( \bar{x} - n\phi \right) \right) + \left( \Psi + \frac{1}{\rho_a^2 \sigma_x^2} \Psi \Psi' \right) f - \rho_a \left( I + \frac{1}{\rho_a^2 \sigma_x^2} \Psi \right) x \quad (\text{A.25})$$

Rearranging terms yields

$$pr = \hat{\Sigma}_a \left( \Sigma^{-1} \mu + \Psi f + \frac{1}{\rho_a^2 \sigma_x^2} \Psi \Psi' \left( f - \rho_a^{-1} x \right) - \rho_a \left( \bar{x} + x - n\phi \right) \right) \quad (\text{A.26})$$

Note that  $(f - \rho_a^{-1} x) = B^{-1}(pr - a)$  is the signal that investors observe from prices. The first three terms in Equation (A26) are equal to the posterior mean of the average investor, as given by the Bayesian updating formulas. Defining

$$\hat{\mu}_a := \hat{\Sigma}_a \left( \Sigma^{-1} \mu + \Psi f + \frac{1}{\rho_a^2 \sigma_x^2} \Psi \Psi' \left( f - \rho_a^{-1} x \right) \right), \quad (\text{A.27})$$

the price function can be rewritten as given in Proposition 1. This ends the proof of Proposition 1.

#### PROOF OF PROPOSITION 5

The CED for investor  $j$  is defined as

$$\text{CED}_j^E := \sup \left[ \xi | \mathbb{E}[U_{\text{dir}}^j(\hat{\mu}_j, \hat{\Sigma}_j) | \mathcal{F}_0] \leq \mathbb{E}[U_{\text{del}}^j(\hat{\mu}_m^j - \xi, \hat{\Sigma}_m^j) | \mathcal{F}_0] \right].$$

We first derive  $\mathbb{E}[U_{\text{dir}}^j(\hat{\mu}_j, \hat{\Sigma}_j) | \mathcal{F}_0]$ . The utility from direct investment in the second period, given optimal portfolio choice  $q_{\text{dir}}^j = \frac{1}{\rho_j} \hat{\Sigma}_j (\hat{\mu}_j - pr)$  and information choice  $\hat{\Sigma}_j$ , is

$$\mathbb{E}[U_{\text{dir}} | \mathcal{F}_0] = \frac{1}{2\rho_j} \mathbb{E}[(\hat{\mu}_j - pr)' \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) | \mathcal{F}_0]. \quad (\text{A.28})$$

At the delegation stage,  $(\hat{\mu}_j - pr)$  is normal with mean  $\mathbb{E}[(\hat{\mu}_j - pr) | \mathcal{F}_0] = (I - B)\mu - a$  and variance

$$\text{Var}[(\hat{\mu}_j - pr) | \mathcal{F}_0] = (I - B)' \Sigma (I - B) + \sigma_x^2 CC' - \hat{\Sigma}_j. \quad (\text{A.29})$$

This follows from the equilibrium price function (A1). The first two terms on the right-hand side of Equation (A29) exhibit the unconditional variance and the

third term is the conditional variance of  $(\hat{\boldsymbol{\mu}}_j - \mathbf{pr})$ . Hence, we write the Period-2 utility as

$$\begin{aligned} \mathbb{E}[U_{\text{dir}}^j | \mathcal{F}_0] &= \frac{1}{2\rho_j} (\text{Tr}(\hat{\Sigma}_j^{-1} ((\mathbf{I} - \mathbf{B})' \Sigma (\mathbf{I} - \mathbf{B}) + \sigma_x^2 CC' - \hat{\Sigma}_j)) \\ &\quad + ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})' \hat{\Sigma}_j^{-1} ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})). \end{aligned} \quad (\text{A.30})$$

Substituting for the correlation structure of the risky assets  $\hat{\Sigma} = \Gamma \hat{\Lambda} \Gamma$  yields

$$\begin{aligned} \mathbb{E}[U_{\text{dir}}^j | \mathcal{F}_0] &= \frac{1}{2\rho_j} (\text{Tr}(\Gamma \hat{\Lambda}_j^{-1} \Gamma' ((\mathbf{I} - \mathbf{B})' \Gamma \Lambda \Gamma' (\mathbf{I} - \mathbf{B}) + \sigma_x^2 CC' - \Gamma \Lambda_j \Gamma')) \\ &\quad + ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})' \Gamma \hat{\Lambda}_j^{-1} \Gamma' ((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})) \end{aligned} \quad (\text{A.31})$$

$$\begin{aligned} &= \frac{1}{2\rho_j} \left( \sum_{l=1}^L \frac{1}{\hat{\lambda}_{j,l}} \left( \underbrace{\lambda_l (1 - \lambda_{B,l})^2 + \sigma_x^2 \lambda_{C,l}^2 + [((\mathbf{I} - \mathbf{B})\boldsymbol{\mu} - \mathbf{a})' \Gamma_l]^2}_{=: X_l^E} \right) - L \right), \end{aligned} \quad (\text{A.32})$$

where  $\Lambda_B$  and  $\Lambda_C$  are the eigenvalues of  $\mathbf{B}$  and  $\mathbf{C}$ . Similarly, by substituting for  $\mathbf{q}_{\text{del}}^j$ , we write  $\mathbb{E}[U_{\text{del}}^j(\hat{\boldsymbol{\mu}}_m^j - \xi, \hat{\Sigma}_m^j) | \mathcal{F}_0]$  as

$$\begin{aligned} \mathbb{E}[U_{\text{del}}^j | \mathcal{F}_0] &= \frac{1}{\rho_m^j} \left( 1 - \frac{1}{2} \frac{\rho_j}{\rho_m^j} \right) \mathbb{E}[(\hat{\boldsymbol{\mu}}_m^j - \mathbf{pr})' (\hat{\Sigma}_m^j)^{-1} (\hat{\boldsymbol{\mu}}_m^j - \mathbf{pr}) | \mathcal{F}_0] \\ &\quad + \left( 1 - \frac{\rho_j}{\rho_m} \right) \boldsymbol{\mu}_\phi - \frac{1}{2} \rho_j \hat{\sigma}_\phi^2 - \xi, \end{aligned} \quad (\text{A.33})$$

where  $\rho_m^j$  is the risk aversion of the manager of investor  $j$ . The above equation is equivalent to

$$\begin{aligned} \mathbb{E}[U_{\text{del}}^j | \mathcal{F}_0] &= \frac{1}{\rho_m^j} \left( 1 - \frac{1}{2} \frac{\rho_j}{\rho_m^j} \right) \left( \sum_{l=1}^L \hat{\lambda}_{m,l}^{j-1} X_l^E - L \right) + \left( 1 - \frac{\rho_j}{\rho_m} \right) \boldsymbol{\mu}_\phi \\ &\quad - \frac{1}{2} \rho_j \hat{\sigma}_\phi^2 - \xi. \end{aligned} \quad (\text{A.34})$$

Combining Equations (A32) and (A34) and using the definition of the CED yield the expression in Proposition 5.



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