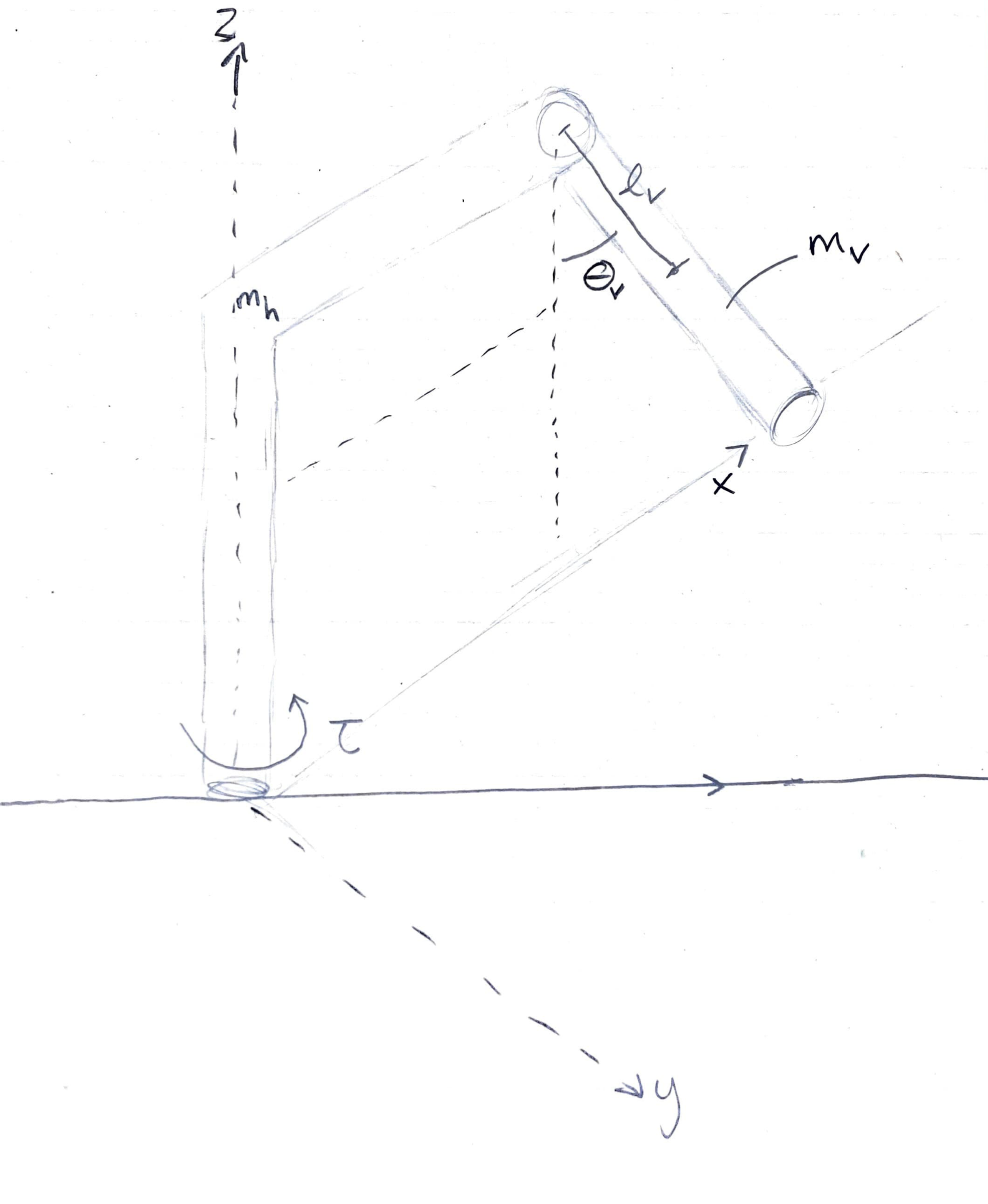
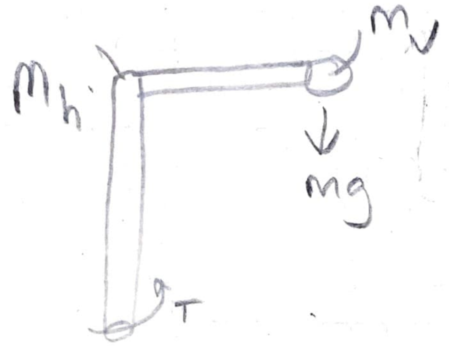
State space model



State Space Equations

m_v = free floating arm

m_h = Pendulum (right angle arm)



$T = J\dot{\omega}$

$T = J\ddot{\theta}$

$T = J\alpha$

θ: angular displacement

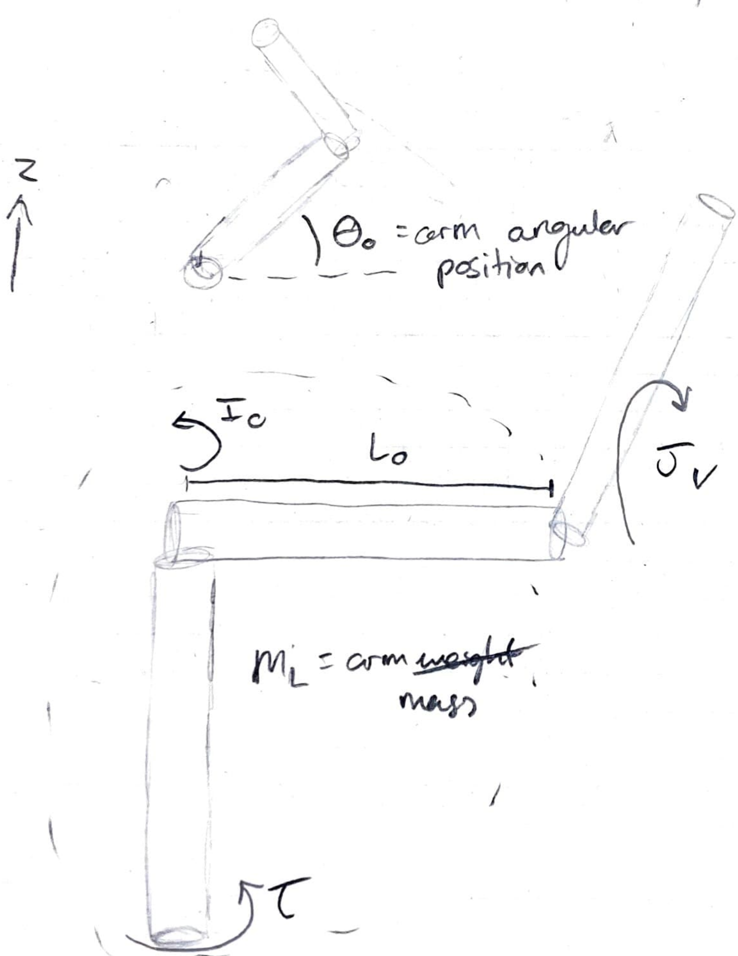
ω: angular velocity

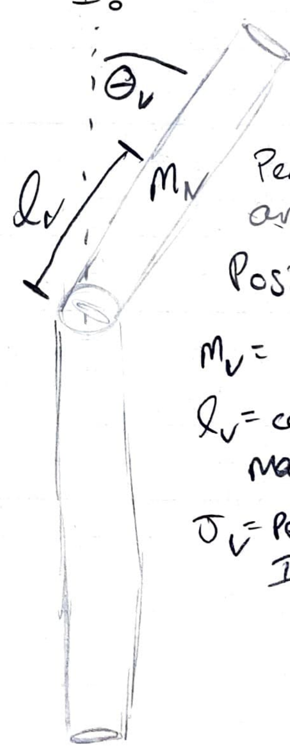
J: inertia

$$T = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

L_o = arm length

I_o = arm inertia



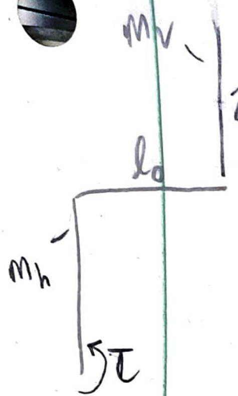


Pendulum angular Position

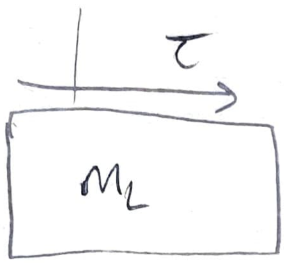
M_v = Pendulem mass

l_v = center of mass

J_v = Pendulem Inertia



$$\tau = \frac{1}{2}(m_h + m_v)\dot{x}^2 + \frac{1}{2} m_h l_v^2 \dot{\theta}_v^2 - m_v l_v \dot{\theta}_v x \cos\theta_v$$
$$- m_v g l_v \cos\theta_v$$



$$l_v^2 m_v \ddot{\theta} + m_v l_v \ddot{x} = 2 m_g \theta l_v$$

$$\tau = (m_h + m_v)\ddot{x} + m_v l_v \ddot{\theta}$$

angular acceleration

$$\ddot{x} = \frac{2 m_v g \theta l_v - m_v \ddot{\theta}}{m_v l_v} \qquad \ddot{\theta} = \frac{2 m_v g \theta l_v - m_v l_v \ddot{x}}{m_v}$$

$$\ddot{x} \; 2g\theta_v - \frac{\ddot{\theta}}{l_v} \qquad \ddot{\theta} = \frac{m_v g \theta l_v - m_v l_v \ddot{x}}{m_v l_v^2}$$

$$\tau = (m_h + m_v)\ddot{x} + m_v l_v \ddot{\theta}_v \cos\theta_v + m_v l_v \dot{\theta}_v^2 \sin\theta_v$$

$$0 = l_v \ddot{\theta}_v - \ddot{x}\cos\theta_v - g \sin\theta_v$$

$$(m_h + m_v)\ddot{x} = m_v l_v \ddot{\theta}_v \cos\theta_v + m_v l_v \dot{\theta}_v^2 \sin\theta_v - \tau$$

$$l_v \ddot{\theta}_v = \ddot{x}\cos\theta_v - g \sin\theta_v$$

$$\ddot{\theta}_v = \frac{\ddot{x}\cos\theta_v - g\sin\theta_v}{l_v}$$

$$\ddot{x} = \frac{m_v g \theta_v l_v - m_v \ddot{\theta}}{m_v l_v} \qquad \ddot{\theta} = \frac{m_v g \theta_v l_v - m_v l_v \ddot{x}}{m_v l_v^2}$$

$$\ddot{x} = g\theta_v - \frac{\ddot{\theta}}{l_v} \qquad \ddot{\theta} = \frac{g\theta_v}{l_v} - \frac{1}{l_v}\ddot{x}$$

$$\tau = (m_h + m_v)\ddot{x} + m_v \cancel{l_v}\left(\frac{g\theta_v}{\cancel{l_v}} - \frac{1}{\cancel{l_v}}\ddot{x}\right)$$

$$\tau = m_h\ddot{x} + \cancel{m_v\ddot{x}} + m_v g\theta_v - \cancel{\ddot{x} m_v}$$

$$m_h\ddot{x} = \tau - m_v g \theta_v$$

$$\tau = m_h\ddot{x} + m_v\ddot{x} + m_v l_v \ddot{\theta}$$

$$= m_h\left(g\theta_v - \frac{\ddot{\theta}}{l_v}\right) + m_v\left(g\theta_v - \frac{\ddot{\theta}}{l_v}\right) + m_v l_v \ddot{\theta}$$

$$m_h l_v \ddot{\theta} = m_h g \theta_v + m_v g \theta_v - \tau$$

$$m_h l_v \theta = (m_h + m_v)(g\theta_v) - \tau$$