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1)

P => ~Q = ~P ∨ ~Q

Q => ~P = ~Q ∨ ~P

P	**Q**	**P => ~Q**	**Q => ~P**
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

P ⇔ ~Q = (P => ~Q) ∧ (~Q => P)
= (~P ∨ ~Q) ∧ (Q ∨ P)

P	**Q**	**P ⇔ Q**	**((P ∧ ~Q) ∨ (~P ∧ Q))**
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	T

2)

(S => F) => (~S => ~F)
= (~S ∨ F) => (S ∨ ~F)
= ~(~S ∨ F) ∨ (S ∨ ~F)
= (S ∧ ~F) ∨ (S ∨ ~F)

S	**F**	**(S ∧ ~F) ∨ (S ∨ ~F)**
T	T	T
T	F	T
F	T	F
F	F	T

The sentence is neither valid nor unsatisfiable since it is true for some worlds, but not all.

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$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

$= (\sim S \vee F) \Rightarrow (\sim(S \vee H) \vee F)$

$= \sim(\sim S \vee F) \vee ((\sim S \wedge \sim H) \vee F)$

$= (S \wedge \sim F) \vee ((\sim S \wedge \sim H) \vee F)$

S	**H**	**F**	**$(S \wedge \sim F) \vee ((\sim S \wedge \sim H) \vee F)$**
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

The sentence is neither valid nor unsatisfiable since it is true for some worlds, but not all.

$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$= (\sim(S \wedge H) \vee F) \Leftrightarrow ((\sim S \vee F) \vee (\sim H \vee F))$

$= ((\sim S \vee \sim H) \vee F) \Leftrightarrow (\sim S \vee \sim H \vee F)$

$= [(\sim S \vee \sim H \vee F) \Rightarrow (\sim S \vee \sim H \vee F)] \wedge [(\sim S \vee \sim H \vee F) \Rightarrow (\sim S \vee \sim H \vee F)$

$= (\sim S \vee \sim H \vee F) \Rightarrow (\sim S \vee \sim H \vee F)$

$= \sim(\sim S \vee \sim H \vee F) \vee (\sim S \vee \sim H \vee F)$

$= (S \wedge H \wedge \sim F) \vee (\sim S \vee \sim H \vee F)$

S	**H**	**F**	**$(S \wedge H \wedge \sim F) \vee (\sim S \vee \sim H \vee F)$**
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

This sentence is valid since it is true for all worlds.

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3)

Variables:

I – Immortal
M – Mythical
A – Mammal
H – Horned
G – Magical

Knowledge Base:

M => I
~M => (~I ^ A)
(I v A) => H
H => M

M => I
= **~M v I**

~M => (~I ^ A)
= M v (~I ^ A)
= **(M v ~I) ^ (M v A)**

(I v A) => H
= ~(I v A) v H
= (~I ^ ~A) v H
= **(H v ~I) ^ (H v ~A)**

H => G
= **~H v G**

CNF:

(~M v I) ^ (M v ~I) ^ (M v A) ^ (H v ~I) ^ (H v ~A) ^ (~H v G)

Prove Mythical (M):

1	**~M v I**	
2	**M v ~I**	
3	**M v A**	
4	**H v ~I**	
5	**H v ~A**	
6	**~H v G**	
7	**~M**	**to prove contradiction**

8	~I	2 & 7
9	A	3 & 7

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10	H	5 & 9
11	G	6 & 10
12	~M v H	1 & 4

Cannot prove contradiction, therefore the unicorn is not always mythical.

Prove Magical (G):

1	**~M v I**	
2	**M v ~I**	
3	**M v A**	
4	**H v ~I**	
5	**H v ~A**	
6	**~H v G**	
7	**~G**	**to prove contradiction**

8	~H	6 & 7
9	~A	5 & 8
10	~I	4 & 8
11	M	3 & 9
12	**I**	**1 & 11 ← Contradiction with 10!**

Since we found a contradiction, we know that unicorns are magical.

Prove Horned (H):

1	**~M v I**	
2	**M v ~I**	
3	**M v A**	
4	**H v ~I**	
5	**H v ~A**	
6	**~H v G**	
7	**~H**	**to prove contradiction**

8	~A	5 & 7
9	M	3 & 8
10	I	1 & 9
11	H	4 & 10 ← Contradiction with 7!

Since we found a contradiction, we know that unicorns are horned.